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Research Paper

Offering strategy of price-maker energy hubs in integrated local electricity and heat markets

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ABSTRACT

Integrated energy systems are considered a promising solution for increasing the flexibility of energy systems and accommodating more renewable energy resources to achieve decarbonization targets. In the context of EU countries, power-to-heat technologies are useful for achieving sector-coupling aspirations. However, designing an appropriate market-based platform that facilitates efficient cooperation and interaction among various power and heat units is crucial. Integrated energy systems in the form of energy hubs play an important role in this market-based framework as interface agents among different energy carriers. To this end, the paper proposes a market design based on a simultaneous market clearing in local power and heat markets, in which a price-maker energy hub participates in an attempt to maximize its revenue by offering in the joint markets. Hence, a bi-level programming method is used, in which, at the upper level of the problem, the focus is on maximizing the entity's profit, while the lower level addresses the local integrated market-clearing process. Second-order stochastic dominance constraints are also imposed on the problem to mitigate the studied energy hub's market participation risks. The problem is solved in two different sections, including the risk-neutral and risk-averse perspectives of the energy hub operator. The results of the proposed risk-averse bi-level scheme are demonstrated to be appropriate for each EH to promote its benefit in the joint power and heat markets while controlling the profit distribution and the risk of market participation based on the operator's market preference. The results indicate that implementing the proposed approach allows EH's owner to adjust its day-ahead offers in real time using flexible units in both power and heat forms. Also, imposing second-order stochastic dominance constraints enables the EH's operator to adjust its profit in the worst scenario based on its preference in a wide range, 873 € to 1737 €, which is not possible in other risk-management methods like CVaR.

Introduction

Environmental concerns, limited fossil fuel resources, and increasing energy demand have paved the way for increasing renewable energy penetration in the power sector. One of the main drawbacks of utilizing renewable resources in energy systems is the intermittent and stochastic production profile of these resources. Accordingly, increasing the flexibility of the power system to cope with renewable energy generation volatility is of paramount importance. In the meantime, power-to-X (PtX) technologies that provide the possibility of storing excess power produced by renewable energy resources in other forms of energy carriers, including heat and gas, can play a significant role in the flexibility provision of the power system [1] and [2]. In this vein, power-to-heat (PtH) units are seen as potential sources for increasing the flexibility of the power system at all voltage levels, especially at the distribution level, as well as fulfilling decarbonization targets in the heat sector [3]. Using the potential synergy between power and heat energy carriers requires integration of the power and heat networks. Connecting the heating and electricity sectors with smart grid technologies not only improves the regulation, monitoring, and optimization of heat and electricity flows but also allows the possibility of recycling heat losses in the integrated energy system and utilizing thermal storage units to add flexibility [4,5]. To this end, simulations are required to evaluate operating conditions and performance in different circumstances [6].

Besides the physical integration of these two energy systems, the design of a local market-based platform to facilitate coordination between diverse players is crucial. Multiple research works have addressed

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Nomenclature

Acronyms

CDF	Cumulative Distribution Functions
CHP	Combined Heat and Power
CO_2	Carbon Dioxide
CVaR	Conditional Value at Risk
DA	Day-Ahead
EB	Electric Boiler
EH	Energy Hub
EHP	Electric Heat Pump
ESS	Electricity Storage System
HSS	Heat Storage System
ILEHM	Integrated Local Electricity-Heat Market
IGDT	Information Gap Decision Theory
KKT	Karush-Kuhn-Tucker
LL	Lower Level
LMPs	Locational Marginal Prices
MPEC	Mathematical Program with Equilibrium Constraints
Obj	Objective Function
Obj ^{UL}	Upper-Level Objective Function
Obj ^{LL}	Lower-Level Objective Function
PM	Price Maker
PtH	Power-to-Heat
PtX	Power-to-X
PT	Price Taker
PV	Photovoltaic System
P2P	Peer-to-Peer
RH	Rival Heat
RP	Rival Power
RT	Real-Time
SDT	Strong Duality Theory
SOSD	Second-Order Stochastic Dominance
ST	Solar Thermal System
SILEHM	Stochastic Integrated Local Electricity-Heat Market
UL	Upper Level
WT	Wind Turbine

Sets and Indices

$c \in C$	Set of CHPs
$e \in E$	Set of EHPs
$es \in ESS$	Set of ESSs
$b \in EB$	Set of EBs
$g \in G$	Set of GFGs
$p \in P$	Set of PVs
$rp \in RP$	Set of ILEHM's RPs
$rh \in RH$	Set of ILEHM's RHs
$sv \in SV$	Set of STs
$s \in S$	Set of Scenarios
$t \in T$	Set of Times
$hs \in HSS$	Set of HSSs
$w \in W$	Set of WTs
υ,υ'	Set of benchmark scenarios used in SOSD

Parameters

CP	EHP's coefficient of performance (constant)
$E_{ESS}^{min}, E_{ESS}^{\max}$	Minimum/maximum electricity stored in ESSs (MWh)
$E_{HSS}^{min}, E_{HSS}^{\max}$	Minimum/maximum thermal energy stored in HSSs (MWh)
$E_{ESS}^{in}, E_{HSS}^{in}$	Initial electricity/heat stored in ESSs/HSS (MW)
H_{Demand}	ILEHM's heat demand (MW)
H_{SV}	Solar thermal produced heat (MW)
$H_{EHP}^{\min}, H_{EHP}^{\max}$	Minimum/maximum produced heat of EHPs (MWh)
$H_{HSS}^{ch,\min}, H_{HSS}^{ch,\max}$	Minimum/maximum charged heat of HSSs (MWh)
HPR	Heat-to-power ratio of CHPs (constant)
HV_{gas}	Natural gas heat value (MWh/m^3)
k	Prefixed value of benchmark scenarios used in SOSD (constant)
P_{Demand}	ILEHM's electricity demand (MW)
$P_{ESS}^{dch,\min}, P_{ESS}^{dch,\max}$	Minimum/maximum discharged power of ESSs (MW)
$P_{ESS}^{ch,\min}, P_{ESS}^{ch,\max}$	Minimum/maximum charged power of ESSs (MW)
$P_{GFG}^{\min}, P_{GFG}^{\max}$	Minimum/maximum produced power of GFGs (MW)
$P_{CHP}^{\min}, P_{CHP}^{\max}$	Minimum/maximum produced power of CHPs (MW)
$P_{EB}^{\min}, P_{EB}^{\max}$	Minimum/maximum consumed power of EBs (MW)
P_{PV}, P_{WT}	Output power of PVs, WTs (MW)
$q_{RP}^{\min}, q_{RP}^{\max}$	Minimum/maximum amount of RPs' offers (MW)
$q_{RH}^{\min}, q_{RH}^{\max}$	Minimum/maximum amount of RHs' offers (MW)
Q_p, Q_H	EH's power/heat quantity offers to LIEHM (MW)
ρ	Scenario probability (constant)
λ_{gas}	Natural gas price (€/MWh)
α_p, α_H	EH's submitted electricity/heat offer prices (€/MWh)
α	Defined confidence level in CVaR(constant)
β	Risk-aversion parameter used to apply CVaR (constant)
η_{EB}^{H}	The efficiency of EBs (%)
$\eta_{CHP}^{E}, \eta_{CHP}^{H}$	Electricity/heat efficiency of CHPs (%)
η_{GFG}^{E}	The efficiency of GFGs (%)
$\eta_{ESS}^{ch}, \eta_{ESS}^{dch}$	Charge/discharge efficiency of ESSs (%)
$\eta_{HSS}^{sb}, \eta_{HSS}^{ch}, \eta_{HSS}^{dch}$	Standby/charge/discharge efficiency of HSSs (%)
τ	Probability of each predefined benchmark scenario (constant)

Decision Variables

CVaR	Conditional Value at Risk (€)
E_{ESS}, E_{HSS}	Electricity/ Heat stored in ESSs/HSS (MWh)
G_{CHP}, G_{GFG}	Gas flow from gas station to CHPs/GFGs (m^3)
$H_{CHP}/\ H_{EHP}/H_{EB}$	Generated heat of CHPs/EHPs/EBs (MW)
$H_{HSS}^{dch}, H_{HSS}^{ch}$	Discharged/charged heat of HSSs (MWh)
OC_{CHP}, OC_{GFG}	Operating cost of CHPs /GFGs (€)
P_{GFG}	Generated power of GFGs (MW)
$P_{ESS}^{dch}, P_{ESS}^{ch}$	Discharged/charged power of ESSs (MW)
P_{CHP}	Generated power of CHPs (MW)
P_{EHP}, P_{EB}	Consumed power of EHPs/EBs (MW)
q_P, q_H	EH's accepted power/heat offers (MW)
q_{RH}, q_{RP}	RH/RP's quantity offers (MW)
$\lambda_{RP}, \lambda_{RH}$	RH/RP's price offers (MW)
μ	Dual variables of LL's non-equality constraints
λ_p, λ_H	DA electricity/heat market clearing prices (€/MWh)
$VaR,\ \xi$	Auxiliary variables used to apply CVaR (constant)
ψ	Auxiliary variable used to apply SOSD constraints (constant)

Binary Variables

U.	Binary variable for the performance of·

market perspectives in the operation of integrated power and heat systems. Since power and heat systems are usually managed by independent entities, decentralized optimization methods are focused on in many studies. In this context, a decentralized multi-period operation approach for interconnected power and heat networks is proposed in [7], while each optimal power flow and optimal thermal flow problem is solved independently considering the operational status of another network.

The evaluation of the design of the potential local interconnected energy markets that facilitate the integration of power and heat distribution systems and provide an extra level of flexibility is very important. In [8], a model for the future interconnected power and heat markets is proposed, in which both electricity and heat sectors are considered equally important energy carriers. In this study, a linear formulation of power market clearing considering AC power flow and achieving exact locational marginal prices (LMPs) is proposed. Furthermore, the optimal thermal flow problem is solved, and heat LMPs are obtained. A fixed point between optimal power flow and optimal thermal flow represents the integrated market equilibrium and is reformulated in the framework of mixed-integer linear programming. In [9], a local market design based on a centralized auction-based trading mechanism is presented, in which integrating multiple energy carriers can facilitate the efficient use of available resources as well as employing flexible solutions. The objective of the market clearing process is to maximize the sum of producers' and consumers' surplus. A double-sided auction-based local market design for the application of integrated power and heat energy systems is introduced in [10], in which the objective of the market clearing process is to maximize social welfare based on the submitted orders in the market. A two-stage integrated market clearing model is presented in [11] to ensure the benefits of market players and establish a fair framework for the competition of market participants. The real-time (RT) operational costs and day-ahead (DA) dispatch costs are minimized in the market-clearing process to mitigate the operational risk of integrated energy systems with high penetration of renewable energy resources.

In order to provide end-users with active participation in a distribution-level market framework, the concept of peer-to-peer (P2P) energy trading is introduced. In this regard, facilitated energy transactions between agents while addressing the mutual interdependencies between diverse energy carriers can incentivize participants to be actively involved in local market transactions. A novel P2P energy market framework is presented in [12], in which mutual interdependencies between power and heat sectors are considered in the settlement of energy transactions in the integrated power and heat market. A linear optimization problem is proposed in this study to achieve optimal strategies for the existing agents in the designed market and obtain optimal operational scheduling for market participants. Finally, a P2P multi-energy market mechanism is developed in [13], in which peers can trade both electricity and heat by joining one of the predefined coalitions, including an electricity-heat trading coalition and an electricity-only trading coalition. Each energy market is cleared separately per energy carrier and per coalition.

