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Evaluating the Time-Varying Benefits of Cryptocurrencies in Portfolio Management

Evidence from European Financial Markets

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ABSTRACT:

This thesis examines the time-varying benefits of incorporating cryptocurrencies – specifically, Bitcoin, Ethereum, and a value-weighted altcoin portfolio – into a traditional portfolio comprising European equity, bond, and commodity indices. The purpose of this study is to evaluate whether cryptocurrencies enhance the diversification and performance of traditional European portfolios, and how the potential benefits persist in different market regimes. Additionally, the results suggest best practices for optimizing portfolios that include cryptocurrencies.

The study utilizes data from 2018 to 2023 to evaluate whether the inclusion of cryptocurrencies enhances the out-of-sample performance and diversification of portfolios under different market conditions. Three different optimization methods are employed: Mean-Variance, Conditional Value-at-Risk, and Omega Ratio. The optimizations are performed in three rebalancing intervals: weekly, monthly, and quarterly, utilizing a rolling window approach that incorporates data from the past year. Furthermore, the optimizations are performed as unconstrained, while still applying long-only and fully invested constraints, as well as under Global Variance-Based Constraints (GVBC).

The empirical findings suggest that the inclusion of cryptocurrencies increased both total return and total risk. The results are mixed in terms of risk-adjusted returns. Most benefits are evident in the risk-focused and higher-moment strategies, while most disadvantages are associated with the return-seeking mean-variance strategies. Generally, the diversification metrics show diversification benefits for all the strategies. However, these findings were highly dependent on the underlying market regime, the benefits being positive in favourable market conditions and absent or adverse in downturns.

These findings suggest that portfolio efficiency can benefit from the inclusion of cryptocurrencies, particularly in bull markets, but it requires careful strategy selection and dynamic risk management. The study contributes to the existing literature by providing a comprehensive out-of-sample evaluation of the recent period, which validates previous findings and highlights unique aspects of portfolio management with cryptocurrencies.

KEYWORDS: Cryptocurrencies, Portfolio Optimization, Asset Allocation, Diversification, Risk-adjusted Returns, Modern Portfolio Theory, Post-Modern Portfolio Theory, Time-Varying Benefits

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TIIVISTELMÄ:

Tämä tutkielma tarkastelee kryptovaluuttojen – erityisesti Bitcoinin, Ethereumin ja markkina-arvopainotetun altcoin-portfolion – tuomia ajan myötä vaihtelevia hyötyjä osana perinteistä eurooppalaista sijoitussalkkua, joka koostuu osake-, korko- ja hyödykeindekseistä. Tutkimuksen tavoitteena on arvioida, parantavatko kryptovaluutat perinteisten eurooppalaisten salkkujen hajautusta ja tuotto-riskisuhdetta, sekä kuinka nämä mahdolliset hyödyt säilyvät erilaisissa markkinatilanteissa. Lisäksi tutkimus esittää suosituksia parhaista käytännöistä kryptovaluuttoja sisältävien salkkujen optimointiin.

Tutkimuksessa hyödynnetään vuosien 2018-2023 dataa arvioidessa, parantaako kryptovaluuttojen sisällyttäminen salkkujen otoksen ulkoista suorituskykyä ja hajautusta eri markkinatilanteissa. Optimoinneissa käytetään kolmea eri menetelmää: keskiarvo-variassi, ehdollinen riskiarvo ja Omega-suhdeluku. Optimoinnit toteutetaan kolmella eri uudelleenpainotus frekvenssillä: viikoittain, kuukausittain ja neljännesvuosittain, hyödyntäen liukuvaa vuoden tarkastelujaksoa. Optimoinnit suoritetaan sekä rajoittamattomina – kuitenkin vain ostopositiioita ja täysimääräisesti sijoitetun salkun ehtojen mukaisesti – että globaaleilla volatilitteettipohjaisilla rajoitteilla.

Empiiriset tulokset osoittavat, että kryptovaluuttojen sisällyttäminen lisäsi sekä salkun kokonaistuottoa että kokonaisriskiä. Riskikorjatuissa tuotoissa tulokset olivat vaihtelevia: suurimmat hyödyt havaittiin riskiorientoituneissa ja korkeamman momentin strategioissa, kun taas haitat keskittyivät tuottohakuisempiin keskiarvo-variassistrategioihin. Hajautusmittarit osoittavat yleisesti parantunutta hajautusta kaikissa strategioissa. Tulokset ovat kuitenkin vahvasti riippuvaisia markkinatilanteesta: hyödyt korostuvat suotuisissa olosuhteissa, mutta heikentyvät tai jopa kääntyvät negatiivisiksi laskumarkkinoilla.

Tutkimus osoittaa, että kryptovaluuttojen sisällyttäminen voi parantaa salkun tehokkuutta erityisesti nousumarkkinoilla, mutta edellyttää harkittua strategian valintaa ja dynaamista riskienhallintaa. Tutkimus täydentää aiempaa kirjallisuutta kattavalla ulkoisen suorituskyvyn analyysillä tuoreelta ajanjaksolta, validoiden aiempia löydöksiä ja tuoden esiin uusia kryptovaluuttojen salkunhallintaan liittyviä erityispiirteitä.

AVAINSANAT: Cryptocurrencies, Portfolio Optimization, Asset Allocation, Diversification, Risk-adjusted Returns, Modern Portfolio Theory, Post-Modern Portfolio Theory, Time-Varying Benefits

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1 Introduction

The field of cryptocurrencies has undergone significant evolution since the introduction of Bitcoin by Satoshi Nakamoto in 2008 and its subsequent launch in 2009. Since then, the cryptocurrency market has expanded, with thousands of new cryptocurrencies emerging. For instance, the number of active cryptocurrencies grew from roughly 2,000 to 9,000, with many more being created but later becoming illiquid or inactive. This growth highlights the dynamic and immature nature of the cryptocurrency asset class. Simultaneously, their importance in global financial markets has increased with rising investor interest and growing institutional adoption. Prominent asset managers have entered the space by launching Exchange-Traded Funds (ETFs) for Bitcoin and Ethereum, with increasing interest in other cryptocurrencies. Other actors, such as payment service providers (e.g., PayPal), have begun integrating cryptocurrencies into their platforms. The growing attraction has broadened investors' focus beyond Bitcoin to other altcoins in search of new opportunities and diversification benefits.

However, the cryptocurrency market is characterized by extreme volatility and distinct cyclical behavior, which pose questions and challenges for portfolio management. Since 2018, the market has experienced various cycles and major crises due to both macroeconomic and crypto-specific events, including the cryptocurrency boom of 2020-2021, the FTX collapse in 2022, and the COVID-19 crash in 2020. Under varying conditions, the performance of cryptocurrencies and their correlation with traditional assets fluctuate significantly, influencing the attractiveness of including them in a portfolio. It raises the question of how their benefits hold up across different market regimes.

Regarding the aforementioned, investors and researchers have shown growing interest in whether various cryptocurrencies can improve portfolio performance through increased performance and diversification. Early empirical studies (e.g., Brière et al., 2015; Wu & Pandey, 2014) provided initial evidence of Bitcoin's potential diversification benefits, characterized by low correlation with traditional assets and improved portfolio efficiency. However, those studies are limited to Bitcoin's early years and did not consider

the broader cryptocurrency landscape or market cycles. Newer studies have included more recent periods and a comprehensive selection of cryptocurrencies (e.g., Petukhina et al., 2021), yielding more nuanced results regarding the benefits of cryptocurrencies.

Nonetheless, the cryptocurrency market is immature and continuously evolving. This creates a need for further analysis of how the benefits have evolved in recent periods and market conditions, and how broader cryptocurrency selection fits into this picture. Due to the dynamic nature of cryptocurrencies, studying their benefits in a time-varying framework is essential. Moreover, from a practical perspective, understanding these dynamics in a European market context is valuable, as European investors typically hold portfolios comprising regional equities, bonds, and commodities.

1.1 Purpose of the study

This thesis aims to evaluate the time-varying benefits of cryptocurrencies in portfolio management, with evidence from European financial markets. In particular, the study examines whether incorporating cryptocurrencies into a traditional European asset portfolio, comprising stocks, bonds, and commodities, improves risk-adjusted returns and diversification over a recent period characterized by diverse market conditions. The analysis spans from January 2019 to December 2023, encompassing one cryptocurrency bull market and two bear market phases. By capturing multiple cycles, the study can observe how the contribution of cryptocurrencies to a traditional portfolio varies between bull and bear markets, as well as during different crises.

Furthermore, rather than focusing solely on the major cryptocurrencies (e.g., Bitcoin and Ethereum), the analysis also includes various altcoins in a value-weighted portfolio. Through this approach, the study examines whether diversification benefits primarily come from Bitcoin or if other cryptocurrencies offer additional value. Moreover, by utilizing multiple rebalancing intervals (weekly, monthly, quarterly) and various portfolio

optimization methods (mean-variance, CVaR, Omega-ratio), this study comprehensively assesses when and to what extent cryptocurrencies contribute to portfolio efficiency.

This thesis extends previous literature by providing a comprehensive, dynamic, and practical perspective on the integration of cryptocurrencies in portfolio management. While many earlier studies relied on older datasets, focused on in-sample performance, or considered narrower asset inclusion, often limited to a couple of cryptocurrencies, this study evaluates the out-of-sample effects of a diversified set of cryptocurrencies across multiple recent market regimes. By incorporating higher-moment optimization techniques and imposing Global Variance-Based Constraints, it offers new insight into how strategy selection influences the benefits of cryptocurrencies. The combination of time-varying performance analysis, broader cryptocurrency exposure, and practical portfolio allocation considerations enables this study to validate prior research, address gaps, and generate findings more applicable to real-world investment decision-making.

1.2 Hypotheses

H1: Portfolios with cryptocurrencies exhibit significantly higher out-of-sample risk-adjusted returns than equivalent traditional portfolios.

The risk-adjusted returns of portfolios composed of Bitcoin, Ethereum, and a value-weighted Altcoin portfolio, as well as European equities, bonds, and commodities, are tested against portfolios optimized solely with traditional assets, using the Sharpe Ratio, Sortino Ratio, and Omega Ratio.

H2: Cryptocurrencies will significantly increase portfolio diversification.

Similarly to H1, the cryptocurrency-inclusive portfolios are tested against portfolios without cryptocurrencies, using Diversification Ratio and Effective Number of Bets.

H3: Performance and diversification benefits are highly dependent on the cryptocurrency market regimes, being elevated in bull markets and indistinguishable or negative in bear markets.

The performance and diversification metrics are presented dynamically, enabling the evaluation of the differences between portfolios with and without cryptocurrencies in different market regimes.

H4: Portfolio optimization with higher-moment methods will deliver better out-of-sample risk-adjusted performance on average than classical mean-variance optimization.

The performance and diversification of portfolios produced by different optimization strategies (mean-variance versus minimum conditional value-at-risk and Omega Ratio optimizations) are compared.

Beyond these four hypotheses, the study examines how imposing Global Variance-Based Constraints and varying rebalancing frequencies impact the crypto-inclusive portfolios. Global Variance-based Constraints enable the testing of whether constraining risk contributions can control extreme cryptocurrency allocations and reduce volatility without substantially sacrificing returns. Likewise, comparing weekly, monthly, and quarterly rebalances addresses whether the intensive nature of the cryptocurrency market necessitates more frequent adjustments to portfolio weights to capture the diversification and performance benefits fully.

1.3 Structure of the study

The rest of the thesis is organized as follows. Chapter 2 reviews the relevant literature on cryptocurrencies in portfolio management, summarizing the prior findings on their benefits on risk-adjusted returns and diversification. The literature review aims to

identify the conclusions and gaps in the research. Chapter 3 presents the theoretical framework for the study. It introduces key financial theories and concepts, such as Efficient Market Hypothesis and Modern Portfolio Theory, and discusses the unique characteristics of cryptocurrency markets that distinguish them from traditional assets. Chapter 4 presents the final data used in the empirical analysis and the methods employed to obtain it. It presents data sources, the asset set, including the value-weighted altcoin portfolio formulation, and the descriptive statistics of the assets. Chapter 5 introduces the methodology behind the findings, explaining the rolling-window portfolio optimization approach, specific portfolio optimization strategies, and performance evaluation measures. Chapter 6 presents empirical results that are time-varying in nature, including correlations, final optimized portfolios, risk-adjusted return ratios, and diversification measures. Chapter 8 discusses empirical results in greater depth, connecting the findings from previous literature with the hypotheses mentioned above. Chapter 9 addresses the study's limitations, such as data and methodology constraints, which should be considered when interpreting the results. Finally, Chapter 10 concludes the thesis by summarizing the main findings and contributions of the research.

2 Literature Review

The integration of cryptocurrencies into portfolio management has attracted academic interest as the asset class matures. Early studies on the role of cryptocurrencies in portfolio management primarily focused on Bitcoin and its diversification benefits, as observed in studies by Brière et al. (2015) and Wu and Pandey (2014). Brière et al. (2015) analyzed Bitcoin's investment potential from the standpoint of a US investor with a diversified portfolio of traditional assets. The study examines the early years of Bitcoin's history, spanning from 2010 to 2013, acknowledging the potential bias that a bullish period and the asset's immaturity may have on the results. The correlation analysis suggests from the outset that there is potential for diversification benefits offered by Bitcoin, as correlations are low between traditional assets.

Brière et al. conducted a mean-variance spanning test and mean-variance optimization to effectively analyze the potential benefits. The spanning tests indicate that incorporating Bitcoin into traditional asset portfolios significantly enhances the efficiency of the portfolios and provides diversification benefits, as the efficient frontier is expanded to include enhanced risk-return trade-off areas. Furthermore, optimizing portfolios at 6% and 12% risk levels with and without Bitcoin demonstrated that, in both cases, Bitcoin-inclusive portfolios resulted in significantly higher risk-adjusted returns, as measured by Sharpe and Sortino ratios, and lower downside volatility, as measured by semi-variance. On the other hand, Bitcoin resulted in a significant fall in the Adjusted Sharpe Ratio, indicating a higher risk of extreme fluctuations or tail risks in the portfolio.

An earlier study by Wu and Pandey (2014) came to similar conclusions during the same period. They also optimize the portfolio with and without Bitcoin using mean-variance optimization, but differentiate themselves from Brière et al. (2015) by utilizing Sortino ratio, Omega ratio, and downside semi-variance optimizations. The study shows that Bitcoin is part of each optimal portfolio, enhancing its performance, especially in the maximum Sortino and Sharpe ratio portfolios, where it constitutes the entire portfolio. Additionally, as the Omega ratio is optimized quarterly, the results demonstrate Bitcoin's

integral role in the portfolios. Finally, Wu and Pandey employ the Black-Litterman approach to evaluate the portfolio formations under pessimistic scenarios for Bitcoin, such as a 50% decline in its value, demonstrating that it can enhance the portfolio's performance even under more unfavorable conditions.

Both studies highlight the promising role of Bitcoin as a diversification tool and a considerable asset to be included in traditional portfolios. However, their findings are limited by several significant limitations, including the immaturity of the asset, the limited availability of data, a lack of out-of-sample testing, and a potential bullish bias in the sample period, among others. These contributions still provide the groundwork for more comprehensive studies of the role of cryptocurrencies in portfolio management.

Furthermore, multiple studies have focused on the dynamics between Bitcoin and traditional assets, evaluating its role as a diversification and risk management tool (Bouri et al., 2017; Corbet et al., 2018; Guesmi et al., 2019). Bouri et al. studied Bitcoin's relationships with other assets from 2011 to 2015, using the dynamic conditional correlation (DCC) model (Engle, 2002), with a focus on its safe haven, hedging, and diversification properties. They conclude that, in most cases, Bitcoin can be an effective diversification tool, while in a few cases, it serves as a hedge or a safe haven. Additionally, the diversification properties may vary over time.

Corbet et al. expanded the topic further by analyzing the relationships between three different cryptocurrencies (Bitcoin, XRP, and Litecoin) and traditional assets from 2013 to 2017, utilizing Diebold and Yilmaz's (2012) methodology to examine return and volatility spillovers as well as Barunik and Krehlik's (2015) methodology for evaluating different investment horizons. The spillovers indicate a disconnect between cryptocurrencies and traditional financial markets, which supports their potential benefits for diversification in traditional portfolios. However, these benefits vary over time: the spillovers remain minimal over longer horizons while increasing over the short term. Lastly, Guesmi et al. complement these studies by investigating the dynamic volatility transmissions

between Bitcoin and other financial assets from 2012 to 2018, strengthening the evidence supporting Bitcoin's potential as a diversification tool.

The aforementioned studies provide solid evidence for the benefits of including Bitcoin in diversified portfolios, but offer limited evidence regarding other cryptocurrencies. Besides, as the cryptocurrency market matures and evolves, the stability and consistency of these results remain a question.

Dorfleitner and Lung (2018) narrowed the research gap by examining the diversification properties of eight cryptocurrencies from 2015 to 2018 using mean-variance spanning tests and efficient portfolio calculations. They found that seven of them offer significant diversification benefits in diversified portfolios. These benefits are almost entirely characterized by elevated returns, rather than risk reduction. Additionally, these benefits are found to vanish in bearish market conditions. The same potential exists for alternative cryptocurrencies as for Bitcoin (e.g., Dorfleitner and Lung, and Corbet et al., 2018). Furthermore, Liu (2019) and Brauneis and Mestel (2019) investigated diversification among cryptocurrencies, showing the potential for significantly reducing risk and enhancing risk-adjusted returns when the asset class is recognized beyond Bitcoin.

Petukhina et al. (2021) further extended the existing literature by examining the benefits of incorporating cryptocurrencies into diversified portfolios from 2015 to the end of 2019, utilizing 55 cryptocurrencies and 16 traditional assets. They include multiple portfolio optimization strategies, rolling-window analysis with varying rebalancing frequencies, liquidity constraints, out-of-sample testing with and without cryptocurrencies, and an evaluation of diversification and overall performance using different metrics. Their general results align with earlier studies, which suggest that cryptocurrencies offer diversification benefits and improve portfolio performance. The out-of-sample outcome supports this finding. For example, the correlation stays generally low with traditional assets, cryptocurrencies expand the efficient frontiers, and the outperformance of traditional portfolios is observed.

Additionally, the study strengthens Liu's (2019) and Brauneis and Mesel's (2019) evidence of considering a broader set of cryptocurrencies, rather than only the top ones. Furthermore, Petukhina et al. find more detailed results. Return-seeking strategies, such as the Maximum Sharpe Ratio, and diversification-focused strategies, like the Equal Risk Contribution, capture the highest benefits. However, the study also emphasizes the importance of cryptocurrency liquidity as differences exist between the portfolios' performances with and without liquidity constraints. Moreover, the rebalancing frequency analysis indicates that monthly rebalancing sufficiently captures the performance-enhancing effects compared to daily or weekly rebalancing, considering transaction costs.

Most of the aforementioned studies' time frame extends until the end of 2017, capturing only results from bull market conditions of cryptocurrencies, except for Dorfleitner and Lung (2018) and Petukhina et al. (2021). Dorfleitner and Lung's study captures half of the 2018 bear market and finds that the diversification benefits disappear entirely. Similarly, Petukhina et al. find that the short-term benefits may be reduced during the market downturns, such as the 2018 cryptocurrency market crash. The efficient frontiers collapsed, and the cryptocurrencies held nearly no weight in the optimal portfolios.

On the other hand, Huang et al. (2023) use nine cryptocurrency asset classes to expand the analysis to pre- and post-COVID-19 periods. The study finds that most cryptocurrency categories enhance portfolio performance, even during periods of high market volatility and uncertainty, such as the post-COVID-19 period, while continuing to act as diversifiers. While aggressive investors tend to gain the most benefits, rather than risk-averse ones, the study still provides robust evidence of cryptocurrencies' diversification benefits, even during economic downturns.

However, Allen (2022) shows that correlations between Bitcoin and Ethereum, as well as traditional assets, increased significantly during the COVID-19 crisis, thereby diminishing the diversification benefits. Similar findings were reported by Maasoumi and Wu (2021), Conlon et al. (2020), and Grobys (2021). The broader scope of Huang et al.'s

methodology and inclusion of diverse cryptocurrency categories might explain the divergence in their findings, leaving the actual diversification benefits of cryptocurrencies still inconclusive.

In conclusion, the broad set of studies agrees that cryptocurrencies offer diversification benefits during normal market conditions. In addition, a more recent study by Han et al. (2024) demonstrates that factor portfolios constructed of cryptocurrencies also provide out-of-sample diversification benefits. However, the evidence of diversification performance during economic downturns remains somewhat inconsistent. This creates a need for more evidence for the diversification benefits of different market conditions, especially when a broader set of cryptocurrencies is considered. Furthermore, there is a need to distinguish the benefits of cryptocurrencies, such as the most common Bitcoin and Ethereum, from those of other alternative cryptocurrencies.

3 Theoretical Framework

In this section, the relevant theories for the background of the study are introduced. These aspects include the Efficient Market Hypothesis, an introduction to the markets analyzed in the study, and portfolio management theories, models, and methods.

