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Optimising tactical production planning with Mixed-Integer Linear Programming

How can the tactical production planning processes be optimised in terms of profitability in Outokumpu Chrome Oy using MILP -based model?

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Master's thesis in Industrial Management
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ABSTRACT:

This paper explores optimization in tactical production planning within resource-constrained environments, where medium- to short-term decisions are essential for operational efficiency. Optimizing production planning under these conditions can confer competitive advantages and improve resource utilization, supporting sustainability and green development goals. Through a theoretical framework, this research identifies effective methodologies suitable for case company for enhancing tactical production planning with a focus on profitability. Guided by this framework, a simulation model based on mixed-integer linear programming (MILP) was developed and tested using historical production data from a case company. Simulation tool was developed with Microsoft Excel, with OpenSolver add-on. Simulation tool was developed in collaboration with the case company and targeted to be a usable tool in production planning. MILP was chosen based on findings in theoretical framework and empirical analysis as simulation tool was developed further and new requirements emerged. The model aims to maximize profitability by improving allocation of production volumes to more profitable period. Tool will generate optimized production plans that increase profit per tonne produced, while still fulfilling certain production restrictions and requirements. Performance was assessed by comparing the simulation model's suggested plans to actual historical production plans over a three-year period. Results demonstrate the model's potential for profitability improvements, while the simulation tool requires further development to fully cover all characteristics of the production environment. Based on these findings, the paper provides recommendations for future research, along with suggestions for further enhancing the simulation tool.

TIIVISTELMÄ:

Tämän tutkielman tarkoituksena on perehtyä taktisen tuotannonsuunnittelun optimointiin resurssi rajoitteisessa ympäristössä, missä keskipitkän ja lyhyen aikavälin päätöksillä on merkittävä vaikutus operatiiviseen tehokkuuteen. Tällaisessa ympäristössä tuotannon suunnittelun optimointi voi tuoda kilpailuetua, parantaa resurssien käytön tehokkuutta, sekä tukea ympäristön kannalta kestävää kehitystä. Teoreettisen viitekehyksen avulla tutkielma esittelee yritykseen sovellettavia keinoja parantaa tuotannon suunnittelun tehokkuutta, etenkin tuottavuuden saralla. Teoreettiseen tutkimukseen pohjautuen, empiirisessä osiossa esitellään tuotannon suunnitteluun soveltuva simulaatiotyökalu, joka hyödyntää mixed-integer-pohjaista lineaarista optimointia. Työkalu kehitettiin käyttäen Microsoft Excel -ohjelmistoa, jossa lisäksi oli OpenSolver -lisäosa. Simulaatiotyökalu on kehitetty yhteistyössä yrityksen edustajien kanssa, tarkoituksena kehittää työkalu tuotannon suunnittelun tueksi. Mixed-integer optimointi valittiin mallin pohjaksi teoreettisen viitekehyksen, sekä empiirisen tutkimuksen pohjalta, kun huomattiin kriittiset tarpeet työkalun kehityksessä. Malli pyrkii parantamaan tonni -kohtaista katetta allokoimalla tarvittavat tuotantomäärät tuottavammille aikakausille, samalla kun tuotannon muut tarpeet ja rajoitteet täytetään. Simulaation suorituskykyä testattiin vertailemalla työkalun ehdottamia optimoituja tuotantosuunnitelmia yrityksen historiallisiin tuotantosuunnitelmiin ja niiden kannattavuuteen kolmen vuoden ajanjaksolla. Tulokset osoittavat, että esitetyllä mallilla on potentiaalia tuottavuuden parantamisen näkökulmasta. Esitetty simulaatiotyökalu itsessään tarvitsee lisää kehitystä, jotta se pystyisi huomioimaan kaikki tuotantoympäristölle ominaiset tekijät. Tutkielman tulosten perusteella annetaan ehdotukset jatkotutkimuksia varten, sekä kehitystarpeet suunnitteluun soveltuvan simulaatiotyökalun parantamiseksi.

KEYWORDS: production planning, aggregate planning, linear programming, mixed-integer programming, optimisation

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1 Introduction

1.1 Background

Optimisation is important aspect of business now and in the future. Efficient and continuously improving utilization of resources is crucial for business, but also for sustainable development and environment. Case organization Outokumpu is forerunner in sustainable stainless-steel manufacturing, which requires sustainable resources and optimised production processes. To be able to keep development ongoing, new ways to improve ways of working should be sought and studied. In this thesis, we are seeking ways to improve tactical production planning processes – ways to improve forecasting and production planning for more efficient utilisation of resources. Due to macroeconomic challenges and uncertainty in the world, the importance of preparation and planning has increased more than ever. Preparedness for different economic scenarios is indispensable for both business and environment.

I have previously studied process performance and measurement through performance metrics, which has deepened my interest in optimization. In this thesis, my goal is to explore quantitative methods to enhance efficiency and identify optimal solutions. I aim to develop models that are not only capable of delivering the best possible outcomes for the case organization but are also adaptable to various scenarios and industries.

1.2 Case study: Ferrochrome production

This case study focuses on the ferrochrome production process at Outokumpu, with product types limited to lumpy products manufactured during the 2021–2023 period. Ferrochrome production relies on various raw materials, electricity, and coke. Upgraded chromite lumpy ores and fine chromite concentrate are transported from the Outokumpu's Kemi Mine to the ferrochrome works. Lumpy ores are directly used in the smelting process, while fine concentrate is pelletized with coke and bentonite.

Ferrochrome is produced in submerged arc furnaces by smelting pellets, quartz, coke, and upgraded lumpy ores into liquid ferrochrome. The liquid ferrochrome is either transported to steel melt shop as raw material of stainless steel or casted and crushed into lumpy form for external sales or internal use. Coke, used as a reductant in the smelting process, is available in three grades: high, medium, and low, based on phosphorus concentration. Additionally, biocoke is employed for producing special "green" ferrochrome. The different grades of coke vary in price and influence product characteristics and sales pricing. Coke and electricity represent the largest cost components in ferrochrome production, playing a critical role in the optimization process.

In addition to raw materials coke, bentonite, quartz, fine concentrate, and lumpy ores, subcontracting costs are included as variable costs. Subcontracting costs include transportation expenses between the Kemi Mine and Ferrochrome plant, as well as contract-related fees. Raw material requirements vary between product types, and these differences are accounted for in the simulation tool described in subsequent chapters. All parameter prices fluctuate on a monthly basis, creating opportunities for optimization.

1.3 Purpose

Purpose of this study is to find how tactical production planning can be optimised in case organization based on literature review and empirical analysis. Aim is to develop simulation tool that helps in forecasting and planning future production and product portfolio by best profitability under certain restrictions set for production. When planning seasonal and complex optimisation problems, simulation is often most effective forecasting tool. Best strategies for different scenarios can be found in by rerunning simulated scenarios with computer program (Nahmias, 2005). Theoretical framework is gathered from field of operations and production management, production planning and optimisation. Simulation tool based on most suitable methodologies is presented with analysis in this study. Objective is to form simulation tool that has capabilities to calculate optimal monthly production level for three different production lines, with current resources and restrictions in mind.

1.4 Delimitations

This study will focus on planning the scope of ferrochrome production. Other functions such as procurement and sales are not in scope. However, if some information regarding empirical data collection requires collaboration with other functions, supporting information may be gathered, but solely to improve production planning. Inventories are not in scope. Optimisation will be done in four-month cycles, with scope of three years from 2021 to 2023.

1.5 Research questions and objectives

1. What methodologies and best practices have been shown to effectively optimise production planning processes in existing literature, particularly in terms of efficiency and profitability?
2. How can Linear Programming -based simulation be applied in the case organisation to enhance production efficiency and profitability?
3. What factors influence optimal profitability in the case organisation, and how should production planning consider risks related to production variables?
4. How well suggested simulation tool performs compared to historical production planning, and how it should be developed further?

1.6 Structure of the thesis

Chapters 2 and 3 will form the theoretical framework for this study. Chapter 4 will present the research methods used in this research. Chapter five will analyse empirical data with previously presented methodologies, and chapter six will present conclusions and recommendations for future research.

2 Operations management and production planning

2.1 Introduction to Operations management

Krajewski et al. (2021) gives following definition for operations management:

> *Operations management = the systematic design, direction, and control of processes that transform inputs into services and products for internal, as well as external, customers.* (Krajewski et al., 2021)

Another way to define operations management according to Kumar & Suresh (2009) is to extend production management to include services management, whereas production management is described as set of interrelated management activities that are related to produce certain product (Kumar & Suresh, 2009). Overall conception is, that operations management is crucial activity that defines how organisation can maintain its main processes that turn inputs into outputs.

Term process is activity or series of activities that turn inputs into outputs. Processes should have objectives, inputs, outputs, activities, and resources defined, and preferably in measurable form. Input defines what is required for the process, activity what and how should be done, and output what is required from the product or service (ISO 9000, 2024). Objective of a process is to create additional value through activities for internal or external stakeholders.

Process can be divided into manufacturing and service processes. In this study, we will focus on manufacturing processes. Manufacturing processes turn raw or already processed materials into products that can be stored, transported or further on refined in another manufacturing process. Distinctive characteristics for manufacturing process are e.g.:

- Physical, durable output that can be inventoried.
- Low customer interaction.
- Long response times.
- Capital intensive.

(Krajewski et al, 2021)

Processes can be also divided based on their role within the organization. In this case three categories for processes can be named: management processes, main or core processes, and support processes. Management processes play significant role in decision making and include often processes such as planning, budgeting, and management of support and core processes. Main or also known as core processes are the core business processes of the organization. For manufacturing organization, manufacturing processes are the core processes. Support processes are crucial to successfully maintain the operation of core processes but are not necessary the core business for the organization (von Rosing et al, 2014).

An operation is an activity where resources are used partially or completely in one or more processes (Krajewski et al, 2021). Organisations have often multiple processes and operations linked together before product or service is finished and received by customer. These linked chains of processes are called supply chains. Supply chains may have multiple companies involved and further on companies can have multiple supply chains. To operate successfully, supply chain management is activity that seeks to ensure efficient flow of resources, activities, processes, and services within organisation’s supply chains (Krajewski et al, 2021).

2.2 Production management

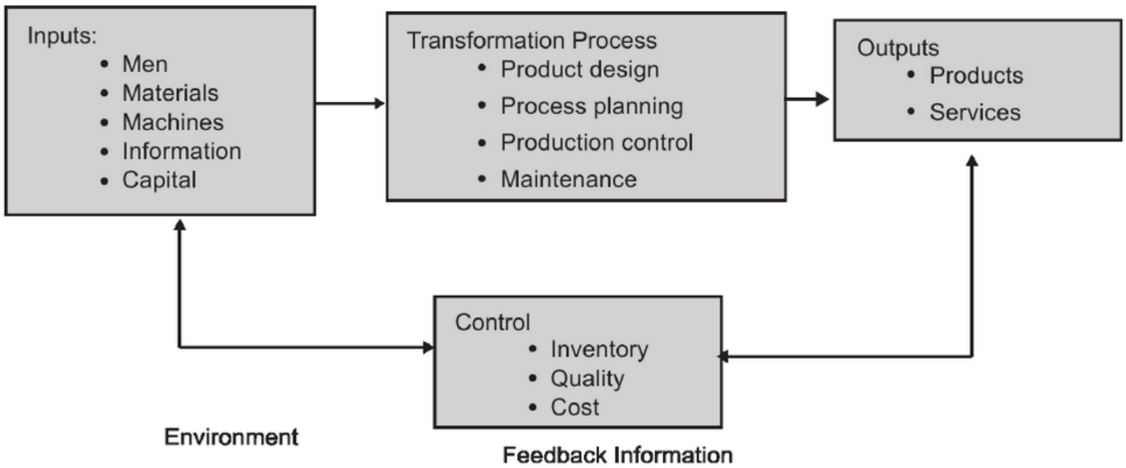
As mentioned in the operations management -chapter, production management is part of operations management focusing solely into production processes and all the activities related to production function. Production management focuses on creating products as planned, in schedule and with minimum cost. Kumar & Suresh (2009) in book *Operations Management* present four objectives of Production Management. These four objectives are, as presented in the book:

1. Right quality - the product should be fulfilling customer’s needs as well as technical characteristics and with reasonable cost.

2. Right quantity – amount of product produces should meet the demand as well as possible. Excess amounts of inventories should be avoided due to higher storage costs while product shortage will mean losses in potential sales.
3. Right time – optimal usage of resources requires right timing in production.
4. Right manufacturing cost – actual manufacturing costs should be as close to pre-established standard cost as possible. Meaning that actual production cost should not overrun previously planned budget.

   (Kumar & Suresh, 2009)

2.3 Production systems and processes

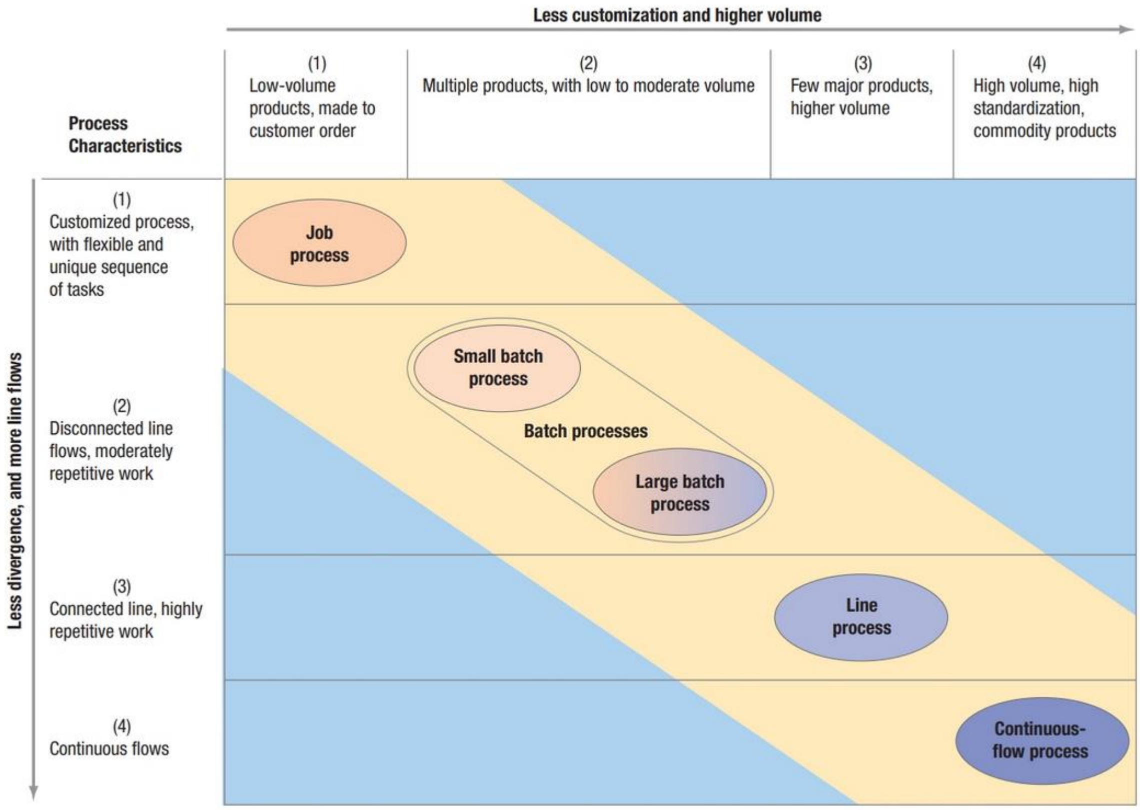


Picture 1 Schematic production system (Kumar & Suresh, 2009)

Production system is the entirety of production function, including production processes and material flow. Every production function has an objective, every system transforms inputs into desired outputs, is connected to other functions to ensure data flow, and includes and feedback functions to improve the production system (Kumar & Suresh, 2009).

Production systems are classified based on their production/operation volume and output/product variety. In the book by Kumar & Suresh, four classes of production systems are identified as, continuous production, mass production, batch production, and job-shop production. Continuous production has the largest production volume, and smallest variety in products while job-shop production has the largest variety in products while production volumes are lowest (Kumar & Suresh, 2009).

Krajewski et al. (2021) presents job process views for different manufacturing processes using same classification as Kumar & Suresh for production systems. These process types are determined with process choice structuring, based on level of customization, volume, divergency and line flows.



Picture 2 Process Matrix for Manufacturing processes (Krajewski, et al. (2021)

Above is the product-process matrix that represents different process characteristics based on different scenarios. Process type should be always determined to be best suitable for different case. Optimal process type makes best usage of resources and maximizes customer value. These are identical with previously mentioned production system types. Job process or job-shop production type is commonly used when level of customization and variety between products is high. Respectively volumes are low and often products are made to order. Process flows in job process can vary a lot, which requires competence and thorough process planning for every job. This type also requires often large space requirements and inventories which increase cost requirements (Kumar & Suresh, 2009) (Krajewski;Malhotra;& Ritzman, 2021). Batch production type or batch process is step towards mass production from job processes by now having multiple batches that are being made often regular intervals, but still having limited volume. Batch production utilizes machinery and resources better than job production, as it can use some line flow and larger quantities. Batch production and processes share same limitations as job production, if not as severe (Krajewski;Malhotra;& Ritzman, 2021) (Kumar & Suresh, 2009).

Mass production is associated with very large quantities of product that are manufactured in standardized production lines. Krajewski et al. (2021) uses line process -term for this process type. This type is between batch production and continuous production. Process steps are highly standardized. Production quantities are high, cycle times short and process inventories low compared to previous types of manufacturing. Production planning and controlling are relatively easy, material handling often automated. Due to high standardised and process step dependency on each other, changes can be challenging to implement and line is vulnerable to breakdowns (Krajewski;Malhotra;& Ritzman, 2021) (Kumar & Suresh, 2009).

Continuous production is completely standardised, and the facilities are built to follow certain sequence of production flow and operations. Products or items will be following standardized process flows, from beginning to finished product. This type of production has the highest level of volume, and lowest level of variability. Material handling is automated, unit cost lowest of the production types. Due to high level of standardization and possibly automation, changes and differentiations to products are difficult to implement if impossible (Kumar & Suresh, 2009) (Krajewski;Malhotra;& Ritzman, 2021). Case organization of this research utilizes continuous production processes in production. Therefore, it is important to forecast and plan production well ahead before possible implementation due to high number of preparations.

