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**Identifying challenges and prospects for advancing
evolutionary approaches in Reliability Redundancy
Allocation Problem**

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ABSTRACT:

In most of the industries, reliability has been an important aspect that determines system efficiency and performance. In this thesis, we seek to identify the prevalent challenges in one of the sub-areas of reliability optimization, i.e., Reliability Redundancy Allocation Problem (RRAP). It focuses on optimizing system reliability while balancing constraints like cost, weight, and volume and finding ways to resolve it using some tested methods such as evolutionary approaches or computational Intelligence methods, demonstrating the accuracy in solving complex engineering problems and thus, addressing the growing complexity of modern systems. Bibliometric analysis of the related publications was captured to get the intrinsic insights from the publications and the current trend of RRAP. In recent times, researchers are deeply involved in using the evolutionary algorithm in solving the RRAP due to its complexity and ability to adapt to changing conditions while exploring a wider solution space to reduce risk. It eventually contributes to the development of more cost-effective systems and dependability. Several works on the issues relating to reliability, redundancy, and the combination of both have been critically accessed and identified in the thesis. The thesis explicitly offers invaluable insights for researchers and engineers in replicating more advanced sustainable systems in driving immense innovation in the reliability engineering field and discuss the comprehensive understanding of the trend and evolution landscape of RRAP research and possible future innovations in the field.

KEYWORDS: Reliability; redundancy; reliability optimization; evolutionary algorithm

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Abbreviation/Notations	Method/Description
ABC	Artificial bee colony
BBO	Biogeography based optimization algorithm
CS	Cuckoo search
CS–GA	Hybrid cuckoo search and genetic algorithm
DE	Differential evolution
EA	Evolutionary algorithm
FGO	Fuzzy global optimization
GA	Genetic algorithms
GA–PSO	Hybrid genetic algorithm–particle swarm optimization
HS	Harmony search
IA	Immune based algorithm
ICS	Improved cuckoo search
IDE	Improved differential evolution
IPSO	Improved particle swarm optimization
MaORRAP	Many-Objective RRAP
MOPSO	Multi-objective particle swarm optimization
MPSO	Modified particle swarm optimization.
NSGA	Non-dominated sorting genetic algorithm
PSSO	Particle-based simplified swarm optimization
RRAP	Reliability redundancy allocation problem
SA	Simulated annealing
SOMA	Self-organizing migrating algorithm
SPEA2	Strength pareto evolutionary algorithm 2
SSO	Simplified swarm optimization
XKL	Analytic algorithms and local strategy.
r_i	Component reliability
w_i	Weight of component
n_i	Number of redundant components
R_s	System Reliability
m	Number of subsystems
V	Maximum volume
C	Cost
W	Weight
α_i & β_i	Physical characteristics of component at stage
T	Operating time during which components remains active
\mathbb{Z}^+	Space of positive integers

1. Introduction

In recent times, reliability optimization has proven to be one of the most important features in automotive industries, communication systems, and much more (Jiang et al., 2023). It is a crucial process aimed at enhancing the dependability and performance of systems, products, and processes. Reliability is the probability that a system will function properly for a longer period under the influence of environmental constraints such as a power surge or current fluctuation. However, failure is an almost unavoidable occurrence in mechanical systems, process plants, and components. As a result, reliability analysis has gained significant importance over the years.

The unending need for highly reliable systems necessitates the quest for the study of reliability optimization as reliability-based design optimization plays an important role in advancing product safety, service performance, reducing operational and maintenance costs for large-scale manufacturing systems. Therefore, designing reliable systems can be challenging due to the presence of multiple, and sometimes conflicting, objectives. Nevertheless, advancing technological and industrial equipment necessitates the need to improve and enhance the reliability of the systems in terms of production so as to maximize efficiency towards reliability while minimizing costs.

In creating a highly reliable system, three methods have been identified for enhancing the system's reliability by adding redundant components, increasing the component reliability and combination of both (reliability redundancy allocation problem). To reduce the risk of component failures and ensure continuous operation of systems, adding redundant components and increasing the reliability of the existing component are strategies used in the industry or companies. The method generally implements a back-up system in enhancing the performance and the effectiveness of critical components or tools in a way to mitigate the possibility of system failures. In the aviation industry, specifically in the design of an aircraft which are equipped with redundant components in advancing the reliability of the system is important for communication, navigation, and flight control (ZIO et al., 2019). For instance,

commercial airplanes often combine several separate hydraulic systems to manage the aircraft's flight surfaces, ensuring continued control even if one hydraulic system fails.

The design and the materials are refined to avert the occurrence of critical component failure. Therefore, increasing the reliability of existing elements is a vital means of enhancing the total reliability of the system. In the automotive factory, manufacturers persistently strive to enhance the reliability of vehicle components, which in turn leads to improved overall efficiency in performance and safety.

1.1. Why Reliability Redundancy Allocation

When systems or devices are designed, the main goal is to keep them reliably active for a long span of time especially in designing the advanced manufacturing setups. Industries are concerned on keeping equipment running and also to reduce the chances of breakdown, hence the need for redundancy is very crucial. It is observed that an increment in the number of redundant components within a subsystem also tends to increase the total reliability of the system.

The reliability redundancy allocation has enabled engineers to design a more reliable systems capable of withstanding components failure by adding extra parts in smarter ways to different parts of the system to keep it dependable. This process involves figuring out where there is component breakdown and deciding ways to provide backup plans to specific parts to keep the manufacturing system running. Adding backups ensures that important parts of each subsystem keep working, even in an emergency. A number of studies have been done regarding the redundancy and reliability of systems in industries. Therefore, reliability redundancy allocation is an important aspect that has a potential to influence the success of industries and same time boost the customer's trust towards using such products.

Garg, (2015) solves the reliability redundancy allocation problem in series-parallel using the penalty guided biogeography-based optimization (BBO). It uses a parameter-free penalty function to maximize system reliability. The penalty-guided BBO-based optimization minimizes system reliability under nonlinear constraints. The proposed approach shows little

advancement in reliability, but these are quite tough achieving in high reliability operations. The algorithm shows promising tool for reliability redundancy optimization. Effective redundancy allocation is crucial for the smooth operation of production lines, enabling the immediate detection and rectification of errors before they lead to system shutdowns. Four benchmark problems demonstrate the approach's superiority over existing literature solutions. Statistical simulations are performed for supremacy. This approach using redundancy makes the system active and prevents a complete shutdown by providing backup components.

Guo et al., (2011) presents the optimization of weapon system reliability, proposing a model for redundancy allocation based on system reliability, and mathematical solving algorithm using Lagrange multipliers method and Kuhn-Tucker condition for nonlinear programming models of reliability redundancy allocation, particularly in equation and constraint conditions. Jin et al., (2017) offers a novel approach to analyzing the balance between redundancy allocation and stocking spare parts within the semiconductor equipment industry. It is more beneficial to prioritize component redundancy over maintaining a large spare parts inventory. The study explores the balance between designing for redundancy and stocking spare parts in a dynamic service setting where the fleet size is expanding.

Strategically balancing reliability and managing redundancy within the subsystems, manufacturers can reduce disruptions and tools downtime. Hence, optimizing productivity and enhancing customer's satisfaction. Its stability and efficiency make it suitable for practical applications in reliability optimization. This leads to an overall enhancement in system performance, particularly in discrete optimization problems. Achieving the desired level of system robustness necessitates a careful balance between redundancy and cost/resource efficiency.

To explain the reliability redundancy allocation, Xing et al., (2013) implements an overview of various computational intelligence (CI) methods for reliability optimization problems, showing successful implementation and describing decision variables in ways to differentiate reliability problem types. CI methods have been chosen due to their adaptation, fault tolerance, high

computational speed, and error resilience, making them suitable for building good reliability systems. Hybridization, which combines the efficacy of individual CI approaches, may offer better solutions. It sums up redundancy as an important approach used in complex and well-detailed technical design in achieving decent reliable systems. Nonlinear programming problems are NP-hard, but CI has emerged as a heuristic search strategy to solve these problems as a reliability optimization process.

1.2. Evolutionary Algorithm

In contemporary times, evolutionary algorithms (EAs) have become essential tools for solving optimization problems, particularly in reliability redundancy allocation. Researchers focus on enhancing these processes to address constraints effectively. Various evolutionary algorithms such as genetic algorithms, bee colony optimization, simulated annealing, particle swarm optimization, biogeography-based optimization, artificial immune system optimization, cuckoo algorithm, and imperialist competitive algorithm have demonstrated efficacy in reliability optimization.

Inspired by natural phenomena, these methods can find nearly optimal solutions for complex systems efficiently. These tactics, which draw inspiration from natural observations, aim to imitate human occurrences (such as immigration and colonization), animal behaviors (such as those of ants, bees, fish, and birds), and even biological interactions such as an immune system (Chowdury et al., 2024a).

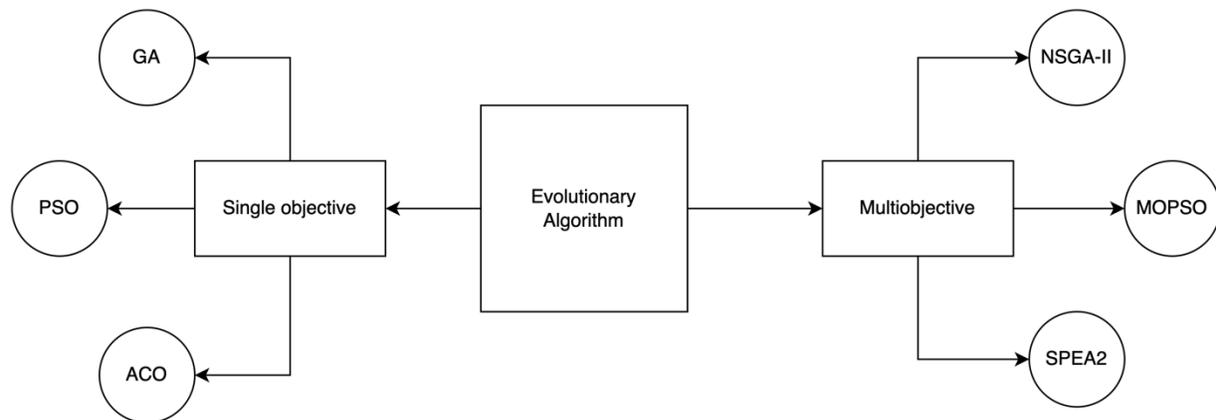


Fig 1: Classification of Evolutionary Algorithm

The efficacy of EA lies in its ability to operate complex interaction and ability to handle large datasets. This implementation of evolutionary algorithm seems valuable in handling diverse issues towards preventive maintenance, manufacturers can reduce downtime and extend the lifespan of critical equipment. The study of EA explores the complexities of RRAP across different scales, from small systems to large-scale engineering applications. Dos Santos Coelho et al., (2009) presents the DE and the Harmony Search (HS). The HS makes use of an evolutionary algorithm inspired by music, which is based on musical improvisation. It provides a link with a modified HS approach to address optimization difficulties in reliability engineering. DE has shown to be successful in reliability redundancy optimization since it is straightforward and simple to use. When utilizing the overspeed prevention system, it is observed that the HSDE supersedes the HS when compared in regard to quality of solutions.

The EA has been used to tackle and solve emerging issues regarding the RRAP, considering system reliability and uniformity of importance measures. Shojaei & Mahani, (2019) proposes a hybrid method using co-evolutionary PSO and HS algorithm to solve RRAP. The result of simulation performed showed new approach outperforms the previous method. The redundancy level and component reliability synchronize in a way that attributes correlation between the subsystems involved in the operation. In Fig.1 above shows the traditional classification of EA, which branches to single objective optimization and multi-objective optimization problems, which are further divided into the specific algorithms.

1.3. Why EA for Reliability Optimization Problems

Reliability optimization aims to identify the optimal design for a system, observing different competing objectives and constraint relating to cost, volume and weight. Reliability optimization makes informed decisions that balance cost and reliability effectively. Reliability improvement can be significant in avoiding disruption and failures especially in critical and complex systems possibly leading to saving costs (Zhang et al., (2023). The EAs are known to be highly effective and useful in solving the RRAP. The major contribution of EAs in tackling RRAP is the capability to adapt to environmental constraints. In solving the RRAP, the relationship among the constraints, the decision variables and objectives poses a challenge for traditional

optimization to manage. EAs excel by exploring complex solutions and adapting to the specific complexities of reliability optimization, making them suitable for such tasks.

Recent researchers using EAs have combined hybrid algorithms to tackle problem regarding reliability as (E. Zhang et al., 2014) proffers active method for solving MORRAP using bare-bones particle swarm optimization and sensitivity-based clustering (BBMOPSO). Striking a balance in EAs in terms of exploration enables engineers to advance solutions across diverse issues in finding solution space. This exploration involves enhancing the reliability while reducing the cost process, ensuring the resulting solutions are viable for reliability engineers and system designers to improve the system performance.

Using the strengths of evolutionary algorithms, researchers can tackle the challenges of redundancy allocation problems and achieve better reliability across various applications. EAs are good at balancing different engineering systems in adjusting to specific requirement as the flexibility in them makes it perfect for improving redundancies in different fields. EAs are sophisticated in handling the variability in component failures and the environmental conditions which ensures the systems remain stable and allows multiple computational resources equally. This is especially important for solving large and complex redundancy problems that have many goals and limitations, which is often needed in modern engineering systems with many interacting parts and subsystems. This also applies to critical infrastructure, transportation networks, and manufacturing processes, ultimately leading to progress in engineering practices and innovation.

Engineers and researchers can design systems that are more resilient, efficient, and reliable by taking advantage of the adaptability and robustness of EAs. Khalili-Damghani et al., (2013) introduces a dynamic self-adaptive multi-objective particle swarm optimization (DSAMOPSO) that adjusts itself automatically and deals with multiple goals in optimizing reliability for systems that switch between two states. This helps make their method robust and competitive compared to other existing techniques in solving these kinds of complex reliability problems. The DSAMOPSO method improves benchmark cases solutions and has been shown to be

preferred over competing approaches in terms of accuracy and diversity metrics. The method can be formulated in resolving issues related to real-life engineering and effective management problems as further analysis can be conducted through experimental and parameter tuning analysis.

1.4. Research Question and Objectives

This thesis aims to scrutinize the work that has been done on the reliability optimization problem and more specifically, to unravel the hurdles relating to RRAP associated with industries in reaching optimal solution with minimal constraint. This study looks at past research to find out where there are gaps and challenges in RRAP. The thesis examines the area of reliability and redundancy in the aspect of engineering and to explore and deepen research on what impact has been made in the last three decades and seek ways to improve on it by providing detailed work regarding addressing the complex issues. The goal is to come up with new ideas and solutions to make the reliability engineering system more dependable and significant. The scope details the decision and complexities involved in ways to add extra resources to make systems more reliable which remains a major concern for engineers and authors in this field.

RRAP is often used in industries like aerospace, automotive, and telecommunications. Recent trends suggest it could also be useful in newer areas like renewable energy, healthcare systems, and cyber-physical systems (Lin et al., 2021). The thesis focuses on understanding how evolutionary techniques function in both small and large engineering systems. It examines real-world cases to see how different methods enhance efficiency, resilience, and scalability of systems. By using these insights, the thesis aims to benefit both academic research and practical applications. This approach aims to connect theoretical concepts with the real challenges engineers encounter when dealing with RRAP in everyday engineering tasks. Studying how RRAP is applied across various fields can help researchers and industries better target their work.

The research question bothers on:

How evolutionary techniques can be useful in addressing the challenges in RRAP for both small- and large-scale engineering system?

The objectives include:

1. Comprehensive analysis of existing literature on RRAP focusing on small- and large-scale systems.
2. Investigate how different evolutionary approaches contribute to addressing distinct aspects of RRAP and related issues. In the same context, study the strengths and limitations of these approaches.
3. Perform a bibliometric analysis to get the intrinsic information about the keywords analysis, publication trend, collaborative network etc. in the field of RRAP with a focus on computational (evolutionary) approaches.

2. Literature Review

Reliability Redundancy Allocation Problem (RRAP) has garnered significant attention in reliability engineering as it addresses the addition or allocation of redundancy to components within a system to optimize and improve reliability while minimizing costs associated. The need for reliability optimization underscores the importance of interdisciplinary approaches that integrate principles from reliability engineering, operations research, and decision variables. By leveraging advanced methodologies such as dynamic programming, constraint optimization, and probabilistic modeling, researchers can strive to develop innovative strategies for achieving optimal reliability outcomes within resource-constrained environments even though this aspect as proven to be difficult.

Nath & Muhuri, (2024) presents a study on the prioritized many-objective optimization problem RRAP (PrMaORRAP) for four different system structures: series, series-parallel, overspeed gas turbine and complex bridge systems based on Genetic Algorithm and compares it with existing approaches. The hybrid method utilizes many-objective optimization and priority relations between objectives. Outcome of the result tested highlighted the proposed approach optimizes all objectives while maintaining reliability and considered superior to others.

Achieving system reliability is a necessity and as such researchers are poised with the motivation to lessen computational time and constraint involves in the process. The study then aims to peruse and document in clearer perspective how techniques involving evolutionary algorithm has been used in solving the RRAP. The process involved in reducing constraints involves a typical thorough process through brainstorming and thoughtful analysis in choosing the right decision.

