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Research paper

Price-increasing competition by heterogeneous marketing strategies☆

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ABSTRACT

We study the price competition between two firms that have to advertise to reach buyers. We show that differences in advertising strategies can result in price-increasing competition. Advertising can be either informative or persuasive; the former informs buyers about the price of the standard good, and the latter increases buyers' willingness to pay of the branded good. We show that when firms engage in different advertising methods, the pricing is in pure strategies, and if branding is effective, the firm using persuasive advertising asks a price that is higher than the monopoly price of the branded good (and, a fortiori, higher than the monopoly price of the standard good).

1. Introduction

A standard economic theoretical insight suggests that competition decreases market prices. However, there is some empirical evidence that this is not always the case. For example, using supermarket scanner data, Ward et al. (2002) show that when private-label food products entered the market, prices of name-brand goods *increased*. Similarly, Perloff et al. (1995) illustrate that prices can *increase* with entry of new products using data from the antiulcer drug market. One theoretical explanation for these empirical evidences is given by Chen and Riordan (2008) who demonstrate that competition can increase prices if there are two differentiated goods on the market, and product differentiation steepens the demand curve. However, Deck and Gu (2012) experimentally test the theoretical predictions of Chen and Riordan (2008) and find that price-increasing competition is rare in the way Chen and Riordan suggest. So, alternative theories are being called for.

In this paper, we offer an explanation for price-increasing competition from the perspective of *advertising*. We construct a duopoly model in which firms have to advertise to reach buyers. Advertising is either informative or persuasive. We call the good promoted by informative advertising a *standard good* and the good advertised persuasively a *branded good*. In informative advertising, a firm tries to make the price of its product known to buyers, and the more intensive it is, the more likely a buyer is to learn about the price and the firm that offers the product. Alternatively, a firm that engages in persuasive advertising sets up a campaign that results in an increase in the buyers' willingness to pay for its product. We assume that persuasive advertising does not mention the price and that buyers simply have an expectation about it; in equilibrium, the expectation is naturally fulfilled.[1]

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[1] In most cases, no price is mentioned in the advertisements (see Anderson and Renault 2006). However, retail advertising (such as newspapers) typically includes a price quote (see Driver 2017, Waldman and Jensen 2016). Essentially, Ward et al. (2002) noticed that companies with branded products tend to engage in promotions without price ever being mentioned, in contrast to private label goods.

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We analyse the following three scenarios: (i) Both firms engage in informative advertising, (ii) Both firms engage in persuasive advertising, and (iii) One firm advertises informatively, while the other advertises persuasively.

The first two cases are mainly to introduce how different advertising methods work, and how they affect pricing. If both firms engage in informative advertising, the firms use mixed strategies in pricing, and the intensity of advertising does not depend on the realised price. In equilibrium, the prices are below the monopoly price and the profits are non-monotone in advertising costs (Proposition 1).[2] If both firms brand their products, we assume that the demand of the branded goods increase, but less than in the case where there is only one firm persuasively advertising.[3] We show that there exists an equilibrium in which both firms demand monopoly prices (Proposition 2). This is due to the assumption that persuasive advertising does not inform about prices, and so there are no profitable deviations from the monopoly price. Consequently, if both firms engage in persuasive advertising, they share monopoly profits.

Our main contribution is to derive equilibrium pricing in the case where one firm advertises informatively, while the other engages in persuasive advertising (Proposition 3). Pricing is in pure strategies, and the firm using persuasive advertising asks a price that is *higher* than the monopoly price of the *branded* good (and, a fortiori, higher than the monopoly price of the standard good). This is a new result, and the heuristic is as follows.

In equilibrium, the firms use pure strategies, and the firm with persuasive advertising asks for a higher price than the firm with informative advertising. There is a stochastic element in informative advertising, and it does not reach all buyers, while persuasive advertising is assumed to reach everyone. The firm with persuasive advertising gets two kinds of buyers: (i) buyers who are not aware of the firm with informative advertising, and (ii) high-valuation buyers who have learned about the price of the informative advertiser.[4] For the first group, the optimal price would be the monopoly price, and if the price is some P, all buyers with a valuation of at least this high are willing to buy. However, the buyers in the latter group have the option of buying the standard good at a low price. Consequently, only buyers who have relatively high valuations (above P) can guarantee a higher surplus than buying the standard good. For these buyers, it would be optimal to raise the price even higher (remember that they find out the price only by visiting the firm). Raising the price, however, means that the firm loses some buyers from the first group. This trade-off leads to an equilibrium price that is at least the monopoly price and, for some parameter values, strictly higher than the monopoly price of the branded good.

Our results have the following interpretation. If there were initially a monopolist selling a branded good and another firm with informative advertising entered the market, this would result in an increase in the price of the branded good. This phenomenon is consistent with empirical evidence of Ward et al. (2002) finding that the entry of the private label (a standard good) is correlated with increased prices of the name brand (a branded good). In contrast to Chen and Riordan (2008), in our model, product differentiation itself does not steepen the demand curve and prices; it is purely the strategic choice of the brand firm to increase the price as a reaction to competition.

In our analysis, we use linear demand curves. This is a simplification, but not necessary for our main results. In order to achieve an equilibrium in which the branded good is priced above its monopoly price, it is necessary that (i) the branding is effective, i.e. the demand curve deepens sufficiently (Proposition 3), (ii) the price of the branded good is not observable (Proposition 4), and (iii) informative advertising does not reach all buyers (Remark 2). These assumptions guarantee that the firms divide the demand so that the branding firm captures the buyers with high valuations, the informatively advertising firm the buyers with low valuations, and all the buyers with intermediate valuations are randomly shared by the persuasively and informatively advertising firms resulting in the situation in which the branded good is priced above its monopoly price (see Fig. 1).[5] By similar arguments, the assumption that the branding campaign reaches all buyers is not a necessary condition. If the branding firm only captured a random share of buyers, it would still be profitable to ask a higher price than the monopoly price if the firms share the demand in the way described above.

1.1. Literature

The literature on advertising is vast; a comprehensive, though not too recent, overview is given by Bagwell (2007) and a more recent one by Renault (2015). Typically, advertising research is focused either on informative or on persuasive advertising. An account from the perspective of public policy is given by Driver (2017). As Driver (2017, p. 3) notes, a formal way of differentiating between the two types of advertising is elasticity: "Formally, informative advertising does not make demand less price elastic, while persuasive advertising does." In our setting, the demand curve for the good that is persuasively advertised is steeper than that for the good that is informatively advertised, which informally corresponds to less elastic demand.

A pioneering work of informative advertising in an equilibrium setting is Butters (1977), where firms send advertisements (ads hereafter) to buyers in a random manner. This results in frictions, as not every buyer gets an ad and some get several ads. The resulting equilibrium pricing is in mixed strategies. Similarly, Stahl (1994) studies the price competition of several firms that can engage in informative advertising. Our results in Section 3.1 replicate the findings of Stahl: the firms use mixed strategies in pricing, but the intensity of advertising is constant. The heterogeneity of production costs leads to a surprisingly rich set of potential outcomes

[2] As in Stahl (1994) and Guimarães (1995).

[3] This assumption takes into account possible deteriorating effects of having two branding campaigns advertising at the same time. That is, we assume that the effectiveness of persuasive advertising is decreasing in the number of firms engaged in branding.

[4] In Fig. 1, the first group of buyers is illustrated by the solid blue and the latter group by the dashed blue line.

