

Application of Machine Learning to GNSS/IMU Integration for High Precision Positioning on Smartphones

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BIOGRAPHY

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Prof. Mohammed S. Elmusrati received his B.Sc. (with honors) and M.Sc. (with high honors) degrees in electrical and electronic engineering, University of Benghazi, Libya, in 1991 and 1995, respectively, and the Licentiate of Science in technology (with distinction) and the Doctor of Science in Technology (D.Sc.) degrees in automation and control engineering from Aalto University Finland, in 2002 and 2004, respectively. Currently, he is Full Professor and Head of the Digitalization Unit at the School of Technology and Innovations – University of Vaasa, Finland. His research interest includes wireless communications, artificial intelligence, machine learning, biotechnology, big data analysis, stochastic systems, and game theory. Elmusrati has published more than 130 papers, books, and book chapters. Prof. Elmusrati is an active member in different scientific societies such as Senior Member at IEEE, Member at Society of Industrial and Applied Mathematics (SIAM), and Member at Finnish Automation Society.

ABSTRACT

This paper describes our solution for the Google smartphone decimeter challenge (GSDC), which was held from May to August 2022. The GSDC is a competition for improving positioning accuracy of smartphones. The global navigation satellite system (GNSS) data from smartphones have lower signal levels and higher noise in GNSS observations compared to commercial GNSS receivers. Therefore, it is difficult to directly apply the existing GNSS high-precision positioning methods like precise point positioning (PPP) and real-time kinematic (RTK). The smartphones used to collect the raw GNSS data have multi-constellation, dual-frequency GNSS receivers, and Inertial Measurement Unit (IMU) sensors. Multi-sensor fusion technology has become very prominent for seamless navigation systems due to its complementary capabilities to GNSS positioning. In this work, we developed a machine learning (ML) based adaptive positioning approach to estimate the positions of the smartphone by utilizing post-processed kinematic (PPK) precise positioning techniques to process the GNSS datasets. The ML model is used to predict the driving paths (highways, tree-lined streets, or downtown areas). Depending on the predicted driving path, PPK technique uses the carrier phase to compute the user position using differential corrections from known GNSS base stations. We then use the Rauch–Tung–Striebel (RTS) smoother, which consists of a forward pass Kalman Filter (KF) and a backward recursion smoother to achieve a loosely coupled integration of GNSS and IMU measurements for positioning estimation of the smartphone. We refer to this method as LC-GNSS/IMU/ML using ML based adaptive positioning (MAP) real-time kinematic (RTK) post-processing algorithm (MAP RTK). This method is validated using reference data from GNSS survey-grade receivers provided with the training datasets. The final validation of this proposed method is done on Kaggle.com, the host of the GSDC competition. Using the proposed method, we estimated the location of the smartphone and tackled the competition. The final public score was 2.61 m, while the final private score was 2.29 m.

Keywords - Adaptive positioning; Machine Learning; Kalman Filter; Rauch–Tung–Striebel smoother; Inertial Measurement Units; Global Navigation Satellite Systems; Post-processed kinematic; Smartphones

I. INTRODUCTION

Multi-constellation global navigation satellite systems (GNSS) provide benefits such as the availability of more visible satellites that can be used to improve user positioning performance [1]. This is useful in challenging environments where GNSS signals could be partially or totally blocked and are affected by multi-path reflections, for example, in urban areas or places with dense foliage, this becomes very useful. Accuracy greater than 100 meters in the worst non-line-of-sight condition can improve to between 5 meters and sub-meter depending on the technique used for data collection and the receiver used (survey-grade or low-cost receivers). In 2016, the Android operating system released an application program interface to access raw GNSS measurement data from GNSS installed in smartphones [2]. As a result, high-precision positioning at the decimeter and centimeter levels on smartphones has attracted much attention [3, 4]. There have been growing interests in the use of low-cost receivers and GNSS chipsets on smartphones for positioning applications. However, they do not provide very precise accuracy as the high grade (survey-grade receivers), especially due to antenna constraints. Differential correction, a data collection technique, can be used to remove errors in GNSS data created by selective availability, ionospheric delay, tropospheric delay, and ephemeris errors. There are other factors that lead to error in positioning, such as multi-path propagation which causes ranging measurement errors [5]. Other factors (less correctable) that create an error in GNSS data include the large distance between the rover and the base station (about 10 mm degradation with every kilometer away from the base station when differential correction is considered), low SNR (signal to noise ratio), and low satellite elevation [5].

