Matching of expenses in financial reporting : a matching function approach

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Matching of expenses in financial reporting: a matching function approach

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Abstract

Purpose – The purpose of this study is to introduce a matching function approach to analyze matching in financial reporting.

Design/methodology/approach – The matching function is first analyzed analytically. It is specified as a multiplicative Cobb-Douglas-type function of three categories of expenses (labor expense, material expense and depreciation). The specified matching function is solved by the generalized reduced gradient method (GRG) for 10-year time series from 8,226 Finnish firms. The coefficient of determination of the logarithmic model (CODL) is compared with the linear revenue-expense correlation coefficient (REC) that is generally used in previous studies.

Findings – Empirical evidence showed that REC is outperformed by CODL. CODL was found independent of or weakly negatively dependent on the matching elasticity of labor expense, positively dependent on the material expense elasticity and negatively dependent on depreciation elasticity. Therefore, the differences in matching accuracy between industries emphasizing different expense categories are significant.

Research limitations/implications – The matching function is a general approach to assess the matching accuracy but it is in this study specified multiplicatively for three categories of expenses. Moreover, only one algorithm is tested in the empirical estimation of the function. The analysis is concentrated on ten-year time-series of a limited sample of Finnish firms.

Practical implications – The matching function approach provides a large set of important information for considering the matching process in practice. It can prove a useful method also to accounting standard-setters and other specialists such as managers, consultants and auditors.

Originality/value – This study is the first study to apply the new matching function approach.

Keywords Financial reporting, Matching function, Finnish firms, Matching principle, Revenue-expense correlation, Matching elasticities

Paper type Research paper

1. Introduction

The matching principle of accounting plays in financial reporting a central role. Matching of expenses to revenues is defined as the process of collecting all revenues which are earned during the accounting period and matching these revenues with the expenses incurred to produce those revenues. In the matching process, revenues are first recognized and expenses

are then matched against these revenues. For the accounting period, matching thus brings in the income statement together expenses and resulting revenue enabling to assess the earnings of the firm as the difference between revenue and expenses. Earnings are regarded as the most important output of the accounting system (Graham *et al.*, 2005). This makes also matching very important, as the accuracy of matching plays a key role in assessing earnings. Therefore, an inquiry into matching can potentially provide valuable insights into the properties of accounting earnings (Dichev and Tang, 2008). Consequently, it is of importance to investigate and improve the methods to assess the quality of matching. The objective of this study is to introduce a novel approach to do that.

Dichev and Tang (2008) follow Paton and Littleton (1940, p. 123) and state that the purpose of accounting is to properly match the expenses against the resulting revenues. If expenses are not properly matched against the resulting revenues, it is defined as a poor matching and is regarded as a noise in the economic relation of advancing expenses to obtain revenues. Matching is considered to be perfect in the case where all expenses can be traced directly and specifically to specific revenues. Perfect matching provides a series of implications. Firstly, in competitive equilibrium earnings tend to gravitate toward the cost of equity capital. Secondly, deviations in earnings from the long-run mean will gradually diminish over time. Thirdly, there exists an economic shock in every period, which is the noise in the matching relation and has a mean of zero. The variance of this economic shock represents the economic volatility of the business environment. Fourthly, in a perfect matching situation, the volatility is driven entirely by economic factors.

Dichev and Tang (2008) describe poor matching introducing a random variable to represent mismatched expense being unrelated to the well-matched expense and revenue. Thus, the mismatched expense acts as a noise. The quality of matching is directly related to the inverse of the noise. Poor matching has also several implications. Firstly, poor matching decreases the time-series contemporaneous correlation between revenues and expenses. In poor matching, some of the perfectly matched expenses get scattered across different periods, which results in a lower synchronal correlation than the underlying economic correlation of advancing expenses to produce revenues. Secondly, poor matching increases the volatility of earnings. The volatility in earnings that are poorly matched is higher because the mismatched expenses act as a noise that is not related to the economic process of creating earnings. Thirdly, the persistence of earnings will decrease with poor matching. Thus, poor matching can seriously distort the quality of earnings.

The contemporaneous correlation between revenues and expenses has in previous studies been used as an indirect measure of the matching quality. Sivakumar and Waymire (2003) used this relationship to assess the effects of new rules for depreciation accruals hypothesizing that new rules will lead to lower correlation due to restrictions on matching. They also hypothesized that a new regulation policy to conservatively measure income will not lead the operating expenses to be less positively correlated with contemporaneous revenues. Dichev and Tang (2008) used the contemporaneous revenue-expense correlation to reflect the quality of matching in large firms showing a significant decline in quality over 40 years. Their results suggested that accounting factors such as the quality of accruals are a substantial determinant of the observed temporal patterns and changes in the real economy play only a secondary role.

Donelson *et al.* (2011) used a similar sample and concluded that none of the accounting standards has significantly affected special items, which they found to have an impact on the correlation. They also tested the effects of specific economic events, which cannot arise from book-keeping practices alone (negative employee growth, merger, acquisition, discontinuing operations, declining sales and operating loss). Using an index measure they concluded that the

economic events associated with special items have increased in organizations that have a medium or high level of competitive pressure. Srivastava (2011) found that the shift in the US economy toward industries with higher period costs and more research and development activities contributed to the decline in matching. He and Shan (2016) investigated the time-series trend and determinants of matching between revenues and expenses in a sample of 42 countries. They found that the decline in matching documented by Dichev and Tang (2008) is not unique to the USA, but is a worldwide phenomenon.

Matching is also found to be associated with accounting standards, audit opinions, and cost of capital. Jin *et al.* (2015) used the revenue-expense correlation to reflect matching accuracy. They showed that the revenue-expense relation has declined in Australia during 2001-2005, but improved following implementation of International Financial Reporting Standards (IFRS). The improvement was found largely attributable to increases in the association of operating expenses and other expenses with contemporaneous revenues. Jin, Shan and Taylor conclude that these results are in sharp contrast to documented declines in matching among US firms, and also highlight a positive outcome associated with Australian adoption of IFRS. However, He and Shan (2016) did not find any evidence of a connection between matching accuracy and IFRS adoptions. Kim and Lee (2016) investigated how firms manage the revenue-expense relationship in the presence of a going-concern audit opinion (GCO). Using Korean data, they found that firms with GCOs both delay and accelerate the recognition of current expenses for current revenues. Kim (2018) showed that firms with high revenue-expense matching enjoy a lower cost of capital, supporting the direct impact of high matching on the cost of capital by increasing the precision of public information signals. Thus, the revenue-expense relation plays an important role in accounting systems.

In previous studies, also different versions of the linear revenue-expense relationship are used. The starting point for the versions is usually the regression equation where the current revenue is explained by expenses from the previous period (lag expense), the current period (contemporaneous expense) and the following period (lead expense). When a firm in an ideal situation has a perfect revenue-expense matching, the contemporaneous correlation between revenues and expenses is equal to one, and the expenses in the previous and following periods do not increase the coefficient of determination in the regression model (Dichev and Tang, 2008). Instead of the correlation, Bushman *et al.* (2016) measured matching using the adjusted coefficient of determination from the cross-sectional regression of revenues on lead, lag and contemporaneous expense as a more direct measure of the random error component of expense recognition to measure matching. Kim (2018), firstly, estimated the regression between revenue and lead, lag and contemporaneous expenses (unrestricted model), and secondly, the regression between revenue and only contemporaneous expenses (restricted model assuming zero coefficients for lag and lead expenses). Then, the matching measure was calculated as one minus the ratio of the coefficient of determination of the restricted model to that of the unrestricted model.

Basu *et al.* (2016) used two different measures for matching. First, they assessed how much expense is being matched to revenues. They measured the percentage of expense recognized (MEXPper cent) as a linear function of contemporaneous revenue. The authors multiplied the estimate for the level of expenses that are being matched to revenues as a percentage of revenue by the ratio of average revenue to average expense over a ten-year period. Basu, Cready and Paek also measured mismatching focusing on the total amount of expense that is explained by the introduction of the adjacent period revenue terms (MISM). That is, they examined the reduction in residual error obtained by supplementing the contemporaneous revenue with the lag revenue variable in regression using the difference of

the mean squared errors. The authors found a more sizeable decline in matching than Dichev and Tang (2008) since the turn of the century.

However, instead of directly measuring expense as a function of revenue, Prakash and Sinha (2013) investigated matching in the context of deferred revenue by examining profit margins. They employed a linear regression equation between change in the current deferred revenue liability and current and future profit margins. This is justified as some costs related to the deferred component of revenue are expensed as incurred while revenue is recognized in subsequent periods. If there is mismatching the deferred component of revenue is associated with incremental period costs. Following this kind of argumentation, Prakash and Sinha (2013) used the parameters of the estimated regression equation to evaluate matching accuracy. They found that profit margins are lower in periods when deferred revenues increase and higher in periods when they decrease. This pattern is consistent with the expenses on deferred revenue being recognized before the revenue is recognized.

In summary, previous studies suggest that the revenue-expense correlation (REC) or its (regression) version provides us with a useful indirect measure of the quality of matching. These studies concentrate however on the effects of different factors on matching quality but they do not pay considerable attention to the intrinsic characteristics of REC as an indicator of mismatching. This important point is greatly omitted in research although REC potentially suffers from several weak points as a measure of the quality of matching. Firstly, it assumes a fixed linear relationship between revenues and expenses, which hardly holds in growing firms. Secondly, this relationship can change over time due to development in the efficiency of expenses to advance revenues, which may diminish REC (Basu *et al.*, 2016). Thirdly, REC only takes account of total expense although there are obvious differences in matching accuracy between different expense categories. This characteristic can lead to hidden mismatching bias when the mismatched items from different expense categories partly cancel each other.

The objective of this study is to introduce a matching function approach to analyze matching and also to assess matching accuracy. This approach will avoid some observed weak points of REC. Firstly, this approach assumes a non-linear relationship between revenues and expenses in the form of a Cobb–Douglas-type matching function. Secondly, it will also take account of potential development in efficiency by a Solow-residual-type growth factor. Thirdly, it forms a function between revenues and several categories of expenses, which give a more accurate view of matching than total expenses alone. In this study, the matching function is specified for three main categories of expenses (labor expense, material expense and depreciation). The coefficient of determination of the function in the logarithmic form (CODL) is suggested as a measure of revenue-expense relationship, and thus, as an indirect measure of matching quality. It is hypothesized that REC is outperformed by CODL (*H1*). Moreover, it is hypothesized that the matching elasticities of the different expense categories are associated with matching quality *(H2)*.

This study provides us with several contributions to contemporary research on matching. First, it introduces a general matching function concept where revenue is described by a multiplicative model of time and different expense categories. This kind of approach provides useful information about the change in efficiency in expenses over time and the sensitivity of different expense categories to matching. Secondly, the coefficient of determination of the matching function provides us with a useful measure of matching the accuracy of revenue and expenses that takes account of the change of the expense efficiency in time. This measure is also constructed to avoid the hidden mismatching bias and it is expected to give a more reliable picture of matching accuracy than the REC based measures.

