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TMM-based study on Sound Insulation Characteristics of Laminated Cylindrical Shell Lined with Porous Materials

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Abstract

This study utilises the Transfer Matrix Method (TMM) to address the acoustic characteristics of multi-layered cylindrical shells lined with porous materials. The TMM theoretical model for the sound transmission loss of composite cylindrical shells with internal porous materials is derived by establishing transfer matrices for the air/composite material interface, composite material/foam interface, foam/air interface and boundary interfaces. The accuracy of the TMM model is validated through a comparison and analysis with experimental results. Building upon this, the impact of porous foam material parameters and types on the structural sound transmission loss is discussed. The results indicate that the use of TMM accurately reflects the acoustic performance of composite structures. Additionally, this model allows for the determination of the influence patterns of porous foam material parameters and types on the acoustic performance of composite structures. In the frequency range of 100-10000 Hz, the sound transmission loss of the melamine foam lined composite structure increases with the increase of flow resistance, porosity and the decrease of the tortuosity factor. The use of the porous lining material significantly enhances the structural sound insulation performance.

Keywords Composite cylindrical shell · Porous materials · Transfer matrix method · Sound insulation characteristics

1 Introduction

Currently, in order to meet the launch needs of large communication satellites and near-Earth orbit spacecraft [1-2], carrier rockets are evolving towards economic, high reliability, fewer stages, greater payload capacity, stronger environmental adaptability and rapid launch capabilities [1]. Consequently, the noise environment inside the rocket fairing cavity, designed to protect the payload, is becoming increasingly harsh. The urgent need to control the noise inside the cavity has led to growing attention and significant research significance in addressing the noise control issues of composite layered cylindrical shell structures.

A great deal of research has been carried out at home and abroad on the sound vibration characteristics of composite structures. Narayanan and Shanbhag et al. [3] analysed the impact of boundary conditions on the noise attenuation of damping composite plates. They found that variations in boundary conditions mainly affect the low-frequency range. The authors derived mathematical expressions based on an infinite damping composite plate model to reflect the interrelationships between sound transmission loss, coincidence frequency, core layer material parameters and the incident angle of the sound field. However, these expressions are only applicable for calculating the sound transmission loss of symmetrically constrained damping plates, lacking generality. Guyader and Leuseur et al. [4] proposed a hybrid model for calculating the sound transmission loss of composite laminates using the regular modal method and statistical energy method. However, obtaining the regular modal numbers and other relevant parameters in the statistical energy method for orthotropic composite laminates can be challenging. Based on the transfer matrix method, Zheng Hui et al. [5] established a transfer matrix model for the sound transmission loss of viscoelastic damped composite laminates. Using this method, the influence of different structural types on the sound transmission loss of composite laminates was discussed and the influence of the parameters of each layer of composite laminates on the overall sound insulation characteristics of the structure was analysed more comprehensively through the calculation results. Wang Qing et al. [6] developed finite element dynamic unit models for two types of composite laminated plates. Based on the displacement field assumptions of the Reissner and Zig-Zag models, they used the subspace iteration method to calculate the natural frequencies and modes of the laminated plates. The sound transmission loss was calculated using the theory of sound transmission in a reverberant field and numerical simulation studies were conducted. The effects of the thickness ratio and elastic modulus ratio between the laminated structural panel and the core panel were considered to explore the applicability of different models. Han Baokun et al. [7] initially conducted numerical calculations on the sound transmission loss of foam aluminum composite structures and validated the results through experiments. They found that the thickness of the foam aluminum layer significantly affects the sound transmission loss of the composite structure. Sheng Li et al. [8] developed a model for calculating sound transmission loss in composite laminates embedded in infinite baffles with low-frequency simple harmonic plane acoustic waves obliquely incident. The model uses a plate cell based on Mindlin theory and a boundary element based on Rayleigh's surface integral equation, taking into account fluid-structure coupling. Narasimman et al. [9] experimentally compared the sound absorption coefficients of linen fibre and glass fibre composite laminates at low frequencies, and the results showed that linen fibre has better sound absorption properties compared to glass fibre. Campouna and Atalla et al. [10] investigated the effect of different sound absorbing materials between thin double layers on the overall sound insulation characteristics when plane waves are incident vertically. Sgard and Castel et al. [11] used finite element simulation to investigate the sound insulation characteristics of a composite sandwich structure with a porous middle sandwich material under a diffuse sound field.

Most of the previous studies have focused on multilayer and sandwich panel structures, while for composite laminated plate structures, only the sound insulation characteristics in the low-frequency range have been considered. Furthermore, the computational methods proposed for sound transmission loss in composite material structures in the aforementioned studies are relatively complex. Despite its limitations, such as potential deviations in simulating wave propagation along curved surfaces, the applicability of the Transfer Matrix Method (TMM) in studying the acoustical performance of composite structures has been validated [12-13]. Its widespread use, simplicity and high computational efficiency enable us to effectively simulate sound transmission loss in multi-layer structures within the frequency range of 100-10000 Hz using this method. Therefore, this paper employs the TMM to derive the sound transmission loss calculation formula for a composite laminated cylindrical shell lined with porous materials. Based on this, the sound insulation characteristics of the composite laminated cylindrical shell are studied in the frequency range of 100-10000 Hz. Building upon the transfer matrices of sound waves on the composite material layer, foam layer and boundary interfaces, a sound transmission loss model for composite structures based on TMM is established. Utilizing this model, the study investigates the impact of porous foam material properties (flow resistance, tortuosity factor and porosity) and the type of porous material on the sound insulation performance of composite structures.

2 Sound transmission loss transfer matrix model

TMM is an effective method for calculating sound transmission loss in multilayered structures. Fig. 1 depicts the three-dimensional model of a composite cylindrical shell structure with a layered porous foam and to illustrate the sound wave propagation within this structure, the configuration shown in Fig. 2 is employed in this paper. Fig. 2 is a schematic diagram illustrating the sound transmission loss in the composite cylindrical shell structure with a layered porous foam. In Fig. 1 and Fig. 2, the solid layer consists of a composite material laminated cylindrical shell with a thickness of *h*, and the foam layer is applied to the inner wall of the cylindrical shell with a thickness of *d* using porous foam material.