Over time, various methods for allocating redundancy in systems have been proposed and refined through research. The way real-world systems are designed and operate depends on these optimization methods. In today's fast-paced technology landscape, systems must constantly adapt to new demands and constraints. Diverse authors in reliability engineering have proffered solutions to improving the reliability while avoiding or minimizing the system constraint. As such, RRAP has been combined with other metaheuristics or component in tackling these challenges, Dobani et al., (2019) introduced component mixing to RRAP (RRAP-

CM) to yield a stronger, dependable structures with higher reliability. With this addition to RRAP, it further complicates the process, and this makes the RRAP difficult to solve, therefore a new algorithm stochastic fractal search (SFS) combined with the GA effectively navigates the solution space of RRAP-CM, delivering enhanced optimization capabilities.

Muhuri & Nath, (2019) tries to solve the RRAP problem by initiating a bilevel RRAP using the bilevel evolutionary algorithm based on quadratic approximations. This method proves very effective in quality of solutions compared to previous method which involves combining the nested GA and PSO. It proposes that the bilevel RRAP may be suitable in the application of water resource control and in over-speed protection system as it reliably solves the optimization issues.

In RRAP studies, the designing of a subsystem usually involves the redundancy component being connected in parallel to either a series-parallel or a bridge method. As Nath & Muhuri, (2022) identified RRAP as a tough challenge related to balancing the multiple objectives involved. The author tackled issues relating to allocation of resource using the branch and bound in breaking the problem into smaller parts and carefully selects the best solutions. Despite the promising results obtained, it is cumbersome solving the large-scale method with the method.

According to (Chern, 1992), the RAP has been identified as highly challenging especially as the system increases, being NP hard makes it strenuous and hard to solve and this further increases the system size. There is a need to address this complexity encountered by innovating solutions that synthesize advanced techniques with practical method in a means to reduce this complexity. This process involves allocating resources efficiently so as to improve efficiency and minimize cost and other constraint. Building effective and functional systems in recent times that withstand any defect involves careful and proper approach in using resources available. RRAP method has been identified as ways to achieve stability and ensure resources are used within specified limits.

The research detailed on how designing the reliability redundancy allocation model improves how reliable and dependable systems are. The conditions for designs to remain viable must be

satisfied as these variables are subjected to constraint. These constraints can include the cost considerations, the physical limitations and requirement for performance. Optimization involves defining the objective's function and the performance of the design. The optimization process achieves a balance between competing objectives in maximizing efficiency and reducing cost. The need to optimize these objectives either by maximization or minimization involves combination of decision variables optimally. The study's contributions include exceptional performance in RRAP calculations and adaptation to large-scale benchmarks and system design.

2.1. Redundancy Allocation Problem

Industries are faced with the need to improve their products in an efficient way to avoid breakdown or downtime, the need to improve on the equipment necessitate that proper care is taken to improve the system reliability. Redundancy in general view prevents total or complete breakdown of equipment thereby allowing addition of extra part to replace the parts failing without stopping operation. Manufacturers has been faced with the bureaucratic involved with redundancy as this procedure add extra cost, weight and volume to equipment. Active and standby redundancy are the two main design strategies that are used to install redundant components and boost system reliability (Kim & Kim, 2017).

Several methods have been employed to solve the redundancy in which (Ha & Kuo, 2006) addresses the efficient branch-and-bound method providing solution regarding redundancy allocation problem. The algorithm eliminates irrelevant solution space and uses a greedy heuristic to solve each branched subproblem. Detailed experiment shown that this particular method outweighs the known exact method in unraveling the redundancy allocation issues and is efficient in terms of computation time. Exact algorithms for solving RAP are usually only possible for small examples due to its NP-hardness.

As the scale of the problem rises, these methods frequently use branch and bound, dynamic programming, or integer programming techniques, all of which can become computationally prohibitive. As a result, heuristic and metaheuristic techniques are widely used by researchers and practitioners to identify nearly ideal solutions in reasonable amounts of time. Reliability

redundancy optimization has developed and has been an attractive research area for decades with techniques categorized into three classes: exact methods, heuristics, and evolutionary computation (Li & Wood, 2011).

To introduce the concept of reliability entropy to address practical reliability growth problems, Han et al., (2019) proposed an important section of electromechanical system of reliability redundancy optimization method, addressing the issue of over-design or under-configuration. It defines reliability entropy, finds the redundancy allocation optimal value, establishes the reliability solution function. To prove the functionality of the redundant systems in meeting the reliability standards, simulation tests are used to show the feasibility of the methods implemented.

RAP problems are considered hard due to inherent factors associated to different constraints and the capacity of resources involved. For instance, the connections between resources and objectives makes solving the problem more difficult. The resources most times cannot be divided and must be allocated in whole numbers.

This research's main achievement documents the challenges in enhancing how system reliability is assessed in redundancy allocation problems. These insights are valuable for modern manufacturing systems and industrial software applications. By incorporating reliability evaluators into optimization models, there is a notable enhancement in performance. As noted by (Yeh et al., 2016), which developed a new method called general multi-level redundancy allocation problem (GMRAP) to overcome limitations and improve existing approaches. To boost the redundancy allocation model and efficiency, the method is applied, and a simplified swarm optimization with modular search (SSO-MS) used in hierarchical design variables was introduced. The proposed SSO-MS outperforms older version of PSO, SSO and GA in regard to the solution quality and running time used in the operation. However, previous models have limitations, losing generality.

2.2. Reliability Redundancy Allocation Problem

Several researchers has argued that RRAP is a NP hard problem and proves difficult solving using the previous known model. When the RRAP is considered a large problem, novel heuristics sought to solves it optimally. Abouei Ardakan et al., (2016) solves the RRAP, showing

its advantages over traditional approaches. The mixed strategy is used in a more complex RRAP and compared to active and standby strategies. The output value indicates the mixed strategy improves system reliability in all benchmark problems. Their numerical conception was extended to calculate system reliability under the mixed strategy. The mixed strategy resulted to higher reliability which differs from the active and standby components, offering more flexibility in system design and improved reliability of complex systems.

RRAP poses challenging but practical nonlinear mixed-integer optimization problem which enhances the reliability of the system through component redundancy and dependability choices. Constant improvement in technology combined with the need to produce high-end products necessitates the need to build systems that are durable and efficient even though this complex systems comes with its own constraint. These constraint such as environmental factor or defect of tools impacts negatively in the production and designs of the systems. Therefore, increment in the system reliability amplifies the need for reliability optimization among researchers and industries to promote dependability and durability.

Therefore, prioritizing reliability regarding design offers several benefits, ranging from creating safer and more dependable products to enhancing competitive positioning through substantial cost reduction. The reliability design has been an important subject that has gained popularity among researchers in the last two decades using mathematical programming or the heuristics optimization approaches. Ha & Kuo, (2005) proposes a diverse-path iterative heuristics in combination to scaling methods for RRAP superseding the previous solutions on heuristics. This method computes sensitivity factors in every decision variables, reducing the number of nonlinear programming (NLP) subproblems and avoiding redundant computation. This heuristic can minimize the NLP problem and enhance solution quality due to its increased exploration with little modification.

This thesis aims to improve how we use evolutionary techniques to solve the RRAP. The evolutionary algorithm application considers useful in handling complex RRAP effectively in both small and large systems. Deb et al., (2021) suggested in creating new algorithms that

solves RRAP, collaboration from different backgrounds to push forward the EA should be a priority by engineers and computer scientists. Evolutionary Algorithm is rated highly in the engineering firm and industry as it can simultaneously optimize multiple objectives. Handling the EA methods solves the RRAP and makes it better in overcoming challenges and adapting faster in achieving stability performance.

The EA method proves efficient in metaheuristic to examine different possibilities and avoiding result that may look less favorable. This process integrates important modifications such as crossover and mutation operators as element of algorithm. This flexibility enables users to choose the solution that best suits their needs based on its performance and how well it meets their specific requirements. Ashraf et al., (2019) uses the PSO and GA to address the fuzzy multi-objective reliability redundancy allocation problem (FMORRAP) in both series-parallel and parallel-series systems. Guilani et al., (2022) used mixed redundancy strategies and variety of components that combines stochastic fractal search and genetic algorithms was used to solve RRAP in complex systems. The objective is to proffer solutions to different challenges posed by RRAP by using different concepts and also determining the optimal sequence of components in each subsystem to model a reliable system.

Najmi et al., (2021) aims to challenge conventional ideas about how to manage reliability in systems by combining different types of components and a specific strategy for redundancy. It uses a mathematical model to figure out the best ways to improve subsystems that run with either cold or active standby systems. The result was tested and compared with previous results using heterogeneous components, the outcome shows a new way to solve problems regarding reliability. Y. Liu & Qin, (2013) created an improved version of differential evolution (IDE) to enhance reliability redundancy optimization, using an adaptive method. IDE boosts efficiency by automatically adjusting important parameters like the scaling factor and crossover rate (CR) during the search for solutions. Bhandari, Kumar, et al., (2023b) investigates the RRAP in fire extinguisher drones (FEDs) to improve reliability while considering limited resources. The findings aid in the development of resource-conserving systems by designers and the result showed increased flexibility and improved reliability.

As addressed above, the RRAP has been pivotal in addressing the system design especially as industries strive towards improving reliability and durability. The recent advancement regarding the evolutionary algorithms shows a unique application for tackling and solving the complex optimization nonlinear problem. Researchers therefore have implemented diverse methods to tackle the multifaceted nature of RRAP, the demand for dependable systems keeps advancing towards exploring innovative solutions not only in meeting the designs but adapting it to ever evolving technological space. It is of paramount importance prioritizing the reliability optimization towards strengthening the competitive and dependable products in the emerging industrial trend.

3. Methodology

To have a better understanding, this thesis sort to explore the mixed methodology in this research, where in, qualitative analysis will investigate how different evolutionary approaches contribute to addressing distinct aspects of RRAP and related problems. Similarly, the thesis will identify the missing gaps, classification of evolutionary approaches used to address RRAP, case studies, and comparative analysis. The quantitative part will utilize the bibliometric analysis to get the intrinsic information about the keywords analysis, publication trends, collaborative networks, etc in the field of RRAP with a focus on evolutionary approaches using VOS viewer and related applications. For the qualitative part we will extensively explore the extracted papers and address the research question and corresponding objectives. This involves systematically reviewing the literature to have a better grasp on the current state and advancements in RRAP.

The data collection and analysis process is methodically structured and follows several steps, illustrated in Fig. 2, which will be detailed later in the document. Initially, we identify relevant databases and define search criteria to ensure a comprehensive collection of pertinent studies. The gathered data will then be processed and analysed using bibliometric techniques, enabling us to identify emerging trends, influential publications, and key researchers in the field. Finally, these insights will be synthesized to provide a holistic view of the research landscape in RRAP using evolutionary algorithms.

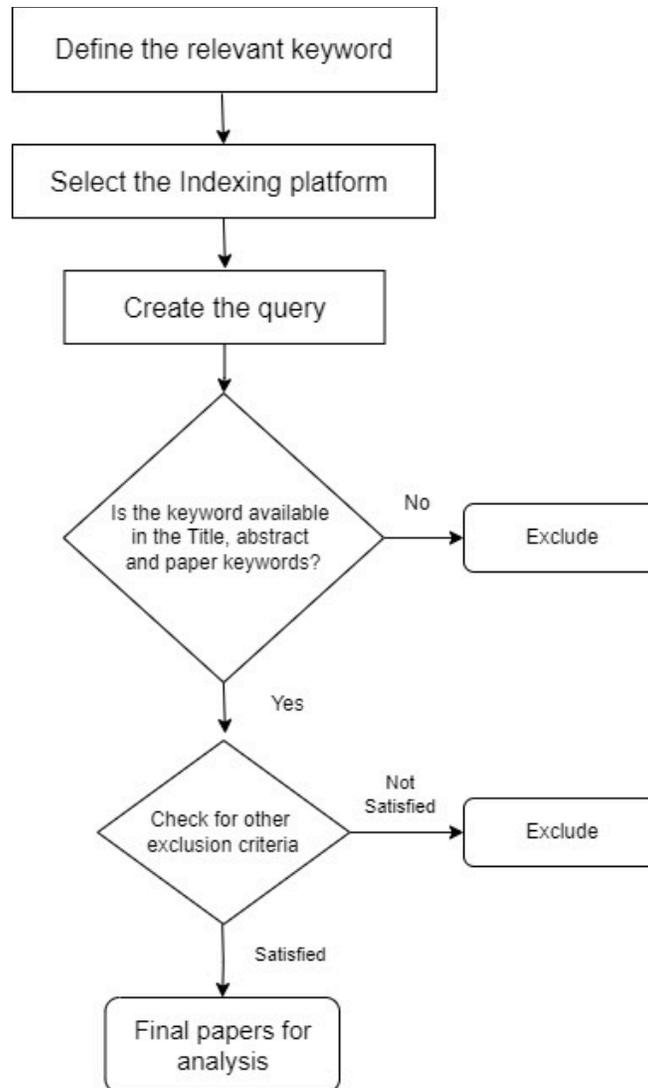


Fig 2: Flowchart of the data collection process

For the data collection, we used the Scopus data base, and used the following keyword to extract the papers: “reliability redundancy allocation”. The data utilized in this research were exclusively extracted from the Scopus database. However, if alternative databases such as Web of Science, Dimensions and Google Scholar had also been considered, it is likely that a significantly larger number of articles would have been identified and included in the analysis. Each of these databases possesses its own unique collection of articles, journals, and conference papers, which may not overlap entirely with Scopus. Consequently, the incorporation of these additional sources could have led to the discovery of diverse insights and perspectives, potentially resulting in different conclusions and a more nuanced understanding

of the subject matter under investigation. Furthermore, the inclusion of a broader array of databases would have enhanced the robustness and comprehensiveness of the study, offering a more holistic view of the research landscape and ensuring that critical developments and varied viewpoints within the field were adequately represented.

The used keyword is utilized to extract all the papers in the domain of the addressed problem of the paper and we have searched this keyword only in the title, abstract and keywords to be specific to the theme of the paper. Some papers were excluded for the analysis such that review papers in conferences and papers with language other than English. The query was executed in the month of March, 2024, and 189 papers were extracted.

This thesis explains the following issues in details:

1. Methods for initializing parameters in reliability redundancy allocation models.
2. Techniques for developing deep models in the context of reliability redundancy allocation.
3. Development of reliability redundancy allocation models for various scenarios, including using fuzzy systems and machine learning.
4. Methods for selecting different parameters in reliability redundancy allocation models, followed by optimization techniques.
5. Approaches for analyzing the impact of hyper-parameters on the generalization performance of reliability redundancy allocation models.
6. Identifying gaps in the current literature on reliability redundancy allocation models and suggesting future research directions.
7. The data used was from some of the papers studied for reference purpose.

4. Result & Analysis

To clarify the approach used in identifying and addressing the RRAP, the result section discusses the bibliometric analysis, presenting results in terms of citations, publication of articles over the years, authors with the most published works, and other bibliometric indicators. This generated analysis gives deeper information in understanding the trend of analysis involved in the RRAP or more specifically the addressed query to perform bibliometric analysis. This analysis shows what research (in terms of publication analysis) has been ongoing in the last three decades on reliability issues and till now. The study provides a comprehensive overview of both the research landscape through bibliometric analysis and a deep dive into the specifics of RRAP by exploring the literature in this area. This structured approach helps researchers understand the current state and future direction of reliability optimization research. It highlights key areas for further exploration and improvement.

4.1. Results of Bibliometric Analysis

The bibliometric analysis used in this thesis seeks to provide thorough overview of research ongoing in the use of RRAP. This analysis details on collaboration networks, the application areas where significant, emerging trend and diverse techniques used in the research. By quantitatively analysing scientific publications, bibliometrics offers valuable insights. Bibliometric analysis enables authors to discover latest methods, cross-disciplinary collaboration, and apply their findings practically. Many online databases collect and share bibliographic data, making detailed statistical and analytical studies possible (Guler et al., 2016).

Exploration used in bibliometric can help researchers, practitioners and authors to adapt and get familiar with the concepts and spread of the studied area of interest. For instance, researchers from diverse background can liaise and work closely in finding solutions to advanced RRAP issues. In finding solutions can propel interdisciplinary innovations in the field as analysis achieved using bibliometric exposes majorly where RRAP application domain is common. Examining co-authorship networks highlights trends in interdisciplinary collaboration within RRAP research. Understanding these changes involved in the process offers

comprehensive perspectives on the methodological emergence associated with research in RRAP. The bibliometric result in this thesis shows relevant detailed knowledge and observation into rapid development of RRAP and the attention it's gained in recent years regarding reliability improvement. Understanding changes involved in this process, offers wider perspectives on methodological concept associated with research of RRAP.

Bibliometric analysis makes it possible to create citation networks that show the ideas and influence that move throughout the RRAP literature. It highlights highly cited papers by other authors and researchers, seminar works, and influential authors can provide valuable guidance for researchers seeking to navigate the vast body of RRAP literature and identify key foundational concepts and methodologies. For clarity's sake, the bibliometric analysis and extensive review here covered over 170 scholarly articles on RRAP published between 1987 to present. In the further sub-sections, we have provided empirical analysis of the bibliometric outcome.