Notably, some entities can trade power and heat in the electricity-heat-integrated local markets. Energy hubs (EHs) as interface agents between diverse energy carriers are one of those actors. In general, the self-scheduling of EHs, including net energy trading between the existing units, the amount of stored energy in storage resources, etc., is manageable within their framework. However, the optimal participation of these entities within local integrated electricity-heat markets and in the presence of multiple uncertain factors, including the behavior of their rivals, market outcomes, and renewable energy resources, is a critical issue and requires precise scheduling. In the following, some studies that have focused on the scheduling of EHs in the presence of power and heat markets have been reviewed. A two-stage risk-constraint optimal operation strategy for an EH is proposed in [14] in the presence of DA and RT electricity and heat markets. The objective function of the problem is not only to maximize the expected profit of the EH but also to minimize the risk level and carbon dioxide (CO2) emission. In this work, the stochastic p-robust optimization approach is executed to manage financial risk. A mathematical program with equilibrium constraints (MPEC) formulation is proposed in [15] for an EH that participates in the distribution-level electricity and heat markets. At the upper level (UL) of the problem, the studied EH submits bids, including quantities and prices, to the markets. The market-clearing process, which is implemented based on optimal power and thermal flows, determines the transaction between the EH and the two mentioned energy markets at the lower level (LL) of the problem. A risk-constrained stochastic scheduling method for an EH participating in power, heat, and gas markets is presented in [16]. The objective function of the problem is to minimize operating and emission costs. The conditional value at risk (CVaR) measure is incorporated with the objective function to manage the risk in the worst scenarios with higher operating costs. The concept of a virtual EH is introduced in [17], which is an integrated energy system with self-scheduling methods to maximize its revenue while participating in different energy markets. In this study, the information gap decision theory (IGDT) is applied to propose a scheduling method for a virtual EH participating in the local power and heat energy markets. When the uncertainties of the problem are considered, the proposed method enables the virtual EH to follow two different strategies, including the risk-seeker strategy and the risk-averse strategy. A scenario-based robust self-scheduling method for a virtual EH that is capable of participating in DA and RT power and heat markets is proposed in [18]. The objective function of the problem is to maximize the profit of the virtual EH and to manage financial risk by considering the downside risk method.

On the other hand, in some studies, the scheduling problem of an EH is addressed only in the presence of power markets. The strategic scheduling of a price maker (PM) multi-energy system in the DA wholesale market is proposed using bi-level optimization [19], in which the UL problem minimizes EH's operation cost, and the LL maximizes public satisfaction in the market. The optimal operation problem of an EH in the presence of the wholesale electricity market is proposed [20], in which the objective function is to minimize the operation cost of the studied EH. Both the robust chance-constrained approach and the robust IGDT method are employed to solve the problem by considering risk level in decision-making. In [21], a two-stage risk-aware stochastic method is proposed for the scheduling of an EH that is capable of participating in DA and RT electricity markets. Value-at-risk is used as a risk metric to restrict the emergence of high operation costs in the worst scenarios. The results determine the components' operating points and the EH's transactions of electricity and natural gas. In [22], a two-stage stochastic programming method is proposed for the scheduling of an EH participating in DA and regulating electricity markets. The CVaR term is used to manage the risk of high operation costs in the worst scenarios. The objective function of the problem is to minimize the total expected cost of the considered EH. Ultimately, a bi-level optimization approach is exploited in [23] to propose the optimal bidding and scheduling of an EH equipped with advanced-adiabatic compressed air energy storage. Accordingly, at the UL of the problem, self-scheduling and power purchase of the EH from the distribution-level electricity market to minimize operation costs are addressed. Meanwhile, in the LL, the market-clearing problem is considered to minimize generation costs and upstream grid payments. A brief summary of reviewed papers is provided in Table 1.

Based on the reviewed papers and Table 1, the following research gaps are identified:

> Although increased sector coupling in the power and heat networks makes them more dependent on each other, none of the reviewed papers in (3-9) consider the ILEHM, where simultaneous market clearing processes in power and heat markets are done. (RG1)

Table 1
Comparison of reviewed studies with the present paper.

Ref.	Market Participation		EH Modeling		Market Modeling				Risk Management	
	Wholesale	Local	PM	PT	Power	Heat	ILEHM	SILEHM	Methods	SOSD By CVaR
[14]	×	✓	×	✓	✓	✓	×	×	CVaR	×
[15]	✓	×	✓	×	✓	✓	×	×	×	×
[16]	✓	×	×	✓	✓	✓	×	×	CVaR	×
[17]	×	✓	×	✓	✓	✓	×	×	IGDT	×
[18]	✓	×	×	✓	✓	✓	×	×	Downside Risk	×
[19]	✓	×	✓	×	✓	×	×	×	Robust	×
[20]	✓	×	✓	×	✓	×	×	×	Robust Chance-Constrained Robust IGDT	×
[21]	✓	×	×	✓	✓	×	×	×	VaR	×
[22]	✓	×	×	✓	✓	×	×	×	CVaR	×
[23]	×	✓	×	×	✓	×	×	×	×	×
This Paper	×	✓	✓	×	✓	✓	✓	✓	×	✓

Although papers (10-14) consider the participation of the integrated energy systems in the form of EH in power and heat markets, none of them consider their participation in ILEHM. (RG2)
Additionally, papers (15-19) only consider EH's participation in a power market. Also, in most of the studies (10), (12-14), and (17-19), EH is not considered as a PM entity, where they can generate a considerable amount of power and heat, resulting in ignoring their effects in the market clearing process. (RG3)
Although in some of the studies, risk measures are used to control the risk of the EH's participation in power and heat markets, none of them used second-order stochastic constraints as risk measures, which provides the EH's operator with more freedom in implementing their preferences in risk-management, which is not possible through common risk measures like VaR and CVaR. (RG4)

According to defined gaps, this paper proposes a stochastic integrated local electricity and heat market (SILEHM), in which not only the market clearing process is done simultaneously for power and heat markets but also the stochastic behavior of power and heat market participants, as well as the uncertainty of demand, are also considered. As an integrated energy system, EH participates in this market to maximize its profit. Additionally, since the studied EH is assumed to be a PM entity, its effect on the market clearing process is also considered. To this end, a two-stage stochastic method is nested in the UL of a bi-level programming method to maximize the profit of the EH participating in ILEHM. In the LL of the problem, the market clearing process is done for power and heat markets simultaneously. The main contributions of this paper are defined as follows:

SILEHM is introduced in this study to give market operators and participants greater flexibility and freedom in energy management, increasing participants' profit and improving the network's overall performance. (Address RG1)
EH is modeled as a PM player participating in the ILEHM and affects the market clearance process via a bi-level programming approach. (Address RG2 and RG3)
Second-order stochastic dominance (SOSD) [20] constraints are added to the problem to control the risk of EH's participation in the proposed local market platform, giving more freedom to the EH's operator in confronting the worst scenarios. (Address RG4)
In the proposed SOSD risk management method, a feasible region for controlling risk is defined by CVaR, which guarantees the feasibility of the problem.

HYPERLINK "SPS:id::Sec1" Methodology

This study aims to propose a scheduling method for the participation of a PM EH in the ILEHM to not only maximize EH's profit but also take into account the market-clearing problem with the objective of operation cost minimization. Hence, to model the participation of the EH as a strategic player in the ILEHM, a bi-level programming approach is proposed, in which the scheduling and operation of the EH is implemented in its UL, while the local market-clearing procedure is implemented in its LL. Studied EH consists of a combined heat and power (CHP) and a gas-fired generator (GFG) as conventional units, renewable energy generators such as wind turbine (WT), photovoltaic system (PV), and solar thermal system (ST), energy storage systems including electricity storage system (ESS) and heat storage system (HSS), as well as PtH technologies including electric boilers (EBs) and electric heat pumps (EHPs), as shown in Fig.1.

As stated, EH participates in the ILEHM as a PM player, so its offers will affect the market clearing process and market clearing prices. The stochastic behavior of EH's power and heat rivals and the uncertainty of market demands are considered, so the market is modeled as a stochastic integrated local electricity and heat market (SILEHM) in which both power and heat market clearing processes are done simultaneously. To address EH's scheduling considering market clearing problems, a bi-level method is used in which the UL maximizes EH's expected profit, which is the difference between the amount of revenue obtained by selling energy and the operation cost of CHPs and GFGs. The LL solves the ILEHM market clearing problem, and its objective function is to minimize the operation cost of the market. EH operator faces a wide range of uncertainty factors while making decisions, including the energy profile of wind turbine (WT), photovoltaic (PV), and solar thermal (ST) units, power and heat demand, as well as offers of its rivals in the power and heat markets. Therefore, applying appropriate risk-management methods is highly important to deal with these uncertainties. Hence, SOSD constraints are implemented in this paper to control the EH's risk of participation in the local market, which is superior to other common risk-management methods such as CVaR. A high-level overview of the methodology and detailed proposed methodology are represented in Figs. 2 and 3. The UL of the problem consists of two stages. The first stage determines the EH's submitted power and heat prices and quantity offered to the ILEHM, as well as the operating points of its CHP and GFG units. Hence, decision variables of this stage are not dependent on scenarios. On the other hand, after realizing the stochastic parameters, the operating points of the EH's flexible units, including EHPs, EBs, ESS, and HSS, are specified in the second stage to maximize this entity's profit. Furthermore, in the LL of the problem, the EH's accepted power and heat offers, power and heat-market clearing prices, as well as accepted offers of the EH's power and heat rivals, are determined with the objective of minimizing the operation cost. In this procedure, EHs submitted and accepted power and heat offers in the ILEHM act as linking variables between these two levels.

Problem Formulation

This section presents the mathematical formulation for scheduling

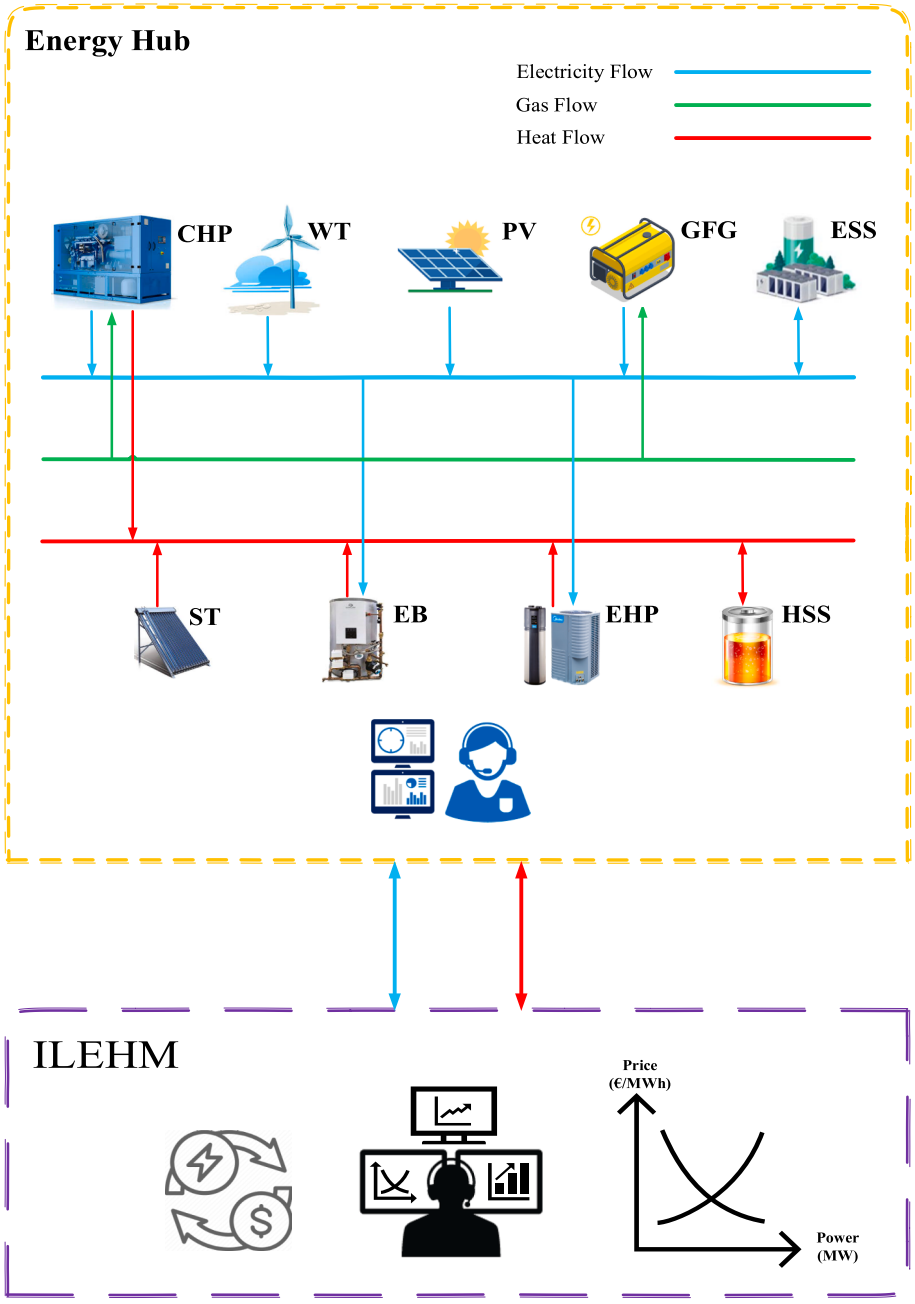

Fig. 1. Detail description of the studied EH considering participating in ILEHM.