Thirdly, this matching function approach is empirically useful in analyzing the effect of different events or factors on matching, such as the introduction of standards, financial difficulties, audit opinions or cost of capital. This approach can be used to show how these events or factors are related to the matching sensitivity of different expense categories. It can also be useful in industrial research where the matching sensitivities of different categories may strongly differ with respect to industries, which obviously leads to notable differences in the accuracy of matching.

The contents of the paper is organized as follows. Firstly, the background and motivation of the study were discussed in this introductory section. Secondly, the matching function approach is analytically presented and discussed in Section 2. At the end of this section, the hypotheses of the study are also drawn. Thirdly, the data and statistical methods are presented in Section 3 while, Section 4 presents empirical results. The final sample of the study includes ten-year time-series from 8226 Finnish firms. For these firms, the matching function is solved using generalized reduced gradient method (GRG). The results clearly supported *H1*, as CODL exceeded for most firms the squared REC (SREC) leading to a higher degree of explanation for the revenue-expense relationship. Thus, taking also an account of its theoretical justification CODL provides us with a stronger measure of matching quality than SREC. Evidence also almost fully supported *H2*, as the matching elasticities were statistically associated with the coefficient. CODL was found to be increasing in the elasticity of material expenses and decreasing the elasticity of depreciations. However, it was insignificant or negative in the elasticity of labor expenses. Consequently, the differences in matching accuracy between industries emphasizing different categories of expenses were found very significant. Finally, the results are discussed and concluded in Section 5.

2. The matching function approach
2.1 Static analysis
The matching of expenses with sales revenue is in accrual accounting one of the most important principles, which affect the quality of earnings. The matching principle requires that revenues and related expenses are recognized together and reported in the income statement of the same period. It is assumed that there exists a fixed contemporaneous economic relation of advancing expenses to obtain revenues, which is used in matching expenses against revenues. In the matching process, this relationship can, however, be complicated and difficult or even impossible to describe mathematically. For convenience, let us simplify the analysis and describe this economic relation by the following multiplicative matching function:

$$S = K \prod_{i=1}^{m} E_i^{\varepsilon \, (i)} \tag{1}$$

where S is sales revenue allocated to the accounting period and K is the constant scale factor, E_i $(i = 1, \ldots, m)$ are matched expenses from m categories and $\varepsilon(i)$ is the constant expense elasticities of sales revenue $(i = 1, \ldots, m)$.

The specified multiplicative form of the matching function [Equation (1)] is similar to the Cobb–Douglas (CD) production function, which has been a very influential contribution to economic theory (Jones, 2005). The CD production function may be the best justified and most widely used function in production economics (Felipe and Adams, 2005, p. 428). It has the advantage of algebraic tractability and of providing a fairly good approximation of the

production process leading to a good fit with data. However, its main limitation is to impose an arbitrary level for substitution possibilities between production factors (Reynes, 2017). This assumption can be relaxed but however only with a corollary of a strong increase in complexity. In production economics, the substitution effects of production factors play an important role. However, matching is an accounting procedure, which is in practice usually carried out separately by expense categories using different methods of expensing. Therefore, substitution effects between the expense categories are expected to play a minor role in matching although there behind the matching process obviously exist interactions with production technology.

The multiplicative matching function [Equation (1)] also assumes constant coefficients (exponents) for the expenses leading to constant expense elasticities of sales. This constancy of elasticities means that it is assumed that matching a given proportional change in a matched expense category is responded by a constant proportional change in revenue for any amount of expenses (in any expense category). This assumption is a simplification of the matching process, which makes the model both mathematically and statistically tractable. When the domain of the matching function is restricted within the relevant range of expenses, this simplification can act as a reasonable but useful approximation of the fixed contemporaneous economic relation of expenses and revenues. Finally, the multiplicative form [Equation (1)] assumes that revenue is zero when at least one of the specified expense categories is zero. However, the relevant expense categories should be specified in the way that the lower boundary of expenses is positive.

Let us further simplify the analysis assuming that there are only three main categories of expense leading to the following expression of S:

$$S(L, M, D) = K L^{\alpha} M^{\beta} D^{\gamma} \tag{2}$$

where L is labor expense, M is material expense, D is depreciation (fixed asset expense) and α, β and γ are the constant elasticities of L, M and D, respectively. This classification of expense categories is useful in this context, as each category can reflect a different method of matching. In practice, labor expenses L are often matched with revenues using a cause and effect method when there exists a clear and direct relationship between revenues and expenditures. Material expenses M have not usually any discernible future benefit and are often expensed by immediate recognition. However, depreciation D cannot be directly linked to specific revenue transactions. Thus, they are typically tied to a span of years and allocated as an expense (depreciation) to each of those years. This kind of method ranking indicates that the matching is expected to be most accurate in firms, which have a lot of expenditures expensed immediately and little fixed expenditures generating revenue for a long investment period.

For simplicity, the elasticities of sales for each expense category are assumed constant. These elasticities can be presented mathematically in the following way:

$$\frac{\partial S}{\partial L} \frac{L}{S} = \alpha \tag{3a}$$

$$\frac{\partial S}{\partial M} \frac{M}{S} = \beta \tag{3b}$$

$$\frac{\partial S}{\partial D}\frac{D}{S} = \gamma \tag{3c}$$

Although the matching function is mathematically seemingly similar to the CD production function, in the matching context the parameters of the function have a different interpretation. In the production context, the elasticities are called the factor elasticities of production referring to the efficiency of the production factors in production. However, matching these elasticities refer to the sensitivity of sales revenue to matched expenses. Therefore, they can in this context be called matching sensitivities or elasticities. Similarly, in the production function context, K is called the total (or multi) factor productivity (TFP). In matching, the matching function does not describe the technical relationship between the factors and the output but only an economic relation showing how sales revenue and expenses are matched periodically with each other. Thus, K can in this matching framework be called the total expense productivity (TEP), measured in terms of matched sales (instead of production).

If matched expenses in each category are increased by the coefficient h, sales revenue will increase to the following degree:

$$K(Lh)^{\alpha}(Mh)^{\beta}(Dh)^{\gamma} = Sh^{\alpha+\beta+\gamma} \tag{4}$$

In the special case, $\alpha + \beta + \gamma$ is equal to unity leading to constant returns to scale (CRS). When $\alpha + \beta + \gamma > 1$, there is increasing returns to scale (IRS) and decreasing returns to scale when $\alpha + \beta + \gamma < 1$ (DRS). In matching, CRS means that if matched expenses of each category increase by coefficient h, the matched sales will also increase by this same coefficient.

Let us assume that $S(L,M,D)$ fulfils the standard assumptions of a typical neoclassical production function. Then, let us define earnings or profit P as a difference between revenues and expenses as:

$$P(S,L,M,D) = S(L,M,D) - L - M - D = S - E \tag{5}$$

where E is total expense defined as $E = L + M + D$. When differentiating P with respect to L, M and D the following partial derivatives are got:

$$\frac{\partial P}{\partial L} = K\alpha L^{\alpha-1}M^{\beta}D^{\gamma} - 1 = 0 \tag{6a}$$

$$\frac{\partial P}{\partial M} = KL^{\alpha}\beta M^{\beta-1}D^{\gamma} - 1 = 0 \tag{6b}$$

$$\frac{\partial P}{\partial D} = KL^{\alpha}M^{\beta}\gamma D^{\gamma-1} - 1 = 0 \tag{6c}$$

which gives the following optimal values for L, M and D maximizing earnings P:

$$L^{*} = E\,\alpha/(\alpha + \beta + \gamma) \tag{7a}$$

$$M^* = E \beta / (\alpha + \beta + \gamma) \tag{7b}$$

$$D^* = E \gamma / (\alpha + \beta + \gamma) \tag{7c}$$

The result expressed in equation (7) indicates that the optimal expenses in different categories are in matching directly related to the corresponding expense elasticities of sales revenue (matching sensitivities or elasticities). Thus, these elasticities are important parameters showing the matching sensitivity of sales to expenses of different categories but they also directly reflect the profit-maximizing (in this sense, optimal) values of those expenses.

The matching sensitivities play an important role in matching expenses with revenues. The importance of these sensitivities can be demonstrated in several ways. First, let us take logarithms from both sides of function $S(L,M,D)$ leading to the expression:

$$\log S(L, M, D) = \log \left[K L^\alpha M^\beta D^\gamma \right] = \log K + \alpha \log L + \beta \log M + \gamma \log D \tag{8}$$

which indicates that in matching the elasticities determine the separate contribution of the logarithmic expenses to the logarithmic sales revenue. This result also indicates that K reflects the separate portion of sales revenue that cannot be attributed to the matched expenses.

Secondly, let us calculate the total differential of $S(L,M,D)$, which gives the following result:

$$dS = \frac{\partial S}{\partial L} \, dL + \frac{\partial S}{\partial M} \, dM + \frac{\partial S}{\partial D} \, dD \tag{9}$$

leading to the result:

$$\frac{dS}{S} = \alpha \frac{dL}{L} + \beta \frac{dM}{M} + \gamma \frac{dD}{D} \tag{10}$$

This result shows that the contributions of proportionate changes in expenses from different categories to the proportionate change in matched sales revenue are directly related to the matching sensitivities. These changes can be understood to reflect the planning of matching in the annual closing of accounts. They show how much matched sales according to the economic relationship should be changed if the expenses from different categories are changed in the matching process.

2.2 Growth analysis
In general, it can be assumed that the expense elasticities are quite stable over time for a firm without any structural changes in the business. However, it is probable that the total expense productivity TEP will not stay constant due to development in the efficiency of expenses to advance revenues. Therefore, let us specify the matching function as a function of time as follows:

$$S_t(L_t, M_t, D_t) = K_t L_t^\alpha M_t^\beta D_t^\gamma \tag{11}$$

where sales, expenses and TEP may change over time t but the expense elasticities of sales revenue are constant.

For this dynamic function, the growth of sales revenue from period t-1 to t can be presented in the following way:

$$\frac{S_t}{S_{t-1}} = \frac{K_t}{K_{t-1}} \cdot \left[\frac{L_t}{L_{t-1}}\right]^\alpha \cdot \left[\frac{M_t}{M_{t-1}}\right]^\beta \cdot \left[\frac{D_t}{D_{t-1}}\right]^\gamma$$

$$= 1 + g_S = (1+k)(1+g_L)^\alpha (1+g_M)^\beta (1+g_D)^\gamma \quad (12)$$

where g_S, k, g_L, g_M and g_D are growth rates of sales, TEP, labor expense, material expense and depreciation, respectively.