4.1.1. Publication growth and citation trends

The graph in Fig. 3 below depicts number of papers published over the years from 1987 to 2024, the number of papers published annually remained relatively low and stable, fluctuating between 0 and 3 papers per year. This indicates limited research activity or interest in the topic during this period. From 2005 onwards, there's a noticeable upward trend in the number of publications. The count starts to exceed 5 papers per year and continues to grow. Between 2005-2007: The number of publications remains relatively steady at around 5 papers per year, indicating a stable but growing interest. In 2008-2009, there is a small increase, with the number of publications reaching around 8-10 papers per year, showing a gradual buildup in research activity. From 2010 onwards, there is a marked increase in the number of papers published each year, peaking at around 2017 and showing substantial fluctuations. The highest number of publications occurs around 2022, with more than 25 papers published in that year.

Despite fluctuations, the overall trend is upward, indicating increasing interest and research activity in recent years. The graph clearly illustrates a rising number of studies on RRAP over

time, particularly since around 2010, with a notable increase in published papers. This suggests that the research has gained substantial attention and importance in the academic and research communities over the past decade causing a monumental rise in research output by scholars and researchers.

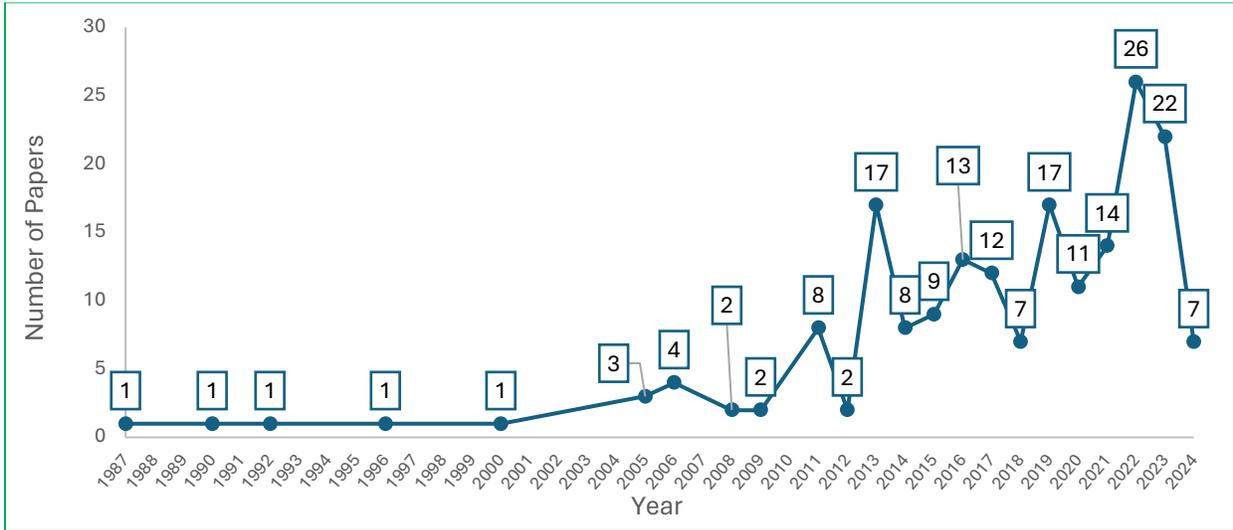


Fig 3: Publication analysis over the years

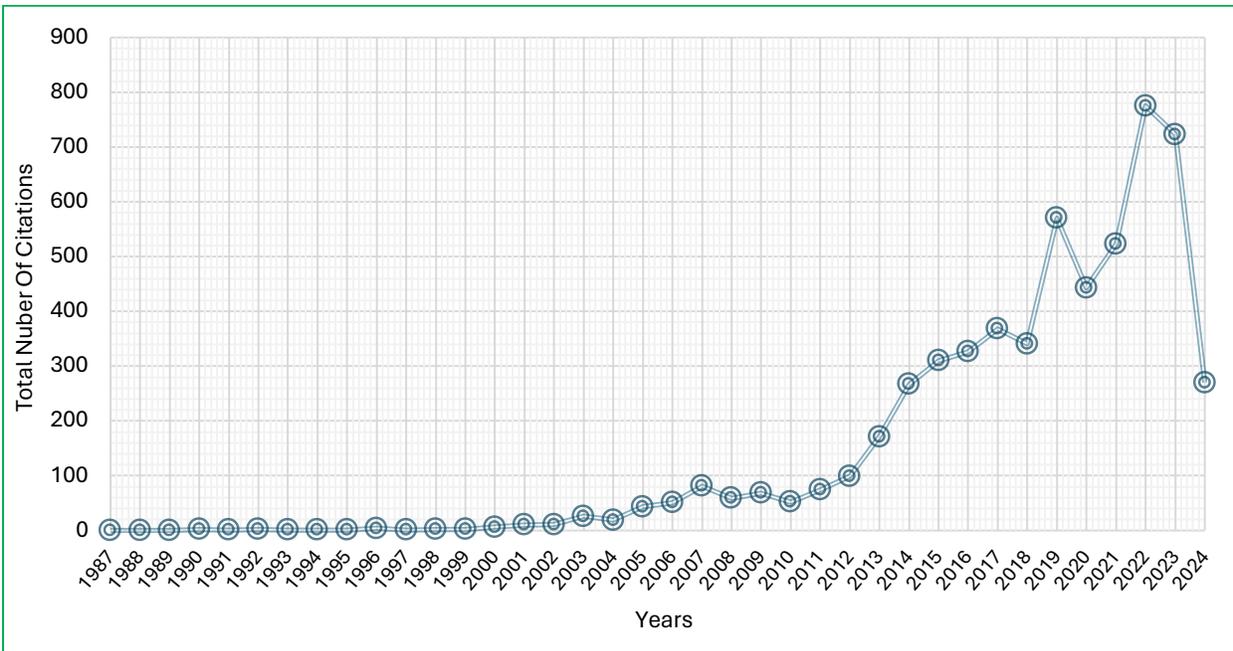


Fig. 4: Citations of papers over the years

Figure 4 above shows the number of citations from 1987 to 2024. Initially, from 1985 to around 2005, citation numbers remain relatively low and stable. Post-2005, there is a noticeable

upward trend, with citations gradually increasing. Since 2010, there has been a noticeable acceleration in growth, peaking around 2020 with nearly 800 citations. This indicates a strong increase in research interest in this field over the years, particularly in recent times, though there has been a slight decline afterward.

4.1.2. Keyword Analysis

In this part, figure 5 below shows the outcomes of analyzing keywords from RRAP publications using the VOSviewer program. The figure illustrates a network of linked phrases connected to reliability redundancy allocation problems (RRAP). It visually presents main ideas, groups of topics, how ideas are linked, specific methods used, emphasis on optimization, and new trends in RRAP research. The keyword such as “redundancy”, “reliability” and “reliability redundancy allocation” are identified and discussed extensively in the academic research. These terms forms the unique topics around which other related ideas and insight are organized.

To explain these concept in clearer terms:

- The red cluster centers on different optimization methods and heuristic approaches.
- The green cluster centers on concepts related to system reliability and redundancy.
- The yellow cluster emphasizes specific optimization techniques like PSO and GA.
- The purple cluster addresses multi-objective optimization and decision-making processes.
- The blue cluster deals with benchmarking and redundancy allocation problem.

These array of interconnected networks illustrates the grasp on RRAP showing various aspects of reliability. The connecting lines in the graph illustrates are often these ideas are studied together, showing a significant network of research across diverse fields. The graph further underscores that solving RRAP demands teamwork and collaboration among researchers and authors. In finding best solution to solving RRAP, the graph shows range of heuristics technique such as PSO, integer programming, SA, fuzzy and EA. These algorithms helps to tackle the RRAP problem and researchers have used these to improve efficiency in reliability optimization and distribute redundancies in systems. In solving RRAP, optimization plays an important aspect as can be notice in the bibliometric graph below showing different terms related to optimization.

These approaches relaxes the complexity associated with reliability issues and ensures system reliability.

The terms like “fuzzy reliability” and “multi-objective optimization” indicates that the field is drifting towards advance computer tools in synthesizing the research process. These trends suggest that the discipline is active and always bringing in new methods to handle complex problems in RRAP.

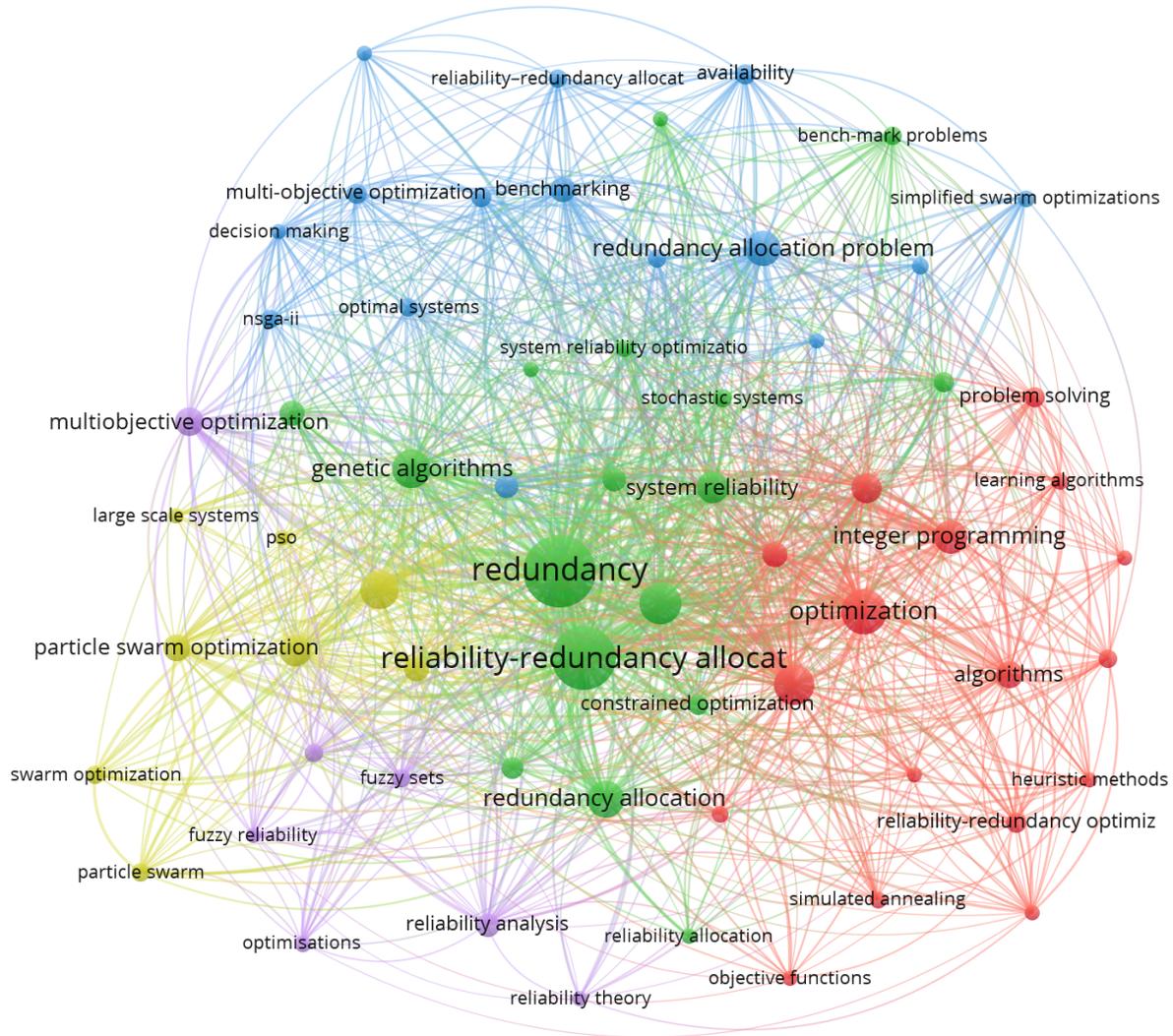


Fig 5: All Keyword Analysis using VOSviewer

4.1.3. Author Analysis

Table 1 below shows chart that lists the authors who have contributed the most on RRAP research. The analysis highlights the varying levels of productivity and impact among

researchers. The table points out both prolific and impactful researchers showcasing an overview of authors based on their research output and impact. Way Kuo with total paper of 6 leads in the number of citations and citations per publication, showing significant impact in this field. This followed by S.P Sharma and Harish Garg which also demonstrated a good number of citations per publication. Wei-Chang Yeh has the highest number of publications, indicating prolific research activity. Mohamed Arezki Mellal and Pranab K. Muhuri balance a good number of publications with considerable impact. Few authors like Tongdan Jin and Dongkui Li have the lowest citations signifying lesser influence in their field.

Table 1: Prolific Authors and their Publications

S No	Authors	TP	TC	CPP
1	yeh, wei-chang	11	373	33.91
2	ram, mangey	10	43	4.30
3	bhandari, ashok singh	9	43	4.78
4	kumar, akshay	9	43	4.78
5	mellal, mohamed arezki	9	238	26.44
6	muhuri, pranab k.	9	207	23.00
7	ashraf, zubair	8	159	19.88
8	ardakan, mostafa abouei	7	162	23.14
9	garg, harish	7	523	74.71
10	nath, rahul	7	74	10.57
11	abouei ardakan, mostafa	6	226	37.67
12	huang, chia-ling	6	163	27.17
13	kuo, way	6	869	144.83
14	adjerid, smail	4	28	7.00
15	davari-ardakani, hamed	4	60	15.00
16	dos santos coelho, leandro	4	232	58.00
17	jin, tongdan	4	6	1.50
18	li, dongkui	4	2	0.50
19	sharma, s.p.	4	378	94.50
20	chebouba, billal nazim	3	26	8.67

Table 2 below shows authors with influential output reflecting their ongoing engagement and contribution to RRAP. The chart underscores the prominent authors who have made considerable contributions to the field, with a notable focus on those who have published

extensively. Out of the 25 sampled authors, Wei-chang Yeh has the number for the most total paper published followed by two others authors having a total of 9 papers. In terms of citation per paper, analysis shows V. Rajendra prasad has total number of 575 which shows how influential the authors is in this field, closely followed by Maw-sheng Chern who has a total of 463 with the lowest being Zubair Ashraf with a total of 19.88 citation per page.

Table 2: Most Influential Authors

S No	Authors	TP	TC	CPP
1	kuo, way	6	869	144.83
2	rajendra prasad, v.	1	575	575.00
3	garg, harish	7	523	74.71
4	chern, maw-sheng	1	463	463.00
5	sharma, s.p.	4	378	94.50
6	yeh, wei-chang	11	373	33.91
7	mellal, mohamed arezki	9	238	26.44
8	khalili-damghani, kaveh	2	234	117.00
9	dos santos lcoelho, leandro	4	232	58.00
10	abouei ardakan, mostafa	6	226	37.67
11	mariani, viviana cocco	3	215	71.67
12	muhuri, pranab k.	9	207	23.00
13	hsieh, tsung-jung	2	205	102.50
14	tavana, madjid	2	199	99.50
15	kim, heungseob	2	181	90.50
16	zio, enrico	3	174	58.00
17	lin, hsin-hui	2	173	86.50
18	xu, zhongkai	2	173	86.50
19	abtahi, amir-reza	1	171	171.00
20	huang, chia-ling	6	163	27.17
21	ardakan, mostafa abouei	7	162	23.14
22	ashraf, zubair	8	159	19.88
23	jawahar, n.	1	156	156.00
24	kanagaraj, g.	1	156	156.00
25	ponnambalam, s.g.	1	156	156.00

4.1.4. Publication Source Analysis

In the analysis of the papers studied, the table below underscores the productivity and varying impact of these journals in the field. Overall, the table highlights journals and conferences that are prolific in publishing research and high-impact publication, with varying degrees of impact as indicated by citation counts. This information underscores the diversity of sources contributing to advancements in reliability engineering and related fields.

Table 3: Most Productive Journals

S. No	Source/Journal	TP	TC
1	reliability engineering and system safety	28	1354
2	ieee transactions on reliability	6	917
3	engineering optimization	5	163
4	proceedings of the institution of mechanical engineers, part o: journal of risk and reliability	5	54
5	computers and industrial engineering	4	432
6	journal of statistical computation and simulation	4	18
7	quality and reliability engineering international	4	27
8	advances in intelligent systems and computing	3	22
9	expert systems with applications	3	148
10	international journal of system assurance engineering and management	3	12
11	nonlinear studies	3	7
12	proceedings - annual reliability and maintainability symposium	3	8
13	2011 international conference on system science, engineering design and manufacturing informatization, icsem 2011	2	2
14	applied mathematics and computation	2	222
15	applied mechanics and materials	2	6
16	applied soft computing	2	21
17	computers and operations research	2	253
18	conference proceedings - ieee international conference on systems, man and cybernetics	2	6
19	european journal of operational research	2	152
20	ieee access	2	0

In table 3, the "Most Productive Journals" table highlights key journals by their total papers (TP) and total citations (TC). "Reliability Engineering and System Safety" tops with 28 papers and 1354 citations, indicating significant influence. "IEEE Transactions on Reliability" follows with 6 papers and 917 citations. "Engineering Optimization" and "Proceedings of the Institution of Mechanical Engineers, part o: Journal of Risk and Reliability" both have 5 papers, with 163 and 54 citations respectively. "Computers and Industrial Engineering" has 4 papers with 432 citations, showing strong contributions. Other notable journals include "Expert Systems with

Applications" (3 papers, 148 citations) and "Applied Mathematics and Computation" (2 papers, 222 citations).