3.1 Efficient Market Hypothesis

The framework of Efficient Market Hypothesis (EMH) was formalized by Fama (1970), and it is closely related to the concept of random walk. A random walk indicates that changes in asset prices are random and unpredictable, and only new and unexpected information can affect them. Consequently, it illustrates the movements of markets that reflect all current information, which is the core concept of the Efficient Market Hypothesis (EMH). (Bodie et al. 2018, p. 334-335). Fama categorized the three forms of market efficiency: weak form, semi-strong form, and strong form. The weak form suggests that the historical market trading information is already reflected in the current prices. In addition to the historical price data, the semi-strong form encompasses all publicly available information, including annual reports, news releases, and other relevant disclosures. Finally, a strong form indicates that all information, public and private, including insider information, is reflected in the prices. (Fama, 1970, p. 388).

EMH makes multiple assumptions about markets and their participants. The sufficient conditions for capital market efficiency, as outlined by Fama (1970), are that there are no transaction costs, information is available to all market participants at no cost, and all participants agree on the price based on the available information. While the absence of these conditions does not necessarily mean that the markets are inefficient, they can be regarded as potential sources of inefficiency. For markets to be efficient, no investor can consistently achieve returns that are better than the market average, such as through arbitrage opportunities created by the irrational behavior of other investors. (Fama, 1970, p. 387-388).

The EMH is widely challenged by numerous studies that introduce different market anomalies, suggesting possible market predictability based on price patterns, valuation parameters, or firm characteristics (Malkiel, 2003, pp. 61-67). The existence of price predictability through price patterns, or technical analysis, is an argument against markets being efficient, even in the weak form sense. For example, Lo et al. (2000) examined technical indicators, such as the head-and-shoulders pattern, and found that they may have some predictive ability. Jegadeesh and Titman (1993) also introduced one of the most widely acknowledged challenges to the weak form of the EMH, the momentum phenomenon. It is an investment strategy for buying stocks that have recently performed well and selling those that have underperformed. Their findings suggest that the strategy yields significant abnormal returns. Furthermore, multiple other anomalies challenge the weak form of market efficiency, including market over- and underreactions, mean reversion, and seasonal and Day-of-the-Week patterns. Similarly, the semi-strong form of EMH is contradicted by fundamental-based anomalies, such as the predictive power of different price multiples or firm characteristics. These are often still perceived as temporary, inaccessible, or premium due to the higher risk associated with them. (Malkiel, 2003, p. 71-72).

Even under the strong form of EMH, a crucial role exists for portfolio management. The EMH is a foundational concept in portfolio management theories, such as Modern Portfolio Theory and the Capital Asset Pricing Model, which rely on the assumption that markets are efficiently priced. Under this assumption, diversification continues to play a significant role. It is a way to eliminate firm-specific risk, leaving only undiversifiable market risk, while allowing investors to adjust their portfolios according to their risk tolerance. (Bodie et al. 2018, p. 342-343). Furthermore, market inefficiencies are widely acknowledged, and even EMH allows for the possibility of inefficiencies as long as they are not systematic or persistent. These inefficiencies can allow portfolio management to benefit from the uncorrelated returns and mispricing.

Market inefficiencies often diverge from traditional market movements, offering wider opportunities for diversification due to uncorrelated movements with other assets and deviations from the overall market. This improves the effectiveness of diversification by reducing the overall portfolio risk. Inefficiencies are often present in alternative markets, such as cryptocurrency, due to their speculative nature. For instance, Urquhart (2016) found that between August 2010 and July 2016, Bitcoin did not satisfy the characteristics of the weak form of market efficiency. Nevertheless, Urquhart points out some evidence that it was becoming more efficient, which is expected with the rising interest in the asset.

3.2 Cryptocurrencies

The first cryptocurrency, Bitcoin, was first introduced in a research paper by an alias for an individual or group called Satoshi Nakamoto in 2008, followed by its launch in 2009. It operates as a decentralized blockchain network and was created as an alternative to traditional fiat currencies, particularly in response to the 2008 global financial crisis. Since then, the growth of the cryptocurrency asset class has been rapid, driven by increasing investor interest and the emergence of thousands of new cryptocurrencies. According to CoinMarketCap (2024), a widely used source for cryptocurrency statistics, there were approximately 1,400 'active' cryptocurrencies at the start of 2018. By the end of 2023, this number had risen to around 9,000. Despite this growth, many are untradeable due to low or non-existent volume, leaving them illiquid. However, the liquidity of the cryptocurrency market has been increasing since 2016, driven by higher investor interest and the emergence of new exchanges. This illustrates the immature and evolving nature of the cryptocurrency market.

Although the cryptocurrency market is still in its early stages, its significance within the broader financial markets is growing rapidly. In addition to retail investors, institutional adoption of the asset class has been growing. For example, some of the world's most prominent asset managers, such as BlackRock, have launched Exchange-Traded Funds

(ETFs) products of Bitcoin and Ethereum (BlackRock, 2024), with increasing interest in other cryptocurrencies. Payment service providers, such as PayPal, are integrating cryptocurrencies into their platforms (Reuters, 2024). Furthermore, Governments have begun to recognize the value of cryptocurrencies by launching or planning to launch strategic reserves of Bitcoin (e.g., the US and Brazil). (Decrypt, 2025). Central banks are introducing blockchain-based digital versions of fiat currencies, such as central bank digital currencies (CBDCs). (MasterCard, 2025). Additionally, there has been no regulatory clearance for cryptocurrencies. However, the situation is changing as governmental entities plan and implement new regulations worldwide, such as the European crypto-assets regulation (MiCA). (EY, 2024).

This chapter aims to define the characteristics of cryptocurrencies, the dynamics and efficiency of the cryptocurrency market, their role in the overall financial markets, and their potential for broader adoption in portfolio management.

3.2.1 Characteristics

Cryptocurrencies are digital assets that use distributed ledger or blockchain technology, which relies on a peer-to-peer (P2P) network, eliminating the need for any centralized party. Decentralization arises from the blockchain's structure, as it is maintained and validated by a distributed network of nodes—devices that contribute to the network's integrity. The nodes must agree on every update through a consensus mechanism, such as Proof of Work (PoW), used by Bitcoin, or Proof of Stake (PoS), used by Ethereum. The nodes run mathematical algorithms based on the blockchain's consensus mechanism to verify transactions, validate blocks, and maintain consensus across the network. For example, the most well-known algorithm type used by Bitcoin's PoW mechanism is the SHA-256 algorithm. (Härdle et al. 2020, p. 181-187).

According to Härdle et al. (2020, pp. 187-188), cryptocurrencies can be categorized into approximately seven distinct classes, each serving a specific purpose within the

blockchain ecosystem. First class is transaction mechanisms to which Bitcoin falls under. The second group is distributed computation tokens, such as Ethereum. These enable the running of computer programs on the network, such as smart contracts, which automatically execute the terms of an agreement when predefined conditions are met. The third class is utility tokens, which offer access to a specific product or service within the blockchain. Render is a utility token that offers GPU computing power contributed by other network participants in exchange for token payments. The fourth class comprises security tokens that represent the ownership or right to a real-world asset (RWA), such as stocks, bonds, or real estate. Fifth and sixth classes are fungible and non-fungible tokens (NFTs). Many cryptocurrencies fall under the fungible token class, as it refers to one unit of token being interchangeable with another unit of the same token. On the contrary, each NFT is unique, and the value of one NFT is not usually the same as that of another (e.g., tokenized artwork). The seventh and final class is stablecoins, which are designed to maintain a steady value, similar to fiat currencies. They are pegged to a reference asset, which is often a fiat currency, such as USDT, which is pegged to the US dollar at a one-to-one ratio. (Härdle et al. 2020, p. 187-188).

The valuation of cryptocurrencies differs significantly from traditional assets. According to Bhambhwani et al. (2023), the intrinsic value of a cryptocurrency stems from its computing power, adoption, and overall network size. They find a statistically significant positive relationship between these factors and the price of a cryptocurrency. Another factor that affects the valuation is the sentiment. For instance, Cretarola and Figa-Talamanca (2017) find a link between the price of Bitcoin and confidence in the underlying technology, while Aste (2019) studies the price movements and social sentiment changes of nearly 2,000 cryptocurrencies, discovering interrelations between sentiment and prices that affect each other. Furthermore, the valuation is also influenced by the intrinsic utility of the token, such as speculation regarding the future adoption of specific functionalities (e.g., Ethereum smart contracts). Additionally, it is essential to note that the total supply, its allocation among parties, and potential deflationary features create scarcity, which impacts valuation. (Härdle et al. 2020, p. 193-197). In conclusion, the

aforementioned are a few characteristics that affect the valuation of cryptocurrencies. They demonstrate how the valuation process differs from that of stock markets.

3.2.2 Market Dynamics and Efficiency

The cryptocurrency market is a new and developing financial ecosystem with different characteristics from traditional markets. Cryptocurrency volatility is exceptionally high compared to traditional assets, such as gold or the S&P 500. For example, the average daily volatility of Bitcoin was more than five times higher than that of the S&P 500 between 2017 and 2019. (Härdle et al. 2020, p. 188-192). Multiple studies have confirmed the higher volatility of the cryptocurrency market. Trimborn et al. (2020) concluded that cryptocurrencies are more volatile than traditional assets, based on a study of 39 cryptocurrencies. This is also supported by Chu et al. (2017) and Nikolova et al. (2020), among others. Furthermore, cryptocurrencies exhibit asymmetric time-varying volatility, where positive events have a greater impact on volatility than adverse events (Dutta & Bouri, 2022; Fakhfekh & Jebiri, 2020; Cheikh et al., 2020). The return distributions are generally abnormal, illustrating high skewness and kurtosis-tail risk. Another highly studied characteristic of the cryptocurrency market is its correlation with traditional assets. Cryptocurrencies often have low or negative correlations with traditional assets during normal market conditions, indicating the benefits in portfolio management. However, the correlations increase considerably during market downturns.

The cryptocurrency market remains immature, characterized by speculation, inefficiencies, and opportunities for arbitrage. This begs the question of how efficient the market is based on Fama's (1970) forms of market efficiency. According to Urquhart (2016), Bitcoin did not achieve weak market efficiency between 2010 and 2016; however, it did become more efficient. Similar results regarding the inefficiency of the cryptocurrency market were reported by Cheah et al. (2018), covering the period up to March 2017, and Noda (2021), covering the period up to September 2019. Additionally, they find that the market's efficiency fluctuates over different periods. On the other hand, some other

studies have found that Bitcoin has reached a weak form of market efficiency. This conclusion is supported, for example, by Tiwari et al. (2018) and Sensoy (2019). Similarly, they also confirm the time-varying effect of market efficiency. Furthermore, a more recent study by Sahoo and Sethi (2024) employs the popular price-volume framework to assess the market efficiency of eight top cryptocurrencies from August 2015 to October 2022. They arrive at the same conclusion about the cryptocurrency market, following the weak form of the EMH in general, which is still experiencing time-specific inefficiencies. These results are supported by other studies, such as Kim and Park (2023). Key takeaways from the studies are that the cryptocurrency market is less efficient than traditional markets, and its efficiency varies across different periods, with some cryptocurrencies being less efficient than others (Kristoufek & Vosvrda, 2019; Sahoo & Sethi, 2024). Nevertheless, it is becoming more efficient due to institutional adoption of the asset class (e.g., high-frequency trading entering the market).

Several factors contributing to inefficiencies include the aforementioned high volatility and liquidity of the cryptocurrency market. Al-Yahyaee et al. (2020) find that the inefficiencies are time-varying due to changes in liquidity and volatility. They find a positive relationship between liquidity and market efficiency, as one improves, so does the other. On the contrary, they find that generally there is a negative relationship between volatility and market efficiency. These findings are consistent with Wei's (2018) and Kim and Park's (2023) findings. Furthermore, behavioral factors exist in cryptocurrency markets that create inefficiencies. The uncertainty surrounding cryptocurrency fundamentals leads to social influence and sentiment having a greater impact on investment decisions, generating conflicting beliefs among investors, speculative trading, bubbles, and overall irrational behavior. Additionally, cryptocurrencies are prone to herding behavior, which amplifies inefficiencies and fosters more bubble-like market behavior. (Almeida and Goncalves, 2023). Ultimately, the cryptocurrency market remains a developing sector, undergoing a significant transformation in its dynamics and efficiency.

3.3 Portfolio Management

3.3.1 Modern Portfolio Theory (MPT)

Markowitz (1952) established the foundation for modern portfolio theory (MPT) by introducing mean-variance optimization (MVO) and the concept of the efficient frontier. MVO focuses on the trade-off between risk, measured by variance, and return. It assumes rational risk-averse investors who aim to either maximize the portfolio return for a given level of risk or minimize its risk for a desired level of return, depending on the investor's risk tolerance. Furthermore, rather than assuming portfolio risk as an average of individual asset variances, Markowitz considered the effect of asset co-movements through the variance-covariance matrix. Recognizing the impact of asset correlations on portfolio risk enables optimization by considering diversification benefits. This leads to the efficient frontier, which comprises portfolios offering the best possible risk-return trade-offs. The selection of the efficient frontier portfolio is solely based on the investor's risk tolerance.

Tobin (1958) extended Markowitz's theory by introducing a risk-free rate. The risk-free rate, combined with a portfolio of risky assets, enables a better risk-return trade-off, leading to the creation of the Capital Market Line (CML). CML is tangent to the efficient frontier, which delimits investors' rational allocation choices among different combinations of the risk-free rate and the optimal portfolio. Sharpe (1964) further developed Markowitz's and Tobin's theory by building the Capital Asset Pricing Model (CAPM), which added the concept of systematic (market) risk and unsystematic (idiosyncratic) risk. While unsystematic risk is part of the portfolio risk that can be reduced or eliminated through diversification, systematic risk affects the whole market rather than individual assets and cannot be diversified away. This led to the development of the Security Market Line (SML), which examines the relationship between an asset's beta (systematic risk) and its expected return. Lintner (1965) expanded on the CAPM by explaining the effects of systematic risk and risk premium on asset pricing, while Mossin (1966) formalized the concept of market equilibrium, suggesting that higher systematic risk should be

rewarded with higher expected returns, whereas unsystematic risk is not. Overall, this framework led to the formulation of CAPM:

$$(1) E(R_i) = R_f + \beta_i[E(R_m) - R_f]$$

The result of Markowitz's theory, along with the contributions of others, is the Capital Allocation Line (CAL). This tool helps make optimal investment decisions and allocate portfolios based on risk tolerance. Capital Market Line (CML) is a CAL where the market portfolio is used as a risky asset. The slope of CAL reflects the Sharpe Ratio (Sharpe, 1966) of the optimal portfolio; a higher slope indicates better risk-return combinations. Therefore, when optimizing for the Sharpe Ratio using MVO, the goal is to maximize the slope of the CAL.

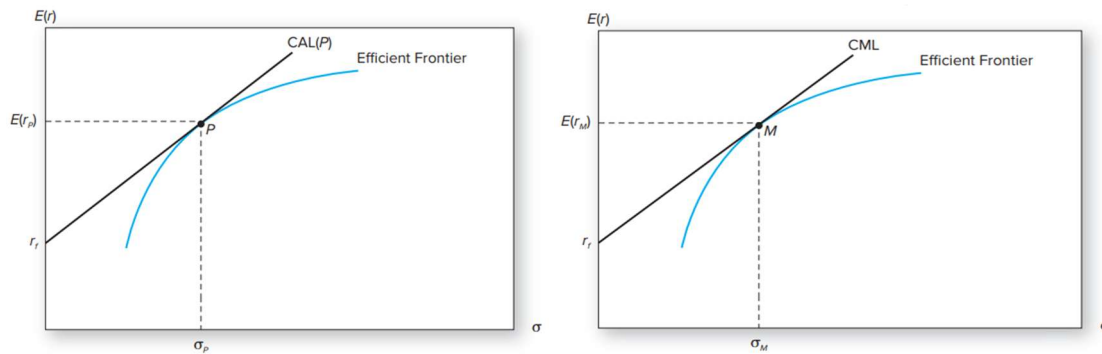


Figure 1. Capital Allocation Line and Capital Market Line

Modern portfolio theory forms the foundation for understanding the risk-return trade-off in creating optimal portfolios and provides the rationale behind portfolio allocation decisions. However, it relies on assumptions about investors being risk-averse and rational, market efficiency, the absence of transaction costs, borrowing at the risk-free rate, short selling, a long time horizon, risk evaluation, and a normal return distribution. The limitations of MPT regarding return distribution and risk evaluation are addressed by extending the post-modern portfolio theory.

3.3.2 Post-Modern Portfolio Theory (PMPT)

MPT assumes that asset returns are normally distributed, which is rarely true for financial time series data. Additionally, MPT focuses on variance as the risk metric rather than downside risk. While variance captures both downside and upside volatility, downside risk metrics focus solely on potential losses, which investors are often more concerned about. Due to these limitations, numerous studies have been conducted to focus specifically on downside risk and to consider higher-order moments, such as skewness and kurtosis, of the asset return distributions. These form the Post-Modern Portfolio Theory (PMPT), which aims to provide a more comprehensive and robust portfolio management approach by recognizing some of the limitations of the Modern Portfolio Theory (MPT).

Both skewness and kurtosis measure the shape of the distribution, which is important for portfolio management because they offer a more comprehensive picture of the risk and return characteristics of the assets and portfolios. Skewness describes the asymmetry of a distribution, with negative skewness indicating a left-skewed tail and positive skewness indicating a right-skewed tail. Regarding return distribution and an asset's risk, negative skewness implies a higher likelihood of significant losses. On the contrary, positive skewness indicates a higher likelihood of significant gains, although these may be infrequent. Thus, skewness risk is a crucial metric for assessing the risk of an asset. For example, Kraus and Litzenberger (1976, 1983) extended the CAPM by incorporating skewness into the valuation model, finding that investors prefer positive skewness. Furthermore, other studies have emphasized the importance of managing the left-tail risk in portfolio management, such as Rockafeller and Uryasev (2000), who studied Conditional Value-at-Risk as a tool for minimizing extreme losses in portfolio optimization.

$$(2) \text{ Skewness} = \frac{\sum_i^n (X_i - \bar{X})^3}{(n-1)\sigma^3}$$

Kurtosis describes the heaviness of the distribution's tails compared to a normal distribution. High kurtosis indicates fat tails or a higher probability for extreme outliers, while low kurtosis indicates thin tails or a lower probability for extreme outliers. In portfolio

management, kurtosis helps to understand the likelihood of extreme events. For example, when kurtosis is low, the returns will likely be more concentrated around the mean, with fewer extreme outcomes; however, high kurtosis suggests a higher probability for both significant gains and losses. For this reason, in the presence of high kurtosis, focusing on minimizing left-tail risk becomes especially important, as it can pose significant risks to an asset or a portfolio (e.g., Rockafeller and Uryasev, 2000). Additionally, Stacey (2008) extends the traditional mean-variance approach by incorporating kurtosis into the optimization. The study emphasizes the importance of understanding the shape of the return distribution in mitigating portfolio risks more efficiently.

$$(3) \text{ Kurtosis} = \frac{\sum_i^n (X_i - \bar{X})^4}{(n-1)\sigma^4}$$

Regarding cryptocurrencies, returns tend to be highly volatile, and distributions are non-normal; therefore, the application of skewness and kurtosis is even more critical. The return distributions of cryptocurrencies often exhibit high kurtosis and are skewed, indicating a higher probability of extreme price movements. Therefore, higher-order moments are important risk factors when optimizing portfolios with cryptocurrencies, especially the left-tail risk. Furthermore, traditional assets do not typically display the same extreme characteristics as cryptocurrencies. This discrepancy between the market behavior may indicate potential diversification benefits. Incorporating cryptocurrencies into traditional portfolios can achieve unique risk-return profiles, enhance overall portfolio performance, and provide resilience against extreme market conditions.

3.3.3 Portfolio Optimization Methods

Generally, MVO focuses on either minimizing the risk for a desired level of return or maximizing the return for a given level of risk, with a common approach being the maximization of risk-adjusted return measures, such as the Sharpe Ratio. As concluded, it considers variance as the risk metric, overlooking higher-order moments and assuming a normal distribution of returns simultaneously. Furthermore, the same conduct for

upside volatility as for downside is problematic. These factors are crucial considerations when selecting a suitable optimization method, particularly when working with volatile cryptocurrencies, which often exhibit asymmetric return distributions and extreme price fluctuations.

Value-at-Risk (VAR) is a risk measure that considers only downside risk, without assuming normality for the distribution, except for a few calculation methods. While VAR does not directly incorporate skewness or kurtosis into its calculation, it can indirectly manage both factors. It measures the maximum loss a portfolio or an asset could encounter in a specific period with a determined confidence level (see Figure 2). In other words, it is a probability that the actual loss will not exceed the VAR threshold. There are multiple ways to calculate VAR, the most common being the historical method. Furthermore, it can be applied to a portfolio optimization framework, for example, Tsao (2010) studied portfolio selection using three different methods for VAR calculation to construct a mean-VAR efficient frontier.

