Comparing the Risk Spillover from Oil and Gas to Investment Grade and High-yield Bonds through Optimal Copulas

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Comparing the risk spillover from oil and gas to investment and high-yield bonds through optimal copulas

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Abstract

This paper compares the tail dependence and risk spillovers from the oil and gas to high-yield and investment grade bond markets. We use time-varying optimal copula framework to examine the dependence and further quantify upside and downside risk spillovers. We also explore how energy futures can be used to hedge risk of high-yield and investment grade bond portfolios. Our results show that the bond returns are more sensitive to risk shocks in the oil market compared to gas market. We find both negative and positive tail dependence between bond and energy pairs and the relationship is stronger during the oil-crunch period. The dependence however is asymmetric across the tails. Finally, hedge ratios of gas are found to be much lower than the hedge ratios of oil indicating that a much smaller position in gas futures is needed to offset the risk in investing in investment grade and high-yield bonds. These results can help credit market participants in managing risk and designing the optimal asset allocation strategies. These might also assist policy-makers in formulating plans and regulations to manage the effects of risk transmissions.

Keywords: Dynamic dependence, risk spillover, bond and energy market, time-varying optimal copula, conditional value-at-risk

JEL classifications : C58, E44, G12, Q43

1. Introduction

This paper examines the tail dependence and risk spillover from the oil and gas market to investment grade (IG) and high-yield (HY) bond markets. The motivation of this paper is three-fold. First, a large literature examines the impact of energy prices particularly oil price shocks on stock markets (Driesprong et al., 2008; Kilian, 2008; 2009; Ready, 2016). However, relatively little attention has been provided on examining the relationship between energy and bond markets (Kang et al., 2014; Gormus et al. 2018). While the impact of recent energy market volatility on aggregate stock market (Mensi et al., 2017), Islamic equity market (Shahzad et al., 2018), exchange rates (Ji et al., 2018), and commodity market (Ji et al., 2018) has been widely examined, energy market's shock spillover to both HI and IG bond markets, particularly during the extreme market conditions, is yet to be explored.

Second, the global bond market is twice as big as the global stock market. In the case of US, the bond market is worth of about 40 trillion US dollars while the stock market value is just under 20 trillion US dollars (Bloomberg, 2016). Additionally, the global demand for bonds has been increasing due to increase in investors' degree of risk aversion and shifting of investment from equity market to relatively safer bond market after the global financial crisis (GFC) and European debt crisis. Low level of interest after the GFC resulting from quantitative easing of respective governments may have attracted investors to a particular segment of bond market (such as HY bond market). Given that the bond market constitutes a large portion of the overall financial market, it is worthwhile to understand the impact of energy price shock, a global risk factor, on the bond market.

Third, with the financialization of commodity markets and increased integration of financial markets in the recent years, new information tends to diffuse quickly across markets. Additionally, the GFC in 2008 characterizes that (i) entire financial market system can be vulnerable to external economic shocks, and (ii) cross-market linkage can be stronger during the periods of crisis due to financial contagion across markets (Liu et al., 2016). To this end, modelling dynamic dependencies across markets particularly during extreme market conditions is increasingly important.

The link between bond and energy markets can arise from different channels. Since same underlying corporate assets represent claims against corporate bonds and stocks, energy price shocks affecting value of these corporate assets may have an impact on both stock and bond prices. This proposition is supported by a large literature showing significant relationship between bond and stock markets (see, Bittlingmayer & Moser, 2014; Hong et al., 2012; Tolikas, 2017). Additionally, energy price shocks may affect firms' future earnings which affects investors' required rate of return and eventually bond prices. It is argued that some segments of bond market (particularly HY bond) behave like stocks, and bond prices show a trend in tandem to stock prices (Hong et al., 2012; Downing et al., 2009). Since there is significant evidence of energy price shock spillover to equity market, same spillover can also be observed in bond market due to bond market's co-movement with stock market. Nonetheless, bond and stock market may exhibit an asymmetric response to common factors (Tolikas, 2017). More specifically, sophisticated institutional investors dominate bond markets who have greater and faster access to relevant information compared to that of individual investors. This phenomenon may contribute to quicker diffusion of common information in bond market than in stock market. On the other hand, number of analysts covering stocks of a firm is typically higher than number of analysts covering bond of the firm as bond-related analysis is mostly conducted by credit rating agencies. Since revision of analyst recommendation is generally more frequent compared to that of credit rating agencies, some common information may be diffused quicker in stock market compared to that in bond market.

This paper makes several contributions to the literature. First, as indicated earlier, a very thin literature provides evidence of return and volatility transmission between energy and bond markets. While Kang et al. (2014) focus on the aggregate US bond index and Gormus et al. (2018) concentrate on the HY bond market, we focus on the risk spillover from the oil and gas market to the HY and IG bond markets. Theoretically energy price shocks may have a different impact on the HY and IG bond markets. Due to the cyclical nature of energy industry and banks' reluctance to provide long-term debt particularly after the GFC, oil and gas firms typically use HY debt to finance most of their expansion projects.[1] Therefore, the HY bond portfolio includes a large amount of energy company debts (Gormus et al., 2018). Energy market uncertainty is likely to have a negative impact on forecasted operating cashflows of these energy firms which in turn can have a negative impact on their bond prices. On the other hand, IG bonds are typically issued by large-cap and blue-chip companies which are less vulnerable to external shocks. Hence, IG bond prices are expected to be less sensitive to energy price shocks compared to HY bond prices.

Second, Kang et al. (2014) and Gormus et al. (2018) respectively use variance decomposition approach and LM-GARCH model which are unable to capture optimal

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[1] Although traditionally energy companies have obtained majority of their financing in the form of bank debt, they have gradually shifted from bank-led financing to capital market-based financing (such as HY bond). This transformation is mainly attributed to heightened political and economic instability following the GFC. The GFC and subsequent implementation of Basel III regulation have led to more stringent regulations on banks and greater capital adequacy requirement that ultimately pressurized banks to reduce their long-term loan exposure. As an alternative source of financing, energy companies rely on HY bonds particularly due to the incurrence (instead of maintenance) covenants. While commonly used maintenance covenants require borrower to maintain certain financial ratios on an ongoing basis, incurrence covenants require that a financial ratio is tested only when issuer intends to borrow further or take any other relevant action. Therefore, the energy sector – cyclical in nature and highly exposed to geopolitical volatility – is benefited from HY bonds particularly from the incurrence covenants. Large energy projects are divided into three stages: planning, construction and fully operational. HY bonds are typically used in the operational phase as due to stable underlying cash flows in this phase, bond financing is an economically appropriate alternative. The common characteristics of the HY bonds issued by energy companies are (i) deep and liquid market, (ii) fixed coupon rate, (iii) single drawdown, (iv) local currency denomination, and (v) absence of mandatory prepayments etc. See, Bahgat and Sokolova (2018) for more details about financing of energy projects.

dependence structure and systemic risk spillover across return distributions under different market conditions. In this paper, we use recently developed time-varying optimal copula (TVOC) model. While the commonly-used static copulas and time-varying copula (TVC) parameters are unable to identify dynamic changes in the nature of dependence structure between time-series, TVOC approach properly models asymmetric dependence across markets and identifies direction, intensity and nature of dependence structure.

Third, we quantify upside and downside risk spillover between the energy and bond markets using value-at-risk (VaR), conditional VaR (CoVaR), and delta CoVaR (Δ CoVaR) measures. While VaR is a widely used measure of downside risk of an investment (Girardi & Ergün, 2013), CoVaR reflects risk in investing in the bond markets conditional on extreme movement in the energy markets. Δ CoVaR captures asymmetric upside and downside risk spillover across markets. These methodologies have not been previously used to examine dependence structure and risk spillovers between bond and energy markets.

On the whole, this study investigates the upside and downside risk spillovers between the energy and bond markets. These measures, firstly, represent a good tool for international investors to assess the extreme investment losses, and secondly, relevant for safety-first investors who want to minimize the likelihood of extreme losses that may drive them out of business (Reboredo et al., 2016). Since the financial and commodity markets tend to integrate more during the periods of financial crises resulting in an increased degree of risk transmission between the markets, exploring the dependence structure between energy and bond markets particularly in the extreme conditions can be of interest to general financial market participants. As indicated earlier, HY bond portfolios hold a large amount of the US energy company debt. Therefore, energy prices are expected to impact energy company bonds. However, any evidence on energy prices' impact the HY and IG bond markets as a whole can have significant implications for firms in designing their debt policy, investors in devising their portfolio decisions, and policy makers in formulating bond and energy markets stabilizing policies.

We report several key findings. First, dependence structure between the bond and energy markets is not static rather time-varying and the direction of dependence (either positive or negative) changes over time. For example, the HY bond returns exhibit a negative dependence on the oil returns from 2010 until 2015, however the direction turns out to be positive afterwards. Second, while the tail risk in the HY bond market is higher compared to that in the IG bond market during the oil-crash period, the IG bond market exhibits higher downside risk than HY bond market in the whole sample period. Moreover, the bond returns are typically more sensitive to risk shocks in the oil market compared to the gas market. Third, we find a co-movement in both the negative and positive tails for all the bond and energy pairs implying that the energy market is not a good hedge against the tail risk of investment in the bond market. However, an asymmetric upside and downside risk spillover is found between the energy and bond markets.

The rest of the paper proceeds as follows: section 2 provides a brief review of relevant literature; methodological aspects are described in section 3, data description and descriptive statistics are provided in section 4; section 5 presents empirical results and analysis; finally, a summary of the paper is provided in section 6.

2. Literature review

Since the seminal paper of Hamilton (1983), a large literature examines the relationship between oil prices and economic growth and the channels to which this relationship arises. Although Hamilton (1983) shows a significant negative relationship between oil prices and future economic growth, subsequent studies show a non-linear relationship between them (for example, Hamilton, 2003).

Motivated by the conventional wisdom that shocks to energy prices particularly oil prices have a spillover effect on entire economy, researchers tend to examine oil price changes' impact on financial markets. These studies demonstrate different channels through which energy prices can affect financial market prices. For instance, Driesprong et al. (2008) argue that oil shocks can lead to higher economic risk, higher required rate of returns and subsequent lower prices in stock market. Ready (2016) claims that a demand-driven oil price rise results in higher revenue for oil producing firms which ultimately leads to positive returns for their stock returns. Kilian (2008) and Baumeister and Kilian (2016) indicate that energy price shocks can be transmitted either through supply (or cost) channel or demand (or consumption) channel. For instance, an increase in oil price increases cost of producing goods and services, and an inability to adjust output prices will result in a reduction in firms' earnings (Baumeister & Kilian, 2016). However, this proposition holds for only those industries that use oil as a major factor of production such as transportation, chemical, rubber, plastic, electricity generation etc. In a case where majority of energy is consumed by final consumers (for example, in the USA), a more important channel is demand (or consumer spending) channel. Another channel through which oil price can affect financial market prices is its effect on consumer sentiment. While Hamilton (2009) shows that an increase in global oil price during the first half of 2008 resulted a reduction in consumer spending followed by a big decline in overall consumer sentiment, Lemmon & Portniaguina (2006) report that consumer sentiment is positively related to capital market prices.