Table 4: Most Influential Journals

S. No	Source/Journal	TP	TC
1	reliability engineering and system safety	28	1354
2	ieee transactions on reliability	6	917
3	operations research letters	1	456
4	computers and industrial engineering	4	432
5	computers and operations research	2	253
6	applied mathematics and computation	2	222
7	engineering optimization	5	163
8	european journal of operational research	2	152
9	expert systems with applications	3	148
10	swarm and evolutionary computation	1	121
11	2011 ieee congress of evolutionary computation, cec 2011	1	117
12	simulation modelling practice and theory	1	104
13	journal of manufacturing systems	2	101
14	international journal of advanced manufacturing technology	2	98
15	isa transactions	1	89
16	ieee transactions on fuzzy systems	1	76
17	iie transactions (institute of industrial engineers)	1	73
18	international journal of industrial engineering computations	1	61
19	proceedings of the institution of mechanical engineers, part o: journal of risk and reliability	5	54
20	journal of quality in maintenance engineering	1	51

In table 4 above, the "Most Influential Journals" table ranks journals by their number of total papers (TP) and total citations (TC). "Reliability Engineering and System Safety" leads with 28 papers and 1354 citations, showing significant impact. "IEEE Transactions on Reliability" has 6 papers with 917 citations, indicating its high influence. "Operations Research Letters" stands out with 1 paper receiving 456 citations. Other notable journals include "Computers and Industrial Engineering" (4 papers, 432 citations), "Computers and Operations Research" (2 papers, 253 citations), and "Applied Mathematics and Computation" (2 papers, 222 citations). Conferences like the "2011 IEEE Congress of Evolutionary Computation" also show significant influence with 1 paper and 117 citations.

Additional influential journals are "European Journal of Operational Research" (2 papers, 152 citations), "Expert Systems with Applications" (3 papers, 148 citations), and "Swarm and Evolutionary Computation" (1 paper, 121 citations). The list highlights key sources contributing

valuable research and insights in the field. The table emphasizes the importance of major conferences and journals in influencing the conversation and advancement of reliability optimization research, highlighting the role of high-impact publications in propelling both scholarly and applied progress.

4.1.5. Influential Publication analysis

Table 5 lists the most influential papers in terms of total citations. The table lists the top 20 most cited publications in the field of reliability redundancy allocation problems (RRAP), with citations ranging from 84 to 569. These techniques are preferred because they can efficiently manage large sets of solutions and effectively address multiple reliability-related objectives. Researchers have observed a growing trend in using combined optimization strategies inspired by biological systems. "An annotated overview of system-reliability optimization," is written by Kuo and Prasad's with 569 citations, reflecting its significance as a key reference in the study of system reliability improvement.

Another notable work is Chern's 1992 study, which has been cited 456 times, titled "On the computational complexity of reliability redundancy allocation in a series system." This study delves into the difficulties of computing reliability redundancy allocation, offering valuable insights. A number of recent studies have shown that top publications frequently address the complexity of RRAP by utilizing cutting edge techniques like hybrid approaches, evolutionary algorithms, and particle swarm optimization. This pattern shows how flexible and constantly evolving the field is.

Table 5: Top 20 most cited publications

S. No.	Authors	Title	Year	TC
1	Kuo W.; Rajendra Prasad V.	An annotated overview of system-reliability optimization	2000	569
2	Chern M.-S.	On the computational complexity of reliability redundancy allocation in a series system	1992	456
3	Garg H.; Sharma S.P.	Multi-objective reliability-redundancy allocation problem using particle swarm optimization	2013	219
4	Yeh W.-C.; Hsieh T.-J.	Solving reliability redundancy allocation problems using an artificial bee colony algorithm	2011	190
5	Khalili-Damghani K.; Abtahi A.-R.; Tavana M.	A new multi-objective particle swarm optimization method for solving reliability redundancy allocation problems	2013	170
6	Kanagaraj G.; Ponnambalam S.G.; Jawahar N.	A hybrid cuckoo search and genetic algorithm for reliability-redundancy allocation problems	2013	154
7	Chen T.-C.	IAs based approach for reliability redundancy allocation problems	2006	140
8	Kim H.; Kim P.	Reliability–redundancy allocation problem considering optimal redundancy strategy using parallel genetic algorithm	2017	129
9	Garg H.	An efficient biogeography based optimization algorithm for solving reliability optimization problems	2015	121
10	Dos Santos Coelho L.; De Andrade Bernert D.L.; Mariani V.C.	A chaotic firefly algorithm applied to reliability-redundancy optimization	2011	117
11	Ardakan M.A.; Hamadani A.Z.	Reliability-redundancy allocation problem with cold-standby redundancy strategy	2014	104
12	Huang C.-L.	A particle-based simplified swarm optimization algorithm for reliability redundancy allocation problems	2015	104
13	Ha C.; Kuo W.	Reliability redundancy allocation: An improved realization for nonconvex nonlinear programming problems	2006	100
14	Abouei Ardakan M.; Rezvan M.T.	Multi-objective optimization of reliability–redundancy allocation problem with cold-standby strategy using NSGA-II	2018	95
15	Ouyang Z.; Liu Y.; Ruan S.-J.; Jiang T.	An improved particle swarm optimization algorithm for reliability-redundancy allocation problem with mixed redundancy strategy and heterogeneous components	2019	94
16	Afonso L.D.; Mariani V.C.; Dos Santos Coelho L.	Modified imperialist competitive algorithm based on attraction and repulsion concepts for reliability-redundancy optimization	2013	93
17	Mellal M.A.; Zio E.	A penalty guided stochastic fractal search approach for system reliability optimization	2016	91
18	He Q.; Hu X.; Ren H.; Zhang H.	A novel artificial fish swarm algorithm for solving large-scale reliability-redundancy application problem	2015	89
19	Xu Z.; Kuo W.; Lin H.-H.	Optimization Limits in Improving System Reliability	1990	89
20	Kuo W.; Lin H.-H.; Xu Z.; Zhang W.	Reliability Optimization With The Lagrange-Multiplier and Branch-And-Bound Technique	1987	84

4.2. Extensive overview on RRAP

To understand how researchers are addressing the RRAP, several authors have researched on identifying the RRAP problems considering what computational intelligence or heuristic (evolutionary) approaches have been used to address these RRAP problems, find how these various heuristics have solved the problems and also the challenges and limitations in the papers, a conclusive study of these papers reviewed are summarized below.

4.2.1. Formal definition of RRAP and system components

The RRAP is generally formulated as (Ben Guedria & Hassine, 2019)

$$\max R_s = f(r, n) \quad (1)$$

$$\text{subject to } g(r, n) \leq 1 \quad (2)$$

$$n_i \in \text{positive integer}, 1 \leq i \leq m, 0 \leq r_i \leq 1 \quad (3)$$

Where the $f(r, n)$ indicates the objective function for total system reliability, n_i and r_i are the number and reliability of redundant system respectively.

In gaining deeper understanding in optimizing the system reliability, there are mostly five different cases are analyzed in the literature which consist of the series, series-parallel, complex, overspeed protection and large-scale systems. Each one of them is mathematically explained below:

4.2.1.1. Series System

In the series system, the system reliability, as the objective to be maximized, can be formulated as (Ouyang et al., 2019a)

$$\text{Maximize } R_t = \prod_{i=1}^5 R_{i(t)}^{sub}, \quad (4)$$

$$\sum_{i=1}^5 w_i v_i^2 (n_i^{active} + n_i^{cold})^2 \leq V, \quad (5)$$

$$\sum_{i=1}^5 \alpha_i \left(\frac{-1000}{\ln R_{i(t)}^{com}} \right)^{\beta_i} ((n_i^{active} + n_i^{cold}) + \exp \left(\frac{n_i^{active} + n_i^{cold}}{4} \right)) \leq C, \quad (6)$$

$$\sum_{i=1}^5 w_i (n_i^{active} + n_i^{cold}) + \exp \left(\frac{n_i^{active} + n_i^{cold}}{4} \right) \leq W \quad (7)$$

Where α_i and β_i are the intrinsic component parameters showing the shaping and scaling factors respectively, equations 5, 6 & 7 indicate the constraints of the volume, cost and weight respectively.

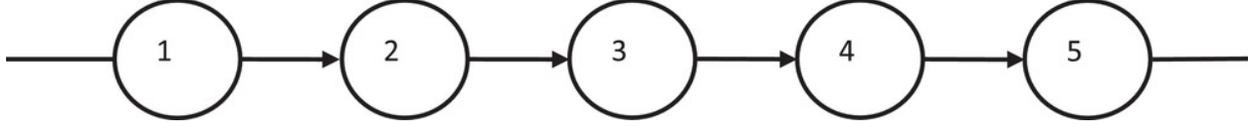


Fig 6: Series connection

The Fig. 6 shows the series connection for RRAP, the components are arranged serially, and the system operates successfully only if all components are functioning. The failure or error in any single element leads to the failure of the entire system. The overall reliability of the series connection is the product of the reliabilities of each components representing the system.

4.2.1.2. Series-parallel system

This arrangement refers to subsystems linked in series, with individual components within each subsystem also connected in parallel (Bhandari et al., 2024).

$$\text{Maximize } f(r,n) = 1 - (1 - R_1 * R_2) [1 - (R_3 + R_4 - R_3 * R_4) * R_5] \quad (8)$$

$$g1(r,n) = \sum_{i=1}^m w_i v_i^2 n_i^2 - V \leq 0 \quad (9)$$

$$g2(r,n) = \sum_{i=1}^m \alpha_i \left(-\frac{1000}{\ln(r_i)} \right) \beta_i [n_i + \exp(0.25n_i)] - C \leq 0 \quad (10)$$

$$g3(r,n) = \sum_{i=1}^m w_i n_i \exp(0.25n_i) - W \leq 0 \quad (11)$$

$$0 \leq r_i \leq 1, n_i \in Z^+, 1 \leq i \leq m$$

The constraint represented by equation 9 indicate the volume; (10) represent the cost constraint and (11) weight constraint. The α_i and β_i represents the physical characteristics of the i th component.

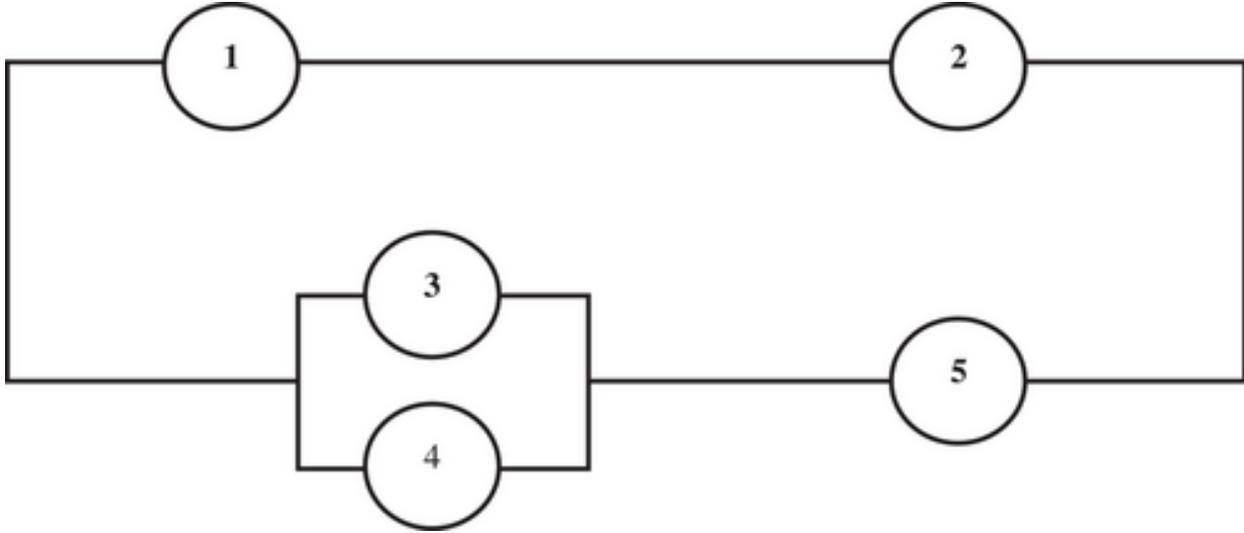


Fig 7: Series-parallel connection

In Fig. 7 above, a series-parallel connection for reliability redundancy allocation, components are arranged in a combination of series and parallel configurations. This setup enhances overall system reliability by combining the benefits of both configurations.

4.2.1.3. Complex (bridge) system

A comprehensive model for the reliability of complex systems is developed, integrating their reliability models with their operational process models. This model considers variations across different operational states, including the reliability structures and component reliability parameters (Abouei Ardakan et al., 2016). The complex (bridge) system can be mathematically explained as follows:

$$\begin{aligned} \text{Maximize } f(r,n) = & R_1R_2 + R_3R_4 + R_1R_4R_5 + R_2R_3R_5 + R_1R_2R_3R_5 - R_1R_2R_4R_5 - \\ & R_1R_3R_4R_5 - R_2R_3R_4R_5 + 2R_1R_2R_3R_4R_5 \end{aligned} \quad (12)$$

$$g_1(r,n) = \sum_{i=1}^m w_i v_i^2 n_i^2 - V \leq 0 \quad (13)$$

$$g_2(r,n) = \sum_{i=1}^m \alpha_1 \left(-\frac{1000}{\ln(r_1)} \right) \beta_1 [n_1 + \exp(0.25n_1)] - C \leq 0 \quad (14)$$

$$g_3(r,n) = \sum_{i=1}^m w_1 n_1 \exp(0.25n_i) - W \leq 0 \quad (15)$$

$$0 \leq r_1 \leq 1, n_1 \in Z^+, 1 \leq i \leq m$$

The constraint $g_1(r,n)$, $g_2(r,n)$ and $g_3(r,n)$ represent the volume, cost and weight equation respectively. The α_1 and β_1 represents the physical characteristics of the i th component.

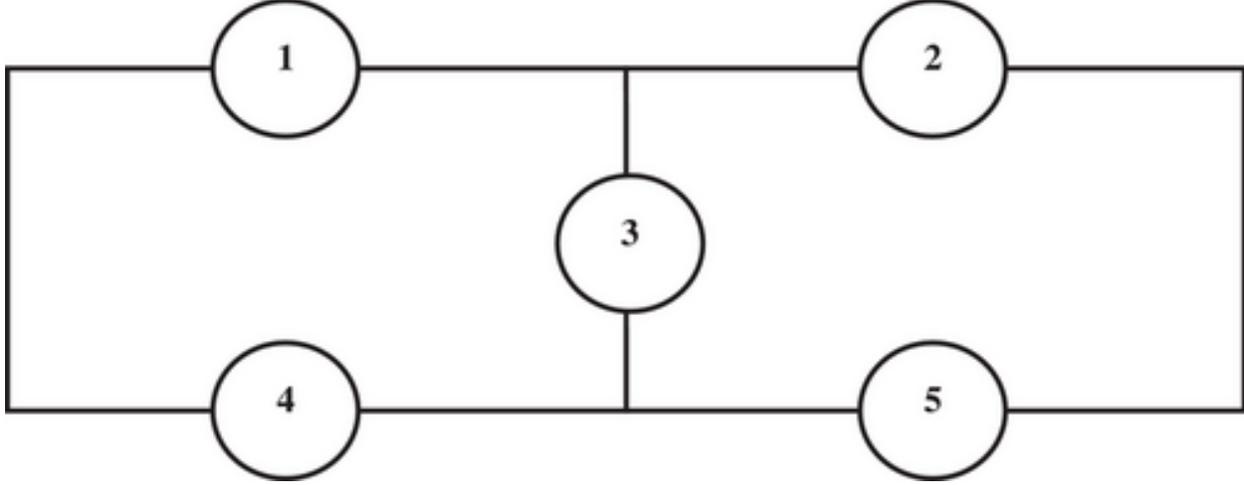


Fig 8: Complex (bridge) system.

4.2.1.4. Overspeed protection for gas-turbine

To ensure the safe operation of gas turbines to avoid failure, overspeed protection systems are implemented. These systems continuously monitor the turbine's rotational speed and take immediate action if it exceeds the predetermined safe limit (Nath & Muhuri, 2022a).

$$\max f(r, n) = \prod_{i=1}^m [1 - (1 - r_i)^{n_i}] \quad (16)$$

$$\min V_s(n) = \sum_{i=1}^m w_i v_i^2 n_i^2 \quad (17)$$

$$\min C_s(r, n) = \sum_{i=1}^m C(r_i) [n_i + \exp(0.25n_i)] \quad (18)$$

$$\min W_s(n) = w_i n_i \exp(0.25n_i) \quad (19)$$

$$\text{s.t. } V_s - V \leq 0, C_s - C \leq 0, W_s - W \leq 0$$

$$\text{with } 0.5 \leq r_i \leq 1 - 10^{-6}, r_i \in \mathbb{Z}^+, 1 \leq n_i \leq 10$$

$$\text{where } C(r_i) = \alpha_i \left(-\frac{T}{\ln(r_i)} \right)^{\beta_i}; \quad (20)$$

The T represent the operating time for the component to stay active, while the α_1 and β_1 represents the physical characteristics of the i th component.