When the logarithms from both sides of the equation are taken, the following result is got:

$$\log(1+g_S) = \log(1+k) + \alpha \log(1+g_L) + \beta \log(1+g_M) + \gamma \log(1+g_D) \quad (13)$$

leading to the solution for the growth rate of TEP as follows:

$$\log(1+k) = \log(1+g_S) - \alpha \log(1+g_L) - \beta \log(1+g_M) - \gamma \log(1+g_D) \quad (14)$$

This kind of result as presented in equation (14) is in the context of production function referred to as the Solow (1956) residual. It is in matching context the portion of the growth of matched sales that cannot be attributed to the growth of matched expenses. This residual is an important indicator of the change in the efficiency (productivity) of expenses to generate sales revenues.

When the expenses from different categories grow at the same rate so that $g_L = g_M = g_D =: g_E$, the proportions of these expenses stay constant over time and also total expenses E_t grow at this same rate denoted as g_E. However, the growth rate of sales g_s depends on the parameters of the model as follows:

$$1 + g_s = (1+k)(1+g_E)^{\alpha+\beta+\gamma} \quad (15)$$

When there are constant returns to scale CRS so that $\alpha+\beta+\gamma = 1$ and $k = 0$, then $g_S = g_E$ and the firm follow a steady growth path maintaining the relationship between total sales S_t and total expenses E_t fixed over time.

2.3 Poor matching: noise
Dichev and Tang (2008) conclude that poor matching decreases the synchronal correlation between revenues and expenses. With poor matching, some of the perfectly matched expenses get scattered across different periods, which results in a lower synchronal correlation than the underlying economic correlation of advancing expenses to produce revenues. Technically, in the steady state framework, the time-series correlation between revenues and expenses is unity only if $g_S = g_E$, as in that case the covariance of (S_t, E_t) equals the product of standard deviations of S_t and E_t. When g_S differs from g_E, the correlation will be lower than unity and depend on the difference. The present model indicates that g_S deviates from g_E when the change rate in TEP k deviates from zero or when the matching relationship between expenses and sales deviates from constant returns to

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scale (CRS). Thus, in addition to poor matching, the low correlation between sales revenue and expenses can result from $k \neq 0$ or $\alpha + \beta + \gamma \neq 1$.

Dichev and Tang (2008) present the case of poor matching, which is based on modeling the equation for expenses. In this equation, a random variable is introduced that represents mismatched expenses being unrelated to the well-matched expense and revenue. Thus, the mismatched expense acts as a noise. The noise variable has a strong negative first-order autocorrelation reflecting the fact that the mismatches of expenses are eventually resolved in the long run because accounting is self-correcting. Dichev and Tang show that matching becomes worse if the noise in the current period is higher. Therefore, they define the quality of matching as the inverse of the noise variable. It is technically closely related to the correlation between sales revenue and total expenses (REC). The higher correlation coefficient is associated with lower noise, and thus, a better quality of matching.

Dichev and Tang (2008) conclude that poor matching increases the volatility of earnings because the mismatched expenses act as a noise that is not related to the economics process of creating earnings. Furthermore, the persistence of earnings will decrease with poor matching, as it brings negative autocorrelation in the time-series of earnings. However, matching expenses against revenues is essentially a time-series phenomenon and the mismatches of expenses are resolved in the long run. Thus, deviations in earnings from the long-run mean will gradually diminish over time. There is an economic shock in every period, which is the noise in the matching relation and has a mean of zero. The variance of this economic shock represents the economic volatility of the business environment. In a perfect matching situation, the volatility is driven entirely by economic factors.

The noise driven by poor matching affects the time-series of earnings through the impact on the time-series of expenses. The variance (volatility) of earnings can be presented as follows:

$$VAR(S_t - E_t) = VAR(S_t) + VAR(E_t) - 2\,COV(S_t, E_t) \tag{16}$$

where VAR refers to time-series variance and COV to time-series covariance respectively. The noise can make an effect on both $VAR(E_t)$ and $COV(S_t, E_t)$. In a simplified deterministic framework, let us assume that S_t and E_t grow at steady rates g_S and g_E, which may, however, differ from each other due to a non-zero change in TEP ($k \neq 0$) or to the inconstant returns to scale ($\alpha + \beta + \gamma \neq 1$).

For the steady deterministic time-series of E_t and S_t the mean of the square over n periods can be presented in the following mathematical form:

$$MSQ(E_t) = \frac{E_0^2(1+g_E)^2\left(1-(1+g_E)^{2n}\right)}{-ng_E(2+g_E)} \tag{17a}$$

$$MSQ(S_t) = \frac{S_0^2(1+g_S)^2\left(1-(1+g_S)^{2n}\right)}{-ng_S(2+g_S)} \tag{17b}$$

where E_0 and S_0 are the initial values of expenses and sales, respectively. In the same way, the mean of these time-series over n periods can be presented as follows:

$$MEAN(E_t) = \overline{E} = \frac{E_0(1+g_E)(1-(1+g_E)^n)}{-ng_E} \tag{18a}$$

$$MEAN(S_t) = \overline{S} = \frac{E_0(1+g_S)(1-(1+g_S)^n)}{-ng_S} \tag{18b}$$

These results can be used to calculate the variance of the time-series as the difference between the mean of the square [Equation (17)] and the square of the mean [Equation (18)], which leads to the following expressions:

$$VAR(E_t) = \frac{E_0^2(1+g_E)^2}{n(-g_E)}\left[\frac{1-(1+g_E)^{2n}}{2+g_E} - \frac{(1-(1+g_E)^n)^2}{n(-g_E)}\right] \tag{19a}$$

$$VAR(S_t) = \frac{S_0^2(1+g_S)^2}{n(-g_S)}\left[\frac{1-(1+g_S)^{2n}}{2+g_S} - \frac{(1-(1+g_S)^n)^2}{n(-g_S)}\right] \tag{19b}$$

where n is the number of observations in the steady time-series. If $E_O = S_O$ and $g_E = g_S$, then the variances of E_t and S_t are equal. It should be noted that variances are affected by the length of period n.

Let us assume that there is an alternating series of constant noise terms r, which change its sign each period so that the noise term r is positive ($+r$) in one period but negative ($-r$) in the following period, and so on. Thus, noise r is independent of E_t but affects its variance. The noise term reflects mismatching of expenses with sales and as an alternating series, it brings negative autocorrelation in the time-series of E_t. Let us assume that n is even so that the mean of r is zero resolving the mismatches of expenses in the long run and leading to self-correcting accounting. As the noise term r is constant, the deviations in expenses due to the noise will gradually diminish over time if $g_E > 0$. On these assumptions, the noise term r makes an additive impact $R(VAR)$ on $VAR(E_t)$ where $R(VAR)$ is expressed as:

$$R(VAR) = r^2 + 2rE_0\frac{(1+g_E)((-1-g_E)^n - 1)}{n(2+g_E)} = r^2 + 2r\overline{E}\frac{g_E}{2+g_E} \tag{20}$$

where $\overline{E}$ is the mean of the time-series E_t. Thus, for an even number of observations n noise r leads to that $VAR(E_t, r) = VAR(E_t) + R(VAR)$ where $VAR(E_t, r)$ is the variance of time-series of E_t with the noise r.

The noise term r also affects the covariance between sales and expenses. Using the same assumptions, the covariance $COV(S_t, E_t)$ can be presented in the following form:

$$COV(S_t, E_t) = E_0S_0\frac{(1+g_E)(1+g_S)}{n}\left[\frac{1-((1+g_E)(1+g_S))^n}{1-(1+g_E)(1+g_S)} - \frac{(1-(1+g_E)^n)(1-(1+g_S)^n)}{ng_Eg_S}\right] \tag{21}$$

with the following additive effect of noise $R(COV)$:

$$R(COV) = r\,S_0 \frac{(1+g_S)\left((-1-g_S)^n - 1\right)}{n(2+g_S)} = r\,\overline{S}\,\frac{g_S}{2+g_S} \tag{22}$$

where $\overline{S}$ is the mean of the time-series of S_t.

Thus, the effect of noise r on the volatility (variance) of earnings $S_t - E_t$ can be presented by the following difference between $R(VAR)$ and $-2\,R(COV)$:

$$R(VAR) - 2\,R(COV) = r^2 + 2r\left[\frac{\overline{E}g_E}{2+g_E} - \frac{\overline{S}g_S}{2+g_S}\right] \tag{23}$$

which is positive for typical values of the parameters. However, the impact of noise on the variance of earnings can be negative for a large difference between average sales and expenses $\overline{S} - \overline{E}$ (average earnings). If there is a steady state so that $g_S = g_E = g$, then this impact is negative for:

$$\frac{\overline{S}_t - \overline{E}_t}{r} > \frac{1}{2} + \frac{1}{g} \tag{24}$$

Therefore, the sign of the effect of noise r on the volatility of earnings depends on this situation on the noise term r, steady growth rate g and average earnings $\overline{S} - \overline{E}$.

The correlation coefficient between S_t and E_t without and with noise r is defined as follows:

$$CORR(S_t, E_t) = \frac{COV(S_t, E_t)}{\left[VAR(S_t)\,VAR(E_t)\right]^{1/2}} \tag{25a}$$

$$CORR(S_t, E_t, r) = \frac{COV(S_t, E_t) + R(COV)}{\left[VAR(S_t)\left(VAR(E_t) + R(VAR)\right)\right]^{1/2}} \tag{25b}$$

where $CORR(S_t, E_t)$ is the original correlation without noise r and $CORR(S_t, E_t, r)$ with noise r. If there is a steady state so that $g_S = g_E = g$, then $CORR(S_t, E_t)$ equals unity and $CORR(S_t, E_t, r)$ is less than unity due to the noise. Table I presents exemplary values for both correlation coefficients assuming, firstly that r is 2.5 per cent (Panel 1) and, secondly, 5.0 per cent (Panel 2) of the average $\overline{S}$. The diagonals of the matrices describe the steady state where $g_S = g_E = g$. The table shows that, in the diagonals, the effect of noise r is stronger, the lower is g. The effect of noise is not sensitive to g_S, as there is only a low variation in the correlations between the rows of the right-hand matrices. However, the effect is stronger, the lower is g_E. These findings indicate that even for a large positive difference between g_S and g_E the noise only makes a relatively weak impact on the correlation. However, when $g_E \geq g_S$, the effect of noise is stronger, the less significant is the difference between the growth rates. In fact, when $g_E = g_S$, the effect is strongest for $g_E \geq g_S$.