2.4 Production planning

Production planning can be divided into three categories based on their level of involvement in the production process and time frame. Characteristics and aspects can overlap between these dimensions, especially between strategical and tactical planning and vice versa.

2.4.1 Strategic planning

Highest level of planning with a time frame often multiple year, and at least one year. Strategical planning seeks to set high objectives for upcoming years and what kind of plan should be adhered to. Strategic planning involves both top-down and bottom-up reflection – high-level strategies based on market demand and set goals of higher management are put into practice, while operational findings are used to improve goals and strategy. Strategic planning gives the framework for tactical and operational planning to operate in (Slack & Lewis, Operations Strategy. Available from: VitalSource Bookshelf, (6th Edition), 2019).

Strategical planning strives to form objectives based on market demand and set the strategy of the organization. The successful strategy seeks to find those competitive factors that improve the competitiveness of the organization and what performance objectives help to measure these factors. Slack et al. (2010) gives examples of competitive factors that should be considered in strategic planning.

Figure 1 Competitive factors in Strategic planning (Slack et al. 2010)

Three factors pictured above determine how a change in performance can increase or decrease competitive benefits. Performance in order-winning factors is directly related to competitive advantage. With qualifying factors, change in competitiveness is big in certain performance levels, but minor after that level has been surpassed. Less important factors have a lower relation from performance improvements to competitiveness, and therefore as named, should not play as important role as the previous two factors (Slack;Chambers;& Johnston, Operations management (6th edition), 2010). Examples of order-winning factors can be quality, price, service etc. Qualifying factors can be certain qualifications or certifications, for example low carbon dioxide footprint of a product. Strategic production planning should determine goals and focus areas, where an organization can achieve the highest possible competitive efficiency.

2.4.2 Tactical / Aggregate planning

Capacity and inventory management are the focus areas in tactical production planning while implementing strategic objectives and production plans into practice. Optimising and improving tactical decisions lead to better resource utilization and improved competitiveness. Tactical planning connects strategical top-level plans to operative bottom-level details. Key themes for tactical planning are capacity planning, inventory management and master scheduling. Aggregate planning is another term often used for tactical planning (Waters, 2003). Nahmias (2005) describes aggregate planning as follows:

> *"To develop techniques for aggregating units of production and determining suitable production levels and workforce levels based on predicted demand for aggregate units" (Nahmias, 2005).*

The key problem related to aggregate planning is to seek the optimal level of quantity and mix of different products that manufacturing firms should have. Also known as macro production planning, the aggregate planning process should always start with demand or forecast. Forecasts can and will vary from actual demand, and therefore aggregate planning is based on the best possible assumption for future demand. Ability to adjust production planning to fluctuations in demand or forecast is one competitive objective that tactical & aggregate planning strives to improve. Changes in demand can lead to challenges regarding workforce – low demand may result to lay-offs while having too large workforce leads to increasing inventories and increased inventory-related costs. Having an optimal stable workforce can be seen as a second competitive objective. Third competitive objective is optimising the production plan where company can maximize profit from production, while limited by different constraints and capacity restrictions (Nahmias, 2005). This third competitive objective is the goal of this thesis case study.
Capacity management is another part of tactical production planning. Capacity is a term that describes the maximum rate of output from a production line, system, or process. Capacity should be adjusted and controlled based on the demand of the production and abilities of the company. Too high capacity can lead to excess inventories that further lead to high inventory costs and other liabilities. Too low capacity where demand surpasses supply, the company loses the opportunity for extra sales (Krajewski;Malhotra;& Ritzman, 2021). Capacity management consists of capacity planning and Constraint management. Capacity planning can be viewed as long-term planning while constraint management is more focused in short-term challenges (Krajewski;Malhotra;& Ritzman, 2021). Constraint management seeks to prevent and mitigate possible bottlenecks that can decrease or even stop normal production processes. However, constraints always exist, and therefore should be an important part of the planning process.

Capacity management can be part of both tactical and production planning. Large, long-term investments regarding expansions in production capacity and production strategy are common with strategic production planning. Mid-term capacity management such as optimising capacity usage and timing production are common with tactical planning.

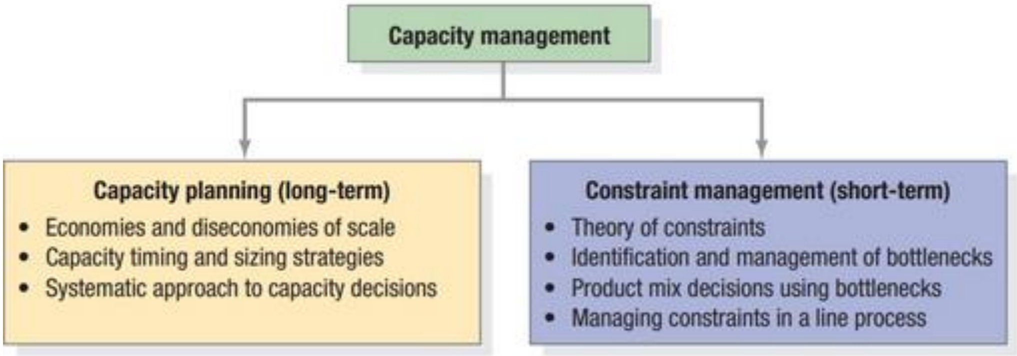


Figure 2 Capacity management (Krajewski et al. 2021)

2.4.3 Operational planning and control

Operational production planning is about planning and controlling production in a short time frame from hours and days to months. Planning and controlling activities are done with different time horizons in scope, which affects to the activities.

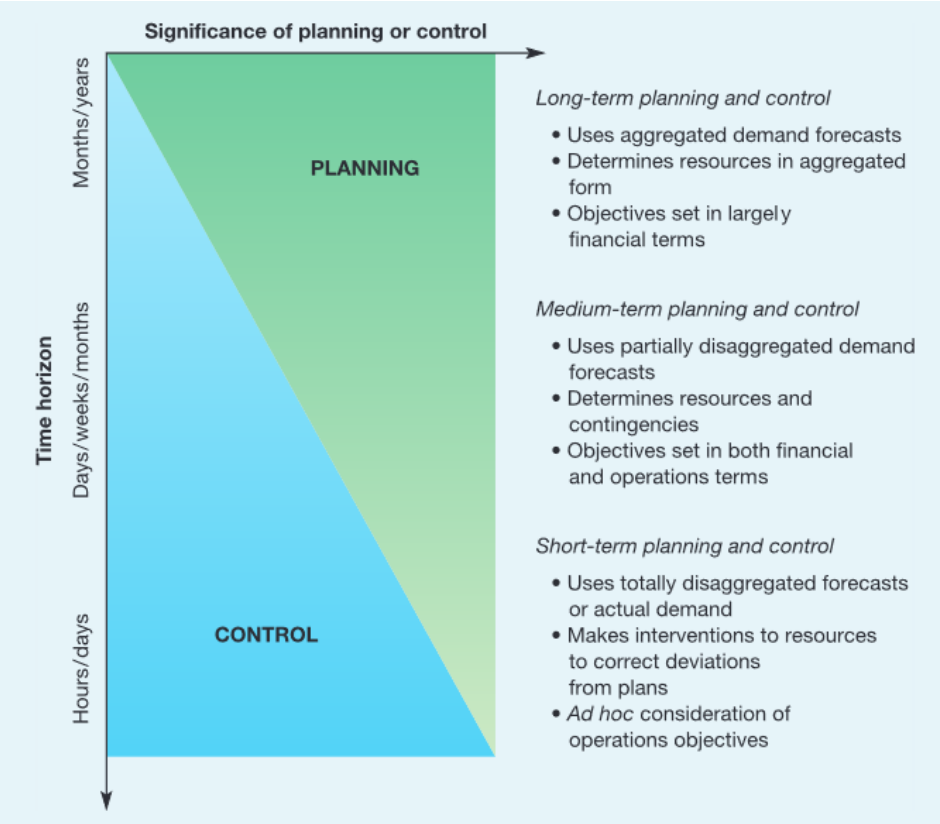


Figure 3 Significance of planning or control (Nigel et al. 2010)

The figure above presents the relation of time horizon with planning & control. In operative planning, importance of control plays in major role, since the activities and results are viewed in shorter time frame (Slack;Chambers;& Johnston, Operations management (6th edition), 2010). Operative production planning focuses on daily production activities, implementing production plans and schedules into practice.

2.5 Operations planning and scheduling

Sales and operations plan (S&OP) seeks to find optimal use of resources that ensure balance between supply and demand. It consists of sales plan that seeks to forecast future demand for products or services and operations plan that consists of constraint management and operations strategy. S&OP is adjusted based on long term business strategy and shorter business plan that should be set at least for the oncoming year. S&OP plan determines product mixes, inventory levels, and production capacities and other relevant information that will build the framework for production (Krajewski;Malhotra;& Ritzman, 2021).

Krajewski et al. (2021) presents Sales and operations planning as the first level of operations planning and scheduling. Second level is resource planning, which specifies use of resources and capabilities to as far as product level descriptions of required materials and capacities. Where S&OP plan has determined what product categories production should focus on based on demand forecasts and other drivers, resource planning seeks to present requirements of resources, production times, and other needs from whole product categories to individual products.

Third level is the scheduling itself, which divides resources plan into operational processes and activities that must take place efficiently and meet the requirements set in S&OP plan. Scheduling seeks to find optimal level of production, which takes constraints and bottlenecks into account.

2.6 Optimising productivity

In this chapter, objective is to investigate, what optimising production planning means, and what are currently known methodologies that aim to improve production planning processes. This paper seeks to find methods that help especially tactical / aggregate planning, while at least partly excluding role of workforce and inventories. Goal is to optimise input to output ratio and use of resources in production planning.

> *Productivity = The value of outputs (services and products) produced divided by the values of input resources (wages, cost of equipment, etc.)*

(Krajewski et al, 2021)

The core definition of productivity is mentioned in the quote above, but productivity can be measured in many ways depending on the situation and setting. As an example, units per certain time frame or value of certain number of inputs are ways to calculate productivity.

Based on the book *Operations Management* by Kumar & Suresh, productivity can be viewed as an ability to utilize resources in production. Productivity can be improved in three ways that are:

1. Improving control of inputs.
2. Improving process efficiency.
3. Improving used technology.

   (Kumar & Suresh, 2009)

When seeking opportunities for productivity improvement, productivity analysis should be conducted first. Four different types of productivity analysis can be utilized. Trend analysis compares organizational productivity over a certain period. Horizontal analysis compares case organization with relevant peer companies. Vertical analysis uses comparison with other industries and organizations of different scales. The last productivity analysis type is budgetary analysis, which uses previously mentioned analysis types in setting certain benchmarks or goals for productivity. After setting goals, budgetary analysis seeks suitable ways and strategies that help to achieve these goals (Kumar & Suresh, 2009). In this case study, we are seeking efficiency in production planning, meaning the objective is to maximize profit for finished output under certain restrictions, resources, and constraints.

2.7 Multi-product multi-period optimisation problem

The case optimisation problem presented in this paper involves a traditional multi-product, multi-period aggregate production plan, where we seek to maximize profit under certain constraints and capacities. Multi-product multi-period (MPMP) production is a process where multiple products are produced over a certain period. As in aggregate production planning theory suggests, the objective is to balance capacity to match demand, with a different mix of products (Vollman;Berry;Whybark;& Jacobs, 2005).

Optimisation will be based on demand. Demand is based on Master Production Scheduling. Master Production Schedule (MPS) determines the production plan for different products over a time frame. Below is the aggregate plan and MPS example, published in *Operations Management* by Kumar et al. (2009).

Aggregate Plan

Month	J	F	M	A	M	J	J	A	S
Number of motors	40	25	55	30	30	50	30	60	40

Master Schedule

Month	J	F	M	A	M	J	J	A	S
AC motors									
5hp	15	–	30	–	–	30	–	–	10
25hp	20	25	25	15	15	15	20	30	20
DC motors									
20hp	–	–	–	–	–	–	10	10	–
WR motors									
10hp	5	–	–	15	15	5	–	20	10

Figure 4 Aggregate plan and Master Production Schedule (Kumar et al. 2009)

In MPMP production, inventory and lot sizing will affect to productivity and optimisation. Lot size or production capacity will determine the size of batches the company can provide.

Optimising lot size and inventory helps to reduce inventory and holding costs, while providing enough supply to fulfil market demand. Multi-period optimisation seeks to find correct lot sizes and inventory levels while considering optimal safety stocks and fluctuating market environment (Nahmias, 2005). This paper acknowledges the importance of lot sizing and inventory management; however, these aspects are beyond the scope of this study as they are more relevant to strategic-level production planning.

Variability and Uncertainty management is an area in MPMP production, which focuses on fluctuations in market demand and variation in production processes. Variability in production or demand forecast is common and can cause challenges for companies due to cost increases or missed supply opportunities. These risks can be mitigated with techniques such as safety stock calculations, improving flexibility and adjustment of production factors (Hopp & Spearman, 2008).

As previously mentioned, Scheduling and Control play a role in multi-product multi-period problems as well. While optimising production, product mix with different demands, requirements and production times must be considered. Depending on the setting and situation, different algorithms and methods can be used to optimise the best possible fit for scheduling & controlling optimisation problems. Hillier & Lieberman suggest Linear Programming as a tool for formulating these optimisation problems. Different spreadsheet models can be used to model these problems, Excel Solver for example (Hillier & Lieberman, Introduction to Operations Research (9th edition), 2010).

2.8 Literature review of methodologies for optimising

A study by Nam & Logendran (1992) conducted a literature review including 140 journal articles and 14 books to form a summary of best methods used in aggregate production planning. Most mentioned methodologies included Linear Programming (LP), Goal Programming (GP), Linear Decision Rule (LDR) and Simulation models (Nam & Logendran, 1992).

2.8.1 Linear Programming

Linear programming is a widely used mathematical approach for solving production allocation problems in environments with limited resources and defined constraints (Metei & Jain, 2019). This methodology is particularly well-suited for organizations that produce or refine specific products from a defined set of input resources. The ability to optimize the transformation of inputs into outputs provides a competitive advantage in dynamic and challenging markets. The case organization serves as an example of a company that refines raw materials into a targeted mix of products, while operating under constraints such as limited availability of critical resources like coke and high resource costs. These challenges make profitable production across the entire production plan more complex and underscore the need for optimization.

Restrictions and constraints are set in forms of equations and inequalities, while all of them being in linear relationship with each other. Linear programming problem can either minimize or maximize objective function while simultaneously fulfilling set restrictions and constraints. When mathematically seeking to solve maximization or minimization problem under certain restrictions, it is called optimisation problem (Metei & Jain, 2019). Linear programming is effective tool when optimising allocation problems with multiple products and periods due to its characteristics of mathematically seeking minimization or maximization solutions while satisfying often complex constraints and restrictions (Hillier & Lieberman, Introduction to Operations Research (9th edition), 2010). Linear programming uses simplex method and can solve large problems efficiently due to linearity of variables (Nahmias, 2005). Linear programming in simplest for mused linear variables. However, sometimes integer or binary variables are required to simulate more complicated aspects. When linear programming model involves multiple different variables, model is called mixed-integer linear programming, MILP. More of linear programming in chapter 4.

2.8.2 Dynamic Programming

Dynamic programming is method used to solve complex, multi-stage decision problems where the issue at hand is divided into different sub-problems. Each sub-problem is solved either by top-down or bottom-up, depending on how the solving process will be conducted. Top-down technique starts the solving process with the main problem and moves to sub-problems if they are encountered. Bottom-up technique is the opposite – sub-problems are solved starting from the smallest and moving into bigger and bigger sub-problems until the main issue is solved (Sniedovich, 2010). Dynamic programming can be efficient tool for small but complex programming problems with ability to restrict redundant work and recursive solutions. Splitting main issue to multiple sub-problems, may lead to curse of dimensionality, where solving main issue becomes overly complicated to solve efficiently due large number of sub-problems.

2.8.3 Goal programming

Goal programming, GP is a multi-objective programming technique based on linear programming, involving multiple objectives. Where linear programming focuses on solving one objective function, goal programming seeks to satisfy multiple objectives. Concept of GP is to find solution where multiple, often conflicting objectives, are satisfied based on pre-set goals, as well as possible. Goal programming seeks to find feasible solutions based on goals, rather than seeking absolute maximum or minimum for objective functions. Different GP techniques include e.g. pareto efficiency detection & realisation, normalisation and redundancy checking (Mehrdad;Jones;& Romero, 1998). Goal programming algorithms are based on two different methods, weights method and pre-emptive method. The weights method is using single objective function which is the weighted sum of the goals of sub-problems that algorithm seeks to solve. The pre-emptive method optimises one goal at a time, in order of their priority to the main issue (Taha, 2017, p. 349-355). GP is one of the most popular techniques in multi-criteria decision-making category, but not always the most suitable as on its own. GP is best suitable for production planning when optimisation includes many conflicting objectives that require balance, e.g. when seeking to balance production line volumes, prioritizing production requirements or minimising deviations. Goal programming involves prioritizing with weights which can increase subjectivity. GP can be used to seek best all-around solution to scenario where achieving every goal is not achievable.

2.8.4 Uncertainty and fuzziness in programming optimisation models DONE

In real-world decision-making, goals and parameters are often not clear-cut or deterministic, which brings the concept of "fuzziness" into relevance. Fuzziness in optimization refers to handling uncertainties that arise from imprecise data or ambiguous objectives, making it particularly important for goal programming (GP). Traditional goal programming, which operates based on well-defined goals, can struggle with uncertainties, leading to the development of Fuzzy Goal Programming (FGP). FGP is a variant of GP that incorporates fuzzy sets, membership functions, and weights, allowing for a broader range of acceptable solutions. As highlighted in an overview by Aouni et al. (2009), FGP requires compromises from the decision-maker, as it cannot simultaneously optimize all conflicting objectives. Instead, it provides the most satisfying solution based on the decision-maker’s preferences, though it may not always achieve pareto efficiency (Aouni, Martel, & Hassaine, 2009). Possibilistic Linear Programming (PLP) is another approach to fuzzy optimization, specifically designed to manage uncertainties by using fuzzy values instead of precise data. Unlike deterministic linear programming, which assumes certain values for inputs, PLP represents uncertain parameters, like demand forecasts or production capacities, as fuzzy numbers, or ranges rather than single, fixed values. This approach is beneficial in complex environments where key data is imprecise or fluctuates due to unpredictable factors. PLP is rooted in fuzzy set theory, as developed by (Zadeh, 1978), and possibilistic theory, which allows uncertain values to be represented through possibility distributions. In applications such as production planning, PLP has been shown to be effective for handling fuzzy parameters like forecast demand, capacity, and cost, which are rarely known with certainty (Wang & Liang, 2005). In this framework, PLP models often aim to minimize costs while balancing other goals, like minimizing inventory and labour fluctuations. The model seeks a compromise solution that minimizes the most likely total costs, maximizes the chance of achieving lower costs, and reduces the risk of higher costs (Inuiguchi & Ramik, 2000) (Wang & Liang, 2005).