GNSS provides raw signals, which the GNSS chipsets in smartphones use to compute a position. Code phase utilization is one processing technique that gathers data via a C/A (coarse acquisition) code receiver, which uses the information contained in the satellite signals (aka the pseudo-random code) to calculate positions. After differential correction, this processing technique can result in 1-5 meter accuracy. Carrier phase utilization is another processing technique that gathers data via a carrier phase receiver, which uses the radio signal (aka carrier signal) to calculate positions. The carrier signal, which has a much higher frequency than the pseudo-random code, is more accurate than using the pseudo-random code alone. After differential correction, this processing technique can result in sub-meter accuracy depending on the GNSS receiver. The data used in this paper is from the Google smartphone decimeter challenge (GSDC). In GSDC, each dataset includes raw GNSS measurements collected in the US San Francisco Bay and other areas by several Android smartphone devices, together with the ground truth trajectories collected by high-grade GNSS and inertial navigation unit (IMU) integration system for reference.

Another point of interest is how machine learning (ML) can be used to improve the positioning of smartphones. ML is a very powerful tool in processing time-series data, as it can be applied in learning time-dependent patterns across multiple models. It is useful in analyzing hidden and unknown patterns and information in GNSS data. GNSS is a source of affordable big data useful for ML exploitation. Also, data storage and the growth of less expensive and more powerful processing capabilities have propelled the growth of ML [6]. The aim of using ML is not to generate an explicit formula for the distribution of the data; however, it is used to train an algorithm to detect the relationships between the features of a data set, directly from the data.

In this paper, we describe our solution used in this competition in which there were over 571 teams from all over the world

working on high-precision positioning of smartphones.

II. STRATEGY

Real-time kinematic (RTK)-GNSS technique, which is usually used for high-precision positioning, is difficult to implement in smartphones due to limitations of their antennas. This is because it is difficult to solve the integer ambiguities in the carrier phase measurements using smartphone antenna. Even in an open-sky environment with an accuracy of 3 m it is still difficult to reach decimeter level since there is a lot of noise and outliers in the observations. Therefore, a robust position estimation method is needed. In our work, we still attempt to make use of RTK-GNSS technique or in our case, post-processed kinematic (PPK) positioning techniques to process the GNSS datasets. A ML model is used to predict the driving paths (highways, tree-lined streets, or downtown areas) for adaptive positioning using PPK. In the PPK technique, the carrier phase is used to compute the user position making use of differential corrections. We then implement loosely coupled integration of GNSS and IMU measurements for position estimation, using Kalman filter (KF) algorithm and Rauch–Tung–Striebel (RTS) smoother. The initial results show improvements in the positioning accuracy. GNSS low-cost receivers often have limited channels and computational resources, therefore, the complexity of the algorithm had to be kept modest. Further validations will be performed to ensure the integrity of the proposed loosely coupled GNSS/IMU/ML (LC-GNSS/IMU/ML) method.

The steps or approach taken are as follows:

1. Data Analysis and Preparation
2. ML based prediction of driving path
3. PPK precise positioning techniques to process the GNSS datasets
4. GNSS/IMU integration making use of KF/RTS-based method

III. DATA ANALYSIS AND PREPARATION

The performance of smartphones satellite observation indicates that they cannot track satellites stably, also they can only observe a small number of satellites continuously with dual-frequency signals. Furthermore, the quality of the satellites observed by smartphones generally have a low carrier-to-noise ratio (C/NR) [7]. Multipath is another factor that significantly impaired the positioning accuracy of smartphones. Therefore, we first analyze the GNSS data of smartphones provided by GSDC to understand the observation quality of GNSS data.