The correlation coefficients presented in Panels 1 and 2 of Table I are calculated for equations (25a) and (25b) using a ten-period time-series ($n = 10$). However, Panel 3 shows the correlation coefficients calculated for a longer time-series ($n = 20$). It shows that the longer the time-series, the lower is the correlation coefficient for different growth rates g_S and g_E. It also shows that the longer the time-series, the lower is the effect of noise r on the correlation.

g_E	Correlation coefficient without noise: g_S					Correlation coefficient with noise: g_S				
	0.01	0.02	0.03	0.04	0.05	0.01	0.02	0.03	0.04	0.05
Panel 1. $n = 10$ and $r = 2.5\%$ of average E_t										
0.01	1.0000	0.9999	0.9997	0.9993	0.9988	0.8009	0.8009	0.8008	0.8006	0.8003
0.02	0.9999	1.0000	0.9999	0.9997	0.9993	0.9277	0.9278	0.9277	0.9276	0.9273
0.03	0.9997	0.9999	1.0000	0.9999	0.9997	0.9636	0.9639	0.9640	0.9639	0.9637
0.04	0.9993	0.9997	0.9999	1.0000	0.9999	0.9778	0.9782	0.9784	0.9785	0.9785
0.05	0.9988	0.9993	0.9997	0.9999	1.0000	0.9845	0.9851	0.9854	0.9857	0.9857
Panel 2. $n = 10$ and $r = 5.0\%$ of average E_t										
0.01	1.0000	0.9999	0.9997	0.9993	0.9988	0.6037	0.6037	0.6036	0.6035	0.6034
0.02	0.9999	1.0000	0.9999	0.9997	0.9993	0.7996	0.7996	0.7996	0.7995	0.7993
0.03	0.9997	0.9999	1.0000	0.9999	0.9997	0.8846	0.8848	0.8849	0.8849	0.8848
0.04	0.9993	0.9997	0.9999	1.0000	0.9999	0.9258	0.9262	0.9264	0.9265	0.9265
0.05	0.9988	0.9993	0.9997	0.9999	1.0000	0.9481	0.9485	0.9489	0.9491	0.9492
Panel 3. $n = 20$ and $r = 2.5\%$ of average E_t										
0.01	1.0000	0.9997	0.9987	0.9972	0.9951	0.9225	0.9223	0.9215	0.9202	0.9184
0.02	0.9997	1.0000	0.9997	0.9988	0.9973	0.9775	0.9778	0.9775	0.9767	0.9753
0.03	0.9987	0.9997	1.0000	0.9997	0.9988	0.9884	0.9894	0.9897	0.9894	0.9886
0.04	0.9972	0.9988	0.9997	1.0000	0.9997	0.9913	0.9928	0.9937	0.9940	0.9938
0.05	0.9951	0.9973	0.9988	0.9997	1.0000	0.9912	0.9934	0.9949	0.9958	0.9961

Notes: g_S = growth rate of sales, g_E = growth rate of total expenses

Thus, in this framework, the length of the time series affects the correlation coefficient, which can be shown analytically in the following way. The correlation coefficient between S_t and E_t can for steady growth rates g_S and g_E presented in the following simplified form:

$$CORR(S_t, E_t) = CORR\left(S_0(1 + g_S)^t, E_0(1 + g_E)^t\right) = CORR\left((1 + g_S)^t, (1 + g_E)^t\right)$$
(26)

which shows that it depends on the difference between g_S and g_E but also on the length of the time series t ($t = 1, \ldots, n$).

It is obvious that the dependence of the correlation coefficient on the length of the time-series n and the difference between g_S and g_E weaken the accuracy of the coefficient as a measure of the quality of matching. However, these problems can be solved replacing the correlation coefficient by the correlation coefficient of the logarithmic time-series. For the steady time-series of S_t and E_t this coefficient of correlation can be simplified as follows:

$$CORR(\log S_t, \log E_t) = CORR\left(\log S_0 + t\log(1 + g_S), \log E_0 + t\log(1 + g_E)\right)$$
$$= CORR\left(t\log(1 + g_S), t\log(1 + g_E)\right) = CORR(t, t) = 1 \qquad (27)$$

Thus, the correlation coefficient between the logarithmic time-series is identically 1 being independent of the difference between g_S and g_E but also of time t or the length of the time-series n. Therefore, this coefficient may provide us with a statistically less biased benchmark on assessing the quality of matching than the ordinary linear correlation coefficient between S_t and E_t (REC).

2.4 Hypotheses

Dichev and Tang (2008) state that if expenses are not properly matched against the resulting revenues, it is defined as a poor matching and is regarded as a noise in the economic relation of advancing expenses to obtain revenues. Dichev and Tang conclude that poor matching decreases the synchronal correlation between revenues and expenses. Poor matching means that some of the perfectly matched expenses get scattered across different periods, which results in a lower synchronal correlation than the underlying economic correlation of advancing expenses to produce revenues. Thus, the correlation coefficient between current sales revenue and contemporaneous total expense (REC) provides us with a useful indirect indicator of poor matching. However, REC is exposed to several drawbacks. Firstly, it only reflects the linear relationship between revenue and expenses assuming a linear matching function. Secondly, REC coefficient does not pay any attention to a potential increase in productivity of expenses, which affects the measure and can be observed as different growth rates of revenue and expense. Thirdly, REC only concentrates on the relationship between total revenue and total expenses without paying attention to the different categories of expenses.

In this study, a non-linear matching function has been introduced. The coefficient of determination of this function in a logarithmic form (CODL) can potentially be used to measure the revenue-expense relationship more accurately than REC. This measure of poor matching can be properly justified and it avoids the obvious drawbacks of a correlation. Firstly, it reflects a non-linear matching relationship-based mathematically on a CD-type function that is well justified and widely used function in production economics (Felipe and Adams, 2005, p. 428). Thus, this kind of non-linear function is better theoretically justified than a linear function. Secondly, the matching function approach can take account of expenses from different categories to strengthen the revenue-expense relationship. In this study, the matching function is specified for three main categories of expenses (labor expense, material expense and depreciation). Thirdly, the matching function approach takes explicitly account of the potential increase in the productivity of expenses (TEP) using a Solow-type of residual. Fourthly, when based on logarithmic time-series, it is practically relatively independent of the length of time-series used in estimation and also of the differences between the steady growth rates of revenue and expenses.

The sensitivity of REC to noise was here numerically investigated in simplified conditions. Table I indicated that on the given simplified conditions REC is sensitive to a random noise reflecting poor matching between revenue and expenses as proposed by Dichev and Tang (2008). However, on these conditions, REC was not found sensitive to the differences between the growth rates of expenses and sales although being more sensitive to lower than higher values of the growth rate of expenses. This finding implies that although REC only reflects the linear dependence between sales and expenses, it may not be very sensitive to a non-zero change in TEP or to the deviations from constant returns on scale making the growth rates of sales and expenses to differ from each other. Therefore, in certain specified cases, the contribution of the matching function can be expected to be less than remarkable. However, the following hypothesis *H1* on the matching quality measures is presented for empirical testing:

H1. The matching function approach gives a stronger explanation of the revenue-
expense relationship in terms of CODL than the correlation coefficient in the
squared form (SREC).

Thus, it is expected that CODL more accurately describes the relationship between revenue
and expenses than SREC. However, it is also probable that these measures are closely
associated with each other.

The distinct nature of the matching function approach is that it takes explicitly
account of different expense components instead of concentrating on total expense
alone. Dichev and Tang (2008) showed that accrual components have lower correlation
than cash components so that the correlation between total revenues and expenses
(REC) is driven down when accounting is based on more accruals. Dichev and Tang
also split the accrual component of expenses into working capital, long-term operating
and financing accruals to search for the differential impacts of these components.
However, they find some variation but failed to uncover meaningful differences in their
relative roles. Moreover, Dichev and Tang showed that when the quality of accruals is
low, firms are likely to be more affected by deteriorating matching quality. Thus,
Dichev and Tang (2008) concluded that accounting-related factors play a substantial
role in the temporal patterns of revenue and expenses.

However, Donelson *et al.* (2011) divided the total expense into six components, which
are the costs of goods sold, selling general and administrative expenses, depreciation
expenses, tax expenses, other expenses and special items. Special items consist mostly of
gains and losses from asset sales, restructuring charges and asset impairments. They
used regression analysis and found that that the revenue-expense relation (REC) is
sensitive to the component of the special item whose importance has increased due to
increased competition over time. Thus, they concluded that the decrease in REC observed
by them and Dichev and Tang (2008) is caused by rather economic events than
accounting-related factors.

In this study, the total expense is divided into three components (labor expense,
material expense and depreciation) according to their expected quality of accruals. It is
expected on the grounds of the analytical results that matching quality is associated with
the matching sensitivities (elasticities) of these components. These expense components
economically form the most important expense categories in business activities.
However, tax expenses, other expenses and special items considered by Donelson *et al.*
(2011) are excluded due to their special nature. The expenses in the selected three
categories strongly differ with respect to the quality of accruals, and thus, potentially
with respect to the accuracy of matching. In accounting practice, material expenses are
usually most accurate to match with sales revenue, followed by labor expenses, and
finally by depreciations. Material expense is the cost of materials used to manufacture a
product or provide a service, excluding all indirect materials. The labor expense is
defined as the salaries and wages paid to the employees, plus related payroll taxes and
benefits. The labor expense is broken into direct and indirect (overhead) costs. These
expenses can usually be matched with sales revenue with an average accuracy but less
accurately than material expenses. Thus, labor expenses may not significantly increase
or decrease the average matching accuracy. However, fixed asset investments generate
revenue for a long period in the future making periodic expenses (depreciation) difficult
to match accurately with current sales revenue. Therefore, the following hypothesis *H2* is
presented for empirical testing:

H2. The matching quality is *H2a* independent of the importance of labor expense, *H2b* positively associated with the importance of material expense and *H2c* negatively associated with the importance of depreciation.

If hypothesis *H2* is supported by empirical evidence, it has important implications. As the importance of the expenses from different categories greatly varies between different industries, the hypothesis implies that there will be found significant differences in the accuracy of matching between industries.

3. Empirical data and methods

3.1 Empirical data

The empirical data of the study are extracted from the Orbis database of Bureau Van Dijk (BvD) under restrictions that the selected firm must be Finnish, industrial firm, have successive financial statements available for at least 10 years, and have total assets at least €1m in each year. Originally, there were 14,296 firms fulfilling the above three criteria set for the sample firms. However, 5,416 firms had missing values in financial data (sales revenue, labor expense, material expense or depreciation) and were excluded from the sample. Moreover, 654 firms were excluded from the sample as outliers due to extreme values obtained in estimating the matching function model. The final sample thus includes all 8,226 firms. Almost all firms have financial statements until 2014 or 2015 covering a period characterized by the 2008 financial crisis that produced a significant economic shock to the global economy. This crisis first touched the US financial sector in 2007, but the effects spread to several national economies, resulting in what has often been called the Great Recession. Therefore, the economic development in Finnish firms was quite negative reflected by low profitability, productivity and growth.