The ability to incorporate fuzziness allows PLP and FGP models to be more flexible and adaptable, enabling decision-makers to achieve practical, robust solutions under uncertain conditions. In contrast to deterministic models, which may offer mathematically optimal solutions based on exact inputs, fuzzy optimization approaches like PLP provide a structured way to address imprecision in real-world scenarios. These models require decision-makers to interpret and adjust fuzzy constraints and objectives, balancing between flexibility and accuracy. Ultimately, PLP and FGP offer robust frameworks for optimizing in environments where uncertainty and variability are inevitable, enabling more realistic and adaptable decision-making.

2.8.5 Heuristic programming

Heuristic programming is employed when other algorithms are not applicable to solve complex problems or would require excessive number of resources. Heuristic programming uses simple, practical direct search techniques to find approximate solutions for problems. Solutions are not optimal but should be reasonable for more efficient ways of simulating. Heuristic techniques encompass methods such as greedy algorithms, local search, and metaheuristics. Greedy algorithms are simple, rule-based approaches that make the locally optimal choice at each step in hopes of finding a global optimum. However, they do not always guarantee the best solution, especially in complex problems. Local search algorithms begin with a random feasible solution and iteratively improve it by making small changes within the neighbourhood of the current solution. The process continues until no further improvements can be made. Local search is effective for finding good solutions quickly but has the limitation of potentially getting stuck in local optima, as it only focuses on incremental improvements within a small region of the solution space.

Metaheuristics provide a higher-level framework for escaping local optima and exploring a broader solution space. They begin from one or several locally optimal points and allow for controlled exploration of the solution space, sometimes accepting slightly worse solutions temporarily to explore new regions. This helps avoid the trap of local optima. For example, Tabu Search is a widely used metaheuristic that tracks solutions already explored and labels them as "tabu" to prevent revisiting them. This avoids cycling back to previously discarded solutions and ensures that the search moves forward (Taha, 2017, p. 397-403).

According to Hamdy Taha's *Operations Research: An Introduction*, these methods are critical for solving complex, large-scale optimization problems where exact methods like Linear Programming or Mixed-Integer Programming become computationally infeasible. Metaheuristics, such as Simulated Annealing and Genetic Algorithms, extend the basic concepts of local search by introducing randomness and diversification strategies to avoid local traps and search the global solution space more effectively. These techniques are particularly valuable in production planning problems, where multiple products, resources, and constraints need to be managed simultaneously across large time horizons (Taha, 2017, p. 404-413).

2.8.6 Economic Production Quantity, EPQ and Economic Order Quantity, EOQ

To further add methodologies to aggregate planning and production planning theme, even though inventory control is not in main focus in this thesis, Economic Production Quantity, EPQ and Economic Order Quantity, EOQ are used to optimise production levels and holding stock.

Economic Order Quantity, EOQ is used to optimise costs related to holding stock. EOQ modelling involves seeking balance between holding larger inventories with less frequent replenishments or vice versa. Total cost is calculated as addition of holding cost and ordering cost. Following formulas are taken from book *Operations Management (6th Ed)* by Nigel Slack (2010).

Holding costs = holding cost per unit * average inventory

$$Holding\ costs = C_h * \frac{Q}{2}$$

Ordering costs = Ordering costs * Demand during period

$$Ordering\ costs = C_o * \frac{D}{Q}$$

$$Total\ cost\ C_i = \frac{C_h Q}{2} + \frac{C_o D}{Q}$$

Total cost formula can be used in table form to seek optimal order size for scenario. Alternatively, one can use EOQ formula to seek needed solution.

EOQ formula:

$$EOQ = \sqrt{\frac{2C_o D}{C_h}}$$

Where:

C_o= order cost per unit, D = demand rate during period, C_h = holding cost per unit

Time between orders would now be determined as $\frac{EOQ}{D}$ and order frequency as $\frac{D}{EOQ}$ (Slack et al. 2010 p. 349-354). EOQ is closely related to production techniques which determines which values are most suitable for certain scenarios. Just in time, JIT as an example is philosophy that seeks to minimise inventories and avoiding excess amounts of work-in-process inventories (Nahmias, 2005). When implementing EOQ, it should be always thoroughly planned to suit strategical and tactical production needs and suitable techniques.

To suit better needs of production planning in terms of optimising production lot sizes, Economical Production Quantity, EPQ is needed. EPQ or depending on the source often referred as a lot size model, is used to determine optimal method for production *n* number of products. EPQ model as follows:

$$EPQ = \sqrt{\frac{2C_s D}{C_h} * \frac{P}{P - D}}$$

Where:

C_o= order cost per unit, D = demand rate during period, C_h = holding cost per unit, C_s = setup cost per production run, P = production rate during period, , $P - D$ = rate of inventory build-up (Nahmias, 2005). EOQ and EPQ are essential techniques for optimizing inventory and production levels. While they are valuable for modelling lot sizes and scaling production, they are often most effective when used in combination with other complementary methodologies.

3 Linear programming as optimisation tool

3.1 Introduction and basics of linear programming

Linear programming is mathematical method that is used when goal is to maximize or minimise a linear function with *n* variables subject to *m* constraints. Linear programming is versatile method that can be used to solve broad number of real problems that include e.g. scheduling of personnel, blending problems, inventory control and production planning, distribution and logistics problems, and assignment problems (Nahmias, 2005).
Linear programming models are presented as linear programming problems (LPP). General LPP can be expressed as shown below (Metei & Jain, 2019).

$$Maximize\ or\ Minimize\ Z = c_1x_1 + c_2x_2 + \cdots + c_nx_n \text{ (Linear Objective function)} \quad (1)$$

Subject to the constraints

$$a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n (\leq, =, \geq) b_1 \quad (2)$$

$$a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n (\leq, =, \geq) b_2 \quad (3)$$

.
.
.

$$a_{i1}x_1 + a_{i2}x_2 + \cdots + a_{in}x_n (\leq, =, \geq) b_i \quad (4)$$

.
.
.

$$a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n (\leq, =, \geq) b_m \quad (5)$$

$$x_1, x_2, \ldots, x_n \geq 0 \text{ (Non-negative restriction)} \quad (6)$$

In the formula, row (1) presents the objective function, and variables a_{ij}, b_i, c_j where $i = 1, 2, \ldots, m, j = 1,2, \ldots, n$ are linear constraints. $c_1, c_2, \ldots, c_n$ present the cost coefficients or unit profits of $x_1, x_2, \ldots\ x_n$ -variables. Constraints, linear equalities or inequalities (rows 2-5) restrict the optimisation with realities that can be as an example, available resources. Row (6) presents non-negative restriction, which checks that all values are positive. (Metei & Jain, 2019). To simplify, each inequality or equality formula is i^{th} functional constraint where b_i is the maximum amount of available resource, $a_{ij}$is coefficient of x_j and presents the amount of used resource by each unit of activity j. As an example, *Z* can present maximum profit, which is led from amount of production units x_n times profit per unit, c_n. If we have three products with profit per unit (c_i) being 1 for product x_1, 2 for product x_2 and 2.5 for product x_3, linear objective function would be as follows.:

$$Maximize\ Z = x_1 + 2x_2 + 2.5x_3 \tag{7}$$

Now, to produce products x_i, resources such as coke and concentrate are required. Example of constraints for coke and concentrate would be as follows. If coke consumption for finished unit of product x_1 is 0.3, for product x_2 is 0.25 and for product x_3 is 0.40, while maximum amount of coke available is 500, constraint can be formalized as below (8):

$$0.3x_1 + 0.25x_2 + 0.4x_3 \leq 500 \tag{8}$$

Constraint for concentrate would follow same formulation. If we assume that consumption for concentrate per finished unit of product x_1 is 0.34, for product x_2 is 0.5 and for product x_3 is 0.6, formula would be as follows: (9):

$$0.34x_1 + 0.5x_2 + 0.6x_3 \leq 1000 \tag{9}$$

Complete LP model of the previous examples would be as follows (10):

$$Maximize\ Z = x_1 + 2x_2 + 2.5x_3 \tag{10}$$

Subject to

$$0.3x_1 + 0.25x_2 + 0.4x_3 \leq 500$$
$$0.34x_1 + 0.5x_2 + 0.6x_3 \leq 1000$$
$$x_1, x_2, x_3 \geq 0$$

Examples above are simplified for the sake of introducing basics of linear programming. Real-world cases require often more complex models that can include large sets of constraints and variables. However, these principles still hold.

When implementing linear programming methods into real-world scenarios, certain assumptions must be made. According to Metei & Jain (2019), following assumptions are when implementing linear programming approach.

1. Certainty – all parameters must be known before simulation can be done. This includes objective function, constraints and coefficients and resources.
2. Linearity – all variables of the LP model must have exponent of one. Meaning that relationship between all variables must be linear. To clarify linearity aspect, Metei & Jain presents three properties for linearity.
   a. Proportionality of objective function and decision variable values. Value of objective function is directly related to objective functions and vice versa.
   b. Relationship of values is additive. Value of objective function variable is additive to constraint variables. If objective function variable is 2.5 x_3 and $x_1 = 2$, contribution is 5.
   c. Divisibility. In theory, variables can have decimal or fractional continuous values, but this is not often practical in real cases. When implementing to real-world case, variables should be rounded to integer values, or

whichever is most relevant for scenario. Nevertheless, it should be acknowledged that rounding may have impact to optimality.
(Metei & Jain, 2019).

The example (11) presented a basic version of product mix resource allocation problem, where we want to maximise, the objective function representing sales profit, by choosing what products and how much should be produced while restricted by the resources available. Available resources are presented on the right-hand side of the equation symbol, or b_m. Left-hand side, $a_{mn}x_n$, of the equation symbol presents the amount of usage of that resource (Hiller & Liebermann, 2015, p.28-29).

3.2 Mixed-integer Linear Programming

Mixed-integer linear programming, MILP, is variant of linear programming apart from including at least one integer variable. Integer variables are implemented to traditional LP-model with additional constraints that accept only integer values (Hillier & Liebermann, 2015, p. 474-476). Integer variables are often required when simulating problems that involve workforce, production, or binary variables that can only be expressed as integers.
To acknowledge, MILP -models include integer and continuous variables. However, pure integer programming can be used where only integer variables are allowed. When only binary integer variables are allowed, binary integer programming, BIP, is used where variables can only get values 1 or 0. Binary integer programming can be used e.g. when programming mutually exclusive investment decisions where investment must meet requirements to be carried out (Hillier & Liebermann, 2015, p. 474-476).
Below is example of Mixed integer variable model. Model will be used to check that production line produces only one product per day in this scenario. j presents days, and i presents if product is produced. We expect that production of one product takes one day, and therefore only one product can be produced per day. We use the previous LP-model (11) with modifications that optimise production for three-day period.

$$Maximize\ Z = \sum_{j=1}^{3} (x_{1j} + 2x_{2j} + 2.5x_{3j}) \tag{12}$$

Subject to

$$x_{1j} + x_{2j} + x_{3j} = 1 for\ j = 1,2,3$$
$$0.3x_1 + 0.25x_2 + 0.4x_3 \leq 500\ for\ j = 1,2,3$$
$$0.34x_1 + 0.5x_2 + 0.6x_3 \leq 1000\ for\ j = 1,2,3$$

and

$x_{ij} \in \{0,1\}\ for\ all\ i = 1,2,3\ and\ j = 1,2,3$

When working with mixed-integer models, convexity becomes a critical consideration, as it significantly influences the ability to efficiently optimize solutions. Basic linear programming -models are often convex models, meaning that all constraints in model are linear and most distinctly, all points within the feasibility region can be connected to each other with straight lines. Non-convex models have non-linear constraints or objectives, such as binary constraints that lead to disjoint feasible region. Figure 5 illustrates examples of convex and non-convex functions. The line segment *A*−*B* lies within the feasible region of the convex function, showing that any line segment between two points in this region remains within the feasible space. In contrast, the line segment *C*−*D* crosses outside the feasible region of the non-convex function, demonstrating non-convexity, where not all line segments between points on the curve remain within the region.
The MILP model used in this thesis, as presented in Chapter 5, is linear in its relaxed form but includes binary constraints. In its relaxed state—without the binary constraints dependent on variable values—the model can be classified as convex. However, the inclusion of binary variables, which restrict production to one product per production line, renders the model non-convex. This is because specific parameter values can trigger the

binary constraints, deviating from the full linearity of the model.

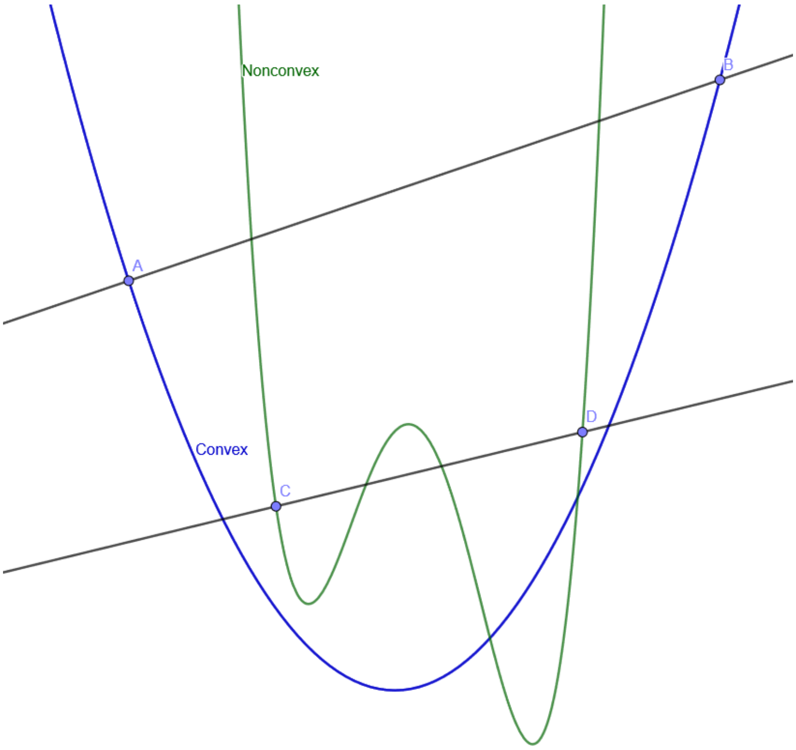


Figure 5 Feasible region of convex vs non-convex function

3.3 Sensitivity analysis

After optimised results have been given from the simulation, it should be noted that the results are always based on estimates that rely on assumptions presented in previous chapter. Sensitivity analysis is used to calculate how result would vary if certain parameter values were changed. As a result of sensitivity analysis, sensitivity parameters can be determined. Sensitivity parameters are parameter values that cannot be changed without changing the optimal solution. For coefficients that are not directly sensitive, it can be also beneficial to seek ranges, where objective function starts to change. Hillier & Liebermann (2015) describes the range where left-hand side coefficient can change value without affecting the objective function, *allowable range of coefficient.* When determining same range for functional constraints, it is called *allowable range for the right-hand side.* When constraint variables of the right-hand side are sensitive meaning that change in values lead to change in optimal solution or often price, term *shadow price* becomes relevant. Shadow price is used to describe the value of additional resource used in the optimisation. It can be also described as what would be the new price if the number of resources would be increased by one unit (Hillier & Liebermann, 2015, p.226). Nahmias (2005) presents definition of shadow price "*as the improvement in the objective function realized by adding one additional unit of a resource*" (Nahmias, 2005, p.172). Shadow price can be used to determine e.g., how beneficial some expansions or additional resources could be in terms of profit generation in theory. A certain shadow price is only valid in the allowable range for right-hand side, meaning that when constraint variable changes enough to affect to objective function, a shadow price will also change. Important factor regarding sensitivity analysis is, that when optimising with integer programming -based models that include binary variables, conducting sensitivity analysis must be made manually since the effect of binary variables to the optimal solution is not linear, and therefore cannot be calculated seamlessly.

3.4 Linear programming in production planning

How Linear Programming has been implemented in relevant production planning applications.

Nahmias (2005) states in book *Production and Operations Analytics* that general linear programming methods are valid and sufficient for most real-world problems production planning problems. However, special tailored versions can be faster for certain large-scale scenarios that may include non-linear characteristics such as economies of scale in production (Nahmias, 2005, p.130-131).

Belil et al. (2018), in their research paper titled "MILP-based Approach to Mid-term Production Planning of Batch Manufacturing Environment Producing Bulk Products," examined how mixed-integer linear programming (MILP) methods can optimize bulk production planning. The study addresses a production planning problem involving multiple continuous manufacturing units, focusing on a weighted bi-objective: maximizing demand satisfaction while minimizing inventory levels. The key components of the analysis are: (1) a set of products, each with unique variants and demand patterns; (2) a manufacturing system comprised of multiple facilities, each with specific capacity constraints; (3) a material handling process featuring a multi-line pipeline for transporting finished

goods to distribution points; and (4) a distribution facility capable of fulfilling end-customer deliveries (Belil;Kemmoé-Tchomté;& Tchernev, 2018).
Each manufacturing line has the ability to produce various products, but certain lines are tailored to specific product requirements. Safety stock is maintained in inventory based on the characteristics of the product and fluctuating demand. Additionally, the demand for each product is time-specific, varying over the planning periods, and each manufacturing line operates under its own capacity limits (Belil;Kemmoé-Tchomté;& Tchernev, 2018).
Problem is modelled as a combinatorial optimisation problem where different scenarios are optimised in contrast to objective function. Since the process varies between product families, model was presented in the form of directed graph *G = (N, A)* where *N* presents set of nodes, and *A* is set of arcs. Nodes present different subsets of modelling e.g. manufacturing lines, inventories, distribution processes. Arcs present different subsets that are moving between nodes, e.g. subset transporting from manufacturing line to transport conveyor. Indexes of the model involves *i* and *j* indexes for nodes and arcs, index for period, products, and storage place. Parameters included e.g. product families, binary parameters that check if production line can produce certain product, maximum daily capacities of arc, demand for product *P* at time *T*, minimum quantity by production line and multiple capacity and binary variables that check if product is produced and transferred through all of the nodes during certain period.
Model used real-world historical data of demand and real values of capacities. Planning horizon was two weeks divided into fourteen planning periods. Fourteen product variants were included, produced by six production lines (Belil;Kemmoé-Tchomté;& Tchernev, 2018). To simplify the MILP-model used in the study by Belil et al., model sought to maximise demand satisfaction rate of product P during period T. Constraints were mainly capacity constraints that checked that flow of products does not exceed capacities of process steps. Optimisation was done using IBM's Cplex.