[5] There might be also other possible explanations than heterogeneous advertising strategies for making the firms to divide the demand in this way.

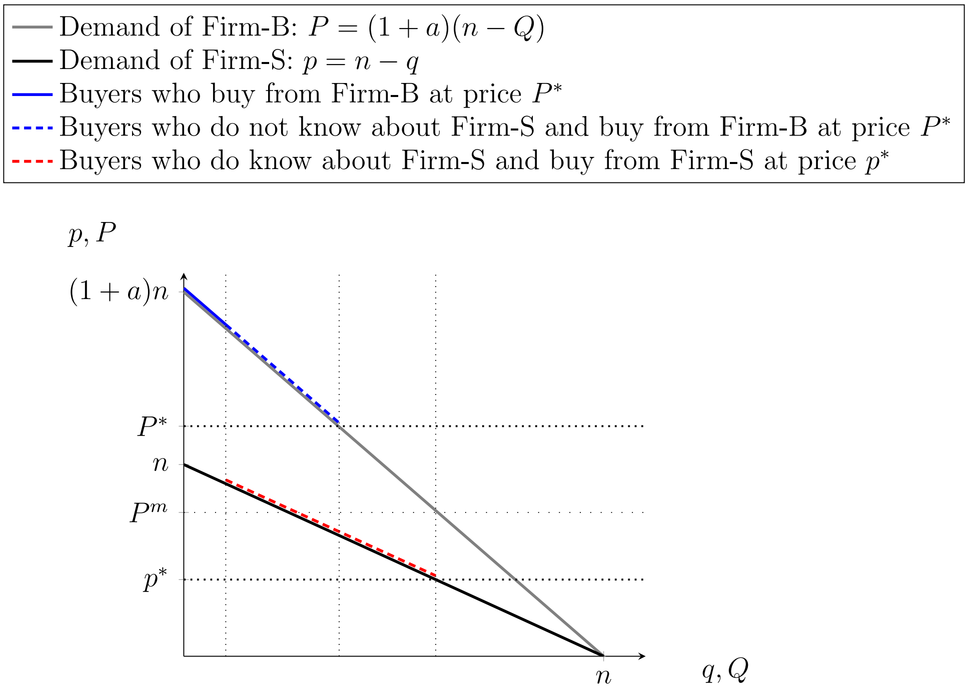

Fig. 1. Inverse demand curves of different goods.

(see Hoernig 2002, Baye and Morgan 1999). Allowing firms to not publicise their prices may sustain equilibria where the firms ask the monopoly price (see Salonen 1992). Baye and Morgan (2001) study a market for homogeneous goods, where firms have the option to post their prices on a gatekeeper's website. Buyers choose whether to buy the goods from the local monopolist's boutique or check the prices in the neighbouring town from the Internet and buy the goods from there. Baye and Morgan find that the advertised prices are lower than the unadvertised monopoly prices in equilibrium. In contrast, Kultti and Pekkarinen (2021) show that in the Butters's (1977) model with concave advertising costs, higher prices are advertised more than lower prices.

In a competitive setting, the persuasive advertising literature finds most applications in a differentiated good case where the modelling utilises the Hotelling-line. The main interest there is between advertising intensity and the increase in utility provided by persuasive advertising; prices are assumed to be public knowledge. An example is Jiang and Srinivasan (2016) where the main result is that favourable shocks, for example in production costs, are complementary to persuasive advertising. They also show that symmetric improvements in advertising efficiency may lead to a decrease in profitability. Our work differs from this in two respects. First, we do not consider horizontally differentiated products, but a situation that is reminiscent of vertical differentiation: the ordering of the buyers as to valuation is identical for the standard and brand good. If one buyer values the standard good more than another buyer, that applies also to the brand good. Second, persuasive advertising is not informative about the price, but buyers find it out only once they contact the firm. To the best of our knowledge, we are the first to study the competition between informative and persuasive advertising.

1.2. Structure

In Section 2, we build our model. In Sections 3.1 and 3.2, we study the case where both firms compete with informative advertising or persuasive advertising, respectively. Our main analysis is given in Section 3.3, where we study the case where one firm engages in informative advertising and the other in persuasive advertising. In Section 3.4, we give conditions under which the situations in Sections Section 3.1 constitute an equilibrium when firms can choose the advertising method. We provide all our proofs in Appendices A and B to improve the readability.

2. Environment

The demand for standard goods in the economy is given by $q = n - p$. The interpretation is that there is a measure n of buyers, each with a unit demand, and their valuations are uniformly distributed in the interval $[0, n]$. A firm that advertises informatively chooses an intensity of advertising $\lambda > 0$ at cost $A\lambda$ where $A > 0$ is the constant marginal cost. The buyers are exposed to a number of ads that is positively related to the intensity. A buyer has to be exposed to at least one ad to become aware of the offer. When a firm chooses intensity λ a buyer is exposed to k ads with probability $e^{-\lambda}\frac{\lambda^k}{k!}$ and he observes at least one ad with probability $1-e^{-\lambda}$.[6]

We assume that if a firm engages in persuasive advertising, it is so intensive that everyone becomes aware of the product. This seems to be the case with many products, but it also simplifies the analysis, as it is enough to assume that a firm has to pay a fixed cost $C > 0$ to complete a persuasive advertising campaign.[7] Given that the other firm advertises informatively, persuasive advertising changes the firm's demand curve into $Q = n - \frac{P}{1+a}$ or $P = (1+a)(n-Q)$ where the parameter $a > 0$ measures the proportional increase in utility.[8] Notice that persuasive advertising affects only the willingness to pay; the measure of buyers remains the same on the higher demand curve. The marginal costs of production are assumed to be zero for both firms, and there are no fixed costs.

If there are two firms that engage in different advertising methods, we name the firms so that Firm-S relies on informative advertising and sells standard goods, while Firm-B engages in persuasive advertising and sells branded goods. The prices and quantities of the demand curve associated with informative advertising are written in the lower case, while the corresponding quantities of the demand curve associated with persuasive advertising are written in the upper case. A buyer whose valuation of the persuasively advertised good is P has a valuation $p = \frac{P}{1+a}$ of the informatively advertised good. Note also that the monopoly prices are $p^m = \frac{n}{2}$ and $P^m = \frac{(1+a)n}{2}$, and the corresponding monopoly quantities satisfy $q^m = Q^m = \frac{n}{2}$. In Fig. 1, we illustrate the demand curves of the different advertising methods.

The order of events is such that the firms first advertise, and then they choose the prices. The choices could also be simultaneous, and there would be a difference only if the form of advertising were a choice variable, as then the possible deviations and responses to them would differ depending on whether the game is sequential or not.

3. Results

In this section, we first derive equilibria when the methods of advertising are fixed. First, we concentrate on the case where both firms use informative advertising and derive equilibrium pricing and advertising strategies, which can also be found in Stahl (1994) with n firms. Second, we derive an equilibrium of both firms branding their goods. After that, we analyse the setup where one firm engages in informative advertising and the other in persuasive advertising. In Section 3.4, we derive conditions under which the equilibria given in Sections 3.1–3.3 constitute equilibria when also the mode of advertising is a choice variable.