At the ION GNSS+ 2022, Google released a total of 206 traces in the challenge, whose collection process is described in a paper published in the proceedings of ION GNSS+ 2020 [2]. The data were released twice, first the training data and next was the test data. The training datasets consisted of GnsLogger files, RINEX observation files, ground truth files, and device IMU/GNSS data files (GNSS intermediate values derived from raw GNSS measurements, provided for convenience). The test datasets include the same types of data and follow the same convention as the training dataset, except for the ground truth data that are not provided. The results of test datasets will be used as the prediction of the expected ground truth and will be evaluated using the unreleased ground truth. In both the training and test datasets, the derived values can be used to compute a corrected pseudorange which is a closer approximation to the geometric range from the phone to the satellite.

The driving paths of the provided GNSS data can be classified into three main categories: highways, tree-lined streets, or downtown areas. The train dataset ground truth track is shown in Figure 1. These pools of GNSS and IMU datasets collected from smartphones can be useful in developing high precision GNSS positioning using the accompanied high accuracy ground truth from high-grade GNSS receivers. Table 1 shows an overview of the weighted least square (WLS) positioning results, provided by the competition organizer for three categories of driving environments. The smartphone observed pseudorange is much noisier compared to commercial GNSS receiver, and GNSS code positioning using only pseudorange has limited positioning accuracy.

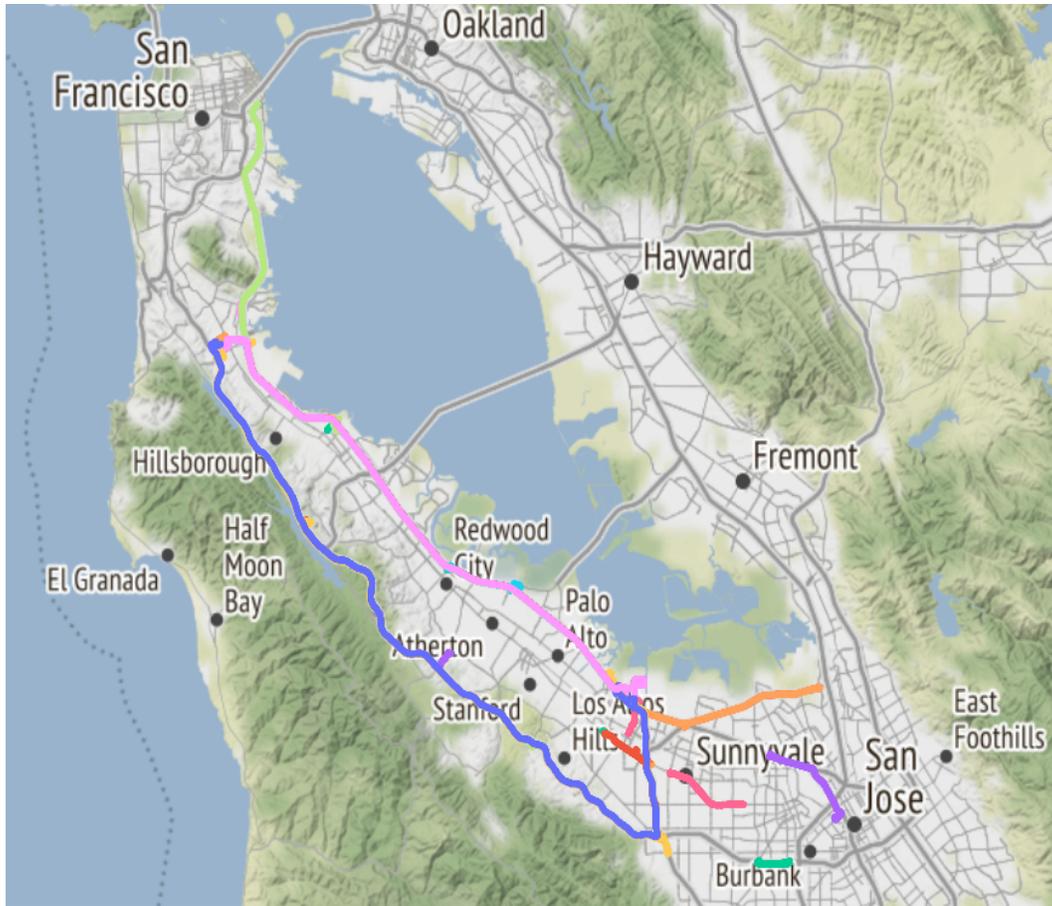


Figure 1: Driving trajectories included in training and some test data provided by GSDC.