EH to participate in ILEHM as a strategic player.

UL Problem: Scheduling of the Strategic EH

The objective function of the UL is to maximize the expected profit of the EH using a two-stage stochastic programming approach, in which q_P and q_H are EH's accepted power/heat offers, ρ is the probability of scenarios, λ_p and λ_H are DA electricity/heat market clearing prices, and OC_{CHP} and OC_{GFG} are operating cost of CHPs /GFGs. The differences between the profit of EH resulting from selling power and heat to the local market, based on the uniform market clearing outcomes, and the operation costs of the available CHPs and GFGs form the objective function at this level.

$$OF^{UP} = Max \sum_{t\in T} \left[\begin{array}{c} +\sum_{s\in S} \rho(s).\left(q_p(t,s)\cdot\lambda_p(t,s) + q_H(t,s)\cdot\lambda_H(t,s)\right) \\ -\sum_{c\in CHP} OC_{CHP}(c,t) - \sum_{g\in GFG} OC_{GFG}(g,t) \end{array} \right] \quad (1)$$

The considered objective function is subject to a set of constraints as follows:

1 Constraints of the EH's power and heat offers in the ILEHM:

The EH's submitted power and heat offers to the ILEHM are non-negative variables that

are denoted in inequalities (2), in which Q_p and Q_H are EH's power and heat quantity offers to LIEHM, and α_p and α_H are EH's submitted electricity and heat offer prices.

On the other hand, as it is demonstrated in Eq (3) and Eq (4), the accepted power and heat offers of the EH need to be restricted to the generation capacity of the aggregated dispatchable as well as non-

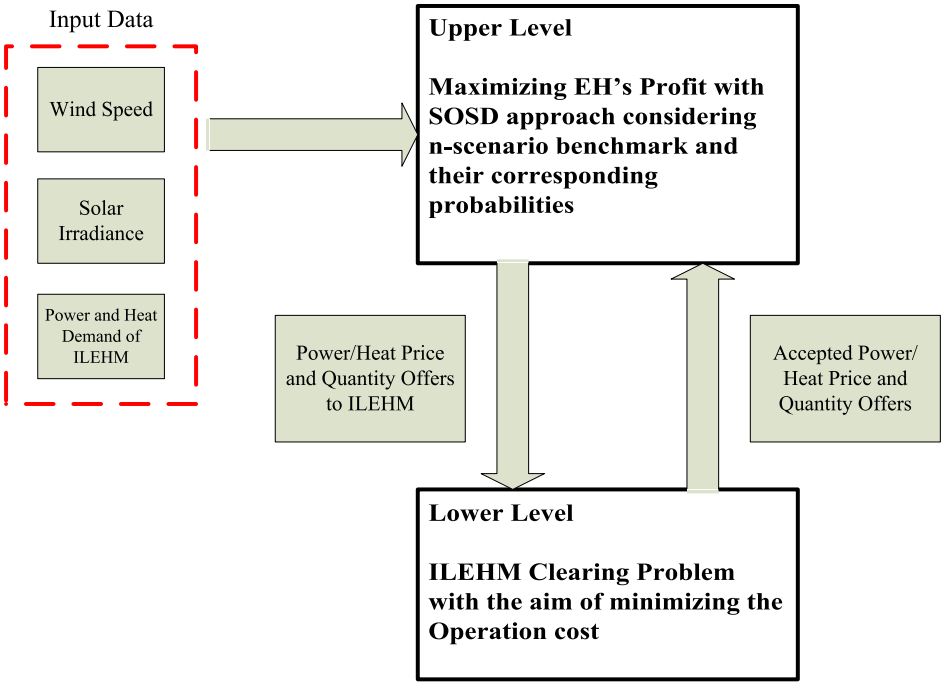

Fig. 2. A high-level overview of the proposed methodology.

dispatchable resources within the studied EH. In these equations, P_{CHP}, P_{GFG}, P_{WT}, P_{PV}, P_{ESS}^{dch}/ P_{ESS}^{ch}, P_{EHP}, P_{EB}, are generated power of CHPs, GFGs, WTs, PVs, discharged/charged power of ESSs, and consumed power of EHPs and EBs, respectively. Also, H_{CHP}, H_{EHP}, H_{EB}, H_{SV}, H_{HSS}^{dch} /H_{HSS}^{ch} are generated heat of CHPs, EHPs, EBs, STs, and discharged/ charged heat of HSSs, respectively.

$$\begin{aligned} &Q_p(t) \geq 0 \quad , \quad \alpha_p(t) \geq 0 \quad ;\forall t \\ &Q_H(t) \geq 0 \quad , \quad \alpha_H(t) \geq 0 \quad ;\forall t \end{aligned} \tag{2}$$

$$\begin{aligned} q_p(t,s) \leq & \sum_{c\in CHP} P_{CHP}(c,t) + \sum_{g\in GFG} P_{GFG}(g,t) \\ & + \sum_{w\in WT} P_{WT}(w,t,s) + \sum_{p\in PV} P_{PV}(p,t,s) \\ & + \sum_{es\in ESS} P_{ESS}^{dch}(es,t,s) - P_{ESS}^{ch}(es,t,s) \\ & - \sum_{e\in EHP} P_{EHP}(e,t,s) - \sum_{b\in EB} P_{EB}(b,t,s) \qquad ;\forall t,s \end{aligned} \tag{3}$$

$$\begin{aligned} q_H(t,s) \leq & \sum_{c\in CHP} H_{CHP}(c,t) + \sum_{e\in EHP} H_{EHP}(e,t,s) \\ & + \sum_{sv\in SV} H_{SV}(sv,t,s) + \sum_{b\in EB} H_{EB}(b,t,s) \\ & + \sum_{hs\in HSS} H_{HSS}^{dch}(hs,t,s) - H_{HSS}^{ch}(hs,t,s) \qquad ;\forall t,s \end{aligned} \tag{4}$$

2 Constraints of the CHP units:

CHP refers to the units that produce power and heat simultaneously. Using these units increases energy efficiency while protecting the environment since they produce less greenhouse gas emissions by 13-18% [24 25]. These units in the structure of the EH are formulated by Eqs (5) to (8), in which the first one demonstrates the relation between output power and units' injected gas, G_{CHP}. In Eq (5), HV_{gas} and η_{ESS}^{ch} are natural gas heat value and electricity efficiency of CHPs. Eq (6) indicates the correlation between the thermal and electrical output of these units, in which HPR and η_{CHP}^{H} are the heat-to-power ratio of CHPs and heat efficiency of CHPs. Also, the power generation of CHPs is restricted by Eq (7), in which $P_{CHP}^{\min}$ and $P_{CHP}^{\max}$ are minimum and maximum produced power of CHPs. Ultimately, the operation costs of CHPs are calculated by Eq (8) [26], in which λ_{gas} is the natural gas price.

$$G_{CHP}(c,t) = \frac{P_{CHP}(c,t)}{HV_{gas}\cdot\eta_{CHP}^{E}(c)} \qquad ;\forall c,t \tag{5}$$

$$H_{CHP}(c,t) \leq P_{CHP}(c,t)\cdot HPR(c)\cdot\eta_{CHP}^{H}(c) \qquad ;\forall c,t \tag{6}$$

$$P_{CHP}^{\min}(c) \leq P_{CHP}(c,t) \leq P_{CHP}^{\max}(c) \qquad ;\forall c,t \tag{7}$$

$$OC_{CHP}(c,t) = G_{CHP}(c,t)\cdot\lambda_{gas}(t) \qquad ;\forall c,t \tag{8}$$

3 Constraints of the GFG units:

GFG units are mathematically formulated by Eqs (9) to (11). The relation of consumed fuel, G_{GFG}, of these units and units' output power is depicted by Eq (9), in which η_{GFG}^{E} is the efficiency of GFGs. Moreover, GFGs' generated power is restricted by Eq (10), which $P_{GFG}^{\min}$ and $P_{GFG}^{\max}$ are the minimum and maximum produced power of GFGs. The operation cost of these units is calculated through Eq (11) [26].

$$G_{GFG}(g,t) = \frac{P_{GFG}(g,t)}{HV_{gas}\cdot\eta_{GFG}^{E}(g)} \qquad ;\forall g,t \tag{9}$$

$$P_{GFG}^{\min}(g) \leq P_{GFG}(g,t) \leq P_{GFG}^{\max}(g) \qquad ;\forall g,t \tag{10}$$

$$OC_{GFG}(g,t) = G_{GFG}(g,t)\cdot\lambda_{gas}(t) \qquad ;\forall g,t \tag{11}$$

4 Constraints of the ESS units:

The mathematical formulation of the ESS is presented in expressions (12) to (16). The charged and discharged powers of the ESS are limited by Eqs (12) and (13), respectively. In these equations, $P_{ESS}^{ch,\min}$, $P_{ESS}^{ch,\max}$,$P_{ESS}^{dch,\min}$, $P_{ESS}^{dch,\max}$ are the minimum/maximum charged power of ESSs, and minimum/maximum discharged power of ESSs. Eqs (14), (15), and (16) are used to express the amount of electrical energy stored in the ESS [27]. In these equations, E_{ESS},E_{ESS}^{min}/$E_{HSS}^{\max}$, η_{ESS}^{ch}/ η_{ESS}^{dch} are electricity stored in ESSs, minimum/maximum electricity stored in ESSs, and charge/discharge efficiency of ESSs.