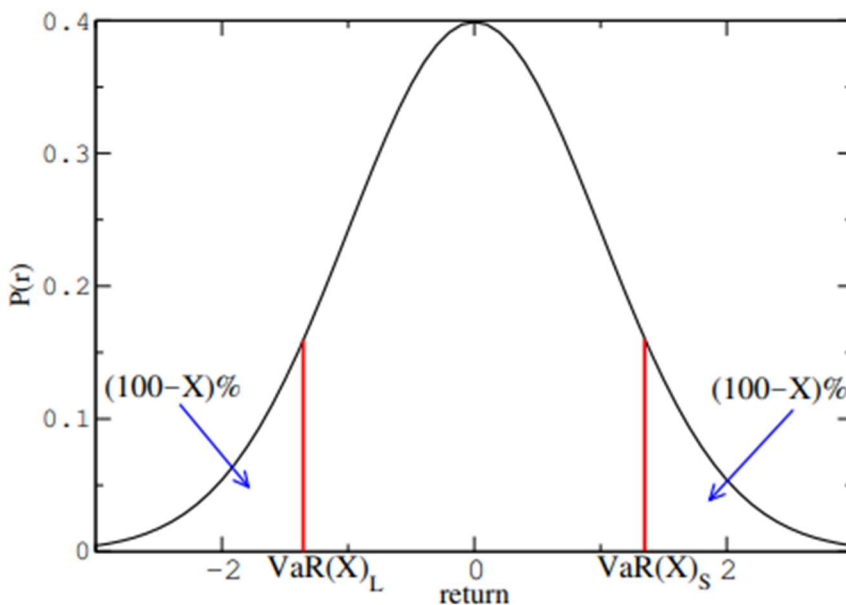


Figure 2. Value-at-Risk (Long & Short)

Rockafeller and Uryasev (2000) introduced an advanced approach to VAR for studying the optimization and hedging of a portfolio called Conditional-Value-at-Risk (CVaR). While VAR focuses on the probability that a specific maximum loss will occur, CVAR measures the expected loss if the VAR threshold is exceeded, making it a more comprehensive method for assessing downside risk. Rockafeller and Uryasev found that CVaR is superior to VAR in capturing risk. Additionally, even though their objective was formally to minimize CVaR, the VAR was also reduced. Xiong and Idzorek (2011) investigated how skewness and kurtosis impact allocation decisions using MVO and Mean-CVaR (M-CVaR) optimization methods, considering both normal and non-normal distributions. They found that MVO and M-CVaR lead to the same allocation decisions when considering a normal distribution. For non-normal distributions, the methods yielded significantly different results, with M-CVaR favoring assets that exhibit higher positive skewness, lower kurtosis, and lower variance. Furthermore, it outperformed MVO throughout the six-month out-of-sample testing period, including during the financial crisis. This supports the notion that when dealing with non-normal return distributions, such as those commonly found in cryptocurrencies, portfolio optimization should incorporate methods that account for higher-order moments, rather than relying solely on traditional mean-variance optimization (MVO) approaches.

The Omega Ratio, first introduced by Shadwick and Keating (2002), is a performance measure that considers the entire return distribution without making any assumptions about it. While it does not directly calculate higher moments such as skewness and kurtosis, it still captures their risks. By comparing the probability of gains to losses based on a threshold return, the Omega Ratio simultaneously accounts for both downside risk and upside potential (see Figure 3). The variable threshold return determines the gain and loss of a return distribution, making it critically important for the optimization's results. Since it has no pre-determined, precise specification, it can be set to a fixed or specific market rate. For example, Yu et al. (2022) considered yields of treasury securities and a negative CVaR value for the Omega thresholds, finding that the Omega Ratio with

floating thresholds resulted in higher performance than fixed thresholds, outperforming CVaR optimization.

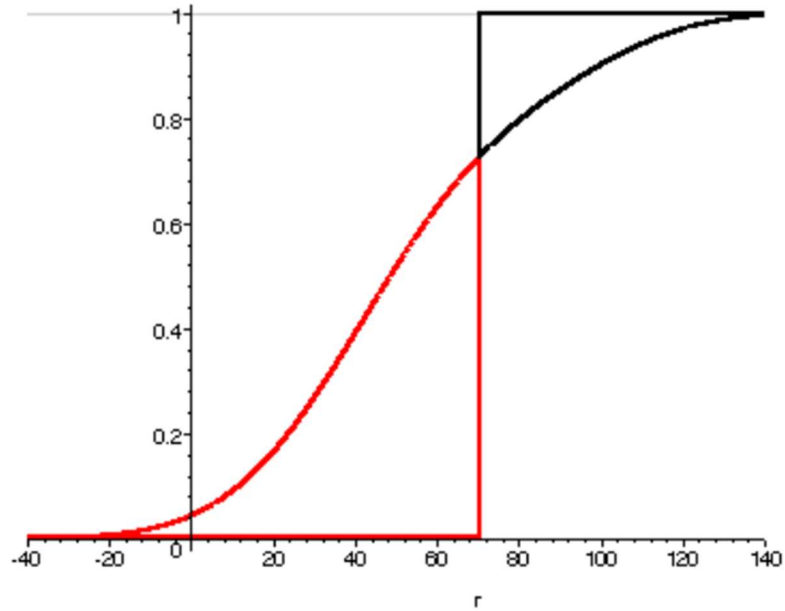


Figure 3. Cumulative Distribution Function and Omega Ratio Threshold (r), y-axis: probability, x-axis: return

4 Data

The data comprises the daily closing prices of cryptocurrencies and indices from January 2018 to December 2023. All the cryptocurrencies in the top 30 based on market capitalization during the period are chosen, excluding Stablecoins. The cryptocurrency investment opportunities are rebalanced at the beginning of every month, and all assets that have fallen off the top 30 list are removed, while new ones are included. The selection method aims to reduce selection bias by following strict criteria and eliminate survivor bias by selecting cryptocurrencies at the start of each period, while also ensuring that the cryptocurrencies remain liquid through low-frequency rebalancing. This results in 88 unique cryptocurrencies being used throughout the period, including, for example, Bitcoin (BTC), Ethereum (ETH), and XRP. For the whole alternative coin (altcoin) list and related periods, see Appendix 1. The cryptocurrency data is retrieved from CoinMarketCap, a widely used source that aggregates prices from multiple exchanges to provide a more universal and accurate value, thereby minimizing exchange biases (Kraaijeveld & De Smedt, 2020, p. 7). Furthermore, the top 30 cryptocurrencies are defined based on the CoinMarketCap historical snapshots from the first week of each month.

The European stock market is represented by the iShares STOXX Europe 600 UCITS exchange-traded fund (EXSA), which replicates the performance of an index composed of the 600 largest companies from European developed countries (Blackrock, 2024). Additionally, the SPDR Bloomberg Euro Aggregate Bond UCITS ETF (SYBA) serves as a proxy for the European bond market, replicating the Bloomberg Euro Aggregate Bond Index. It tracks the performance of the fixed-rate, investment-grade Euro-denominated bond market, including the treasury, corporate, and government bonds. (State Street, 2024). Lastly, the commodity market is represented by the Invesco Bloomberg Commodity UCITS ETF (CMOD), which tracks the Bloomberg Commodity Index through futures contracts on physical commodities, including energy, precious metals, grains, and industrial metals (Invesco, 2024). The index price data is retrieved from the Yahoo Finance API. These indices provide a comprehensive and diversified representation of European equity, bond, and commodity market dynamics, enabling the isolation of the specific

diversification benefits of cryptocurrencies. Later, these assets are referred to as stocks, bonds, and commodities.

The logarithmic returns are used for asset return calculations (formula 4). All asset data is denominated in EUR, except for Altcoin data. The Altcoin log returns are translated to EUR-based returns using the Finnish Central Bank's (similar to the European Central Bank's) daily EUR/USD exchange rate log returns. This helps to eliminate the impact of exchange rate fluctuations on returns and correlations, ensuring that the analysis of diversification benefits remains unaffected. The three-month Euribor rate is used as the risk-free rate. The original cryptocurrency dataset contained between 2,182 and 2,186 data points, while the dataset for other assets ranged between 522 and 1,526 data points. The difference in the number of data points between cryptocurrencies and other assets is primarily due to the continuous trading of cryptocurrencies. In contrast, other assets are limited to traditional market hours. All the price data is reindexed to a common date range, and any dates with missing values for any assets are removed from the cleaned dataset. The final data dates are 2.1.2018 to 29.12.2023. As the assets in a portfolio are rebalanced simultaneously, removing the dates that are not tradable for every asset is justifiable. The final log return dataset consists of 1,502 data points.

$$(4) R_{log} = \ln \left(\frac{r_t}{r_{t-1}} \right)$$

Where \ln represents the natural logarithm function and r_t is the price of an asset at time t .

The cryptocurrency data is divided into three groups: Bitcoin, Ethereum, and other Altcoins, to distinguish the role of each in traditional portfolios. Bitcoin is considered separately due to its significant market dominance within the cryptocurrency asset class, which ranges from approximately 36% to 74% of the total market capitalization during the analyzed period. For the same reason, Ethereum is considered separately as its market dominance ranges from approximately 13% to 26% during the period. (TradingView,

2024). Furthermore, this enables a more nuanced segregation of diversification benefits across different cryptocurrencies. On the other hand, a value-weighted portfolio is constructed from Altcoins, with each Altcoin weighted according to its market capitalization, reflecting its relative size. The weights are rebalanced daily, and Altcoins included in the top 30 in a specific month are used, excluding ETH. The portfolio's assets range from 21 to 26, with an average of approximately 23. Since most indices are value-weighted, the aim is to replicate the overall altcoin market in a more realistic and representative manner. (Malladi and Fabozzi, 2017). It encompasses a range of categories, including smart contract platforms, DeFi, utility, payment, privacy, meme, and storage network tokens. Formula 1 represents the weight of an asset in a value-weighted portfolio.

$$(5) \textit{Weight} = \frac{\textit{Market Capitalization (Component)}}{\textit{Total Portfolio Market Capitalization}} \times 100\%$$

Figure 1 illustrates the weights obtained for the value-weighted Altcoin portfolio over time. The weight of an individual altcoin during the period ranges from 0.0 % to 46.65 % (XRP). The maximum weight allocated to a single altcoin does not exceed the maximum of 46.65 % during the period. XRP receives the highest allocation, with an average weight of 20.07%. The second- and third-highest allocations are BNB and BCH, with average weights of 12.95% and 7.28%, respectively. Overall, the portfolio remains diversified and effectively captures the price development of the altcoin market throughout the entire period. Refer to Appendix 2 for more detailed information on the weight of each cryptocurrency in the portfolio.

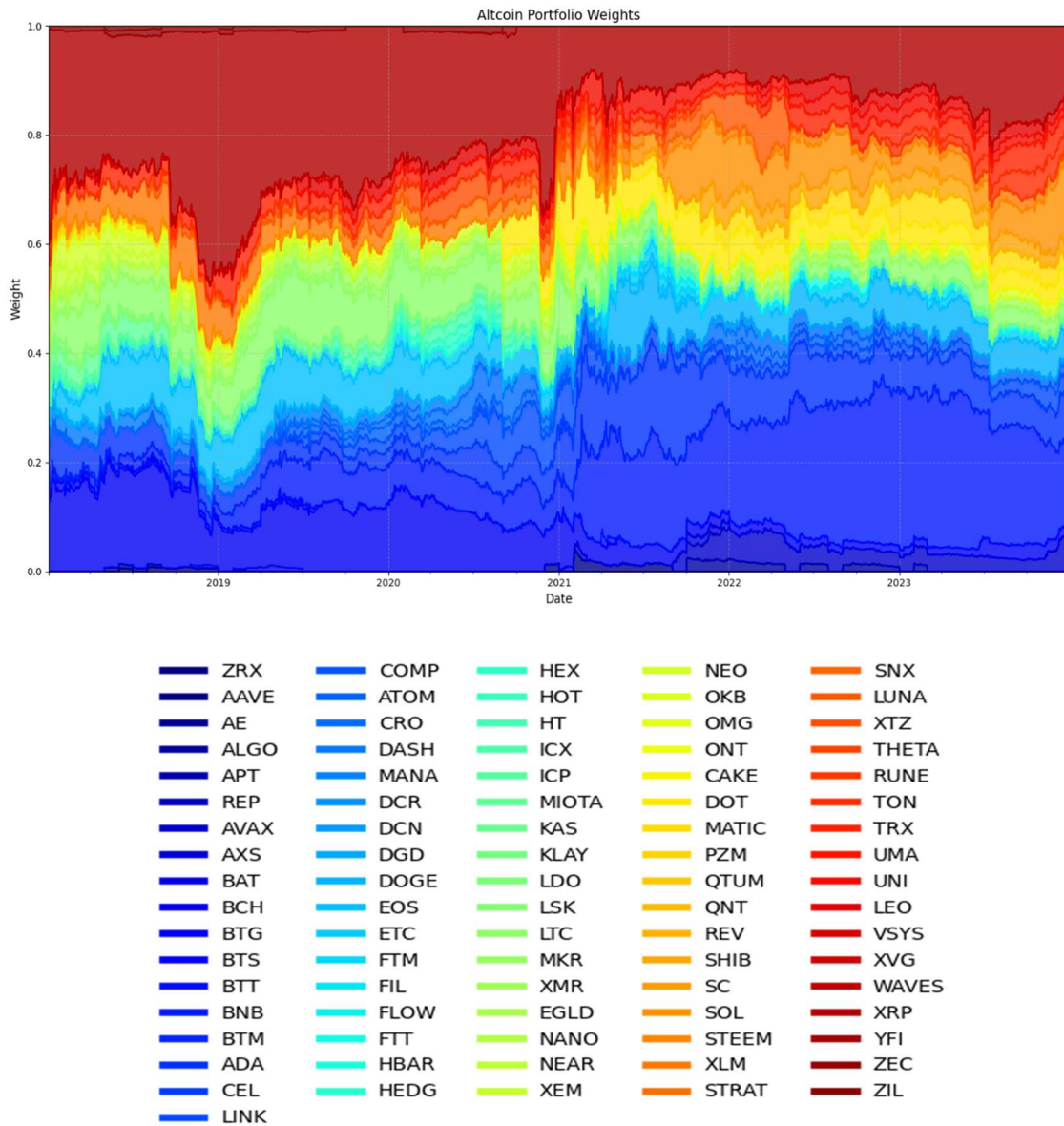


Figure 4. Value-Weighted Altcoin Portfolio Weights

The empirical research includes three ETFs (EXSA, SYBA, and CMOD), the risk-free rate (three-month Euribor), the cryptocurrencies Bitcoin and Ethereum, and a value-weighted portfolio of the remaining Altcoins.

Figure 2 presents the cumulative log returns of each asset over the period, highlighting the high volatility and return potential of cryptocurrencies compared to traditional assets. Table 1 provides descriptive statistics that offer a deeper insight into the characteristics of each asset. As expected, cryptocurrencies have higher mean log returns than

traditional assets, reflecting their higher risk and return profile. The same characteristics are illustrated by the low minimum and high maximum log returns, as well as significantly elevated standard deviations. Equity and commodity indices offer similar statistics, are much less volatile than cryptocurrencies, and have lower log returns. Finally, the bond index is predictably the most risk-averse asset.

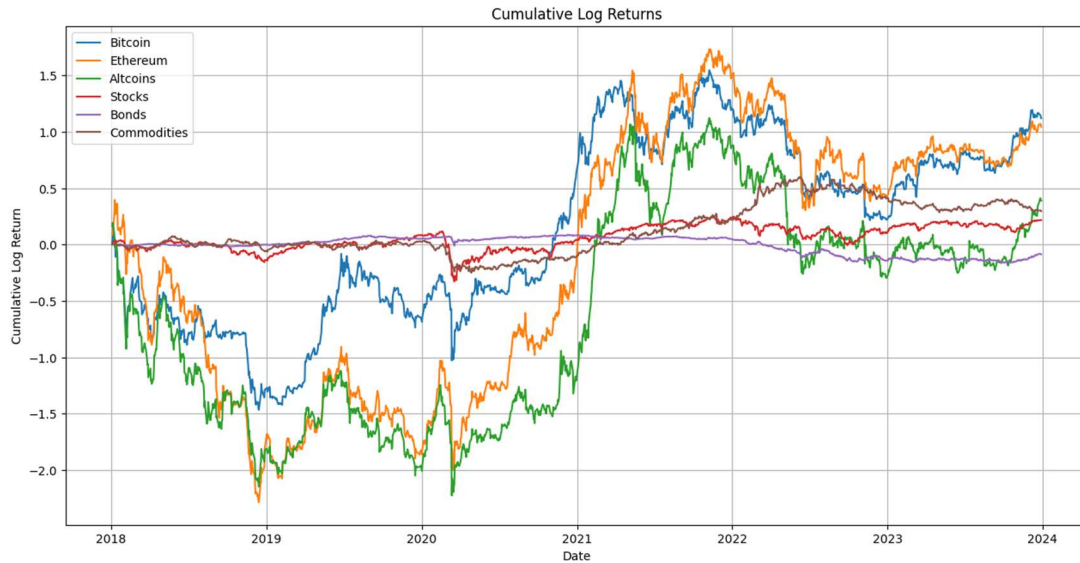


Figure 5. Cumulative Log Returns

Table 1 presents the descriptive statistics, which show that the return distributions of each asset are left-skewed, indicating the presence of more negative outliers. The most negatively skewed are the equity index (-1.47), Bitcoin (-0.98), Ethereum (-0.82), and Altcoin Portfolio (-1.00). The high kurtosis values also indicate heavy-tailed distributions, suggesting a higher likelihood of extreme outliers. The highest kurtosis is present in the same assets: Bitcoin (11.11), Ethereum (9.20), Altcoin Portfolio (7.82), and equity index (18.84). The negative skewness and high kurtosis values indicate that the distributions are non-normal. The Jarque-Bera test is used to assess the normality of a distribution, and in this case, it confirms that none of the assets' distributions are normal. The most significant deviation from normality is observed in the distribution of the Euro Stoxx 600 returns. However, the Commodity Index has the lowest skewness (-0.19) and kurtosis (5.23) values, making it the asset closest to a normal distribution.

	Bitcoin	Ethereum	Altcoins	Eurostoxx 600	Bond Index	Commodity Index
Mean	0,07 %	0,07 %	0,03 %	0,01 %	-0,01 %	0,02 %
Median	0,06 %	0,12 %	0,36 %	0,10 %	0,00 %	0,00 %
Maximum	20,34 %	32,89 %	28,94 %	8,19 %	2,09 %	6,89 %
Minimum	-45,79 %	-54,39 %	-45,69 %	-13,08 %	-2,91 %	-5,62 %
Sum	112,04 %	104,32 %	38,92 %	21,75 %	-8,72 %	29,57 %
Std. Dev.	4,38 %	5,67 %	5,50 %	1,10 %	0,35 %	0,98 %
Skewness	-0,98	-0,82	-1,00	-1,47	-0,27	-0,19
Kurtosis	11,11	9,20	7,82	18,84	9,07	5,24
Jarque-Bera	7 906,29	5 421,08	4 048,44	22 603,02	5 129,03	1 715,54
Probability (JB)	0,00	0,00	0,00	0,00	0,00	0,00
Observa- tions	1502	1502	1502	1502	1502	1502

Table 1. Descriptive Statistics

5 Methodology

Ultimately, the methodology of the study is derived from Markowitz's (1952) modern portfolio theory, Tobin's additions (1958), Sharpe's (1964) theory of market equilibrium, and Litner's (1965) extension of these concepts. However, the methodology incorporates various attributes and recent developments in portfolio theory to yield more comprehensive results regarding the diversification benefits. In addition to evaluating the diversification benefits, the aim is to capture their alteration during the period. More precisely, the methodology aligns with approaches used in similar studies such as Petukhina et al. (2021).

In comparative analysis, data is processed with the rolling-window approach. For the window size, a calendar year is used, which, in terms of trading days, corresponds to $K = 252$ days. Three different rebalancing increments are considered: weekly ($k = 5$ days), monthly ($k = 21$ days), and quarterly ($k = 63$ days). At each rebalancing point, the window advances k days, using the most recent K days of data until the entire dataset ($T = 1502$ observations) is processed. This results in approximately 251 weekly rebalances, 60 monthly rebalances, and 20 quarterly rebalances. The required statistics are calculated at each rebalancing point to optimize the portfolios based on the specific asset allocation strategy. In cases where portfolio optimization fails due to insufficient or poorly conditioned data in specific rolling windows, the window is extended in monthly intervals. This extension incorporates older data while maintaining a fixed end date. The primary objective is to stabilize the covariance matrix in case of singularity or instability. In this study, the extensions generally vary between none and one, except in some cases, where up to three extensions are allowed.

$$(6) \text{ Number of Rebalances} = \frac{T-K}{k} + 1$$

The correlation analysis between the assets is done similarly. They are calculated as a monthly rolling correlation between the assets to illustrate the dynamic relationship between traditional assets and cryptocurrencies. The correlations are later used in the

interpretation of the diversification results. This enables a deeper understanding of how assets move in tandem under various market conditions and how this affects the performance and overall role of cryptocurrencies in portfolio management.