The driving trajectories or paths are from the training data reference positions (ground truth) of the training data. Some new driving paths were included in the test data that are not present in the training data.

Table 1: Horizontal positioning error of the WLS baseline position in each driving path

Path	Highway	Tree-lined Street	Semi-Downtown
WLS baseline score	4.3134 [m]	8.2553 [m]	8.0413 [m]

IV. RELATED RESEARCHES

Precise point positioning using carrier phase measurements has been actively studied in the GNSS field. In [8] precise point positioning using carrier phase measurements with graph optimization is implemented and compared with Kalman filter based implementation. The use of RTK position estimation combined with IMU and other sensors have been studied [9, 10]. These papers showed an improvement in positioning accuracy compared to least-squares-based positioning. GNSS position estimation is highly dependent on the environment where the GNSS receiver is operating (open sky or urban canyons). In [3], the area of data collection was considered during position estimation. This paper is unique because it uses ML to predict the driving path of the data for an adaptive RTK position estimation. The Rauch–Tung–Striebel (RTS) smoother is used to achieve a loosely coupled integration of the GNSS carrier-phase and IMU measurements for positioning estimation in the smartphone.

V. PROPOSED METHOD

1. ML based Adaptive processing

Machine learning (ML) is applied to improve the positioning estimation of the KF/RTS-based GNSS/IMU integration using the ground truth data provided with GSDC training datasets. These ground truth data were collected using high-end survey-grade GNSS receivers. The driving paths of the provided GNSS data can be classified into three categories: highways (open sky) with line-of-sight (LOS), tree-lined streets, and downtown areas with multi-path/Non-LOS (NLOS). The training dataset is used to train the ML model to predict the driving paths. Different approaches has to be used in different areas. Especially in the downtown area where multipath signals caused by reflections and diffractions can significantly degrade the GNSS observations. Based on the predicted driving path, different PPK configurations are used to process the data for improved solutions. Parameters such as elevation angle of the satellite, C/NR, etc are considered based on the predicted driving path. This is done because GNSS measurements are easily disturbed by external influence such as multi-path which can be experienced in tree-lined streets, and downtown areas.

2. PPK precise positioning techniques

Real-time kinematic positioning (RTK) is the application of surveying to correct for common errors in current satellite navigation (GNSS) systems. It uses carrier phase measurements and the information content of the signal and relies reference station or interpolated virtual station to provide real-time corrections, which can give up to centimetre-level accuracy. In our case we use the post-process kinematic (PPK) for position estimation since the data is not collected in real-time. After the prediction of the driving path using the ML model, the data are then processed using PPK precise positioning techniques. The way the data is processed depends on the predicted driving path. First, we convert each phone GNSSLogger raw data file into a RINEX file that can then be processed with RTKLIBS programs [11]. RTKPOST [11] was first used to get an understanding of a couple of datasets and the right configurations to used to get a good PPK solution. Afterward, we batch process all of the data sets to get the position estimation. In the PPK technique, carrier phase data is used to compute the user position. The use of PPK requires differential correction, therefore some base observation were used to download raw observation data for the appropriate dates and times from some nearby CORS stations using NOAA National Geodetic Survey website. The SLAC, P222, and VDCY stations are used because they were reasonably close to all of the data collection rides. The satellite broadcast navigation data (BRDM files) for each data set was also downloaded from the International GNSS Service website including the clock navigation data, and satellite precise orbits [12]. These are used during the post-processing of the GNSS data.