The effect of the financial crisis on the stability of the time-series was assessed splitting the 10-year data into two sub-samples with 5-year time-series (first and second 5-year periods). For both periods, the revenue-expense correlation coefficients (REC) were calculated and compared with each other. The results showed that in general, both sub-periods lead to similar results. However, there are exceptions where correlations in the sub-periods behave in different ways. The Spearman rank correlation coefficient between the sub-period and 10-year period revenue-expense correlations was 0.688 for the first sub-period and 0.691 for the second. However, this rank correlation between the sub-period correlations was only 0.353 referring to different types of matching behavior in the sub-periods. There were found also some inconsistencies in the behavior. In 3.0 per cent of firms, the revenue-expense correlation was negative in the first sub-period and positive in the second. Similarly, in 1.8 per cent of the firms, this correlation was negative in the second sub-period and positive in the first. However, for the whole 10-year period the percent of negative correlations was only 0.5 per cent. Thus, it is obvious that in some firms the matching behavior has strongly changed during the 10-year period which should be taken into account when generalizing the results. Figure 1 shows however that the large majority of firms show similar matching behavior in both sub-periods, as most observations are concentrated on the upper-right corner.

Table II presents the size and industry distributions of the sample firms. Panel 1 shows that the majority of the firms employed (in the last reporting year) less than 50 employees (5,841 firms or 71.0 per cent). There are only 395 (4.8 per cent) firms having more than 250 employees. The average number of employees in the last reporting year was 84 but the median only 11 clearly indicating a skewed size distribution. In the last reporting year, the average total assets were 21,647.2 thousand euro while the median was only €2,068.4

Revenue-expense correlation in the second 5-year period
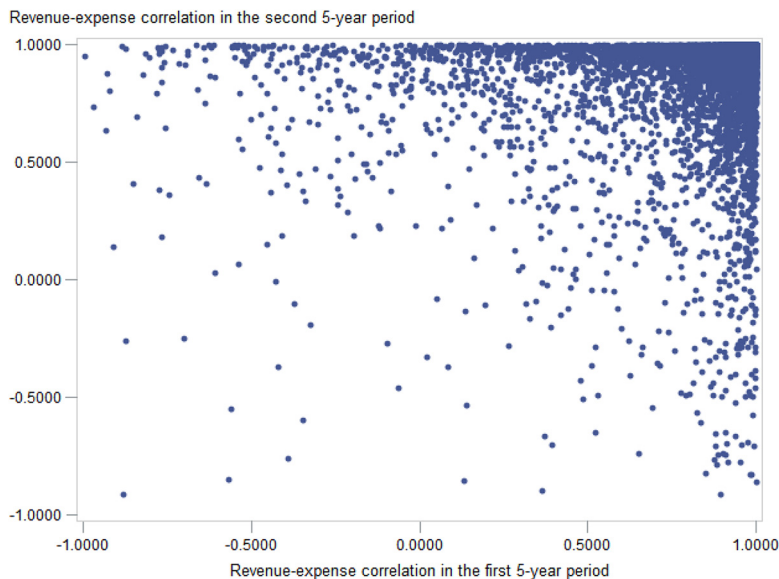
Figure 1.
The relationship
between the revenue-
expense correlation
coefficients in the
first and second five-
year sub-periods

No. of employees	Frequency	(%)	Cumulative frequency	Cumulative (%)
Panel 1. Number of employees (last reporting year)				
No. of employees				
0-50	5841	71.01	5,841	71.01
51-99	649	7.89	6,490	78.90
100-249	436	5.30	6,926	84.20
250-	395	4.80	7,321	89.00
Missing	905	11.00	8,226	100.00
Panel 2. Industry				
industry				
Manufacturing	2129	25.88	2,129	25.88
Construction	1224	14.88	3,353	40.76
Trade	2426	29.49	5,779	70.25
Transport	831	10.10	6,610	80.35
Service	1272	15.46	7,882	95.82
Other	344	4.18	8,226	100.00

Table II.
Statistical
distribution of the
sample firms
($n = 8,226$)

thousand. This distribution corresponds to the skew size distribution of Finnish firms in general (very small firms are excluded) but is very different from the sample used by Dichev and Tang (2008) and Donelson *et al.* (2011). Similarly, industrial distribution is statistically representative (Panel 2). The majority of firms are either manufacturing (25.9 per cent) or trade (29.5 per cent) firms. This diverse industrial distribution makes it possible to assess the effect of the importance of expenses from different categories. The sample includes only 41 (0.5 per cent) listed firms and 43 (0.5 per cent) firms using IFRS instead of local GAAP.

Finally, there are only 96 (1.2 per cent) bankrupt firms in the sample. This percent, however, corresponds to the average percent of bankrupt firms in Finland.

3.2 Statistical methods
The most important task of the empirical analysis is to estimate the parameters of the matching function. First, the matching function is presented with a standard error term in a logarithmic form as follows:

$$S_t = K_t L_t{}^\alpha M_t{}^\beta D_t{}^\gamma \varepsilon_t \tag{28a}$$

$$log\, S_t = \log K + t\log(1+k) + \alpha\log L_t + \beta\log M_t + \gamma\log D_t + \log\varepsilon_t \tag{28b}$$

where K, k, α, β and γ are the parameters of the matching model to be estimated. As it is logical to set non-negativity constraints for the estimates of the matching elasticities, the estimation task includes the following inequalities:

$$\alpha \geq 0 \tag{29a}$$

$$\beta \geq 0 \tag{29b}$$

$$\gamma \geq 0 \tag{29c}$$

As the objective of the model is to provide a measure that is comparable with SREC, the estimation task is defined as on the non-negativity constraints to minimize the squared sum of the random residual terms log ε_t ($SS(\log \varepsilon_t)$) over the period of n years. Then, the resulted minimum squared sum is divided by n times the variance of the logarithmic sales revenue ($VAR(\log S_t)$) over the same period, and deducted from unity to give CODL for the logarithmic matching model as $R^2 = 1 - SS(\log \varepsilon_t)/(n(VAR(\log S_t))$. This coefficient describing the degree of explanation of the variance of the logarithmic sales revenue will be used in the further empirical analysis comparatively to SREC. It is relatively insensitive to the length of the estimation period and to the difference between g_S and g_E.

For the minimization of the objective function for each of the 8,880 sample firms, the model variables were calculated for a period of ten years ($n = 10$) until the last reporting year. First, sales revenue S_t was measured by the Orbis variable *Sales* (in thousands of euro), labor expense L_t by *Cost of employees*, material expense M_t by *Material cost*, depreciation D_t by *Depreciations and amortizations*, and finally time t was defined as 0-9. Then, the minimization problem was solved by the generalized reduced gradient method (GRG) in Microsoft Solver. GRG is a generalization of the reduced gradient method by allowing nonlinear constraints and arbitrary bounds on the variables (Lasdon *et al.*, 1974). It can handle equality constraints and also inequality constraints, which are converted to equalities by the use of slack variables. GRG uses a combination of the gradient of the objective function and a pseudo-gradient derived from the equality constraints. It is an iterative method using a search procedure where the search direction is found in the way that any active constraint remains precisely active for some small move in this direction. In the further analyses, 654 solutions with extreme values (outliers) were excluded leading to

the final sample of 8,226 firms. Usually, the outliers were originated from a structural break in the business of the firm.

The hypotheses of the study will be tested using simple statistical tests. Firstly, the statistical difference between the distributions of CODL and SREC is tested by location tests based on the paired difference (*H1*). These tests are useful in this analysis, as they either increase the statistical power in comparison to unpaired tests or reduce the effects of potential confounders. Thus, the paired *t*-test (normally distributed differences) and the non-parametric Wilcoxon signed-rank test are used to test *H1*. Secondly, ordinary least squares (OLS) regression analysis is used to test the impact of the importance of the expenses from different categories on the matching quality (*H2*). In general, this regression can be presented as follows:

$$MA = a_0 + a_1 I(L) + a_2 I(M) + a_3 I(D) + \varepsilon \qquad (30)$$

where *MA* is matching accuracy, *I(L)* is the importance of labor expense, *I(M)* is the importance of material expense, *I(D)* is the importance of depreciation, ε is the standard error term, a_O is the intercept and a_1, a_2 and a_3 are regression coefficients. In the regression equations, matching quality is measured by CODL and by SREC, for comparison. The importance of the expenses from different categories (labor expense, material expense, and depreciation) is measured by the estimates for the matching elasticities allowing us to test hypotheses *H2a*, *H2b* and *H2c*, respectively.

4. Empirical results
4.1 Descriptive statistics
Table III presents descriptive statistics of the matching model variables for the sample firms ($n = 8,226$). The growth rate k in the scale factor is on average very small indicating only negligible improvement in the efficiency of expenses in generating revenue. Empirically, this rate is very close to the Solow residual calculated using the steady growth rates of sales revenue and expenses from different categories. The Spearman rank correlation coefficient between these estimates exceeds 0.7. The matching elasticities are on average highest for the material expense (β) and clearly lowest for depreciation (γ). The distribution of the depreciation elasticity of sales revenue (γ) is actually very skew and the median value is low. In fact, 5,712 firms (69.4 per cent) got an estimate of zero for the elasticity indicating that very often depreciations are not matched with current sales revenue at all. The average sum of expense elasticities exceeds unity referring to increasing returns on the scale. However, the median sum is very close to unity indicating constant returns to scale. Both CODL and REC are on average very high, and the median values exceed 0.96. The average values of these matching accuracy measures are close to each other. However, in the comparison SREC should be used to make the measures comparable with each other. The low values of the Durbin–Watson statistics indicate that on average successive error terms of the matching equation are positively correlated.