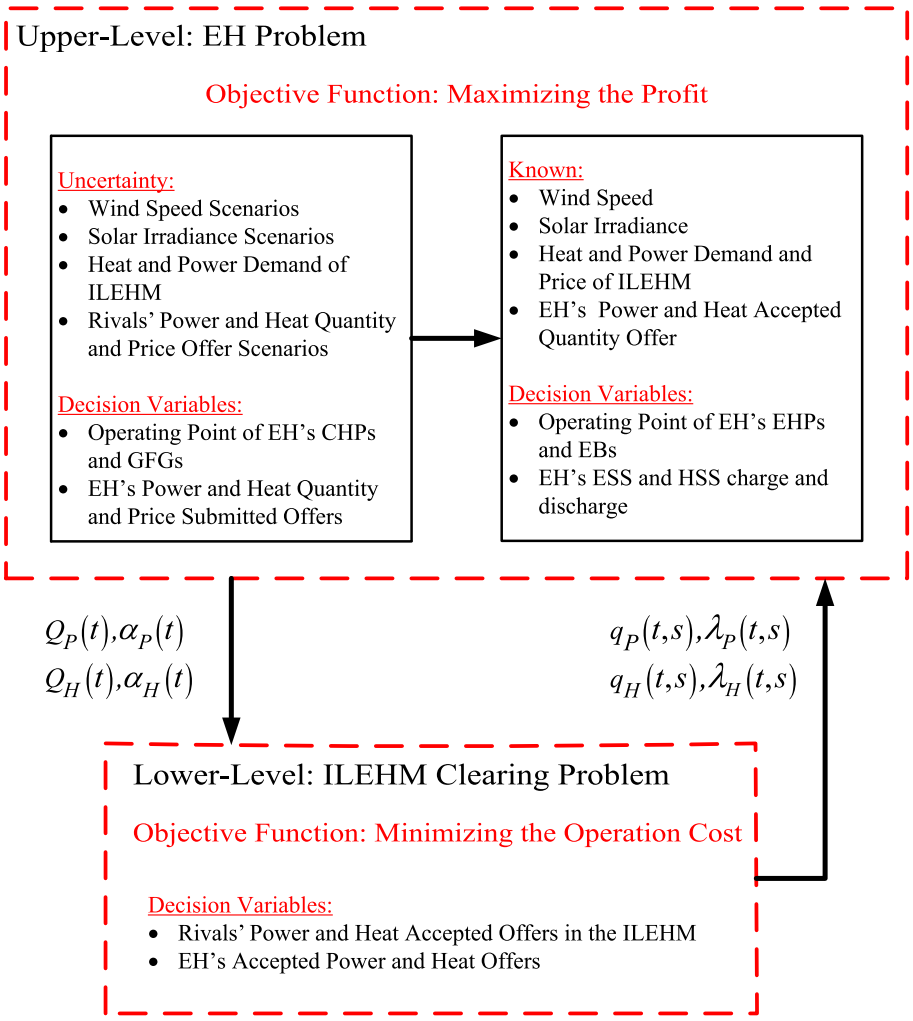

Fig. 3. Detailed proposed methodology for scheduling of the studied EH participating ILEHM.

$$P_{ESS}^{ch,\min}(es)\cdot U_{ESS}(es,t,s) \le P_{ESS}^{ch}(es,t,s) \le P_{ESS}^{ch,\max}(es)\cdot U_{ESS}(es,t,s) \quad ;\forall es,t,s \tag{12}$$

$$P_{ESS}^{dch,\min}(es)\cdot(1-U_{ESS}(es,t,s)) \le P_{ESS}^{dch}(es,t,s) \le P_{ESS}^{dch,\max}(es)\cdot(1-U_{ESS}(es,t,s)) \quad ;\forall es,t,s \tag{13}$$

$$E_{ESS}(es,t,s) = E_{ESS}^{in}(es) \quad ;\forall es,t=1,s \tag{14}$$

$$E_{ESS}(es,t,s) = E_{ESS}(es,t-1,s) + P_{ESS}^{ch}(es,t,s)\cdot\eta_{ESS}^{ch}(es) - P_{ESS}^{dch}(es,t,s)\cdot\eta_{ESS}^{dch}(es) \quad ;\forall es,t>1,s \tag{15}$$

$$E_{ESS}^{min}(es) \le E_{ESS}(es,t,s) \le E_{ESS}^{\max}(es) \quad ;\forall es,t,s \tag{16}$$

5 Constraints of the HSS units:

Expressions from (17) to (21) are used to model HSS Units. The charged and discharged heat of these units are limited by Eqs (17) and (18), respectively, in which $H_{HSS}^{ch,\min}$, $H_{HSS}^{ch,\max}$, $H_{HSS}^{dch,\min}$, $H_{HSS}^{dch,\max}$ $P_{ESS}^{dch,\max}$ are minimum/maximum charged heat of HSSs , minimum/maximum discharged heat of HSSs.

The amount of thermal energy stored in HSS is expressed through Eqs (19) and (20). Finally, Eq (21) is utilized to limit the amount of heat stored in HSS [28]. in these equations, E_{HSS},E_{HSS}^{min}/$E_{HSS}^{\max}$, η_{HSS}^{ch}/ η_{HSS}^{dch}/η_{HSS}^{sb} are heat stored in HSSs, minimum/maximum heat stored in HSSs, and charge/discharge/ standby efficiency of HSSs.

$$H_{HSS}^{ch,\min}(hs)\cdot U_{HSS}(hs,t,s) \le H_{HSS}^{ch}(hs,t,s) \le H_{HSS}^{ch,\max}(hs)\cdot U_{HSS}(hs,t,s) \quad ;\forall hs,t,s \tag{17}$$

$$H_{HSS}^{dch,\min}(hs).(1-U_{HSS}(hs,t,s)) \le H_{HSS}^{dch}(hs,t,s) \le H_{HSS}^{dch,\max}(hs).(1-U_{HSS}(hs,t,s)) \quad ;\forall hs,t,s \tag{18}$$

$$E_{HSS}(hs,t,s) = E_{HSS}^{in}(hs) \quad ;\forall hs,t=1,s \tag{19}$$

$$E_{HSS}(hs,t,s) = E_{HSS}(hs,t-1,s)\cdot\eta_{HSS}^{sb}(hs) + H_{ESS}^{ch}(hs,t,s)\cdot\eta_{HSS}^{ch}(hs) - H_{HSS}^{dch}(hs,t,s)\cdot\eta_{HSS}^{dch}(hs) \quad ;\forall hs,t>1,s \tag{20}$$

$$E_{HSS}^{min}(hs) \le E_{HSS}(hs,t,s) \le E_{HSS}^{\max}(hs) \quad ;\forall hs,t,s \tag{21}$$

6 Constraints of the EHP units:

EHP units consume electricity to produce heat, as shown in Eq (23), which *CP* is EHP's coefficient of performance. Also, the produced heat of EHPs is limited by Eq (22) [29], in which $H_{EHP}^{\min}$ and $H_{EHP}^{\max}$ are minimum and maximum produced heat of EHPs.

$$H_{EHP}^{\min}(e) \le H_{EHP}(e,t,s) \le H_{EHP}^{\max}(e) \quad ;\forall e,t,s \tag{22}$$

$$H_{EHP}(e,t,s) = P_{EHP}(e,t,s)\cdot CP(e) \quad ;\forall e,t,s \tag{23}$$

7 Constraints of the EB units:

EBs utilize electricity to produce heat, as shown in Eq (24), in which η_{EB}^{H} The efficiency of EBs. Generated heat by EBs is limited by Eq (25)

[26], in which $P_{EB}^{\min}$ and $P_{EB}^{\max}$ are minimum and maximum consumed power of EBs.

$$H_{EB}(b,t,s) = P_{EB}(b,t,s)\cdot\eta_{EB}^{H}(b) \quad ;\forall b,t,s \tag{24}$$

$$P_{EB}^{\min}(b) \le P_{EB}(b,t,s) \le P_{EB}^{\max}(b) \quad ;\forall b,t,s \tag{25}$$

Constraints of the risk management:

CVaR:

In this part, the constraints of the CVaR as a common risk measure are provided from Eq (29) to Eq (32). In order to apply this measure to the EH's problem, the objective function of the UL problem should be written according to Eq (29) to set a trade-off between the obtained expected profit and its risk level [30]. In these equations, VaR, ξ are auxiliary variables used to apply CVaR. α and β are defined as confidence level and risk-aversion parameters.

$$Maximize(1-\beta)\cdot OF^{UL} + \beta\cdot CVaR \tag{29}$$

$$CVaR = VaR - \frac{1}{1-\alpha}\cdot\sum_{s}^{S}\rho(s)\cdot\xi(s) \tag{30}$$

$$VaR - \sum_{t\in T}\left[\begin{array}{c} q_p(t,s)\cdot\lambda_p(t,s) + q_H(t,s)\cdot\lambda_H(t,s) \\ -\sum_{c\in CHP} OC_{CHP}(c,t) - \sum_{g\in GFG} OC_{GFG}(g,t)\end{array}\right] \le \xi(s) \quad ;\forall s \tag{31}$$

$$\xi(s) \ge 0 \quad ;\forall s \tag{32}$$

SOSD:

The constraints from Eq (33) to Eq (35) are added to the problem to use SOSD constraints as a risk-management method. The preference of this approach over other risk-management methods is that it directly controls the expected profit of the worst scenario by imposing benchmark distribution through the definition of $k(\upsilon)$ or $k(\upsilon')$ and their related probability $\tau(\upsilon)$ or $\tau(\upsilon')$[31]. In these equations,k and ψ are prefixed values of benchmark scenarios and auxiliary variables to apply SOSD constraints.

$$k(\upsilon) - \sum_{t\in T}\left[\begin{array}{c} q_p(t,s)\cdot\lambda_p(t,s) + q_H(t,s)\cdot\lambda_H(t,s) \\ -\sum_{c\in CHP} OC_{CHP}(c,t) - \sum_{g\in GFG} OC_{GFG}(g,t)\end{array}\right] \le \psi(s,\upsilon) \quad ;\forall s,\upsilon \tag{33}$$

$$\sum_{s\in S}\rho(s)\cdot\psi(s,\upsilon) \le \sum_{\upsilon'}\tau(\upsilon')\cdot\mathbf{max}(k(\upsilon)-k(\upsilon'),0)\,; \quad \forall\upsilon \tag{34}$$

$$\psi(s,\upsilon) \ge 0 \quad ; \quad \forall s,\upsilon \tag{35}$$

To sum up, the set of decision variables of the UL is shown as follows:

$$\begin{aligned}\Delta^{UL} = \{&Q_p(t), Q_H(t), \alpha_p(t), \alpha_H(t), P_{CHP}(c,t,s), P_{GFG}(g,t,s),\\ &P_{EHP}(e,t,s), P_{EB}(eb,t,s), P_{ESS}^{ch}(es,t,s), P_{ESS}^{dch}(es,t,s), H_{CHP}(c,t,s)\\ &H_{EB}(eb,t,s), H_{EHP}(e,t,s), H_{HSS}^{ch}(hs,t,s), H_{HSS}^{dch}(hs,t,s),\\ &G_{GFG}(g,t,s), G_{CHP}(c,t,s), E_{HSS}(hs,t,s), E_{ESS}(es,t,s), U_{HSS}(hs,t,s)\\ &U_{ESS}(es,t,s), VaR, \xi(s), k(\upsilon), \psi(s,\upsilon)\}\end{aligned}$$

LL Problem: Integrated Local Market-clearing Process

As noted, the ILEHM clearing process is performed in the LL of the proposed bi-level problem from the EH viewpoint, considering the stochastic behavior of EH's power and heat rivals participating in the market. Accordingly, the EH, as well as its rivals, submit their power and heat price and amount of offers to the ILEHM. Then, after performing the market-clearing procedure, clearing prices for both power and heat, as well as accepted offers of the EH and rivals, are determined with the aim of minimizing the operating cost of the market. q_{RH}/q_{RP}, $\lambda_{RP}/\lambda_{RH}$ are RH/RP's quantity and price offers.