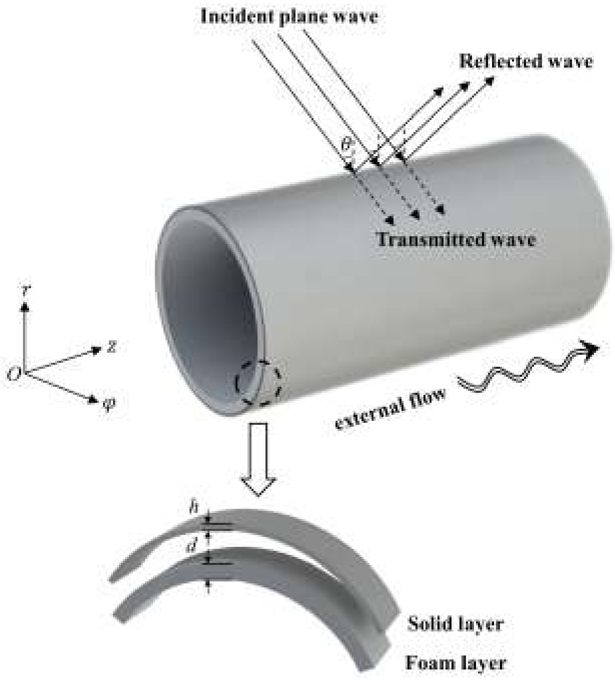


Fig. 1 Three-dimensional model diagram of a cylindrical shell structure with a layered porous foam

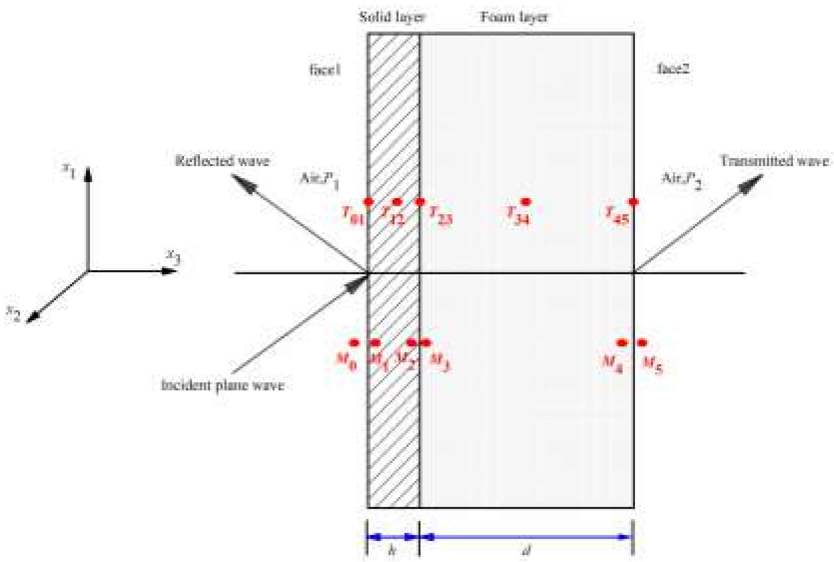


Fig. 2 Schematic diagram illustrating the sound transmission loss in a composite cylindrical shell structure with a layered porous foam

In Fig. 1, z、 φ and r represent the axial, circumferential and radial directions respectively. The cylindrical shell structure is assumed to be subjected to an incident plane wave at an angle of incidence θ from the outside air. The sound wave undergoes reflection by the cylindrical shell and returns to the outside air. Additionally, part of the sound wave is absorbed by the cylindrical shell and the porous foam material. The remaining portion of the sound wave penetrates the cylindrical shell and the porous foam, entering the inner cavity and forming a transmitted wave. In Fig. 2, the propagation of sound through the structure can be expressed by the following transfer matrix:

$$V\left(M_0\right)=\left[T\right]V\left(M_5\right) \tag{1}$$

where M_0 and M_5 are the points on the outer and inner foam of the cylindrical shell respectively, V is the quantity describing the characteristics of the point sound field and $[T]$ is the transfer matrix of the whole system.

The general idea of the transfer matrix method is to use the transfer matrices on the single layer and boundary surfaces to form the transfer matrix of the whole system, and then use the overall transfer matrix of the system to calculate the sound pressure P_1 and P_2 at both ends. As illustrated in Fig. 2, for composite panels laid with porous foam, the acoustic wave transmission matrices in each single layer are: solid layer T_{12} and foam layer T_{34}, respectively. The transfer matrix of the boundary layer contains: the fluid-solid boundary transfer matrix T_{01} for air and shell, the solid-foam boundary transfer matrix T_{23} for shell and foam and the foam-fluid transfer matrix T_{45} for foam and air. In order to calculate the overall transfer matrix of the structure, the transfer matrix of each layer and the transfer matrix of each boundary will be solved for one by one.

2.1 Transfer matrix of the solid layer

Assuming that the composite cylindrical shell has a certain degree of elasticity, the incident and reflected waves in an elastic solid contain mainly two forms of longitudinal and transverse waves, so this paper uses longitudinal and transverse waves to describe the sound field in an elastic solid. In the coordinate system of Fig. 2, the displacement potential functions for the longitudinal and transverse waves incident into the cylindrical shell structure are

$$\varphi = e^{\left[j\omega t - jk_1 x_1\right]}\left(A_{1s} e^{\left[-jk_{13}^s x_3\right]} + A_{2s} e^{\left[jk_{13}^s x_3\right]} \right)$$
$$\psi = e^{\left[j\omega t - jk_1 x_1\right]}\left(A_{3s} e^{\left[-jk_{33}^s x_3\right]} + A_{4s} e^{\left[jk_{33}^s x_3\right]} \right) \tag{2}$$

where ω is the angular frequency, φ is the displacement potential function of the longitudinal wave, A_{1s} and A_{2s} are the longitudinal wave amplitudes in the incident and reflected waves respectively. ψ is the displacement potential function of the transverse wave, A_{3s} and A_{4s} are the transverse wave amplitudes in the incident and reflected waves respectively. k_{13}^s and k_{33}^s are the number of waves in the x_3 direction for the longitudinal and transverse waves in the sound wave, respectively.