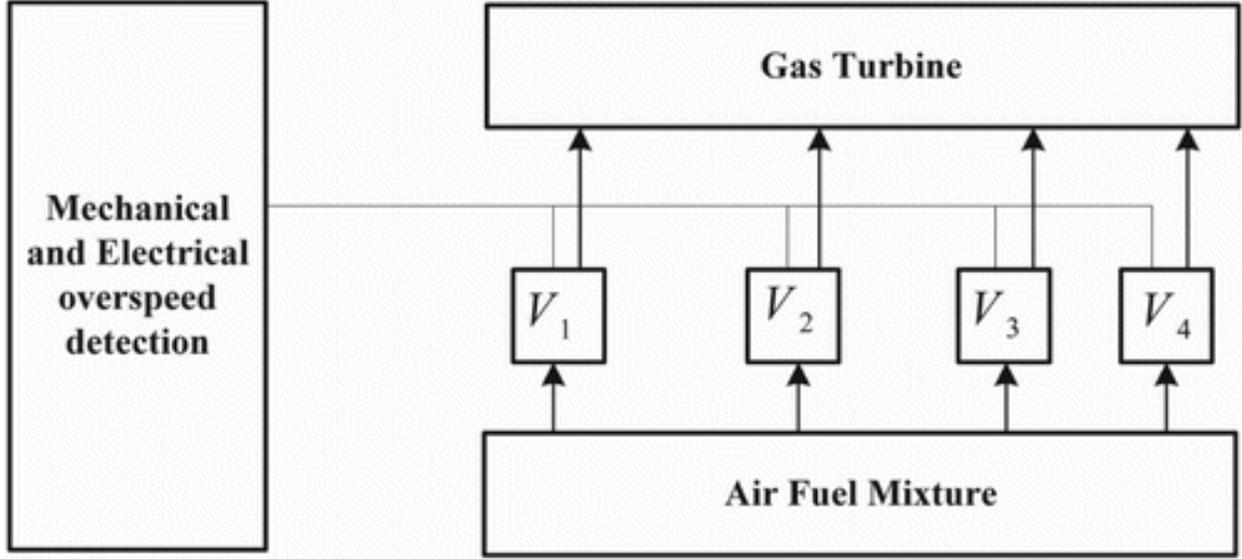


Fig 9: Overhead protection for gas-turbine

The figure 9 above shows a gas turbine overspeed protection consisting of four valves which provides fuel for the system. Overspeed protection systems help ensure the safe and reliable operation of gas turbines in various industrial applications.

4.2.1.5. Large-scale system

This large-scale system has been considered in Mellal & Zio, (2016) & Huang et al., (2019). This mathematical description is formulated below:

$$\text{Maximize } R_s(r, n) = \prod_{i=1}^{15} [1 - (1 - r_i)^{n_i}] \quad (21)$$

Subject to:

$$g_1(r, n) = \sum_{i=1}^{15} v_i n_i^2 \leq V \quad (22)$$

$$g_2(r, n) = \sum_{i=1}^{15} \alpha_i (-T / \ln r_i)^{\beta_i} [n_i + \exp(\frac{n_i}{4})] \leq C \quad (23)$$

$$g_3(r, n) = \sum_{i=1}^{15} w_i n_i \exp(\frac{n_i}{4}) \leq W \quad (24)$$

$$0.5 \leq r_i \leq 1, r_i \in [0, 1] \subset \mathbb{R}^+; 1 \leq n_i \leq 10, n_i \in \mathbf{Z}^+; i = 1, 2, \dots, 15$$

The constraint $g_1(r, n)$, $g_2(r, n)$ and $g_3(r, n)$ represent the volume, cost and weight equation respectively.

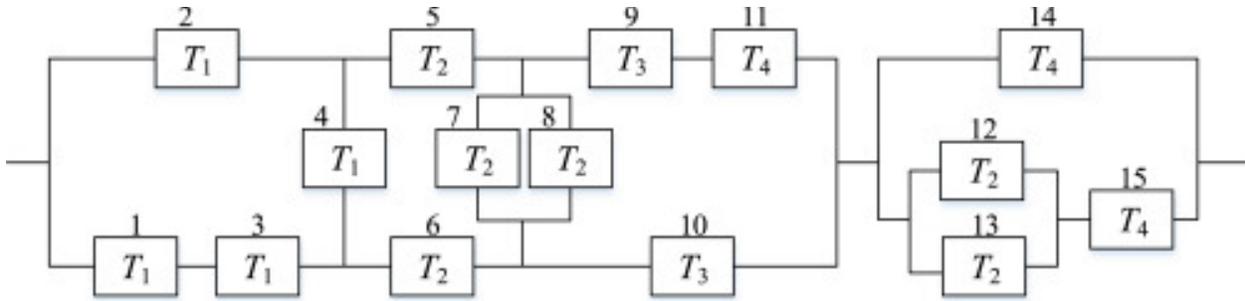


Fig 10: Large-scale system

In fig. 10 above shows a large-scale system of 15 different interconnected components and subsystems, each critical to the overall functionality. Large-scale systems can achieve high reliability and minimize downtime.

4.2.2. Taxonomy of RRAP

(a) Small scale RRAP

In simpler terms, a small-scale system in RRAP deals with a handful of useful components needed for a larger system to function correctly. The goal is to strategically incorporate additional resources or parts so that the smaller system can continue operating if certain components fail or experience downtime. This method ensures reliability by preventing interruptions and ensuring that essential operations proceed smoothly. Small scale system is known for their limited number of components, it has its own specific challenges and constraints. Systems and subsystems which comprises of lesser components are regarded as small-scale system.

The small-scale system characterizes limited resources on cost, weight and volume. Small scale systems need to be reliable in a way that it continues working should any part fails. Introducing redundancies to small scale systems keeps the system running as these methods keeps it going should any part fails. In addressing the small-scale RRAP, many authors have tried solving this problem using various methods like exact, mathematical algorithm, dynamic programming and mixed integer methods.

To understand the concept of small-scale systems using 5 subsystems, Chowdury et al., (2024b) provide novel method for evolutionary multi-task optimization in RRAP using the multifactorial

evolutionary algorithm (MFEA). The method is tested on two test sets, comparing two optimization tasks: complex and series-parallel systems. The MFEA method supersedes other approaches and significantly improves computation time, with 28.02% and 14.43% improvements compared to the genetic algorithm. The method's percentage improvements increase significantly compared to other approaches.

Tian et al, (2008) presented a 3-stage multi-state series-parallel system which demonstrate their proposed RRAP approach. The result suggested that the RRAP resulted in lower cost consumption. Zheng et al (2023) adopts a direct position updating-based trying mutation PSO (DTPSO) which proposed to improve global optimization of PSO and enhance performance in complex, high-dimensional problems. This proposed DTPSO shows excellent and stable performance in reliability optimization, achieving corresponding optimization of reliability and system cost.

Yeh et al., (2015) implemented the use of SSO algorithm to improve computation efficiency and solve RRAP. The RRAP is NP-hard, requiring simultaneous decision-making with nonlinear constraints. Four common RRAP problems including the series-parallel systems, complex (bridge) systems, and overspeed protection of gas turbine systems are used to demonstrate its applicability and effectiveness. Guo et al (2011) discusses this approach using the Lagrange multipliers method and Kuhn-Tucker for nonlinear programming models for solving the redundancy allocation.

(G.-S. Liu, 2006) describes a reliability-redundancy optimization approach that uses H–J pattern search and dynamic programming with dominating sequences. The approach is more efficient and flexible than typical combination approaches. Yalaoui et al., (2005) addresses the RRAP in parallel-series systems, focusing on cost minimization under minimum reliability constraints. The authors developed a dynamic programming method, the pseudopolynomial YCC algorithm. The algorithm consists of two steps: resolution of subproblems for each system stage and global resolution using previous results. Dynamic programming is the most adapted method for solving sub-problems and can deal with more constraints, but with increased computation time.

Maneckshaw & Mahapatra, (2022) proposes a multi-objective RRAP model, enhancing the system reliability and cost reduction through feasible redundancy. It uses multi-objective evolutionary algorithm (MOEA) for analyzing results in various interval alternatives. The need to figure out how many redundancy components, within a certain system weight, are required to improve system reliability while lowering costs.

Yeh & Hsieh, (2011) introduces the penalty guided ABC algorithm for solving nonlinearly mixed-integer reliability design problems, including series systems, series-parallel systems, complex systems, and overspeed protection systems. ABCs offer advantages like memory, multi-character, local search, and solution improvement mechanisms. The algorithm effectively and efficiently finds near-optimal solutions, with numerical examples showing its effectiveness and comparing favorably with previous algorithms. They achieve global or near-global solutions in mixed-integer reliability problems.

Meta-heuristics, such as simulated annealing, genetic algorithm, tabu search etc. have been incorporated and used in reliability-redundancy optimization. The ICA is an evolutionary algorithm based on imperialistic competition. The AR-ICA approach, which incorporates attraction and repulsion concepts, is an efficient optimization technique for reliability redundancy allocation. Afonso et al., (2013) uses the Imperialist Competitive Algorithm (ICA) and Attraction and Retraction (AR)-ICA to solve reliability redundancy allocation problems. AR-ICA uses an attraction and repulsion concept for better exploration of search space. Simulation results show AR-ICA performs well in solution quality and improves.

Three cases are considered:

Case 1 Series parallel system

$$\prod_{j=1}^n [1 - (1 - r_j)^{x_j}] \geq R_s^L \Rightarrow \sum_{j=1}^n (1 - r_j)^{x_j} \leq Q_s^U (Q_s^U = 1 - R_s^L)$$

Case 2 n stage standby element system

$$\prod_{j=1}^n \left[r_j 1 - \ln r_j + \frac{(\ln r_j)^2}{2!} - \dots + (-1)^{x_j-1} \frac{(\ln r_j)^{x_j-1}}{(x_j-1)!} \right] \geq R_s^L$$

$$\Rightarrow \sum_{j=1}^n \left[1 - \left[r_j \left(1 - \ln r_j + \frac{(\ln r_j)^2}{2!} - \dots + (-1)^{x_j-1} \frac{(\ln r_j)^{x_j-1}}{(x_j-1)!} \right) \right] \right]$$

Case 3 n stage partially redundant system

$$\left[\prod_{j=1}^n \left(1 - \binom{x_j}{0} (1 - r_j)^{x_j} + \binom{x_j}{1} r_j (1 - r_j)^{x_j-1} + \dots \right. \right. \\ \left. \left. + \binom{x_j}{k_j-1} r_j^{k_j-1} (1 - r_j)^{x_j-k_j+1} \geq R_S^L \right] \right. \\ \Rightarrow \sum_{j=1}^n \left[\binom{x_j}{0} (1 - r_j)^{x_j} + \binom{x_j}{1} r_j (1 - r_j)^{x_j-1} + \dots \right] \\ \left. + \binom{x_j}{k_j-1} r_j^{k_j-1} (1 - r_j)^{x_j-k_j+1} \leq Q_S^U \right]$$

In small-scale system, deciding where to add backup options requires careful decision-making. Factors like how critical each component is, how cost-effective the solution is, and what the system needs to accomplish all influence these decisions. It involves using calculations and specialized methods to figure out the best places to put these backup options and the need to ensure everything runs smoothly, keeping maintenance costs manageable, and reducing any negative effects if something goes wrong with the system. The table 6 shown below seeks to identify small-scale systems used in the papers and to explain how researchers and industries has addressed the problem.

Table 6: Overview of small-scale systems

Ref.	CI Approach	Addressed Problem	Result/Remark
Kuo et al. (1987)	Lagrange Multiplier & Branch and bound	Pure integer, mixed integer	4 & 5 stage series system; LMBB resulted in reliability values of 0.929
Xu et. Al (1990)	XKL	Series system	5 stage series; R= 0.77939, 0.87183, 0.90288, 0.71139, 0.78779
Ha & Kuo (2005)	Diverse-path iterative heuristics	Series system, Complex system	Minimize NLP problem
Yalaoui et al., (2005)	Pseudopolynomial YCC algorithm	Series system	One stage problem; Solutions converge quickly towards the optimum
Ha & Kuo, (2006)	Efficient branch-and-bound method	Bridge system	5 subsystems; Global optimum without assumptions
H.-G. Kim et al., (2006)	Simulated annealing algorithm	Series system, Series-parallel system, complex system	4, 5 subsystems; SA algorithm outperforms others
Chen, (2006)	Artificial Immune Algorithm	Series system, Series-parallel system, Complex system, Overspeed protection system	5 subsystems; IAs provides superior solution than GA and achieves global solution
Dos Santos Coelho et al., (2009)	Differential equation (DE) and the Harmony search (HS)	Overspeed protection system for a gas turbine	4-stage; HSDE's best performance was 0.99995467
dos Santos Coelho, (2009)	Self-organizing migrating algorithm (SOMA)	Series system, series-parallel system, Overspeed protection system	5-subsystems; SOMA outperform the previously best-known solutions
Dos Santos Coelho et al., (2011)	Firefly Algorithm (FA), inspired by firefly flashing behavior	Overspeed protection system for a gas turbine	4-stage; Simulation results show FA and FAC are effective
Ebrahimipour et al., (2011)	Multi-objective particle swarm optimization	Complex system, Series system	4, 5 subsystems; MOPSO satisfies the reliability and cost criteria
Hsieh & You (2011)	Two-phase approach using Immune algorithm	Series system, Series-parallel system, complex system	5 subsystems; Phase I solutions exhibiting higher system reliabilities
Yeh & Hsieh, (2011)	Penalty guided artificial bee colony algorithm (ABC)	Series system, series-parallel system, complex systems, Overspeed protection system	5 subsystems; The algorithm effectively and efficiently finds feasible optimal/near optimal solutions
Sun et al., (2011)	Linear programming and extreme learning machine (LP-ELM)	semi-Markov model	ELM is trained to predict optimal sojourn time distribution parameters

Guo, Jiang, et al., (2011)	Lagrange multipliers method and Kuhn-Tucker condition	Complex system connected to series with multiple stage subsystem	Lagrange multipliers method is $[R1, R2, R3, R4] = [0.999735, 0.999494, 0.999294, 0.999388]$
Ebrahimipour et al., (2013)	Emotional learning in Sugeno-type fuzzy inference systems (ELFIS)	Series-parallel system	ELFIS model demonstrated superior performance
Khalili-Damghani et al., 2013	Dynamic self-adaptive multi-objective particle swarm optimization (DSAMOPSO)	Series-parallel system	DSAMOPSO method showed relative preference
Garg & Sharma, 2013	Particle Swarm Optimization (PSO) and compared with Genetic Algorithm (GA)	Series-parallel system	Produces a bias among Pareto optimal solutions
Najafi et al., 2013	Simulated Annealing (SA) and Genetic Algorithm (GA) are proposed	Series-parallel system	GA performed better than SA in computational experiments, but average computational time was longer
Xing et al., 2013	Computational intelligence	Series-parallel system	Hybridization combines with CI approaches may offer better solutions
Sheikhalishahi et al., 2013	Hybrid approach based on GA and PSO	Series system, Series-parallel system, Complex system	4, 5 subsystems; The result better than other heuristic and meta-heuristic
Kanagaraj et al., (2013)	Hybrid cuckoo search and genetic algorithm (CS-GA)	Series system, Series-parallel system, Complex system, Overspeed protection system	5 subsystems; CS-GA is more efficient in finding optimal solutions than existing methods
Zhang et al., 2014	Bare-bones multi-objective particle swarm optimization algorithm (BBMOPSO)	Series system	5 subsystems; BBMOPSO generate best approximation for Pareto optimal front
Caserta & Voß, 2015	Exact dynamic programming	Series-parallel system	Exact dynamic programming obtains numerical results
Soltani, (2014)	Binary state non-repairable systems	Series system, Series-parallel system, Complex system, Overspeed protection system	New models and methods in reliability optimization
Garg et al., 2014	Particle swarm optimization	Complex system, Overspeed protection system	4, 5 subsystems; Solved the MORRAP in a fuzzy environment
Ardakan & Hamadani, (2014)	Genetic Algorithm	Series system, Series-parallel system, Complex system	5 subsystems; Offers greater flexible for system design
Ashraf et al., 2015	Particle swarm optimization	Series-parallel system	Approach outperforms other recently reported results