Table 3. Average demand satisfaction rate per product

Product	Average demand satisfaction rate	Product	Average demand satisfaction rate
1	92,86%	8	99,79%
2	100%	9	100%
3	86,66%	10	86,36%
4	28,57%	11	92,86%
5	96,16%	12	98,85%
6	28,57%	13	100%
7	100%	14	50%

Table 4. Capacity used of manufacturing lines

Manufacturing line	Total Capacity used in tons	Capacity of Line	% of Capacity used
1	27140	30240	89,75%
2	28097	30240	92,91%
3	59160	60480	97,82%
4	35304	40320	87,56%
5	31769	40320	78,79%
6	36394	40320	90,26%
Average manufacturing line utilization			89,52%

Figure 6 Results of MILP optimisation in study by Belil et al. 2018

Results of the optimisation were following; average demand satisfaction rate was 92.1% while manufacturing line utilisation was 89.5% (Figure 5).

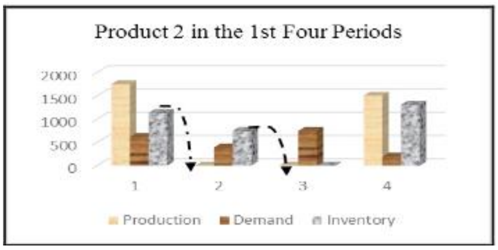


Figure 7 Results of MILP optimisation in study by Belil et al. 2018

Further on results (Figure 6) suggest that to match demand of period 2 and 3, first period requires excess amounts of inventories.

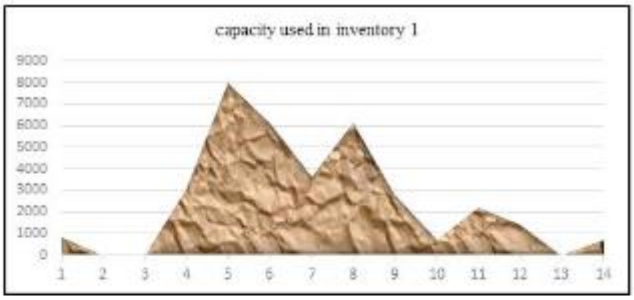


Figure 8 Results of MILP optimisation in study by Belil et al. 2018

Figure 7 shows the change in inventory levels across periods. Belil at al. states that proposed model is promising in analysing trade-offs between production and inventory level (Belil;Kemmoé-Tchomté;& Tchernev, 2018).

Conclusions of the study states that MILP is valid tool in tactical production planning and able to solve real size optimisation problems related to multi facility production, inventory, and distribution system in reasonable time. Additionally, model provides room for development in terms of additional periods and products. For future work, researchers suggest improving decision making tool for tactical planning which takes demand uncertainty into account (Belil;Kemmoé-Tchomté;& Tchernev, 2018).

Letelier et al. (2020) studied how a mixed-integer programming (MIP) approach can improve production scheduling for strategic open-pit mine planning. The research problem involved scheduling decisions on which blocks to extract, when to extract them, and whether they should be processed, all aiming to maximize the net present value of the materials. This problem was unique due to the exceptionally large datasets involved, ranging from 20,000 to 5,000,000 blocks over 20 to 50 time periods. Traditional mixed-integer linear programming (MILP) models are not efficient for datasets of this magnitude.

To address this large-scale scheduling problem, the researchers developed several methods, including preprocessing (eliminating variables that do not appear in the optimal solution), cutting planes (adding constraints to limit the feasible region), and modified heuristic methods (Letelier et al., 2020). Cutting plane techniques enhanced the efficiency of both relaxed and heuristic solutions in approaching the optimal solution. The study demonstrated ways to optimize larger datasets using various techniques and heuristic models. It also proved that with thorough modelling, heuristic methods can achieve solutions relatively close to the optimal with better efficiency.

Another approach to heuristic MILP models was presented by Tonelli et al. (2013) in their research paper "Production Planning of Mixed-Model Assembly Lines: A Heuristic Mixed Integer Programming-Based Approach." The paper highlights the importance of flexibility and the capability to implement tactical and operational production planning techniques into master scheduling methods. The researchers proposed a MILP-based approach to introduce more flexibility into master scheduling by considering due date setting, capacity sequencing, and production sequencing, allowing different production lines to produce various product variants.

The case study focused on an assembly factory producing a high volume and variety of agricultural machinery. The goal was to integrate a MILP model into an Advanced Planning System (APS) to enhance production planning flexibility. According to the researchers, the APS is enterprise resource management software enhanced with additional valuable interfaces. The MILP-based model is part of a proposed three-stage optimizer framework for the APS (Figure 8). The problem involved a large dataset, including 20,000 units produced annually with 100 variants and 4,000 parts required in production. Linear programming was utilized in three ways:

1. Initial Linear Relaxation: Due to the large amount of data, the first optimization was performed as a relaxed optimization, allowing continuous variables. While continuous variables are not feasible in the real-world scenario, this approach makes optimization more efficient and provides boundaries to aid further planning.
2. Iterative Rolling-Horizon Strategy: Following the relaxed optimization, the linear programming model was refined, and optimization was conducted in time-wise sub-intervals. In this phase, the relaxed reference plan served as a guideline while integer variables were introduced into the LP model, and constraints were set accordingly. This process was repeated until all constraints were incorporated, and the continuous variables from the relaxation phase were replaced with integer variables.
3. Heuristic Approach: Subproblems related to the case were managed using mixed-integer programming, while larger optimization tasks were managed

through LP relaxation, enhancing overall optimization efficiency (Tonelli et al., 2013).

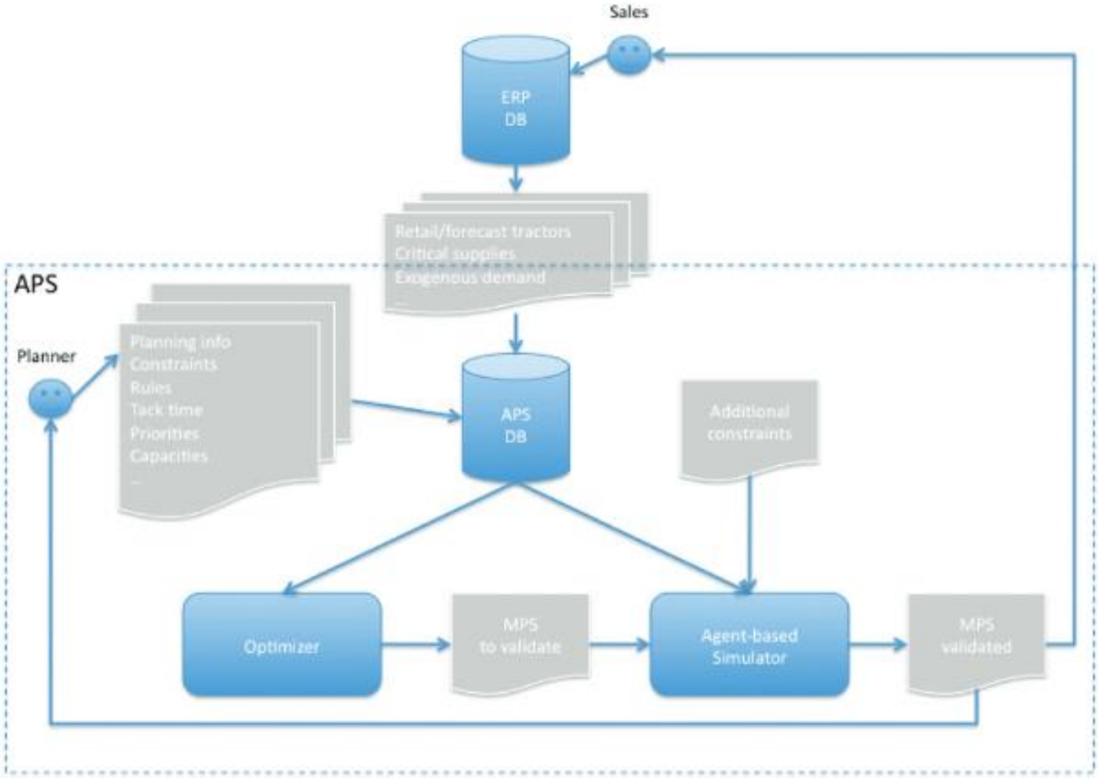


Figure 9 Suggested APS solution by Tonelli et al. (2013)

The research paper provided a method to apply variations of linear programming to large-scale problems. The key takeaway is the demonstration of the possibilities that linear optimization offers; approximate solutions for large-scale problems using relaxed and heuristic models, and more detailed solutions for small-scale problems with thoroughly planned LP models. Both approaches are relevant for different purposes and can be combined within a larger solution to gain synergistic benefits.

Tonelli et al. (2013) provide an insightful example of using Mixed-Integer Linear Programming (MILP) in a multi-product, multi-period production problem, referred to as "multi-item, multi-level" in their study. The case organization operates a three-stage assembly system, progressing from a three-line initial production phase to a two-line assembly phase, and finally, a three-line assembly phase. The optimization timeframe is based on work weeks (typically five days), accounting for holidays and stoppages. The optimisation constraints are divided into hard and soft categories. Hard constraints are mandatory and include production line capacities, subset production limits for specific groups, relative production bounds between different product groups, levelled production constraints to ensure stability during production rate changes. Soft constrains are more flexible, and include target production rate, smoothing production rates, and preferred line assignments. The optimisation itself is multi-objective aiming to minimize several goals: tardiness, early production (to reduce inventory costs), unmet demand, deviations from target production rates, and stability in both production rates and line assignments. The solution uses MILP with a rolling-horizon approach, allowing for periodic updates and adjustments (Tonelli;Paolucci;Anghinolfi;& Taticchi, 2013). The study highlights that multi-objective optimization requires weighting and balancing different objectives, which are determined by the production manager or decision maker. This introduces potential risk if the decision-maker lacks expertise in setting the optimal weights. Tonelli et al. (2013) suggest that the production manager's goal is to balance the sometimes-conflicting objectives of maximizing plant efficiency and meeting customer demand and satisfaction.

In their 2005 study, Wang and Liang explored how possibilistic linear programming (PLP) can enhance multi-product aggregate production planning when demand, related operating costs, and capacities are uncertain. PLP, as discussed in the chapter 2.8, is a linear programming technique where variables and parameters in the optimization are treated as fuzzy and uncertain, leading to optimization based on certain assumptions. Wang and Liang formulated the optimization problem as follows: a case company manufactures n types of products over a planning horizon *T*, with forecast demand, operating costs, and labour and machine capacities subject to uncertainty. The researchers developed a PLP model aimed at optimizing the aggregate production plan to meet forecasted demand by setting appropriate production rates, inventory and labour levels, and managing subcontracting, backordering, and other controllable variables (Wang & Liang, 2005).

This study is interesting in the light of this thesis since it involves many of the characteristics of the case study of this thesis. However, this Wang & Liang use the uncertainty and fuzziness as key element of the study, and model is PLP instead of MILP. Especially problem formulation is useful when formulating MILP model for this thesis' case study. Below is objective function and key constraints to the PLP model, with key variables presented.

$$Min\ Z = \sum_{n=1}^{N} \sum_{t=1}^{T} \begin{matrix} (a_{nt}Q_{nt} + b_{nt}O_{nt} + c_{nt}S_{nt} \\ +d_{nt}I_{nt} + e_{nt}B_{nt}) \end{matrix} + \sum_{t=1}^{T} (k_t H_t + m_t F_t) \tag{13}$$

Where:

Z = total cost, *N* = types of products, *T* = planning horizon, a_{nt}= regular time production cost per unit of *n*th product in period t, Q_{nt}= regular time production, b_{nt}= overtime production cost per unit, O_{nt}= overtime production, c_{nt} = subcontracting cost per unit, S_{nt} = subcontracting volume, d_{nt} = inventory carrying cost per unit, I_{nt}= inventory level, e_{nt} = backorder cost per unit, B_{nt} = backorder level, k_t = cost to hire one worker in period t, H_t = workers hire in period t, m_t = cost to layoff one worker, F_t = workers laid off (Wang & Liang, 2005).

Constraints of model are divided into three parts, carrying inventory, labour levels, and machine capacity and warehouse space.

Constraint on carrying inventory as below, row (14). Checks that previous total inventory does not exceed forecasted demand.

$$I_{nt-1} - B_{nt-1} + Q_{nt} + O_{nt} + S_{nt} - I_{nt} + B_{nt} = D_{nt} \quad for\ n, t \quad (14)$$

Where:

I_{nt} = inventory level in period t of product *n,* B_{nt} = backorder level, Q_{nt} = regular time production, O_{nt} = overtime production, S_{nt} = subcontracting volume, D_{nt} = forecast demand (Wang & Liang, 2005).

Labor level constraints below.

$$\sum_{n=1}^{N} i_{nt-1}(Q_{nt-1} + O_{nt-1}) + H_t - F_t - \sum_{n=1}^{N} i_{nt}(Q_{nt} + O_{nt} \quad for\ t \quad (15)$$

$$\sum_{n=1}^{N} i_{nt}(Q_{nt} + O_{nt}) \leq W_{t\,max} \ for\ t \quad (16)$$

Where:

i_{nt} = hours of labour per unit of *n*th product in period *t,* $W_{t\,max}$ = maximum labour level available in period *t.* Row (15) checks that labour levels of *t* equals labour level of *t - 1* and possible layoffs or hires. (16) checks that labour level does not exceed maximum possible labour level for period t (Wang & Liang, 2005).

Constraints regarding production machine capacity and warehouse space:

$$\sum_{n=1}^{N} r_{nt}(Q_{nt} + O_{nt}) \leq M_{t\,max}\ for\ t \tag{17}$$

$$\sum_{n=1}^{N} v_{nt} I_{nt} \leq V_{t\,max}\ for\ t \tag{18}$$

Where:

r_{nt}= hours of machine usage per unit of *n*th product in in period t, $M_{t\,max}$ = machine hour maximum capacity available for period t, v_{nt} = warehouse space, $V_{t\,max}$ = maximum warehouse space available. Row (17) constraint checks that machine usage does not exceed maximum capacity, and row (18) constraint checks that warehouse capacity is not exceeded. Besides these constraints, non-negativity constraint is also included for every decision variable (Wang & Liang, 2005).

After presenting the PLP model, researchers introduced a triangular possibility distribution to manage the inherent uncertainty in cost coefficients, demand, and capacity. Each imprecise parameter, such as demand D_{nt} and cost components $a_{nt}b_{nt}$ etc., were defined by a triangular distribution with three key values: pessimistic, most possible, and optimistic. This distribution allows for a structured representation of uncertainty, enabling the model to consider varying degrees of likelihood for each parameter. After the triangular distribution is presented, researchers transformed it into multi-objective linear programming model with three objectives, 1. Minimise the most possible value of total cost, 2. maximise the possibility of obtaining lower total cost, 3. minimise the risk of obtaining higher total costs. Each sub-objective was converted into auxiliary crisp objective function using the weights of the triangular distribution while allowing the model to integrate decision-maker preferences regarding risk and cost expectations. With fuzzy decision-making techniques, three objective functions were transformed into single-goal linear programming model. For each objective, positive and negative ideal solutions were specified. With use of membership functions that presented satisfaction level of each solution, decision-maker was able to adjust parameters until desired solution is achieved (Wang & Liang, 2005).

Wang and Liang presented a case study done in industrial tool manufacturer where uncertainty is common issue. PLP based model was implemented with four-month planning horizon. Results stated that PLP proved to be flexible tool that satisfied decision-makers goals in aggregate planning. Overall, researchers stated that proposed approach can be superior tool in industry where excessive amounts of uncertainty is involved in crucial parameters for aggregate planning (Wang & Liang, 2005).

Dohale et al. (2022) explored the use of linear programming as an optimization tool to enhance aggregate production planning in an automobile components firm. The researchers adopted multi-objective linear programming (MOLP) as the LP technique due to the presence of multiple objectives in the optimization problem, which were identified using a group decision-making approach. The objectives were assessed, compared, and weighted, with the most significant ones being minimizing lead time, minimizing total cost, and maximizing resource utilization. These were selected as the primary objective functions for the aggregate production plan. MOLP was then applied to implement the aggregate production plan, ensuring that the objective functions operated within acceptable performance ranges. The study noted some limitations, including assumptions made during the MOLP development and a potential for bias in objective selection due to the group decision-making process (Dohale, Ambilkar, Gunasekaran, & Bilolikar, 2022).

Many of the presented research papers have implemented multi-objective techniques in linear programming-based optimization. This approach is logical for aggregate planning, as its scope is often broad and dynamic. Optimizing for a single objective may not yield the best results or even be realistic, given the interconnected nature of decisions in production planning. Figure 10 presents the conceptual framework of aggregate production planning as presented by Dohale et al. (2022). In this framework, decision options include strategies for modifying demand and supply, covering functions from sales and inventory management to subcontracting. Given this diverse range of options, optimization problems that seek benefits across multiple functional areas require multi-objective optimization to effectively address the varied needs of each function. Additionally, as the optimisation problem expands, uncertainty increases. This explains the use of fuzzy variables and parameters with multi-objective optimisation. Broader optimisation problem increases the uncertainty of the outcomes and therefore to mitigate risk, multiple options with different fuzzy parameters can help in the planning and decision making.