$$\begin{aligned}&Min\sum_{rp\in RP} q_{RP}(rp,t,s)\cdot\lambda_{RP}(rp,t,s)\\ &+\sum\nolimits_{rh\in RH} q_{RH}(rh,t,s)\cdot\lambda_{RH}(rh,t,s)\\ &+ q_P(t,s)\cdot\alpha_p(t) + q_H(t,s)\cdot\alpha_H(t) \quad ;\forall t,s\end{aligned} \tag{36}$$

The above-mentioned objective function is subject to a set of constraints as follows:

1 Power and heat balances in the ILEHM:

The accepted offers of the EH and its power and heat rivals should satisfy the local market's forecasted power and heat demands. In this regard, the offers are accepted in the market-clearing process from the lowest price until satisfying the market's demand based on Eqs (37) and (38). Notably, dual variables associated with each constraint of the LL are indicated after a semicolon. The dual variables associated with the power and heat balance constraints represent the power and heat local market-clearing prices. P_{Demand} and H_{Demand} are ILEHM's power and heat demand.

$$\begin{aligned}&q_P(t,s) + \sum_{rp\in RP} q_{RP}(rp,t,s) =\\ &P_{Demand}(t,s) \quad ;\forall t,s \quad ; \quad \lambda_P(t,s)\end{aligned} \tag{37}$$

$$\begin{aligned}&q_H(t,s) + \sum_{rh\in RH} q_{RH}(rh,t,s) =\\ &H_{Demand}(t,s) \quad ;\forall t,s \quad ; \quad \lambda_H(t,s)\end{aligned} \tag{38}$$

2 Power and heat offer limitations:

Accepted offers of ILEHM participants, including the EH and its power and heat rivals, should be lower than their submitted power and heat offers to the market, as shown in Eqs (39) to (42). μ represents dual variables of LL's non-equality constraints.

$$0 \le q_P(t,s) \le Q_p(t) \quad ;\forall t,s \quad ; \quad \mu_1(t,s),\ \mu_2(t,s) \tag{39}$$

$$0 \le q_H(t,s) \le Q_H(t) \quad ;\forall t,s \quad ; \quad \mu_3(t,s),\ \mu_4(t,s) \tag{40}$$

$$\begin{aligned}0 \le q_{RP}^{\min}(rp) \le q_{RP}(rp,t,s) \le q_{RP}^{\max}(rp) \quad &;\forall rp,t,s\\ ;\mu_5(rp,t,s), \quad &\mu_6(rp,t,s)\end{aligned} \tag{41}$$

$$\begin{aligned}0 \le q_{RH}^{\min}(rh) \le q_{RH}(rh,t,s) \le q_{RH}^{\max}(rh) \quad &;\forall rh,t,s\\ ;\mu_7(rh,t,s), \quad &\mu_8(rh,t,s)\end{aligned} \tag{42}$$

The set of decision variables related to LL is represented as follows:

$$\begin{aligned}\Delta^{LL} = \{&q_{RP}(rp,t,s),\ q_{RH}(rh,t,s), q_P(t,s), q_H(t,s)\\ &\lambda_P(t,s), \lambda_H(t,s), \mu_1(t,s), \mu_2(t,s), \mu_3(t,s)\\ &\mu_4(t,s), \mu_5(rp,t,s), \mu_6(rp,t,s), \mu_7(rh,t,s)\\ &\mu_8(rh,t,s)\}\end{aligned}$$

Reformulation of the Bi-level Problem to a Single-level Problem

Since the LL of the presented framework is linear and continuous, it is a convex problem. As a result, Karush-Kuhn-Tucker (KKT) conditions can be used to reformulate the bi-level model to a single-level mathematical program with equilibrium constraints (MPEC). To this end, the LL problem can be substituted with its KKT conditions. The formulation of the bi-level problem and their transformation to MPEC using KKT conditions are provided in [32] with great detail. After developing the Lagrange function of the LL, the KKT conditions associated with it that have been demonstrated from Eq (43) to Eq (54) are replaced with the LL problem.

1 Stationary constraints:

Table 2
Technical characteristics of EH's power and heat units.

CHP					
# Unit	P^{min}_{CHP}	P^{max}_{CHP}	η^{E}_{CHP}	η^{H}_{CHP}	*HPR*
1	0.5 (MW)	4.2 (MW)	0.45	0.55	1.0
GFG					
# Unit	P^{min}_{GFG}		P^{max}_{GFG}		η^{E}_{GFG}
1	0.25 (MW)		2.5 (MW)		0.75
WT, PV, ST					
# Unit	P^{max}_{WT}		# Unit	P^{max}_{PV} , H^{max}_{SV}	
1	2.4 (MW)		1	3.0 , 2.0 (MW)	

$$\lambda_{RP}(rp,t,s) - \lambda_P(t,s) - \mu_5(rp,t,s) + \mu_6(rp,t,s) = 0 \quad ;\forall rp,t,s \tag{43}$$

$$\lambda_{RH}(rh,t,s) - \lambda_H(t,s) - \mu_7(rh,t,s) + \mu_8(rh,t,s) = 0 \quad ;\forall rh,t,s \tag{44}$$

$$\alpha_P(t) - \lambda_P(t,s) - \mu_1(t,s) + \mu_2(t,s) = 0 \quad ;\forall t,s \tag{45}$$

$$\alpha_H(t) - \lambda_H(t,s) - \mu_3(t,s) + \mu_4(t,s) = 0 \quad ;\forall t,s \tag{46}$$

2 Complementary constraints:

$$0 \le q_P(t,s) \perp \mu_1(t,s) \ge 0 \quad ;\forall t,s \tag{47}$$

$$0 \le [Q_P(t) - q_P(t,s)] \perp \mu_2(t,s) \ge 0 \quad ;\forall t,s \tag{48}$$

$$0 \le q_H(t,s) \perp \mu_3(t,s) \ge 0 \quad ;\forall t,s \tag{49}$$

$$0 \le [Q_H(t) - q_H(t,s)] \perp \mu_4(t,s) \ge 0 \quad ;\forall t,s \tag{50}$$

$$0 \le [q_{RP}(rp,t,s) - q^{min}_{RP}(rp)] \perp \mu_5(rp,t,s) \ge 0 \quad ;\forall rp,t,s \tag{51}$$

$$0 \le [q^{max}_{RP}(rp) - q_{RP}(rp,t,s)] \perp \mu_6(rp,t,s) \ge 0 \quad ;\forall rp,t,s \tag{52}$$

$$0 \le [q_{RH}(rh,t,s) - q^{min}_{RH}(rh)] \perp \mu_7(rh,t,s) \ge 0 \quad ;\forall rh,t,s \tag{53}$$

$$0 \le [q^{max}_{RH}(rh) - q_{RH}(rh,t,s)] \perp \mu_8(rh,t,s) \ge 0 \quad ;\forall rh,t,s \tag{54}$$

Therefore, the presented bi-level problem is transformed into a single-level problem, and its objective function is the objective function of the UL problem, Eq (1), with constraints of Eqs (2) to (35) and (43) to (54).

Linearization of Non-linear Terms

The existence of non-linearity sources in the final MPEC of the proposed bi-level problem, including terms $q_p(t,s)\cdot\lambda_p(t,s)$, $q_H(t,s)\cdot\lambda_H(t,s)$, and complementary constraints in Eqs (47)-(54) makes the MPEC problem nonlinear. To linearization the non-linear terms following procedures are applied.

1 Linearization of complementary constraints:

The Big-M method is utilized in this study for the linearization of complementary constraints [33]. The following set of equations, in which U is an auxiliary binary variable and M is a very large number, are replaced to non-linear complementary constraints.

$$If \quad 0 \le \tau \perp \upsilon \ge 0 \quad \Rightarrow \begin{cases} \tau \le M\cdot U \\ \upsilon \le M\cdot(1-U) \end{cases} \tag{55}$$

2 Linearization of the non-linear terms in the UL's objective function:

The Strong Duality Theorem (SDT) is used for the linearization of the non-linear terms existing in the UL's objective function [34]. According to this theory, in a convex problem, the value of the optimal point in primal and dual problems is the same. As a result, the dual of the LL's objective function is written as Eq (56).

Table 3
Technical characteristics of EH's PtH technologies.

EHP			
# Unit	H^{min}_{EHP}	H^{max}_{EHP}	CP_{EHP}
1	0.0 (MW)	1.0 (MW)	2.5
EB			
# Unit	P^{min}_{EB}	P^{max}_{EB}	η^{H}_{EB}
1	0.0 (MW)	2.0 (MW)	0.8

Table 4
Technical characteristics of EH's energy storage systems.

ESS				
# Unit	E^{min}_{ESS} , E^{max}_{ESS}	$P^{ch,min}_{ESS}$, $P^{ch,max}_{ESS}$	$P^{dch,min}_{ESS}$, $P^{dch,max}_{ESS}$	η^{ch}_{ESS} , η^{dch}_{ESS}
1	0.4 , 4 (MWh)	0.0 , 1.0 (MW)	0.0, 1.0 (MW)	0.95 , 0.95
HSS				
# Unit	E^{min}_{HSS} , E^{max}_{HSS}	$H^{ch,min}_{HSS}$, $H^{ch,max}_{HSS}$	$H^{dch,min}_{HSS}$, $H^{dch,max}_{HSS}$	η^{ch}_{HSS} , η^{dch}_{HSS} , η^{sb}_{HSS}
1	0.0 , 3.0 (MWh)	0.0 , 2.0 (MW)	0.0 , 2.0 (MW)	0.98 , 0.98 , 0.95

$$\begin{aligned} Min \sum_{rp\in RP} & q_{RP}(rp,t,s)\cdot\lambda_{RP}(rp,t,s) \\ & + \sum\nolimits_{rh\in RH} q_{RH}(rh,t,s)\cdot\lambda_{RH}(rh,t,s) \\ & + q_P(t,s)\cdot\alpha_p(t) + q_H(t,s)\cdot\alpha_H(t) \\ & = P_{Demand}(t,s)\cdot\lambda_P(t,s) + H_{Demand}(t,s)\cdot\lambda_H(t,s) \\ & - Q_P(t)\cdot\mu_2(t,s) - Q_H(t)\cdot\mu_4(t,s) \\ & + \sum\nolimits_{rp\in RP}\left[q^{min}_{RP}(rp)\cdot\mu_5(rp,t,s) - q^{max}_{RP}(rp)\cdot\mu_6(rp,t,s)\right] \\ & + \sum\nolimits_{rh\in RH}\left[q^{min}_{RH}(rh)\cdot\mu_7(rh,t,s) - q^{max}_{RH}(rh)\cdot\mu_8(rh,t,s)\right] \quad ;\forall t,s \end{aligned} \tag{56}$$

From Eqs (45) and (47):

$$\alpha_P(t)\cdot q_P(t,s) - \lambda_P(t,s)\cdot q_P(t,s) + \mu_2(t,s)\cdot q_P(t,s) = 0 \tag{57}$$

From Eqs (46) and (49):

$$\alpha_H(t)\cdot q_H(t,s) - \lambda_H(t,s)\cdot q_H(t,s) + \mu_4(t,s)\cdot q_H(t,s) = 0 \tag{58}$$

From Eqs (57) and (48):

$$\alpha_P(t)\cdot q_P(t,s) = \lambda_P(t,s)\cdot q_P(t,s) - Q_P(t)\cdot\mu_2(t,s) \tag{59}$$

From Eqs (58) and (50):

$$\alpha_H(t)\cdot q_H(t,s) = \lambda_H(t,s)\cdot q_H(t,s) - Q_H(t)\cdot\mu_4(t,s) \tag{60}$$

By replacing Eqs (59) and (60) in the LL's dual problem, the linear form of UL problem's non-linear terms can be achieved, shown in Eq (61).