Based on the theoretical arguments presented in the previous paragraph, a large literature examines the relationship between oil and stock prices. Jones and Kaul (1996) show that an increase in oil prices depress aggregate stock returns particularly in the postwar period. Sadorsky (1999) shows a negative relationship between positive oil price shocks and real stock returns. Brzeszczyński et al. (2019) report consistent impact of crude oil price on socially responsible energy companies' stock prices. Nonetheless, oil price changes are found to have no ability to predict aggregate stock returns by Chen et al. (1986) and Huang et al. (1996), among others. This mixed-finding puzzle is explained by Kilian (2008; 2009) and Ready (2016). The authors establish that oil price changes are driven by demand and supply shocks and oil price changes' impact on aggregate economy as well as on stock market depends on whether the price change is demand or supply shock driven. Ready (2016) reports a strong positive relationship between demand shocks and stock returns while a negative relationship between supply shocks and stock returns. Apart from the contemporaneous relationship, several papers find a lead-lag relationship between oil and stock returns (Driesprong et al., 2008; Fan & Jahan-Parvar, 2012; Xu, 2015). Degiannakis et al. (2018) provide a good review of the literature on oil prices and stock markets.

Empirical findings discussed in the previous paragraph motivate researchers to examine if energy price shocks have an influence on the other segments of financial market i.e. bond market. This motivation is also linked to the contemporaneous or lead-lag relationship between bond and stock markets reported by several papers (Tolikas, 2017; Gebhardt et al., 2005).[2] These studies generally argue that the interaction between these two segments of financial market arises due to their joint response to market-wide common information factors. Additionally, the relationship between bond and energy prices may arise as a substantial portion of bond market is comprised of bonds issued by energy companies. A rising (declining) energy price is likely to result in higher (lower) revenues for these companies and higher (lower) risk-adjusted bond yields. In line with the theoretical expectation, oil price shocks are found to have

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[2] For instance, Tolikas (2017) find that aggregate stock returns lead IG and HY bond returns however the lead-lag relationship does not hold for the safest (Aaa) and least safe (Ca – D) bonds. This result implies that common information affects both bond and stock markets however, stock market incorporates information quicker than the bond market. Similar result is found by Gebhardt et al. (2005) and Hong et al. (2012), among others.

an important impact on the US bond market. For instance, Kang et al. (2014) show that 30.6% of the long-term variation in the broad-based US bond returns can be explained by demand and supply shocks in the oil market. The authors further report that structural oil price shocks exhibit an ability to explain 28.2% of the long-term variation of corporate bond returns while oil market-specific demand shocks interpret 24.4%, 13.2%, 11.1%, and 16.1% return variations of 1-, 5-, 10- and 30-year government bonds. Gormus et al. (2018) examine price and volatility transmission between energy (oil, natural gas, and ethanol) and HY bond markets using Toda-Yamamoto (TY) approach and LM-GARCH based volatility transmission approach. The TY approach reveals a bidirectional information flow while modified TY approach shows a unidirectional information flow from energy to HY bond market. The authors however do not find any price-level interaction between HY bond and natural gas prices.

Overall, previous studies provide significant evidence of oil prices' impact on entire economic activity as well as on stock market. However empirical research on risk transmission from energy to bond market is surprisingly low. Moreover, our predecessors do not examine the dynamic dependencies and systemic risk spillover between energy and bond market particularly during extreme market conditions. We aim to fill-up this gap.

3. Methodology

3.1. Time-varying optimal copula modelling

In this paper, we use recently developed time-varying optimal copula (TVOC) model (Liu et al., 2016) to identify the optimal dependence structure between the energy and bond returns over time. Copulas are powerful tools for effectively capturing cross-market dependence structure under extreme conditions. The copula theory is based on the theorem that two continuous random variables X and Y's joint distribution [F_{XY} (x,y)] can be expressed as a copula function [C(u,v)] and their marginal distribution functions [F_X (x), F_Y (y)] can be expressed as $F_{XY}(x,y) = C(u,v)$ where u and v respectively denotes $F_X(x)$ and $F_Y(y)$. Copula is, therefore, a multivariate function that has uniform marginals. To measure the degree of interdependence between variables, first, marginals to a multivariate distribution function can be connected through copulas, and second, the multivariate distribution function can be disentangled into univariate marginal distribution function and a copula. The later represents interdependence between variables. Copulas can measure dependence in the upper and lower tails of distributions which reflects extreme upward and downward co-movement. We consider both symmetric (normal copula and student-t copula) and asymmetric copula (Clayton copula, Rotated Clayton copula, Gumbel copula, and Rotated Gumbel copula) specifications to capture different dependence structures between the energy and bond returns. Internet Appendix A provides technical details of copula estimation and Internet Appendix Table A1 reports a summary of these copula specifications.

For the copula functions mentioned above, the time-varying dependence is modeled by allowing copula dependence parameter to vary with respect to an evolution equation. While the early studies mostly use the static copula (Junker et al., 2006; Miller & Liu, 2006; Kole et al., 2007), more recently, it has been extended to the form of time-varying copula (Giacomini et al., 2009). However, the existing time-varying parameter (TVP) copula models have one major limitation. Since they are mainly based on time-varying copula-dependence parameters, they cannot capture the dynamic changes in the cross-market dependence structure (Liu et al., 2016). To address this problem, we apply the data-driven TVOC method. This approach is the combination of TVP with changes in the family of copula. Therefore, it is capable of capturing asymmetries in the dependence across markets. Additionally, this method can identify dynamic changes in the direction, strength and type of the dependence across markets. We use a fixed-window length of 250-day which approximately corresponds to a one-year trading period.

The TVOC process consists of two parts: optimal copula (OC) modelling and time-varying (TV) modelling. Liu et al. (2016) introduce a distribution-free test for independence into the copula model. On the basis of such testing procedure, the OC dependence structure of the corresponding time point will be identified by the data. OC modelling involves five algorithmic steps. In the first step, we use VARX-GARCH (1, 1) model with skewed-t distribution to filter the return series and derive standardized residuals. Kendall's τ is computed for the subsample of time point t in the second step. The third step involves ascertaining dependence relationship by statistical inference. In the fourth and fifth steps, optimal copula for bond and energy returns is selected based on the statistical inference derived in step 3. The details of these steps are explained in Internet appendix A.2.

After selecting the OC for time point t, we apply a time-varying modelling. While the OC captures dependence structure at a given point in time, TVOC presents dynamic characteristics of the dependence structure. We use the rolling window method to perform the TV modelling. More specifically, we take an initial window of 250 days from January 1, 2009 to December 29, 2009 and estimate Kendall's τ. Then moving one day forward, we take a window of 250 days and re-estimate Kendall's τ. This process is continued until the last observation of the data series is used. In this way, we get a yearly measure of Kendall's τ. The use of 250-day window length is consistent with the literature (Harris & Shen, 2003).[3]

3.2. VaRs, CoVaRs and Delta CoVaRs risk measures

After examining the dynamic dependence structure between the bond and energy returns, value-at-risk (VaR), conditional VaR (CoVaR) and delta CoVaR (Δ CoVaR) are estimated to quantify tail risk. VaR measures degree of potential loss from an unconditional investment in a single

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[3] We check robustness of our result using alternate window lengths [(for example, 66-day (3-month), 130-day (6-month), 196-day (9-month)] and obtain qualitatively similar results. These results are not collated in this paper; however, they can be obtained from the corresponding author on request.

asset (such as HY or IG bond) that may arise for taking a long position (downside risk) or a short position (upside risk) in the asset. Investors are interested about both measures of risk as they tend to minimize the likelihood of extreme losses from an investment (Reboredo et al., 2016). Additionally, both the risk must be priced in the market, as investors must get sufficient compensation for assuming the risk. The downside VaR at time t and for a confidence level $1-\alpha$ is given by $\Pr(r_t \leq \mathrm{VaR}_{\alpha,t}) = \alpha$ while the upside VaR is given by $\Pr(r_t \geq \mathrm{VaR}_{1-\alpha,t}) = \alpha$. The downside and upside VaR can be determined by the following marginal models:

$$\mathrm{VaR}_{\alpha,t}^{\mathrm{Downside}} = \mu_t + t_{\upsilon,\eta}^{-1}(\alpha)\sigma_t. \quad (1)$$

$$\mathrm{VaR}_{\alpha,t}^{\mathrm{Upside}} = \mu_t + t_{\upsilon,\eta}^{-1}(1-\alpha)\sigma_t \quad (2)$$

where μ_t and σ_t are the conditional mean and standard deviation of the return series and $t_{\upsilon,\eta}^{-1}(\alpha)$ denotes the α quantile of the skewed-t distribution.

As the literature reports evidence of dependencies between energy and bond markets (Kang et al., 2014; Gormus et al., 2018), in line with Adrian and Brunnermeier (2011), we estimate CoVaR that measures the extent of probable losses from an investment in single asset (such as HY or IG bond) conditional on its risk exposure to other market (such as oil or gas). CoVaR is essentially a measure of the impact of distressed condition in one market (measured by its VaR) on the VaR of another market. A higher interdependence between two markets (for example, HY bond and oil) will lead to high CoVaR representing high degree of probable losses and high risk of investment. The CoVaR for the asset *i* and *j* is the VaR for asset *i* conditional on the fact that asset *j* exhibits an extreme movement. The downside (upside) CoVaR for the bond returns for an extreme downward (upward) energy price movement is as follows:

$$\Pr\left(r_t^b \leq \mathrm{CoVaR}_{\beta,t}^{b,\mathrm{Downside}} \middle| r_t^e \leq \mathrm{VaR}_{\alpha,t}^{e,\mathrm{Downside}}\right) = \beta \quad (3)$$

$$\Pr\left(r_t^b \geq \mathrm{CoVaR}_{\beta,t}^{b,\mathrm{Upside}} \middle| r_t^e \geq \mathrm{VaR}_{1-\alpha,t}^{e,\mathrm{Upside}}\right) = \beta \quad (4)$$

where $\mathrm{r}_\mathrm{t}^\mathrm{b}$ and $\mathrm{r}_\mathrm{t}^\mathrm{e}$ are the bond and energy returns, respectively. $\mathrm{VaR}_{\alpha,\mathrm{t}}^\mathrm{e}$ is the α-quantile of the energy return distribution and $\Pr\left(\mathrm{r}_\mathrm{t}^\mathrm{e} \leq \mathrm{VaR}_{\alpha,\mathrm{t}}^\mathrm{e}\right) = \alpha$ measures the potential loss that energy returns may experience for a confidence level $1-\alpha$ and for a specific time horizon. $\mathrm{VaR}_{1-\alpha,\mathrm{t}}^\mathrm{e}$ quantifies the potential loss by considering a short position for a confidence level $1-\alpha$ and for a specific time horizon. The CoVaR in eq. (3) and eq. (4) can be represented in terms of copulas, since the conditional probabilities can be re-written, respectively, as:

$$\mathrm{C}\left(\mathrm{F}_{\mathrm{r}_\mathrm{t}^\mathrm{b}}\left(\mathrm{CoVaR}_{\beta,\mathrm{t}}^\mathrm{b}\right), \mathrm{F}_{\mathrm{r}_\mathrm{t}^\mathrm{e}}\left(\mathrm{VaR}_{\alpha,\mathrm{t}}^\mathrm{e}\right)\right) = \alpha\beta \tag{5}$$

$$1 - \mathrm{F}_{\mathrm{r}_\mathrm{t}^\mathrm{b}}\left(\mathrm{CoVaR}_{\beta,\mathrm{t}}^\mathrm{b}\right) - \mathrm{F}_{\mathrm{r}_\mathrm{t}^\mathrm{o}}\left(\mathrm{VaR}_{1-\alpha,\mathrm{t}}^\mathrm{e}\right) + \mathrm{C}\left(\mathrm{F}_{\mathrm{r}_\mathrm{t}^\mathrm{b}}\left(\mathrm{CoVaR}_{\beta,\mathrm{t}}^\mathrm{b}\right), \mathrm{F}_{\mathrm{r}_\mathrm{t}^\mathrm{e}}\left(\mathrm{VaR}_{1-\alpha,\mathrm{t}}^\mathrm{e}\right)\right) = \alpha\beta \tag{6}$$

where $\mathrm{F}_{\mathrm{r}_\mathrm{t}^\mathrm{b}}$ and $\mathrm{F}_{\mathrm{r}_\mathrm{t}^\mathrm{e}}$ are the marginal distributions of the bond and energy returns, respectively. In line with Reboredo and Ugolini (2015), we follow a two-step procedure to compute the CoVaR. In Step 1, we solve Eq. (5) or Eq. (6) in order to obtain the value of $\mathrm{F}_{\mathrm{r}_\mathrm{t}^\mathrm{b}}\left(\mathrm{CoVaR}_{\beta,\mathrm{t}}^\mathrm{b}\right)$, given the significance levels for the VaR and CoVaR, and for specific forms of the copula function. In Step 2, we use the distribution function for the energy and bond returns and compute the CoVaR for the bond returns as $\mathrm{F}_{\mathrm{r}_\mathrm{t}^\mathrm{b}}^{-1}\left(\mathrm{F}_{\mathrm{r}_\mathrm{t}^\mathrm{b}}\left(\mathrm{CoVaR}_{\beta,\mathrm{t}}^\mathrm{b}\right)\right)$.