3.1. Informative vs informative advertising

Assume that there are two competing firms that both engage in informative advertising. The equilibrium pricing is in continuous mixed strategies on some interval $[p^L, p^m]$ where $p^m = \frac{n}{2}$ is the monopoly price.[9] Denote by G the mixed strategy on $[p^L, p^m]$ and by $R(p) = p(n-p)$ the revenue from price p. Assume that one firm draws the price x from G and chooses the intensity of the advertising according to $\lambda(x)$. That is, in principle, there is a unique advertising intensity $\lambda(p)$ for each price $p \in [p^L, p^m]$. Then if the other firm chooses price $p \in [p^L, p^m]$ and an arbitrary intensity λ, its expected profit is

$$R(p)\left(1-e^{-\lambda}\right)\left\{\int_{p^L}^{p} e^{-\lambda(x)}dG(x) + (1-G(p))\right\} - A\lambda. \tag{1}$$

The firm gets revenue $R(p)$, but it is proportionally reduced as not every buyer learns about the offer or gets a better offer from the other firm. The share $e^{-\lambda}$ of buyers does not receive any ads (uninformed buyers), and the proportion $1-e^{-\lambda}$ receives ads and thus are potential buyers (informed buyers, but they still might buy from the other firm). The first term in the curly brackets aggregates the probability that the other firm chooses a price $x < p$; the proportion of buyers who do not learn about this is $e^{-\lambda(x)}$, and they only know about the firm with price p. The second term is the probability that the other firm chooses a price higher than p; in this case, it does not matter whether buyers learn about it because those who know about the firm with price p choose it. The final term is the cost of advertising.

Using this notation, we introduce our first result, whose proof is in Appendix A.

Proposition 1. *Assume that advertising costs are relatively low: $A < \left(\frac{n}{2}\right)^2$. In a mixed strategy equilibrium, the price is determined by distribution*

$$G(p) = (1-\Gamma)^{-1}\left(1 - \frac{R(p^m)}{R(p)}\Gamma\right),$$

where $p \in [p^L, p^m]$ such that

$$p^L = \frac{n\left(1-\sqrt{1-\Gamma}\right)}{2} < \frac{n}{2} = p^m,$$

[6] One interpretation is that the firm buys advertisements in the street, newspapers and internet, and a buyer spots them in a random manner leading to a Poisson-distribution with parameter λ.

[7] We assume that C is small enough for the firm to participate. Assuming that the persuasive advertising is not perfect and randomly reaches a positive share of buyers would not change the main insights.

[8] See Dai and Koh (2024) for a model where the monopolist can flexibly manipulate the demand curve by choosing an advertisement plan.

[9] In equilibrium, the mixed strategy is atomless and its support is an interval, see the proof from Stahl (1994).

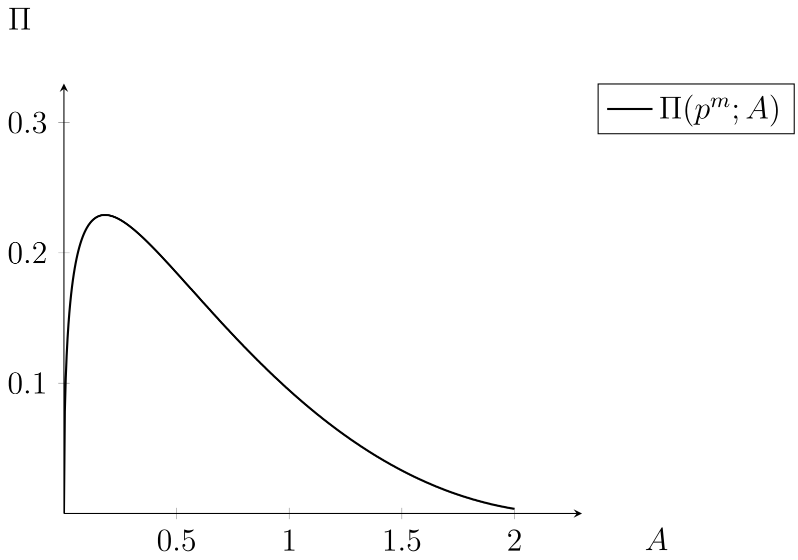

Fig. 2. Equilibrium profits as a function of marginal costs of advertising when $n = 3$.

and the advertising intensity is given by

$$\lambda = \ln \Gamma^{-1},$$

where $\Gamma \equiv \sqrt{\frac{4A}{n^2}}$.

Because informative advertising does not reach all buyers, each firm reaches some buyers who do not know about the other firm. This gives some monopoly power to the firms in these cases, which leads to pricing in mixed strategies, G. If the demand parameter n is high, the price interval $[p_L, p^m]$ is wide — the lower bound p_L shrinks and the upper bound p^m grows as n increases. Furthermore, the mixed strategy G with the demand parameter n' is first-order stochastically dominated by the strategy with the demand parameter $n'' < n'$. Similarly, the mixed strategy G with the marginal cost parameter A' is first-order stochastically dominated by the strategy with the cost parameter $A'' > A'$. That is, an increase in demand results in lower prices on average, and high advertising costs lead to higher prices on average.

The equilibrium advertising intensity is independent of prices, and in a symmetric equilibrium both firms choose the same intensity. In a mixed strategy equilibrium, each price yields the same expected return, and it is the return that determines intensity. The intensity is decreasing in the costs of advertising. That is, once the costs of ads become cheaper, the firms send more ads, and more buyers become informed about the products.

Although the lowest price in the support of the mixed strategy is increasing in A, the profit is not monotone in A. For small values of A the profits increase in A first, and then start to decrease, reaching zero level when $A = \frac{n^2}{4}$. In other words, if the costs of advertising are sufficiently large, $A \geq \left(\frac{n}{2}\right)^2$, then both firms operating in the market would make negative profits, and hence there is no duopoly equilibrium, where both firms are active. These observations are illustrated in Fig. 2.

3.2. Persuasive vs persuasive advertising

Suppose that if both firms engage in persuasive advertising, the demand curve is steepened by $(1 + a')$ for $0 \leq a' \leq a$. That is, if both firms are branding, there may be some deteriorating effects on how effective the persuasion is compared to the case where only one firm brands its good. In the polar case, where $a' = 0$, there can be only one good with a high valuation on the market, and if another firm also engages in persuasive advertising, both goods become standard goods. In the other polar case, where $a' = a$, both firms can equally benefit from a branding campaign and achieve high valuation.

As firms do not advertise prices, there exists an equilibrium such that both firms ask for the monopoly price $\frac{(1+a')n}{2}$ and the buyers flip a fair coin on which firm to contact. When the cost of persuasive advertising satisfies $C < \frac{(1+a')n^2}{8}$ both firms make positive profits in the equilibrium. It is clear that any deviation from the monopoly price is not profitable in this case. This reasoning gives us the following result.

Proposition 2. *Assume that the costs of persuasive advertising are relatively low* $C < \frac{(1+a')n^2}{8}$. *Then there exists an equilibrium where both firms engage in persuasive advertising and ask for the monopoly price* $\frac{(1+a')n}{2}$.

3.3. Informative vs persuasive advertising

Suppose that Firm-S relies on informative advertising and sells the standard good, while Firm-B engages in persuasive advertising and sells the branded good. That is, a buyer who values the standard good from Firm-S at p, values the branded good from Firm-B at $P = (1 + a)p$.

The pure-strategy equilibrium is given by the following result.