5.1 Optimization Methods

Three portfolio optimization strategies are considered. Firstly, mean-variance optimization (MVO) aims to minimize volatility, maximize returns, or to find the optimal balance between the two. The objective function for a minimum-variance portfolio:

$$(7) \min_w w^T \Omega w, \sum_{i=1}^N w_i = 1 \text{ and } w_i \geq 0 \forall i$$

Where w is the vector of portfolio weights, Ω is the covariance matrix of asset returns, and N is the number of assets. The formula for optimal minimum-variance portfolio weights:

$$(8) w_{min-var} = \frac{\Omega^{-1} \mathbf{1}}{\mathbf{1}^T \Omega^{-1} \mathbf{1}}$$

Where $\mathbf{1}$ is a vector of ones. The objective function for a maximum-return portfolio:

$$(9) \max_w w^T \mu, \quad \sum_{i=1}^N w_i = 1 \text{ and } w_i \geq 0 \forall i$$

Where μ It is the return vector of the assets. The formula for the Sharpe Ratio is as follows:

$$(10) \quad \text{Sharpe Ratio} = \frac{w^T \mu - r_f}{\sqrt{w^T \Omega w}}$$

Where r_f is the risk-free rate. The objective function for the maximum Sharpe Ratio (or maximum risk-adjusted return) portfolio:

$$(11) \quad \max_w \frac{w^T \mu - r_f}{\sqrt{w^T \Omega w}}, \sum_{i=1}^N w_i = 1 \text{ and } w_i \geq 0 \forall i$$

The optimal weights for the maximum Sharpe Ratio portfolio are calculated as follows:

$$(12) \quad w_{max-sharpe} = \frac{\Omega^{-1}(\mu - r_f \mathbf{1})}{\mathbf{1}^T \Omega^{-1}(\mu - r_f \mathbf{1})}, \sum_{i=1}^N w_i = 1 \text{ and } w_i \geq 0 \forall i$$

If we deal with normally distributed returns, mean-variance optimization would be sufficient. However, this is not the case, as higher portfolio moments indicate that asset returns do not follow a Gaussian distribution. These deviations from normality justify the inclusion of more advanced optimization methods. Considering higher-order risks, the Conditional Value-at-Risk (CVaR) optimization is introduced. Although it does not directly reflect skewness and kurtosis, it focuses on downside risk (or the left tail), capturing the negative skewness and higher kurtosis from the left tail of the distribution. The formula for CVaR at a confidence level β is as follows:

$$(13) \quad CVaR_{\beta}(w) = \frac{1}{1-\beta} \int_{L(w,r) \geq \alpha(w)} L(w,r) p(r) dr$$

Where w represents the vector of portfolio weights, r the vector of portfolio returns, $p(r)$ the probability distribution, $L(w,r)$ the loss of the portfolio ($-w^T r$), α is the Value-at-Risk with confidence β . The optimization problem for CVaR:

$$(14) \quad \underset{w, \alpha}{\text{minimise}} \quad \alpha + \frac{1}{1-\beta} \frac{1}{T} \sum_{i=1}^T u_i$$

subject to $u_i \geq 0, u_i \geq -w^T r_i - \alpha, w_i \geq 0 \forall i$

Where u_i represents the VaR α exceeding losses, T is the number of return scenarios.

Lastly, as Conditional-Value-at-Risk does not account for the positive skewness and higher kurtosis in the right-side tail, an Omega ratio-based (OR) optimization is employed.

Omega ratio evaluates the entire return distribution, acknowledging the effects of all portfolio returns' distribution moments. The Omega ratio formula is defined as follows:

$$(15) \quad \Omega_{\tau} = \frac{\int_{\tau}^{+\infty} [1-F(r)]dr}{\int_{-\infty}^{\tau} F(r)dr} = \frac{w^T \mu - \tau}{\sum_j [\tau - w^T r_j]^+ p_j}$$

Where τ is the threshold return, μ is the portfolio return vector, w is the portfolio weight vector, r_j which are the returns in different scenarios, p_j represents the probability of r_j returns. The optimization problem can be described as follows:

$$(16) \quad \underset{w}{\text{maximise}} \quad \frac{w^T \bar{r} - \tau}{\sum_j [\tau - w^T r_j]^+ + p_j}$$

subject to $\sum w_i = 1, \underline{w} \leq w \leq \bar{w}, w_i \geq 0 \forall i$

Where $w^T \bar{r} - \tau$ are the returns beyond the threshold τ , $[\tau - w^T r_j]^+ p_j$ represents the p_j weighted excess losses below τ . In this case, the threshold is based on the risk-free rate.

The final result comprises five distinct strategy portfolios: minimum variance (Min-Var), maximum return (Max-Ret), maximum Sharpe ratio (Max-Sharpe), minimum CVaR (Min-CVaR), and maximum Omega ratio (Max-OR). Short-selling constraint is applied to all strategies, ensuring that portfolio weights remain non-negative. The optimizations are performed in two stages: with and without cryptocurrencies, to compare the impact of cryptocurrencies on the overall diversification and performance. Additionally, out-of-sample returns are used to assess the portfolio performance.

$$(17) \quad \text{Portfolio return on date } t = \sum_i \text{Weight}_{t-1,i} \times \text{Asset return}_{t,i}$$

5.2 Performance Evaluation

Two types of measures are used explicitly for assessing portfolio outcomes: performance and diversification. The simplest way to evaluate the portfolio's performance is to analyze its cumulative log returns. While it provides a high-level picture of the performance, it does not capture the risk-adjusted nature of returns. Therefore, performance measures such as the Sharpe ratio, Sortino ratio, and Omega Ratio are calculated from the out-of-sample returns of each portfolio as a rolling monthly metric. Sortino ratio is calculated as follows:

$$(18) \quad \textit{Sortino Ratio} = \frac{R_p - R_f}{\sigma_d}$$

Where R_p is the return of the portfolio, R_f is the risk-free rate, and σ_d is the downside deviation, which captures the standard deviation of negative returns.

The diversification benefits are evaluated using the Diversification Ratio (DR) and the Effective Number of Bets (ENB), which complement each other. Similar to performance metrics, diversification measures are calculated from out-of-sample returns using a rolling monthly metric. The formula for diversification ratio:

$$(19) \quad \textit{Diversification Ratio} = \frac{w^t \sigma}{\sqrt{w^t \Sigma w}}$$

Where $w^t \sigma$ is the weighted sum of individual asset volatilities, σ is the vector of individual asset volatilities, w^t is the transpose of the portfolio weight vector, and $\sqrt{w^t \Sigma w}$ is the portfolio standard deviation. It represents how efficiently a portfolio reduces risk through diversification, with higher values indicating greater diversification.

The Efficient Number of Bets is calculated in the following way:

$$(20) \quad \textit{Effective Number of Bets} = \exp \left(- \sum_{i=1}^N p_i \ln p_i \right)$$

Where p_i is the proportion of total portfolio risk contributed by the asset i , derived from the normalized risk contributions such that $\sum p_i = 1$. It represents the effective number of equally weighted assets that would produce the same level of diversification, with higher values indicating more balanced risk distribution in the portfolio.

The paired sample t-test is conducted for each performance and diversification metric to evaluate the statistical significance of the differences between portfolios with and without the cryptocurrencies. It analyzes whether the differences between the portfolios are significantly different from zero. This study evaluated all tests at a 5 % significance level.

5.3 Portfolio constraints

The optimization process in this study starts with the basic constraints, focusing on long-only portfolios that are fully invested. These constraints are applied to unconstrained optimization scenarios. Additionally, constrained optimization scenarios are implemented. The goal of constrained optimization is to address the issues in unconstrained optimization, which often result in unrealistic, concentrated allocations and overweighting specific assets during certain periods (see Wu and Pandey, 2014). Further, as the limited one-year window of historical data is used, the estimation risk is highlighted even more. There are various possibilities for weight constraints, including the classical constraints that define fixed upper and lower bounds for weights. However, they introduce some problems for the optimization. As strict bounds are implemented, the optimization problem changes since the weight bounds must be satisfied in addition to the optimization objective, resulting in weights that are farther from the accurate optimal weights. This justifies the use of more relaxed constraints. By constraining weights with flexible weight constraints, the goal is to obtain more realistic and less concentrated optimized portfolios, while maintaining focus on the actual optimization objective. To

overcome these limitations, Global Variance-Based Constraints (GVBC) proposed by Levy and Levy (2014) are employed:

$$(21) \quad \sum_{i=1}^n (x_i - \frac{1}{N})^2 \frac{\sigma_i}{\bar{\sigma}} \leq \alpha$$

Where x_i represents the optimized weight of an asset, $\frac{1}{N}$ is the weight of an asset based on a naïve or equally-weighted portfolio, $(x_i - \frac{1}{N})$ is the deviation of the optimized weight from the naïve weight, σ_i is the standard deviation of an asset, $\bar{\sigma}$ is the mean of the assets' standard deviations, $\frac{\sigma_i}{\bar{\sigma}}$ scales each asset's deviation from naïve weight by its relative volatility, and α is the global threshold that the sum of the assets' scaled deviations cannot exceed. The larger the α is, the more flexibility there is to deviate from equal weights.

GVBC is a flexible and dynamic weight constraint that applies to the portfolio as a whole, rather than constraining individual assets. It penalizes deviations from equal-weighting based on relative volatility, leading to higher-volatility assets contributing more heavily to the total penalty. In practice, the optimization process seeks an optimal allocation solution that satisfies the GVBC while maximizing the objective. This allows the optimizer more flexibility to adjust the weights within the imposed constraint. For example, hard individual asset constraints limit the optimizer's flexibility in weight allocation, which can potentially lead to suboptimal solutions. In contrast, GVBC allows the optimizer to explore a broader range of possible allocations to find the optimal portfolio. An asset can have a significant weight if it is optimal, and the overall portfolio stays within the GVBC threshold. (Levy & Levy, 2024, pp. 372-376).

In this study, a relatively large α is selected for the GVBC to allow the optimizer greater flexibility in finding the optimal weights, while still avoiding the concentration issues that can arise from unconstrained optimizations. The larger α ensures that the optimization is primarily influenced by the objective function rather than the weight restrictions. Consequently, α between 0.1 and 0.5 are tested, and 0.2 offers the best balance between

satisfying the objective function and avoiding overly concentrated portfolios. This is in line with Levy and Levy's (2014) findings that strict constraints are not necessary when dealing with already diversified assets, such as the indices.

6 Empirical Results

In this chapter, the study's empirical results are presented. First, the correlations between the assets are illustrated in an overall and time-varying manner. Later, the portfolio compositions are introduced unconstrained and under Global Variance-Based Constraint (GVBC) as dynamic weights. Lastly, the descriptive statistics, risk-adjusted return, and diversification metrics are presented.

6.1 Correlations

Table 2 presents the correlations between assets across the entire period. Cryptocurrencies exhibit high positive correlations with each other, with correlations exceeding 0.80 for each pair, indicating that they tend to move closely together. Furthermore, they exhibit low correlations with the traditional assets. On average, Bitcoin has a correlation of 0.16, Ethereum has a correlation of 0.18, and the altcoin portfolio has a correlation of 0.15 with stocks, bonds, and commodities. The bonds exhibit the lowest correlation with other assets, particularly with commodities (0.03) and the altcoin portfolio (0.09). Excluding the correlations between cryptocurrencies, the highest correlation is observed between stocks and commodities (0.28). Overall, cryptocurrencies offer potential benefits for a portfolio due to their low correlations with traditional assets.

	Bitcoin	Ethereum	Altcoins	Stocks	Bonds	Commodities
Bitcoin	1,00					
Ethereum	0,81	1,00				
Altcoins	0,81	0,86	1,00			
Stocks	0,24	0,27	0,24	1,00		
Bonds	0,12	0,12	0,09	0,19	1,00	
Commodities	0,13	0,14	0,12	0,28	0,03	1,00

Table 2. Correlations

Table 2 provides a high-level overview of correlations. However, more insightful results are presented in Figures 6, 7, and 8, which plot the dynamic correlations between the assets during the period. These figures illustrate the evolution of correlations between traditional assets and cryptocurrencies over the specified period. For most of 2018, correlations ranged approximately between -0.2 and 0.2, until the end of the year when they rose closer to 0.5. The increase in correlations was due to a decline across asset classes, driven by macroeconomic trends and crypto-specific events. Macroeconomic factors included the U.S. Federal Reserve (FED) tightening its monetary policy, for example, by raising interest rates, which negatively impacted risk-on assets such as stocks and cryptocurrencies, while also fueling fears of an economic downturn. (Bankrate, 2025) Additionally, the cryptocurrency market faced multiple shocks, the most notable being the Mt. Gox exchange Bitcoin sell-off (Cointelegraph, 2019), the Initial Coin Offering (ICO) bubble bursting (CoinDesk, 2023), and the U.S. Securities and Exchange Commission (SEC) labeling many of them as unregistered securities (SEC, 2018). Despite these events, cryptocurrencies rebounded in 2019, leading to a decoupling from traditional assets and a decline in correlations.

In early 2020, the correlation between cryptocurrencies spiked significantly upwards to around 0.80 due to the COVID-19 crash. During the crash, investors sold off all risk assets, moving into cash, which resulted in simultaneous declines across asset classes. However, after the crash and Bitcoin's halving in mid-2020 (Bitbo, n.d.), a Bitcoin bull market began. Macroeconomic conditions became favorable due to looser monetary policies, as central banks implemented economic stimulus measures to support markets (Federal Reserve, 2020). Additionally, institutional adoption has been evident, with MicroStrategy, currently one of the largest holders of Bitcoin, purchasing the cryptocurrency as part of its capital allocation strategy (KuCoin, 2025). These factors contributed to declining correlations with the traditional markets. Overall, the correlation trended downward towards the end of 2021, indicating a more substantial decoupling of cryptocurrencies from traditional financial markets.

In late 2021 and early 2022, correlations between assets began to surge again, except for bonds. This period is characterized by increasing inflation, which prompted interest rate hikes, resulting in the selling off of speculative assets and high-risk investments. On the contrary, it led to buying into safe-haven assets, such as bonds, which explains the spike in a high negative correlation with bonds. The decoupling from traditional markets around mid-2022 is related to crypto-specific events. First was the collapse of Terra's UST stablecoin and Luna coin, which led to a market-wide crypto crash while traditional markets remained stable (CoinDesk, 2024). The collapse spread to crypto hedge funds and lenders, such as Celsius Network, Three Arrows Capital, and Voyager Digital, resulting in liquidity issues and halted withdrawals, which further eroded trust in the cryptocurrency market. Finally, Three Arrows Capital, Voyager Digital, and Celsius Network filed for bankruptcy, which intensified the collapse and fear in the crypto market (CGAA, 2024). The traditional markets were less affected by these incidents and rebounded more quickly, whereas cryptocurrencies continued to remain in a downtrend.

In late 2022, correlations increased due to crypto-specific events, the FTX collapse (CGAA, 2024), as well as macroeconomic tensions, interest rate hikes, and recession fears (Forbes, 2022). These factors contributed to the alignment between cryptocurrencies and traditional assets. The early 2023 decoupling occurred because traditional assets continued to struggle in the face of economic uncertainty, as Bitcoin's status shifted from an alternative asset. Additionally, among other factors, the Silicon Valley Bank collapse raised concerns about a potential banking crisis and prompted a need for a hedge against the traditional banking system (Wikipedia, 2023). Cryptocurrencies also received a bullish catalyst from positive news, such as the filing of the BlackRock Bitcoin ETF (BlackRock, 2024). Lastly, the 2023 fall in the correlation can be attributed to AI-related stock rallies, SEC lawsuits against exchanges Binance and Coinbase, and speculation surrounding the approval of Bitcoin ETFs (Reuters, 2023).

These events and factors help to understand the shifts in the correlations between cryptocurrencies and traditional assets. Furthermore, these are closely linked to the

subsequently introduced diversification results. More detailed information about the correlations can be found in the yearly correlation matrices in Appendix 3.

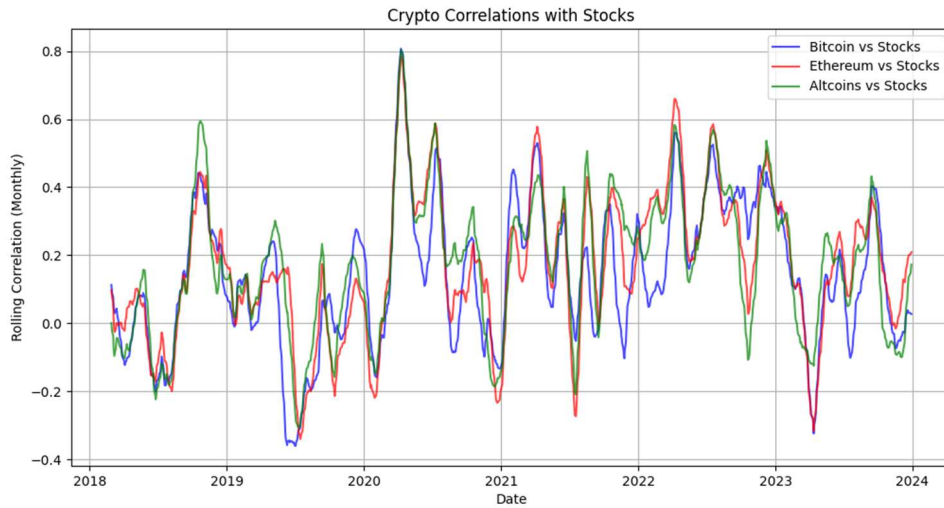


Figure 6. Rolling Monthly Correlation Between Cryptocurrencies and Stocks

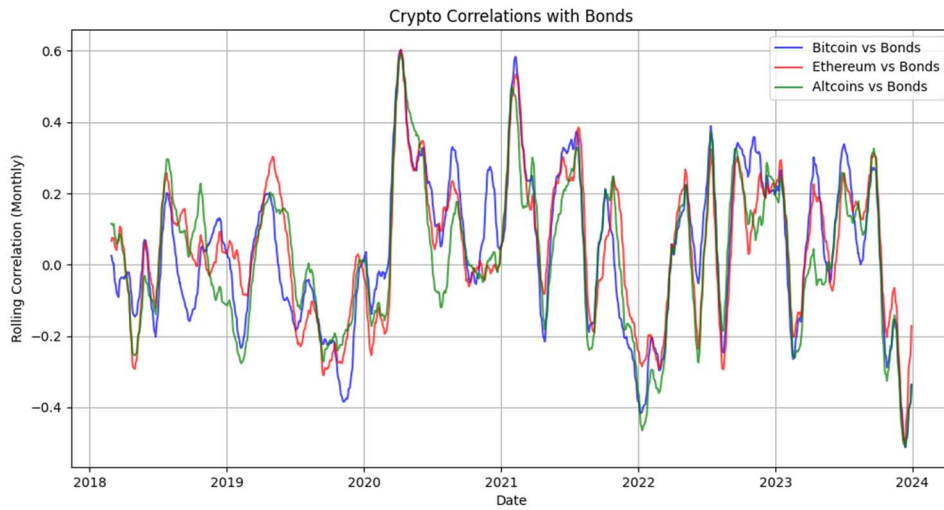


Figure 7. Rolling Monthly Correlation Between Cryptocurrencies and Bonds

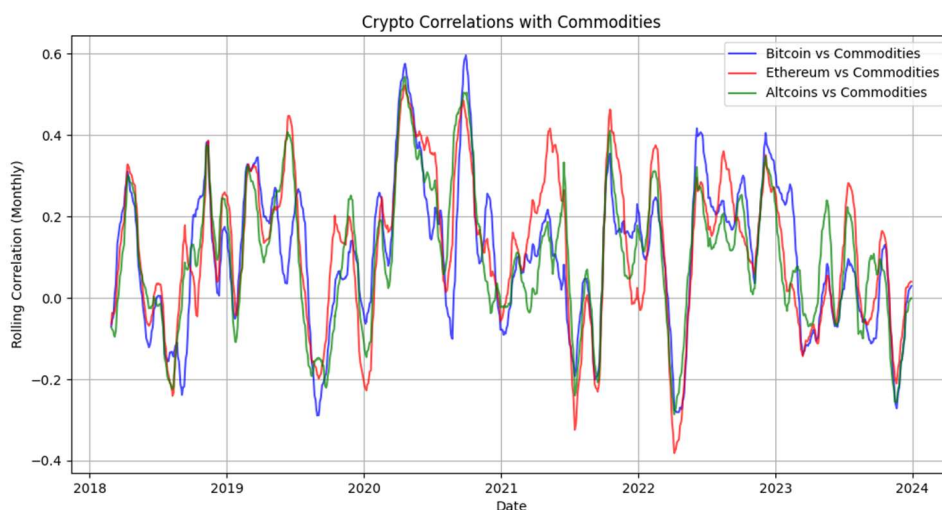


Figure 8. Rolling Monthly Correlation Between Cryptocurrencies and Commodities

6.2 Portfolio Formulation

In this section, the optimized portfolio weights for each strategy are introduced. These optimization weight compositions are introduced with and without the GVBC constraints.

6.2.1 Mean-Variance Optimization

The following figures illustrate the weekly rebalanced weights of three portfolios without GVBC constraints, obtained through mean-variance optimization using the Min-Var, Max-Sharpe, and Max-Ret approaches. The weight compositions of other rebalancing intervals are similar to the weekly interval, except that the higher interval makes them smoother (see Appendix 4). The Min-Var portfolio primarily comprises stocks, bonds, and commodities, with cryptocurrencies accounting for a small portion of the portfolio for most of the period (Figure 9). The weight of cryptocurrencies ranges from approximately 0% to 2.14%. The weight of cryptocurrencies is generally composed of Bitcoin, apart from some short periods, where altcoins and Ethereum hold the maximum weights of 0.39% for Ethereum and 0.20% for Altcoins. Even though cryptocurrencies are highly volatile assets, they have a small weight in the minimum variance portfolio due to the

low correlations with traditional assets. For example, when examining periods of higher cryptocurrency weights (e.g., 2019), the correlations between traditional assets and cryptocurrencies have been low or negative, with notable spikes of significant decoupling. On the contrary, when examining periods of lower or no weight in cryptocurrencies (e.g., 2020), the correlations between the assets have generally been high, with occasional upward spikes. (See Figures 6, 7, and 8). Still, the structure of the portfolio allocation changes with the inclusion of cryptocurrency (see Appendix 5). The inclusion of cryptocurrencies allows the optimizer to shift from a heavy allocation in bonds to a higher allocation in stocks.