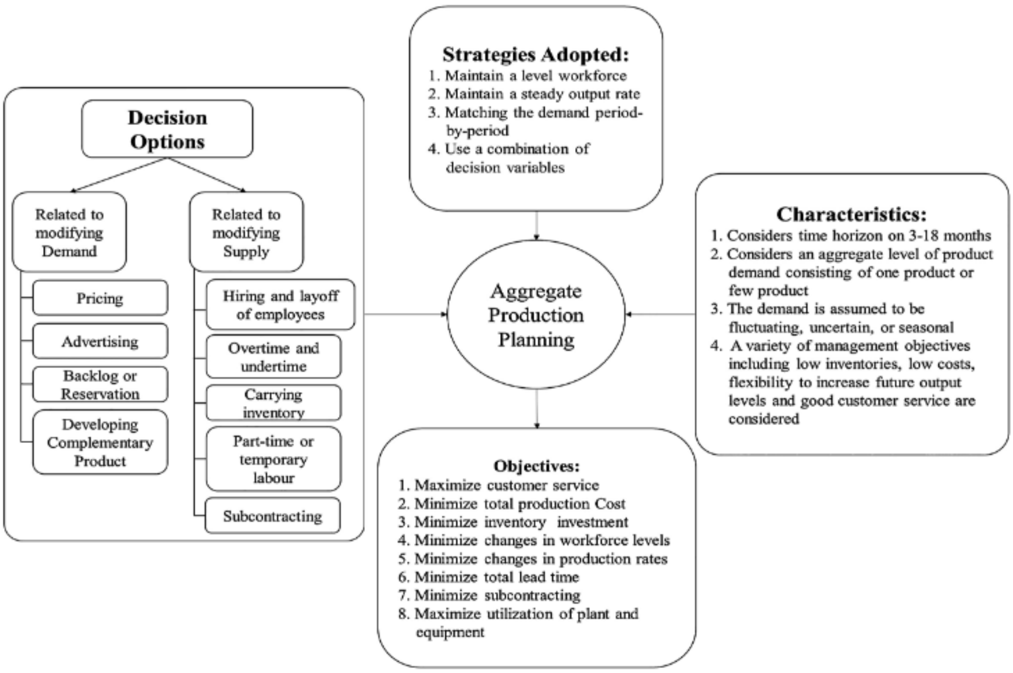


Figure 10 Aggregate planning concept by Ambilkar et al. (2022)

Within this context, optimization should also include smaller-scale efforts within specific aggregate planning functions. These focused optimizations may not consider the entire scope but instead target single objectives within individual functions, as the case simulation in this research demonstrates. Implementing all options across a multi-functional scope and considering every aspect simultaneously demands substantial resources and careful planning to ensure balanced consideration of all factors. Large-scale optimization models can compromise efficiency and validity, potentially making the results less actionable. Thus, single-objective, smaller-scale optimization within functions is often a practical approach, as it enables easier control over validity and utility, aligning well with the needs of users and responsible decision-makers. However, even in single-goal optimizations, it remains essential to account for the broader organizational context and potential stakeholders, at least indirectly, since no function operates in complete isolation.

3.5 Validity of linear programming and delimitations

Optimal solutions for optimisation problems are not integers, which requires rounding the variables. Depending on the implementation, this may result infeasible solution due to rounding difficulties. Cases where rounding may lead to difficulties can be situations where variables present workforce and cannot be treated with any other form than integers, or large and complex sets where rounding makes the modelling much more complex. However, in most cases where relationship with constraints is linear, this method will find feasible solution (Nahmias, 2005). When modelling the optimisation problem, form of variables should be thoroughly considered, and possible effect of rounding should be calculated.

Linear programming parameters include certainty assumptions, as presented in chapter 4.1. Values given for parameters are assumed to be certain, even though in real-world implementations there are often deviation between predicted values and actuals. This is common especially when optimisation is based on forecast of future values. Assumption based risk can be mitigated by conducting sensitivity analysis for optimisation results. This way, sensitive parameters that have bigger impact to solutions can be found and acknowledged (Hillier & Lieberman, 2015, p. 43).

4 Research Methods

In this chapter, research methods, approach and strategy are presented. Chapter will go through processes regarding how data should be gathered and then how it should be analysed.

4.1 Research methods

In this study mixed method is used in collection and analysis of data. Qualitative methods include collecting information about current and previous production data e.g. resource requirements, material costs, overhead and fixed costs. Qualitative methods include interviews with personnel from organization that are either directly involved in production planning or otherwise relevant for this purpose. Interviews are semi-structured. Questionnaires or surveys are not relevant in this context as the number of similar, relevant participants will not be large, and sampling would be too small. Whereas data collection is based on qualitative and quantitative methods, the analysis itself is mostly based on simulation as a key method. Collected data is used in simulation, and analysis will be done based on findings from the simulation. Analysis will be based on comparative scenario analysis between historical performance data of production, compared to simulation tool's suggested production plan. Core of the analysis is profitability in terms of profit generated from produced tonnes. Additionally, sensitivity analysis is conducted to simulation tool parameters to determine which production inputs and factors should be focused on the production planning.

4.2 Research approach and strategy

The research approach in this study will be deductive. The objective is to build a simulation tool that is used to develop production planning using the mixed-integer linear programming, MILP. The tool is based on the MILP concept and theoretical framework presented in this text but also adopts collected feedback from the organization as well as possible.

Selected research strategy will be a case study following action research methods. Action research was chosen due to its flexible and dynamic nature which allows continuous improvement as the study progresses. Figure 11 presents the research process of Action research.

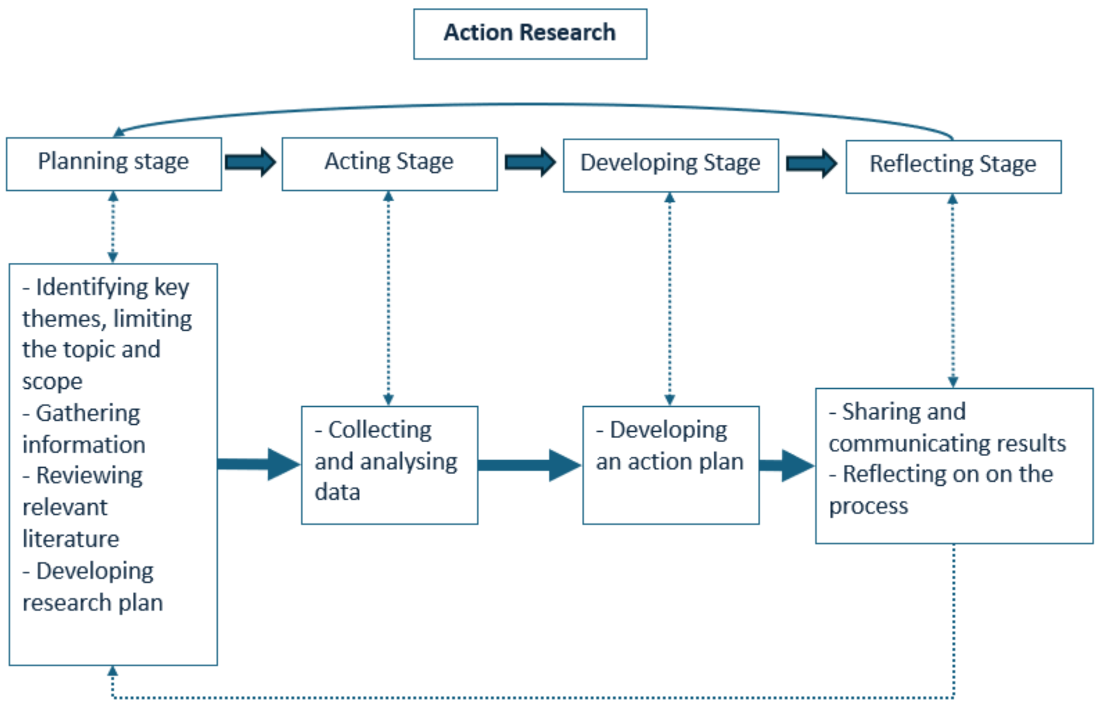


Figure 11 Action research -method

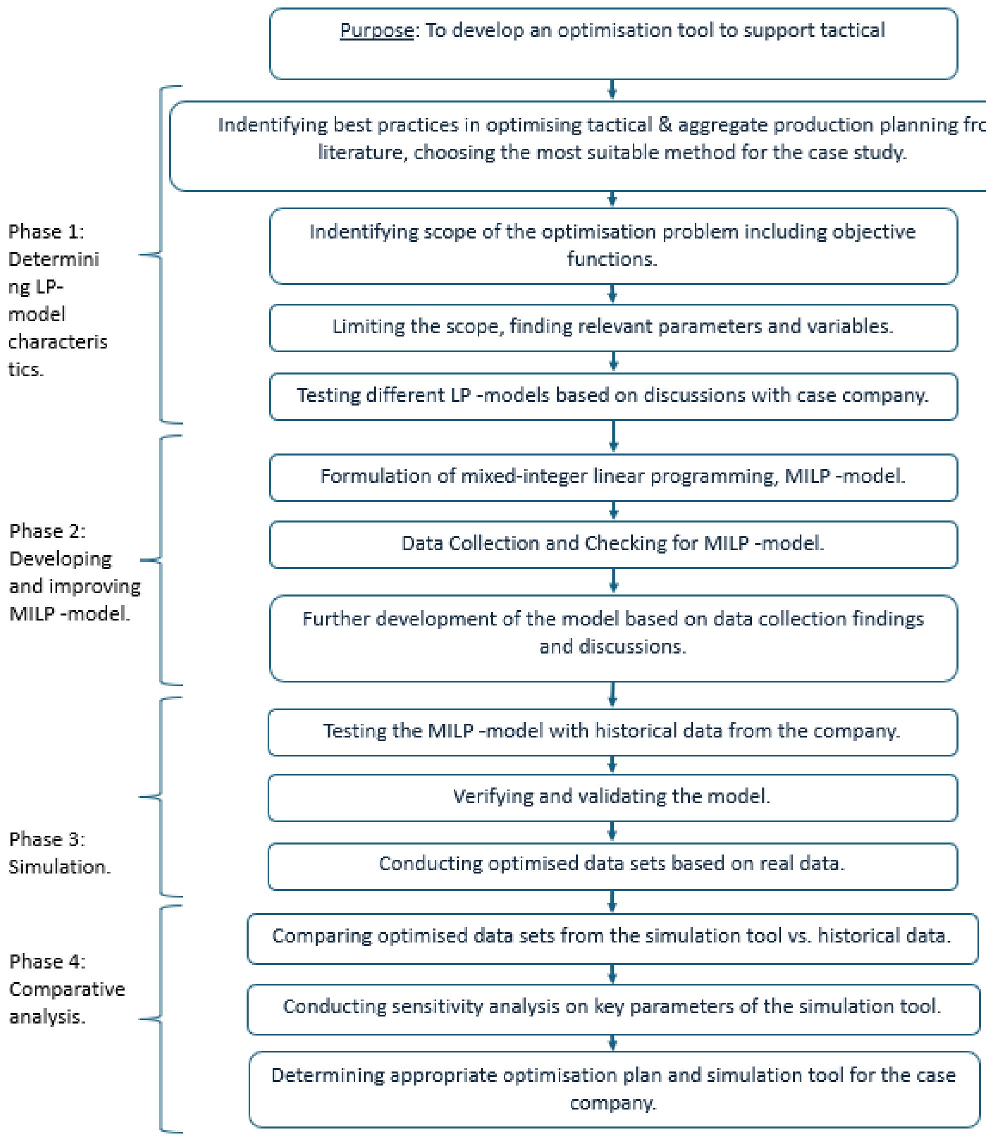


Figure 12 Research methods in more detail

4.3 Data collection and analysis

Data is collected though semi-structured interviews with key stakeholders. Data is collected to simulation tool's database. Collected data mainly includes historical production information including material prices, production volumes, and resource requirements. Data will be verified with key personnel, to ensure the validity of the data.

4.4 Reliability and Validity

Due to technical and scope restrictions, collected data will include adjustments and averaging values, in order to support feasibility in the simulating phase. Regular meetings with stakeholders are held, in order to ensure reliability and validity of findings and recommendations as well as functionality of the simulation tool. Reliability and validity will be discussed further in the results and conclusions.

4.5 Sensitive data

The empirical data used in this thesis is based on the company's historical production data. To protect the company’s competitive advantage, certain details have been adjusted or excluded, including specific figures related to cost efficiency and pricing. Historical data was utilized in the optimization process, with the results presented as percentage differences between the original unoptimized data and the newly optimized production suggestions.

5 Empirical Analysis

5.1 Introduction and Background

Empirical analysis can be divided into two sections: 1. gathering sufficient history database and 2. developing Excel Solver -based simulation tool which is used to test potential of mixed-integer linear programming in tactical production planning. Chapter 5.2. will focus on data collection while 5.3 describes the characteristics of simulation model.

5.2 Collection of historical data

To evaluate the performance of the simulation tool against the case organization's existing production planning processes, multiple datasets were required for comparison. Data was gathered through workshops and semi-structured interviews with personnel from tactical and strategic production planning, production engineering, and financial controlling.

The collected data covered production from 2021 to 2023, with monthly granularity. Compiling the datasets involved three main stages, each focusing on collaboration with different departments within the case company. First, the entire production planning process was thoroughly reviewed with personnel from strategic and tactical production planning. The scope of production, including process steps, products, and other relevant details, was defined, although collaboration with supervisors from the production planning team continued throughout the thesis process.

Financial information, including fixed costs, raw material prices, and other cost-related variables, was gathered in close collaboration with the financial controlling team. Additionally, sales data was collected with their support. However, to optimize production and compare profitability across different products and scenarios, detailed product-specific material requirements were necessary. This data was compiled with the help of production engineers, who provided the required input specifications for each product.

As collected data increased, it was updated to simulation tool and actively tested through the simulation. After collection, storing and testing, validity of data was tested with original stakeholders that helped to gather datasets. More detailed description of datasets will be presented in chapter 5.3.4.

5.3 Simulation tool

Simultaneously as historical data was collected, simulation tool was developed. Tool went through multiple development phases and was modified through the thesis process as new requirements and needs arose with workshops and interviews. Original plan was to use basic linear programming -based solution for optimisation tool, but due to requirements that involve binary variables, mixed-integer linear programming was chosen as the base algorithm for optimisation model.

5.3.1 Background

Simulation tool is built with Microsoft Excel, using Solver tool with add-on open-source software OpenSolver. OpenSolver is required due to large amounts of data processed in the simulation. Linear programming is done with Excel, since the visualization of results and other data is most convenient compared to other solutions. Excel Solver is also easily available and does not require any licenses compared to other solver softwares. Excel Solver is also presented as suitable software for linear programming in production planning presented in book *Production and Operations Analysis* by Nahmias (2005) and *Operations Research – An Introduction* by Taha (2017).

5.3.2 Parameters and constraints

To formulate as realistic simulation as possible with chosen tools and method, correct parameters and constraints must be in place. In simulation, we are using adjusted production plans to determine which parameters are used to produce which product. In simulation, we have six different products, that each have require different inputs. Input parameters include electricity consumption and cost, coke type and consumption, concentrate, quartz, bentonite and lumpy ore consumption and cost. Price of each input parameter varies in time, and this brings the room for optimisation – when to produce and what to produce to maximise profit per tonne in production planning. Optimisation tool suggests complete optimised production plan, including product mix and volumes.

Parameter prices are variable costs that are linear to produced amounts. Addition to variable cost, fixed costs are also included in the simulation. Fixed overhead costs include salaries, maintenance, and other costs.

Simplified optimisation model would only suggest maximising production of the product which has best profitability. However, in simulation we assume that there is certain amount of demand that must be fulfilled. Demand is calculated as a four-month total, which creates a minimum requirement of total production for the model. Monthly production level is constrained by fixed costs – profit from monthly production must exceed fixed cost of the corresponding month.

Model includes three production lines that each have their own capacity. Each production line can produce one product at a time. Here binary variables are required: if line produces product p_1 *at* period t_1, binary variable b_{11} gets value 1, otherwise 0. Below is the summary of variables used in the model.

Variable	Description	Unit, if available & details
p	$= product$	*T*, tonnes of finished product.
t	$= time$	*t* presents one month in this model. Time period in this model is four months total.
l	$= production\ line$	Production line where products are produced. Three production lines total, each can product one product at a time.

C_{pt}	$= cost\ of\ production\ per\ finished\ tonne\ per\ product\ per\ time$	*€/t,* combined cost of raw materials required to produce one tonne of finished product. Values are based on historical data.
P_{pt}	$= selling\ price\ of\ product\ per\ tonne\ per\ time$	*€/t,* selling price of product during the time *t*. Values based on historical data.
$Prof_{pt}$	$= profit\ per\ product\ per\ time$	€, profit per product, subtraction of selling price and production cost.
$Prod_{pt}$	$= amount\ of\ production\ per\ product\ per\ time$	*T, tonnes.* Decision variables.
PPT_{pt}	$= profit\ per\ tonne\ per\ product\ per\ time$	*€/t,* calculated as $\frac{Prof_{pt}}{Prod_{pt}}$
B_{pt}	$= binary\ helper\ variable, used\ restrict\ production\ to\ only\ one\ product\ at\ a\ time\ per\ production\ line$	Value 1 if $Prod_{pt} < 0$ Value 0 if $Prod_{pt} > 0$
$Demand_{pt}$	$= demand\ per\ product\ per\ time$	*T, tonnes*. Estimated demand for product. Used to calculate production level.

Cap_{lt}	$= production\ capacity\ per\ production\ line\ per\ time$	*T, tonnes.* Maximum production constraint per production line. Fixed parameter by default, but utilisation rate can be adjusted if necessary.
Min_{Prod}	$=\ required\ minimum\ production\ level\ per\ product\ per\ cycle\ (four\ months)$	*T, tonnes.* Minimum production constraint per product per cycle of four months.
FC_t	$= fixed\ costs\ per\ month$	€, monthly fixed costs serve as a minimum profit constraint for model.

5.3.3 MILP -model

Objective function as follows:

$$Maximize\ Z = \sum_{p=1}^{P}\sum_{t=1}^{T}(PPT_{p,t} * Prod_{p,t}) \tag{13}$$

Or simply:

$$Maximize\ Z = \sum_{p=1}^{P}\sum_{t=1}^{T}(Prof_{p,t})$$

Where:

- Z = total profit
- $PPT_{p,t}$ = profit per tonne for finished product *p* in month *t*

- $Prod_{p,t}$ = Production amount of product *p* in month *t*

Objective function calculates total production volume for every product which is multiplied by the profit per tonne for corresponding product. Calculation is done for every product using four-month (t_1-t_4) totals.