3. Hardware clock bias

In GSDC dataset there are a couple of data sets that have corrupted carrier phase measurements. The initial RTKLIB solutions for these datasets are quite poor and are typically worse compared to the included Google baseline solutions which uses the WLS for pseudo-range measurements. These data sets can be identified by the *HardwareClockDiscontinuityCount* field in the raw Android log files [13]. If the final value in this field is larger than the initial value, then the carrier phase measurements will be corrupted by the clock discontinuities. The log files of the datasets are scanned using an algorithm to identify any with greater than one discontinuity[12]. The solutions for the listed dataset use the robust WLS solution in place of the poor RTK solutions [14].

4. GNSS/IMU integration using RTS smoothing algorithm

The Inertial Measurement Unit (IMU) is frequently used in robotics, vehicles, and, of course, in mobile phones. An IMU is usually made up of an accelerometer, gyroscope, and magnetometer. Tactical grade IMU is fully calibrated compared to low-cost ones which come with some factory calibrations stored in the IMU registers. The calibration only covers the scaling factors of each axes. The accelerometer measures the acceleration of a movement, the gyroscope measures the angular speed of the sensor, and the magnetometer measures the Earth's local magnetic field direction and magnitude.

Sensor fusion also referred to as data fusion, information fusion, or multi-sensor data fusion is used in the fusion of data from the accelerometer, magnetometer, and gyroscope. The fusion is performed to obtain position estimates and mitigate overall errors. Kalman filter is used for IMU and data fusion whereas numerical integration is required to obtain the position estimates. Basically, accelerometer values are fed to a dynamic model where they were numerically time-integrated twice to get the position, with the discretization and linearization (approximation) phases. The angular velocities measured from the gyroscope can be integrated with respect to time used to obtain the pitch and roll angles, which are later employed in the dynamic model to serve in the general state vector as fine-tuning values for the IMU localization.

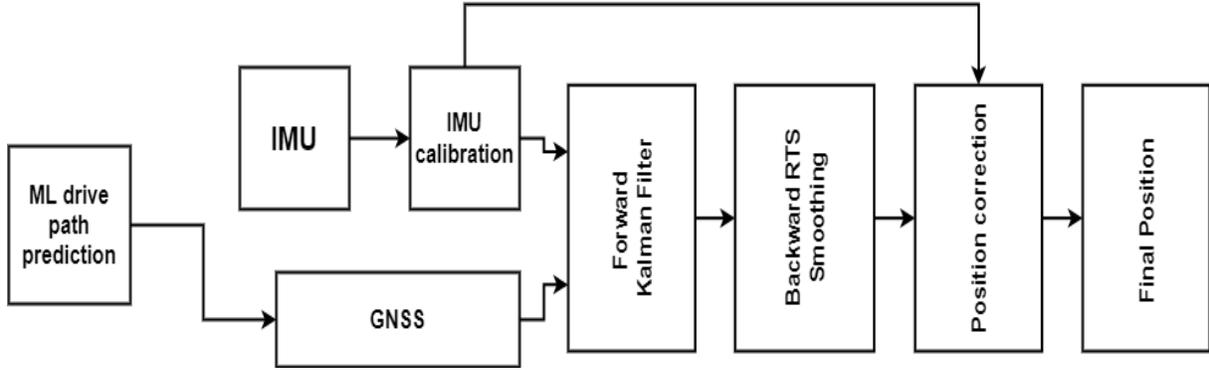


Figure 2: The flow diagram of MAP RTK post-processing algorithm.

GNSS measurements are easily disturbed by external influence such as multipath but the integration of GNSS with IMU is beneficial as the advantages and disadvantages of GNSS and IMU complement each other to enable accurate measurements in challenging areas. They are three types of IMU and GNSS integration namely, loosely coupled, tightly coupled, and ultra-tight coupled integration [15]. In this research we used the loosely coupled integration with KF/RTS, to integrate GPS and IMU in one system. We use the acceleration measured by IMU for inertial navigation calculation to get the data of position. We use them together with the information of GNSS for integration. The result is used as a prior observation of Kalman filtering while the covariance matrix and state transition matrix gained from forward Kalman filtering process is used for the reverse RTS smooth to the state estimate. The Rauch–Tung–Striebel (RTS) smoother, consists of a forward pass Kalman Filter (KF) and a backward recursion smoother to achieve the loosely coupled integration of GNSS and IMU measurements for positioning estimation in the smartphone. The forward classical Kalman filter is used to estimate the state of each moment, while the backward filter is to obtain more accurate state estimate on the basis of forward filter [16]. The flow diagram of MAP RTK post-processing algorithm is shown in Figure 2.