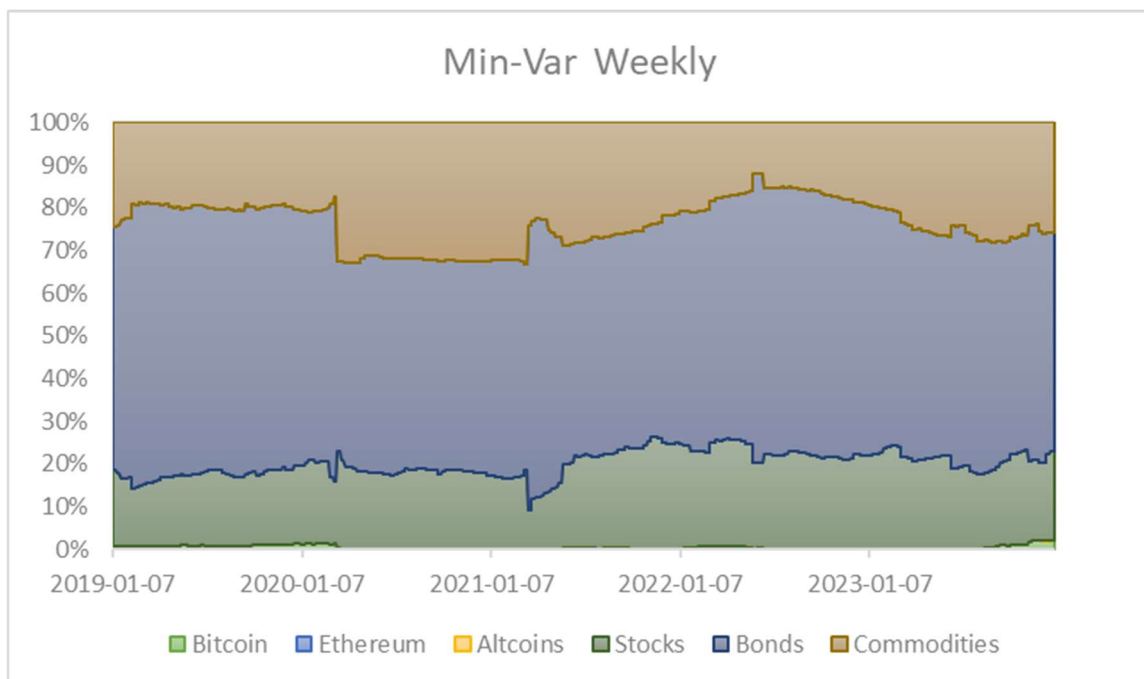


Figure 9. Min-Var Portfolio Weekly Weights

The composition of the Max-Sharpe portfolio is significantly more concentrated than that of the Min-Var portfolio, with periods of nearly all weight in a single asset (Figure 10). For instance, the overall poor performance of assets other than bonds in 2018 (see Figure 5) led to their dominance in the allocation throughout 2019. Additionally, the high performance of commodities in the first half of 2022, combined with the poor performance of other assets, led to the Max-Sharpe portfolio being composed primarily of commodities in the second half of 2022. Furthermore, when cryptocurrencies perform

well, they hold a significant weight in the portfolio. Their high weight reflects the extreme performance of cryptocurrencies that began in early 2020, continued into the second half of 2020, and accelerated at the start of 2021. Afterwards, the volatile cryptocurrency performance in 2021 can be seen as a declining weight in the portfolio, while the less volatile uptrend of commodities and stocks results in a rising allocation to them. Appendix 5 displays the optimized weights when cryptocurrencies are excluded, resulting in a similar portfolio where the weights of cryptocurrencies are redistributed to the next best-performing asset.

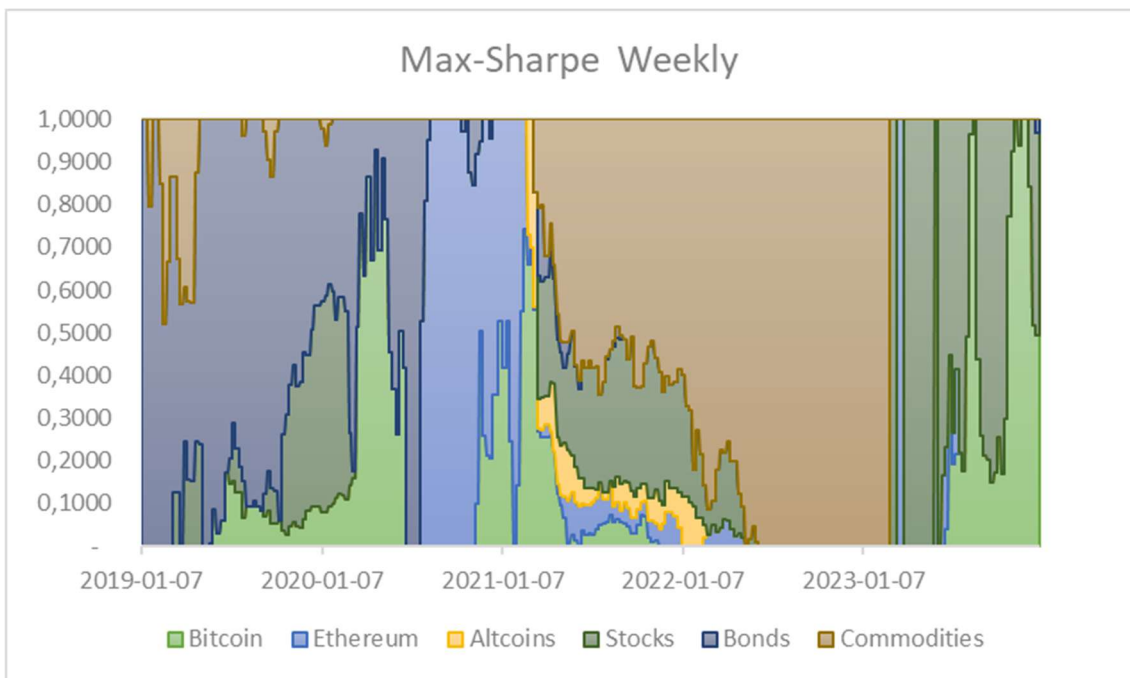


Figure 10. Max-Sharpe Portfolio Weekly Weights

The Max-Ret portfolio is composed of the best-performing assets at each rebalancing period (Figure 11). The high performance of Bitcoin in the first half of 2019 is reflected in the Max-Ret portfolio, which is composed of it through the second half of 2019. Additionally, Ethereum's performance in 2020 is reflected in the Max-Ret portfolio from the second half of 2020 to the start of 2021, which is also evident in the weights of the Max-Sharpe portfolio.

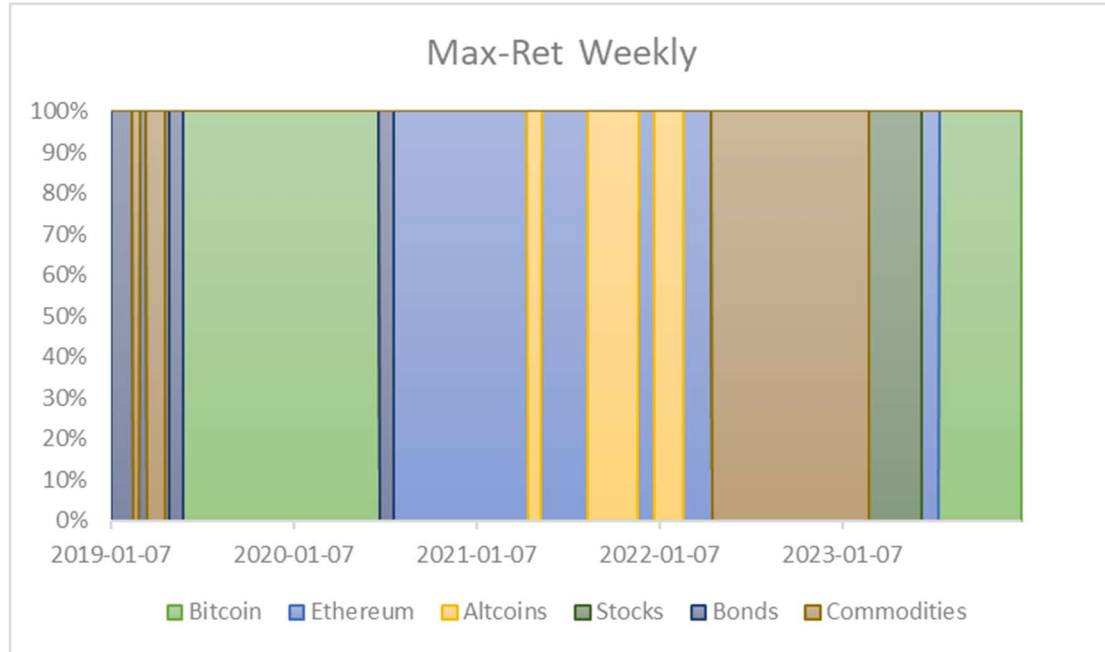


Figure 11. Max-Ret Portfolio Weekly Weights

6.2.2 Conditional Value-at-Risk Optimization

The following figures illustrate the weekly rebalanced weights of portfolios without the GVBC constraint, obtained by the Min-CVaR optimization. The portfolio purely minimizes risk measured by CVaR. The allocation of the Min-CVaR portfolio is predominantly composed of bonds throughout the period, suggesting that they remain the least risky asset in terms of downside risk. Additionally, the portfolio exhibits near-zero exposure to cryptocurrencies. They exhibit high volatility, combined with high kurtosis and negative skewness, resulting in high CVaR values and undesirable assets for the optimization goal. Similarly, stocks and bonds have a low allocation in the portfolio, ranging approximately between 0% and 10%. Even though stocks have high kurtosis and negative skewness, the lower volatility makes them more suitable for the Min-CVaR portfolio, hence the occasional small allocations. Overall, the Min-CVaR optimization prioritizes minimizing downside risk over diversification or return, resulting in highly concentrated weights in bonds and a preference for avoiding riskier asset classes.

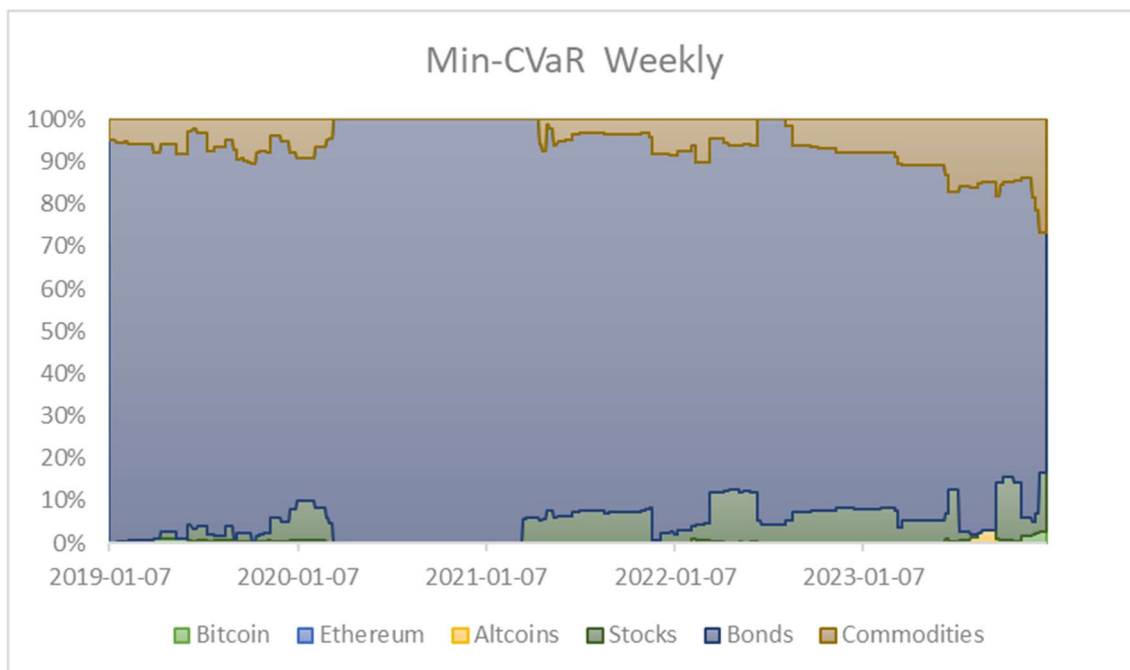


Figure 12. Min-CVaR Portfolio Weekly Weights

6.2.3 Omega Ratio Optimization

Omega ratio maximization tends to select assets that offer significant upside above the risk-free rate threshold, relative to their downside. This can lead to a concentration of the portfolio during specific periods, such as the dominance of bonds from 2019 to mid-2020, and subsequently, a heavy allocation in cryptocurrencies until the second half of 2021 (see Figure 13). In the first two years of the dataset, other assets experienced high volatility and downside, which explains the bond allocations. Furthermore, as cryptocurrencies began to rebound, the portfolio's composition shifted towards them. Again, after mid-2021, as cryptocurrency volatility increased and shortfalls below the risk-free rate became more frequent, the optimizer shifted to commodities and stocks, maintaining this allocation until mid-2023. They provided steadier returns above the threshold. Similarly, after mid-2023, the portfolio began to shift towards cryptocurrencies, particularly Bitcoin, due to the uptrend and limited downside seen over the past year.

The composition of the Max-OR portfolio resembles that of the Max-Sharpe portfolio, as the same trends influence it in asset returns, and because the threshold remains zero or

close to it throughout the period. However, the consideration of partial moments adds sensitivity to tail events, which at times switches the allocation further from the Max-Sharpe. For instance, in 2020, the Max-OR portfolio had more conservative weights in riskier assets and more weight in bonds. The riskier assets had a greater downside in the previous rolling years when returns fell below the threshold, which hurt the Omega Ratio; this is why it weighed more heavily in the steady returns over the threshold provided by bonds.

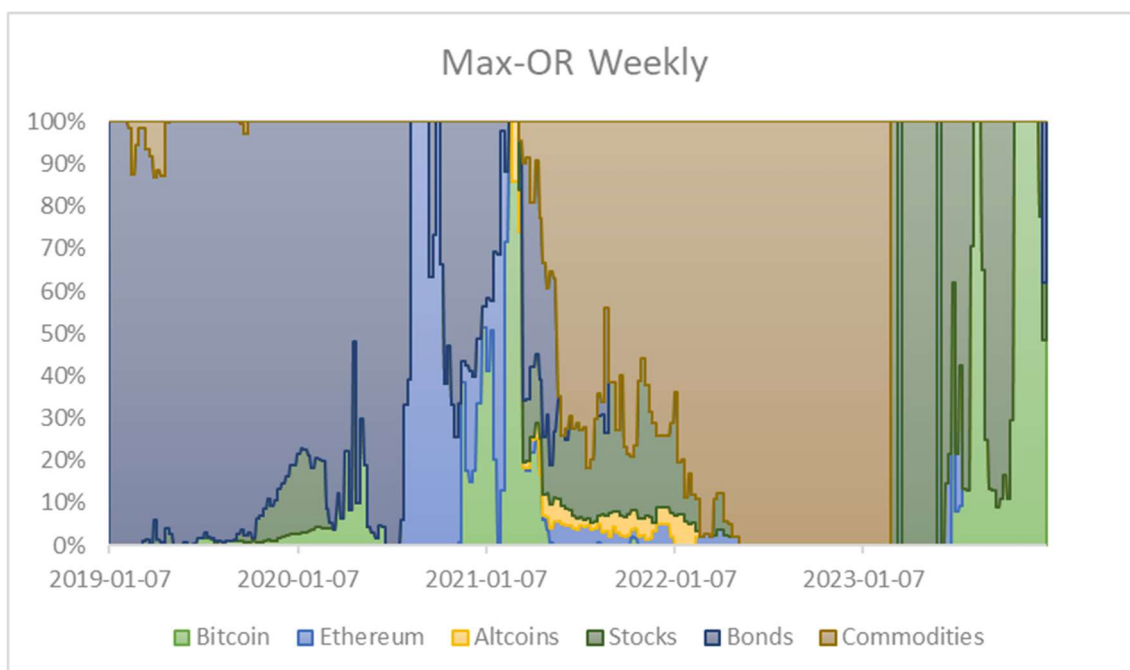


Figure 13. Max-OR Portfolio Weekly Weights

6.2.4 Portfolio Formulation with Global Variance-Based Constraint

Previous unconstrained optimizations are more susceptible to various estimation risks, including reliance on historical data, limitations of the sample size, and the sensitivity of optimization. For instance, small changes in expected returns and covariances derived from the historical data can lead to overweighting or underweighting of assets. Furthermore, a year is a relatively small sample size for each window, resulting in each slight shift in asset dynamics having a greater impact on the optimizations. The following portfolio optimizations with GVBC are implemented to mitigate estimation risk and achieve

more realistic portfolios. It will help provide a practical understanding of the performance and diversification impact of cryptocurrencies.

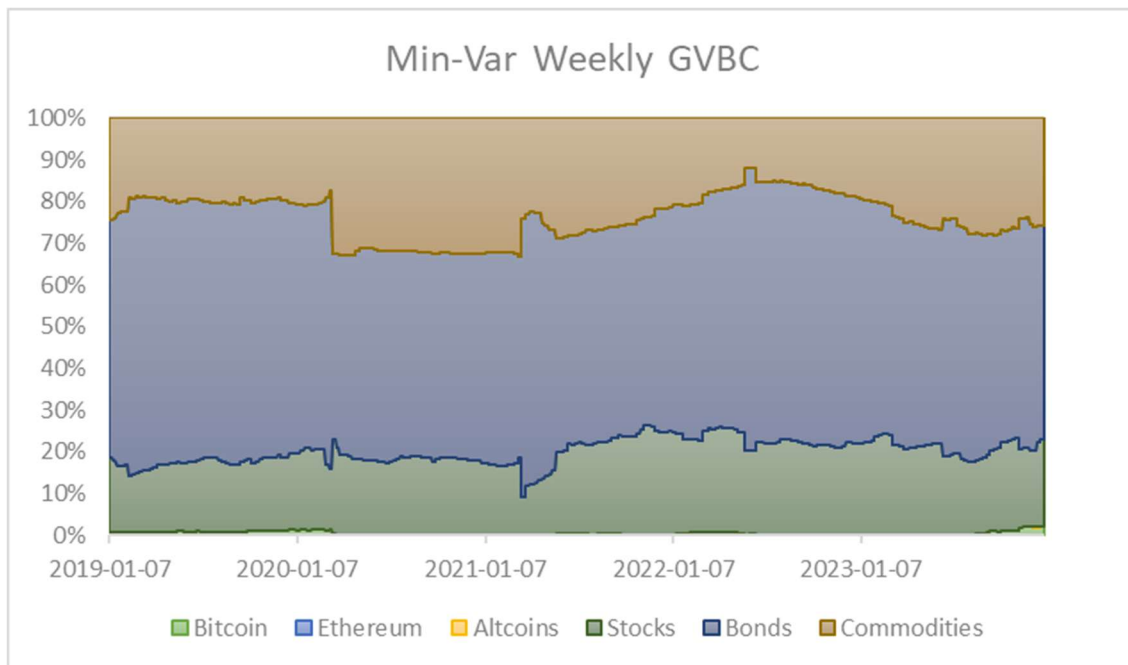


Figure 14. Min-Var Weekly Portfolio Weights with GVBC

The constrained Min-Var portfolio is identical to the unconstrained one. The reason for this is that unconstrained optimization does not result in weights that breach the GVBC, as it naturally diversifies when no single asset dominates in terms of volatility reduction. The Max-Ret portfolio is excluded from the optimization with GVBC as it fully invests in the best-performing asset, and it is not meaningful to constrain it under GVBC. Furthermore, the GVBC affects the allocation of the Max-Sharpe portfolio due to its nature to concentrate on assets with the best Sharpe ratio. At times when a specific asset class is outperforming the others, unconstrained Max-Sharpe optimization is fully invested in that asset class, whereas the constrained one limits the overweighting. It forces diversification into other assets. For example, the constraints significantly influenced the allocation in 2020, limiting the weight on cryptocurrencies. From the second half of 2022, they also limited the weights on commodities, stocks, and cryptocurrencies. Additionally, it mitigates the extreme fluctuations in the portfolio, thereby smoothing out the changes in the allocation.