Proposition 3. *Assume that both firms are active in the market and $n > 4\sqrt{\frac{aA}{1+a}}$. The equilibrium price of the firm with persuasive advertising is*

$$P^*(a) = \begin{cases} P^m & \text{if } a \le \frac{1}{2} \\ \bar{P} & \text{if } a > \frac{1}{2}, \end{cases}$$

the price of the firm with informative advertising is

$$p^*(a) = \begin{cases} \left(\frac{1-a}{1+a}\right) P^m & \text{if } a \le \frac{1}{2} \\ \left(\frac{1}{2(1+a)}\right) \bar{P} & \text{if } a > \frac{1}{2}, \end{cases}$$

and the optimal level of informative advertising is

$$e^{-\lambda(a)} = \begin{cases} \frac{4A}{n^2(1-a^2)} & \text{if } a \le \frac{1}{2} \\ \frac{4a(1+a)A}{\bar{P}^2} & \text{if } a > \frac{1}{2}, \end{cases}$$

where $P^m = \frac{(1+a)n}{2}$ and $\bar{P} = \frac{2a}{1+2a}\left(P^m + \sqrt{(P^m)^2 - \frac{1+a}{a}A(4a^2-1)}\right)$.

The interpretation of the result is as follows.

Ineffective persuasive advertising. If the persuasive advertising does not affect the buyers' valuations relatively much, $a \le 1/2$, then Firm-B asks the monopoly price P^m, and Firm-S asks lower price than its monopoly price $\frac{n}{2}(1-a) < \frac{n}{2} = p^m$. The reason is as follows. Since persuasive advertising increases the buyers' willingness to pay relatively little, it is profitable for Firm-B to focus on only those buyers who are not informed about the standard good. Since informative advertising captures buyers randomly along the demand curve, the best strategy for Firm-B is to ask the monopoly price P^m. If Firm-S asked the monopoly price $\frac{n}{2}$, it would not get any buyers, since all buyers know about the branded good. Lowering the price from the monopoly price and capturing all the informed buyers up to buyer with valuation n maximises the profits of Firm-S. These pricing strategies are not affected by the cost of informative advertising; for any parameter value A, it is optimal for Firm-B and Firm-S to ask the prices P^m and $\frac{n}{2}(1-a)$, respectively, and for Firm-S to adjust the intensity of informative advertising accordingly. As the effectiveness of persuasive advertising gets higher (but still less than half), the monopoly price $P^m(a)$ increases. In response to this, Firm-S lowers its price so that all informed buyers buy the standard good. This implies that there is less revenue for Firm-S and hence it is optimal to engage less in informative advertising.

Effective persuasive advertising. When $a > 1/2$, the persuasive advertising is effective. In equilibrium, Firm-B asks more than its monopoly price: the equilibrium price of Firm-B satisfies $\bar{P} > P^m$ under the assumption $n > 4\sqrt{\frac{aA}{1+a}}$ (and as long as Firm-S is active in the market). As a grows, the price of Firm-B, $\bar{P}$, increases and is between the monopoly price P^m and $2P^m$ assuming that the marginal cost of advertising for Firm-S, A, goes simultaneously down such that Firm-S stays in the market.[10] It is closer to P^m when A is so large that Firm-S finds it barely profitable to stay in the market, and closer to $2P^m$ when A is close to zero such that Firm-S advertises intensively. The latter result, in particular, may be surprising, as one might expect that as a grows, the firm with persuasive advertising can ignore the other firm and charge the monopoly price. But this is not the case when the marginal cost of advertising for Firm-S is low. To see this, assume that informative advertising is very intensive so that almost all buyers are informed also about the standard good. With prices intact, if a grows slightly, all buyers are willing to pay a little more for the branded good. Raising the price of the branded good is profitable for Firm-B because (i) there are more high-value buyers who want to buy from Firm-B, and (ii) Firm-B loses only informed buyers with relatively low valuations. As a result the firms split the demand: high-value buyers get the branded good and low-value buyers get the standard good.

As branding becomes more effective (a increases), Firm-B increases its price more above the monopoly price and the informatively advertising firm reacts by increasing both its prices and advertising intensity. This is due to the fact that once Firm-B concentrates on a smaller group of buyers who have high valuations, there are more potential buyers to capture for Firm-S.[11] However, as a grows, the price of Firm-B increases faster than the price of Firm-S. In other words, the more effective the persuasive advertising, the more spread are the prices of the branded and standard good. We illustrate the equilibrium pricing in Fig. 3.

[10] Since P^* is decreasing in A for $a > 1/2$, the upper bound $2\frac{2a}{1+2a}P^m$ can be achieved by setting $A = 0$. For high a, we know that $\frac{2a}{1+2a} \approx 1$ and therefore the upper bound is close to $2P^m$.

[11] Note that if one focuses on the equilibrium prices and forgets about the underlying competition of the firms, then the firm with persuasive advertising always has its price on the unit elastic point when $a \le \frac{1}{2}$ and on the elastic part when $a > \frac{1}{2}$, while the firm with informative advertising always has its price on the inelastic part of the demand curve.

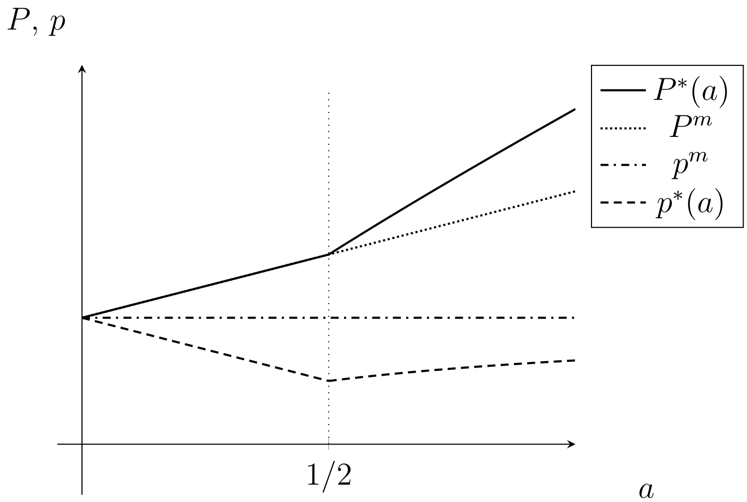

Fig. 3. Equilibrium pricing under persuasive and informative advertising as functions of the effectiveness of persuasive advertising a.

We highlight our main finding by the following remark.

Remark 1. *If branding is effective, the firm using persuasive advertising asks a price that is higher than the monopoly price of the branded good (and, a fortiori, higher than the monopoly price of the standard good).*

So far, we have not studied whether it is profitable for both firms to be active in the market. This naturally requires that the costs of advertising are relatively low. For informative advertising costs, we have the following result. This can be derived using the equilibrium profits given in the proof of Proposition 3.

Corollary 1. *The firm with informative advertising is active in the market as long as the marginal cost of advertising satisfies*

$$A < \begin{cases} \frac{n^2(1-a^2)}{4} & \text{if } a \leq \frac{1}{2} \\ \frac{n^2(1+a)}{16a} & \text{if } a > \frac{1}{2}. \end{cases}$$

It is almost immediate that as a grows the maximum possible value for A goes down. It is clear that there always exist values for $C > 0$ such that the firm with persuasive advertising makes positive profits. In other words, we can always find parameter values A and C such that the profits of both firms are positive. However, if the cost of informative advertising is zero, Firm-S reaches all buyers, and the persuasively advertising firm does not get any buyers.