Garg, (2015)	Penalty guided biogeography-based optimization (BBO)	Series system, Series-parallel system, Complex system, Overspeed protection system	5 subsystems; The best solutions outperform well-known heuristic methods
C.-L. Huang & Yeh, (2015)	Simplified Swarm Optimization (SSO)	Series-parallel system, Complex system, Overspeed protection system	The algorithm successfully solves three RRAP problems
E. Zhang & Chen, (2016)	Multi-objective particle swarm optimization (MOPSO)	Series system	5 subsystems; MOPSO adopts pareto for solving the problem
Haiying & Yubao, (2016)	Mixed tabu-search and interior search algorithm (TS-ISA)	Series system, Complex system, Overspeed protection system	5 subsystems; TS-ISA proves efficient and superior to other methods
Y. Liu, (2016)	Improved bat algorithm (IBA)	Series system, Series-parallel system, Complex system	5 Subsystems; Simulation results show enhanced computational efficiency
Khorshidi, Gunawan, Sutrisno, et al., 2016	Imperialist competitive algorithm (ICA)	Series system, Bridge system	Parameters used improved the ICA
Khorshidi, Gunawan, & Ibrahim, 2016	Genetic Algorithm	k-out-of-n systems	Multi-state weighted k-out-of-n system optimization model was developed
Abouei Ardakan et al., 2016	Mixed strategy	Series-parallel system, Complex system, Overspeed protection system	5 subsystems; Mixed strategy improves system reliability
Sahoo, 2017	Genetic Algorithm	Complex system	5 subsystems; The method tackled defuzzification of fuzzy number
Wang & Wang, 2017	Adaptive particle swarm optimization algorithm with foraging behavior	Series system, Series-parallel system, Bridge system	5 subsystems; The PSO improves the convergence rate
Jin et al., 2017	Integrated product-service mechanism	k-out-of-m system	Two models are developed
H. Kim, 2017	Structured continuous-time Markov chain (CTMC)	Series system, Series-parallel system, Complex system	5 subsystems; Active and standby redundant subsystems
Chebouba et al., 2018	NSGA-II algorithm	Overspeed protection system	4 subsystems; Providing the decision maker with the optimal solution space
Xiao & Pan, 2018	Pareto solutions using the NSGA-II algorithm	Series system	The most efficient scheme is 0.99
Juybari et al., 2018	cold standby redundancy strategy and a penalty guided water cycle algorithm	Small	Superiority over previous studies
Abouei Ardakan & Rezvan, 2018	Multi-objective evolutionary algorithm (NSGA-II)	Series system, Series-parallel system, Complex System	5 subsystems; the proposed algorithm outperform the best active structure

Samanta & Basu, 2019a	Penalty guided fuzzy adaptive particle swarm optimization (FAPSO)	Series system, Series-parallel system, Complex system, Overspeed protection system	Series-parallel system	5 subsystems; Outperforms standard PSO and previous adaptive approaches
Han et al., 2019	Reliability entropy			Optimal value of redundancy allocation
Shojaei & Mahani, 2019	Hybrid method using co-evolutionary PSO and harmony search algorithm	Series system, Series-parallel system, Bridge System		5 subsystems; Better performance in solving three different designs
Yeh, 2019b	Elite selection and boundary search (SEB)	Series system, Series-parallel system, Complex system		5 subsystems; It outperforms the current best algorithm
Dobani et al., 2019	Hybrid SFS-GA algorithm	Series system, Series-parallel system, Complex system, Overspeed protection system		5 subsystems; Better structures with higher reliability values in all test problems
Yeh, 2019a	Boundary simplified swarm optimization (BSO)	Series system, Series-parallel system, Complex system		5 subsystems; Best solution based on boundary conditions
Ben Guedria & Hassine, 2019	Adaptive simplified (ASPSO) based algorithm	Series system, Series-parallel system, Complex system, Overspeed protection system		5 subsystems; New approach outperforms existing methods in achieving optimal solutions
Samanta & Basu, 2019b	Multi objective Genetic Algorithm (MOGA) and Multi-Objective Particle Swarm Optimization (MOPSO)	Bridge network system, Space-capsule system		5 subsystems; MOGA and MOPSO to determine optimal system reliability
Ouyang et al., 2019	Stochastic perturbation particle swarm optimization (SPPSO)	Series system, Series-parallel system, Complex system		SPPSO proves superior when compared with previous result
Rakhi & Pahuja, 2020	Grey Wolf Optimization algorithm	Series system, Series-parallel system, Complex system, Overspeed protection system		5 subsystems; Enhancement in overall system reliability
Mahato et al., 2020	Real coded elitist genetic algorithm (RCEGA)	Crisp model, fuzzy model		5 subsystems; System reliability is closer to crisp models
Mahdavi-Nasab et al., 2020	Continuous time Markov chain (CTMC) and Water cycle algorithm (WCA)	Series system, Series-parallel system, Complex system		5 subsystems; Model corrects approximation errors and improves structures
Taghiyeh et al., 2020	Fuzzy parametric programming (FPP)	Overspeed protection system		4 subsystems; Improvement with PSO
Park, 2020	Mixed integer linear programming (MILP) and Aggregate and Iterative Disaggregate (AID)	Series system, Series-parallel system, Complex system		AID algorithm improves the solution quality
Yeh et al., 2021	Multi-objective SSO	Series system, Series-parallel system, Complex system,		5 subsystems; MOSSO outperforms the NSGA-II & MOPSO

Nazim Chebouba et al., 2020	NSGA-II	Overspeed protection of a gas turbine system Overspeed protection system	4 subsystems; Large number of optimal solutions
Chebouba et al., 2020	Non-dominated sorting genetic algorithm (NSGA-III)	Overspeed protection system	4 subsystems; Generate non-dominated solutions
Chebouba et al., 2021	Non-sorting genetic algorithms II and III	3-stage series system	Identify the best Pareto front
Najmi et al., 2021	Parallel stochastic fractal search algorithm	Series system, Series-parallel system, Complex system	5 subsystems; Results showed increased flexibility and improved reliability
Ashraf et al., 2021	Gradient-based optimizer (GBO)	Series-parallel system	The GBO showing a 0.044% superior performance
Song & Li, 2021	Differential evolution	Complex system, Series system, Series-parallel system, Overspeed protection system	5 subsystems; Individual evaluation and constraint handling integrated
Lin et al., 2021	Simplified swarm optimization (SSO) algorithm	Series system, Series-parallel system, Complex system, Overspeed protection of a gas turbine system	5 subsystems; The proposed algorithm solves the proposed fuzzy constraints models of RRAP
J. Zhang et al., 2021	Artificial bee colony algorithm	Overspeed protection of a gas turbine system	4 subsystems; Solved the nonlinear mixed-integer programming
Z. Wang et al., 2021	Genetic Algorithm	Series system, Series-parallel system, Complex system	5 subsystems; IFCM in RRAP results in different system design solutions
Shukla et al., 2021	Multi-factorial Evolutionary Algorithm (MFEA)	Complex system, Series system	5 subsystems; MFEA framework is proposed, generates better solution to GA and PSO
C.-L. Huang et al., 2021	Particle-based simplified swarm optimization (PSSO)	Series system, Series-parallel system, Complex system, Overspeed protection system	5 subsystems; PSSO outperforms traditional SSO and PSO in system reliability
Guilani et al., 2022	Hybrid stochastic fractal search-genetic	Series-parallel system; Complex (bridge) system	5 subsystems; Improved structures with higher dependability values
T.-J. Hsieh, 2022	Simplified swarm optimization (SSO)	Series system, Parallel-series-parallel system, Complex system	5 subsystems; Enabling better system reliability
Yeh et al., 2022	Simplified swarm optimization, SS3OA, BAT, and penalty function	Series system, Series-parallel system, Complex system, Overspeed protection system	5 subsystems; SS3OA improves solution quality and reduces runtime by half
Sheikhpour et al., 2022	Teaching learning-based optimization (TLBO) algorithm	Series system, Series-parallel system, Complex system,	5 subsystems; Effectively finds system optimal configuration

Maneckshaw & Mahapatra, 2022	Multi-objective evolutionary algorithm (MOEA)	Series-parallel system	Enhance system reliability with reduced cost
Mahdavi-Nasab et al., 2022	Mixed redundancy strategy	Series-parallel system, Complex system, Overspeed protection system	5 subsystems; Estimating exact reliability values
Ramezani Dobani et al., 2022	Fitness-distance balance stochastic fractal search algorithm (FDBSFS)	Series system, Series-parallel system, Complex system	5 subsystems; Evaluate its reliability optimization problems
Zavieh et al., 2022	K-mixed redundancy strategy	Series-parallel system, Complex system	Strategy outperformed other strategies for switch reliability values below 0.95
H. Zhang et al., 2022	Mixed-integer	Series-parallel system	Proposed method outperforms a meta-heuristic algorithm
Paramanik et al., 2022	Real coded elitist genetic algorithm	Complex system	5 subsystems; The algorithm solves the intuitionistic fuzzy environment
Ashraf et al., 2023	Interval Type-2 Fuzzy Multi-objective RRAP (IT2FMORRAP)	Series-parallel system	IT2FMORRAP generates superior Pareto Fronts compared to T1FMORRAP
Jiang et al., 2023b	Binary-addition simplified swarm optimization (BSSO)	Series-parallel system	Performed best when compared to others
M. Sharifi et al., 2023	Immune Algorithm	Series-parallel system	4 subsystems; The model reduces the system's cost
De, Roy, et al., 2023	NF-MOPSO and NSGA-II algorithms	Overspeed protection of gas turbine system	5 subsystems; NF-MOPSO performs better in terms of objective function satisfaction than NSGA-II
Zheng et al., 2023	Direct position updating-based trying-mutation PSO (DTPSO)	Bridge system	5 subsystems; Proposed DTPSO shows excellent and stable performance in reliability optimization
De, Rakshit, et al., 2023	Tuning and Neighborhood-based Fuzzy Multi-Objective Swarm Optimization (TNF-MOPSO)	Overspeed protection system of gas turbine	5 subsystems; Proposed algorithm is more stable to solve the fuzzy MOOP
Bhandari et al., 2023	Particle Swarm Optimization (PSO)	Complex system	5 subsystems; Best outcome in redundancy levels
Dahiya et al., 2023	Dahiya-Garg Heuristic Algorithm	Pharmaceutical plant system	3 subsystems; DG-Alg has highest system reliability
Zhang et al., 2023	Reliability Calculation Method based on Functionality Multi-graph (RCMFM)	Series system, Series-parallel system, Bridge system	5 subsystems; Proposed method supersede other previous studies on RRAP

Attar et al., 2023	Particle swarm optimization	Series system, Series-parallel system, Complex system	Model performs better when using a metaheuristic algorithm
Bhandari et al., 2023	Whale optimization algorithm	Series system	5 subsystems; Cold standby approach is better for solving RRAP
Bhandari, Kumar, et al., 2023a	Hybrid PSO-GWO	Series system, Series-parallel system, Complex system, Overspeed protection system	4, 5 subsystems; PSO Shows promising results when compared
Zhu & Jia, 2023	Genetic Algorithm	Series-parallel system	The model improved reliability and flexibility
Aghamohammadi et al., 2023	Markov chain and the particle swarm optimization (PSO)	Series system, Series-parallel system, Complex system	PSO technique for optimal issue solutions
J. Zhang et al., 2023	Pseudo-parallel genetic algorithm (PPGA)	Series system, Series-parallel system, Complex system, Overspeed protection system	4, 5 subsystems; Model also outperformed previous method
Bhandari et al., 2023	Hybrid particle swarm grey wolf optimizer (HPSGWO)	Series-parallel system	The proposed algorithm is tested against other algorithm and proves effective.
Bhandari et al., 2024	Hybrid GWO-PSO	Series system, Series-parallel system, Complex system	5 subsystems problem; High reliability by ensuring backups
Gholinezhad, 2024	Genetic Algorithm (GA) and Particle Swarm Optimization (PSO)	Series-parallel system, Series system, Complex system	5 subsystems problem; GA's performance outperforms PSO
Chowdury et al., 2024b	Multifactorial evolutionary algorithm (MFEA)	Complex(bridge) system, Series-parallel system, Over-speed protection system, Life support system in a space capsule	5 subsystems problem; MFEA method supersedes other approaches
Nath & Muhuri, 2024	Prioritized many-objective optimization problem RRAP (PrMaORRAP)	Series system, Series-parallel system, Complex system, Overspeed protection system	5 subsystems; Proposed approach is superior in most cases

(b) Large Scale RRAP

A large-scale problem consists of several interconnected components or subsystems.

Large-scale systems often operate in dynamic environments where conditions and requirements change over time. The system needs to adapt to these changes in real-time or near real-time. A system involving vast number of components in the range of 10 or more subsystems can be regarded as a large-scale RRAP and as this system becomes more complex and harder, solving them becomes crucial and important, therefore researchers applied the GA, PSO and ACO to optimize the system allocation.

In solving the RRAP, authors have made several approaches both novel and heuristics method, few of the papers have used these various algorithms like the Mellal et.al (2023) opined that PSO is more effective than GWO in solving a 15-subsystems system. He et al., (2013) proposes a hybrid genetic algorithm (HGA) for multilevel redundancy allocation problem (MLRAP), which is NP-hard and a hierarchy optimization problem. The HGA use hybrid dimensional array to represent solutions, with different row representing the redundancy information of a unit in the system and a simulation annealing strategy is used to maintain feasible solutions which generates new populations for the next generation.

Mellal & Salhi, (2022) explores the implementation of the Plant Propagation or Strawberry Algorithm (PPA/PPS) for system reliability redundancy allocation. The algorithm is a stochastic evolutionary optimization method inspired by strawberry plants and focuses on solution space exploration. The algorithm evaluates each member of the population and assigns a fitness function, which influences the number of runners and distance traveled. PPA proved functional when tested including integrated energy systems design for off-grid mining operations using 10 subsystems. Its superiority is highlighted through benchmark problems. Q. He et al., (2015) introduces a novel artificial fish swarm algorithm (NAFSA) for solving large-scale RRAP using 20 subsystems. The best solutions found by NAFSA are better than or tied to known best solutions by other heuristic methods.

SOMA is a stochastic evolutionary algorithm inspired by social behavior of co-operating individuals and competitive-cooperative strategies. SOMA was found to outperform the previously best-known solutions available. The gaussian operator (GSOMA) effect of diversity

control mechanisms should be monitored in solving optimization problems in power systems and reliability (dos Santos Coelho, 2009).

The gaussian moderator for GSOMA defined by

if $ud < 0.5$ then

$$x_{i,j}^{ML+1} = x_{i,j,start}^{ML} + (x_{L,j}^{ML} - x_{i,j,start}^{ML}) \cdot t \cdot PRTV_{ector j}$$

else

$$x_{i,j}^{ML+1} = x_{i,j,start}^{ML} + G_{i,j} \cdot (x_{L,j}^{ML} - x_{i,j,start}^{ML}) \cdot t \cdot PRTV_{ector j}$$

end if

where the $G_{i,j}$ is a random Gaussian number with the mean value μ set to zero, ud is uniformly distributed random value from zero to one. The paper evaluates the approximating error using RRAP benchmark problems and recommends the proposed reliability model for large-scale or high reliability systems and proposes new examples.

To further advance our understanding, the table 7 below depict researchers who have used large-scale system in solving RRAP.

Table 7: Overview of large-scale system

Ref.	CI Approach	Addressed Problems	Result/Remark
Pan et al., 2013	Hybrid genetic algorithm (HGA)	Two multi-level serial systems	HGA outperforms the state-of-the-art approaches
H. Zhang et al., 2013	Improved particle swarm optimization (IPSO) with harmony search (HS) and LXPM	Complex systems, series system, series-parallel system, large-scale system	5, 20 subsystems; LXPM-IPSO-GS effectively avoids prematurity and improves convergence performance
He et al., (2013)	Hybrid genetic algorithm (HGA)	Multilevel serial system	Two-dimensional array-based HGA outperforms state-of-the-art approaches
Valian & Valian, 2013	Cuckoo Search and Lévy flight	Series system, series-parallel system, Complex systems, Overspeed protection system, Large-scale system	5 & 20 subsystems; CS algorithm provides the best Solutions
He et al., 2015	Novel artificial fish swarm algorithm (NAFSA)	Series system, series-parallel system, Complex systems, Overspeed protection system, Large-scale system	5 & 20 subsystems; NAFSA shows better performance related to computational accuracy
Talafuse & Pohl, 2016	Bat Algorithm (BA)	Series-parallel system	14 subsystems; The BA provides quality solutions over others
Chambari et al., 2016	Improved continuous genetic algorithm (CGA)	Series-parallel system	14 subsystems; The algorithm improves performance by parameter tuning
Mellal & Zio, 2016	Penalty guided stochastic fractal search	Series system, series-parallel system, complex system, overspeed protection, large-scale system, 10-unit structure system, 15-unit structure system, pharmaceutical plant	Penalty guided stochastic fractal search shows stability And a lower number of function evaluations (NFE)
Mellal & Zio, 2017	Genetic algorithm, cuckoo optimization algorithm with penalty function, and penalty guided stochastic fractal search (PSFS)	Pharmaceutical plant	10, 30 subsystems; The PSFS outperforms the two other method relating to best performance and system reliability
Süle et al., 2018	Process graph	Reforming reaction system	6 subsystems; proposed algorithm is useful for risk-based priority resource allocation
Süle et al., 2019	P graph	Reforming reaction system	9 subsystems; Improve energy system reliability
X. Huang et al., 2019	Penalty function is proposed, followed by a stochastic fractal search algorithm	8-unit structure system, 15-unit structure system	The survival signature is highly efficient
Ashraf et al., 2019	Particle swarm optimization (PSO) and genetic algorithm (GA)	Series-parallel FMORRAP, Parallel-series FMORRAP	10 subsystems; Proposed PSO-based solution approach outperforms GA
Cheng et al., 2019		Series-parallel system	14 subsystems; Improved NSGA-II has the best stability