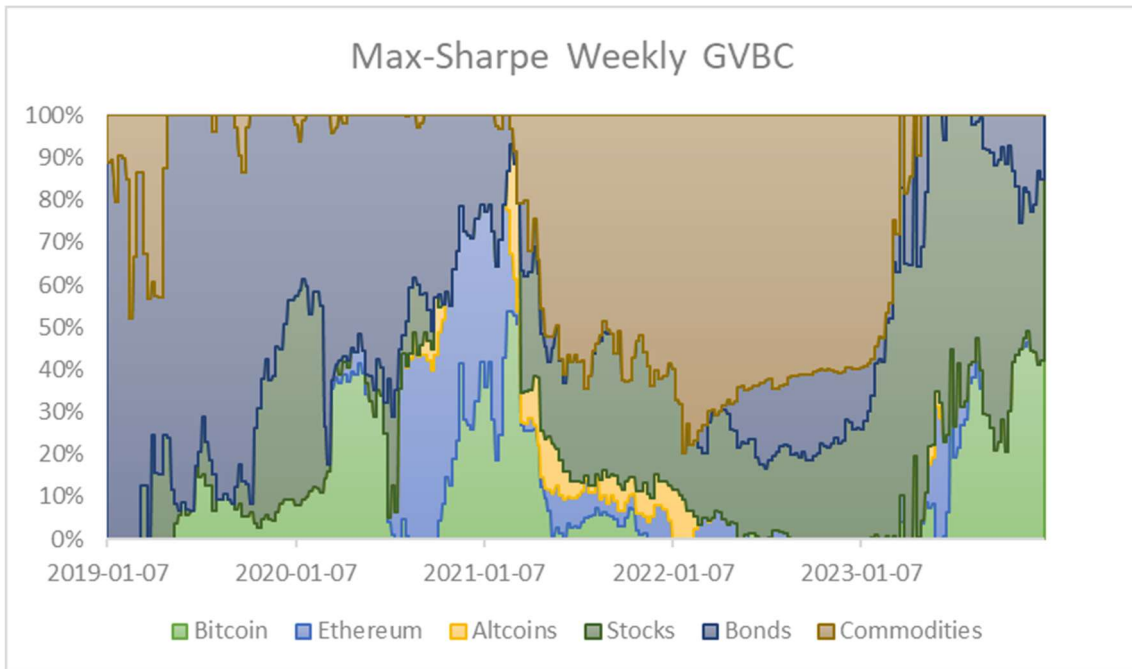


Figure 15. Max-Sharpe Weekly Portfolio Weights with GVBC

The unconstrained Min-CVaR portfolio was highly concentrated on bonds with minimal allocations to other assets. The cryptocurrency allocation is almost zero, except for occasional small weights. Commodities generally have a consistent allocation of approximately 5% to 10%. Stocks have varying weights, ranging from approximately 0% to 10%. With GVBC implementation, the bonds do not dominate as much. Cryptocurrency allocations have increased slightly but remain relatively small; stocks are included more consistently, and the presence of commodities is higher, reaching approximately 30% at times. The GVBC reduces the extreme concentration of the portfolio and increases the diversification. Similarly to Max-Sharpe, the allocations are more consistent and stable, limiting the extreme shifts in the portfolio composition.

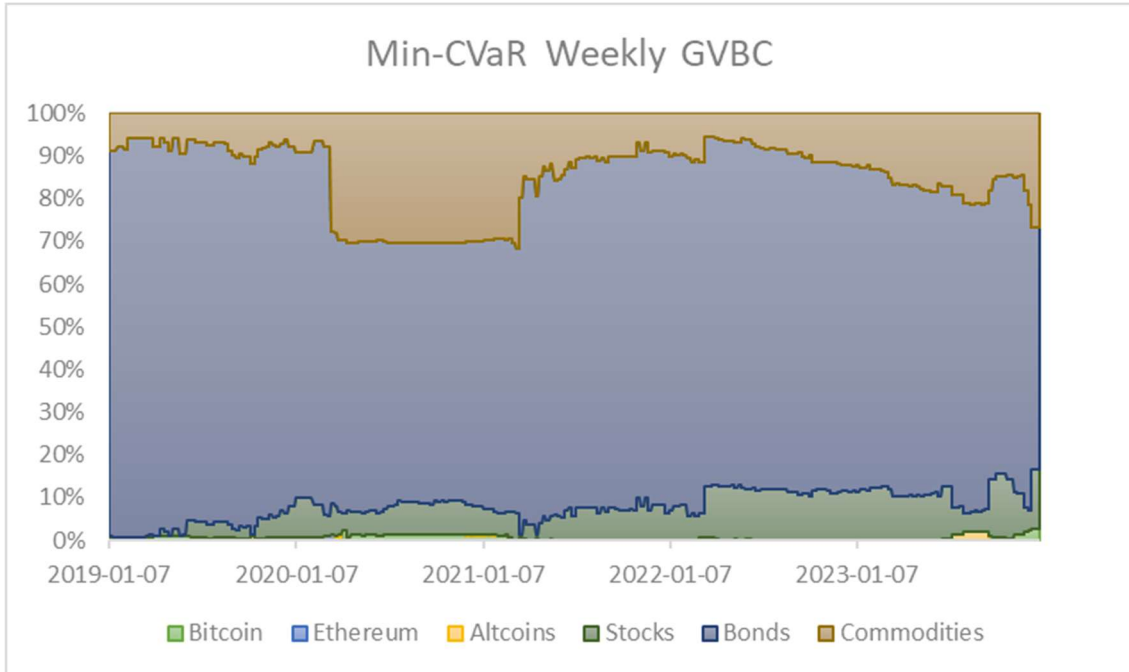


Figure 16. Min-CVaR Weekly Portfolio Weights with GVBC

Unconstrained Omega Ratio optimization is highly concentrated at specific periods, and it experiences extreme shifts in weights. For instance, there are multiple periods when the weight of a particular asset spikes to 100% of the portfolio allocation. At times, the weights change dramatically from one asset to another, holding one asset for a couple of weeks and exchanging it for another. Following the previous GVBC portfolios, the Max-OR with GVBC is more stable and diversified, with smoother weight transitions. Additionally, the allocations in cryptocurrencies are more controlled, and there is more consistent exposure to each asset. The GVBC addresses the Omega Ratio optimization trend of maximizing upside by concentrating on assets that have performed well recently. It mitigates excessive risk exposure by enforcing a more diversified composition and balanced risk-taking approach.

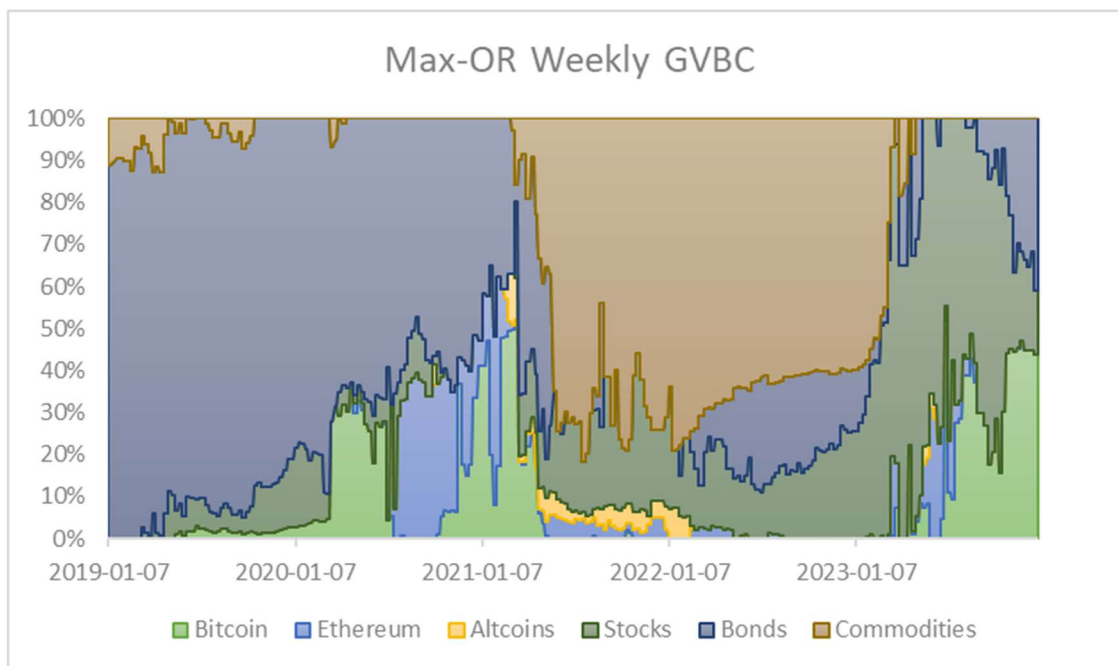


Figure 17. Max-OR Weekly Portfolio Weights with GVBC

In conclusion, across different optimization methodologies, GVBC consistently enhances diversification, reduces extreme allocations, and stabilizes weight transitions. It prevents the optimizer from overreacting to short-term estimation changes by constraining high deviations from equal weighting. Furthermore, the portfolios still maintain the structure of the unconstrained portfolios, as the GVBC optimizations have sufficient room to satisfy the optimization objective effectively.

6.3 Performance

The following section presents the performance implications of incorporating cryptocurrencies into a traditional portfolio, using metrics such as cumulative log return, volatility, Sharpe ratio, Sortino ratio, and Omega ratio, calculated from the out-of-sample returns of each portfolio. The performance metrics are introduced as monthly rolling measures to capture the time-varying dynamics, except for the cumulative log return. The focus is on the differences between portfolios that include cryptocurrencies and those that

exclude them. First, the results from weekly rebalancing are used, which will later be compared to other rebalancing frequencies.

6.3.1 Unconstrained Portfolios

	Max-Sharpe Weekly		Min-Var Weekly		Min-Ret Weekly		Min-CVaR Weekly		Max-OR Weekly	
	With CCs	Without CCs	With CCs	Without CCs	With CCs	Without CCs	With CCs	Without CCs	With CCs	Without CCs
Mean	0,17 %	0,02 %	0,01 %	0,00 %	0,19 %	0,03 %	-0,01 %	-0,01 %	0,10 %	0,03 %
Std Dev	2,43 %	0,81 %	0,47 %	0,36 %	4,47 %	0,91 %	0,36 %	0,36 %	1,75 %	0,83 %
Min	-19,67 %	-4,90 %	-4,63 %	-3,34 %	-45,79 %	-5,62 %	-2,91 %	-2,91 %	-13,49 %	-5,62 %
Max	21,16 %	6,89 %	3,00 %	2,45 %	32,89 %	6,89 %	2,09 %	2,09 %	9,29 %	6,89 %
Skewness	0,64	-0,24	-1,46	-1,25	-0,86	-0,42	-0,68	-0,66	-0,42	-0,28
Kurtosis	17,15	10,35	17,07	15,56	15,77	8,16	10,66	10,57	12,53	10,34
25% Quantile	-0,46 %	-0,22 %	-0,20 %	-0,16 %	-1,14 %	-0,26 %	-0,17 %	-0,17 %	-0,38 %	-0,22 %
Median	0,07 %	0,04 %	0,03 %	0,01 %	0,08 %	0,04 %	0,00 %	0,00 %	0,06 %	0,04 %
75% Quantile	0,69 %	0,29 %	0,25 %	0,17 %	1,57 %	0,35 %	0,17 %	0,17 %	0,52 %	0,30 %
Cumulative Return	213,73 %	30,51 %	11,44 %	-4,82 %	243,43 %	36,53 %	-7,86 %	-8,09 %	121,42 %	32,53 %

Table 3. Portfolios' Descriptive Statistics

Table 3 presents the descriptive statistics of unconstrained portfolios, categorized into two columns: "With CCs" and "Without CCs". Across the strategies, cryptocurrency-inclusive portfolios exhibit higher mean and median results as well as significantly higher cumulative log returns, except for the Min-CVaR portfolio, which has relatively similar statistics for both portfolios. For example, the Max-Sharpe portfolio yields a cumulative return of 213.73% with cryptocurrencies, while it yields 30.51% without them, and the Min-Var portfolio yields 11.44% with and -4.82% without. This indicates strong long-term growth potential from the inclusion of cryptocurrency. On the other hand, the volatility of these portfolios is also notably higher; for example, the Max-Sharpe at 2.43% versus 0.81%, or the Max-OR at 1.75% versus 0.83%. The skewness generally turns more negative with cryptocurrencies, except for the Max-Sharpe portfolio, where it becomes positive (0.64 with and -0.24 without). This suggests a right-skewed distribution for cryptocurrency-inclusive Max-Sharpe and a left-skewed distribution for others. The kurtosis increases with cryptocurrencies, indicating fatter tails. In conclusion, the descriptive

statistics confirm an increase in return potential; however, this comes with increased risk in terms of volatility and tail risks, which is particularly evident from the higher kurtosis.

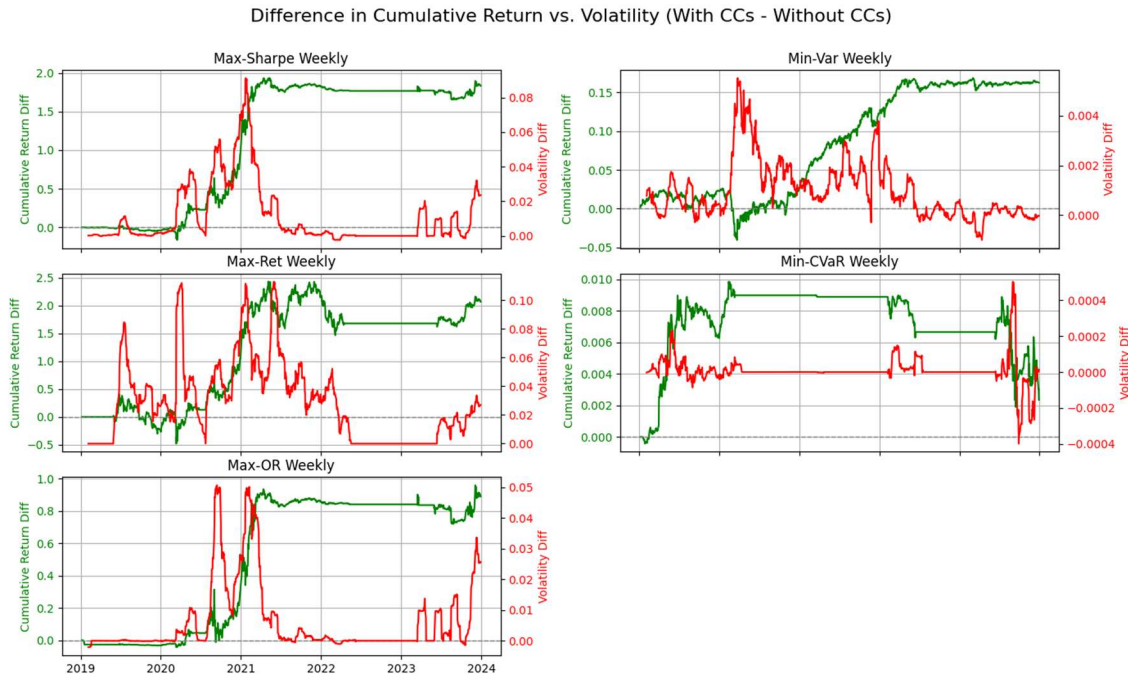


Figure 18. Difference in Cumulative Return and Volatility with and without Cryptocurrencies

Figure 18 illustrates the difference in cumulative log returns (green line) between portfolios that include and exclude cryptocurrencies, paired with the monthly rolling volatility (red line), which complements the observations made in Table 3. In general, cryptocurrency-inclusive portfolios exhibit significantly higher long-term cumulative log returns, accompanied by increased short-term volatility. The periods with greater allocation in cryptocurrencies are visible in the charts, characterized by a soaring difference in returns as well as a spiking difference in volatility. Otherwise, the charts are flat-lined, indicating no differences in allocations between the two portfolios, as both exclude cryptocurrencies. For instance, the Max-Sharpe portfolios from the first half of 2022 to early 2023 were both dominated by commodities, resulting in a flat line in Figure 18 (see Figure 10 and Appendix 5, Figure 43).

The strategies most affected by the addition of cryptocurrencies are Max-Sharpe, Max-Ret, and Max-OR, with the most considerable differences in returns and volatility. These

strategies exhibit the most significant disparity in allocation to traditional portfolios due to their substantial allocation to cryptocurrencies. On the other hand, the magnitude of differences between the two portfolios is the lowest for the risk-minimizing strategies, as they have only small allocations in cryptocurrencies, with Min-CVaR being closest to the traditional portfolio.

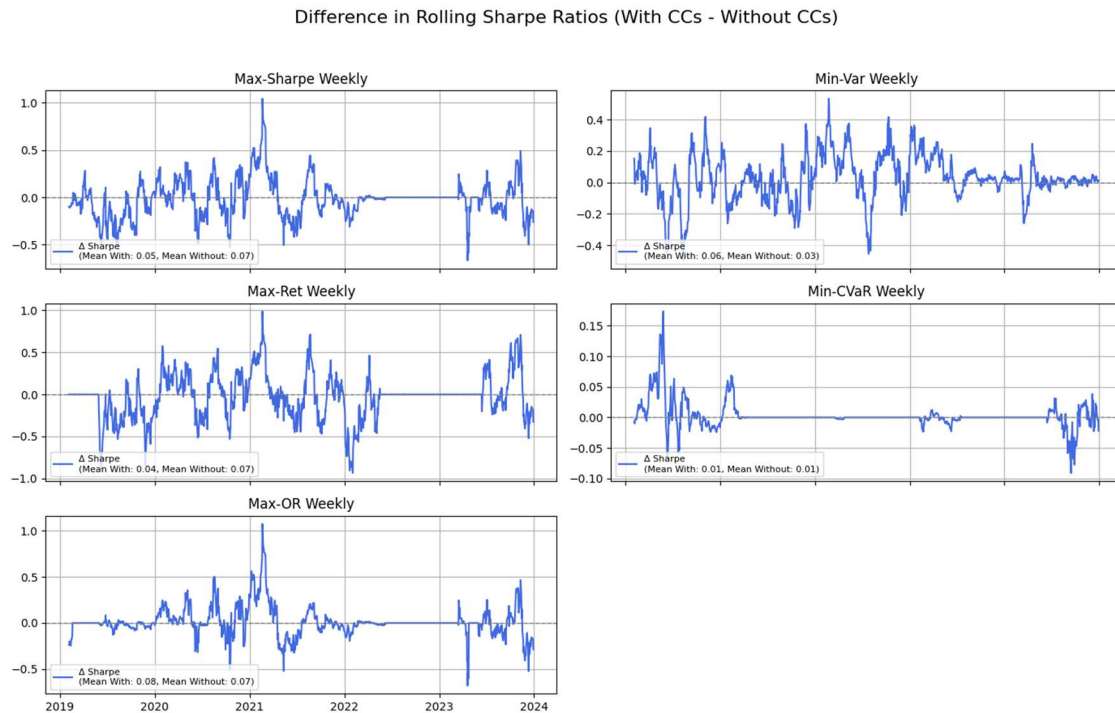


Figure 19. Difference in Rolling Sharpe Ratio with and without Cryptocurrencies

Figure 19 illustrates the rolling Sharpe ratio differences between the portfolios with and without cryptocurrencies over the specified period. A line above zero indicates that portfolios with cryptocurrencies experienced better risk-adjusted returns, as measured by the Sharpe ratio. A line below zero indicates worse risk-adjusted returns. Some strategies, such as Min-Var and Max-OR, consistently benefit from cryptocurrency exposure, which is also marked by higher means (0.06 for Min-Var and 0.08 for Max-OR) compared to those without cryptocurrency exposure (0.03 for Min-Var and 0.07 for Max-OR) (see the legends). These two strategies are the only ones with a higher mean in the rolling Sharpe ratio when considering cryptocurrency exposure compared to no exposure. Other strategies are more timing-sensitive, such as the Max-Sharpe, which exhibits more frequent

and greater fluctuations between positive and negative Sharpe differences. Furthermore, Min-CVaR shows nearly no difference between the portfolios, showcasing the minimal cryptocurrency exposure. These findings highlight that the impact of including cryptocurrencies on Sharpe ratios is highly dependent on the underlying portfolio strategy.

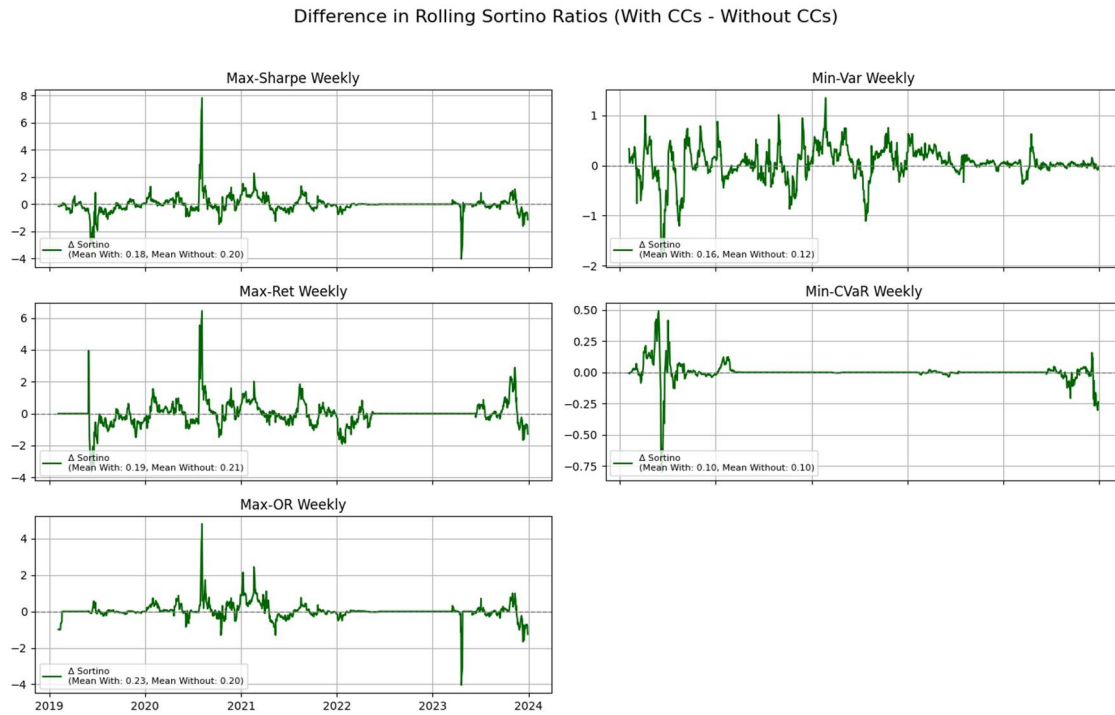


Figure 20. Difference in Rolling Sortino Ratio with and without Cryptocurrencies

Similarly, Figure 20 presents the difference between cryptocurrency-inclusive and traditional portfolios, but using the rolling Sortino ratio. Overall, the fluctuations in downside-risk-adjusted return indicate that performance is not constant over time. The Max-OR and Min-Var strategies yield the most benefits from cryptocurrencies, aligning with the findings based on the Sharpe ratio. They display consistent positive differences and higher average Sortino ratios when cryptocurrencies are included. Although the other strategies do not consistently outperform with cryptocurrency exposure, the differences between the portfolios are relatively minor compared to the Sharpe ratio when only downside risk is considered. This suggests that the better performance of traditional portfolios under the Sharpe ratio is partly due to penalizing growth.