Subject to constraints

1. Production line assignment constraint.

Since one production line can produce only one product at a time, a constraint is needed which assigns one product per one production line per month.

$$\sum_{p=1}^{P} B_{p,t,l} \leq 1$$

Where:

- $B_{p,t,l}$ = binary variable which gets value of 1 if production line *l* produces product *p* during month *t*

2. Production line capacity constraint.

Each of three production lines has a maximum capacity for production volume. Following constraint restricts production not to exceed this limit.

$$Prod_{p,t,l} \leq Cap_{lt} * B_{pt} \quad for\ p,t$$

Where:

- Cap_{lt} = maximum capacity for corresponding product during month in question
- B_{pt} = binary variable, given value 1, if there is production for product *p* in month *t*, or 0 if there is no production.

3. Minimum production constraint.

Constraint which checks that production meets the minimum demand for production requirements for every product.

$$Prod_{pt} \geq \ Min_{Prod}\ for\ p,t$$

Where:

- $Prod_{pt}$ = total amount of production in tonnes for product during month

- Min_{Prod} = total minimum requirement for production for product

4. Monthly fixed cost coverage constraint for profit generation.

Constraint ensures that profit generated from production covers the fixed cost total of corresponding month.

$$\sum_{p=1}^{P} Prof_{p,t} \geq FC_t \quad for\ t$$

Where:

- Sum of $Prof_{p,t}$ presents total profit generated from all products in one month.
- FC_t presents total amount of fixed costs during that month.

To summarise, below is the general version of the MILP -model. Generalised version

$$Maximize\ Z = \sum_{p=1}^{P} \sum_{t=1}^{T} (PPT_{p,t} * Prod_{p,t})$$

$Subject\ to$

$$\sum_{p=1}^{P} B_{p,t,l} \leq 1$$

$$Prod_{p,t,l} \leq Cap_{lt} * B_{pt} \quad for\ p,t$$

$$Prod_{pt} \geq\ Min_{Prod}\ for\ p,t$$

$$\sum_{p=1}^{P} Prof_{p,t} \geq FC_t \quad for\ t$$

And

$$B_{pt} \quad is\ binary\ integer\ for\ p,t,l$$

5.3.4 Excel -tool

Simulation model was built on basic Excel with OpenSolver -add-on. Optimisation concept is based on three different sheets which are data -sheet, input -sheet, and calculation -sheet. Figures are masked due to their confidentiality.

Price data	Unit	1/2021	2/2021	3/2021	4/2021
Electricity (€/mWh)	€	***********	***********	***********	***********
Electricity cost per finished FeCr tonne	€	***********	***********	***********	***********
Coke P High (€/tonne)	€	***********	***********	***********	***********
Coke P Medium (€/tonne)	€	***********	***********	***********	***********
Coke P Low (€/tonne)	€	***********	***********	***********	***********
Biocoke price (€/tonne)	€	***********	***********	***********	***********
FeCr Concentrate price (€/tonne, from mine to production)	€	***********	***********	***********	***********
Quartz (€/tonne)	€	***********	***********	***********	***********
Bentonite (€/tonne)	€	***********	***********	***********	***********
Upgraded lumpy ore (€/tonne)	€	***********	***********	***********	***********
Subcontracting services, allocated cost per produced FeCr tonne	€	***********	***********	***********	***********
Cost allocation to Lumpy production from total		***********	***********	***********	***********
Production Cost (fixed, per month)					
Overhead (total, allocated to Lumpy production)	000€	***********	***********	***********	***********
Salaries TOTAL FERROCHROME	000€	***********	***********	***********	***********
Maintenance TOTAL FERROCHROME	000€	***********	***********	***********	***********
Other fixed costs TOTAL FERROCHROME	000€	***********	***********	***********	***********
Salaries, allocated	000€	***********	***********	***********	***********
Maintenance, allocated	000€	***********	***********	***********	***********
Other fixed costs, allocated	000€	***********	***********	***********	***********

Figure 13 Excel -tool: Data -sheet

Figure 8 presents a portion of the data sheet where all the required input cost information is listed (data modified for the illustration). The information on this data sheet was gathered from historical records. In everyday use, the data sheet could be based on both actual historical data and forecasts. It includes all the information used in the simulation, such as raw material prices, fixed costs, sales data (including product related volumes and prices for both internal and external sales).

The input sheet (Figure 9) pulls data from the data sheet. Users can input a desired date for optimization, such as 8/2023, and the relevant data is automatically fetched from the data sheet into the input sheet. This allows users to make quick comparisons between different periods without manually adjusting datasets. The data is divided into three four-month periods per year: 1-4, 5-8, and 9-12. These intervals were chosen because a month is the shortest available timeframe for real data, and a four-month period provides enough scope for optimization while remaining manageable within Excel Solver.

Input data	Month, *t*	1	2	3	4	
Set month for optimisation (form m/yy)		tammi.21	helmi.21	maalis.21	huhti.21	
Price data						
Electricity (€/mWh)		*************	*************	*************	*************	
Coke P High (€/tonne)		*************	*************	*************	*************	
Coke P Medium (€/tonne)		*************	*************	*************	*************	
Coke P Low (€/tonne)		*************	*************	*************	*************	
Biocoke price (€/tonne)		*************	*************	*************	*************	
Concentrate (€/tonne, from mine to production)		*************	*************	*************	*************	
Quartz (€/tonne)		*************	*************	*************	*************	
Bentonite (€/tonne)		*************	*************	*************	*************	
Upgraded lumpy ore (€/tonne)		*************	*************	*************	*************	
Subcontracting services, allocated cost per produced FeCr tonne		*************	*************	*************	*************	
Monthly utilization rate from historical data	SAF3	*************	*************	*************	*************	************
	SAF2	*************	*************	*************	*************	************
	SAF1	*************	*************	*************	*************	************

Figure 14 Excel -tool: Input -sheet

Product, *p* 1	**10-50mm**				
Electricity consumption per finished tonne	*************	*************	*************	*************	**mWh**
Electricity cost per finished tonne	*************	*************	*************	*************	€
P High Coke consumption (per finished tonne)	*************	*************	*************	*************	**T**
P High Coke(cost per finished tonne)	*************	*************	*************	*************	€
P Med Coke consumption per finished tonne	*************	*************	*************	*************	**T**
P Med Coke cost	*************	*************	*************	*************	€
P Low Coke consumption	*************	*************	*************	*************	**T**
P Low Coke cost	*************	*************	*************	*************	€
Fine concentrate consumption	*************	*************	*************	*************	**T**
Fine concentrate cost	*************	*************	*************	*************	€
Biocoke consumption	*************	*************	*************	*************	**T**
Biocoke (cost per finished tonne)	*************	*************	*************	*************	€
Quartz consumption per finished tonne	*************	*************	*************	*************	**T**
Quartz cost per finished tonne	*************	*************	*************	*************	€
Bentonite consumption per finished tonne	*************	*************	*************	*************	**T**
Bentonite price per finished tonne	*************	*************	*************	*************	€
Lumpy ore consumption	*************	*************	*************	*************	**T**
Lumpy ore cost per finished tonne	*************	*************	*************	*************	€
Subcontracting cost	*************	*************	*************	*************	€
Total cost per finished tonne € / tonne	***********	***********	***********	***********	**€**

Figure 15 Excel -tool, Input -sheet: Production card

Figure 10 shows the production card for product $p = 1$. A similar table has been created for each product. Raw material prices are imported from the data sheet, as shown in Figure 9, and these prices are linked to the product-specific tables. Since the consumption of raw materials varies between products, an itemized cost structure table is necessary.

Each row alternates between two key details: the raw material consumption per tonne of finished product and the corresponding cost. The consumption row indicates how much raw material (in tonnes) is required to produce one tonne of the finished product. The row below calculates the cost by multiplying the consumption by the raw material price or the price of electricity. After the cost for each input is calculated, the final row, *Total cost per finished tonne*, presents the sum of all production costs for one tonne of finished product, denoted as variable C_{pt}.

The Calculation sheet is where the optimization process takes place. Due to the limitations of Excel Solver, all data used in the optimization must be located on the same sheet. To prevent alterations to the original data, a separate sheet is used as a safeguard.

The Calculation sheet consolidates all variables: product-specific cost per tonne, selling price, monthly profit per tonne per product, fixed costs, required production levels, and production capacities for each of the three production lines. From the user's perspective, the Calculation sheet primarily serves to activate the optimization process and display results, which are then copied into the production plan.

5.4 Optimisation

This chapter evaluates the performance of the MILP-based simulation by comparing its results to actual historical data. The comparison is conducted in four-month cycles, as the optimization model operates on four-month periods to achieve optimized results. Data from 2021 to 2023 is used for this comparison.

Following this comparison, a tailored sensitivity analysis is performed to support the MILP model, conducted manually. In this analysis, production levels and variables remain constant over each four-month period, while raw material prices and other relevant parameters are individually adjusted. For instance, to assess the impact of electricity prices on total profitability, electricity prices are adjusted incrementally by +5%, +10%, and +15%, as well as -5%, -10%, and -15%. This approach enables a detailed analysis of each input's effect on production, informing production planning and risk management.

5.5 Simulation results compared to historical data

In this chapter, historical production data is compared against the optimized production plan suggested by the developed optimization model. The comparison is presented in tabular form, displaying differences between optimised and non-optimised historical data in percentages regarding costs and profits per produced tonnes. Production volumes are omitted, as they remain constant across both datasets for the four-month cycle. The results are presented as production plans for a four-month cycle. The simulation tool allocates production volumes for the next four consecutive months based on constraints, requirements, and profitability. These results assume that parameter prices and production requirements for the following four months are known in advance. This was chosen method based on most realistic real-world simulation this optimisation tool capabilities could provide. It should be noted that the historical production plan was not developed using the same four-month cycle principle. However, for the purpose of comparison, this study assumes that the necessary information is available at the start of each cycle, enabling production planning to be conducted as four-month totals. In practice, production planning typically relies on forecasts to some extent. The true benefits and potential outperformance of the optimization tool could be realized by applying it to future production planning. The comparison could then be conducted by evaluating the tool's recommendations alongside traditional production planning methods, both using the same set of forecasted data.

For 2021, the profit-per-tonne achieved with the MILP-based optimization model was 15% higher than the historical, non-optimized results. The model showed superior performance in each four-month cycle, with respective improvements of 28%, 10%, and 12% in cycles 1-4, 5-8, and 9-12. These figures are calculated as the total profit over four months divided by production volume, providing weighted profit-per-tonne values. Figure 16 illustrates the monthly profit-per-tonne trajectory, with weighted four-month values included for reference. Monthly values exhibit high volatility, attributed to their unweighted nature.

The optimization model's approach to production scheduling strategically emphasizes the most profitable month, while allocating only the minimum required production in months with lower profitability—covering variable and fixed costs.
Main drivers behind differences of simulation and original were fluctuations in product sales prices. Especially in first cycle, simulation model heavily relied on last two months of the four-month cycle where selling price – and therefore profit was higher. Table 1 presents the cycles in more detail.

Table 1 2021 PPT: Simulation tools production plan vs historical data

4-month cycle	**Total cost**	**Profit Per Tonne, PPT**	**Notes**
1-4/2021	Difference, % -4,5%	Difference, % +27,8%	Simulation tool's costs were 4.5% less, and overall profitability was 27.8% better compared to non-optimised historical data. Over 30% difference in product selling prices between January and April. Simulation emphasized production in March & April, while historical production/selling volumes were relatively stable, and April had the lowest volume of the four-month period.
5-8/2021	Difference, % -3,5%	Difference, % +10,4%	While the cost structure parameters differed only slightly, efficient month-to-month production allocation significantly increased profitability over the four-month period. The simulation model suggested minimal production for July 2021, covering only essential costs, while maximizing production levels in June and August.
9-12/2021	Difference, % -10,2%	Difference, % +11,9%	10 % difference in selling prices during the period gave room for optimisation. High volatility of electricity prices equalised differences between selling prices, while optimised solution still outperformed by 12 %. December especially differentiated between optimised and non-optimised due to high production costs. This can be seen in figure 16.

Year 2021 totals	Difference, % -6,3%	Difference, % +15%	Total 2021 weighted Profit per tonne by simulation tool outperformed historical by 15%, while costs were 6% less than non-optimised.

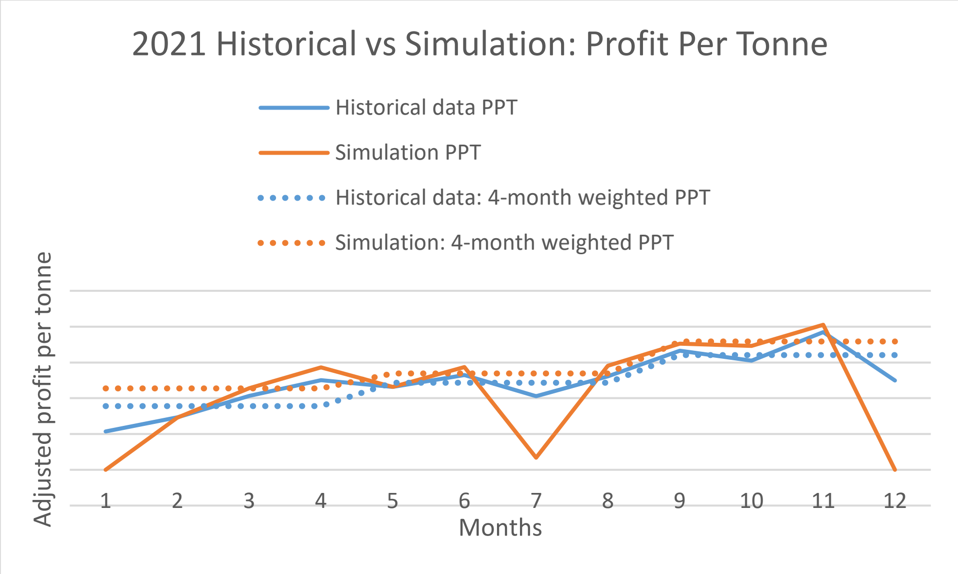


Figure 16 2021 Simulation vs Historical – PPT

Figure 16 presents four graphs comparing monthly profit-per-tonne values for the optimized production plan generated by the simulation tool with non-optimized production data from the case company. The y-axis represents profit-per-tonne values, which are removed for privacy reasons, while the x-axis shows the timeline from January 2021 to December 2021. Optimization was performed in four-month cycles, with weighted values displayed accordingly.

As demonstrated in Figure 16, the volatility in profitability is notably higher in the production plan suggested by the simulation tool due to more intense changes in production volume allocations. January, July, and December exhibited the lowest profitability within their respective four-month cycles, primarily due to elevated electricity costs and fluctuations in selling prices. The model's production plan strategically limits output in these months, maintaining only the essential production levels needed to cover key production and overhead costs.

Table 2 2022 PPT: Simulation tools production plan vs historical data

4-month cycle	**Total cost**	**Profit Per Tonne, PPT**	**Notes**
1-4/2022	Difference, % -5,6%	Difference, % +17,1%	Significant volatility in electricity prices resulted in substantial differences in production costs per tonne between the first two and last two months. Optimisation tool emphasized last two months due to higher profitability per tonne, which resulted 5.6% less costs and 17% increase in profitability.
5-8/2022	Difference, % -7,6%	Difference, % +21%	A similar trend continued as in the first cycle of 2022, though in reverse: production costs in May and June were over 20% lower than in July and August, creating additional profitability opportunities for the simulation tool.
9-12/2022	Difference, % -0,5%	Difference, % +17%	Parameter prices remained relatively stable compared to earlier in the year, except in December, which saw the highest production costs within the three-year period. While the percentage difference in profit per tonne (PPT) was significant at 17%, the absolute profit figures were considerably lower, approximately one-quarter of the previous cycle’s total profit.
Year 2022 totals	Difference, % -4,8%	Difference, % +18,8%	The total weighted profit per tonne for 2022 achieved by the simulation tool outperformed historical data by 18%. In absolute numbers, 2022 was best-performing year for the optimization tool. High production/sales volumes provided greater flexibility to allocate production to more profitable months, while significant volatility in raw material costs—especially electricity—created opportunities to enhance profitability.

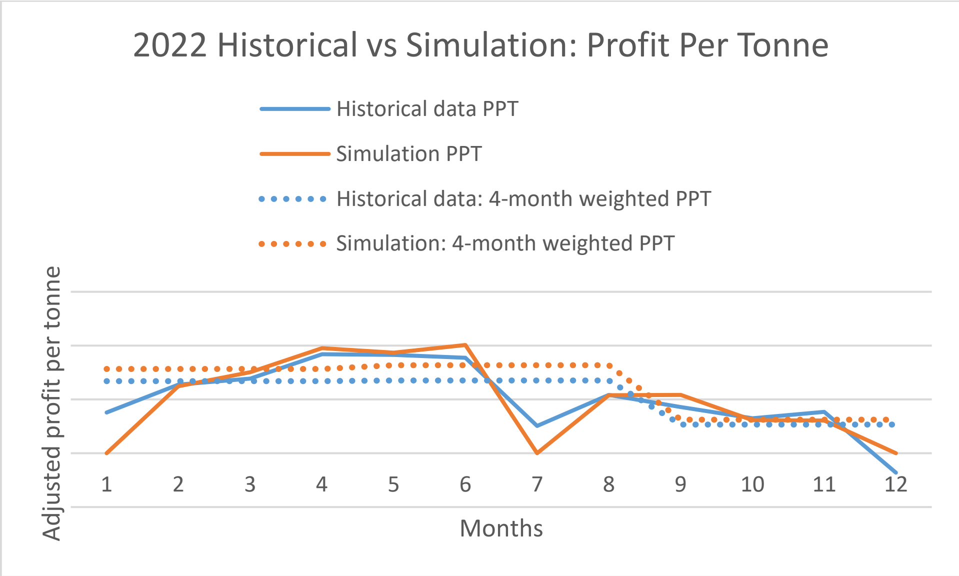


Figure 17 2022 Simulation vs Historical – PPT

In 2022, the simulation model outperformed the baseline data with a 18.8 % increase in weighted profit per tonne. The optimized production plan exceeded historical, non-optimized data by 17%, 21%, and 17% across different periods. From the figure 17 graph we can notice same trend as in 2021, January, July and December being the most difficult months in terms of profitability per tonne. However, compared to 2021, beginning of the year (January-July) was more profitable than rest of the year, as in 2021 this was opposite, as weighted profitability trend increased towards end of the year.