From equation (2), the potential function of the sound field at any point in the cylindrical shell structure can be described by the four unknown amplitudes $\left[A_{1s}, A_{2s}, A_{3s}, A_{4s}\right]$ of the incident and reflected waves. Based on the work of Folds and Loggins [14], the sound field at any point in a cylindrical shell structure is described using four mechanical variables $\left[v_1, v_3, \sigma_{33}, \sigma_{13}\right]$. Where v_1, v_3, σ_{33} and σ_{13} are the velocities at any point in the cylindrical shell structure in the x_1 and x_3 directions and the positive and tangential stresses at that point respectively.

As shown in Fig. 2, assuming the point M_2 on the cylindrical shell as the origin, the state of the sound field at the point M_1 and the point M_2 on the cylindrical shell can be expressed in terms of four mechanical variables $\left[v_1, v_3, \sigma_{33}, \sigma_{13}\right]$, which can be written in the following form

$$V^s\left(M_1\right) = \Gamma\left(-h\right) A, V^s\left(M_2\right) = \Gamma\left(0\right) A \tag{3}$$

As a result, combining the definition of the transmission matrix in equation (1) for a composite plate structure with laid porous foam, the transmission matrix of acoustic waves in the solid layer of the composite structure transmission matrix T_s is given by

$$T_s = [\Gamma(-h)][\Gamma\left(0\right)]^{-1} \tag{4}$$

Where h is the thickness of the solid layer.

2.2 Transfer matrix of the foam layer

The structure of the porous material is made up of an interlocking foam skeleton, forming a complex mesh structure. The structural form of its internal pores contains mainly: closed holes, semi-through holes and through holes. When sound waves propagate between the framework and voids of porous materials, numerous reflected waves are generated. Due to the simultaneous presence of incident and reflected waves, describing the acoustic field inside porous materials directly becomes challenging. Therefore, the TMM is widely used to calculate the sound transmission loss (absorption coefficients) of porous materials [15]. According to Biot theory, the acoustic waves propagating in porous materials consist of two longitudinal waves (incident and reflected waves) and one transverse wave (incident wave) [16-17]. The displacement potential function of the porous material framework caused by the two longitudinal waves inside porous materials are

$$\varphi_1^s = A_{11} e^{\left[j\left(\omega t - k_{13}^p x_3 - k_t x_1\right)\right]} + A_{11}' e^{\left[j\left(\omega t + k_{13}^p x_3 - k_t x_1\right)\right]}$$
$$\varphi_2^s = A_{12} e^{\left[j\left(\omega t - k_{23}^p x_3 - k_t x_1\right)\right]} + A_{12}' e^{\left[j\left(\omega t + k_{23}^p x_3 - k_t x_1\right)\right]} \tag{5}$$

The displacement potential function for the porous material skeleton under the action of transverse acoustic waves inside the porous material is

$$\psi_2^s = A_{13}\,e^{\left[j\left(\omega t - k_{33}^p x_3 - k_t x_1\right)\right]} + A_{13}^{'}\,e^{\left[j\left(\omega t + k_{33}^p x_3 - k_t x_1\right)\right]} \tag{6}$$

Where φ_1^s and φ_2^s are the displacement potential functions of the porous material skeleton under the action of the incident and reflected longitudinal waves respectively. ψ_2^s is the displacement potential function of the porous material skeleton under the action of the transverse wave. A_{11}, $A_{11}^{'}$, A_{12}, $A_{12}^{'}$, A_{13} and $A_{13}^{'}$ are the amplitudes of the acoustic waves acting inside the porous material. k_t is the number of waves in the x_1 direction, k_{13}^p, k_{23}^p and k_{33}^p are the number of waves in the x_3 direction for the three types of sound waves inside the porous foam respectively.

For porous materials, the displacement potential function of the air inside them is linked to the displacement potential function of the skeleton of the porous material as follows

$$\begin{gathered} \varphi_i^f = \mu_i \varphi_i^s, i = 1,2 \\ \psi_2^f = \mu_3 \varphi_2^s \end{gathered} \tag{7}$$

Where μ_1, μ_2 and μ_3 represent the ratio of the fluid potential to the displacement potential function of the rigid skeleton of the porous material.

At this point, the sound field at any point in the foam layer can be described by the six unknown amplitudes $A_{11}, A_{11}^{'}, A_{12}, A_{12}^{'}, A_{13}$ and $A_{13}^{'}$ of the incident and reflected waves. Allard [18] used six independent variables: the velocities v_1^s and v_3^s of the foam skeleton structure in the x_1 and x_3 directions, the velocity v_3^f of the air inside the porous material in the x_3 direction, the stresses σ_{33}^s and σ_{13}^s of the porous material structure and the stress σ_{33}^f of the fluid inside the foam to describe the vibration of the porous material structure and the propagation of the internal sound field.

As a result, based on Allard's study, using six mechanical variables $\left[v_1^s, v_3^s, v_3^f \sigma_{33}^s, \sigma_{13}^s, \sigma_{33}^f\right]$ to express the sound field inside the porous foam, it is first necessary to use six unknown amplitudes $A_{11}, A_{11}^{'}, A_{12}, A_{12}^{'}, A_{13}$ and $A_{13}^{'}$ for the six mechanical variables $\left[v_1^s, v_3^s, v_3^f \sigma_{33}^s, \sigma_{13}^s, \sigma_{33}^f\right]$, so that from equations (5), (6) and (7) it follows that

$$\begin{bmatrix} v_1^{ps} \\ v_3^{ps} \\ v_3^{pf} \\ \sigma_{33}^{ps} \\ \sigma_{13}^{ps} \\ \sigma_{33}^{pf} \end{bmatrix} = \left[\Gamma'\left(x_3\right)\right] \begin{bmatrix} A_{11} + A_{11}^{'} \\ A_{11} - A_{11}^{'} \\ A_{12} + A_{12}^{'} \\ A_{12} - A_{12}^{'} \\ A_{13} + A_{13}^{'} \\ A_{13} - A_{13}^{'} \end{bmatrix} \tag{8}$$