Ghavidel et al., 2018	Non-dominated sorting genetic algorithm (NSGA-II) with a constraint Pareto dominance to handle design constraints Improved hybrid Jaya algorithm based on time-varying acceleration coefficients (TVACs) and the learning phase introduced in teaching-learning-based optimization (TLBO) named the LJaya-TVAC algorithm	Series system, Series-parallel system, Complex systems, Overspeed protection system, Large-scale system	when the crossover probability is 0.98 5, 20 subsystems; Jaya-TVAC algorithm provide acceptable optimization performance compared to the original Jaya algorithm
Mellal & Zio, 2020	Enhanced nest cuckoo optimization algorithm (ENCOA)	Series system, Series-parallel system, Complex systems, Large-scale system	5, 20 subsystems; ENCOA outperform other algorithm
Mellal et al., 2020	Hosted cuckoo optimization algorithm (HO-COA)	Series system, series-parallel system, Complex systems, Overspeed protection system, Large-scale system	5, 20 subsystems; HO-COA outperforms the other Methods
Wang et al., 2020	NSGA-II	Large-scale system, Series system	5, 10 subsystems; The proposed model achieve higher reliability
Mellal & Salhi, 2022	Plant Propagation or Strawberry Algorithm (PPA/PPS)	Pharmaceutical plant	10 subsystems; Superiority is highlighted through benchmark problems
S. Li et al., 2022	Improved particle swarm optimization algorithm with a specific particles initialization approach (called PIPSO)	Series system, series-parallel system, Complex systems, Overspeed protection system, Large-scale system	5, 20 subsystems; Solves the mixed-integer issues
T. Kundu et al., 2022	Hybrid salp swarm algorithm with teaching learning-based optimization (HSSATLBO)	Series system, Series-parallel system, Bridge system, Overspeed protection system, Convex quadratic reliability problem, Mixed series-parallel system, Large-scale system	5, 36 subsystems; HSSATLBO approach performs better than
T. Kundu & Garg, 2022b	Improved neural network algorithm (INNA)	Series system, Series-parallel system, Bridge system, Overspeed protection system, Convex quadratic reliability problem, Mixed series-parallel system, Large-scale system	5, 36 subsystems; Solve complex optimization problems
T. Kundu & Garg, 2022a	Hybrid TLNABC algorithm connects the features of ABC, NNA, and TLBO algorithms	Series system, Series-parallel system, Bridge system, Overspeed protection system, Convex quadratic reliability problem, Mixed series-parallel system, Large-scale system	5, 36 subsystems; Showing better results with less standard deviation
Nath & Muhuri, 2022	Non-dominated sorting genetic algorithm-III (NSGA-III)	Series-parallel systems, Overspeed gas turbine system, Large-scale system	NSGA-III proves superior when compared to others

X. Li & Li, 2022	Improved Hybrid Swarm Optimization (HSO) algorithm	Series system	14 subsystems; Modified HSO algorithm is utilized to solve the Fyffe
AL-Saati, 2022	Wild horse optimization (WHO) algorithm	Series-parallel system, Complex (bridge) system, Overspeed protection system, Pharmaceutical plant	5, 10 subsystems; WHO algorithm outperformed other optimization methods
Mellal et al., 2023 Thymianis et al., 2023	PSO and GWO Bat Algorithm and Firefly Algorithm (BA-FA)	Series system Series system, Series-parallel system, Bridge system, Overspeed protection system, Large-scale system, Life-support system in a space capsule, Ten-unit structure system, Fifteen-unit structure system, Pharmaceutical plant	15 subsystems, PSO is more effective than GWO 4, 5, 10, 15 subsystems; the BA-FC outperforms the other nine compared algorithms
Z. Zhao et al., 2024	Interval multiobjective particle swarm optimization algorithm based on probability dominance (PD-IMOPSO)	Scada system	6 subsystems; The proposed model determine the dominant relationship of intervals.
H. Ouyang et al., 2024	Dual population collaborative harmony search algorithm with adaptive population size (DPCHS)	Series system, Series-parallel system, Bridge system, Overspeed protection system, Large-scale system, Convex quadratic reliability problem, Mixed series-parallel system, Incomplete fault detecting the switching	5, 36 subsystems; DPCHS outperforms other HS variants and algorithms

4.2.3. Computational approaches for addressing RRAP

RRAP has been a focal point in the optimization domain, with numerous computational approaches employed to derive efficient solutions. These methods are diverse, leveraging both traditional optimization techniques and advanced heuristic algorithms to navigate the complex solution space inherent in RRAP. Other than that, there are many computational intelligence (CI) approaches which have been used to address RRAP.

4.2.3.1. Fuzzy sets approaches used to solve RRAP's

Fuzzy sets in RRAP have apparently become increasingly significant in areas where imprecision and uncertainty are common. The objective associated with fuzzy set involves satisfying the constraint that may be fuzzy. Ebrahimipour & Sheikhalishahi, (2011) used the triangular fuzzy numbers and discount rates for components which address the MOPSO approach used to solve the model. The results indicate the proposed approach helps make informed decisions about redundant components and their reliability, ensuring system meets both demands.

The research shows method for continuous assessment and improvement of redundancy allocation problem, focusing on redundancy level and component reliability. Garg et al., (2014) addresses the bi-objective RRAP for series-parallel systems, considering reliability of the system and designing cost in a fuzzy environment. A fuzzy model is converted to a crisp model, based on fuzzy numbers and decision maker preferences. Particle swarm optimization is used, and statistical simulations are performed for supremacy. The resulting decision support system helps plant maintenance personnel decide their future strategies for optimal system performance. The general idea can be applied to series-parallel, k-out-of-n system, and circular structure systems.

$$\mu_{\Lambda}(x) = \begin{cases} \frac{x-a}{b-a}; & a \leq x \leq b \\ 1 & ; x = b \\ \frac{c-x}{c-b}; & b \leq x \leq c \\ 0 & ; \text{otherwise} \end{cases}$$

Sahoo, (2017) presents a method for transforming complex system components into crisp nonlinear programming problems applying the triangular fuzzy number (TFN). It presents a GA for solving RRAP in fuzzy environments. The algorithm is real-coded and sensitivity analyses are performed. Taghiyeh et al., (2020) suggests ways to deal with uncertainty in reliability-redundancy allocation for overspeed protection systems using the fuzzy parametric programming (FPP). The FPP can be synthesized with PSO to enhance system reliability and easier to navigate. (Ashraf et al., 2014) considered MORRAP with fuzzy type-2 uncertainty, a first in research. The NSGA-II algorithm was used, obtaining Pareto-optimal solutions with non-dominated fronts for different components. The paper provides numerical examples to demonstrate and highlights the importance of considering type-2 fuzzy uncertainties in system design.

4.2.3.2. Machine Learning used to solve RRAP's

Innovation in research regarding RRAP improvement has gained attention in recent times and industries are concerned on ways reliability can be improved and dependable. Therefore, there is the need to shift away from conventional methods used by previous researchers such as the exact or mathematical methods. As applications continue to get complicated, machine learning has impacted positively in improving the RRAP problems. Application of machine learning has tried to solve advanced algorithm, or some cumbersome methodology used in the past by enhancing the quality of result and also the search efficiency.

By using machine learning approach, Ebrahimipour et al., (2013) proposed how emotional learning-based fuzzy inference system (ELFIS) can aid the reliability optimization in RRAP. ELFIS propelled by machine learning provides a means for managing complex or large systems. The machine learning leverage on the data by training an intelligent model with ELFIS to provide accurate and reliable solutions.

Using machine learning to identify patterns and optimize solutions offers a new way to address RRAP. By analyzing performance metrics, failure rates, and redundancy strategies, these algorithms can determine the best configuration for backup components. This ensures systems stay reliable while meeting cost, weight, and space constraints. This study looks at how effective these algorithms are in different situations. With the rapid progress in medical

technology fueled by machine learning, procedures that were once considered impossible are now becoming possible.

Bhandari, Rawat, et al., (2023) studied the Total Artificial Heart (TAH) because it's designed to replace failing human hearts and improve the lives of people with heart failure. To ensure the TAH works reliably for a long time, a smart method called the whale optimization algorithm was used. This technique helps us manage complex systems like the TAH. By evaluating and adjusting the algorithms enhances the process to keep the TAH functioning smoothly. The capability to self-improve and adapt enables them to learn from past errors, thereby enhancing their performance over time. Therefore, the implementation of machine learning algorithms goes beyond merely optimizing search efficiency, it also ensures the generation of superior quality solutions.

4.2.3.3. Types of Evolutionary Algorithms used to solve RRAP

There are several EAs used in solving RRAP in the literature as have been explained in the above sections. This thesis covers few of them here which are most commonly used EAs such as: Genetic Algorithms (GA), Particle Swarm Optimization (PSO), Ant Colony Optimization (ACO), Differential Evolution (DE) and the non-dominated sorting genetic algorithm. The importance of each algorithm with respect to RRAP is addressed as follows:

- i. **Genetic Algorithm:** The concept of genetic algorithm was premised on Darwin's theory of evolution which imbibes on the belief of survival of the fittest. According to (Sahoo, 2021) the GA uses the crossover, mutation and selection in natural process to produce quality solutions. Genetic algorithms provide several advantages over conventional optimization algorithms. The two most noteworthy are parallelism and the capacity to handle complicated issues. Whether the goal (fitness) function is linear or nonlinear, GA can handle a wide range of optimization problems though its time consuming in large scale problems (Kuo & Wan, 2009).

According to research, GA has proven very effective in terms of cost-effectiveness of parts selections and allocation of system redundancy. (Najafi et al., 2013) implemented the GA as an active metaheuristics technique to optimize the system's mean time to

failure (MTTF) and the results of GA was compared to another algorithm, simulated annealing (SA) in which the GA outperforms the SA in terms of the quality of solution.

- ii. **Particle Swarm Optimization:** It is commonly regarded as a strong optimization which can be used in solving many optimization issues. Bhandari et al., (2023) explained that PSO have maximum tendency to search solutions locally but unable to unravel global optimum. Several papers which have used the modified version of PSO like (Ouyang et al., 2019b) introduces heterogeneous components to RRAP and proposes an improved PSO algorithm with stochastic perturbation nature (SPPSO). Comparative studies show efficiency and greater system reliability compared to previous methods, with switching reliability significantly influencing optimal system configuration.

PSO combines with another known algorithm to solve the RRAP. For instance, Sheikhalishahi et al., (2013) introduces hybrid GA-PSO approach for RRAP in series and complex systems. It aims to reduce the nonlinear constraints related to cost, weight and volume and in turn increase the system reliability and in turn reduces computational time and variance, benefiting engineers and managers. The approach maximizes system reliability while improving computation efficiency. (Zheng et al., 2023) utilizes the PSO approach in trying-mutation strategy and a direct position updating method, this improves the global optimization capabilities of the algorithm. The PSO when compared to other algorithms provide superior optimization performance in handling high-dimensional problems, making it an effective tool for reliability optimization in engineering applications.

- iii. **Ant Colony Optimization:** It particularly effective for tackling challenging problems regarding the combinatorial optimization problems such as the vehicle routing or traveling salesman problem. It is very common in learning the search path and the possible shortest route, when compared to other algorithms. It is more adapted to path optimization which determines the likelihood of an ant choosing a particular path based on pheromone concentration and heuristic information (Zhao et al., 2024). ACO's performance is influenced by several factors, including pheromone evaporation rate, heuristic information accuracy, and the balance between exploration and exploitation.

Comparative studies indicate that ACO often outperforms traditional optimization algorithms, especially in dynamic and stochastic environments.

- iv. **Differential Evolution:** It is a powerful optimization technique used in different engineering sections mainly to reduce constraint and improve productivity. It works with the principle of improving candidate solutions over successive generations. It does this through processes similar to natural selection. Some of the authors have applied this method in solving complex problem like Song & Li, (2021) used the principle of DE to improve performance for constrained optimization problems, individual evaluation and constraint handling techniques integration. The algorithm shows its effectiveness and is expected to be adopted for real-world problems.
- v. **Non-dominated sorting genetic algorithm:** This models a unique and dynamic sorting to evaluate and rank solutions. It handles multiple objectives simultaneously and works by evaluating and evolving a population of potential solutions over successive generations. The solution is considered non-dominated once the objective in a solution supersedes other solutions and at least as good in one objective. (Cheng et al., 2019) uses NSGA-II with a constraint Pareto dominance to construct the design constraints. It represents component mixing in subsystems with integers and uses a crossover and mutation matrix. Chebouba et al., (2020) uses the NSGA-III for the generation diverse set of optimal solutions, known as the Pareto front, which informs researchers on evaluating the trade-offs between competing objectives more effectively.

4.2.4. Domains of Applications

Applications relating to RRAP has been discussed and utilized in several articles as manufacturers are interested in the reliability of systems and researchers are constantly working on achieving distinct progress in the advancement of RRAP. Applications of this domain related to RRAP has been modeled in the transportation system as this sector is important in daily life and economies which requires systems to be highly reliable and robust in services provided. The RRAP design implemented in the transportation sector ensures thorough safe operations in optimizing and using the redundant systems to aid signals and regulate the traffic management ensuring higher lifetime as proposed by Sun et al., (2011) using semi-Markov

model (SMM) to optimize the transportation system lifetime. The model obtains approximate optimal transient probabilities using the Linear Programming algorithm, involving multiple-input-multiple-output regression. The application of RRAP into real-life has been evident as corroborated by (Muhuri & Nath, 2019) suggested that the introduction of bilevel to RRAP application has tremendous use in the engineering industry such as in electric power systems, water resource control and heat exchanger network.

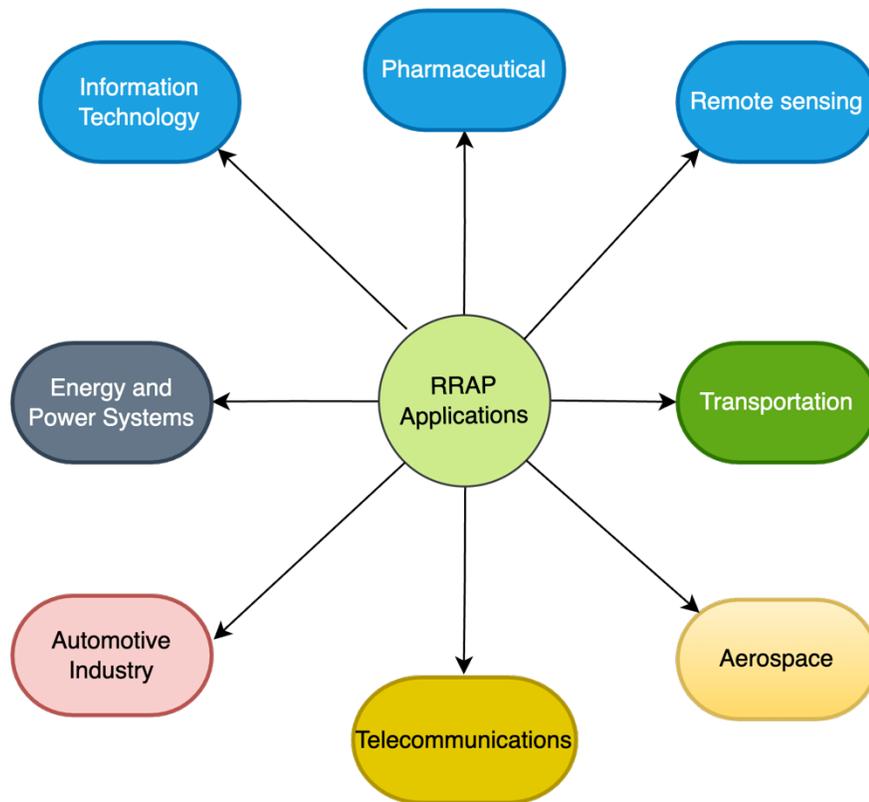


Fig. 11: RRAP Applications

According to fig. 11 shown above, RRAP applications has been very instrumental in today's world as many sectors implements the application. RRAP applications has been applied in the automotive industry, pharmaceutical, aerospace(Muhuri & Nath, 2019). Researchers continue to harmonize RRAP into many applications. For instance, in the reliability of aircraft to avoid air mishap or disaster, ZIO et al., (2019) opined the implementation of reliability in the design and development of an aircraft remain paramount. It discusses various approach used in certifying an aircraft like reliability design and analysis, certification of reliability requirements and

routine maintenance. This reliability method and prospect are utilized in different phases in the life cycle of a modern aircraft. They argued that airplane dependability cannot be achieved by advanced reliability technologies alone; rather, a knowledgeable, thorough organizational structure for reliability-related operations is needed. W. Wang et al., (2020b) also presented a new model that processes parts of production using sequentially ordered subsystems. The system switches between production lines based on detection results. A NSGA-II algorithm is modeled for Pareto optimal solutions. The model and reliability optimization methodology improve reliabilities of different production lines for the system and its industrial use.

In the foregoing, RRAP has garnered significant importance to how it solves issues relating to enhancing reliability. Most industries have gained from the effects of implementing reliability optimization. Industries employing reliability optimization encompass aviation, healthcare, and others(Xing et al., 2013). According to Boeing Airline engineering support, the integration of advanced algorithms and data into airline operations is crucial for enhancing reliability and minimizing unplanned maintenance. By leveraging managed data analytics and prognostics services, airlines can significantly improve reliability performance. Fleet reliability offerings utilize advanced analytics to identify trends within an operator's fleet data, incorporating managed Analytics to create prognostics aimed at preempting unplanned maintenance events in next-generation aircraft.