The state vector of the Kalman filter comprises both GNSS (longitude, latitude) and IMU (accelerometer) epochs, as follows in equation 1:

$$\mathbf{x} = [p^x \quad v_x \quad a_x \quad p^y \quad v_y \quad a_y]^T \quad (1)$$

where p^x, p^y are the (longitude, latitude) positions gained from GNSS. v_x, v_y are the non-measured speeds, and a_x, a_y are the IMU accelerations, respectively. Hence, the measurements are p^x, p^y and a_x, a_y . This makes the attributes of Kalman filter as follows in equation 2:

$$\mathbf{H} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (2)$$

The Newtonian equations of motion for predicting the iterative position, velocity and acceleration are:

$$x_{k+1} = x_k + \Delta t \dot{x}_k + \frac{\Delta t^2}{2} \ddot{x}_k + \frac{\Delta t^3}{6} \dddot{x}_k$$

$$\dot{x}_{k+1} = \dot{x}_k + \Delta t \ddot{x}_k + \frac{\Delta t^2}{2} \dddot{x}_k$$

$$\ddot{x}_{k+1} = \ddot{x}_k + \Delta t \cdot \dddot{x}_k$$

Where Δt is the time step, the state transition F matrix becomes:

$$\mathbf{F} = \begin{bmatrix} 1 & \Delta t & \frac{\Delta t^2}{2} & 0 & 0 & 0 \\ 0 & 1 & \Delta t & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & \Delta t & \frac{\Delta t^2}{2} \\ 0 & 0 & 0 & 0 & 1 & \Delta t \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (3)$$

The Kalman algorithm proceeds as follows:

$$\begin{aligned} \mathbf{x}_{k+1} &= \mathbf{F}_k \mathbf{x}_k + \mathbf{Q}_k \\ \mathbf{y}_k &= \mathbf{H}_k \mathbf{x}_k + R_k \end{aligned} \quad (4)$$

where $\mathbf{Q}_k, \mathbf{R}_k$ are the process and measurements covariance matrices, and x_{k+1} and y_k are the new states update and the generated measurements update, respectively.

These filtered a-priori and a-posteriori state estimates $\hat{X}_k|k-1, \hat{X}_k|k$ and the covariances $\hat{P}_k|k-1, \hat{P}_k|k$ are saved for use in the backwards pass (for retrodiction). The smoothed state estimates $\hat{X}_k|n$, and covariances $\hat{P}_k|n$ are computed for the backward pass. The below recursive equations are used starting from the last time step and proceeding backwards in time [17].

$$\hat{X}_k|n = \hat{X}_k|k + C_k(\hat{X}_{k+1}|n - \hat{X}_{k+1}|k) \quad (5)$$

$$P_k|n = P_k|k + C_k(\hat{X}_{k+1}|n - \hat{X}_{k+1}|k) \times C_k^T \quad (6)$$

where $C_k = P_k|k F_{k+1}^T P_{k+1}^{-1}|k$

$X_k|k$ is the a-posteriori state estimate of timestep k and $X_{k+1}|k$ is the a-priori state estimate of timestep $k+1$ which also applies to the covariance.