Table IV shows the steady growth of OLS estimates for the model variables. The growth rate of labor expense (g_L) on average exceeds that of sales revenue (g_S), whereas the growth rate of depreciation (g_D) is lower. For each steady growth rate, the lower quartile is negative reflecting the difficult economic situation of Finnish firms in the research period. The average coefficient of determination R^2 for the growth equations is highest for labor expenses exceeding 0.50. For other expense categories, R^2 is somewhat less than 0.50. The actual weighted average percentages of expense categories are comparable with their estimated average elasticities of sales revenue. This result

Variable	Description	Mean	Median	SD	Lower quartile	Upper quartile
$ln\,K_0$	Logarithmic scale factor (Total cost productivity)	1.0070	0.9990	1.6221	0.5071	1.8394
K_0	Scale factor (Total cost productivity)	7.4811	2.7156	13.4168	1.6604	6.2930
$ln\,(1+k)$	Logarithmic growth rate of scale factor	0.0031	0.0017	0.0400	−0.0081	0.0151
k	Growth rate of scale factor	0.0039	0.0017	0.0412	−0.0081	0.0152
α	Sales elasticity of labor expense	0.3845	0.3049	0.3686	0.0653	0.6048
β	Sales elasticity of material expense	0.5140	0.5494	0.3116	0.2728	0.7614
γ	Sales elasticity of depreciation	0.1453	0.0330	0.2288	0.0000	0.2066
$\alpha+\beta+\gamma$	Sum of elasticities	1.0438	1.0048	0.2862	0.8971	1.1403
Model R^2	R^2 of the estimated matching model	0.9050	0.9647	0.1488	0.8911	0.9896
D-W	Durbin-Watson statistics	0.6308	0.5985	0.2731	0.4256	0.7968
Solow residual	Estimate of Solow residual	0.0028	0.0014	0.0367	−0.0098	0.0143
S/E	Average sales per average total expense	1.4469	1.3309	0.4772	1.1963	1.5464
CORR($S_t - E_t$)	Correlation between sales and total expense	0.9068	0.9673	0.1639	0.9010	0.9907

Table III.
Descriptive statistics of the model variables

Variable	Description	Mean	Median	SD	Lower quartile	Upper quartile
g_S	Growth rate of sales revenue	0.0367	0.0279	0.1099	−0.0205	0.0819
$R^2(S_t)$	R^2 of the growth model for sales revenue	0.4774	0.5004	0.3163	0.1710	0.7681
g_L	Growth rate of labor expense	0.0489	0.0378	0.1119	−0.0047	0.0887
$R^2(L_t)$	R^2 of the growth model for labor expense	0.5339	0.5959	0.3170	0.2373	0.8225
$L/(L+M+D)$	Weighted percentage of labor expense	0.3528	0.3269	0.2181	0.1741	0.4891
g_M	Growth rate of material expense	0.0406	0.0249	0.1568	−0.0321	0.0864
$R^2(M_t)$	R^2 of the growth model for material expense	0.4295	0.4250	0.3035	0.1352	0.7045
$M/(L+M+D)$	Weighted percentage of material expense	0.5819	0.6081	0.2461	0.4117	0.7898
g_D	Growth rate of depreciation	0.0308	0.0197	0.1478	−0.0545	0.0961
$R^2(D_t)$	R^2 of the growth model for depreciation	0.4467	0.4533	0.3035	0.1576	0.7164
$D/(L+M+D)$	Weighted percentage of depreciation	0.0653	0.0351	0.0827	0.0135	0.0846

Table IV.
Descriptive statistics of the sales and expense variables

indicates that the actual percentages are on average relatively close to the theoretically optimal percentages. The (Pearson and Spearman rank) correlation coefficients between the actual and optimal percentages (not showed in the table) are over 0.6 for the material expense, about 0.5 for the labor expense, and less than 0.3 for depreciation being all

statistically very significant. Material expense makes on average almost 60 per cent of actual total expense whereas the percentage for labor cost is about 35 per cent and for depreciation, it is only less than 7 per cent (the median value being less than 4 per cent). Thus, material expenses which are the most accurate to match, play the central role in total expense while depreciations being the most inaccurate to match, have got only a negligible role.

4.2. Testing hypotheses
4.2.1 Hypothesis H1. Hypothesis *H1* assumes that CODL gives a stronger explanation for the revenue-expense relation as compared with SREC. Empirically, these measures are distributed in a very similar way. The Pearson coefficient of correlation between them is 0.612 (*p*-value < 0.0001) while the Spearman coefficient of rank correlation is even higher, 0.766 (*p*-value < 0.0001) indicating a significant non-linear relationship. These high correlation coefficients refer to a high internal consistency between the measures leading to standardized Cronbach Alpha of 0.759 (acceptable level). Thus, it is obvious that at least on average they produce similar results for the matching quality. For 6344 out of 8226 firms (77 per cent), CODL gives a higher figure than SREC. The mean of CODL is 0.9050 that clearly exceeds the mean of SREC (0.8488). The average difference between the measures is thus 0.0562. These findings indicate that in general CODL is stronger associated with the revenue-expense relationship than SREC.

Table V shows the quantiles of the measures and of their difference. For each quantile, CODL exceeds SREC indicating a stronger degree of explanation. At the 0.50 level (median) the quantile of CODL exceeds that of the squared correlation by about 0.03. However, the quantiles show that CODL gives significantly higher figures especially for low values of the measures. For example, at the 0.10 level the quantile of CODL is 0.737, whereas that of SREC is only 0.563. Thus, SREC is very clearly outperformed by CODL especially when the relationship between revenue and total expense is weak. Both the paired *t*-test (30.84) and the signed-rank test (10247744) indicate that CODL gives higher figures (*p*-value < 0.0001). The scatter plot for the measures is presented by Figure 2 which graphically confirms the conclusion. Thus, empirical evidence strongly supports *H1*. The figure also shows that the

Level	Quantiles R^2	$CORR^2$	Differences R^2-$CORR^2$	Quantiles: R^2-$CORR^2$
100% Max	1.0000	1.0000	0.0000	0.9562
0.99	0.9996	0.9995	0.0001	0.6592
0.95	0.9984	0.9975	0.0009	0.3677
0.90	0.9968	0.9944	0.0024	0.2297
75% Q3	0.9896	0.9814	0.0082	0.0843
50% Median	0.9647	0.9357	0.0291	0.0185
25% Q1	0.8911	0.8119	0.0792	0.0006
0.10	0.7365	0.5634	0.1731	−0.0279
0.05	0.5846	0.3598	0.2247	−0.1050
0.01	0.2538	0.0547	0.1990	−0.4516
0% Min	0.0044	0.0000	0.0044	−0.9529

Notes: R^2 = Coefficient of determination of the matching model, $CORR^2$ = Squared coefficient of revenue-expenditure correlation

Table V.
The quantiles of the matching quality measures

Squared revenue-expense correlation coefficient
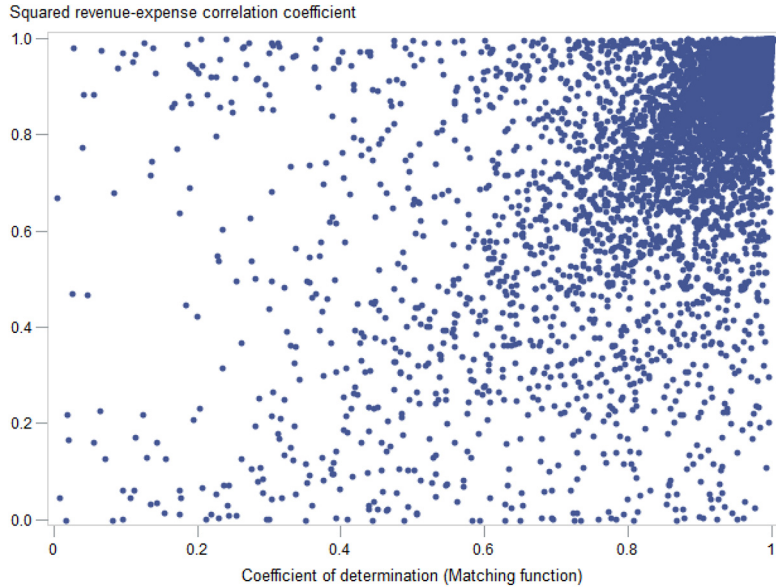
Figure 2.
The relationship
between the
coefficient of
determination of the
matching model and
the squared revenue-
expense correlation

Coefficient of determination (Matching function)

values of both CODL and SREC are highly concentrated on the upper-right corner where the values close to unity are located.

4.2.2 Hypothesis H2. Table VI presents the regression analysis results for the CODL (Panel 1) and REC (Panel 2) used to test hypothesis *H2*. Panel 1 shows that expense elasticities of sales explain about 15 per cent of the variation in CODL. Low variance inflation factors (VIFs) indicate that there does not exist significant multicollinearity in the model. Firstly, the regression results support *H2a*, as the estimate of the coefficient of the labor expense elasticity (α) reflecting the importance of the expenses from this category, is negative but statistically insignificant (p-value = 0.67). The standardized estimate is very close to zero emphasizing the insignificance of the variable. Secondly, the results also support *H2b* because the coefficient of the material expense elasticity (β) is positive and statistically very significant (p-value < 0.0001). The standardized estimate is high indicating a strong positive effect on CODL. Thirdly, empirical findings also give support to *H2c*, as the estimate of the depreciation elasticity (γ) is negative and statistically very significant (p-value < 0.0001). The standardized estimate is in absolute terms about the same height as the estimate of the material expense elasticity also indicating a strong effect. Thus, the evidence clearly supports *H2a*, *H2b* and *H2c*.

Panel 2 of Table VI shows that the three expense elasticities explain together about 10 per cent of the total variation in SREC. The VIFs for this regression equation have the same low values as before due to the identical independent variables indicating tolerable multicollinearity. However, the estimates of the regression coefficients differ from each other. Firstly, the estimate of the coefficient of the labor expense elasticity (α), is negative and statistically significant (p-value < 0.0001) although the t-values and the standardized estimate are in absolute terms lower than for other elasticities. Therefore, for SREC empirical evidence does not support *H2a*. Secondly, the evidence clearly supports *H2b*, as material expense elasticity (β) has obtained a positive and

Panel 1. Coefficient of determination of the matching model explained by expense elasticity estimates

Model	F value	p-value	R^2	Adjusted R^2		
	487.7000	<0.0001	0.1511	0.1508		
Parameter	Estimate	Standard error	t-value	p-value	Standardized estimate	Variance inflation
Intercept	0.8703	0.0063	137.2500	<0.0001	0.0000	0.0000
Labor expense elasticity (α)	−0.0025	0.0058	−0.4300	0.6687	−0.0062	2.0295
Material expense elasticity (β)	0.1130	0.0073	15.4100	<0.0001	0.2365	2.2810
Depreciation elasticity (γ)	−0.1540	0.0075	−20.6400	<0.0001	−0.2366	1.2725

Panel 2. Squared coefficient of revenue-expense correlation explained by expense elasticity estimates

Model	F value	p-value	R^2	Adjusted R^2		
	320.9400	<0.0001	0.1048	0.1045		
Parameter	Estimate	Standard error	t-value	p-value	Standardized estimate	Variance inflation
Intercept	0.8102	0.0091	89.3400	<0.0001	0.0000	0.0000
Labor expense elasticity (α)	−0.0468	0.0084	−5.6000	<0.0001	−0.0833	2.0295
Material expense elasticity (β)	0.1404	0.0105	13.3900	<0.0001	0.2110	2.2810
Depreciation elasticity (γ)	−0.1067	0.0107	−10.0000	<0.0001	−0.1177	1.2725

Table VI.
Regression analysis
results explaining
matching accuracy
by the importance of
expense categories

statistically very significant coefficient (p-value < 0.0001). Again, the standardized estimate indicates a strong positive effect on the dependent variable. Thirdly, the results support *H2c*, as the depreciation elasticity (γ) has a negative and statistically very significant coefficient ($p < 0.0001$). However, the standardized estimate is in absolute terms remarkably lower than the estimate of the material expense elasticity indicating a weaker impact on SREC. Thus, evidence on SREC clearly supports *H2b* and *H2c* but contradicts with *H2a*. These hypotheses were also tested using REC instead of SREC as the dependent variable. The results were similar but the statistical significance of the model was lower.