Overall, the optimization tool's production plan for 2022 demonstrated strong performance, similar to the results achieved in 2021. More detailed information of 2022 in table 2.

Table 3 provides a detailed breakdown of the 2023 results. The optimisation outperformed historical data in 2023 by 43% in terms of profit per tonne. However, this year was exceptionally difficult in terms of cost environment, and therefore absolute numbers in terms of total profits from the year, are below half of profits from the years 2021 and 2022. Therefore, even though differences in percentages were notable during 2023, absolute differences were not as great.

Another challenge for the simulation tool was the low utilization rate observed in 2023. During January, February, October, and November, Outokumpu operated at very low utilization rates, with the third production line sometimes completely idle. The simulation tool relies on average historical utilization rates to calculate maximum capacities, which led to further difficulties. In months where utilization was significantly below the average, the calculated capacity was insufficient to meet production requirements. This issue was particularly evident in the second cycle of 2023, where the simulation could not identify a feasible solution that satisfied all constraints. Specifically, the model encountered conflicts due to exceeding maximum capacity for certain months and production lines while failing to meet the profit threshold in August.

Table 3 2023 PPT: Simulation tools production plan vs historical data

4-month cycle	**Total cost**	**Profit Per Tonne, PPT**	**Notes**
1-4/2023	Difference, % -9,8%	Difference, % +91,9%	January proved to be the most favourable month for the optimization tool, primarily due to significantly lower electricity costs—over 65% cheaper than in February, March, and April. Despite relatively high prices for other parameters, the optimization model prioritized production in January, resulting in excellent profitability for the period. While the percentage differences in profitability were significant, the absolute differences were small due to the overall low profit levels in this cycle. The optimization tool recommended minimal production for the mid-cycle months, February and March, while emphasizing production in January and April. Although a similar trend was observed in the historical data, the fluctuations were considerably more stable.
5-8/2023	Difference, % -7,6%	Difference, % +66,4%	The percentage difference between the unoptimized and optimized plans was notable, despite the absolute profitability figures being relatively low. The cost environment in May and June showed less variation compared to the end of the cycle. The optimization tool allocated production

			primarily to May and June, while July was assigned minimal production, and no production was scheduled for August. In August, the cost of production exceeded sales revenue for all products, making production unprofitable. Additionally, the four-month production requirements were met earlier in the cycle, allowing the tool to avoid unprofitable production in August.
9-12/2023	Difference, % -3,9%	Difference, % +10,4%	This cycle followed a more traditional pattern, with increased production volume compared to the previous period. The simulation tool performed comparably to the historical non-optimized data, achieving a modest improvement of 10% in profitability.
Year 2023 totals	Difference, % -7%	Difference, % +42,96%	The year 2023 presented possibilities for more profitable allocation, although high costs and relatively low volumes compared to previous years brought challenges for the tool.

The year 2023 presented a significantly more diverse and challenging environment for the simulation tool compared to the previous two years. However, simulation tool was able to allocate production in way that profitability was improved in contrast to unoptimized data, even though absolute differences were not as high as in 2021 and 2022. As illustrated in Figure 18, the trendlines for 2023 differ notably from those of prior years.

While years 2021 and 2022 showed occasionally quite volatile changes in profitability per produced tonne, 2023, overall weighted profitability remain within more static range.

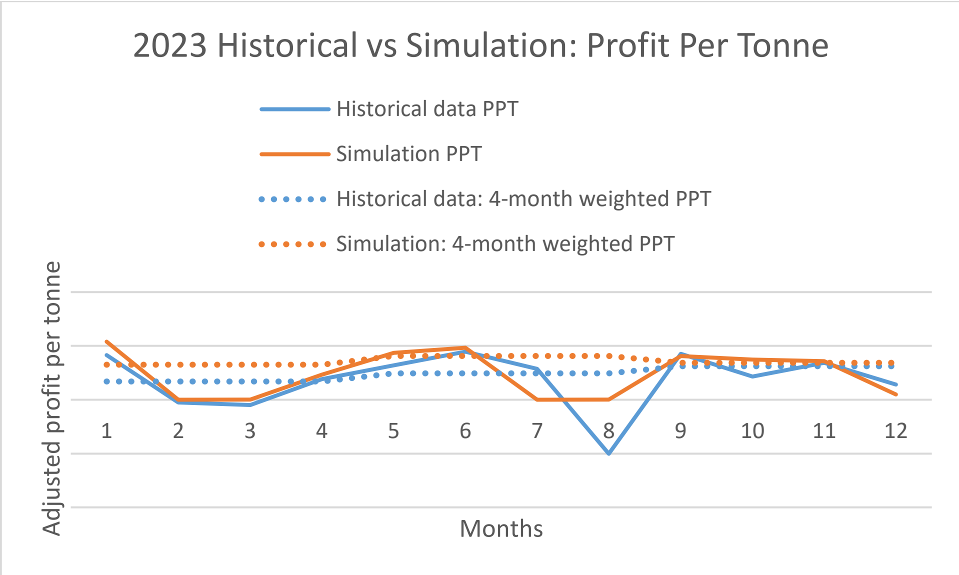


Figure 18 2023 Simulation vs Historical – PPT

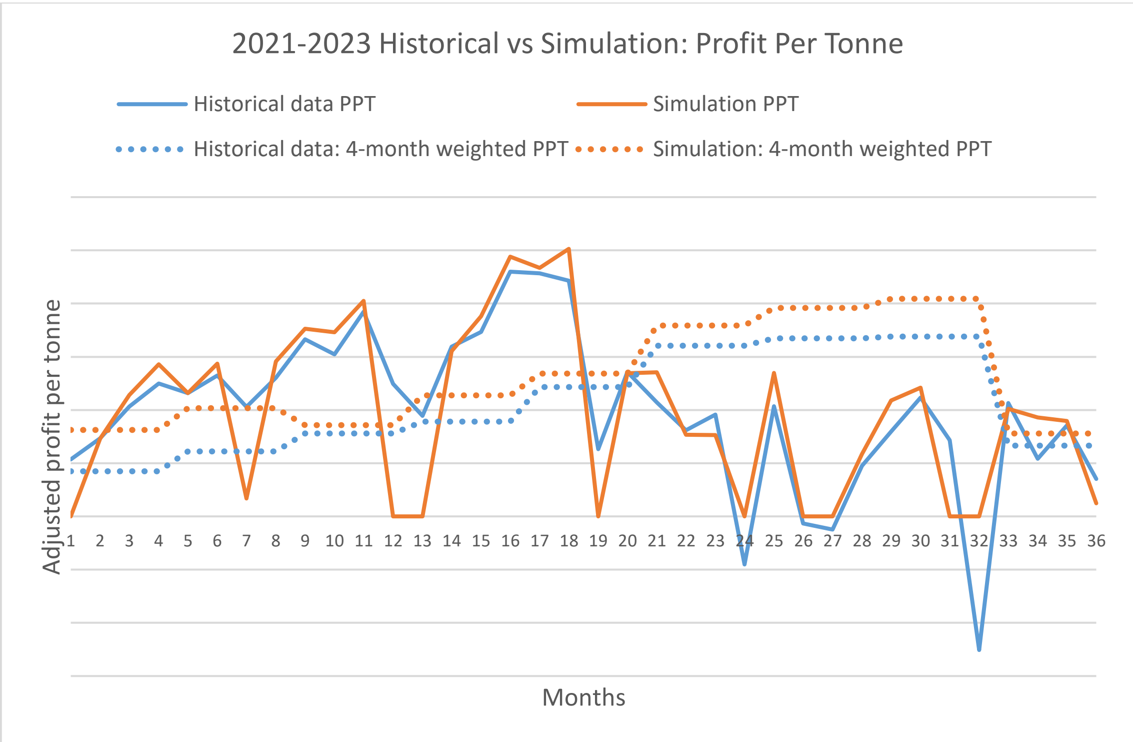


Figure 19 21-23 Simulation vs Historical – PPT

In conclusion, the comparison between the historical unoptimized production plan and the optimization tool's suggested production plan demonstrates an improvement in profitability per tonne. The optimization tool achieved a 15% increase in profitability in 2021, 18.8% increase in 2022, and 43% increase in 2023. These results were possible due to tool's ability to allocate production within the four-month cycles to months where profitability was highest. It is important to note that this test was theoretical, with results adjusted to three four-month cycles within a 12-month period due to technical limitations of the simulation tool. Also unoptimized data that was used in comparison, did not have the advantage of knowing future raw material prices as certain values, which gave optimisation tool an advantage. Clear benefit margins cannot be calculated as direct comparison cannot be made with equal data.

The findings in improved profitability from 2021 to 2023 indicate that MILP-based optimization can effectively enhance profitability in terms of profit per tonne. However, to clarify the effectiveness of the simulation tool, equal data set based on same future forecasts should be tested, in order to prove equal opportunities for both methods.

5.6 Sensitivity analysis of key parameters

To compare the performance of the simulation model's proposed production plan against the historical production plan, a sensitivity analysis was conducted. Sensitivity analysis is often employed in linear programming-based optimization problems to assess the influence of specific parameters on outcomes. For the case company, this analysis provides valuable insights into the impact of various production inputs on the cost structure.

OpenSolver does not support automatic sensitivity analysis when binary variables are used, as is the case in this simulation. Binary variables make the solution non-continuous, meaning that small parameter changes can trigger significant shifts in binary constraints, potentially restricting certain production activities entirely. In this simulation, a binary constraint ensures that only one product can be produced on each production line at any given time.

The sensitivity analysis was performed manually by testing different scenarios in which the pricing of key parameters—electricity, coke, and raw materials—varies. A custom dataset was created to ensure a consistent testing environment. All parameter values were based on the average values from the 2023 historical dataset, and production volumes were held constant across all products to isolate the impact of parameter changes on profitability. It is worth noting that, while assessing the impact of product pricing on profitability would be valuable, this study relies on static historical data, making it difficult to realistically simulate product pricing effects due to the absence of a market relationship factor.

The sensitivity analysis explored scenarios with incremental price changes of ±5%, ±10%, and ±15%. After each price adjustment, the effect on profit per tonne was recorded. The simulation tool was used to perform each test, with a new optimization executed after every price change.

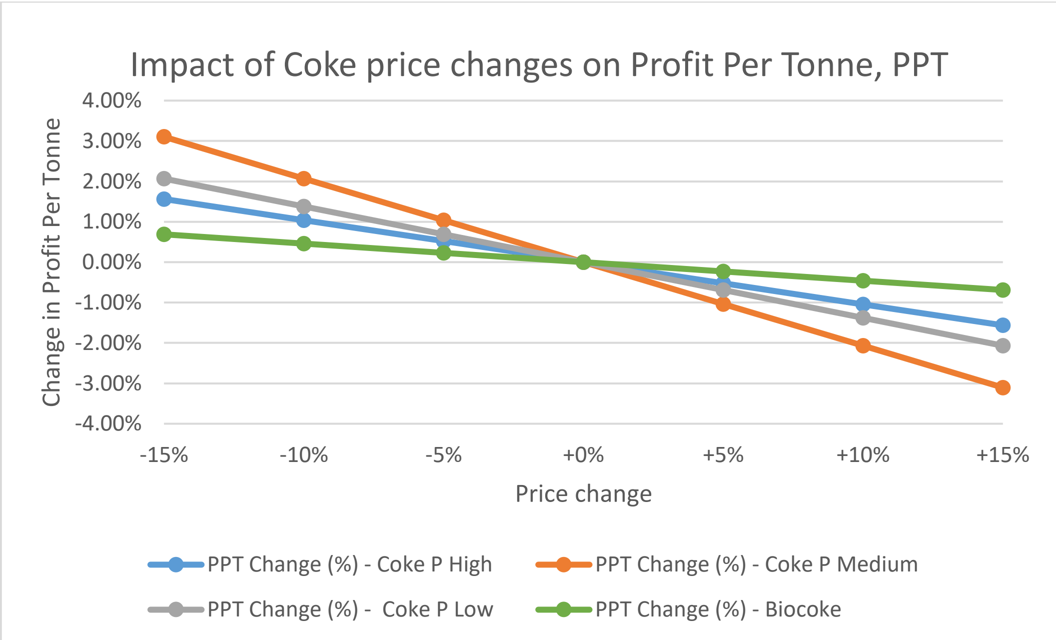


Figure 20 Impact of Coke price changes on PPT

Preliminary insights suggest that electricity and coke costs represent the largest variable costs across all products. However, the linearity of these cost impacts is uncertain, as fixed costs are also embedded within the profit-per-tonne margins. The first sensitivity test was conducted on the coke grades used in production, as shown in Figure 20.

Figure 20 illustrates the effects of coke price changes on profitability. The x-axis represents price changes from -15% to +15%, while the y-axis indicates the impact on total profit per tonne. Each coke grade was tested independently to assess how sensitive profitability is to price fluctuations for each type. Among them, Coke P Medium had the most significant impact on profitability, as it is the most commonly used in production. Biocoke, currently used in smaller quantities and primarily driven by specific customer demands, had the smallest impact on profitability; however, its role is expected to grow in the future.

With sensitivity analysis data set, effects of all coke prices were completely linear to profitability.

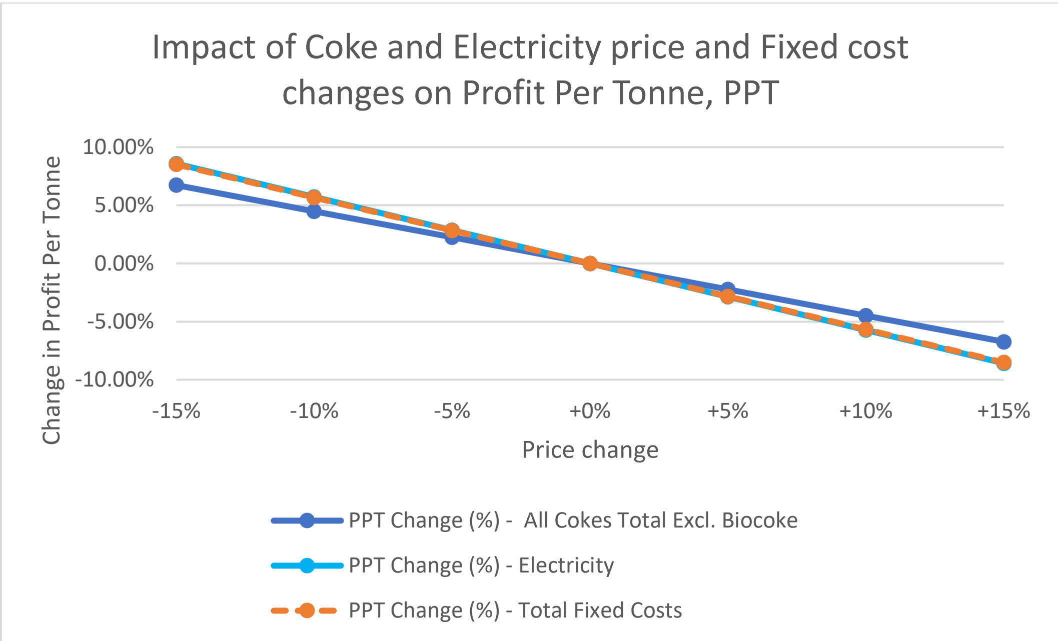


Figure 21 Impact of Coke and Electricity price and Fixed Cost changes on PPT

Figure 20 illustrates the primary factors influencing profitability, as identified in prior analyses. The continuous purple line represents the impact on profit per tonne when prices for all coke grades (excluding biocoke, due to its smaller contribution) are increased or decreased simultaneously. The blue line shows the effect of electricity price changes, and the orange dotted line reflects the impact of incremental increases in fixed costs, as indicated on the x-axis.

As shown in Figure 21, the effects of each factor on profitability are linear. Fixed costs and electricity prices have nearly identical impacts on profit per tonne, with both showing a consistent linear relationship across price increases and decreases. Notably, changes in electricity prices have a greater influence on profitability than changes in fixed costs or coke prices within the selected range of ±15%.

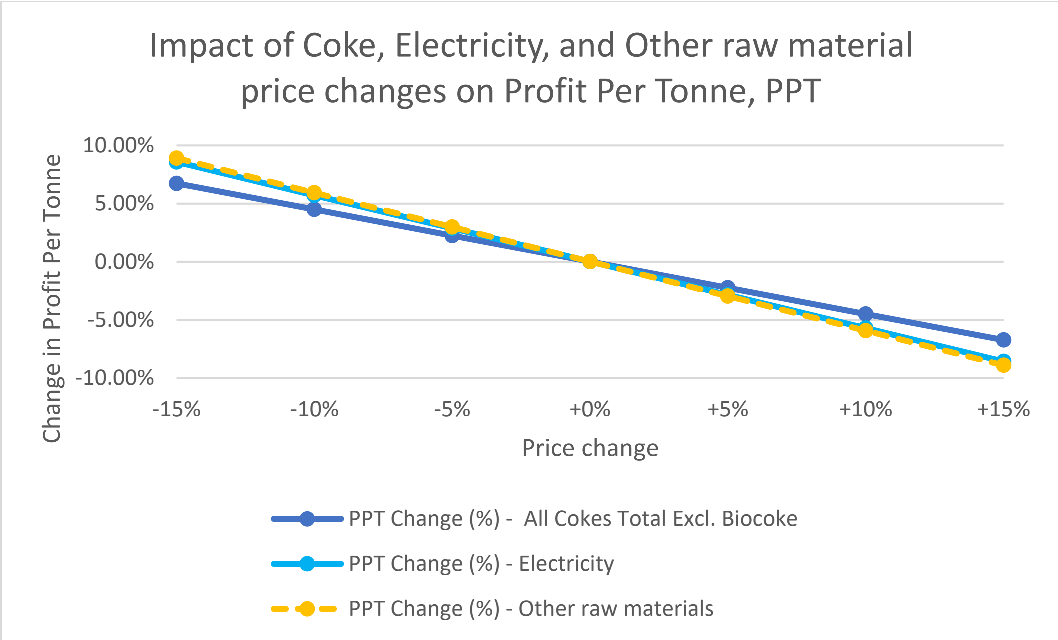


Figure 22 Impact of variable cost factors on PPT

Next scenario examined the effects of all variable costs included in production planning, including all coke grades (excluding biocoke), electricity, and other raw materials. The latter category includes ferrochrome fine concentrate, upgraded lumpy ore, quartz, and bentonite. Prices for all raw materials were increased or decreased simultaneously. Figure 22 illustrates the impact of these variables on profitability.