According to equation (8), the state of the sound field at points M_3 and M_4 on the porous foam can be expressed by the following equation

$$V^s\left(M_4\right) = \Gamma'\left(0\right) A, V^s\left(M_3\right) = \Gamma'\left(-d\right) \tag{9}$$

From this, the transfer matrix T_P of acoustic waves in the foam layer of the composite structure, in combination with the definition of the transfer matrix in equation (1) for a composite panel structure with laid porous foam, is

$$T_P = \left[\Gamma'(\text{-}d)\right]\left[\Gamma'(0)\right]^{-1} \tag{10}$$

Where d is the thickness of the foam layer.

2.3 Transfer matrix on the boundary surface

For the transfer of acoustic waves between two different media, this section details the specific forms of the transfer matrix at the fluid-solid interface, the solid-foam interface and the foam-fluid interface, based on the boundary conditions satisfied by the acoustic waves at the different boundaries.

(1) Transfer matrix at the fluid-solid interface

As shown in Fig. 2, the sound waves satisfy equal velocities in the x_3 direction as they propagate over the boundary between the air and the cylindrical shell. In addition, the positive and tangential stresses at any point on the boundary also satisfy the continuity condition. From this, the continuity condition on the boundary from the air to the cylindrical shell, written in matrix form, gives

$$\left[I_{sf}\right]V^{s}\left(M_{1}\right)+\left[J_{sf}\right]V^{f}\left(M_{0}\right)=0 \tag{11}$$

Where

$$\left[I_{sf}\right]=\begin{bmatrix}0&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix},\ \left[J_{sf}\right]=\begin{bmatrix}0&-1\\1&0\\0&0\end{bmatrix}$$

(2) Transfer matrix for solid and foam layers

When the sound waves propagate on the boundary between the solid layer of the cylindrical shell and the foam layer, the sound waves only propagate in the air inside the foam in the x_3 direction because the foam layer contains both the foam skeleton and the air. Thus, in the x_1 and x_3 directions, the velocity of the mass at the side adjacent to the solid layer is equal to the velocity at the boundary layer near the foam layer. In the x_3 direction, the velocity of the mass at the side of the solid layer is equal to the velocity of the boundary layer close to the air cavity of the foam layer. Furthermore, the positive stress in the boundary layer near the foam layer is the sum of the positive stress in the skeleton and the positive stress in the foam air, and its shear stress is equal to the shear stress in the foam skeleton. This leads to the boundary conditions for the solid and foam layers, which are written in matrix form as

$$\left[I_{sp}\right]V^{s}\left(M_{2}\right)+\left[J_{sp}\right]V^{p}\left(M_{3}\right)=0 \tag{12}$$

Where

$$\left[I_{sp}\right]=\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix},\left[J_{sp}\right]=-\begin{bmatrix}1&0&0&0&0&0\\0&1&0&0&0&0\\0&0&1&0&0&0\\0&0&0&1&0&1\\0&0&0&0&1&0\end{bmatrix}$$

(3) Foam-air boundary conditions

The velocity, positive and tangential stresses on the left and right side of the boundary layer should satisfy the continuity condition when the sound waves are transmitted from the foam layer to the air layer. It follows that in the structure shown in Fig. 2, the velocity of point M_4 in the x_3 direction should be equal to the velocity of point M_5 in the x_3 direction. Additionally, the positive stress on the skeleton of point M_4 should be equal to the positive stress on the skeleton of point M_5, and the positive stress in the air cavity at point M_4 should be equal to the positive stress in the air cavity at point M_5. Furthermore, since no tangential stress is considered in the air layer, the tangential stress at point M_4 should be equal to 0. This leads to the foam-air boundary condition, which is written in matrix form as

$$\left[I_{pf}\right]V^{p}\left(M_{4}\right)+\left[J_{pf}\right]V^{f}\left(M_{5}\right)=0 \tag{13}$$

Where

$$\left[I_{pf}\right]=\begin{bmatrix}0 & (1-\phi) & \phi & 0 & 0 & 0\\ 0 & 0 & 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 0 & 0 & 1\end{bmatrix},\left[J_{pf}\right]=-\begin{bmatrix}0 & -1\\ (1-\phi) & 0\\ 0 & 0\\ \phi & 0\end{bmatrix}$$

ϕ is the porosity of the porous material.

2.4 Overall transfer matrix

As shown in Fig. 2, the sound waves are first transmitted through the air-solid boundary at M_0 to M_1, then inside the solid layer from M_1 to M_2, then through the solid-foam boundary to M_3, then inside the foam layer from M_3 to M_4 and finally, through the foam-air boundary to M_5. From this, coupling the transfer matrices of the layers of material as well as the inter-layer transfer matrices, in parallel with vertical (4), (10), (11), (12) and equation (13) yields the overall transfer matrix of the system as follows:

$$\begin{aligned}&\left[J_{sf}\right]V^f(P_1)+\left[I_{sf}\right]\left[T_s\right]V^s(M_2)=0\\ &\left[I_{sp}\right]V^s(M_2)+\left[J_{pf}\right]\left[T_p\right]V^p(M_4)=0\\ &\left[I_{pf}\right]V^p(M_4)+\left[J_{pf}\right]V^f(P_2)=0\end{aligned} \tag{14}$$

Writing equation (14) in matrix form, the matrix form of the overall system transfer matrix is

$$D_0V_0=0 \tag{15}$$

Where

$$D_0=\begin{bmatrix}\left[J_{sf}\right] & \left[I_{sf}\right]\left[T_s\right] & 0 & 0\\ 0 & \left[I_{sp}\right] & \left[J_{pf}\right]\left[T_p\right] & 0\\ 0 & 0 & \left[I_{pf}\right] & \left[J_{pf}\right]\end{bmatrix},V_0=\begin{bmatrix}V^f(P_1)\\ V^s(M_2)\\ V^p(M_4)\\ V^f(P_2)\end{bmatrix}$$