VI. EXPERIMENTS

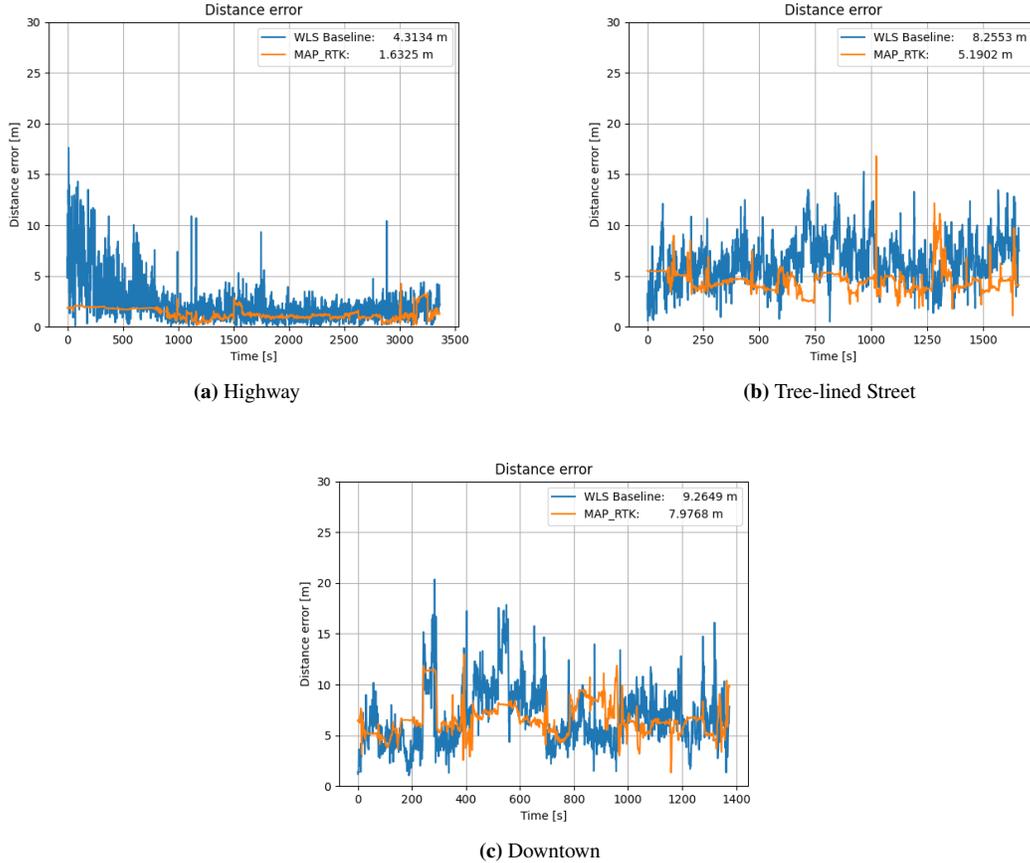


Figure 3: Comparison of positioning error between the WLS baseline and proposed method for each driving paths

The proposed method was evaluated using reference locations provided with the training dataset. In our studies, we made use of the carrier phase information of the data. The results of our model compared to the WLS solutions provided for the GSDC with respect to the ground truth is shown below. We can see an improvement in the estimated position for the respective driving paths as shown in Figure 3.

VII. RESULTS

We estimated the location of the smartphone and made evaluation of our method on the Kaggle platform. In our studies, we used the carrier phase information of the data. As a starting point, using GNSS only solution, initial results using our ML adaptive PPK positioning techniques on the 36 GSDC test datasets gave an accuracy of 2.62 meters for the public score on Kaggle platform. After the loosely coupled integration of the IMU sensor with GNSS navigation solutions, the score became 2.29 meters. From this result, we have shown that the proposed method (LC-GNSS/IMU/ML using MAP RTK) can be used to improve the position estimation of smartphones with relatively high accuracy.

VIII. CONCLUSION

Global Navigation Satellite System (GNSS) provides raw signals, which the GNSS chipsets in smartphones use to compute a position. The positioning accuracy provided by mobile phones is about 3-5 meters of positioning accuracy. This may be useful in some applications but in many cases, this can create a “jumpy” experience for many mobility applications relying on localization. In this paper, we employ a new LC-GNSS/IMU/ML method which makes use of the MAP RTK post-processing algorithm to process carrier-phase information, IMU sensors to improve the position estimates of smartphones. The results obtained show improvements in the positioning accuracy. Using the proposed method, we estimated the location of the smartphone and tackled the competition. The final public score was 2.61 m, while the final private score was 2.29 m. GNSS low-cost receivers and smartphones often have limited channels and computational resources, therefore, the complexity of the algorithm had to be kept modest. Further validations will be performed to ensure the integrity of the proposed method. In the future, we will look into factor graph and robust outlier detection for improvement of our solution.

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