Finally, we estimate systemic risk contribution of the energy returns to the bond returns as $\Delta\,\mathrm{CoVaR}$. $\Delta\,\mathrm{CoVaR}$ of a pair of assets (such as HY bond and oil) is a measure of systemic risk contribution of distressed state of one market (such as oil) on investment in other market (such as HY bond). More specifically, it is the difference between the VaR of the bond return conditional on the distressed state of energy market $\left(\mathrm{R}_\mathrm{t}^\mathrm{e} \leq \mathrm{VaR}_{\alpha,\mathrm{t}}^\mathrm{e}\right)$ and the VaR of the bond return conditional on the benchmark state of the energy market, or the median of the return distribution of energy market (the VaR for $\alpha = 0.5$). The systemic risk contribution of energy market is thus defined as:

$$\Delta\mathrm{CoVaR}_{t}^{b/e} = \frac{\left(\mathrm{CoVaR}_{\beta,t}^{b/e} - \mathrm{CoVaR}_{\beta,t}^{b/e,\alpha=0,5}\right)}{\mathrm{CoVaR}_{\beta,t}^{b/e,\alpha=0,5}} \quad (7)$$

ΔCoVaR is useful as it captures the marginal contribution of the energy market to the bond's systemic risk.

4. Data and Descriptive Statistics

4.1 Data

In this paper, energy market is represented by crude oil futures and natural gas futures prices that are traded on the New York Mercantile Exchange. Previous studies document that futures markets perform relatively better in terms of price discovery compared to spot markets (Uddin et al.2018). To represent high-yield bonds, we consider the S&P U.S. High Yield Corporate Bond Index (S&P HY) and S&P U.S. High Yield Energy Corporate bond index (EN HY). The S&P HY tracks the performance of high-yield, U.S. dollar-denominated corporate bonds that are issued by companies whose country of origin use official G-10 currencies.[4] The securities included in this index must have maturities of one or more months and they must be below-investment-grade rated according to S&P Global Ratings, Moody's, and Fitch. The EN HY is a sub-index of the S&P HY Index. This index represents high-yield bonds issued by energy sector companies within the constituents of S&P HY index. To represent investment grade bonds, we consider the S&P 500 Investment Grade Corporate Bond Index (S&P IG) and S&P 500 Investment Grade Energy Corporate Bond Index (EN IG). S&P IG tracks the performance of U.S. corporate debt issued by the S&P 500 companies with an investment-grade rating. On the other hand, EN IG is a manifestation of the performance of investment grade bonds issued by the energy sector within the constituents of S&P500 bond index.

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[4] The official G10 currencies are the Australian Dollar, the British Pound Sterling, the Canadian Dollar, the Euro, the Japanese Yen, the New Zealand Dollar, the Norwegian Krone, the Swedish Krona, the Swiss Franc, and the US Dollar.

The sample period considered in this paper spans from January 1, 2009 till May 31, 2019. The starting of the sample period is dictated by the availability of daily data on the bond indices. All data are collected from Bloomberg Professional. Daily returns are calculated as the logarithmic difference of prices at time t and t- 1.

4.2 Descriptive statistics

Statistical properties of the return series are presented in Table 1. Descriptive statistics and correlation matrix are respectively in Panel A and Panel B. We first focus on Panel A. Mean returns for both the HY and IG bonds are positive. This result holds irrespective of aggregate S&P HY and IG bond indices and energy sector HY and IG bond indices. Mean returns for oil and gas are positive and negative respectively. Difference between minimum and maximum returns and standard deviation indicate that the bond returns are more volatile compared to the energy returns. Additionally, EN HY and EN IG bond returns exhibit higher volatility compared to S&P HY and S&P IG bond returns. The bond (energy) returns are negatively (positively) skewed implying a higher probability of extreme negative (positive) returns. Kurtosis for all the returns series are higher than three representing leptokurtic distributions. For all the return series, the null hypothesis of normality, unit root, no serial correlation and homoscedasticity is rejected respectively by the Jarque-Bera test, the Augmented Dicky-Fuller and Philip-Perron tests, the Ljung-box test and the Lagrange Multiplier test.

From Panel B, we observe moderate to no correlation between bond and energy returns. For instance, both S&P HY and EN HY returns are positively correlated with oil returns while S&P IG and EN IG returns are negatively correlated with oil returns. HY bond returns' positive relationship with oil returns is consistent with Gormus et al. (2018). However, although the authors find a statistically significant negative relationship between HY bond and gas returns, we find that none of the correlation coefficients between bond and gas returns are statistically significant. As expected, S&P HY and EN HY returns are highly correlated. Similar result is found for S&P IG and EN IG bond returns. Oil and gas returns are positively correlated, and the correlation coefficient is statistically significant at the 1% level.

[INSERT TABLE 1 ABOUT HERE]

We also report time-trends of the variables in Internet Appendix Figure A1. Overall, although a clear relationship between the bond and energy prices is not observed during the early part of the sample period, they tend to move together in the recent years (2014 – 2017). This result may indicate that bond and energy markets have become more integrated over time which also may increase potential risk spillover between the markets. We also present current international conditions in the global oil and gas market in Internet Appendix B.

4. Empirical Results and Discussion

4.1 Tail dependence between the energy and bond prices

We first estimate the marginal models based on the VARX-GARCH with skewed-t innovations. VARX-GARCH is applied on both the HY and IG bond returns, where oil and gas returns are added as exogenous variables. Conditional volatility models are then applied on the residuals of VARX for the bond returns whereas they are directly applied on the gas and oil returns. This is important because the bond returns exhibit significant serial correlation, and auto-regressive moving average (ARMA) models remained unable to remove serial correlation. The optimal lag order of the model is determined by the Akaike information criterion (AIC). We filter the return series via the VARX-GARCH (1, 1) model to obtain approximately independent and identically distributed (i.i.d) standardized residuals. The estimation results for mean and variance equations, and diagnostic tests are reported in Panel A, B, and C of Table 2, respectively.

From Panel A, we observe that the S&P HY bond returns respond positively to its lagged returns and S&P IG lagged returns, and respond negatively to lagged EN IG bond returns.

Although the S&P IG bond returns are insensitive to its own lagged returns, they exhibit a positive relationship with lagged S&P HY bond returns and a negative relationship with EN IG bond returns. We find almost a similar result for EN HY and EN IG bond returns except that EN HY bond returns are insensitive to S&P IG, and both EN HY and EN IG bond returns respond positively to lagged EN HY bond returns. The lagged oil returns have statistically significant impact on the HY and IG bond returns implying the presence of return spillover from the oil to bond market. Nonetheless, the bond returns (both HY and IG) appear to be insensitive to the lagged gas returns implying lack of return spillover from the gas market to the bond markets.

As we move to Panel B, we find that the coefficients of GARCH term are positive and range between 0.805 (S&P HY) to 0.958 (S&P IG) for all the bond and energy returns indicating volatility persistence in these markets. ARCH components are positive and statistically significant for all the returns series implying that one-period lagged shocks have significant impact on current-period conditional volatility. The asymmetries and tails are statistically significant at the 1% level in all the markets.

Panel C of Table 3 reports results of different diagnostic tests which are indicators of the goodness-of-fit of the marginal models. The p-values in the square brackets for different goodness-of-fit tests are mostly greater than 0.05 indicating that the null hypothesis of the correct specification of the marginal models cannot be rejected at the 5% significance level. Overall, the goodness-of-fit tests results imply that our marginal distribution models are correctly specified.

[INSERT TABLE 2 ABOUT HERE]

In order to demonstrate that the TVOC model outperforms other benchmark static and time-varying copula specifications, we compare their log-likelihood values with that of four commonly used copula models [time-varying parameter (TVP) and static Normal and student t copula as proposed by Patton (2006)]. Overall, we find that the TVOC model is optimal in all the cases. The detail result is available in Internet Appendix Table A2.

We now present the characteristics of time-varying copula estimates for energy and bond return pairs in Figures 1, 2, 3, 4. In each Figure, (a) shows time-varying dependence represented by Kendal's tau which is derived from tail dependence parameters, (b) displays histogram of optimal or best-fitted copula and (c) to (f) present time-plots of tail dependence parameters for different tail combinations of the variables.

We first concentrate on Figure 1. Panel A and Panel B respectively present time-varying copula estimates for oil – S&P HY and oil - S&P IG bond return pairs. From Panel A, we find that the time-varying Kendall's τ remains commonly negative from the start of the sample period until 2015 (a) and the best-fitting copulas for the period are *t* copulas rejecting the presence of asymmetries in the dependence structure (b). However, the aforementioned relationship becomes positive after January 2015 until the middle of 2017 which corresponds to oil price crash. During this period, the OCs are mainly the rotated Gumbel confirming an asymmetric positive dependence. Overall, this result may indicate that although the oil and HY bond returns exhibit a negative dependence during a normal state in the oil market, they co-move positively during oil market crisis. This result further indicates downside risk in these markets during a distressed condition in the oil market. The dependence structure turns negative from the middle of 2017 until the end of the sample period. Moreover, from (c) to (f), it is found that the upper – upper or lower – lower tail dependence is generally smaller than the lower – upper or upper – lower tail dependence. This result indicates that firstly, the oil and HY bond pairs exhibit both downside and upside risk, and secondly, the tail dependence is stronger when the variables are in different tails.

As we move to panel B, we find that dependence between the oil – IG bond pair is negative between 2009 – 2013 period when normal copula accounts for the maximum proportion of total number of best-fitting copulas indicating absence of tail dependence. However, the dependence is mostly positive afterwards and best-fitting copulas are dominated by Gumbel and rotated Gumbel implying asymmetric extreme positive dependence. (c) to (f) indicate the presence of lower – upper dependence between 2009 – 2013 while upper – upper dependence from 2013 onwards. The lower – lower and upper – lower dependence is negligible throughout the sample period. This result indicates a co-movement between the pair in the recent years when both of them are in extreme positive tail. However, there is lack of dependence in the other tail combinations.