2.5 Sound transmission loss

From section 2.4 it can be seen that the overall transfer matrix of the system is a matrix of order 12 × 14. In order to calculate the sound transmission loss of the structure, the surface acoustic impedance of the composite plate is introduced as a parameter. When a plane wave is incident on the surface of the composite plate at an angle θ, the acoustic impedance of the composite plate surface is $Z_s=P(M_0)/v_3^f(M_0)$. By writing the acoustic impedance of the composite plate surface in matrix form and associating it with equation (15), a new 13 × 14 matrix equation can be obtained as follows:

$$\begin{bmatrix}-1 & Z_s & 0 & \cdots & 0\\ & & [D_0] & & \end{bmatrix}V_0=0 \tag{16}$$

Let the determinant value of the coefficient matrix in equation (16) be zero, then Z_s can be calculated by the following equation:

$$Z_s=-\frac{\det[D_1]}{\det[D_2]} \tag{17}$$

where $\det[D_1]$ is the value of the new matrix determinant after removing column 1 of $[D_0]$ and $\det[D_2]$ is the value of the new matrix determinant after removing column 2 of $[D_0]$.

The transmission coefficient T and the reflection coefficient R of an acoustic wave are related as follows:

$$P_1 T = P_2 \left(1 + R\right) \tag{18}$$

Where P_1 and P_2 are the sound pressure on the outside and inside of the cylindrical shell structure respectively, the reflection coefficient $R = \left(Z_s \cos\theta - Z_0\right) / \left(Z_s \cos\theta + Z_0\right)$ and the air characteristic impedance $Z_0 = 415N \cdot \text{s/m}^3$.
Combining equation (14) with equation (15) yields a matrix equation of order 13 × 14 which contains the transfer coefficient T of the structure and for which the transfer coefficient T can be solved, in the following form:

$$\begin{bmatrix} [T \quad 0] \quad 0 \quad 0 \quad [-(1+R) \quad 0] \\ [D_0] \end{bmatrix} V = 0 \tag{19}$$

where the new coefficient matrix is a matrix of order 13 × 14. Let the determinant of the coefficient matrix be equal to 0. The transmission coefficients can be calculated from the following equation:

$$T = -\left(1 + R\right) \frac{\det\left[D_{14}\right]}{\det\left[D_1\right]} \tag{20}$$

Where $\det[D_{14}]$ is the value of the new matrix determinant after removing the 14th column from $[D_0]$ and $\det[D_1]$ is the value of the new matrix determinant after removing the 1st column from $[D_0]$.
Thus, the sound transmission loss of a cylindrical shell structure under reverberant sound field excitation can be calculated by the following equation:

$$TL = -10log\left[\int_0^{\pi/2} \left|\tau\left(\theta\right)\right|^2 \sin\left(2\theta\right) d\theta\right] \tag{21}$$

Where $\tau\left(\theta\right) = \left|T^2\left(\theta\right)\right|$ is the transmission coefficient when incident at angle θ.

3 Calculation of sound transmission loss

According to the equivalent fluid model (Johnson-Champoux-Allard, JCA), it is known that the sound absorption characteristics of porous materials is mainly influenced by their own parameters (flow resistance, tortuosity factor, porosity, viscous characteristic length and thermal characteristic length) [19-23]. This section first validates the above TMM theoretical model for calculating sound transmission loss from composite laminated cylindrical shells, and then uses this theoretical model to discuss the effect of porous material parameters and type on structural sound transmission loss.

3.1 Model validation

In order to verify the correctness of the TMM-based calculation of the sound transmission loss of laminated cylindrical shell lined with porous materials, a noise test rig was built in-house to obtain accurate sound transmission loss values for the cylindrical shells and the experimental results were compared with the TMM calculation results. The composite cylindrical shell is placed in the centre of a 5m × 5m × 3.5m empty room and its bottom is supported by four rubber mats to simulate a free state with a height of 50 mm. Two speakers are placed diagonally in the laboratory. Firstly, a section of Gaussian white noise is generated by the computer and passed into the equaliser, which is filtered and then passed into the speakers by the power amplifier, and finally the sound waves are emitted by the speakers as the sound source. In the experimental tests, the room temperature was kept at 20°C, the noise uniformity in the laboratory was controlled within ±3 dB, the deviation of the noise control spectrum was controlled within ±5 dB, the signal sampling rate was 32000 Hz and the sampling time was 8s, after which the internal and external sound pressure response values of the cylindrical shell were obtained via microphone.

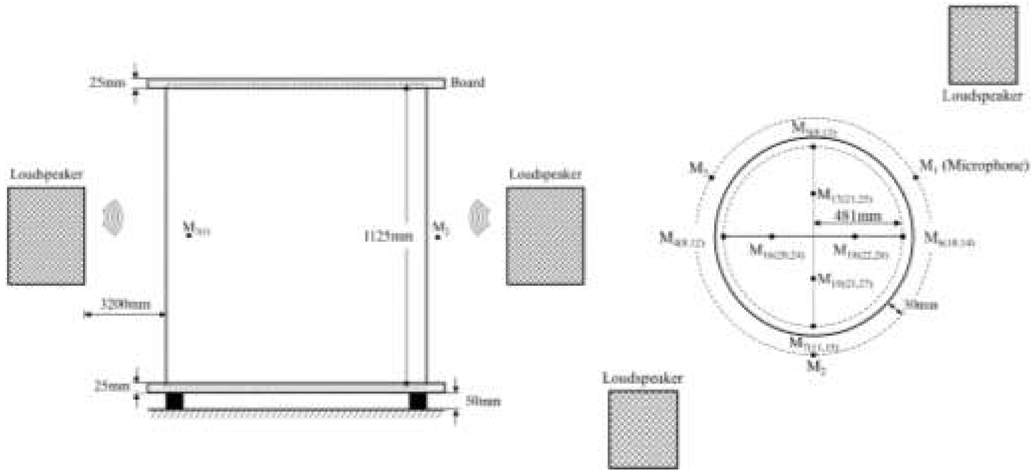
The structure studied in the experiments was a composite laminated cylindrical shell lined with melamine foam, as shown in Fig. 3. The shell is made of glass fibre coarse sand (PPG2400tex) immersed in epoxy resin filled with hardener, where the fibre lay-up is $[0/90/0/90/0]_s$ and the total thickness of the glass fibre epoxy resin cylindrical shell is 4 mm. The outer wall of the cylindrical shell has a diameter of 1050 mm, a length of 1125 mm and a total mass of 14 kg. The upper and lower ends of the cylindrical shell are sealed with MDF boards with a diameter of 1180 mm and a thickness of 25 mm, with a columnar depression of 1050 mm in the middle and a depth of 12.5 mm. The total mass of the upper and lower boards is 15.19 kg. Melamine foam with a thickness of 40 mm was laid on the inner wall of the cylindrical shell. The properties of the glass/epoxy composite cylindrical shell, the wood panels and the melamine foam material properties are shown in Table 1.