Remark 2. *If the cost of informative advertising goes to zero, the informatively advertising firm reaches all buyers, and the persuasively advertising firm does not get any buyers. Consequently, the persuasively advertising firm is not active in the market, and the configuration given in Proposition 3 is not an equilibrium.*

To see this remark, consider the equilibrium given in Proposition 3. Let the costs of informative advertising go to zero, $A \to 0$. Now, Firm-S reaches all buyers since the optimal informatively advertising intensity goes to infinity. In this case, the persuasively advertising firm's price goes to $P^* = \frac{4a}{1+2a}P^m > P^m$ for $a > \frac{1}{2}$ and the informatively advertising firm's price becomes $p^* = \frac{P^*}{2(1+a)}$. Now, the buyer who is indifferent between P^* and p^* is the one who has the highest valuation on the demand curve. That is, all buyers buy standard goods and, consequently, the persuasively advertising firm does not get any buyers. This implies that for any positive branding costs $C > 0$, Firm-B is not active in the market if the informative advertising reaches all buyers.

Lastly, we show that, under perfect information, the price of the branded good is not higher than the monopoly price. In other words, for price-increasing competition, it is indeed necessary that persuasive advertising not only steepens the demand curve but also does not inform about the price.

Proposition 4. *If the price in a persuasive advertisement is observable, then in equilibrium the price of the branded good is at most the monopoly price.*

3.4. Extension

In the previous sections, we take the form of advertising as given and do not analyse advertising as a strategic choice. This gives us an understanding of how different forms of advertising affect pricing. In this section, we assume that the firms can choose their

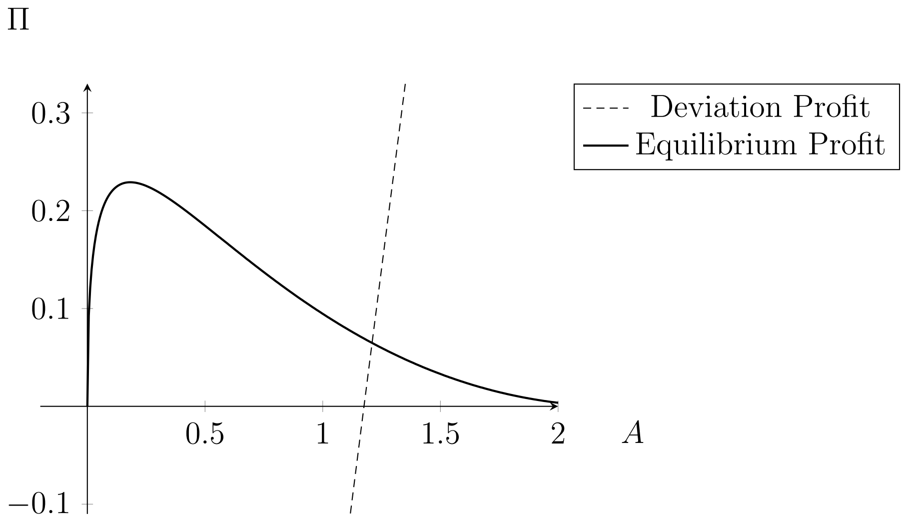

Fig. 4. Equilibrium and deviation profits of an informative advertiser as a function of marginal costs of advertising, A, when $n = 3$, $a = 2/5$, and $C = 6/5$.

advertising methods, and derive conditions under which the equilibria given in Propositions 1–3 constitute pure strategy equilibria in a setting where the firms choose the form of advertising. Here, we assume that the firms make the advertising and pricing decisions simultaneously so that a non-deviating firm cannot react to a deviation.

Next, we give the equilibrium conditions in the following order. We start with the case where both firms choose informative advertising, then we consider the case where both choose persuasive advertising, and, lastly, we study the case where one firm chooses informative advertising and the other persuasive. Since the upcoming derivations are uninspiring, we provide only the conditions in this section and leave the calculations to Appendix B. Let the vector of all the primitives of the model be denoted by $\Theta = (a, a', n, A, C)$ hereafter.

Informative vs informative advertising. If both firms choose informative advertising, it should not be profitable to choose persuasive advertising. In equilibrium, both firms earn the same profits:

$$\frac{n^2}{4}\Gamma(1-\Gamma) - A\ln\left(\Gamma^{-1}\right), \tag{2}$$

where $\Gamma = \sqrt{\frac{4A}{n^2}}$. Now, if one firm deviates to persuasive advertising, the deviator's profits are

$$\frac{n}{2}\sqrt{A}(1+a) + \frac{n-2\sqrt{A}}{2}(1+a)\frac{a-1}{2a}n$$
$$+\ \frac{n-2\sqrt{A}}{2}\frac{1+a}{a}\left[-\frac{2}{n} - \frac{1}{n}\ln\left(p^L\right) + \frac{1}{n}\ln\left(n-p^L\right) + \frac{1}{n-p^L}\right] - C. \tag{3}$$

If parameters Θ are chosen such that (2) is greater than the maximum of zero and (3), then there exists an equilibrium in which both firms engage in informative advertising.

Making the cost of persuasive advertising C very large certainly makes a deviation not profitable, but here we can be slightly more explicit. In Appendix B, we show that whenever $a < \frac{1}{2}, C > \frac{n^2}{8}$, and $\frac{n^2}{4} > A > \left(1-a^2\right)^2\frac{n^2}{4}$, there are no profitable deviations. Since this condition is somewhat tedious to derive, we content ourselves to give just an example here that illustrates this phenomenon. Suppose that $a = 2/5$ and $A < 6/5$. The former being small tells us that persuasive advertising does not increase the willingness to pay very much. The latter being small means that the other firm advertises a lot and many buyers are aware of its product. Choosing parameter values $n = 3$ and $C = 6/5$ guarantees that a deviation is not profitable. Fig. 4 depicts the profit of a deviator and the profit in equilibrium for particular parameter values.

Persuasive vs persuasive advertising. Next, we analyse the situation in which both firms engage in persuasive advertising. We keep assuming that if both firms are branding, then the demand is increased by $1 + a'$ for $0 \le a' \le a$. In equilibrium, each firm earns

$$\frac{(1+a')n^2}{8} - C. \tag{4}$$

Deviating to informative advertising in the case where the other firm engages in persuasive advertising and asks the monopoly price $(1+a')n/2$ yields

$$\frac{(1+a')n}{16} - \frac{4a'(1+a')^2A}{n} - A\ln\left(\frac{n^2}{64a'(1+a')A}\right). \tag{5}$$

If parameters Θ are chosen such that (4) is greater than the maximum of zero and (5), then both firms choosing persuasive advertising with the monopoly price $(1+a')n/2$ is an equilibrium. Clearly, if the cost of informative advertising A is relatively high and C relatively low, then the persuasive advertising is the optimal choice of advertising. For example, the equilibrium condition is satisfied by choosing $a' = 1$, $n = 8$, $C < 15 + 2A - A\ln(2A)$, and $A \leq \frac{1}{2}$.