The robustness of the empirical findings was assessed by splitting the sample into two equal parts according to the value of the matching accuracy proxy. Then, the regression models were again estimated for the sub-samples. Panel 1 of Table AI presents the results for CODL by sub-samples. For the lower half of the sample, the evidence is similar to the whole sample. The findings support hypotheses *H2a*, *H2b* and *H2c*. The estimate of the coefficient of the labor expense elasticity (α) is now positive, but low and statistically insignificant (p-value = 0.48). For the upper half of the sample, evidence for the material expense elasticity (β) and the depreciation elasticity (γ) is similar as for the lower sub-sample supporting *H2b* and *H2c*. However, the coefficient of the labor expense elasticity (α) is negative and statistically significant at p-level of 0.019 although the impact of the coefficient on CODL is quite low. Thus, *H2a* is not supported by the evidence from this upper sub-sample. For SREC, the results for both sub-samples are similar and support hypotheses *H2b* and *H2c*, and for the upper sub-sample also *H2a*. However, for the lower sub-sample, the labor expense elasticity (α) has got a negative estimate that is significant at p-level of 0.017. Thus, the sub-sample results fully support *H2b* and *H2c* but only partially *H2a*.

4.3 Further evidence

The systematic differences between CODL and SREC were further assessed by a logistic regression model applied to explain the conditional probability of these differences. Table VII presents the binary logistic regression results for the conditional probability of whether SREC exceeds CODL (1) or not (0). This table shows that there are at least four important systematic factors, which increase the conditional probability leading SREC to overestimate CODL. First, the higher the change in the scale factor is, the higher is the probability. Thus, positive development in the efficiency of expenses to advance revenues (in terms of matching) may lead to overestimation. Secondly, the higher the matching elasticity (importance) of depreciation is, the higher is the probability. As SREC only takes account of the total expense, it may overestimate matching accuracy when the total expense includes a lot of depreciation. This kind of overestimation can happen due to the hidden mismatching bias where mismatching in different expense categories partly cancels mismatching of depreciations. Thirdly, the larger is the size of the firm, the more systematic is an overestimation. For larger firms, both CODL and SREC are very high and, consequently, the absolute differences between them are relatively small. The non-linearity of the matching function may in these circumstances lead SREC (as a measure of linear sales-expense relationship) slightly to exceed CODL. Finally, the probability is higher if the firm belongs to the trade industry. In this industry, material expenses play the dominant role, which may lead to a very high SREC due to the accuracy of matching.

	Likelihood ratio	χ^2	p-value	Rescaled R^2
Model fit	578.57450	14.00000	<0.0001	0.10310
	Percent concordant	Somers D	Hosmer and Lemeshow	p-value
Model fit	67.10000	0.34200	88.07910	<0.0001
Parameter	Estimate	Standard error	Wald χ^2	p-value
Intercept	−2.46110	0.44590	30.46430	<0.0001
Logarithmic scale factor $\ln K_0$	0.10980	0.05460	4.04490	0.04430
Logarithmic growth rate $\ln(1+k)$	6.21070	0.79830	60.53360	<0.0001
Labor expense elasticity (α)	0.51140	0.34140	2.24390	0.13410
Material expense elasticity (β)	0.10780	0.37730	0.08170	0.77510
Depreciation elasticity (γ)	2.26010	0.25650	77.61940	<0.0001
Employees < 50§	0.05650	0.09120	0.38330	0.53580
Employees ≥ 50 & < 100§	0.39730	0.12480	10.13040	0.00150
Employees ≥ 100 & < 250§	0.47880	0.13960	11.76830	0.00060
Employees ≥ 250§	0.67940	0.14010	23.53340	<0.0001
Manufacturing industry§	0.30690	0.14980	4.19550	0.04050
Construction industry§	−0.19180	0.16040	1.43060	0.23170
Trade industry§	0.86160	0.15040	32.81300	<0.0001
Transport industry§	−0.04400	0.16590	0.07030	0.79090
Service industry§	0.09200	0.15790	0.33990	0.55990

Notes: & = Dependent binary variable: 0 = SREC does not exceed CODL and 1 = SREC exceeds CODL. § = Binary dummy-variable (0 or 1)

Table VII.
Logistic regression analysis results for the probability that the squared correlation coefficient SREC exceeds the coefficient of determination CODL &

The findings on hypothesis *H2* have obvious implications that can be assessed statistically. First, the findings show that matching elasticities are important factors affecting matching accuracy. The additional tests showed (not reported here) that they in a statistical model lead to a higher degree of explanation of matching accuracy than for example the actual percentages or the growth rates of different expense categories. Secondly, as these elasticities are associated with the importance of different expense categories, they may cause significant differences in matching accuracy between industries. These potential differences can be found in Table AII that presents descriptive statistics of the model variables for different industries. This Appendix shows that the differences in matching accuracy between industries are statistically very significant. Trading firms seem to have a very high matching accuracy, which can be due to the very high material expense elasticity but low labor expense and depreciation elasticities. It is also remarkable that construction firms have a relatively low matching accuracy. However, it may be due to the project-type business with its own entry rules in book-keeping rather than to the expense elasticities.

Table AIII presents descriptive statistics of the model parameters for the different size classes. The differences in the accuracy between different size classes are not very remarkable although being statistically significant. Thus, size itself does not strongly affect matching accuracy because the values of the expense elasticities are not closely associated with the size. The differences in matching accuracy were also investigated for other types of firms (not presented here). Firstly, the comparison of firms with a different legal form showed that limited partnership firms had an exceptionally low matching accuracy (mean CODL = 0.769), which may be due to the special nature of those firms in determining labor expenses and profit in book-keeping. Consequently, the average labor expense elasticity of sales in these firms was exceptionally high (0.542), whereas that of material expense was low (0.315). Secondly, low matching quality (mean CODL = 0.868) was found for financially

distressed firms in insolvency proceedings (status). Thirdly, firms using IFRS showed lower matching quality (mean GODL = 0.873) than firms using local GAAP (mean GODL = 0.905) (accounting system). This result may be due to that IFRS is based on the balance sheet approach where earnings are not defined as the difference between revenues and expenses but viewed as a change in net assets leading to higher volatility and lower persistence (Dichev, 2008).

5. Concluding remarks

The matching principle is one of the basic underlying guidelines in accounting affecting the quality of financial reporting. This principle directs a firm to report its expenses in the income statement in the same period as the corresponding revenue. Thus, the principle requires a firm to match expenses with related revenues to report earnings (profitability) during a specified accounting period. Poor matching of expenses will cause several drawbacks to financial reporting, as it makes a noise in the time-series of expenses distorting continuity and predictability of earnings due to increased volatility. High quality of matching can mostly be observed as a high time-series REC. The lower the quality of matching due to the noise, the lower is REC. Therefore, this correlation and its different squared versions intuitively seem to be simple and useful indirect measures of matching quality. However, besides matching quality REC is affected also by several economic factors, which affect the height of the measure. Therefore, in certain circumstances, REC may be a biased measure. The founding idea of this study was to introduce a matching function concept that can better take account of these kinds of economic factors and provide us with a more accurate measure of matching quality.

The present approach was based on a matching function that is multivariate having a similar mathematical (multiplicative) form as the well-known CD production function. This kind of multivariate function can describe the economic relationship between sales revenue and matched expenses more accurately than REC for several reasons. Firstly, it can be generalized for several expense categories instead of total expenses. As the matching accuracy may vary between different expense categories, it is important to divide total expenses into homogenous parts. In this way, the function takes account of the matching accuracy of each expense category and avoids the hidden mismatching bias. This bias is originated from that mismatching in different expense categories may cancel each other indicating erroneously matching quality higher than in reality. In this study, the matching function was specified for three main expense categories, namely, labor expense, material expense and depreciation. These categories obviously differ from each other with respect to matching method and accuracy. Secondly, the growth rates of expenses and sales revenue may be different over time which obviously decreases REC irrespective of matching accuracy. In the matching function approach, this important point was taken into account.

The matching function included a Solow-residual-type growth rate for total expense productivity (TEP) explaining a part of the differences in the growth rates between revenue and expenses. Furthermore, the function took account of returns to scale. If these returns are not constant to scale, it will also lead to different growth rates for revenues and expenses. In summary, it was expected that CODL gives a more accurate measure of matching quality than REC. Therefore, it was hypothesized that CODL leads to a stronger explanation of the economic relationship between revenue and expense than SREC (*H1*). The matching function is also useful as giving information about the matching elasticities or sensitivities referring to the elasticities of sales revenue with respect to matched expenses from different categories. The higher the elasticity with respect to an expense category, the higher is the importance of the category in the matching process. Thus, it was hypothesized that CODL is relatively insensitive to labor expense elasticity, as these expenses can usually be matched

with sales with an average accuracy (*H2a*). However, material expenses are more accurate to match so that a positive impact was hypothesized for its elasticity (*H2b*). Finally, it was hypothesized a negative impact for depreciation elasticity, as depreciations are most difficult and inaccurate to match with current sales (*H2c*).

These hypotheses (*H1* and *H2*) were tested using ten-year time-series data from a sample of 8,226 Finnish firms. For each firm, the matching function was estimated for the ten-year period using the GRG method setting non-negativity constraints for the matching elasticities. Empirical evidence strongly supported *H1*. CODL and SREC were, however, found closely associated with each other leading to correlated rank orders of firms. CODL got higher values than SREC especially for the lower values of the coefficient. However, as the actual degree of mismatching cannot be directly observed from the external time-series, it was not possible to compare directly the empirical validity of CODL and SREC. CODL and SREC are both indirect measures of matching accuracy. However, the characteristics of CODL are theoretically better justified than those of SREC. Therefore, it was suggested that CODL is less biased than SREC. Thus, although the differences between CODL and SREC are empirically relatively small, SREC may be biased if the values of these measures systematically differ from each other. Statistical analyses showed that there were systematic differences between CODL and SREC caused by differences in the scale factor, rate of depreciation, size and industry.

In summary, the present study introduced a novel matching function approach to analyze and measure the quality of matching. This approach provided us with several implications. In practice, a matching function approach may provide us with a more accurate measure of matching accuracy than an ordinary REC. Moreover, this approach will bring plenty of additional information about the matching elasticities of different expense categories and the development of revenue-expense relationships over time. The matching elasticities reflect the importance of the expense categories to matching accuracy. The elasticities strongly differ by the expense categories implying that also the accuracy of matching significantly differs from each other. In general, depreciations proved to be insensitive to periodic sales revenue. Most firms did not match depreciations with current revenue at all leading to zero matching elasticity. In financial reporting, this kind of behavior is negative, as it impairs the quality of earnings. However, material expenses were generally very sensitive to sales revenue which implies a high matching accuracy. Thus, matching accuracy is high for firms emphasizing the importance of material expenses.