Panel A and Panel B of Figure 2 respectively present the results for gas – S&P HY and gas – S&P IG bond return pairs. We first focus on Panel A. The Kendal's tau measure of time-varying dependence indicates that the relationship changes over time between positive and negative without any secular trend implying that the associations are not stable during the sample period. Normal copula accounts for the largest proportion of the total number of best-fitting copulas implying the absence of tail dependence for most part of the sample period. For the other part of the sample period, we do find some evidence of tail dependence (both symmetric and asymmetric), however, OC changes frequently over time. (c) to (f) present that lower – lower and upper – lower dependence is negligible throughout the sample period. The dependence is mostly observed in the lower – upper tail combination while some evidence of extreme dependence is found in upper – upper tail combination (between 2010 – 2011 and during 2016 and 2018).

From Panel B, we find that the dependence structure tends to vary over time. The dependence structure is mostly negative from the start of the sample period in 2009 until 2013 and normal copula accounts for the largest proportion of best-fitting copulas. However, the dependence is mostly positive during 2013 – 2016 and best-fitting copulas are Clayton and rotated Gumbel. The dependence again turns negative from 2017 onwards and t copula accounts for the majority of best-fitting copulas representing the presence of symmetric tail dependence. We further observe that lower – lower and upper – lower dependence is negligible while the upper – upper tail dependence is strong over the sample period (except 2011 – 2012, 2013 – 2014, and 2016 – 2017 periods).

To summarize, the dependence between the oil and bond returns are found to be mostly negative during the earlier part of the sample period (particularly between 2009 – 2013) while the dependence turns out to be positive in the later part (particularly between 2015 – 2017 in the case of oil and HY bond while 2013 – 2015 in the case of oil and IG bond). This result may imply a positive co-movement between the oil and bond returns during the recent oil price crisis. This finding has important implication for portfolio strategies. The positive dependence indicates that the bond market becomes vulnerable to oil price shocks particularly during the crisis period, and oil futures do not provide a good hedging option for the bond portfolios. This result is in line with the argument that oil price shock is considered to be an economic risk which, in one side, affects firms' (particularly energy producing firms and firms that use energy as the main factor of production) ability to pay interest and principal back to bond holders, and on the other side, increases investors' degree of risk aversion and required rate of return. We, however, cannot make an obvious distinction in the above-mentioned result for the HY and IG bond returns which is at odds with our theoretical expectation that the HY bond returns is likely to be more sensitive to oil price shocks. The dependence structure between the gas and bond returns oscillates between positive and negative without any secular trend and the dependence is generally weaker compared to that of oil and bond returns. Overall, the optimal copulas are found to vary over time which justifies the use of time-varying optimal copula model.

[INSERT FIGURE 1 AND 2 ABOUT HERE]

We also explore the time-varying copula estimates for oil – EN HY, oil – EN IG, gas – EN HY, and gas – EN IG bond pairs and report them in Internet Appendix Figure A2 and A3 to conserve space. In line with our previous finding with regard to oil – S&P HY and oil – S&P IG bond pairs, we find that time-varying Kendall's τ remains negative in the early part of the sample period for both oil – EN HY, oil – EN IG pairs while the dependence turns positive particularly from 2015 onwards. However, for gas – EN HY and gas – EN IG pairs, the dependence structure changes between positive and negative over the sample period without any particular pattern.

4.2 Spillover effect between the energy and bond prices

Since tail dependence structure between the bond and energy markets discussed in the previous subsection has important implication for investors in terms of risk spillover, in this subsection, we examine the spillover effect between the energy and bond returns using VaR, CoVaR, and ΔCoVaR. This analysis is done for both full sample and oil-crash subsample periods. The oil-crash subsample ranges from 15 July 2014 till 31 March 2017 when oil prices kept decreasing due to an oversupply of oil due to higher production. This analysis is worthwhile as we report time-varying tail dependence in the previous subsection and the literature also demonstrates different degree of volatility transmission across asset classes over time (Diebold & Yilmaz, 2012; Mensi et al., 2017). Table 3, 4, and 5 respectively displays VaRs, CoVaRs and ΔCoVaRs (standard deviation in parenthesis). In each of these tables, the whole sample and oil-crash subsample results are respectively presented in Panel A and Panel B.

From Table 3, we observe that both downside and upside VaRs are statistically different from zero. This result holds for all the bond returns irrespective of the sample period. This finding indicates significant tail risk for the bond returns. In the whole sample (Panel A), average value of both downside and upside VaR for the S&P IG bond returns is greater than that of the S&P HY bond returns indicating higher upside and downside risk in the S&P IG bond market compared to that in S&P HY bond market. The difference is statistically significant at the 1% level. This result also holds in the oil-crash subsample (Panel B). Although according to the conventional wisdom, we may expect that investment in the HY bonds is likely to carry higher downside risk compared to that of the IG bonds, we do not get support for this notion. This result may be attributed to the fact that HY bonds typically carry incurrence covenants (instead of maintenance). While commonly used maintenance covenants require borrower to maintain certain financial ratios on an ongoing basis, incurrence covenants require that a financial ratio is tested only when issuer intends to borrow further or take any other relevant action. Therefore, HY bonds are less exposed to cyclical and geopolitical volatility.

In the case of energy subsector of the bond markets, in line with our previous result, both downside and upside VaRs of EN IG bond are significantly higher than that of EN HY bond in the full sample period (Panel A). However, during the oil-crash subsample (Panel B), we find that both downside and upside risk of EN HY bond is higher that of EN IG bond. This result is in line with our expectation that HY bond market should exhibit higher downside risk compared to that of IG bond market. These results have important implication as investors may tend to rotate their investment between the HY and IG bond due to their asymmetric exposure to tail risk. Moreover, portfolio choice may be different for S&P HY and S&P IG bond markets and their energy subsectors. For instance, in the case of S&P HY and S&P IG bond markets, investors may like to shift their investment from the HY bond market to the IG bond market when the latter is in extreme positive quantile. On the other hand, investors may invest more in HY bond when the bond market is in extreme low quantile. This approach will expose them to low tail risk in relation to the tail risk exposure of IG bond. In the case of EN HY and EN IG bond market, investors may like to invest more in HY bond market during a normal condition in the oil market. However, when oil market faces a crisis, investors may shift their investment from EN HY bond to EN IG bond market as EN HY bond market becomes more risky during an oil market crisis condition.

We further find that VaRs of EN HY and EN IG are relatively higher compared to that of S&P HY and S&P IG respectively. This result, firstly, holds for both downside and upside VaRs, and secondly, statistically significant at the conventional level. This finding indicates that probability of extreme losses/gains is higher for an investment in energy subsector of HY and IG bond portfolios than that of the aggregate HY and IG bond portfolios. This result is expected as energy companies are typically cyclical in nature and highly exposed to geopolitical volatilities and technological innovation. Additionally, except the case of EN IG, VaRs in the oil-crash subsample are significantly higher than that in the whole sample period. This result implies high tail risk during the oil-crash subsample compared to the tail risk exhibited by the bond returns in the whole sample period. This result is consistent with the notion that tail risk increases during a crisis period due to higher volatility and panic sale of financial assets arising from investors' higher degree of risk aversion and so called flight to safety.

[INSERT TABLE 3 ABOUT HERE]

Now, we focus on CoVaRs (Table 4). We find that the CoVaR statistics are statistically significant at the 1% level indicating significant tail movement between the bond and energy returns. This result holds for all the combinations of bond and energy returns, for both upside and downside tail risk, and irrespective of the whole sample period and oil-crash subsample. From Panel A, for S&P HY bonds, the downside CoVaR from oil is significantly higher (negative values) compared to the downside CoVaR from gas. However, the upside CoVaR from gas is significantly higher than the upside CoVaR from oil. These findings imply that in the down market, the S&P HY bond returns are more sensitive to risk shocks in the oil prices compared to the gas prices. However, in the upmarket, S&P HY bond returns turn out to be more sensitive to the changes in gas prices. In the case of S&P IG bonds, both the upside and downside CoVaRs from oil are significantly higher compared to the corresponding CoVaRs from gas. In the cases of EN HY and EN IG bonds, although we find almost identical downside CoVaRs from oil and gas, the upside CoVaRs from oil are higher than the upside CoVaRs from gas. These results in general manifest higher risk spillover from the oil to aggregate HY and IG bond markets. However, the energy subsector of HY and IG bond markets are similarly affected by risk spillover from both the oil and gas markets, particularly in the extreme low tails. While Gormus et al. (2018) report significant volatility transmission from the oil market to the HY bond market and a weaker information flow from the natural gas to the HY bond market, our finding shows that this result holds for both HY and IG bonds (with few exceptions). Relatively lower risk spillover from the gas market indicates that the gas market is considered to be more local and less integrated to global financial market (Gormus et al., 2018), therefore having a less pronounced impact on the aggregate HY and IG bond markets. Furthermore, shock to the gas market is perceived as a less important risk factor compared to oil price shocks by financial market investors resulting a low level of risk spillover from the gas to the bond market. We further find that the CoVaR statistics of EN HY and EN IG bonds are relatively higher compared to that of S&P HY and S&P IG bonds. This result holds for both oil and gas. This finding is expected as energy subsector of HY and IG bond market is likely to exhibit a higher co-movement with energy prices compared to the aggregate HY and IG bond markets.

As we move to panel B and observe results pertaining to the oil-crash subsample, we find a clearer picture that CoVaRs from oil is higher compared to the CoVaRs from gas. This result generally holds for both upside and downside CoVaRs and for S&P HY, S&P IG, EN HY, and EN IG bonds. We further observe that both the downside and upside CoVaRs are generally higher in the oil-crash subsample (except few cases of EN IG bond) compared to that in the full sample period. This result is particularly evident in the cases of S&P HY and EN HY bond returns. This result implies higher level of risk in the HY bond market compared to the IG bond market, and higher level of risk spillover from the energy market to the bond market between 15 July 2014 till 31 March 2017 when oil price saw a large decline from US$115 per barrel in July 2014 to as low as under $35 per barrel at the end of February 2016. These results are intuitively appealing. The IG bond index, considered in this paper, comprises of higher credit rated bonds issued by financially sound S&P500 companies, while the HY bond index is comprised of lowed credit rated bonds issued by companies with relatively weak financial health. This point explains the higher risk associated with the HY bond returns. Furthermore, a larger risk spillover during the recent oil price crash may be attributable to the fact that a large decline in oil price was taken as an indication of lower ability to service debt particularly by energy firms (that constitute a substantial portion of HY bond index) that ultimately depressed prices of bonds issued by these firms. We also find that upside CoVaRs are typically higher than downside CoVaRs. This result generally holds for all the bond return and irrespectively of the whole sample period and oil-crash subsample. This finding implies that the bond and energy markets become highly integrated and they exhibit a co-movement when they are in extreme positive tails. However, the co-movement in the negative tail is relatively weaker. This result may indicate that during an upmarket condition, investors similarly investment in HY and IG bonds and energy futures. However, during an extreme down market condition, investors shun these investment away. Nonetheless, they exhibit a heterogeneous behavior due to these investments' asymmetric downside risk exposure that leads to a relatively weaker negative tail movement.