Informative vs persuasive advertising. Assume next that one firm engages in informative advertising and the other in persuasive advertising. From the proof of Proposition 3 we get that the equilibrium profits for the informatively advertising firm are

$$\frac{(P^*)^2}{4a(1+a)} - A - A\ln\left(\frac{(P^*)^2}{4a(1+a)A}\right), \tag{6}$$

and for the branding firm:

$$nP^* + 2A - \frac{(P^*)^2(1+2a)}{2a(1+a)} - C. \tag{7}$$

Now, if the informatively advertising firm deviates to persuasion, we assume that the buyers observe the deviation and expect the deviator to ask a lower price than the other persuasive firm. In this case, the deviator captures the whole demand, and hence its optimal price is the monopoly price $\frac{(1+a')n}{2}$. Consequently, the deviator's profits are

$$\frac{(1+a')n^2}{4} - C \tag{8}$$

The persuasively advertising firm finds it profitable to deviate either to p^* or p^m. Therefore, the deviation to informative advertising with price p^* yields

$$\frac{P^*(2(1+a)n - P^*)}{4(1+a)^2} - A - A\ln\left(\frac{P^*(2(1+a)n - P^*)}{4(1+a)^2 A}\right), \tag{9}$$

and the deviation to the monopoly price yields

$$\frac{n^2 a(1+a)A}{(P^*)^2} - A - A\ln\left(\frac{a(1+a)n^2}{(P^*)^2}\right), \tag{10}$$

where P^* is given by Proposition 3. It is an equilibrium if one firm engages in persuasive advertising and the other in informative advertising, if parameters Θ are chosen such that (6) is greater than the maximum of zero and (8), and (7) is greater than the maximum of zero, (9), and (10).

Giving explicit conditions for the parameters Θ that satisfy all these conditions is not straightforward. One way to find suitable parameter values is to set C such that (8) is zero, and then focus on the case in which the equilibrium profits are positive and the persuasively advertising firm does not want to deviate to informative advertising. Again, it is clear that if a is high and A is chosen so that both firms are active in the market, but the informative advertising is not very intensive, then the deviation is not profitable. As an example, let us set $n = 10$, $a' = 1/2$, $a = 1$, and $C = 75/2$. Then for all $A \in [12, 15]$ the equilibrium conditions are satisfied. In this case, we have effective branding and so the persuasively advertising firm asks higher price than the monopoly price. For example, for $A = 12$, we have $P^* = \frac{10+8\sqrt{7}}{6} > 10 = P^m$.

When branding is not effective, $a < \frac{1}{2}$, the firm with informatively advertising chooses the monopoly price, and the optimal deviation is to a price just below it such that the deviator catches the whole market. In Appendix B, we calculate the profit of the deviator and show that when costs of advertising satisfy $A < 4n^2\left(1-a^2\right)^2$ and $C < \frac{A}{1-a} - \frac{n^2}{4}\left(1-a^2\right) + A + A\ln\frac{n^2\left(1-a^2\right)}{4A}$, the persuasive advertiser does not find it profitable to deviate. As an example, if we set $n = 3$, $a = 2/5$, and $C = 6/5$, then having (approximately) $A > 19/20$ the deviation is not profitable. This situation is depicted in Fig. 5.

4. Conclusion

We study pricing under asymmetric advertising methods. We show that there exists an equilibrium where one firm advertises informatively, while the other advertises persuasively. In this equilibrium, the pricing is in pure strategies, and the persuasively advertising firm asks for a price higher than its monopoly price. Essentially, this follows from the different advertising methods, leading the firms to share the demand such that the persuasively advertising firm targets high-value buyers, and the informatively advertising firm sells to low-value buyers. Our analysis offers an explanation for price-increasing competition from the perspective of advertising.

CRediT authorship contribution statement

Klaus Kultti: Formal analysis, Writing – original draft, Writing – review & editing. **Teemu Pekkarinen:** Formal analysis, Writing – original draft, Writing – review & editing.

Declaration of competing interest

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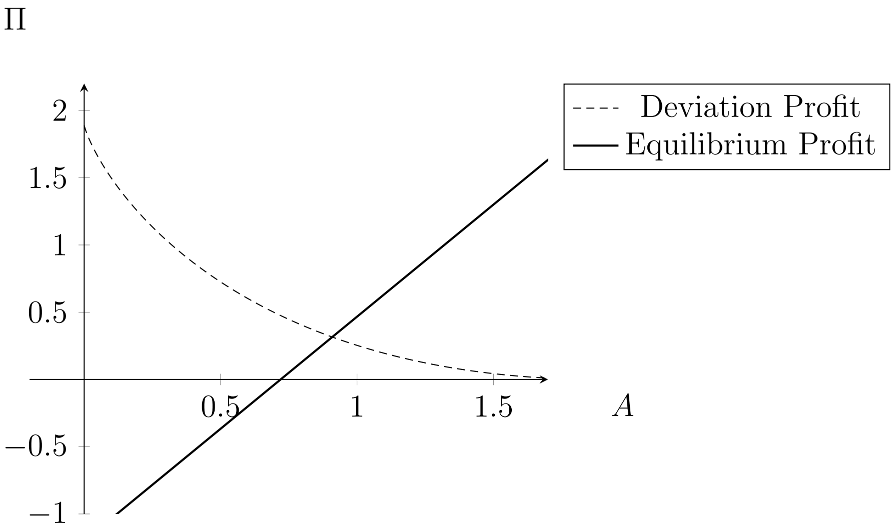

Fig. 5. Equilibrium and deviation profits of a persuasive advertiser as a function of marginal costs of advertising, A, when $n = 3$, $a = 2/5$, and $C = 6/5$.

Appendix A. Proofs

A.1. Proof of Proposition 1

Take the derivative of (1) with respect to λ to determine the optimal choice, $\lambda(p)$, from

$$R(p)\left\{\int_{p^L}^{p} e^{-\lambda(x)}dG(x) + (1-G(p))\right\} e^{-\lambda} - A = 0.$$

Denote $K(p) \equiv \int_{p^L}^{p} e^{-\lambda(x)}g(x)dx$ where we assume that there exists density $g(x) = dG(x)$. Inserting the optimal choice $\lambda(p)$ into a firm's profit function (1), it can be expressed as

$$\Pi(p) = R(p)\,(K(p)+1-G(p)) - A - A\ln\frac{R(p)\,(K(p)+1-G(p))}{A}. \tag{11}$$

In equilibrium the profit is constant for $p \in \left(p^L, p^m\right)$ or the derivative of the profit function is zero, $\frac{\partial \Pi(p)}{\partial p} = 0$. Or more explicitly

$$\left\{1 - \frac{A}{R(p)\,(K(p)+1-G(p))}\right\} \left\{(n-2p)\,(K(p)+1-G(p)) - R(p)\left(1-e^{-\lambda(p)}\right)g(p)\right\} = 0. \tag{12}$$

The first term is strictly greater than zero since $R(p)\,(K(p)+1-G(p)) - A$ is the revenue at the optimal advertising intensity, i.e., profit plus the cost of advertising (see (11)). Consequently, it is the second term that is zero.

Totally differentiating the first order condition (12) one finds that

$$\frac{\partial \lambda(p)}{\partial p} = \frac{(n-2p)\,(K(p)+1-G(p)) - R(p)\left(1-e^{-\lambda(p)}\right)g(p)}{R(p)\,(K(p)+1-G(p))}.$$

But this is zero because the numerator is zero; the intensity is independent of the price, and in a symmetric equilibrium both firms choose the same intensity.

This simplifies the analysis considerably as is evident below. The intensity does not depend on the price as in a mixed strategy equilibrium each price yields the same expected return, and it is the return that determines intensity.