$$\begin{aligned} & \lambda_P(t,s)\cdot q_P(t,s) + \lambda_H(t,s)\cdot q_H(t,s) \\ & = P_{Demand}(t,ds)\cdot\lambda_P(t,s) + H_{Demand}(t,ds)\cdot\lambda_H(t,ds) \\ & - \sum\nolimits_{rp\in RP} q_{RP}(rp,t,ds)\cdot\lambda_{RP}(rp,t,ds) - \sum\nolimits_{rh\in RH} q_{RH}(rh,t,ds)\cdot\lambda_{RH}(rh,t,ds) \\ & + \sum\nolimits_{rp\in RP}\left[q^{min}_{RP}(rp)\cdot\mu_5(rp,t,s) - q^{max}_{RP}(rp)\cdot\mu_6(rp,t,s)\right] \\ & + \sum\nolimits_{rh\in RH}\left[q^{min}_{RH}(rh)\cdot\mu_7(rh,t,s) - q^{max}_{RH}(rh)\cdot\mu_8(rh,t,s)\right] \end{aligned} \tag{61}$$

Case Study and Simulation Results

In the following of the article, a typical case study is presented to

Table 5
Probabilities of each reduced scenario

# Scenario	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10
Probability	0.042	0.064	0.091	0.143	0.184	0.161	0.143	0.084	0.056	0.033

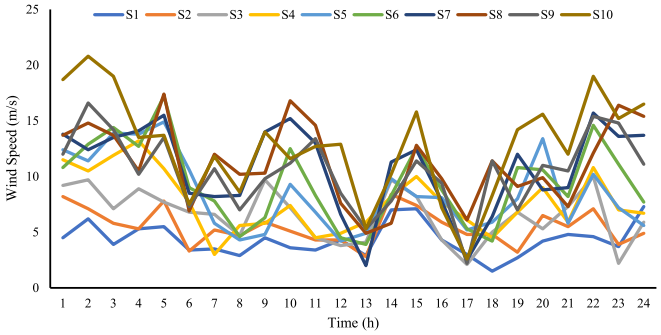

Fig. 4. Wind speed in different scenarios.

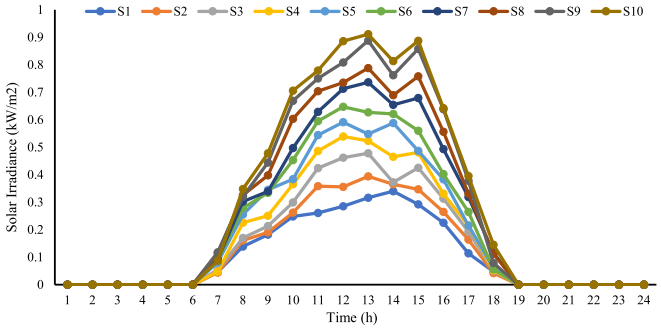

Fig. 5. Solar Irradiance in different scenarios.

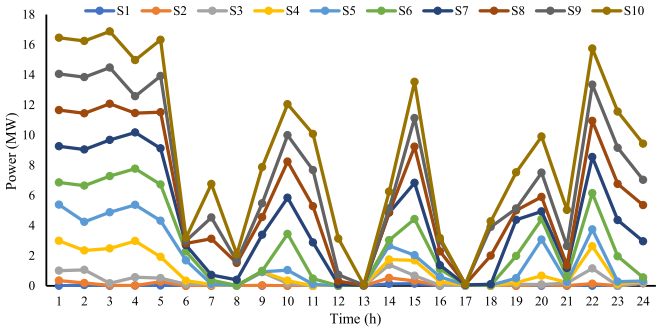

Fig. 6. WT's produced power in different scenarios.

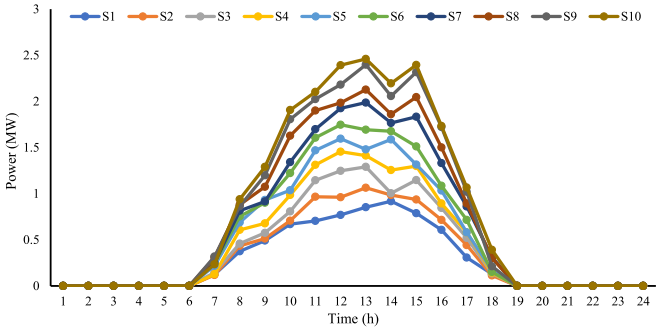

Fig. 7. PV's produced power in different scenarios.

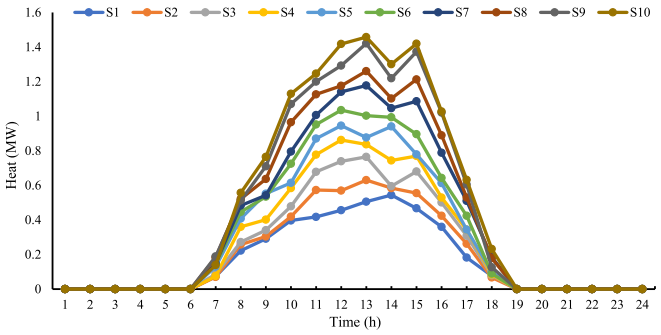

Fig. 8. ST's produced heat in different scenarios.

demonstrate the effectiveness of the proposed methodology for the optimal participation of a strategic EH in an ILEHM, considering the stochastic behavior of power and heat rivals in the market, as well as management of this entity's risk via SOSD risk-management method.

Input data

As shown in Fig. 1, the studied PM EH consists of various units namely CHP, GFG, WT, PV, ST, ESS, HSS, EB, and EHP. Technical specifications of these components are provided in Tables 2, 3, and 4.

To model the uncertainty of stochastic parameters, including wind speed, solar irradiance, ILEHM's power and heat demands, as well as offers of the strategic EH's power and heat rivals, in the proposed model, 10 scenarios are considered, and their associated probabilities are represented in Table 5.

Furthermore, wind speed and solar irradiance in these 10 scenarios as well as the available WT, PV, and ST's produced power and heat are represented in Figs. 4, 5, 6, 7, and 8, respectively. The data relating to these figures are provided in Tables 1, 2, 3, 4, and 5, respectively, in the supplement material.

As described in section 2, in the proposed framework, the existing uncertainties in the ILEHM have been modeled as well. In this context, power and heat demand of the designed local market in 10 scenarios are displayed in Fig. 9 and Fig. 10, respectively. The data relating to these figures are provided in Tables 6 and 7, respectively, in the supplement material.

On the other hand, in the ILEHM, it is assumed that the strategic EH has 4 power rivals and 3 heat rivals as price-taker players in the market. For the sake of space, only the submitted offers of RP3 and RH3 in 10 scenarios are depicted in Fig. 11 and Fig. 12, respectively. The data relating to these figures are provided in Tables 8, 9, 10, and 11, respectively, in the supplement material.

Risk-neutral Scheduling of EH considering SILEHM

In this part of the paper, the results of the obtained risk-neutral scheduling of the studied PM EH in the presence of the SILEHM are represented. In this regard, the obtained quantity and price offers of the EH in the integrated power and heat market are shown in Fig. 13. The EH's offers to the local market depend on various parameters, including power and heat rivals' offers, market's power and heat demands, as well as EH's local productions, which can be analyzed in different scenarios. Moreover, the power and heat offers of the strategic EH in the market are dependent on each other due to coupling elements between the power

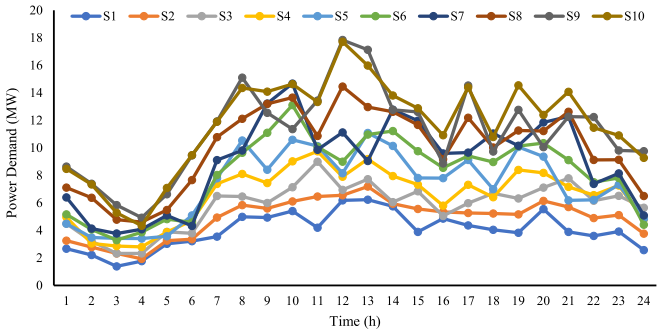

Fig. 9. ILEHM's power demand in different scenarios.

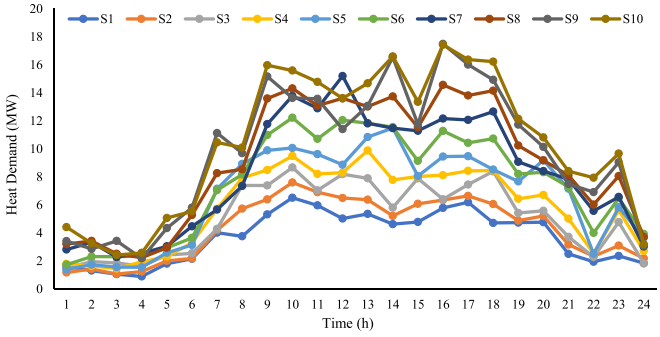

Fig. 10. ILEHM's heat demand in different scenarios.

Table 6
Submitted power quantity and price offers in scenario 1 and at hour 16.

	Submitted Quantity (MW)	Submitted Price (€/MWh)
EH	0	0
RP1	2.2	126.0
RP2	1.3	60.9
RP3	2.0	43.8
RP4	1.3	45.1

Table 7
Submitted heat quantity and price offers in scenario 1 and at hour 13.

	Submitted Quantity (MW)	Submitted Price (€/MWh)
EH	1.2	23.8
RH1	2.3	67.9
RH2	2.1	84.1
RH3	1.8	59.1

and heat sectors via PtH technologies. Therefore, analyzing the EH's offers to the market is a challenging task. In the following, the output results of scenario 5 are explained in more detail to better clarify the relation of different parameters in the scheduling and offering process of the EH.

The power dispatch of the EH's available resources in scenario 5 is depicted in Fig. 14. In this figure, the power produced by resources is represented in the DA and RT stages. Accordingly, the power production of GFG and CHP units is equal in all scenarios, including scenario 5. However, the operator of the EH could utilize its dispatchable resources, including PtH and storage units, to adjust its offers in the RT stage in each scenario. Additionally, this figure shows the dispatchable units' involvement in the power supply. According to the obtained results, when the power production of the EH is higher than its accepted power offers, the EH's operator transfers a part of its surplus power to the heat sector via PtH technologies, i.e., EB and EHP, or stores excess energy in the ESS.

In a similar way, the heat dispatch of the EH's resources in the

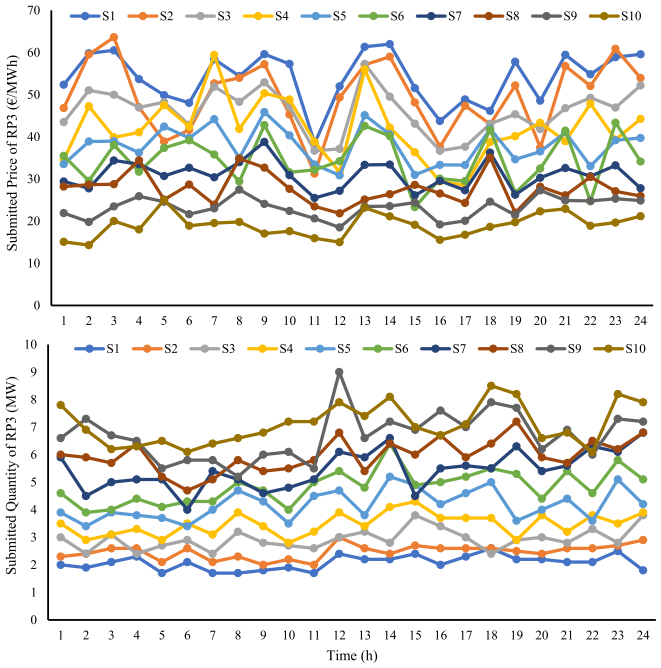

Fig. 11. Submitted price and quantity offer of RP3 in different scenarios.