The results discussed in the preceding paragraph are in line with previous literature. Gormus et al. (2018), for instance, find a strong volatility transmission from the energy to HY bond market after the oil crisis period while no evidence of volatility transmission is found during the pre-crisis period. Likewise, Kang et al. (2014) report significant spillover between oil and bond market during the global financial crisis. In addition, Diebold and Yilmaz (2012) show that volatility spillovers from oil to bond markets increased in 2003 just before and during the invasion of Iraq by US forces, and at the end of 2004 and early 2005, when the surge in the Chinese demand for oil and metals surprised investors and resulted in higher oil prices. Diebold and Yilmaz (2012) further demonstrate that during the episode of global financial crisis in 2008, the volatility shocks in the oil market transmitted mostly to the bond market. Our results can also be used by investors in forming and rebalancing portfolios. Given the evidence of time-varying risk spillover (higher spillover during the oil-crash subsample), an anticipation of an extreme downturn in oil price may inform investors to shift their investment from bond market to an investment vehicle which is less exposed to energy market.

[INSERT TABLE 4 ABOUT HERE]

As we look at ΔCoVaRs (Table 5), we find that they are typically significant at the conventional significance level indicating significant risk spillover from the energy to bond markets. Both the downside and upside ΔCoVaRs from oil is significantly higher than the corresponding ΔCoVaRs from gas. This result indicates that a distressed condition in the oil market exerts a more severe impact on the bond markets compared to the impact of a distressed condition in the gas market. This result holds for all the bond indices (S&P HY, EN HY, S&P IG and EN IG) considered in this paper. Our results are also consistent with Kang et al. (2014) and Gormus et al. (2018), among others, who find significant risk spillover between oil and bond markets. We further find that although the ΔCoVaRs for S&P HY – oil and S&P IG – oil are positive, the ΔCoVaRs for EN HY – oil and EN IG – oil are negative. This result indicates that EN HY and EN IG bond returns are negatively correlated to oil returns. In such case, even if oil market becomes stable and return improves, the bond market may continue to worsen.

The ΔCoVaRs in the oil-crash subsample (Panel B) is typically higher than the ΔCoVaRs in the full sample period implying that the consequence of a distressed condition in the energy market on bond markets is more severe during the oil-crash period. This result supports our findings presented earlier in this subsection. This result is also consistent with Balli et al. (2019) who show a higher risk spillover between commodities during the oil-crash subsample. We further observe that the upside ΔCoVaRs are higher compared to the downside ΔCoVaRs (with one exception). This result holds both in the full sample and oil-crash subsample, for all the bond indices and for both the energy returns. This finding implies that firstly, the spillover effect during the extreme upward and downward movements are not identical, and secondly, the spillover effect increases during the extreme upward movement. However, the downside ΔCoVaR is significantly higher compared to the upside ΔCoVaR for S&P HY – oil pair. This result indicates a high risk spillover during the down market condition compared to the upmarket condition.

[INSERT TABLE 5 ABOUT HERE]

We also explore the time-varying dynamics of risk spillover from the energy markets to bond markets. More specifically, we extract daily measures of VaR, CoVaR and ΔCoVaR and plot them against time. Internet Appendix Figure A4 presents downside and upside VaR and CoVaR and Figure A5 displays downside and upside ΔCoVaR from the energy markets to bond markets. Figure A4 presents couple of key findings. First, in the case of oil – IG and gas – IG bond pairs, downside $CoVaR$ is lower than downside VaR throughout the sample period indicating dependence in the lower tail and co-movement during the extreme downtrend. Similar result for energy – HY bond is particularly evident during the oil-crash subsample. Second, with regard to oil – IG and gas – IG bond pairs, upside $CoVaR$ is higher than upside VaR particularly since 2013 representing co-movement during the extreme up condition in the market. Once again, such co-movement between oil and bond prices is observed during the oil-crash subsample. We find two key results from Figure A5. First, we observe that ΔCoVaR from gas to bond is typically lower compared to ΔCoVaR from oil to bond. This result supports our previous finding of higher risk spillover from oil market to bond compared to the degree of risk spillover from gas market bond market. Second, downside ΔCoVaR is higher compared to upside ΔCoVaR particularly between 2015 – 2017 which is in line with our previous argument that risk spillover increases during the extreme down-market condition. These results generally hold for both the aggregate and energy sector bond indices.

Thus far in this subsection, we get an idea about the riskiness of investment in the bond market and risk spillover from the energy market to bond market. Now we examine the statistical significance of risk spillover and asymmetric effect of the energy returns on bond returns. To this end, we test the following hypotheses:

Hypothesis 1: $H_0: \text{CoVaR}(D) = \text{VaR}\,(D);\ H_1: \text{CoVaR}(D) < \text{VaR}\,(D)$

Hypothesis 2: $H_0: \text{CoVaR}(U) = \text{VaR}\,(U);\ H_1: \text{CoVaR}(U) > \text{VaR}\,(U)$

Hypothesis 3: $H_0: \frac{\text{CoVaR}}{\text{VaR}}(D) = \frac{\text{CoVaR}}{\text{VaR}}(U);\ H_1: \frac{\text{CoVaR}}{\text{VaR}}(D) > \frac{\text{CoVaR}}{\text{VaR}}(U)$

The first and second hypotheses test the statistical significance of downside and upside risk spillovers from the energy to bond markets. The third hypothesis examines asymmetries in downside and upside risk spillover. The hypotheses are tested using the bootstrap Kolmogorov – Smirnov (K-S) test proposed by Abadie (2002).[5] The K-S test is defined as:

$$KS_{mn} = \left(\frac{mn}{m+n}\right)^{1/2} \sup_{x} |F_m\,(x) - G_n\,(x)| \tag{8}$$

where $F_m\,(x)$ and $G_n\,(x)$ respectively represent cumulative CoVaR and VaR distribution function. *m* and *n* are respective sample sizes.

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[5] This test has recently been used by Reboredo et al. (2016), Ji et al. (2019), Liu et al. (2017), Ji et al. (2018), among others, to validate similar hypothesis.

The results of hypothesis tests are reported in Table 6. Columns 1 to 3 respectively present results pertaining to hypotheses 1 to 3. The results in column 1 indicate that the downside risk spillover from (downside CoVaR is less that than downside VaR) energy markets to bond market is statistically significant at the conventional level. This result, firstly, implies a co-movement between the bond and energy returns in the negative tails, and secondly, indicates that with an extreme drop in the oil returns, investors may tend to take their capital out from the HY and IG bond market and may tend to invest in other alternatives. From column 2, we find that the upside CoVaR values are significantly greater than upside VaR values indicating a clear evidence of upside risk spillover from the energy market to bond market. This result further implies that an extreme upturn in energy returns may lead investors to invest in the HY and IG bond markets due to their co-movement in the extreme positive tails. The results is columns 1 and 2 are also consistent with our previous finding of statistically significant CoVaRs reported in Table 4. Column 3 shows that the downside CoVaR/downside VaR are significantly greater than the upside CoVaR/upside CoVaR as the test statistics are found to be significant at the 1% level. Overall, the results provide evidence of asymmetry in downside and upside risk spillover. More specifically, the bonds are subject to greater downside risk spillover compared to upside risk spillover from energy markets. This result may be explained in terms of asymmetric real and financial flows during an extreme down and extreme up market (see Reboredo et al., 2016). Investors may overreact to an extreme down market, triggering capital outflows from the bond market to elsewhere, compared to capital inflow in the bond market during an extreme up market.

Our findings are consistent with those reported by Mensi et al. (2017), Shahzad et al. (2018), and Reboredo et al. (2016), among others, in different asset markets. Mensi et al. (2017) and Shahzad et al. (2018) find evidence of asymmetric upside and downside risk spillovers between oil and stock markets. Reboredo et al. (2016) document similar results for exchange rates and stock markets. It is noteworthy that all these papers reveal greater downside risk spillover compared to the degree of upside risk spillover. Investors should be aware of the asymmetric risk spillovers between oil and bond markets for both the short and long positions to protect their portfolios against the adverse effects of extreme market movements.

[INSERT TABLE 6 ABOUT HERE]

4.3 Hedging effectiveness between energy and bond markets

In this subsection, we explore the effectiveness of oil and gas futures for hedging investment in HY and IG bonds. While numerous papers examine the usefulness of oil as a hedging instrument for equities (see for example, Arouri et al., 2011; Mensi et al., 2013), our paper is a pioneer in investigating energy-bond hedging effectiveness. Hedge ratio represents the magnitude of a short position required in an asset (for example, oil futures) to hedge a given long position in another asset (for example, HY bond). Hedging effectiveness is estimated as the variance differential of unhedged and hedged portfolio scaled by variance of unhedged portfolio. In this paper, hedge ratios are calculated using (i) volatility estimates from the marginal models as reported in table 2, and (ii) correlation estimates based on TVOP framework. We consider the oil and gas futures from 1-month to 12-month maturities. Table 7 shows the results relating to hedge ratios and hedging effectiveness.

In the case of oil futures, we find that the average values of hedge ratios are negative for S&P IG and EN IG bond indices. The negative value indicates an inverse relationship between oil and bond returns and implies that a hedge can be formed by taking a short or long position in both the assets (for example, bond indices and energy futures). More specifically, a \$1000 long position in S&P IG bonds can be hedged by taking another \$16 long position in oil futures (in the case of 1-month maturity). In the case of EN IG bond, the hedge ratios are much smaller and close to zero indicating that much smaller amount of investment in oil futures is needed to hedge a position in EN IG bond. For instance, a $1000 long position in EN IG bonds can be hedged just by taking another $6 long position in oil futures (in the case of 1-month maturity).

In the cases of S&P HY and EN HY, the hedge ratios are positive indicating a positive relationship and hedge can be formed by taking an opposite position in oil and bond. For instance, a $1000 long position in EN HY bonds can be hedged by taking a $56 short position in oil futures. Hedge ratios for S&P HY bonds are much smaller compared to that of EN HY bonds indicating that a smaller hedge position in oil futures is necessary to offset risk in investing S&P HY bonds. We find a similar result when we look at the hedge ratios of gas and bonds. Consistent with our results pertaining to oil, hedge ratios for S&P IG and EN IG are negative while hedge ratios for S&P HY and EN HY are positive. However, hedge ratios of gas are much lower compared to the hedge ratios of oil indicating that a much smaller position in gas futures is needed to offset the risk in investing in IG and HY bonds.

Now we turn to explain results relating to hedging effectiveness (HE). In general, a higher HE index value manifest a higher hedging effectiveness. In the case of HY bonds' combination with oil and gas, we find that oil produces higher HE values compared to that of gas. This result holds for both S&P HY and EN HY bonds, and for alternative maturities (such as 1, 3, 6, and 12 months maturities). However, we find a mixed result in the case of IG bonds' combination with oil and gas. For instance, for 1-month maturity, gas offers higher hedging effectiveness compared to oil. This result is robust for both S&P IG and EN IG bonds. However, in the case of relatively longer maturity (for instance 12-month), oil generates a higher HE compared to that of gas. Overall, we find that oil is a more valuable hedge for HY bonds compared to the hedging benefits offered by gas. However, for IG bonds, oil (gas) is a valuable hedge for longer (shorter) maturity contracts.

As we consider HE for different contract maturities, we find that in the case of S&P IG – oil pair, highest hedging benefit can be obtained for 3-month maturity contracts as HE is the highest in this case. The HE coefficient decreases when the maturity increases to 6 and 12 months. However, in the case of S&P HY – oil, EN IG – oil, and EN HY – oil, the most effective hedge can be obtained for 12-month maturity contracts and HE decreases with the decline in maturity. We find somewhat similar result for gas – bond return pairs. This result in general indicates that longer maturity contracts provide more effective hedge compared to shorter maturity futures. This finding may arise as longer maturity futures exhibit low volatility compared to shorter maturity futures and longer maturity futures are less responsive to market-wide information. Our result is also consistent with Lee et al. (1987) and Shalen (1989), among others.