The results also emphasize the importance of the change of expense efficiency, which affects the traditional linear revenue-expense relation. If there is a significant increase or decrease in efficiency, the values of REC should be used to reflect matching only cautiously. In this kind of situation, it would be better to use CODL that takes account of change in efficiency. The findings also imply that when assessing matching between different kinds of firms by means of REC, differences in total expense productivity TEP, depreciations, size and industry may weaken the accuracy of the measurement. The statistical results also implied that there are several factors (for example, project-type business, IFRS accounting system, financial distress and the limited partnership legal form), which in practice seem to affect matching accuracy. Financial analysts and other stakeholders should pay attention to these kinds of firm-specific factors when assessing matching accuracy or earnings quality in different types of firms.

Although this study has provided novel findings and implications for matching accuracy research, it is also exposed to several limitations that can be relaxed in further studies. The matching function was in this study specified to follow the CD-type multiplicative form with

three expense categories as the arguments. In further studies, different types of functions (for example, CES function) and different expense categories (such as R&D expenses) should be applied to compare the results. Furthermore, the parameters of the matching model were in this study solved using GRG. New advanced estimation methods should be applied and tested. In this study, CODL was used to measure the accuracy of matching and compared only with REC. The performance of this measure should in future research be further compared with coefficients of different types of matching models (for instance, linear models) besides the simple revenue-expense correlation. Furthermore, economic and accounting factors, which besides the matching elasticities affect the accuracy, should be investigated. For example, financial distress, the type of business, accounting system or the legal form could be examples of these factors. Finally, this study was concentrated on a sample of Finnish firms. In the future, it would be useful to compare results from different countries, too.

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Appendix

Panel 1. Coefficient of determination of the matching model explained by expense elasticity estimates

Lower half (lowest values): average coefficient of determination 0.8229

Model	F value 120.4300	p-value <0.0001	R² 0.0808	Adjusted R² 0.0802		
Parameter	Estimate	Standard error	t-value	p-value	Standardized estimate	Variance inflation
Intercept	0.7963	0.0093	85.5900	<0.0001	0.0000	0.0000
Labor expense elasticity (α)	0.0060	0.0085	0.7100	0.4772	0.0139	1.7026
Material expense elasticity (β)	0.1126	0.0115	9.7700	<0.0001	0.1955	1.7908
Depreciation elasticity (γ)	−0.1077	0.0105	−10.2500	<0.0001	−0.1685	1.2073

Upper half (highest values): average coefficient of determination 0.9782

Model	F value 119.4700	p-value <0.0001	R² 0.0802	Adjusted R² 0.0796		
Parameter	Estimate	Standard error	t-value	p-value	Standardized estimate	Variance inflation
Intercept	0.9853	0.0008	1,233.1000	<0.0001	0.0000	0.0000
Labor expense elasticity (α)	−0.0018	0.0007	−2.3600	0.0185	−0.0570	2.6119
Material expense elasticity (β)	0.0054	0.0009	6.1600	<0.0001	0.1571	2.9089
Depreciation elasticity (γ)	−0.0102	0.0011	−9.0300	<0.0001	−0.1518	1.2610

Panel 2. Squared coefficient of revenue-expense correlation explained by expense elasticity estimates

Lower half (lowest values): average coefficient of squared correlation 0.7207

Model	F value 91.3900	p-value <0.0001	R² 0.0625	Adjusted R² 0.0619		
Parameter	Estimate	Standard error	t-value	p-value	Standardized estimate	Variance inflation
Intercept	0.6916	0.0121	57.3300	<0.0001	0.0000	0.0000
Labor expense elasticity (α)	−0.0257	0.0108	−2.3800	0.0174	−0.0463	1.6574
Material expense elasticity (β)	0.1333	0.0150	8.9100	<0.0001	0.1787	1.7612
Depreciation elasticity (γ)	−0.0900	0.0145	−6.2000	<0.0001	−0.1020	1.1870

Upper half (highest values): average coefficient of squared correlation 0.9769

Model	F value 60.7600	p-value <0.0001	R² 0.0425	Adjusted R² 0.0418		
Parameter	Estimate	Standard error	t-value	p-value	Standardized estimate	Variance inflation
Intercept	0.9723	0.0016	598.2100	<0.0001	0.0000	0.0000
Labor expense elasticity (α)	−0.0019	0.0016	−1.2200	0.2231	−0.0320	2.9523
Material expense elasticity (β)	0.0097	0.0018	5.4600	<0.0001	0.1551	3.4588
Depreciation elasticity (γ)	−0.0054	0.0018	−3.0300	0.0025	−0.0556	1.4473

Table AI.
Regression analysis results for lower and upper half sub-samples of the matching accuracy measures

Variable	Statistic	Industry						BF/K-W	p value
		Manufacturing	Construction	Trade	Transport	Service	Other		
$\ln K_0$	Mean	0.87404	0.81257	1.02194	1.19610	1.33549	0.74516	21.15896	0.00000
	Median	0.96015	1.00952	0.96168	1.05269	1.27147	1.00678	119.86877	0.00000
$\ln(1+k)$	Mean	0.00157	0.00433	0.00253	0.00566	0.00539	−0.00275	3.94778	0.00141
	Median	0.00111	0.00243	0.00102	0.00310	0.00362	0.00311	42.48453	0.00000
α	Mean	0.39964	0.53397	0.20898	0.40454	0.52642	0.42385	214.60083	0.00000
	Median	0.34598	0.47479	0.12456	0.36983	0.51282	0.36910	974.11343	0.00000
β	Mean	0.52283	0.40217	0.70798	0.38497	0.33236	0.47187	422.22945	0.00000
	Median	0.55854	0.39447	0.76956	0.36390	0.27903	0.48533	1789.57298	0.00000
γ	Mean	0.14228	0.18728	0.08015	0.23755	0.14448	0.25436	95.53422	0.00000
	Median	0.04408	0.07438	0.00606	0.15629	0.04519	0.12429	431.34413	0.00000
$\alpha + \beta + \gamma$	Mean	1.06475	1.12342	0.99711	1.02706	1.00326	1.15008	50.80786	0.00000
	Median	1.03892	1.06359	0.97821	1.02176	0.98393	1.04510	198.69822	0.00000
Model R^2	Mean	0.90308	0.85497	0.94205	0.90622	0.89730	0.85972	67.47545	0.00000
	Median	0.95825	0.92988	0.98628	0.96175	0.95914	0.91990	893.74827	0.00000
D-W	Mean	0.63315	0.62878	0.63770	0.62346	0.62788	0.60394	1.16946	0.32146
	Median	0.60341	0.59716	0.60783	0.59181	0.59635	0.54678	7.18777	0.20705
Solow residual	Mean	0.00131	0.00438	0.00196	0.00398	0.00254	0.01004	4.28189	0.00068
	Median	0.00101	0.00322	0.00058	0.00340	0.00269	0.00490	33.34416	0.00000
S/E	Mean	1.40515	1.64890	1.26398	1.51984	1.61212	1.48893	169.00555	0.00000
	Median	1.34278	1.52050	1.20426	1.33522	1.48029	1.38884	1547.92233	0.00000
$CORR(S, E_i)$	Mean	0.91421	0.84681	0.95842	0.87930	0.88267	0.86569	103.60020	0.00000
	Median	0.96030	0.92200	0.98950	0.95930	0.95735	0.92685	1284.96286	0.00000

Notes: For the variables see Table II. K-W = Kruskal–Wallis test statistic; BF = Bonferroni F-test

Table AII.
Distribution of the
model estimates by
the industry of the
firm

Variable	Statistic	No. of employees				BF/K-W	p-value
		0-50	51-99	100-249	250-		
$ln\,K_0$	Mean	1.00985	0.96835	0.95660	0.97870	0.28314	0.83761
	Median	1.00515	0.97462	0.97277	0.97007	3.32305	0.34445
$ln\,(1+k)$	Mean	0.00295	0.00696	0.00283	0.00579	2.60579	*0.05002*
	Median	0.00154	0.00347	0.00119	0.00210	12.99090	*0.00466*
α	Mean	0.37715	0.42795	0.41797	0.43301	7.21709	*0.00008*
	Median	0.29140	0.40293	0.37799	0.41033	47.53124	*0.00000*
β	Mean	0.52566	0.47037	0.48403	0.44342	15.38286	*0.00000*
	Median	0.56166	0.51593	0.52801	0.48650	44.72159	*0.00000*
γ	Mean	0.13972	0.15254	0.13573	0.16494	2.20552	*0.08527*
	Median	0.03066	0.03944	0.03182	0.05490	10.49104	*0.01482*
$\alpha+\beta+\gamma$	Mean	1.04253	1.05085	1.03772	1.04137	0.23532	0.87180
	Median	0.99898	1.02928	1.01907	1.03810	21.57423	*0.00008*
Model R^2	Mean	0.90620	0.90153	0.92164	0.89424	2.64730	*0.04731*
	Median	0.96432	0.96512	0.97517	0.97207	17.06518	*0.00069*
D-W	Mean	0.63325	0.62338	0.62832	0.62019	0.51278	0.67346
	Median	0.60496	0.59287	0.57214	0.58390	3.17149	0.36593
Solow residual	Mean	0.00279	0.00618	0.00230	0.00582	2.57754	*0.05196*
	Median	0.00117	0.00340	0.00161	0.00240	19.99259	*0.00017*
S/E	Mean	1.44750	1.45353	1.41613	1.40536	1.55826	0.19732
	Median	1.32603	1.35522	1.32184	1.33502	3.83100	0.28030
CORR(S_t, E_t)	Mean	0.90419	0.92051	0.93214	0.92813	7.78177	*0.00003*
	Median	0.96450	0.97250	0.98025	0.98350	87.31108	*0.00000*

Table AIII.
Distribution of the model estimates by the size of the firm

Notes: For the variables see Table II. K-W = Kruskal–Wallis test statistic; BF = Bonferroni F-test

About the author
Erkki Kalervo Laitinen is an Emeritus professor of Accounting at the University of Vaasa, Finland. He obtained his PhD degree at the University of Jyväskylä in 1980 and taught at the same university and at the Lappeenranta University of Technology before joining the University of Vaasa in 1987. He has published several textbooks and a large number of scientific articles including papers in *Journal of Business Finance and Accounting, Management Accounting Research, Long Range Planning, International Small Business Journal, Scandinavian Journal of Management, Journal of International Review of Financial Analysis, International Journal of Management Science (Omega), European Journal of Operational Research, The European Accounting Review, Accounting, Auditing and Accountability Journal, Review of Accounting and Finance, International Journal of Accounting, Auditing and Performance Evaluation, The Finnish Journal of Business Economics and Critical Perspectives on Accounting, Journal of Entrepreneurial Finance, and Journal of Financial Reporting and Accounting*, among others. Erkki Kalervo Laitinen can be contacted at: erkki.kalervo.laitinen@gmail.com