(Bhandari, Rawat, et al., 2023) uses advancements in medical technology in making Artificial Hearts practical for heart failure patients. The proposed work considers the total artificial heart (TAH) using a metaheuristic whale optimization algorithm. In the power plant systems, Chebouba et al., (2018) focuses on power plant's overspeed protection system using multi-objective optimization algorithm for solving reliability-redundancy allocation in a system by considering the overall system's reliability and cost measures. The NSGA-II algorithm is used to identify optimal solutions, providing the decision maker with the optimal solution space. Also in the pharmaceutical unit, algorithms have played a role as researchers are using the knowledge of algorithm to optimize productivity as Dahiya et al., (2023) introduced the heuristic Dahiya-Garg Heuristic Algorithm (DG-Alg), its unique selection factor enhances its

effectiveness and efficiency. The algorithm was applied to a pharmaceutical manufacturing plant problem, ensuring redundancy allocation within cost constraints. The analysis shows that DG-Alg has highest system reliability, at 0.9690, and enhances stability in liquid industrial system by 25.29%. The paper suggests that integrating DG-Alg with metaheuristics can lead to faster convergence and better reliability under various constraints.

4.2.5. Parameters Configurations and Dataset used in RRAP

In reliability redundancy allocation problems, several parameters and datasets are crucial for effective modeling and solution generation. Key parameters typically include component reliability, redundancy levels, cost constraints, and system reliability targets. Component reliability refers to the likelihood of a component performing its intended function without failure nor breaking down over a specified number of times. Few of the papers reviewed according to (Shojaei & Mahani, 2019) showed some of the parameters used in series-parallel and bridge system.

Table 8. Parameters used in series & bridge systems

i	$10^5 \alpha_i$	β_i	v_i	w_i	C	V	W
1	2.330	1.5	1	7	175	110	200
2	1.450	1.5	2	8			
3	0.541	1.5	3	8			
4	8.050	1.5	4	6			
5	1.950	1.5	2	9			

Table 9: Parameters used in series-parallel system.

i	$10^5 \alpha_i$	β_i	v_i	w_i	C	V	W
1	2.500	1.5	2	3.5	175	180	100
2	1.450	1.5	4	4.0			
3	0.541	1.5	5	4.0			

4	0.541	1.5	8	3.5
5	2.100	1.5	4	4.5

Table 10: Parameters used in the overspeed system

i	$10^5 \alpha_i$	β_i	v_i	w_i	V	C	W	T
1	1	1.5	1	6	250	400	500	1000h
2	2.3	1.5	2	6				
3	0.3	1.5	3	8				
4	2.3	1.5	2	7				

Table 11: Parameters used in Large-scale System

i	$10^5 \alpha_i$	β_i	v_i	w_i	V	C	W
1	0.6	1.5	2	8	600	700	900
2	0.1	1.5	5	9	600	700	900
3	1.2	1.5	5	6	600	700	900
4	0.3	1.5	4	10	600	700	900
5	2.9	1.5	4	8	600	700	900
6	1.7	1.5	1	9	600	700	900
7	2.6	1.5	1	9	600	700	900
8	2.5	1.5	4	7	600	700	900
9	1.3	1.5	4	9	600	700	900
10	1.8	1.5	3	8	600	700	900
11	2.4	1.5	3	9	600	700	900
12	1.3	1.5	1	8	600	700	900
13	1.2	1.5	1	7	600	700	900
14	2.1	1.5	3	10	600	700	900
15	0.9	1.5	4	6	600	700	900
16	1.3	1.5	5	7	600	700	900
17	1.9	1.5	1	7	600	700	900
18	2.7	1.5	4	8	600	700	900
19	2.8	1.5	2	9	600	700	900

i	$10^5 \alpha_i$	β_i	v_i	w_i	V	C	W
20	1.5	1.5	1	9	600	700	900

The parameters used in several of the articles suggested different components are evaluated based on their reliability, cost, and importance within the overall system. This type of analysis with detailed figures showing the number of times for each turn helps in making informed decisions about where to invest resources for maintenance, replacement, or improvement of the system components.

4.2.6. Results outcome from literature and Evaluation metrics

In this sub-section we have covered and discussed few of the outcomes in RRAP with EAs and also provided some performance metrics which can be considered to evaluate the novel proposed approaches.

1. Results outcome

Redundancy levels indicate the number of redundant components included to enhance system reliability. Table 12 data used in the (Khalili-Damghani & Amiri, 2012) proposes a method using the data envelopment analysis (DEA) in solving the binary-state MORRAP. The method generates qualified non-dominated solutions on the Pareto front and improving the benchmark instance's upper bound.

Table 12: Pay-off of original MORRAP.

Empty Cell	Reliability	Cost	Weight
<i>Ideal calculations</i>			
Reliability	0.9999985	128	105
Cost	0	0	0
Weight	0	0	0
<i>Anti-ideal calculations</i>			
Reliability	0	0	0
Cost	1	284	160
Weight	0.9999998	144	192

The upper bound of reliability objective has been improved upon and the DEA model prunes the generated non-dominated solutions, simplifying decision-making. System reliability targets set the desired reliability level for the overall system, guiding the allocation process to meet these objectives. Tan et al., (2013) proposes new algorithm called CDEPSO, involving integration of PSO, DE and a chaotic local search. Table 13 below shows the performance of the algorithm which was investigated on four reliability redundancy allocation problems, revealing better convergence speed and robustness than PSO and CPSO. CDEPSO also exhibited more robust performance compared to other improved meta-heuristics.

Table 13: Experimental results obtained by PSO, CPSO and CDEPSO

Problem	Algorithm	Best	Worst	Mean	Standard deviation
Problem 1	PSO	0.871 096 69	0.310 915 54	0.649 605 06	0.126 2
	CPSO	0.931 666 92	0.741 470 18	0.909 483 91	0.034 4
	CDEPSO	0.931 682 39	0.931 527 11	0.931 655 73	3.072 4×10 ⁻⁵
Problem 2	PSO	0.999 929 60	0.995 004 82	0.998 532 06	0.001 9
	CPSO	0.999 976 64	0.999 870 22	0.999 961 19	2.321 9×10 ⁻⁵
	CDEPSO	0.999 976 65	0.999 967 71	0.999 975 99	1.802 0×10 ⁻⁶
Problem 3	PSO	0.999 775 25	0.991 407 80	0.997 820 20	0.002 1
	CPSO	0.999 889 57	0.999 669 95	0.999 855 98	4.514 5×10 ⁻⁵
	CDEPSO	0.999 889 64	0.999 889 34	0.999 889 46	1.605 0×10 ⁻⁷
Problem 4	PSO	0.999 927 31	0.969 319 83	0.997 745 23	0.004 7
	CPSO	0.999 954 67	0.999 881 79	0.999 931 09	1.975 1×10 ⁻⁵
	CDEPSO	0.999 954 67	0.999 946 15	0.999 954 63	1.204 9×10 ⁻⁶

CDEPSO also has the strongest robustness among all compared algorithms. To differentiate the algorithms, an index performance indicated shows CDEPSO as highly efficient than the rest. Advanced computational techniques, such as heuristic algorithms, may also incorporate simulated datasets to evaluate different redundancy configurations and optimize reliability under given constraints. Mellal et al., (2023) explores the reliability of manufacturing plants

using System Reliability optimization concept. It evaluates the PSO and GWO techniques, revealing that the PSO is more effective than GWO for a 15-subsystem system.

Table 14: Statistical Comparison.

τ	n	R_s	NFE	CPU	σ
PSO					
(0.8, 0.749998, 0.649996, 0.8, 0.849996, 0.829999, 0.779999, 0.659997, 0.779998, 0.899999, 0.789999, 0.769999, 0.699998, 0.789999, 0.669999)	(3, 4, 6, 3, 3, 3, 3, 5, 4, 3, 3, 4, 5, 4, 5)	0.93319	1.86x10 ⁴	1.235	6.1629x10 ⁴
GWO					
(0.797501, 0.748950, 0.65, 0.791910, 0.85, 0.83, 0.78, 0.658206, 0.78, 0.897605, 0.79, 0.77, 0.698285, 0.79, 0.668822)	(4, 4, 5, 4, 3, 3, 3, 5, 4, 3, 3, 4, 4, 3, 5)	0.92901	2x10 ⁴	0.592	5.9928x10 ³

In table 14 above compares the statistical analysis in the result, detailing significant impact in accessing and validating the performance of these evolutionary algorithms in solving RRAP. As shown above, it's concluded that the PSO is more active than GWO, in relation to the function evaluations, standard deviation, and CPU time consumption with future work focusing on hybrid approaches with GWO consuming less CPU time. Few of the datasets used comprise reliability data of components, including best, worst and mean. The dataset accurately models the reliability behavior of each component and the overall system, enabling more precise and effective redundancy allocation.

From the analysis of results computed highlights the uniqueness of techniques such as t-tests, ANOVA, and other statistical methods in arriving at the optimization solutions. In comparing the performance of few algorithms like GAs, PSO, and NSGA-III encountered in this work, the statistics of test results shown improvement in the cost optimization and reliability which depicts they are statistically significant in the optimization problem. These analyses provide yardstick for continued use and development of evolutionary algorithms in reliability engineering.

2. Evaluation metric

The maximum possible improvement (MPI) measures the improvement of newer method in comparison to best solutions found before. The MPI formula calculates and shows comparison to grasp how effective the new solution is compared to previous one.

$$\text{MPI (\%)} = \frac{R_s(\text{this study}) - R_s(\text{prior study})}{1 - R_s(\text{prior study})} * 100$$

The MPI calculation details how much better the new method performs when compared with the best-known results to obtain overall improvement.

5. Open Challenges, Discussion and Conclusion

In the quest to enhance system reliability, identifying challenges and exploring prospects for advancing evolutionary approaches in the Reliability Redundancy Allocation Problem remains a crucial endeavor. By addressing complexities, optimizing trade-offs, and embracing uncertainty, researchers pave the way for more robust and cost-effective solutions. The growing demand for dependable systems necessitates the exploration of reliability optimization. The open challenges related to RRAP has to do with

- The application of evolutionary programming to real-time adaptive systems, where programs continuously evolve in response to changing environmental conditions and objectives.
- Applications include scheduling, data categorization, transportation, and multi-objective optimization.
- Complexity of large-scale systems, which increases computational demands.

The research is anchored on the changes in technology as demand for new advancements aimed at addressing the real-world challenges encountered in the field. The research's contribution aims to provide in-depth insights into the challenges and future opportunities to the problem. The work provides a comprehensive report and recommendations to the researchers and industrial professionals on suitability of specific methods on addressing system reliability in engineering problems. The bibliometric analysis of RRAP research identified highly cited papers and influential authors, offering valuable insights into the foundational works, emerging trends within the field and information regarding recent research was provided.

The bibliometric analysis provides a comprehensive understanding of the landscape of Reliability Redundancy Allocation Problem (RRAP) research detailing how researchers use RRAP to advance their research network. As noted earlier, over 170 scholarly articles were examined especially from 1987 to current date. The analysis showed the collaboration networks and methodologies used highlighting several insights through citation counts, publication trends, and author contributions detailing the evolution and current state of RRAP research.

Understanding these essentially build upon existing knowledge and make important and useful contributions to reliability engineering. In accessing the co-authorship network shows the dynamics involved in collaboration which makes it important in solving RRAP issues.

Collaboration between authors and expertise in the field brings about quality, innovative features and techniques that improves optimization and reliability through diverse fields. For researchers and authors aiming to enhance and upgrade reliability in engineering, a good step is to study well-known papers and authors in the field and in extension, improve on existing knowledge and stay abreast with current or latest advancements.

It is also beneficial to explore how experts collaborate through co-authorship networks as this explain in details how different specialties combine knowledge to solve complex RRAP.

Collaborating brings individuals with disparate backgrounds and perspectives together to share a common goal leading to innovative solutions that improve reliability and effectiveness across a range of industries. By using bibliometric analysis, researchers can discover potentially novel methods such as constraint optimization and dynamic reliability allocation.

Research in RRAP has opened up collaboration in the field of reliability engineering and operation research which seems beneficial for everyone's safety towards the use of technology and in the advancement of infrastructure. These problems involve figuring out the right balance in having enough backup parts to keep the system running smoothly, making sure the system works reliably, and keeping costs under control. This essential role in RRAP has made systems operate seamlessly and reliable among daily users whether in using a smartphone, boarding a plane or using the essential services. By optimizing redundancy, systems become stronger, more cost-efficient, and better for the environment. It focuses on making infrastructures reliable in adding extra backups to equipment to prevent partial or complete breakdown. Understanding the evolving landscape of RRAP applications can inform targeted research efforts and industry partnerships. Beyond the reliability enhancement, the RRAP methodology has impacted the industries positively in terms of renewable energy, healthcare, transportation and also in advance manufacturing as it considers cost-effectiveness and sustainability in operation.

By testing out various solutions multiple times, evolutionary algorithms can find solutions for reliability redundancy allocation problems (RRAP) that are very close to the best possible ones in there. The reliability result has improved significantly in recent years as evolutionary algorithm turned out efficiently in tackling large-scale and uncertain problems using the natural processes. Therefore, advancing the algorithmic input to improve the learning of complex structural patterns with accuracy and efficiency is of necessity as authors and researchers are delving deeper into discovering more robust and efficient ways to tackle structure defects in datasets using advanced algorithm development. Researchers are focused on how EA can be combined with other AI techniques to explore innovative techniques that are capable of handling large-scale and multidimensional data while maintaining computational efficiency.

From the foregoing, it is an established fact that EA advances reliability engineering. It has evolved from the traditional approach to several evolutionary algorithm ranging from the mathematical algorithm to the computational intelligence. EA regarded as a powerful tool has solved the problem inherent in combinatorial and complex issues related to RRAP. The field of EA constantly evolving and researchers are keen in developing new algorithms and also improving on similar ones to be able to solves all kinds of issues bothering on small and large complex problem. Advances in this field aim to solve problems more efficiently, reducing computational time and resources while increasing the accuracy and dependability of solutions. The surge in data accessibility and the integration of machine learning aims to analyze innovative techniques that are capable of withstanding the computational efficiency while handling large-scale systems. As systems continues to grow in complexity and designs, the prospective studies would need to study and address the system compatibility, harnessing the constraint and altering the operational conditions.

In conclusion, this thesis has researched over 170+ articles in the area of reliability redundancy allocation problem. Basically, research work done from 1987 till date has been succinctly captured in this work ranging from reputed journals like the reliability engineering and safety systems, IEEE and quality and reliability engineering international etc. A number of well reputed authors has contributed significantly in the RRAP and how evolutionary algorithm has solved the problem related to reliability. The research here detailed the small and large scale systems

which is dominant in the scope of RRAP, and it is observed that various algorithms has been successfully applied in unraveling issues related to constraint and downtime involved. Authors have relied on using the effective algorithm such as the GA, PSO and the NSGA to tackle these challenges. According to the study, researchers worked on diverse pattern using the series, series-parallel, gas turbine method, k-out-n systems, and mixed redundancy etc. Different methodology concept like exact methods, fuzzy, heuristics and meta-heuristic, and complex systems has been applied for optimization purposes. Technology advancement permits for more improvement using advanced algorithms to tackle issue related to reliability and enable the engineer to design systems than enhance dependability and utilization. The research provides detailed work and gives future directions on RRAP for researchers and engineers to design more reliable system in this field.

A promising development in solving RRAP is using machine learning and data-driven methodology. These technologies when in use improves the RRAP's efficiency and precision at tackling challenges and hard constraint. The recent implementation of machine learning technique shows the relevance RRAP research has across many industries. The thesis detailed how EA and RRAP application has enhanced the system reliability in reviewing existing research to identify trends, insight and methods, providing a clear, detailed understanding of subject area. Evolutionary algorithms are favoured because they can adjust more easily to changes and can be applied to both large and small issues. From the thorough analysis, we have observed that research on RRAP has evolved from applying traditional optimization problem to several evolutionary algorithm because of the inherent problem due to complex and combinatorial problem in solving the RRAP.

The AI techniques which include the deep learning can be integrated into the EA in a means to enhance and deepen the performance related to examining the evolutionary process. The prospect centers on evaluating the constraint involve in the large-systems, solving increasingly complex systems and its positive effects towards system reliability. Exploring data patterns and visualization techniques will provide clear representations of detected structures, making interpretation and decision-making easier (Xu & Saleh, 2021). The RRAP adopts the data-driven

methods for developing reliability. To boost the structure of efficiency in different units needs the implementation of the existing algorithm and also exploring new approach using the deep-learning based model. Utilizing the deep-learning methods can be very useful as it creates more efficient and advanced algorithm relevant in tackling challenges in RRAP. Future research could explore more advanced ways to blend different parts of programs or improve how algorithms are designed. Another promising area for investigation is combining reinforcement learning with evolutionary programming. This approach could allow systems to learn and adjust in real-time situations, which could enhance their performance dynamically (Nazim Chebouba et al., 2020).

The future methodology will incorporate machine learning such as deep learning procedure in the analysis involving large datasets as it's observed from the article review that researchers are seemingly interested in exploring hybrid algorithm to integrate the combination of evolutionary algorithm with other advanced techniques to upscale the real-time performance and handle more complex model. Matlab's application in machine learning can develop models that not only predict the reliability outcomes but identify complex patterns that traditional methods might seem improbable leading to more advanced solutions that generally improve reliability and system's performance. Collaboration with the industry to test novel algorithms in a real-world setting would be crucial as these practical applications can help provide important insight and data sufficient to refine algorithms based on their performances and system's constraint. When implemented, this method will significantly improve the decision-making process in reliability engineering.

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