[INSERT TABLE 7 ABOUT HERE]

5. Conclusion

Despite a large literature focusing the interaction between stock market and energy market, the latter's dynamic interaction with bond market has received less attention from academic researchers. The main objective of this paper is two-fold. First, to examine the less-explored HY and IG bond markets and their dynamic dependence on oil and gas markets. Second, to investigate systemic risk contribution of energy markets on bond market investment. To achieve the first objective, we use TVOC model which can adequately capture the characteristics of cross-market dynamic dependence structure during extreme market conditions. With regard to the second objective, we quantify downside and upside VaRs for the bond and energy markets, and downside and upside CoVaRs and ΔCoVaRs for the bond markets conditioned on the energy markets. This analysis is specially designed to reflect risk in the bond market investment conditional on the extreme condition in the energy markets. We also statistically test the difference between these upside and downside measures to indicate asymmetric risk spillover during extreme upward and downward movement in the bond and energy markets.

We report several key findings in this paper. First, we find evidence of time-varying tail dependence between the bond and energy markets. The degree and direction of dependence between the two markets are found to be sensitive to market conditions. Second, while the tail risk in the HY bond market is higher compared to that in the IG bond market during the oil-crash period, the IG bond market exhibits higher downside risk than HY bond market in the whole sample period. Third, the bond returns are typically more sensitive to risk shocks in the oil market compared to the gas market. Third, we find a co-movement in both the negative and positive tails for all the bond and energy pairs implying that the energy market is not a good hedge against the tail risk of investment in the bond market. However, an asymmetric upside and downside risk spillover is found between the energy and bond markets.

Our results have important implications for devising portfolio strategies, setting financial market stabilizing policy and developing overall energy landscape. Given the evidence of downside risk spillover and changing risk profile over time, conventional portfolio performance measures (such as Sharpe ratio) may be misleading. Therefore, our results provide valuable information in terms of measuring risk-adjusted portfolio performance. An extreme downward and upward co-movement between the bond and energy markets indicates that the bond market is not immune to global energy shocks. Therefore, while investing in the bond market, to hedge against the downside (upside) risk spillover from the energy market, investor may consider taking a short (long) position in energy futures when they anticipate a downturn (upturn) in these markets. A time-varying dependence structure also indicates that investors can rotate their investment in the bond market based on their forecasting of states of the energy market.

Our findings have important implications for policymakers as well. The nature of dependence between these two markets must be incorporated in deriving energy policy because of its potential impact on stability of the bond market. Given that information on energy price changes can improve predictability of IG and HY bond prices, policymakers should be aware of the global oil price risk when designing bond market schemes. It is also worth mentioning that policymakers should take into account the asymmetric risk spillover between the markets in different conditions which should improve the policy response to shocks in the markets.

Future studies can further investigate the energy prices links with energy sector bond index while controlling for the overall bond market conditions. This could be done either by simply regressing energy sector bond index returns against overall bond market returns and applying the copula on regression residuals or by implementing a canonical vines (c-vines) copula. The use of energy volatility index, as a measure of investor sentiment or degree of risk aversion, could also reveal more information with regard to energy – bond markets linkage.

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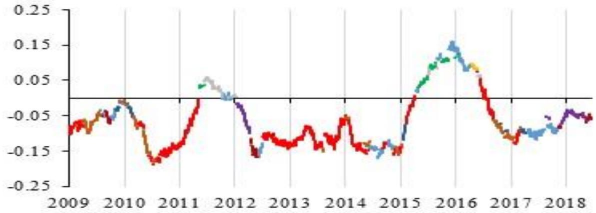
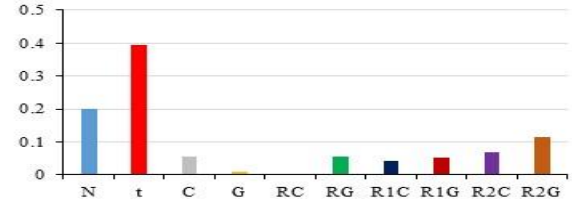
Figure 1: Time varying optimal copula estimates for oil and bond returns

Panel A: Time varying optimal copula estimates for oil and S&P HY bond returns

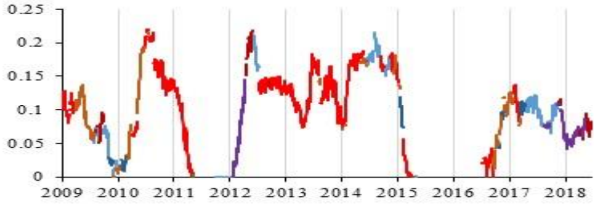
a). Kendall's tau



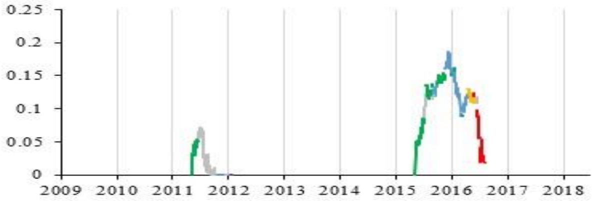
b). Histogram of best fitted copulas



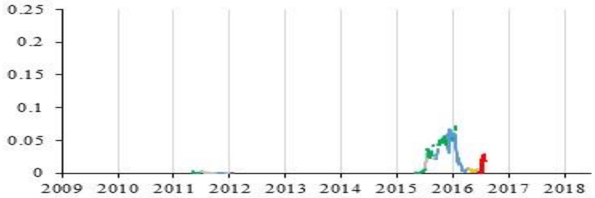
c). Lower – Upper (TDF)



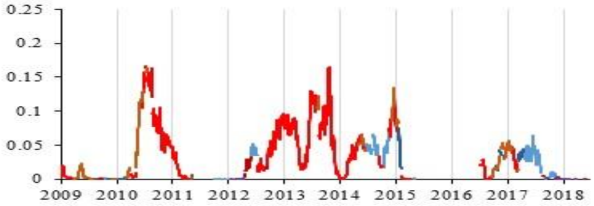
d). Upper – Upper (TDF)



e). Lower – Lower (TDF)

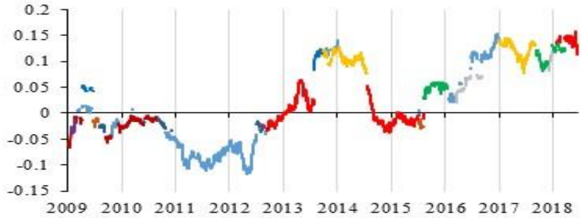


f). Upper – Lower (TDF)

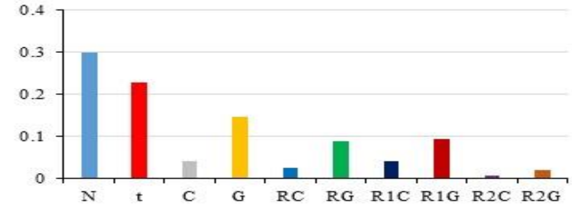


Panel B: Time varying optimal copula estimates for oil and S&P IG bond returns

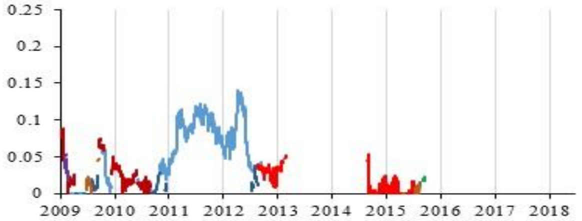
a). Kendall's tau



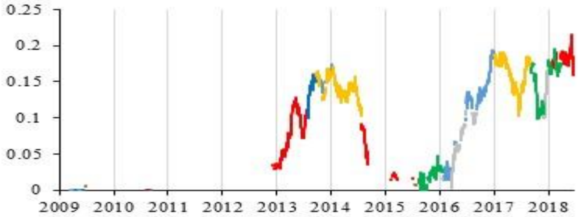
b). Histogram of best fitted copulas



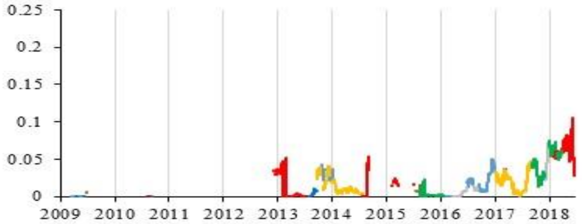
c). Lower – Upper (TDF)



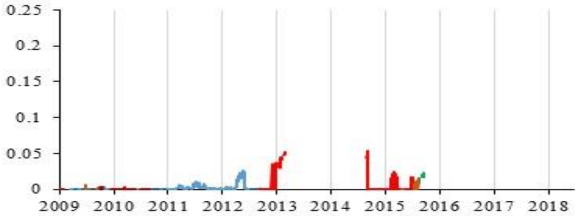
d). Upper – Upper (TDF)



e). Lower – Lower (TDF)



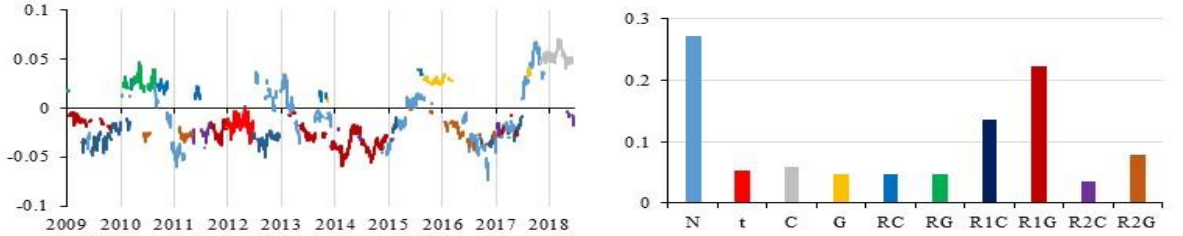
f). Upper – Lower (TDF)

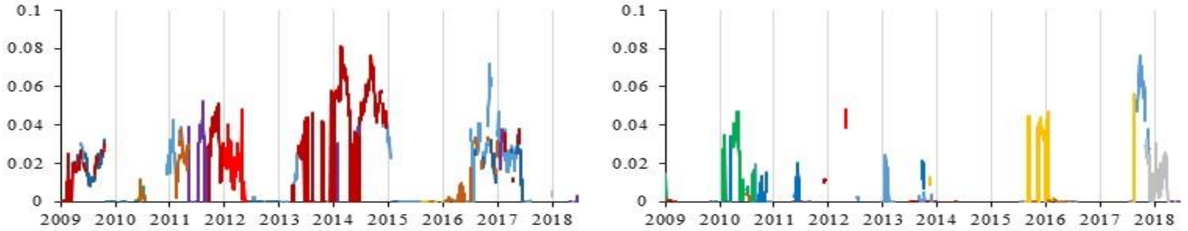


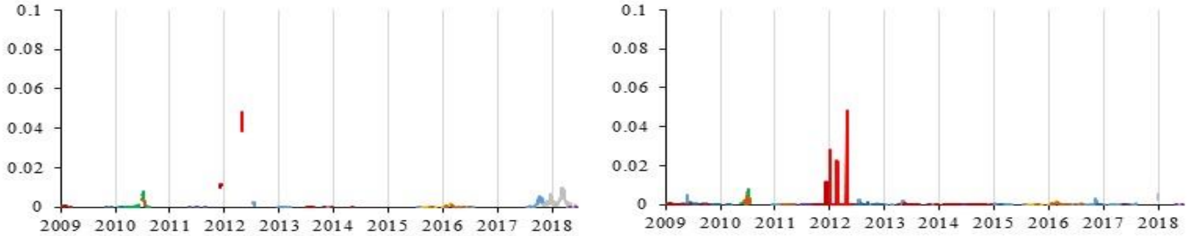
Note: In each Panel, (a) shows the Kendal's tau derived from the tail dependence parameters, (b) displays the percentages of best-fitted copula, and (c)-(f) present the time-varying tail dependence parameters. N: normal; t: student's t; C: Clayton; G: Gumbel; RC: 180° Rotated Clayton; RG: 180° Rotated Gumbel; R1C: 90° Rotated Clayton; R1G: 90° Rotated Gumbel; R2C: 270° Rotated Clayton; R2G: 270° Rotated Gumbel. TDF stands for tail-dependence function. Source: Authors' own estimation.