Determine the optimal intensity by considering a firm that chooses price p^m:

$$\max_{\tilde{\lambda}} \left(R\left(p^m\right)e^{-\lambda}\left(1-e^{-\tilde{\lambda}}\right) - A\tilde{\lambda}\right),$$

where λ is the intensity chosen in equilibrium by the other firm. Taking the first-order condition and solving at point $\tilde{\lambda} = \lambda$ we find that the advertising intensity in equilibrium is given by $e^{-\lambda} = \sqrt{\frac{A}{R(p^m)}}$ or $\lambda = \ln\left(\sqrt{\frac{n^2}{4A}}\right)$.

To simplify notation let us denote $\sqrt{\frac{A}{R(p^m)}} = \sqrt{\frac{4A}{n^2}} \equiv \Gamma$. With this notation we have $\lambda = \ln \Gamma^{-1}$, and $\Pi\left(p^m\right) = R\left(p^m\right)\Gamma\,(1-\Gamma) - A\ln\Gamma^{-1}$.

To determine the lowest price in the support of the mixed strategy G, we use the fact that the expected profit is the same for all prices in the support:

$$\Pi\left(p^m\right) = \Pi\left(p^L\right) = R\left(p^L\right)(1-\Gamma) - A\ln\Gamma^{-1}. \tag{13}$$

As demonstrated above the cost of advertising is the same for both firms, and the only differences in the profits are the revenues, and that the lower price firm gets to sell to all buyers who become aware of it. Simplifying (13) yields a second degree equation in p^L with the solution

$$p^L = \frac{n\left(1-\sqrt{1-\Gamma}\right)}{2}.$$

Next we derive the mixed strategy G by studying a firm that chooses price $p \in \left(p^L, p^m\right)$. Its profit is given by

$$\Pi(p) = R(p)\Gamma(1-\Gamma)G(p) + R(p)(1-\Gamma)(1-G(p)) - A\ln\Gamma^{-1}.$$

From condition $\Pi(p) = \Pi\left(p^m\right)$ one can solve the equilibrium mixed strategy given in Proposition 1. This finishes the proof.

A.2. Proof of Proposition 3

To keep track who is willing to buy from which firm, we first establish the indexing of the buyers by their willingness to buy the standard good. If a buyer is willing to buy the brand good at price P, then the same buyer is willing to buy the standard good at price $p = \frac{P}{1+a}$. If all the buyers whose willingness to buy the brand good exceeds P actually buy, the demand for the brand good is $n - p$.

Denote the buyers' expectation about Firm-B's price by P^*. A buyer in the demand curve for the standard good, indexed by $\tilde{p}$, who is aware of both goods, is indifferent between them if $\tilde{p} - p = (1+a)\tilde{p} - P^*$ from which we can solve $\tilde{p} = \frac{P^*-p}{a}$. All buyers whose index is above $\tilde{p}$ buy from Firm-B, and those with a lower index, but above p, buy from Firm-S. In Fig. 1, a buyer $\tilde{p}$ is given by the demand curve of the standard good at the left end of the red dashed line. All buyers whose index is above $\tilde{p}$ are given by the solid blue line on the demand curve for the branded good. Buyers with an index below $\tilde{p}$ are given by the dashed red and blue lines on both demand curves.

There is no guarantee that $\tilde{p}$ is less than n, and we have to study different cases. It is clear that the price of Firm-B must be at least the monopoly price. It gets those buyers who do not learn of Firm-S's offer as well as those whose valuation is above $\tilde{p}$. For the first group, the optimal price would be the monopoly price, and if $\tilde{p}$ is less than the monopoly price, the same applies to the latter group as well. If $\tilde{p}$ is more than the monopoly price, then Firm-B does not lose any buyers from the second group by raising the price to the monopoly level, but gets more from each of them. Let us proceed by assuming that $\tilde{p} < n$. The profit of Firm-S is given by

$$\Pi_S\left(p, P^*, \lambda\right) = \left(1-e^{-\lambda}\right)p\left(\tilde{p}-p\right) - A\lambda.$$

Taking the first-order conditions with respect to p and λ one finds the following

$$p = \frac{P^*}{2(1+a)} \qquad \text{and} \qquad e^{-\lambda} = \frac{4a(1+a)A}{\left(P^*\right)^2}.$$

where we have utilised the first condition in the second expression. The first-order conditions are necessary and sufficient if $P^* > \sqrt{4a(1+a)A}$. It turns out that $P^* \geq P^m$ in equilibrium and hence a sufficient condition is $n > 4\sqrt{\frac{a}{1+a}A}$.

Firm-B's profit is given by

$$\Pi_B\left(p, P, P^*, \lambda\right) = e^{-\lambda}P\left(n - \frac{P}{1+a}\right) + \left(1-e^{-\lambda}\right)P\left(n-\tilde{p}\right) - C. \tag{14}$$

The first term is the profit from those buyers who have not been exposed to Firm-S's advertising. Then everyone with valuation at least P^* for the brand good contacts Firm-B but nothing prevents Firm-B from asking some other, higher, price (asking a lower price would not be sensible). Asking this price P results in only buyers whose valuation for the brand good is above P to buy the product, i.e, demand is $n - \frac{P}{1+a}$. The second terms is the price times demand from those buyers who are aware of Firm-S's advertised price; the demand consists of the buyers with a valuation at least as high as the indifferent one, $\tilde{p}$.

In equilibrium, the optimal price has to satisfy $P = P^*$, and the first-order condition with respect to P evaluated at $P = P^*$ is given by

$$P^*\left[2ae^{-\lambda} + (1+a)\left(1-e^{-\lambda}\right)\right] = na(1+a) + \left(1-e^{-\lambda}\right)(1+a)p.$$

Above we have utilised the expression for $\tilde{p}$. Further utilising the first-order conditions for Firm-S we can solve a candidate price

$$P^* = \frac{2a(1+a)n + \sqrt{4a^2(1+a)^2n^2 + 16a(1+a)(1+2a)(1-2a)A}}{2(1+2a)}. \tag{15}$$

For small values of $a \leq \frac{1}{2}$ it turns out that the price P^* in (15) is less than P^m, and for these values of a the equilibrium has to feature price P^m for Firm-B. Another way to see this is that for small values of a even if the price were raised to $P = P^m$, making $\tilde{p}$ smaller, and $p = \frac{p^m}{2}$ the indifferent buyer's formula produces $\tilde{p} > n$. Consequently, (15) gives the equilibrium price only when

$a > \frac{1}{2}$, and for the complementary case the optimal advertising intensity and price of Firm-S have to be determined from a different objective function. For the price it is, however, more instructive to proceed a little differently. If Firm-S quotes price $p = \frac{n}{2}$ it does not get any buyers. Lowering the price to $p = \frac{n}{2}(1-a)$ guarantees that Firm-S gets all the informed buyers from $\frac{n}{2}(1-a)$ up to buyer-n, and this is the profit maximising choice. For $a < \frac{1}{2}$ the optimal level of advertising is then given by the first-order condition to

$$\Pi_S(p, P^m, \lambda) = \left(1 - e^{-\lambda}\right) \frac{n}{2}(1-a)\left(n - \frac{n}{2}(1-a)\right) - A\lambda.$$

Firm-B then gets the monopoly price from those buyers who are not informed about Firm-S's price.