Fig. 3 Experimental model of composite laminated cylindrical shell

Table 1 Glass/epoxy composite cylindrical shell and foam properties

	Composite cylindrical shells	Plank	Foam
Thickness (m)	0.004	0.025	0.04
Density (kg/m^3)	2700	800	30
Young's modulus (Pa)	7.1×10^{10}	2×10^{9}	800800
Shear modulus (Pa)	2.67×10^{10}	/	286000
Poisson's ratio	0.3	0.4	0.4
Structural damping	0.007	/	0.265
Porosity	/	/	0.9
Curvature factor	/	/	25000
Sticky feature length	/	/	7.8
Flow resistance	/	/	2.26×10^{8}

The layout of the test equipment and measurement points is shown in Fig. 4. The three extra-cavity sound pressure measurement points of the composite cylindrical shell are located at 30 mm from the outer axial middle of the cylindrical shell at 120° intervals. The location of the sound pressure measurement points in the 24 cavities of the composite cylindrical shell is divided into three layers along the axis, upper, middle and lower, each layer being 1050 mm, 700 mm and 45 mm away from the bottom surface of the cylindrical shell respectively. There are 8 sound pressure measurement points on each level, each at 90° intervals and evenly distributed along the ring, with 4 points at the radial midpoint and 4 points at the radial endpoints.



(a) **(b)**

Fig. 4 Schematic diagram of cylindrical shell sound pressure measurement points. (a) Side view, (b) top view

The TMM calculations for the sound transmission loss of the cylindrical shell structure when lined with 40 mm melamine foam are compared with the experimental results, as shown in Fig. 5. As can be seen from the graph, the results of the TMM used to calculate the sound transmission loss of this structure in the frequency bands 125-400 Hz and 500-1250 Hz are slightly greater than the experimental results, with a maximum difference of 3.76 dB between the two, probably because in reality sound propagates along the sides of the cylindrical shell, but this is not taken into account in the theoretical model. At frequencies above 1250 Hz, the sound transmission loss curve calculated using TMM has the same trend as the curve obtained from experimental tests, and the error is small, with a maximum difference of 1.35 dB between the two, indicating that TMM still has high accuracy and reliability in the high frequency band. Therefore, the results of the TMM for calculating the sound transmission loss of melamine foam lined cylindrical shell structures are more accurate and the method can be used for the calculation of sound transmission loss of similar structures.

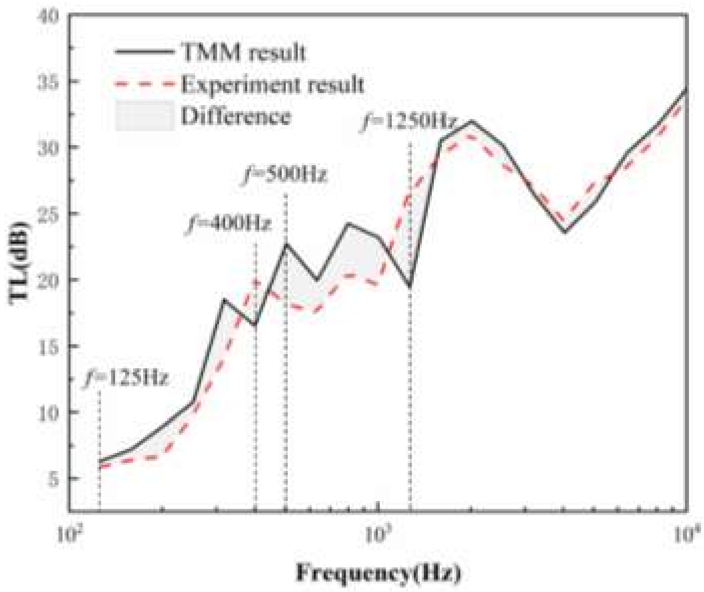


Fig. 5 Comparison of experimental results and TMM calculations

3.2 Effect of foam properties on structural sound transmission loss

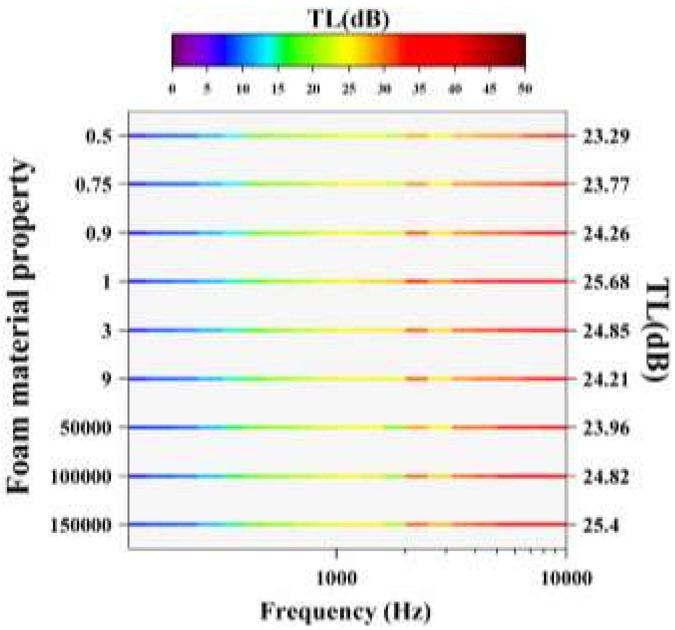


Fig. 6 Influence of porous foam material parameters on structural sound transmission loss