Figure 2: Time varying optimal copula estimates for gas and bond returns

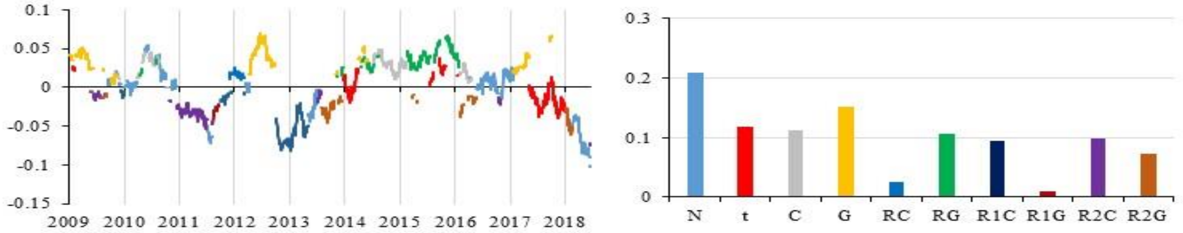
Panel A: Time varying optimal copula estimates for gas and S&P HY bond returns

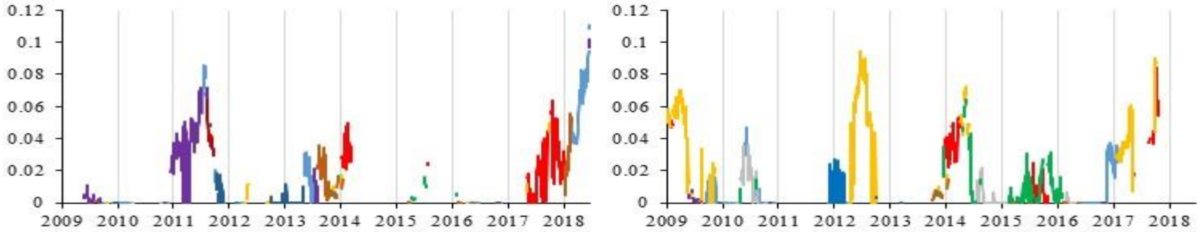

a). Kendall's tau **b)**. Histogram of best fitted copulas

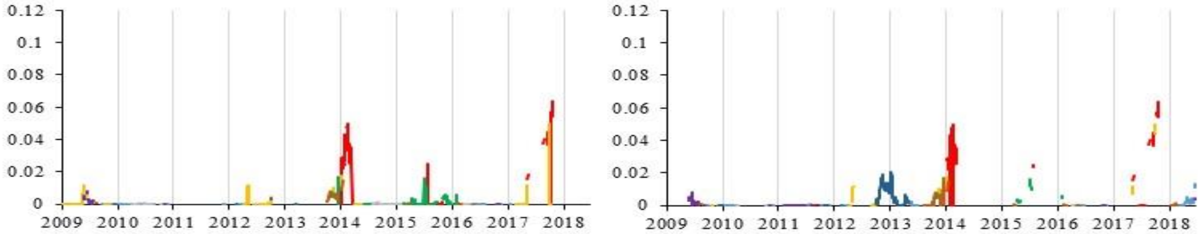

c). Lower – Upper (TDF) **d)**. Upper – Upper (TDF)


e). Lower – Lower (TDF) **f)**. Upper – Lower (TDF)

Panel B: Time varying optimal copula estimates for gas and S&P IG bond returns


a). Kendall's tau **b)**. Histogram of best fitted copulas


c). Lower – Upper (TDF) **d)**. Upper – Upper (TDF)


e). Lower – Lower (TDF) **f)**. Upper – Lower (TDF)

Note: See notes to Figure 1.

Table 1: Statistical properties of return series.

	S&P IG	S&P HY	EN IG	EN HY	Oil	Gas
Panel A: Descriptive statistics						
Mean	0.005	0.016	0.006	0.001	0.007	-0.031
Median	0.005	0.022	0.000	0.017	0.000	0.000
Maximum	2.016	1.952	2.305	2.797	13.136	26.771
Minimum	-1.658	-1.374	-2.537	-3.659	-13.065	-18.055
Std. Dev.	0.291	0.203	0.356	0.359	2.179	2.991
Skewness	-0.133	-0.197	-0.063	-1.378	0.032	0.576
Kurtosis	5.463	12.322	6.490	23.186	7.045	8.715
J-B	694.92***	9856.30***	1380.97***	46989.42***	1852.85***	3847.55***
ADF	-53.71***	-21.11***	-53.41***	-12.92***	-54.18***	-56.65***
PP	-53.73***	-33.95***	-53.45***	-32.51***	-54.19***	-56.65***
Q(20)	35.82*	967.87***	47.92***	1365.37***	24.70	54.43***
Q^2(20)	772.32***	844.19***	1112.14***	2461.66***	1476.32***	444.44***
ARCH (20)	278.10***	416.33***	412.63***	933.12***	428.33***	428.33***
# of Obs.	2717	2717	2717	2717	2717	2717
Panel B: Correlation matrix						
	S&P IG	S&P HY	EN IG	EN HY	Oil	Gas
S&P IG	1.000					
S&P HY	0.427***	1.000				
	(24.59)					
EN IG	0.911***	0.362***	1.000			
	(115.0)	(20.22)				
EN HY	0.190***	0.766***	0.247***	1.000		
	(10.07)	(62.02)	(13.30)			
Oil	-0.217***	0.179***	-0.212***	0.258***	1.000	
	(-11.59)	(9.499)	(-11.30)	(13.92)		
Gas	-0.029	0.015	-0.021	0.025	0.140***	1.000
	(-1.533)	(0.800)	(-1.076)	(1.318)	(7.348)	

Notes: S&P IG, S&P HY, EN IG, and EN HY respectively indicate daily returns of S&P500 investment-grade corporate bond index, S&P U.S high-yield corporate bond index, S&P500 investment-grade energy corporate bond index, and S&P U.S. high-yield energy corporate bond index. Oil and gas respectively represent oil and gas futures returns. J-B is the test statistic of Jarqe–Bera normality test. ADF and PP are the test statistics of the Augmented Dickey and Fuller (1979) and the Phillips and Perron (1988) unit root tests, respectively. Q(20) and Q^2(20) refer to the test statistics of the Ljung-Box test for autocorrelation of the returns and squared returns series, respectively. The ARCH-LM (20) test checks the null hypothesis of the presence ARCH effect in the return series. Panel B presents bivariate correlation coefficients and t-statistics within parenthesis. *,**,*** denote statistical significance at the 1%, 5%, 10% level, respectively. Source: Authors' own estimation.

Table 2: Marginal model estimations (VARX-GARCH with skewed t innovations).

	$R_t^{S\&P\ IG}$	$R_t^{S\&P\ HY}$	$R_t^{EN\ IG}$	$R_t^{EN\ HY}$	R_t^{Oil}	R_t^{Gas}
Panel A: VARX equation						
C	0.0019	0.0086***	0.0047	-0.0029		
	(0.0054)	(0.0033)	(0.0066)	(0.0056)		
$R_{t-1}^{S\&P\ IG}$	0.0745	0.0956***	0.0423	0.0194		
	(0.0512)	(0.0315)	(0.0627)	(0.0537)		
$R_{t-1}^{S\&P\ HY}$	0.2521***	0.4875***	0.1082***	0.2205***		
	(0.0488)	(0.0301)	(0.0598)	(0.0512)		
$R_{t-1}^{EN\ IG}$	-0.1655***	-0.1844***	-0.1195**	-0.1119***		
	(0.0394)	(0.0243)	(0.0483)	(0.0414)		
$R_{t-1}^{EN\ HY}$	0.0020	0.0227	0.1432***	0.4373***		
	(0.0258)	(0.0159)	(0.0316)	(0.0270)		
R_{t-1}^{Oil}	-0.0284***	0.0183***	-0.0338***	0.0460***		
	(0.0024)	(0.0015)	(0.0030)	(0.0026)		
R_{t-1}^{Gas}	0.0000	-0.0007	0.0013	-0.0004		
	(0.0018)	(0.0011)	(0.0022)	(0.0019)		
Panel B: Variance equation						
Constant	0.001***	0.001***	0.001***	0.001***	0.025**	0.141***
	(0.000)	(0.000)	(0.000)	(0.000)	(0.011)	(0.044)
ARCH (alpha)	0.033***	0.176***	0.040***	0.176***	0.059***	0.066***
	(0.003)	(0.040)	(0.003)	(0.033)	(0.011)	(0.011)
GARCH (beta)	0.958***	0.805***	0.955***	0.823***	0.937***	0.918***
	(0.001)	(0.043)	(0.001)	(0.030)	(0.011)	(0.013)
Asymmetry	0.929***	0.936***	0.931***	0.976***	0.902***	1.069***
	(0.021)	(0.024)	(0.024)	(0.021)	(0.021)	(0.027)
Tail	9.732***	6.024***	9.529***	5.108***	6.804***	6.668***
	(1.653)	(0.663)	(1.683)	(0.491)	(0.915)	(0.896)
Panel C: Diagnostic tests						
LL	-280.17	633.05	-651.119	691.7598	-5530.52	-6497.52
AIC	0.163	-1.133	0.483	-0.506	4.076	4.788
ARCH (20)	[0.468]	[0.481]	[0.621]	[0.317]	[0.643]	[0.793]
Q (20)	[0.480]	[0.835]	[0.681]	[0.522]	[0.780]	[0.780]
$Q^2(20)$	[0.441]	[0.582]	[0.426]	[0.698]	[0.898]	[0.512]
QQ2	[0.855]	[0.102]	[0.195]	[0.197]	[0.518]	[0.649]
McLeod-Li (20)	[0.807]	[0.108]	[0.420]	[0.335]	[0.718]	[0.275]
Hosking (20)	[0.846]	[0.216]	[0.510]	[0.919]	[0.792]	[0.185]
K-S Test	[0.608]	[0.171]	[0.245]	[0.215]	[0.480]	[0.014]

Notes: Panel A and Panel B respectively reports the Maximum Likelihood estimates and the standard deviations (within parenthesis) for the parameters of the marginal distribution model. Panel C presents the diagnostic test results for the goodness-of-fit of the marginal models. Optimum lag length is determined by AIC. ARCH represents the Engle LM test for the ARCH effect in the residuals; Q (20) and $Q^2(20)$ are the Ljung-Box statistics for testing serial correlation in the model residuals and squared residuals, respectively; and Hosking (1980) and McLeod and Li (1983) are the autocorrelation tests. These tests are conducted considering up to lag 20. K–S denotes the Kolmogorov–Smirnov test representing the adequacy of the Student-t distribution model. The *p* -values [in the square brackets] below 0.05 indicate the rejection of the null hypothesis. ∗∗∗, ∗∗ and ∗ represent statistical significance at the 1%, 5% and 10% levels, respectively. See notes to Table 1 for further details. Source: Authors' own estimation.