Among these factors, raw material prices had the most significant impact on profit per tonne, with just a margin difference to electricity price. Coke was third yet still showed big influence on profitability. All of the cost factors were linear in terms of effect to the profitability.

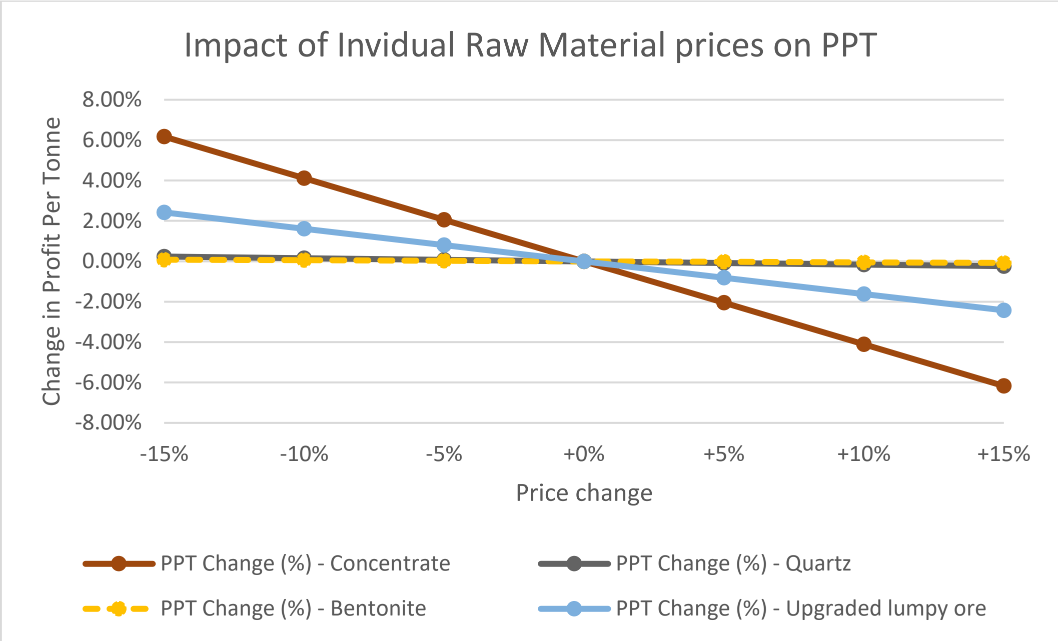


Figure 23 Impact of Raw material prices on PPT

Figure 23 illustrates the impact of various parameter prices on profit per tonne (PPT). While the effects across all parameters were linear, their magnitudes varied significantly. Among the variable costs, electricity had the largest impact on profitability, followed by coke prices, particularly the most commonly used grade, P Medium. Fixed costs also showed a notable influence, closely aligning with electricity in terms of overall impact.

Among raw materials, fine concentrate demonstrated the highest impact on profit per tonne (PPT), followed by upgraded lumpy ores. In contrast, bentonite and quartz had minimal influence on profitability due to their relatively low consumption levels in the production process. Biocoke, while currently having a smaller overall impact, is expected to play a more significant role in the future with the rising demand for "green" ferro-chrome. The effects of all parameters are presented in Figure 24 below.

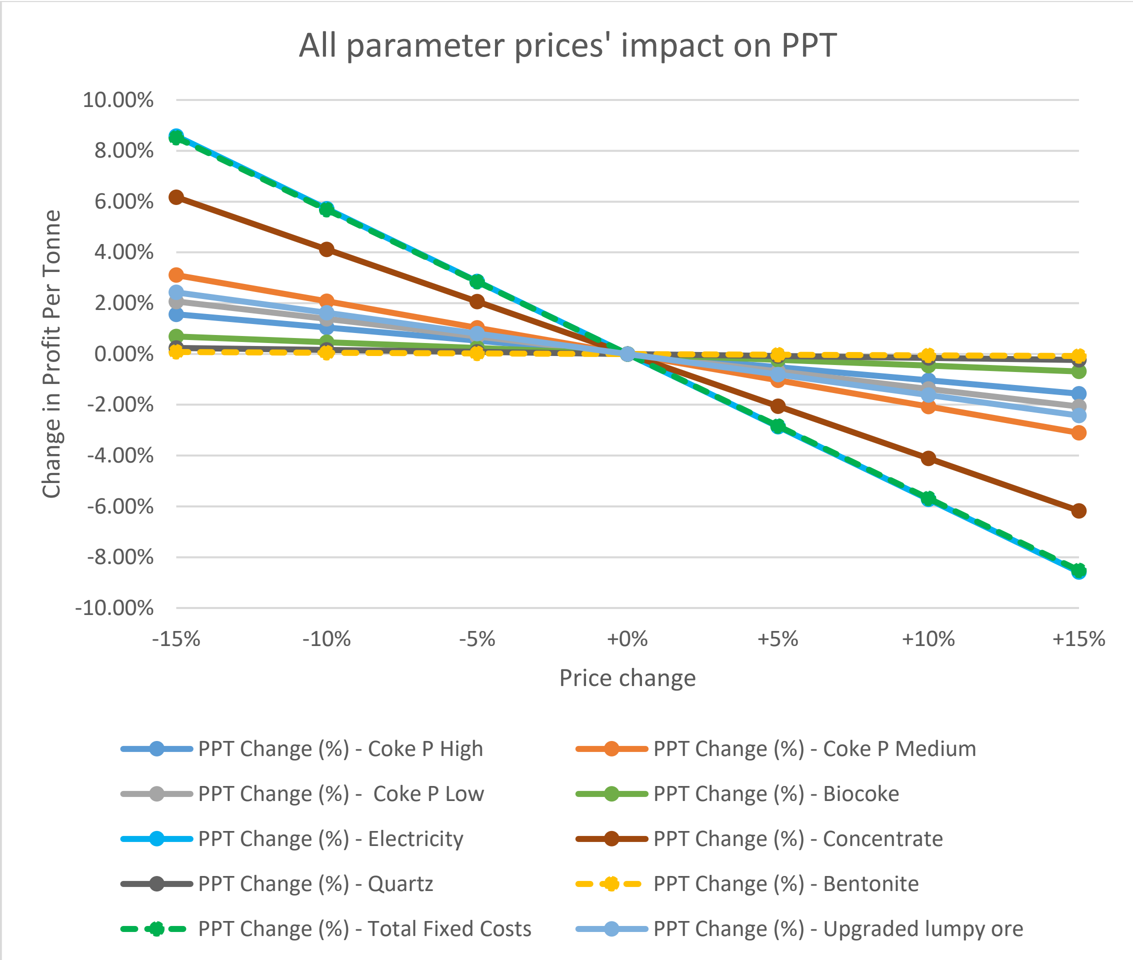


Figure 24 All parameters - Impact on PPT

In conclusion, electricity costs emerged as the dominant variable cost factor impacting profitability, followed by coke prices. Fixed costs also play a crucial role and should be carefully prioritized in production planning. Among raw materials, fine concentrate and upgraded lumpy ores remain the most critical factors for cost optimization, while bentonite and quartz have negligible effects.

This sensitivity test was conducted using a custom dataset based on average historical values from Q4 2023. However, some adjustments were made to the dataset to ensure feasible optimization during testing. It is important to note that the dataset does not fully reflect realistic production conditions, as it assumed very low production volumes and equal sales volumes across all products. Additionally, the relative weight of parameters may vary depending on the product mix in production. Nevertheless, these results provide insight into their impact in a static, equalized scenario.

For instance, in producing greener ferrochrome, the impact of biocoke will increase. The insights from this graph can guide decision-makers in understanding parameter effects when considering different ferrochrome grades and product mixes. Sensitivity analysis proves to be a valuable tool for tactical production planning, especially when integrated into the simulation tool. If automated updates are implemented within the simulation tool, sensitivity analysis could dynamically assess the effects of various parameters as optimization is conducted. This integration would allow decision-makers to evaluate different production plans and evaluate how profitability is affected by changes in parameters, product mix, volumes, and pricing, ultimately enhancing the decision-making process.

Sensitivity analysis can serve as a valuable tool in tactical production planning. However, relying solely on sensitivity analysis to develop production plans requires a deep understanding of both the production planning process and the surrounding operational environment. This thesis does not provide a comprehensive evaluation of the effectiveness of using sensitivity analysis as a standalone method, and therefore no definitive recommendations can be made due to the lack of sufficient evidence.

The implementation and usage of sensitivity analysis vary widely depending on the approach used. Its value in tactical production planning is determined by how effectively the analysis addresses the specific needs of decision-makers and supports their decision-making process.

6 Conclusions

6.1 Best practices for optimising production planning

Optimizing production planning processes covers a broad and versatile field where the term "optimization" can imply a range of objectives. In this study, we focused on enhancing profitability, minimizing costs, and improving forecasting accuracy and resource utilization within a constrained environment. Linear programming (LP) has emerged as a powerful tool for optimizing production planning when resources are limited, and operational constraints apply. LP's adaptability to diverse scenarios makes it particularly valuable in the context of production planning. For more complex optimization problems, techniques such as multi-objective linear programming, fuzzy linear programming, and mixed-integer linear programming (MILP) help to cover more complex characteristics of production planning, and examples of these techniques were presented in literature review.

6.2 Reflecting literature methods with case example

Many of the research papers mentioned in the literature review included inventories as a part of the optimisation. Impact of including inventories in optimisation of this thesis' presented case can only be hypothesized, since it was scoped out from the simulation tool. However, including inventories as a part of the simulation tool can at least achieve more realistic presentation of real-world planning environment, in terms of resource requirements and usage, as well as pricing raw materials as based on inventory value. Available resources are relevant constraint when production is planned, and therefore could be recommended as a part of complete planning tool.

Optimisation in this case was based on historical data, but in real scenario, planning relies on forecasts of future demand and cost environment. Since forecasts inherently involve uncertainty, the use of fuzzy variables is recommended. In this context, fuzzy variables could represent ranges for parameter pricing, such as electricity costs. Production planning could be conducted using fuzzy parameters that account for high, medium, and low-cost scenarios within a specific period. Optimization could then align with strategic decisions that reflect the tactical planning objectives—whether to avoid risk, prioritize the most probable scenario, or pursue another strategy.

Fuzzy variables could be implemented in several ways in this case study. For instance, during an earlier phase of the simulation tool's development, a scenario involving potential extra demand was considered. This extra demand, beyond the required levels for contract customers, often arises from uncertain or estimated market opportunities. If this additional demand proves profitable, it could be included in the production plan. Treating this extra demand as a fuzzy variable would allow decision-makers to incorporate it into the optimization process based on its perceived feasibility and profitability.

Another significant theme in the literature review was the benefits of multi-objective linear programming (MOLP). In this case, MOLP aligns well with the use of fuzzy variables, as scenarios characterized by uncertainty and variability could be addressed through multi-objective optimization. MOLP could identify optimal solutions based on selected strategies, incorporating trade-offs between competing objectives. For instance, it could evaluate the feasibility of various product campaigns, each with distinct cost structures, and provide a mathematical basis for decision-making aligned with strategic priorities.

MOLP can be implemented in multiple ways, but specific recommendations cannot be made without rigorous testing to evaluate its benefits in this context. Multi-objective optimization requires assigning weights to different objectives, which introduces a high degree of uncertainty and a potential risk of biases. Therefore, successful implementation of MOLP requires extensive research to ensure overall objectivity and robustness in the optimization process.

While MOLP offers the flexibility to address complex, multi-faceted problems, it is important to note that single-objective optimization remains the most effective approach for addressing clearly defined production planning problems, as it yields the most precise solutions under specific conditions.

6.3 Implementation of linear programming to case study

This case study examined the potential of a MILP-based model to serve as a simulation tool for production planning within the case organization. MILP was chosen as an optimisation model, due to findings in literature review and natural progression during the development phase of the optimisation tool. Basic linear programming algorithm was used initially, but as the need for binary variables arose, model was modified to present mixed-integer linear programming model.

In literature review, multi-objective linear programming, MOLP was also discovered. Benefits of optimising multiple objectives could have been beneficial in context of this case production planning problem, but due to technical restrictions of used software, MOLP could not be assessed thoroughly enough.

A three-year historical dataset provided a realistic testing environment to evaluate the capabilities of a MILP -based tool, operated in Excel Solver. Results indicated that the model could improve profitability, measured as profit per tonne, across various historical scenarios — especially in high-volume conditions where clear monthly differences in raw material costs and input parameters were present.

The simulation tool's capabilities should be evaluated using the same forecast data currently employed in production planning to ensure a fair comparison without granting the simulation an information advantage. To enable the simulation tool to calculate feasible optimization solutions, certain adjustments were made, and some details were omitted. The simulation tool can be utilized to explore and compare different production planning alternatives at a basic level. Users can modify parameter values, such as raw material prices, to observe their impact on the tool's optimized solutions and conduct manual sensitivity analyses. By adjusting these parameters, decision-makers can evaluate how changes in cost factors influence production planning outcomes. While the tool has certain simplifications and limitations, it can still provide useful insights into cost structures and the mathematical allocation of production for a selected four-month period.

In its current state, the simulation tool lacks the precision needed to accurately model and present critical elements such as inventory usage, valuation, and production cycles. These limitations reduce its value as a practical planning tool. The tool is insufficient for modelling and optimizing all scenarios required in production planning and suffers from several technical restrictions. These restrictions include limited operational reliability, the time-intensive process of developing and modifying algorithms, and overall usability challenges.

Many of these issues are tied to the use of Excel Solver as the software platform in this context. The scale and complexity of the optimization problem exceed the capabilities of Excel Solver, making it impractical for everyday production planning. To address these limitations, alternative platforms should be explored and evaluated to ensure the tool's functionality, scalability, and feasibility for regular use.

6.4 Factors influencing optimal profitability

The sensitivity analysis evaluated the impact of various cost factors on profitability. The results confirmed the initial hypothesis that electricity prices are the most influential factor, having the greatest impact on profitability when adjusted by ±15% in a static environment. Coke prices ranked as the second-largest variable cost factor, followed by fine concentrate, and upgraded lumpy ore in third and fourth place, respectively. Bentonite and quartz, however, showed negligible effects on profitability in this sensitivity test. In addition to variable costs, fixed costs were also found to significantly influence profitability, though their impact on optimization is static. These findings highlight the importance of considering key cost factors when determining product mixes in different cost environments. Products that rely heavily on cost factors with high profitability sensitivity should be prioritized during periods when parameter prices are more predictable in forecasts. Hypothetically, product campaigns with higher cost structures and, consequently, higher risks should be prioritized during periods when forecasts indicate a stable cost environment. Based on historical data, months such as January, December, June, and August exhibited greater volatility in parameter pricing. In contrast, mid-season periods, such as spring and autumn, tend to represent a more static cost environment, making them more suitable for such campaigns. However, this should be studied more thoroughly to provide certain recommendations.

6.5 Validity of results

Certain elements of the tactical production planning process had to be adjusted for this simulation tool, primarily due to four-month simulation cycle. Optimization results were derived from historical monthly data within the organization, with a four-month planning cycle used as the maximum level of detail achievable in this context. Although a four-month cycle may not fully represent real-world planning practices of case company, this approach was necessary due to tool limitations.

Consequently, parameters such as electricity and coke prices were applied as monthly averages, despite intra-month fluctuations in electricity prices and inventory-based variations in coke pricing driven by replenishment cycles. Inventories were out of scope in this research. Additionally, simulation tool allowed production of one product per line per month due to technical constraints restricting the real-world utilization representation. For future research, it would be valuable to incorporate more realistic production line utilization that accounts for line-specific startups and shutdowns, enhancing the model’s fidelity to actual production dynamics.

6.6 For future research and development

This thesis presented a tentative test simulation of linear programming for case organisation, and if it can prove to be valuable tool to support tactical production planning and profitability. To ensure successful implementation of linear programming-based optimisation tool and releasing full potential, following things should be considered and studied further. Things to consider:

1. Detail and scope of parameters used in the model – optimisation requires fluctuations in parameters such as electricity and coke price changes. To be able to optimise in certain time frame, for example monthly production, data needs to be in weekly or bi-weekly level at least. Only then can optimisation be done in shorter cycles. Best outcomes would be when data can be actively verified and updated based on latest information. This would require certain database and interfaces for relevant stakeholders involved in the process.

2. Suitable software – Excel Solver is not capable of handling larger datasets efficiently or yet capable of easy modifications to optimisation algorithm. Even minor changes to algorithm have to be copied into every index and line and as the amount of used data increases, need for manual work multiplies. Versatile and dynamic optimisation model requires valid software, where separate interfaces for users and developers can be found.
3. Tailored algorithm to cover case specific details – Mixed-integer linear programming can help to improve solving tactical production planning problems. However, best outcome could be achieved with modified algorithms that consider all relevant case -specific details. In this case model should include e.g. start-up and run-down of production, advantages of larger production quantities and quantity discounts in costs, inventory control and valuation.
4. Risk management when planning involves uncertainties – If production planning seeks to test more special planning scenarios, or certain parameter data is not available for optimisation, fuzzy optimisation becomes relevant. Fuzzy linear optimisation involves different techniques to mitigate risks and seek optimal outcomes that acknowledge uncertainty. These techniques include setting weights on which things to avoid or emphasize based on risk structure, and multi-objective optimisation where goal is to find best overall outcome for optimisation. In case environment, risks and so-called fuzzy variables could include e.g. future demand and pricing of products and materials, and success ratio of producing different product grades.

Completely own theme and field of future research can be found in the use of artificial intelligence and machine learning. Machine learning based solutions that use existing historical data could be used to improve forecasting accuracy with improved prediction of future material costs and selling prices as well as demand patterns. Optimisation algorithms such as MILP itself could be improved to be more adaptive as AI could identify different case related patterns or restrictions. This field would require another research to cover full potential in case organisation’s production planning context.

Covering mentioned things, linear programming could be successfully implemented to tactical production planning as a valuable tool to support decision making. However, models always include assumptions and adjustments and can not completely replace traditional planning processes. As Hillier & Lieberman (2015) stated, mathematical models are intended to be an idealized representation of the real problems, and certain amount of simplifying assumptions are required if the model is set to be solved (Hillier & Lieberman, Introduction To Operations Research 10th Ed, 2015).

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