Building upon the verification of the correctness of the TMM theoretical model, this method is used to discuss the impact of porous foam material parameters on the sound transmission loss of composite layered cylindrical shells with porous material linings. The parameters of the porous material flow resistance, tortuosity factor and porosity in the TMM theoretical model are successively changed, and the results are shown in Fig. 6. As observed in the figure, within the frequency range of 100-10000 Hz, as the flow resistance of the foam material increases, the sound transmission loss of the composite structure with melamine foam lining also increases. This is due to the higher impedance and increased friction encountered by sound waves during propagation inside the porous material, resulting in greater energy dissipation. Conversely, as the tortuosity factor of the foam material increases, the sound transmission loss of the structure decreases. When Knapen et al. [24] conducted experiments to investigate the sound absorption properties of cement mortars. The results indicate that the sound absorption coefficient decreases as the tortuosity factor of the material increases, which is consistent with the findings presented in this paper. Additionally, the sound transmission loss of the lined melamine foam composite structure gradually increases as the porosity of the melamine foam material increases from 0.5 to 0.9. As the porosity of the melamine foam increases, the contact area of the sound waves with the porous material skeleton also increases. This results in more energy being converted into heat and dissipated as the sound waves propagate inside the porous material.

3.3 Influence of different types of porous materials on the sound transmission loss of structures

For a glass fiber-reinforced epoxy laminated cylindrical shell with a radius of 0.5 m, length of 1 m and thickness of 4 mm, three different types of porous materials are selected as lining materials in this study to investigate the impact of different lining materials on the sound transmission loss of composite structures. The parameters of different types of porous materials are shown in Table 2. For different types of porous materials, we analyze the effect of different lining materials on the sound transmission loss of composite structures by changing the parameters related to the properties of porous materials and the structure in the TMM theoretical model, considering the same lining thickness and the same lining mass.

Table 2 Properties of different types of porous foams

	MF	EPS	PUF
Porosity / ϕ	0.99	0.97	0.96
Flow resistance / $\sigma(\mathrm{N}^{-4}\mathrm{s})$	10900	88000	5000
Curvature factor /a	1.02	2.52	1.24
Skeleton density / $\rho(kg/m^3)$	8.8	31	22
Young's modulus /E	8×10^4	1.4×10^5	4.65×10^4
Poisson's ratio / ν	0.4	0.3	0.4
Structural damping / η	0.17	0.055	0.14

3.3.1 Same lining thickness

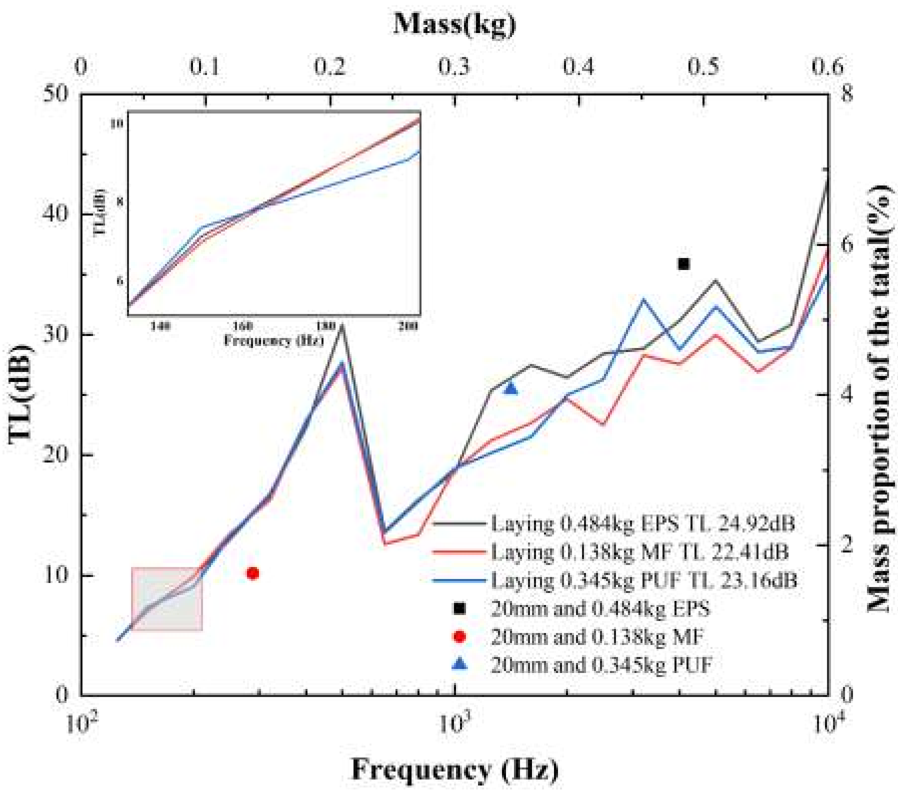


Fig. 7 Sound insulation characteristics of different foam structures for the same thickness of the lining

As can be seen from Fig. 7, the total sound transmission loss values of the structure when laying expanded polystyrene (EPS), melamine foam (MF) and polyurethane foam (PUF), all with a thickness of 20mm, in the frequency band of 100-10000 Hz, are 24.92 dB, 22.41 dB and 23.16 dB respectively, the sound transmission loss of the EPS is greater than the other two materials. This is due to the fact that EPS has the greatest flow resistance and tortuosity factor as well as a large porosity, while EPS is the densest of the three materials, a feature that helps EPS to isolate noise. In addition, the MF lining results in the least increase in structural mass, which is only 1.63% of the weight of the cylindrical shell, and the EPS lining results in a proportional increase in structural mass of 5.74%. Therefore, the effect of mass still needs to be taken into account when considering structural sound transmission loss.