Table 3: Value at risk (VaR) of the bond returns

	Downside VaR	Upside VaR	*t*-statistic (Downside – upside)
Panel A: Whole sample period			
S&P HY	-0.241***	0.228***	-101.48***
	(0.118)	(0.111)	
S&P IG	-0.430***	0.406***	-287.80***
	(0.074)	(0.070)	
EN HY	-0.364***	0.356***	-60.17***
	(0.301)	(0.294)	
EN IG	-0.507***	0.480***	-185.80***
	(0.136)	(0.128)	
t-statistic (S&P HY - S&P IG)	67.34***	-67.28***	
t-statistic (S&P HY – EN HY)	18.93***	-20.30***	
t-statistic (EN HY - EN IG)	21.57***	-19.16***	
t-statistic (S&P IG - EN IG)	24.85***	-25.12***	
Panel B: Oil-crash subsample			
S&P HY	-0.284***	0.269***	-58.99***
	(0.141)	(0.133)	
S&P IG	-0.432***	0.408***	-223.60***
	(0.054)	(0.051)	
EN HY	-0.545***	0.533***	-36.49***
	(0.438)	(0.428)	
EN IG	-0.459***	0.435***	-149.93***
	(0.104)	(0.098)	
t-statistic (S&P HY - S&P IG)	30.04***	-30.01***	
t-statistic (S&P HY – EN HY)	17.93***	-18.61***	
t-statistic (EN HY - EN IG)	-6.19***	7.17***	
t-statistic (S&P IG - EN IG)	13.24***	-13.47***	
t-statistic (S&P HY: whole sample – oil-crash subsmple)	9.75***	-9.75***	
t-statistic (S&P IG: whole sample – oil-crash subsmple)	8.18***	-8.18***	
t-statistic (EN HY: whole sample – oil-crash subsmple)	13.73***	-13.73***	
t-statistic (EN IG: whole sample – oil-crash subsmple)	-1.17	1.17	

Note: This table presents the mean and standard deviation (in parenthesis) of the value-at-risk (VaR) metrics. The downside VaR and upside VaR are calculated under the quantile of 5% and 95%, respectively. The oil-crash subsample ranges from 15 July 2014 till 31 March 2017. ***, ** and * represent statistical significance at the 1%, 5% and 10% levels, respectively. See notes to Table 1 for further details. Source: Authors' own estimation.

Table 4: CoVaR between energy and bond returns

	Spillover from	Downside CoVaR	Upside CoVaR	t-statistic (Downside CoVaR – Upside CoVaR)
Panel A: Whole sample period				
S&P HY				
	Oil	-0.218*** (0.130)	0.207***(0.117)	-87.05***
	Gas	-0.166***(0.081)	0.286***(0.144)	-100.53***
	t-stat (oil – gas)	-16.63***	-21.05***	
S&P IG				
	Oil	-0.295***(0.051)	0.615***(0.195)	-201.51***
	Gas	-0.293***(0.052)	0.539***(0.141)	-225.69***
	t-stat (oil – gas)	-1.38*	15.62***	
EN HY				
	Oil	-0.256***(0.233)	0.501***(0.551)	-48.38***
	Gas	-0.260***(0.218)	0.458***(0.413)	-56.71***
	t-stat (oil – gas)	0.61	3.11***	
EN IG				
	Oil	-0.340***(0.083)	0.670***(0.236)	-179.44***
	Gas	-0.347***(0.091)	0.627***(0.181)	-186.65***
	t-stat (oil – gas)	2.67***	7.19***	
Panel B: Oil-crash subsample				
S&P HY				
	Oil	-0.282***(0.171)	0.255***(0.145)	-50.20***
	Gas	-0.195***(0.096)	0.333***(0.169)	-59.39***
	t-stat (oil – gas)	-13.38***	-10.05***	
S&P IG				
	Oil	-0.305***(0.039)	0.652***(0.148)	-140.90***
	Gas	-0.300***(0.041)	0.539***(0.100)	-159.42***
	t-stat (oil – gas)	-0.706	19.81***	
EN HY				
	Oil	-0.412***(0.333)	0.966***(0.695)	-38.77***
	Gas	-0.391***(0.317)	0.746***(0.599)	-36.76***
	t-stat (oil – gas)	-1.05	7.09***	
EN IG				
	Oil	-0.330***(0.075)	0.763***(0.164)	-120.87***
	Gas	-0.321***(0.077)	0.568***(0.125)	-134.04***
	t-stat (oil – gas)	-1.48*	21.55***	

Note: This table presents the mean and standard deviation (in parenthesis) of the conditional CoVaR metrics. The downside and upside CoVaRs are calculated under the quantile of 5% and 95%, respectively. The oil-crash subsample ranges from 15 July 2014 till 31 March 2017. ***, ** and * represent statistical significance at the 1%, 5% and 10% levels, respectively. See notes to Table 1 for further details. Source: Authors' own estimation.

Table 5 ΔCoVaR between energy and bond returns

	Spillover from	Downside ΔCoVaR	Upside ΔCoVaR	t-statistic (Downside ΔCoVaR – Upside ΔCoVaR)
Panel A: Whole sample period				
S&P HY				
	Oil	0.156***(0.307)	0.118***(0.286)	11.38***
	Gas	0.001***(0.019)	0.019***(0.128)	-7.73***
	t-stat (oil – gas)	24.87***	15.67***	
S&P IG				
	Oil	0.014***(0.032)	0.205***(0.297)	-34.48***
	Gas	0.008***(0.025)	0.083***(0.188)	-21.62***
	t-stat (oil – gas)	6.80***	17.23***	
EN HY				
	Oil	-0.004*(0.109)	0.153***(0.424)	-23.48***
	Gas	-0.002***(0.020)	0.008***(0.128)	-4.41***
	t-stat (oil – gas)	-0.81	16.19***	
EN IG				
	Oil	-0.001(0.059)	0.159***(0.346)	-121.11***
	Gas	0.009***(0.026)	0.082***(0.201)	-19.38***
	t-stat (oil – gas)	-7.91***	9.66***	
Panel B: Oil-crash subsample				
S&P HY				
	Oil	0.185***(0.330)	0.097***(0.261)	5.99***
	Gas	-0.006***(0.008)	0.007***(0.109)	-5.20***
	t-stat (oil – gas)	18.99***	11.13**	
S&P IG				
	Oil	0.035***(0.016)	0.254***(0.213)	-37.64***
	Gas	0.020***(0.018)	0.060***(0.120)	-11.59***
	t-stat (oil – gas)	4.26***	23.41***	
EN HY				
	Oil	0.044***(0.013)	0.402***(0.234)	-44.69***
	Gas	0.007***(0.008)	0.092***(0.153)	-14.03***
	t-stat (oil – gas)	42.78***	36.97***	
EN IG				
	Oil	0.039***(0.011)	0.310***(0.182)	-36.58***
	Gas	0.019***(0.015)	0.046***(0.085)	-7.72***
	t-stat (oil – gas)	11.24***	26.26***	

Notes: This table presents the mean and standard deviation (in parenthesis) of the ΔCoVaR metrics. The downside and upside ΔCoVaR are calculated under the quantile of 5% and 95%, respectively. The oil-crash subsample ranges from 15 July 2014 till 31 March 2017. ***, ** and * represent statistical significance at the 1%, 5% and 10% levels, respectively. See notes to Table 1 for further details. Source: Authors' own estimation.

Table 6: Hypothesis testing of VaR, CoVaR, ΔCoVaR

	$\boldsymbol{H_0: CoVaR(D) = VaR\ (D)}$ $\mathbf{H_1: CoVaR(D) < VaR\ (D)}$	$\mathbf{H_0: CoVaR(U) = VaR\ (U)}$ $\mathbf{H_1: CoVaR(U) > VaR\ (U)}$	$\mathbf{H_0: \frac{CoVaR}{VaR}(D) = \frac{CoVaR}{VaR}(U)}$ $\mathbf{H_1: \frac{CoVaR}{VaR}(D) > \frac{CoVaR}{VaR}(U)}$
	(1)	(2)	(3)
From Oil to S&P HY	0.766***	0.604***	1.000***
From Gas to S&P HY	0.205***	0.179***	0.100***
From Oil to S&P IG	0.590***	0.473***	0.847***
From Gas to S&P IG	0.379***	0.164***	0.756***
From Oil to EN HY	0.767***	0.459***	1.000***
From Gas to EN HY	0.448***	0.287***	1.000***
From Oil to EN IG	0.548***	0.379***	1.000***
From Gas to EN IG	0.325***	0.186***	0.998***

Notes: This table presents test statistics of the Kolmogorov–Smirnov (K-S) test. This test examines the validity of the null hypothesis involving symmetric risk spillover between bond and energy markets. VaR(D) and CoVaR(D) are the downside value-at-risk and conditional value-at-risk; VaR(U) and CoVaR(U) are the upside value-at-risk and conditional value-at-risk. *** represents statistical significance at the 1% level. See notes to Table 1 for further details. Source: Authors' own estimation.

Table 7: Summary statistics of hedge ratios and hedge effectiveness of oil and gas futures for HY and IG bonds.

		Oil				Gas			
	Maturity (Month)	Mean	Min	Max	HE	Mean	Min	Max	HE
S&P IG	1	-0.016	-0.026	-0.007	0.004	-0.003	-0.011	0.001	0.018
	3	-0.015	-0.027	-0.006	0.006	-0.002	-0.012	0.003	0.013
	6	-0.016	-0.028	-0.006	0.005	-0.004	-0.013	0.001	0.005
	12	-0.018	-0.046	-0.003	0.004	0.000	-0.011	0.016	-0.004
S&P HY	1	0.014	0.007	0.040	0.063	0.001	-0.002	0.008	-0.005
	3	0.016	0.008	0.042	0.071	0.002	-0.002	0.009	-0.009
	6	0.018	0.008	0.049	0.074	0.003	0.000	0.010	-0.002
	12	0.020	0.010	0.062	0.074	0.005	0.000	0.025	0.005
EN IG	1	-0.006	-0.020	0.002	-0.011	-0.003	-0.013	0.005	0.015
	3	-0.007	-0.023	0.002	-0.011	-0.001	-0.012	0.008	0.004
	6	-0.007	-0.023	0.002	-0.011	-0.002	-0.011	0.008	0.000
	12	-0.008	-0.024	0.003	-0.014	0.001	-0.007	0.008	-0.003
EN HY	1	0.056	0.027	0.142	0.226	0.004	0.000	0.017	-0.013
	3	0.059	0.030	0.157	0.238	0.005	0.000	0.020	-0.015
	6	0.059	0.030	0.157	0.238	0.005	0.000	0.020	-0.015
	12	0.064	0.030	0.214	0.253	0.013	0.003	0.036	0.007

Notes: Hedge ratios are calculated using volatility estimates from the marginal models as reported in table 2 and correlation estimates based on time-varying optimal copula framework. See notes to Table 1 for further details. Source: Authors' own estimation.