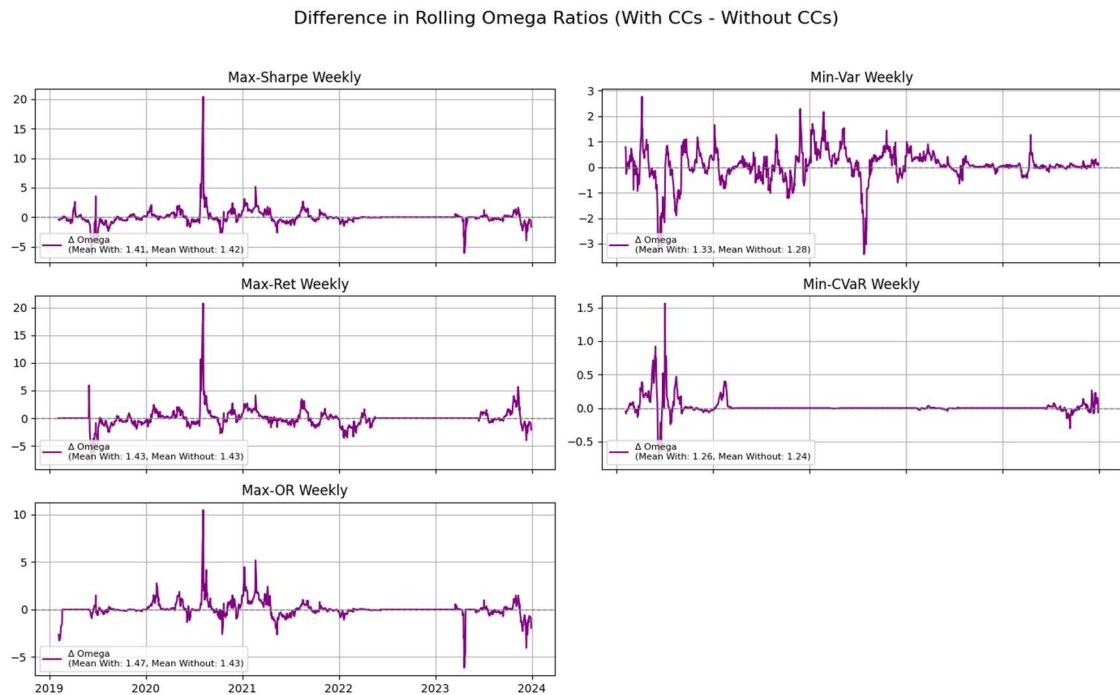


Figure 21. Difference in Rolling Omega Ratio with and without Cryptocurrencies

Similar to previous figures, Figure 21 displays the differences between portfolios with and without cryptocurrencies across the strategies, measured by the rolling Omega ratio. The results largely align with the earlier findings: return-focused strategies with greater exposure to cryptocurrencies exhibit the most substantial fluctuation in the Omega ratio. During specific periods, the difference is positive, while in others, cryptocurrencies harm the ratio. These fluctuations are generally attributed to significant allocation to cryptocurrencies. Notably, the Omega ratio reveals exceptions for Max-Ret and Min-CVaR strategies. The average difference for Max-Ret is comparable to that of the traditional portfolio, and for Min-CVaR, the average performance is superior with cryptocurrencies. Overall, the spikes observed across all three performance ratio figures suggest that although performance can be substantially increased by cryptocurrencies in short windows (e.g., early 2021), these benefits are not consistent over time.

	Cumulative Return Δ	Volatility Δ Sum	Sharpe Δ Sum	Sortino Δ Sum	Omega Δ Sum
Max-Sharpe Weekly	1,832	13,544*	-18,718*	-20,704	-14,667
Min-Var Weekly	0,163	1,195*	34,391*	50,251*	60,850*
Max-Ret Weekly	2,069	35,219*	-30,081*	-15,144	-9,248
Min-CVaR Weekly	0,002	0,008*	3,678*	3,341	22,662*
Max-OR Weekly	0,889	7,669*	14,319*	36,356*	55,758

Table 4. The Cumulative Differences with Significance Levels

Table 4 presents the cumulative differences in performance metrics across all five portfolio strategies, along with significance levels indicated by asterisks for metrics that differ at the 5% level (excluding cumulative log returns). It enables the comparison of the total magnitude of differences in return, risk, and risk-adjusted performance resulting from the inclusion of cryptocurrency, as well as the statistical significance of these differences. Cryptocurrencies consistently increase the cumulative log returns for each strategy, with Max-Sharpe and Max-Ret strategies exhibiting the most considerable improvements, at 1.83 and 2.07, respectively. The most minor improvement is observed in Min-CVaR, with a difference of 0.002, aligning with its relatively low allocation to cryptocurrencies. The higher returns come with higher risk, as evidenced by the statistically significant positive volatility differences across all strategies. Furthermore, the magnitude of the cumulative volatility difference generally reflects the returns, the largest being for Max-Sharpe and Max-Ret, and the lowest for Min-CVaR.

The risk-adjusted performance measures present more nuanced results. Min-Var, Min-CVaR, and Max-OR strategies show statistically significant improvements in the Sharpe ratio with cryptocurrencies. On the other hand, Max-Sharpe and Max-Ret experience a statistically significant reduction in the Sharpe ratio, indicating that volatility increased disproportionately to returns. The same trend continues, as measured by Sortino and Omega ratios; both Max-Sharpe and Max-Ret suffer from a decline in the ratios, although the differences are not statistically significant. The Sortino and Omega ratios further support the findings for the Min-Var strategy, which are significantly positive. Additionally,

Min-CVaR and Max-OR both display positive differences, but their significance in the results is mixed. For Min-CVaR, the Omega ratio is statistically significant, and for Max-OR, the Sortino ratio suggests moderate but less consistent performance gains from cryptocurrency inclusion. These findings emphasize that the benefits of cryptocurrencies are highly dependent on the portfolio objectives. While growth-seeking strategies may yield higher total returns, they do not consistently outperform traditional portfolios in a risk-adjusted manner. In contrast, more moderate allocations in cryptocurrencies often yield better risk-adjusted returns.

6.3.2 Global Variance-Based Constraint Portfolios

This section examines the impact of incorporating cryptocurrencies into a traditional portfolio under the Global Variance-Based Constraint (GVBC). Although the GVBC influences allocation behavior, the performance metrics generally follow similar patterns to those observed in unconstrained portfolios. The Min-Var portfolio is identical to the unconstrained one for the cryptocurrency-inclusive portfolio, but without the cryptocurrencies, there are slight differences in the allocations (see Appendix 5). Overall, there are still some key differences in the findings as a consequence of GVBC.

	Max-Sharpe Weekly		Min-Var Weekly		Min-CVaR Weekly		Max-OR Weekly	
	With CCs	Without CCs	With CCs	Without CCs	With CCs	Without CCs	With CCs	Without CCs
Mean	0,13 %	0,03 %	0,01 %	0,00 %	0,00 %	0,00 %	0,09 %	0,02 %
Std Dev	1,72 %	0,67 %	0,47 %	0,37 %	0,39 %	0,37 %	1,27 %	0,67 %
Min	-13,66 %	-4,45 %	-4,63 %	-3,47 %	-3,68 %	-3,37 %	-12,58 %	-4,45 %
Max	16,70 %	4,54 %	3,00 %	2,48 %	2,79 %	2,47 %	8,49 %	4,51 %
Skewness	0,72	-0,82	-1,46	-1,35	-1,16	-1,25	-0,45	-0,84
Kurtosis	20,53	8,06	17,07	15,95	16,37	15,04	14,84	7,91
25% Quantile	-0,39 %	-0,21 %	-0,20 %	-0,17 %	-0,18 %	-0,18 %	-0,33 %	-0,21 %
Median	0,09 %	0,05 %	0,03 %	0,02 %	0,02 %	0,02 %	0,06 %	0,05 %
75% Quantile	0,60 %	0,31 %	0,25 %	0,18 %	0,19 %	0,19 %	0,47 %	0,30 %
Cumulative Return	159,27 %	32,22 %	11,44 %	-0,59 %	3,85 %	-0,01 %	116,82 %	31,17 %

Table 5. Portfolios' Descriptive Statistics with GVBC

Table 5 shows the descriptive statistics of the portfolios optimized under GVBC. Compared to the unconstrained portfolios, the mean returns are slightly reduced for Max-Sharpe (0.13% vs. 0.17%) and Max-OR (0.09% vs. 0.10%) with cryptocurrencies. For Min-Var, the mean return remains almost identical. For Max-Sharpe and Max-OR without cryptocurrencies, as well as Min-CVaR, the returns are slightly higher. In general, the volatilities are lower or nearly similar, indicating the effectiveness of GVBC in limiting portfolio volatility. For instance, the Max-Sharpe portfolio's volatility decreases from 2.43% to 2.12% with cryptocurrencies. The impact on skewness varies across strategies, e.g., for Max-Sharpe with cryptocurrencies, it increases, but without it decreases. The kurtosis generally increases for the cryptocurrency-inclusive portfolios and decreases for the traditional portfolios.

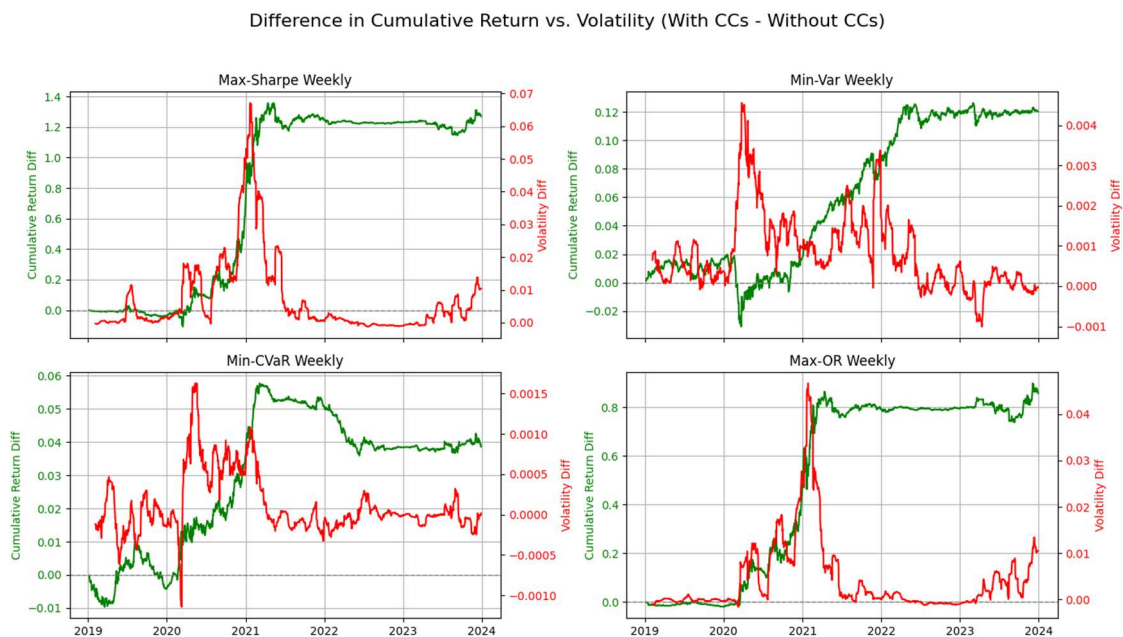


Figure 22. Difference in Cumulative Return and Volatility with and without Cryptocurrencies (GVBC)

Figure 22 presents the cumulative log return and volatility differences between portfolios with and without cryptocurrencies under the GVBC. Compared to the unconstrained case in Figure 18, the cumulative log return differences experience a reduction, especially for the Max-Sharpe portfolio. Additionally, the volatility differences appear to be lower and smoother, thereby limiting the number of extreme spikes. These observations

suggest that GVBC contributes to more stable portfolio behavior and reduces the gap between traditional and cryptocurrency-inclusive portfolios, except for Min-CVaR. While Figure 22 confirms that GVBC reduces both return and risk differences, it is not visually clear whether these effects are due to reduced exposure in the cryptocurrency-inclusive portfolios, increased risk and return in traditional portfolios, or a combination of both. Considering the portfolio allocations as well, it is clear that the restrictions on cryptocurrency allocations play a significant role in these changes.

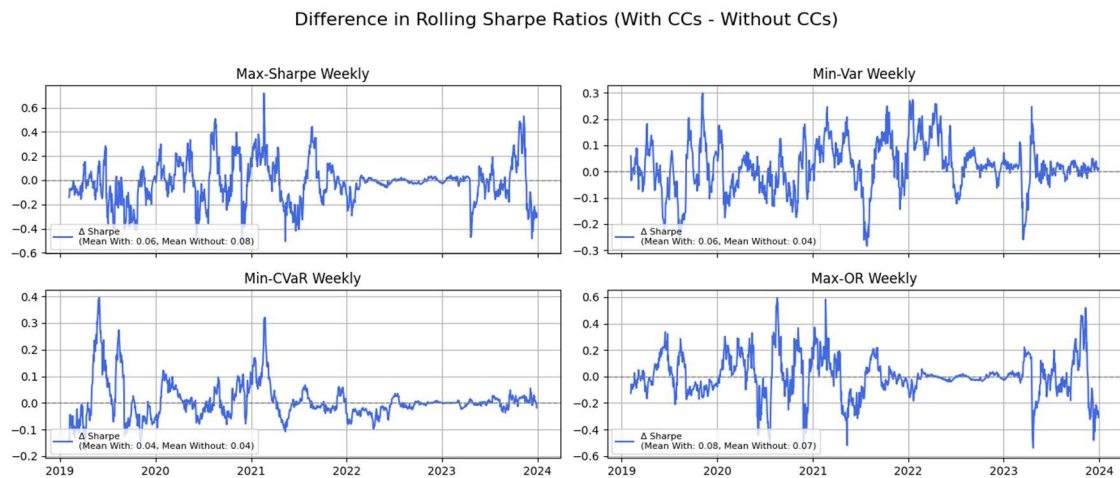


Figure 23. Difference in Rolling Sharpe Ratio with and without Cryptocurrencies (GVBC)

Similar to Figure 19, Figure 23 illustrates the rolling Sharpe ratio differences for the portfolios under GVBC. Across nearly all strategies, the rolling Sharpe ratio oscillates in narrower bands compared to the differences in the unconstrained portfolios. For instance, the Max-Sharpe strategy exhibits differences ranging approximately from -0.4 to 0.6, which corresponds to a range of around -0.5 to 1.0 for the unconstrained counterpart. The only strategy that sees an increase in the differentials is Min-CVaR, as it is forced to move from concentrating in bonds to more volatile assets. Overall, GVBC promotes more stable and lower differences between traditional and cryptocurrency-inclusive portfolios.

Figures 24 and 25 present the differences in rolling Sortino and Omega ratios, respectively, under the GVBC. These figures exhibit similar trends to those in Figure 23, indicating more stable and narrower differences between the portfolios. However, there is one

notable exception in Max-Sharpe and Max-OR strategies. Both exhibit a considerable negative divergence from the traditional portfolio at the end of the examined period, which is not present in the unconstrained portfolios. Furthermore, the mean of rolling Omega ratios for Max-OR becomes equal regardless of whether cryptocurrencies are included under GVBC. In contrast, the unconstrained portfolios had shown a clear advantage for the cryptocurrency-inclusive portfolio.

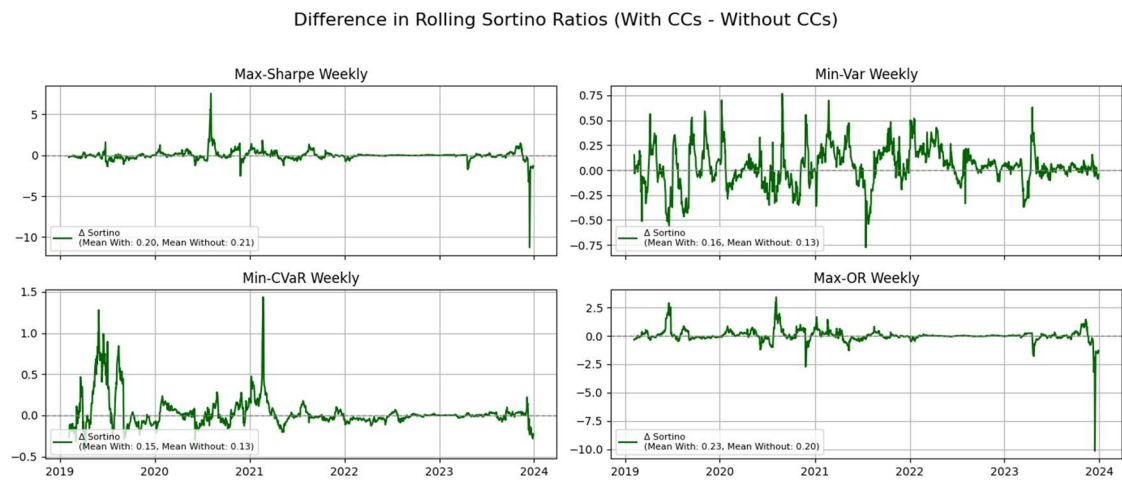


Figure 24. Difference in Rolling Sortino Ratio with and without Cryptocurrencies (GVBC)

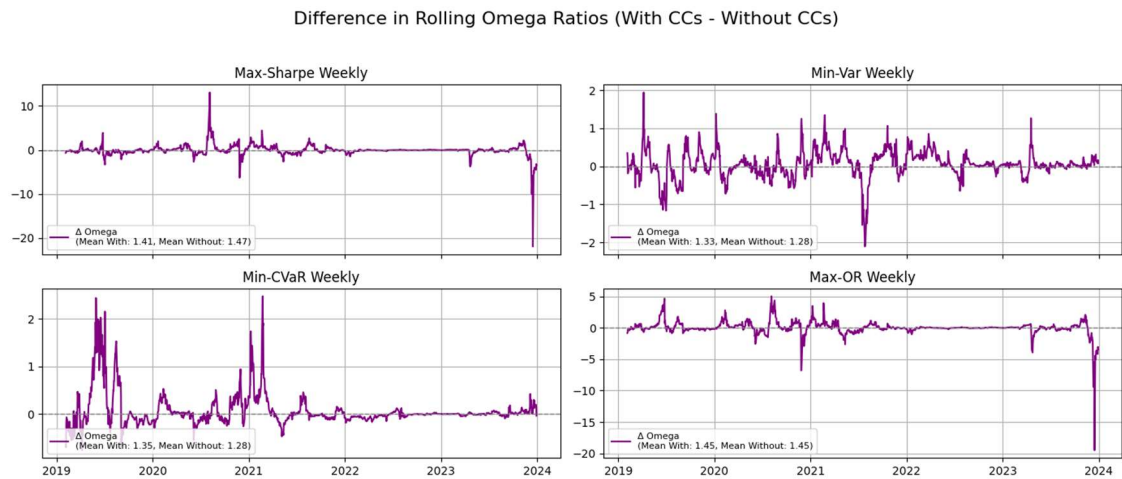


Figure 25. Difference in Rolling Omega Ratio with and without Cryptocurrencies (GVBC)

Table 6 presents the cumulative differences in performance metrics between portfolios with and without cryptocurrencies under the GVBC, along with the significance levels indicated by asterisks for metrics that are statistically different at the 5% level (excluding

cumulative log returns). Similarly to Figure 22, the table confirms the reduction in both cumulative returns and volatility compared to the unconstrained case, which continues to exhibit a rise with the inclusion of cryptocurrency. The reduction can be observed across all strategies, except for Min-CVaR, which experiences an increase in both due to its lower concentration in bonds. For example, for Max-Sharpe, the cumulative return declines from 1.83 to 1.27, and the volatility from 13.54 to 8.71. The differences in volatility are statistically significant for all strategies.