It is worth noting that at frequencies above 500 Hz, the foam lining material significantly increases the sound transmission loss of the structure. This is due to the longer wavelengths and lower energy of the sound waves at lower frequencies, making them lose less energy after reflection and refraction. In contrast, at high frequencies, the shorter wavelength and higher acoustic energy of the sound waves lead to multiple reflections and refractions within the pores of the foam material. This results in an increased non-elastic contact between the sound waves and the skeleton of the porous material, significantly increasing the energy dissipation of the sound waves.

3.3.2 Same lining quality

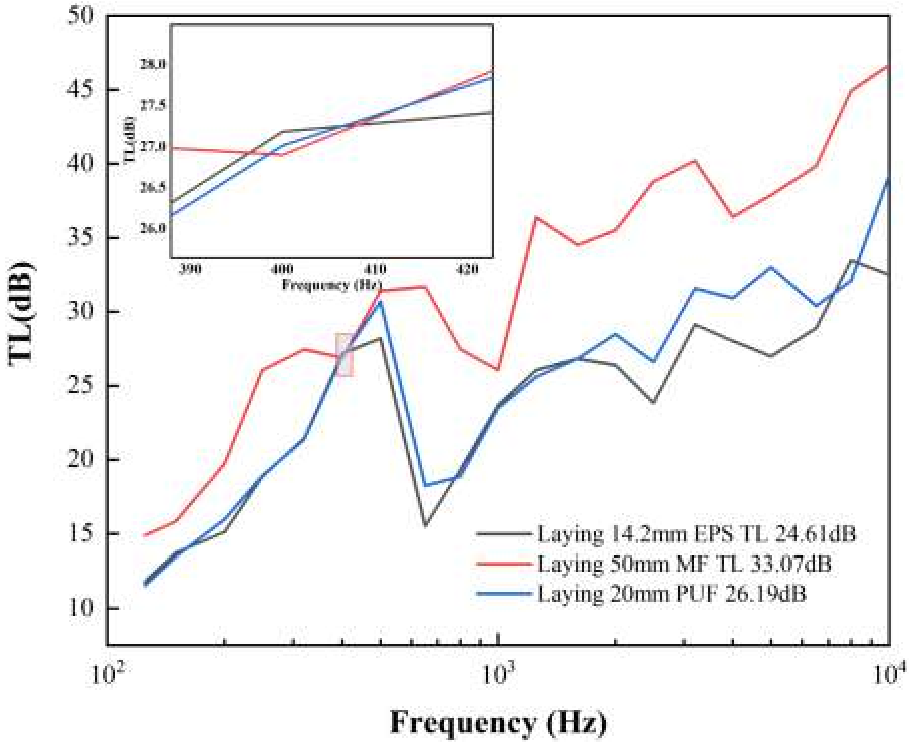


Fig. 8 Sound insulation characteristics of different lining materials for the same mass

As shown in Fig. 8, firstly, the thickness of the MF was selected as 50 mm. Then, the thicknesses of the EPS and PUF were determined to be 14.2 mm and 20 mm respectively, while keeping the masses equal. the total sound transmission loss values of the cylindrical shell when laying different thicknesses of EPS, MF and PUF in the frequency band of 100-10000 Hz were 24.61 dB, 33.07 dB and 26.19 dB, the structure has the highest sound transmission loss when lined with MF. This is because MF has a smaller skeleton size, allowing for more refraction and reflection of sound waves during propagation, thus converting more vibrational energy into heat and dissipating it. This also results in the material having higher flow resistance and a higher absorption coefficient. It is worth noting that it is not that the smaller the pore size of porous foam material, the better the sound absorption performance. When the pore size of porous material is too small, it hinders the entry of sound waves into the pores, leading to a decay in the material's sound absorption effectiveness.

4 Conclusion

In this paper, the transfer matrix method (TMM) is used to calculate the sound transmission loss of the structure in the frequency band of 100-10000 Hz, and a theoretical TMM model of the composite structure is established. On this basis, the porous foam material parameters and the influence of different porous materials on the sound transmission loss of the structure are discussed, providing useful references for the noise control of related structures. The results of the study show that

(1) In the frequency range of 100-10000 Hz, the sound transmission loss curves obtained through TMM for the foam composite laminated cylindrical shell and the experimental test curves exhibit the same trend, with a small discrepancy. In the frequency ranges of 125-400 Hz and 500-1250 Hz, the maximum difference between them is 3.76 dB, and when the frequency is above 1250 Hz, the maximum difference in sound transmission loss between the two is 1.35 dB. This indicates that TMM can accurately reflect the sound insulation performance of the composite structure in the frequency range of 100-10000 Hz. Therefore, TMM can be used to predict the sound transmission loss of multilayer structures.

(2) In the frequency range of 100-10000 Hz, the sound transmission loss of the MF composite structure increases with the increase in flow resistance and porosity. Additionally, the sound absorption coefficient of the porous foam material decreases with the increase in the material's tortuosity factor, while keeping the other parameters of the shell and foam material constant.

(3) In the frequency range of 100-10000 Hz, the EPS provides the structure better sound insulation characteristics for the same thickness of the porous lining material. It results in an increase in sound transmission loss of 2.51 dB and 1.76 dB compared to the use of MF and PUF, respectively. On the other hand, the MF has the smallest increase in mass of the structure, which is only 1.63% of the weight of the cylindrical shell. It has the best sound insulation characteristics for the same mass of porous lining material, and the sound transmission loss of the structure is 8.46 dB and 6.88 dB higher compared to the use of EPS and PUF, respectively.

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Author contributions B. L., proposed the ideas, steps and details of the experiment, most of the experiments were done by N. W., W. J. Y., Y. T., Z. Q. Z., W. J. K., where N. W. was instrumental in the proper conduct of the experiments and wrote the article together with B. L., and all the authors analyzed the data, discussed the conclusions.

Declarations

Conflicts of interest The authors declare that there are no competing interests regarding the publication of this article.

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