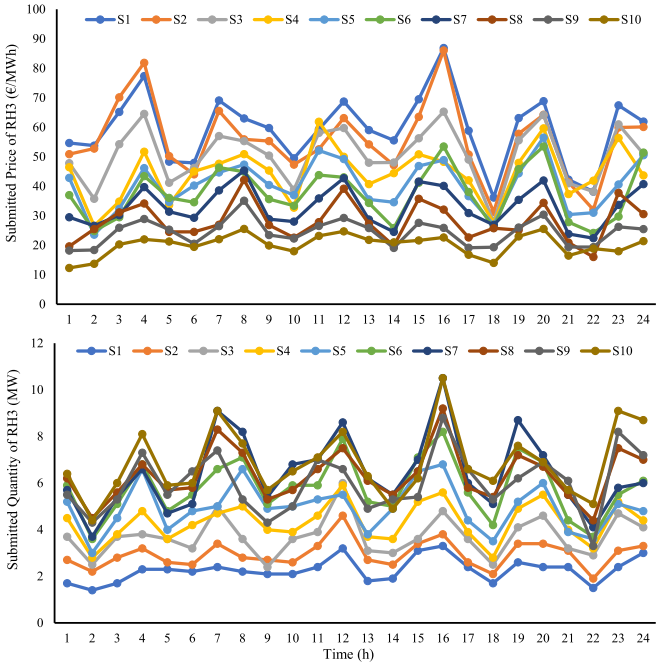

Fig. 12. Submitted price and quantity offers of RH3 in different scenarios.

studied day and in scenario 5 is shown in Fig. 15. In this figure, the CHP's produced heat is equal in all scenarios since it is one of the DA stage variables in the proposed scheduling model. In addition to this, the operator of the EH has the capability to adjust the production of heat resources based on its offers in the ILEHM by utilizing the existing flexible units. For instance, the EH's resources produce more power than its accepted offer by the market in hours 21 and 22, so the operator stores the excess produced power in HSS, which is discharged immediately at hours 23 and 24.

To investigate the integrated local market-clearing process, the EH's accepted power offers in different scenarios are depicted in Fig. 16. On the other hand, the power market-clearing prices in the studied day and

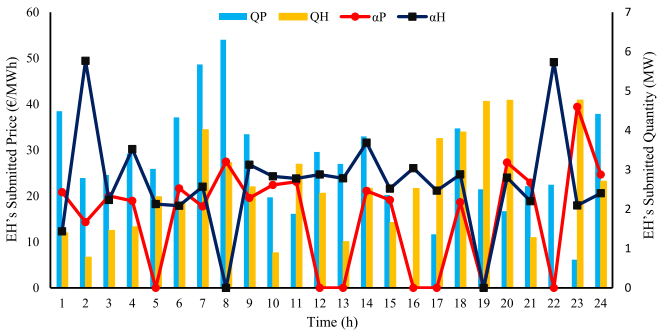

Fig. 13. EH's submitted price and quantity offers in the ILEHM.

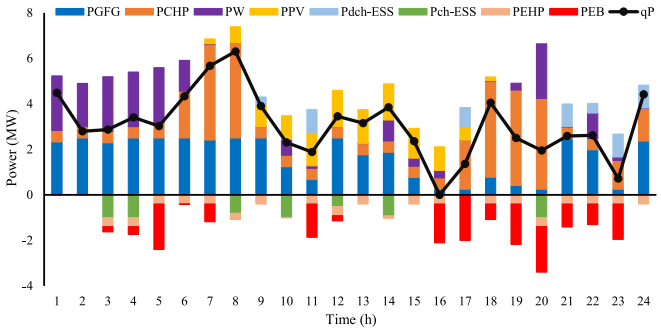

Fig. 14. EH's power balance in scenario 5.

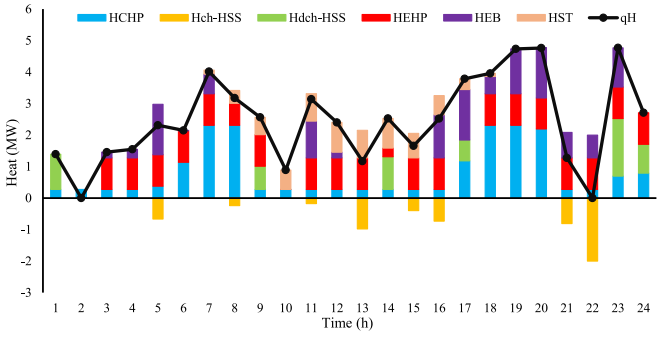

Fig. 15. EH's heat balance in scenario 5.

in the considered scenarios are represented in Fig. 17.

By comparing output results in Fig. 16 and Fig. 13, it is evident that the EH's DA submitted offers are equivalent to its accepted offers in the ILEHM in the middle hours of the studied day. However, in the early and late hours of the day, i.e., off-pick hours, the operator of the ILEHM accepts only some parts of the EH's submitted offers.

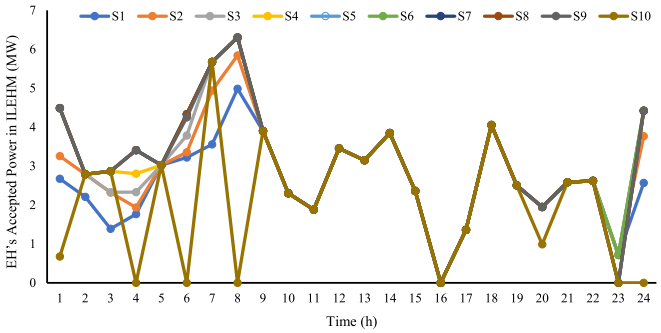

Fig. 16. EH's accepted power offers in the ILEHM and in different scenarios.

As shown in Fig. 17, in some scenarios and hours of the day, the market operator faces peak prices in the ILEHM. For instance, at hour 16 and in scenario 1, the power price of the market is 126 €/MWh. To evaluate this matter, submitted power and price offers of the market players in this scenario and hour are investigated. According to Fig. 9, the total power demand of the ILEHM at this hour is equal to 4.9 MW. Also, the submitted power and price offers of the market players in this scenario and at this hour are represented in Table 6. Based on the information in this Table, whole submitted offers of RP2, RP3, and RP4, which are equal to 4.6 MW, are accepted by the market; as a result, RP1, with a submitted price offer of 126, becomes a marginal producer and determines the market's power price.

Similarly, in the heat sector, accepted heat offers of the EH in 10 different scenarios are shown in Fig. 18. Like the power sector, in the middle hours of the day, the submitted heat offers of the EH is equal to the accepted heat offers by the ILEHM; however, a partial of submitted heat offers of the EH is accepted in the earlier and late hours of the studied day.

Fig. 19 demonstrates the heat market-clearing prices in all scenarios for the studied day. Based on Fig. 19, the maximum heat market price is 84.1 €/MWh at hour 13 and in scenario 1, where the submitted heat offers of market players are represented in Table 7.

Based on Fig. 10, the total heat demand of the ILEHM at this hour is equal to 5.4 MW. Hence, the cheapest submitted heat offers to the market, which are related to offers of EH, RH1, and RH3, are accepted to satisfy 5.3 MW of the heat demand. Consequently, RH2, with a submitted price offer of 84.1 €/MWh, becomes a marginal producer and determines the heat market-clearing price at hour 13.

Finally, Fig. 20 and Fig. 21 illustrate the accepted power and heat offers of the EH's RP3 and RH3 in different scenarios. The accepted power and heat offers of the mentioned rivals are dependent on power and heat market-clearing prices, which are reported in Fig. 17 and Fig. 19, and the amount of submitted offers in the market, Fig.11 and Fig. 12. To illustrate, if the submitted price offers of the rivals is lower than the market-clearing prices, their total submitted quantities are accepted. In addition, in the case that price offers become marginal, the partial quantity offers are accepted in the market.

Risk Management of EH

The main aim of risk-management methods is to ensure the operator will not encounter lower defined amounts of profit. In other words, the amount of profit in the worst scenario will get better while the obtained profit in the best scenario will be decreased. Based on common risk management methods, like CVaR, it is impossible to determine the amount of the profit in the worst scenario based on the operator's preferences. However, using SOSD constraints as a risk management method enables the operator to directly decide the amount of profit in the worst scenario. Notably, the most critical point in imposing SOSD constraint is selecting the feasible region, in which benchmark scenarios are determined based on the operator's preference. By defining

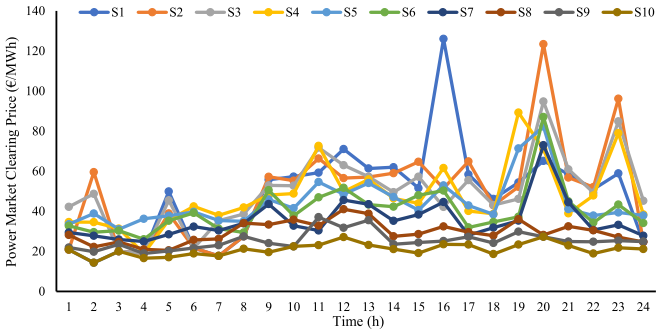

Fig. 17. Power market-clearing prices in different scenarios.

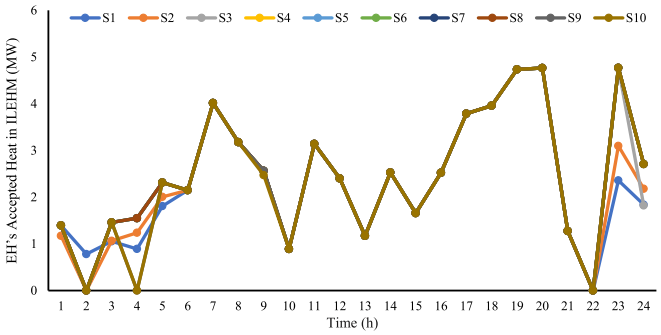

Fig. 18. EH's accepted heat offers in the ILEHM and in different scenarios.

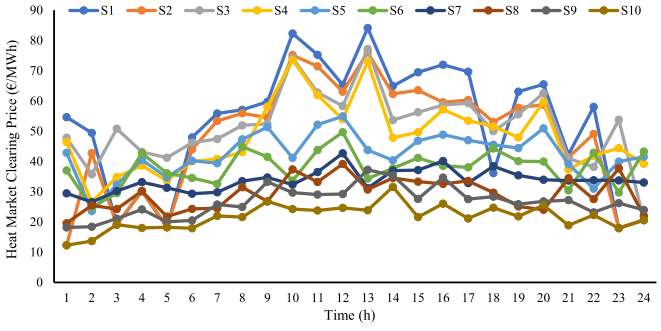

Fig. 19. Heat market-clearing prices in different scenarios.

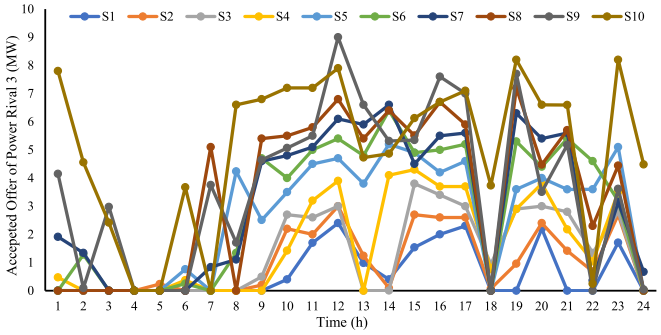

Fig. 20. Accepted power offers of RP3 in different scenarios.

benchmark scenarios out of the feasible region, the optimal problem will be infeasible. Both the risk-neutral problem and the risk-averse problem considering the CVaR risk management method with $\beta = 1$ and $\alpha = 99\,\%$ are solved to determine the feasible region of imposing benchmark scenarios. Then, optimal profit cumulative distribution functions (CDFs) of each problem are obtained considering the probability of obtained profit in each scenario. Left and right-hand-side borders of a feasible region of imposing benchmark scenarios are determined based on the amount of profit in the worst scenarios, which is illustrated in Fig. 22.