A.3. Proof of Proposition 4

It is enough to consider the case where $a > \frac{1}{2}$. The first order conditions for Firm-S are the same as in our main setting. The difference in Firm-B's objective (14) (in Appendix A.2) is that the expression for the indifferent buyer $\tilde{p}$ does not contain the expectation of its price but its actual price. The first order condition for Firm-B becomes after some simplification

$$P^2(3+4a) - 2a(1+a)nP - 12a(1+a)A = 0.$$

From this we can solve the equilibrium price

$$P = \frac{a(1+a)n + \sqrt{a^2(1+a)^2n^2 + 12a(1+a)(3+4a)A}}{(3+4a)}.$$

Requiring that $P < \frac{(1+a)n}{2}$ gives a condition for A, namely $A < \frac{1+a}{16a}n^2$. Inserting the first order conditions of Firm-S in its expression for profit one shows that the profit is positive if $A < \frac{P^2}{4a(1+a)}$. Next inserting the expression for P in this inequality and solving for A one gets condition $A < \frac{1+a}{16a}n^2$, which is identical to the one that guarantees that the equilibrium price is less than the monopoly price. Thus, as long as $A < \frac{1+a}{16a}n^2$ Firm-S is in the market, and the price of Firm-B is less than the monopoly price.

Appendix B. Deviation payoffs

Here we present the calculations for the equilibrium conditions that we stated in Section 3.4.

Informative vs informative advertising. Assume that $a < \frac{1}{2}$, and assume that Firm-B deviates to persuasive advertising from the setting where both firms are supposed to advertise informatively. Then Firm-B chooses price $P = \frac{(1+a)n}{2}$. When the other firm chooses price p the indifferent buyer is given by $\tilde{p} = \frac{1+a}{a}\frac{n}{2} - \frac{1}{a}p$. We focus on the case where $\tilde{p} \le n$; equality is guaranteed when $p = \hat{p} := \frac{(1-a)n}{2}$. As the non-deviating firm uses a mixed strategy we want that the lowest price in the support is greater than $\hat{p}$ or $p^L = \frac{n}{2}\left(1 - \sqrt{1 - \sqrt{\frac{4A}{n^2}}}\right) > \hat{p} = \frac{(1-a)n}{2}$ which is equivalent to $A > \frac{n^2}{4}\left(1-a^2\right)^2$, and this is what we assume in this exercise.

Now the deviating firm's profit is given by

$$e^{-\lambda}\frac{n^2}{4}(1+a) + \int_{p^L}^{\frac{n}{2}} \left(1 - e^{-\lambda}\right) \frac{n}{2}(1+a)(n - \tilde{p})\, g(p_1) dp_1,$$

where $G'(p) = g(p) = \frac{\Gamma}{1-\Gamma} R(p^m) \frac{n-2p}{p^2(n-p)^2}$ and p^L are given in Proposition 1. To figure out the integral we first notice that in the above expression there is p in the integrand (ignoring g) only in $\tilde{p}$. Ignoring most of the constants we need to integrate $\frac{1}{a}pg(p) = p\frac{n-2p}{p^2(n-p)^2} = n\frac{1}{p(n-p)^2} - 2\frac{1}{(n-p)^2}$. The first term can be written as $n\frac{n^{-2}}{p} + n\frac{n^{-2}}{n-p} + n\frac{n^{-1}}{(n-p)^2}$. Integrating these and the second term and simplifying gives

$$-\frac{2}{n} - \frac{1}{n}\ln\left(p^L\right) + \frac{1}{n}\ln\left(n - p^L\right) + \frac{1}{n - p^L}.$$

Consequently, the expected profit from the deviation is given by

$$\frac{n}{2}\sqrt{A}(1+a) + \frac{n - 2\sqrt{A}}{2}(1+a)\frac{a-1}{2a}n$$
$$+ \frac{n - 2\sqrt{A}}{2}\frac{1+a}{a}\left[-\frac{2}{n} - \frac{1}{n}\ln\left(p^L\right) + \frac{1}{n}\ln\left(n - p^L\right) + \frac{1}{n - p^L}\right] - C.$$

We show that this is less than the profit in the mixed strategy equilibrium in which both firms advertise informatively. The profit in that case is given just before (13) as $\frac{n^2}{4}\Gamma(1-\Gamma) - A\ln\left(\Gamma^{-1}\right)$. To honour the conditions under which the above derivations are made we require the following: $C > \frac{n^2}{8}$, $a < \frac{1}{2}$ and $\frac{n^2}{4} > A > \left(1-a^2\right)^2 \frac{n^2}{4}$.[12]

[12] Here we have assumed that the mode of advertising and price are chosen simultaneously. One can show that there are parameter values that support the equilibrium also for a sequential choice where the non-deviator observes that the deviator chooses persuasive advertising.

Persuasive vs persuasive advertising. We observe from the proof of Proposition 3 that the deviator solves

$$\max_{\lambda,p}(1-e^{-\lambda})\left(\frac{1+a'}{a'}\right)p\left(\frac{n}{2}-p\right)-A\ln\lambda,$$

which has the following solution

$$p=\frac{n}{4}\qquad\text{and}\qquad e^{-\lambda}=\frac{64a'(1+a')A}{n^2},$$

for $n^2\geq 64a'(1+a')A$. The profits then are

$$\frac{(1+a')n}{16}-\frac{4a'(1+a')^2A}{n}-A\ln\left(\frac{n^2}{64a'(1+a')A}\right).$$

Informative vs persuasive advertising. Here, the persuasive advertiser deviates to informative advertising. There are two candidates for the optimal deviation. Either the deviator chooses the monopoly price and the associated advertising intensity, or the deviator chooses a price just under p^* and the associated advertising intensity.

Assume first that $a>\frac{1}{2}$. With price p^* the deviator maximises $R(p^*)\left(1-e^{-\lambda}\right)-A\lambda$ with respect to λ. The optimal level of advertising is given by $e^{-\lambda}=\frac{A}{R(p^*)}$ and the profit becomes

$$R\left(p^*\right)-A-Aln\frac{R\left(p^*\right)}{A}.$$

With price p^m the deviator maximises $R\left(p^m\right)\left(1-e^{-\lambda}\right)e^{-\lambda_i}-A\lambda$ with respect to λ, where λ_i is the other firm's level of advertising. The optimal level of advertising is given by $e^{-\lambda}=\frac{\left(P^*\right)^2}{n^2a(1+a)}$ and the profit

$$\frac{n^2a(1+a)AnP^*}{(P^*)^2}-A-Aln\frac{R\left(p^m\right)4a(1+a)}{(P^*)^2}.$$

Then assume that $a\leq\frac{1}{2}$. If the deviator asks for a price just below $\frac{n}{2}(1-a)$, its profit turns out $\frac{A}{1-a^2}-A-A\ln\frac{1}{1-a^2}$, and with the monopoly price, the profits are $\frac{n^2}{4}\left(1-a^2\right)-A-A\ln\frac{n^2\left(1-a^2\right)}{4A}$. The latter one is greater if $A<4n^2\left(1-a^2\right)^2$, and we proceed with this assumption. In the suggested equilibrium the profit of the firm with persuasive advertising is given by $\frac{A}{1-a}-C$ as can be seen by inserting the equilibrium values into (14); notice that $p=\frac{n}{2}(1-a)$ implies that $\tilde{p}=n$, and the latter term in the firm's profit vanishes. That is, whenever $a\leq 1/2$, $A<4n^2\left(1-a^2\right)^2$ and $C<\frac{A}{1-a}-\frac{n^2}{4}\left(1-a^2\right)+A+A\ln\frac{n^2\left(1-a^2\right)}{4A}$, the persuasive advertiser does not find it profitable to deviate.

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