According to the EH operator's risk preference, the number of benchmark scenarios, their prefixed values, and their probabilities are determined. The CDFs of applied benchmarks to the problem, including one-scenario, two-scenario, and four-scenario benchmarks, are represented in Fig. 23. Imposing more benchmark scenarios provides the operator with better risk management; however, it increases the computational burden of the problem.

The one-scenario benchmark and resulting corresponding profit distribution are represented in Fig. 24. As evident in this figure, the amount of profit in the worst scenario is adjusted to proceed with the predefined value of the one-scenario. Nevertheless, the amount of profit

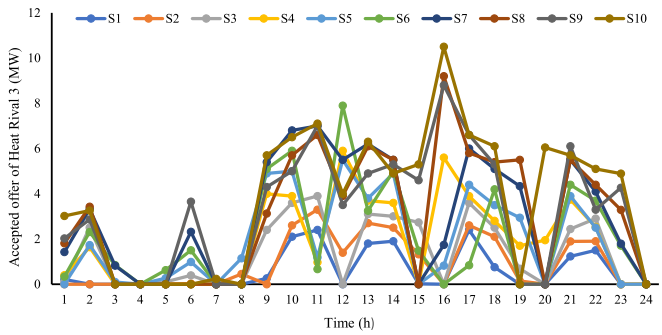

Fig. 21. Accepted heat offers of RH3 in different scenarios.

in the best scenario is decreased considerably.

The two-scenario benchmark and resulting corresponding profit distribution are depicted in Fig. 25. As shown in this figure, not only is the amount of profit in the worst scenario adjusted to exceed the predefined value of two-scenario benchmarks but also the amount of profit in the best scenario is approached to the amount of profit in the risk-neutral problem, resulting in increasing expected profit.

The four-scenario benchmark and obtained profit distribution resulting from imposing it are demonstrated in Fig. 26. The amount of profit in the worst scenario is determined to exceed the least amount of the four-scenario benchmark. Notably, the amount of the profit in the best scenario gets closer to the amount of profit in the risk-neutral problem.

To clarify the difference between CVaR and SOSD risk-management methods based on Fig. 22, imposing CVaR risk-management results in defining the profit in the worst scenario based on the input data of the risk measurement, which is 1737 €, and reducing the amount of the profit in the best scenario, which is 2435 €, which is far less than the amount of the profit in the best scenario in the risk-neutral approach, 4172 €. Based on the results in Fig. 24, defining a one-scenario benchmark at 1690 € forces the profit in the worst scenario to go higher than this point while the profit in the best scenario is 2816 €, which is higher than that of in the CVaR method. Additionally, as seen in the results of Figs. 24 and 25, increasing the benchmark scenarios results in efficient risk management of the problem since the profit in the best scenario has not considerably reduced.

Conclusion

The presented article aimed to offer an efficient bi-level risk-averse scheduling method for the optimal participation of a strategic EH in the ILEHM, considering the uncertainties dominated in the local market. The UL of the proposed framework consisted of a two-stage stochastic model, in which the first stage determined the EH's submitted power and heat offers to the ILEHM as well as the operating points of its CHP and GFG units, while the second stage determined the operating points

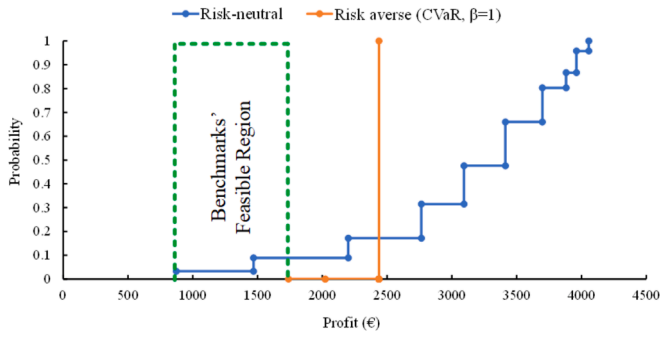

Fig. 22. Definition of benchmark's feasible region.

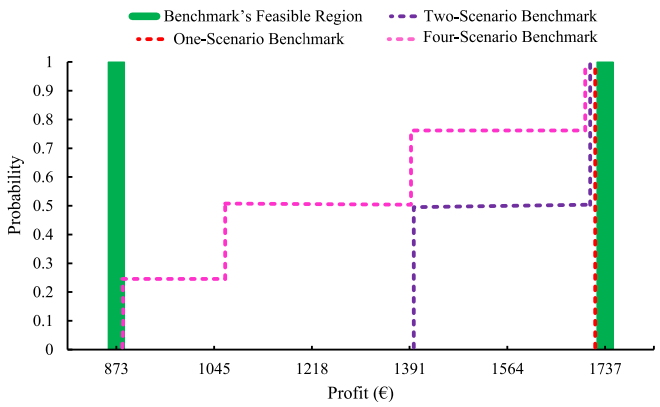

Fig. 23. Imposed benchmark scenarios.

of the EH's flexible PtH and storage units. At the same time, the local market-clearing process was addressed in the LL of the model to minimize the system's operation costs. In the raised framework, the uncertainty in the behavior of EH's power and heat rivals, as well as the market's power and heat demands, were also considered. The results indicate that EH's owner can adjust its DA offers in RT by using flexible units, such as ESS, EHP, HSS, and EB, in both power and heat forms, bringing more flexibility to EH's operator. Additionally, the results showed that market SILEHM clearing price for power and heat is associated with different factors, including SILEHM's forecasted power and heat demand, market participants' submitted price, and quantity offers in both electricity and heat markets. For another point, it was observed that in the cases where the submitted price offers of the participants were lower than the market clearing prices, the total submitted quantity offers associated with them were accepted. In contrast, when the price offers became marginal, a part of their submitted quantity offers was accepted in the market.

On the other hand, SOSD constraints were added as a risk measure to the considered methodology to mitigate the EH's risk through participation in the ILEHM. The most challenging matter in imposing SOSD constraints as a risk measure was determining the feasible region for defining the value and probabilities of benchmark scenarios. To this end, the CVaR risk measure was used to determine the feasible region and guarantee the feasibility of the problem. The risk-averse problem was solved by considering different benchmark scenarios. According to the obtained results, increasing the number of benchmark scenarios can increase the expected profits while controlling the amount of expected profit in the worst scenario.

For future work, the authors intend to investigate the impact of the PM EH's participation in SILEHM, aiming to maximize its benefit while increasing the resiliency in the distribution network.

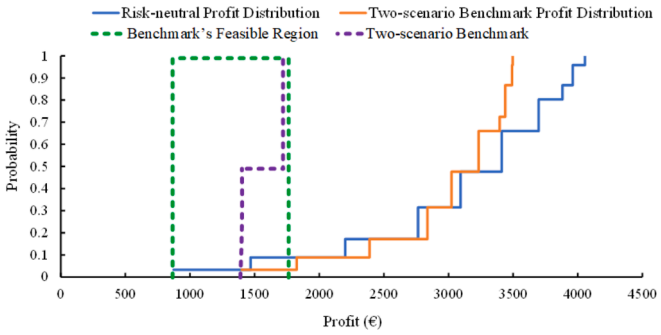

Fig. 25. Two-Scenario benchmark.

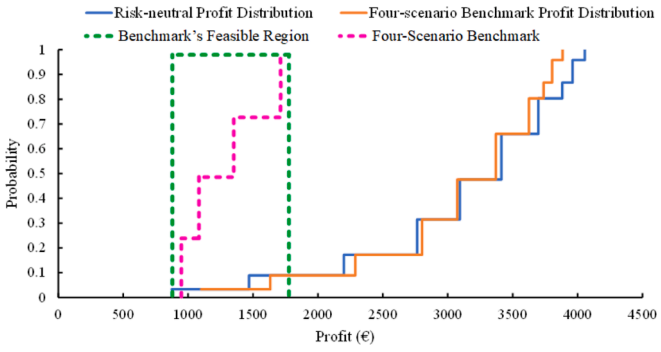

Fig. 26. Four-Scenario benchmark.

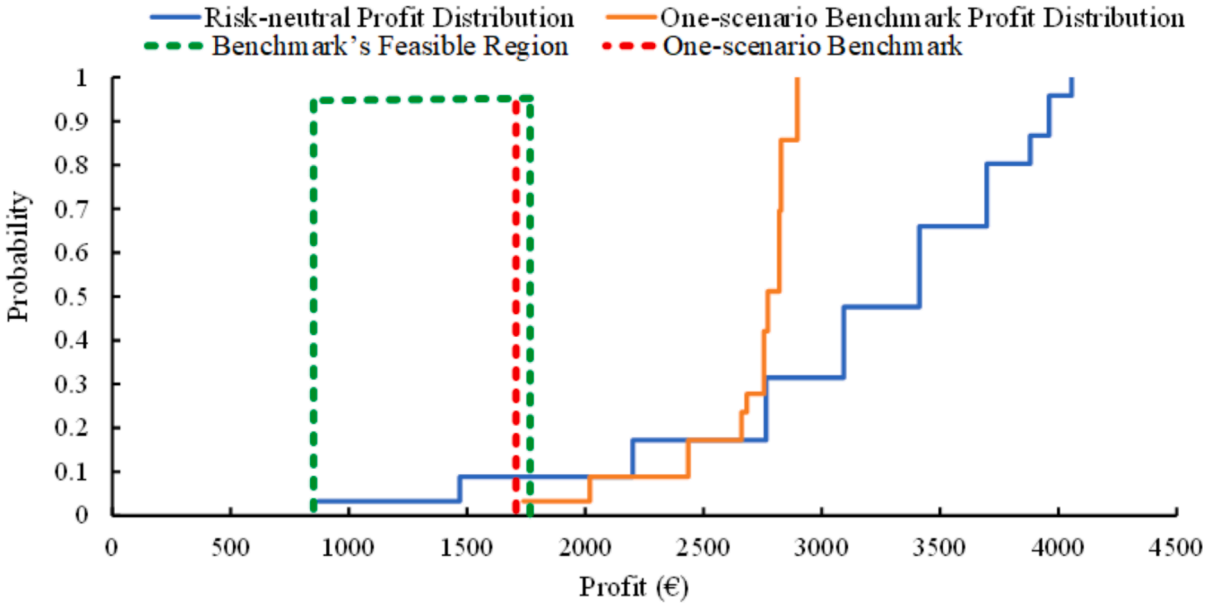

Fig. 24. One-Scenario benchmark.

CRediT authorship contribution statement

Saba Norouzi: Writing – original draft, Visualization, Validation, Software, Methodology, Formal analysis, Data curation, Conceptualization. **Mojtaba Dadashi:** Writing – review & editing, Visualization, Validation, Software, Resources, Methodology, Investigation, Formal analysis, Data curation, Conceptualization. **Sara Haghifam:** Writing – review & editing, Visualization, Validation, Software, Resources, Methodology, Investigation, Formal analysis, Data curation, Conceptualization. **Hannu Laaksonen:** Writing – review & editing. **Kazem Zare:** Writing – review & editing, Supervision, Project administration, Investigation, Formal analysis.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Appendix A. Supplementary data

Supplementary data to this article can be found online at https://doi.org/10.1016/j.ref.2025.100689.

Data availability

Data will be made available on request.

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