	Cumulative Return Δ	Volatility Δ Sum	Sharpe Δ Sum	Sortino Δ Sum	Omega Δ Sum
Max-Sharpe Weekly	1,270	8,710*	-23,633*	-11,125	-68,447
Min-Var Weekly	0,120	1,006*	22,368*	42,946*	61,446*
Min-CVaR Weekly	0,039	0,139*	8,052*	35,434*	88,329*
Max-OR Weekly	0,857	5,315*	2,656	36,177	-2,281

Table 6. The Cumulative Differences with Significance Levels (GVBC)

The cumulative risk-adjusted return metrics reveal a different aspect of the GVBC's effect. The differences between rolling Sharpe ratios indicate a decline in the ratio for the return-seeking strategies compared to the unconstrained case. For Max-Sharpe, the difference declines from -18.72 to -23.63, remaining statistically significant, and for Max-OR, it decreases from 14.32 to 2.66, losing its statistical significance. A similar trend is observed for the rolling Sortino and Omega ratios. Considerable change is observed in the Max-OR strategy's Omega ratio differences, which shift from 55.76 to -2.28 with GVBC, but remains statistically insignificant.

The more conservative and risk-focused strategies remain strong performers under the GVBC. There is a slight reduction in the Sharpe and Sortino ratios for Min-Var, but a slight increase in the Omega ratio, all of which are statistically significant. For Min-CVaR, the Sharpe and Omega ratios increase notably, while the Sortino ratio decreases slightly, yet it reaches statistical significance. Overall, these findings highlight how GVBC restricts the allocation of cryptocurrencies in performance-seeking strategies, while in risk-averse strategies, especially Min-CVaR, it forces allocation toward relatively riskier assets.

6.4 Diversification

In this section, the diversification benefits of cryptocurrencies in traditional portfolios are evaluated by using the Diversification Ratio and the Effective Number of Bets. The findings are presented similarly to the performance section, by using monthly rolling metrics to compare the difference between portfolios with and without cryptocurrencies.

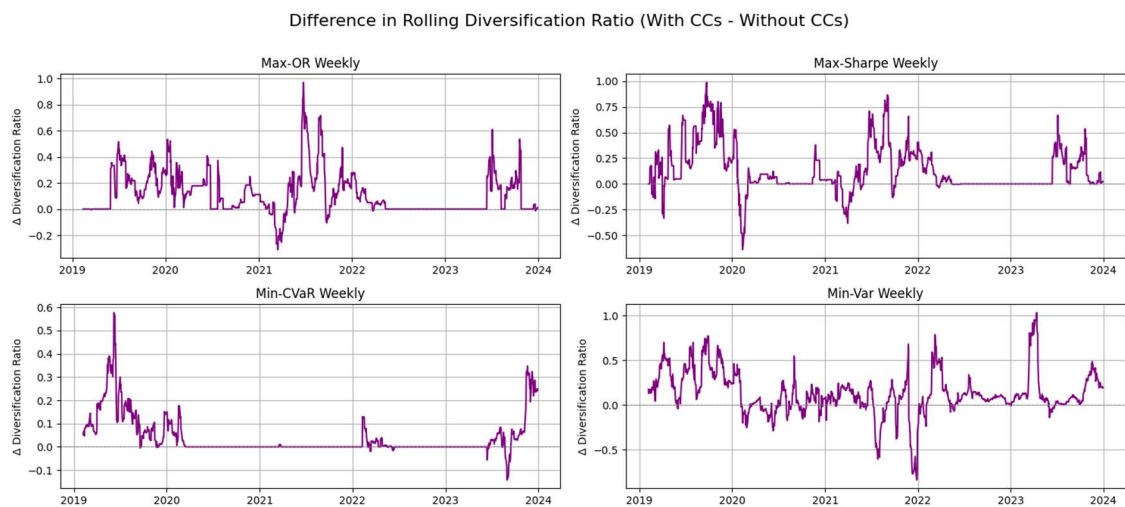


Figure 26. Difference in Rolling Diversification Ratio with and without Cryptocurrencies

Figure 26 illustrates the difference in rolling Diversification Ratio between portfolios that include and exclude cryptocurrencies. Across all strategies, the rolling DR indicates that, generally, cryptocurrencies enhance portfolio diversification. The Max-OR, Max-Sharpe, and Min-CVaR strategies exhibit consistently positive differences throughout most of the sample period, with only a few brief periods of negative values. In contrast, the Min-Var strategy exhibits more inconsistent diversification benefits, with the DR difference frequently oscillating around zero and negative. Comparing the strategies' allocations with the DR differences, periods with concentration in cryptocurrencies more frequently result in negative diversification impacts. In contrast, moderate allocations tend to correspond with sustained positive DR differentials. The differences between portfolios for all the strategies are positive and statistically significant.

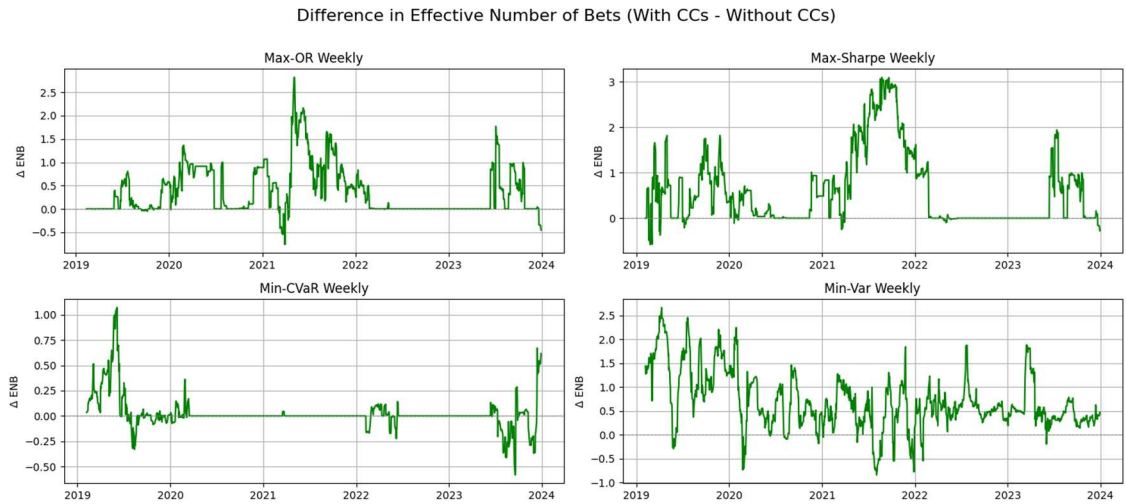


Figure 27. Difference in Rolling Effective Number of Bets with and without Cryptocurrencies

Figure 27 presents the difference in rolling Effective Number of Bets between portfolios with and without cryptocurrencies. Consistent with the DR results, cryptocurrencies generally increased ENB across the strategies, indicating a higher amount of independent risk sources in the portfolios. Max-OR, Max-Sharpe, and Min-CVaR strategies show consistent positive differentials, aligning with the positive DR differences. Similarly to DR differences, the Min-Var strategy exhibits more variability, oscillating more closely around zero. The differences between the portfolios are statistically significant and positive for all strategies. Overall, the findings reinforce the interpretation of Figure 26 by confirming that the improvements observed in DR are not solely due to reduced volatility, but rather genuine enhancements in diversification.

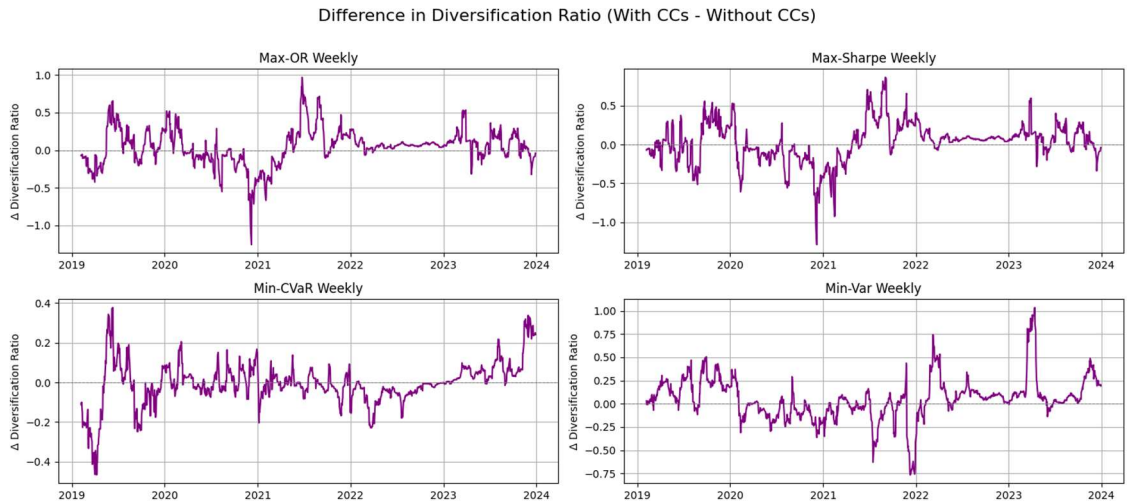


Figure 28. Difference in Rolling Diversification Ratio with and without Cryptocurrencies GVBC

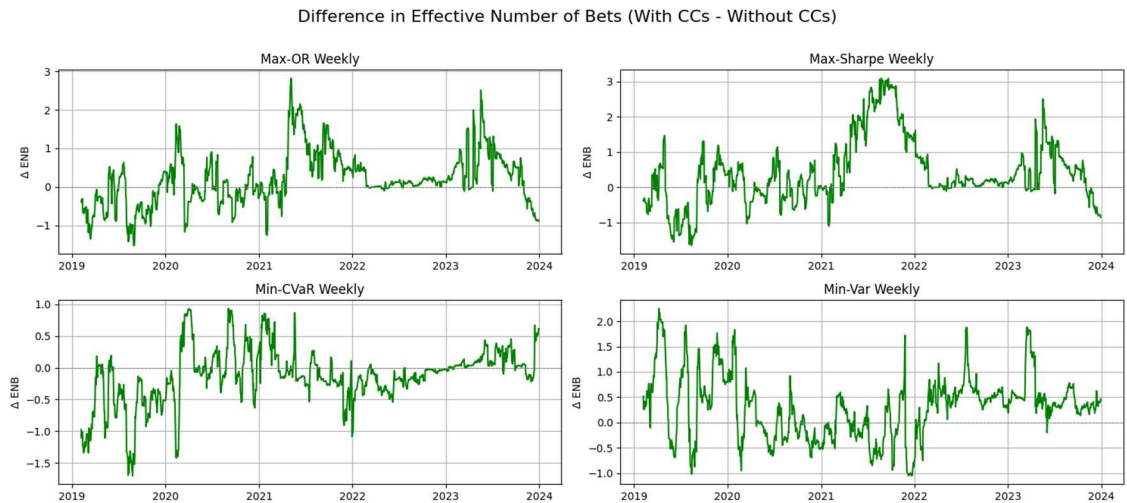


Figure 29. Difference in Rolling Effective Number of Bets with and without Cryptocurrencies GVBC

Figures 28 and 29 present the differences in rolling DR and ENB between portfolios with and without cryptocurrencies under GVBC. Compared to the unconstrained case, cryptocurrencies' diversification benefits seem less consistent. The figures indicate higher volatility, with more frequent negative differences, suggesting that during these periods, cryptocurrencies harmed the portfolio's diversification. Yet, at times, they still offer substantial positive diversification benefits.

For approximately the first two years, the Max-Sharpe and Max-OR strategies oscillate between 0.5 and -0.5 in DR and 1 and -1 in ENB. For the remainder of the studied period, the benefits are consistently above zero for both metrics, a trend that was also observed in the unconstrained case throughout the entire period. In contrast, the Min-Var strategy exhibits persistent volatility in DR and ENB differences, with lower or even negative values, suggesting that under GVBC, cryptocurrencies provide weaker diversification benefits for risk-minimization portfolios. The Min-CVaR strategy demonstrates greater variability around zero, likely due to GVBC causing more divergence between the two portfolios' allocations. For all the strategies, the differences are positive and statistically significant, except for Min-CVaR, where they are negative, and DR is not statistically significant. These findings suggest that diversification benefits become more conditional when both portfolios are optimized under GVBC.

6.5 Rebalancing Intervals

A paired sample t-test is conducted on the returns of cryptocurrency-inclusive portfolios to determine whether the rebalancing interval has a statistically significant impact on portfolio performance. The weekly rebalancing results are compared against monthly and quarterly intervals, and the corresponding t-values and p-values are reported in Table 7. The results indicate that rebalancing frequency does not significantly impact the returns of any strategy, whether unconstrained or constrained under GVBC. These findings suggest that further analysis of the other portfolio performance metrics is unnecessary, as the returns exhibit no significant sensitivity to rebalancing frequency.

	Weekly vs Monthly (t)		Weekly vs Monthly (p)		Weekly vs Quarterly (t)		Weekly vs Quarterly (p)	
	Unconstrained	GVBC	Unconstrained	GVBC	Unconstrained	GVBC	Unconstrained	GVBC
Max-Sharpe	0,600	1,258	0,549	0,209	0,740	1,852	0,459	0,064
Min-Var	0,449	0,449	0,653	0,653	-0,035	-0,035	0,972	0,972
Max-Ret	1,470		0,142		1,235		0,217	
Min-CVaR	0,801	-1,129	0,423	0,258	0,366	-0,668	0,714	0,505
Max-OR	0,244	0,058	0,807	0,954	0,103	0,852	0,918	0,394

Table 7. Statistical Comparison of Portfolio Rebalancing Intervals Returns

Similarly, a paired sample t-test is conducted for the diversification metrics of the cryptocurrency-inclusive portfolios. Table 8 presents the results of a paired sample t-test, comparing the weekly rebalancing interval against monthly and quarterly intervals, and reports the corresponding t-values and p-values. Different strategies exhibit statistically significant differences in the diversification metrics across the rebalancing intervals in the unconstrained case. For Min-Var, both DR and ENB, and Min-CVaR, the DR shows statistical significance against both rebalancing intervals. Additionally, this holds under GVBC, with the addition of a statistically significant ENB for Min-CVaR when compared against the quarterly interval. For Max-Sharpe, only the DR difference is statistically significant in the unconstrained case against quarterly rebalancing. For Max-OR, both metrics show significance in the unconstrained case against monthly rebalancing, while under GVBC, the only DR is significant against quarterly rebalancing.

Strategy	Metric	Weekly vs Monthly (t)		Weekly vs Monthly (p)		Weekly vs Quarterly (t)		Weekly vs Quarterly (p)	
		Unconstrained	GVBC	Unconstrained	GVBC	Unconstrained	GVBC	Unconstrained	GVBC
Max-Sharpe	DR	1,161	1,756	0,246	0,079	3,226	0,809	0,001	0,419
	ENB	1,717	-0,186	0,086	0,853	1,566	-1,902	0,118	0,057
Min-Var	DR	11,116	11,116	0,000	0,000	15,051	15,051	0,000	0,000
	ENB	8,491	8,491	0,000	0,000	7,795	7,795	0,000	0,000
Min-CVaR	DR	5,394	2,828	0,000	0,005	10,474	6,798	0,000	0,000
	ENB	0,603	-0,191	0,547	0,848	0,254	3,881	0,799	0,000
Max-OR	DR	2,486	1,069	0,013	0,285	0,324	2,296	0,746	0,022
	ENB	4,151	0,815	0,000	0,415	-0,401	-1,681	0,689	0,093

Table 8. Statistical Comparison of Portfolio Rebalancing Intervals Diversification Metrics

Table 8 findings indicate that the rebalancing interval can influence the realized diversification benefits of cryptocurrency-inclusive portfolios, particularly in risk-minimization-

focused strategies. On the other hand, the return-seeking strategies showed limited statistically significant differences, mainly in specific metrics or intervals. Additionally, in all statistically significant cases, the t-values were positive, suggesting that weekly rebalancing intervals offered higher diversification benefits. This aligns with the volatile dynamics of cryptocurrencies, which can lead to rapid fluctuations in their relationships with other assets.

7 Discussion

7.1 Overall Results

Over the full 2019-2023 sample, including cryptocurrencies, a European equity-bond-commodity portfolio raised total returns while also increasing volatility. Return-seeking strategies saw cumulative log returns soar roughly 0.8-2.0, whereas risk-focused strategies recorded more modest gains, approximately a 0-0.15 increase, depending on the strategy. Each strategy also saw a statistically significant increase in volatility. These out-of-sample results are consistent with those of Brière et al. (2015), who found that cryptocurrencies expand the efficient frontier by boosting both returns and risk. Additionally, Dorfleitner and Lung's (2018) findings show that the benefits of cryptocurrency are primarily from increased returns rather than reduced volatility.

The results are mixed once adjusted for risk. Conservative, risk-focused strategies experienced significant improvements in the Sharpe Ratio, while the aggressive, return-seeking strategies experienced statistically significant declines in the Sharpe Ratio. The Max-OR strategy is an exception, as the Sharpe Ratio increased; however, it is only statistically significant in the unconstrained case. Generally, the same findings are observed with Sortino and Omega ratios, with both showing statistically significant increases for Min-Var and Min-CVaR, and non-significant declines for Max-Sharpe and Max-Ret. Again, Max-OR mostly sees ratios increase, only Sortino being statistically significant in the unconstrained case. These out-of-sample results are partly in line with the in-sample findings of Wu and Pandey (2014) and Brière et al. (2015), demonstrating increased risk-adjusted returns from Bitcoin. However, out-of-sample performance varies by strategy and risk metric in the real world.

On average, the correlations between cryptocurrencies and traditional assets are low throughout the sample period, indicating potential diversification benefits. Additionally, the diversification ratio and effective number of bets are higher overall for all cryptocurrency portfolios, except for Min-CVaR under GVBC. These out-of-sample diversification

metrics demonstrate that cryptocurrencies have resulted in a new, uncorrelated return stream, as well as increased the number of portfolios' risk sources, offering true diversification benefits, in line with studies such as Bouri et al. (2017), Corbet et al. (2018), and Guesmi et al. (2019). Although the overall results show benefits, it is evident that when studying market downturns, they tend to shrink and even turn negative (e.g., Allen, 2022).

These findings partially support Hypothesis 1 and confirm Hypothesis 2 for all strategies except Min-CVaR under GVBC. Cryptocurrencies enhance the risk-adjusted returns for risk-oriented portfolios, significantly improving nearly all out-of-sample risk-adjusted return measures. As the return-seeking portfolios are examined, the results are more mixed. The unconstrained Max-OR portfolio was the only portfolio that saw significantly higher out-of-sample risk-adjusted returns. Otherwise, the inclusion of cryptocurrency resulted in worse portfolio performance. The diversification benefits, measured by DR and ENB, show a significant improvement in diversification with the inclusion of cryptocurrency across the portfolios.

7.2 Market Regimes

For most of 2018, rolling correlations between cryptocurrencies and traditional assets were low, ranging from approximately -0.2 to 0.2, until the market decline at year-end, which led to an upward spike in correlation. This, along with the overall poor performance of cryptocurrencies, resulted in near-zero cryptocurrency exposure in the optimal allocations at the start of 2019, reflecting nonexistent benefits in terms of both performance and diversification. The extreme underperformance of cryptocurrencies contributed to this outcome during the 2018 bear market, as well as the temporary rise in correlations during market sell-offs, which eroded their appeal as a diversification tool.

Throughout 2019, cryptocurrencies held an average weight of 0.5-5.0% in the optimal portfolios, increasing toward the end of the year. Max-Sharpe portfolios had the highest

cryptocurrency allocations, while Min-CVaR maintained the lowest. Risk-adjusted return metrics indicate that the Max-Sharpe, Min-Var, Max-Ret, and Max-OR portfolios underperformed compared to traditional portfolios. In contrast, the Min-CVaR portfolio outperformed, benefiting from its limited exposure to cryptocurrencies. Under GVBC, the Max-OR portfolio underperformed in terms of the Sharpe Ratio but achieved better Sortino and Omega ratios. Furthermore, the diversification metrics showed higher diversification for all cryptocurrency-inclusive unconstrained portfolios. Under GVBC, the results were mixed: the Diversification Ratio was negative only for Min-CVaR, whereas the Effective Number of Bets was positive only for Min-Var. The underperformance is primarily due to low initial cryptocurrency allocations resulting from the poor 2018 returns, which limited the portfolios' ability to benefit from the rebound in 2019. The positive diversification results indicate the low correlations between cryptocurrencies and traditional assets during these normalized market conditions.

In 2020, the year of the COVID-19 crash, cryptocurrency exposure in portfolios increased notably, ranging from 0.1% (in the unconstrained Min-CVaR portfolio) to 61.2% (in the unconstrained Max-Sharpe portfolio) on average. All the portfolios outperformed their traditional counterparts in terms of risk-adjusted return metrics, except for the Min-Var portfolios. The ENB was higher for all portfolios with cryptocurrencies, as was the DR for the unconstrained portfolios. Furthermore, the DR for the GVBC portfolios was positive only for Min-CVaR. The COVID-19 crash led to a temporary spike in correlations between cryptocurrencies and traditional assets, as markets declined across asset classes, hindering early-year diversification. However, as the year progressed, cryptocurrencies experienced an extreme price rally, decoupling from the traditional assets and driving the outperformance of portfolios with cryptocurrency exposure. The time-varying correlations explain why the diversification benefits turned positive for parts of the portfolios despite the initial crash.

The cryptocurrency weights varied from 0.0% (unconstrained Min-CVaR portfolio) to 34.9% (unconstrained Max-Sharpe portfolio) in 2021. Furthermore, all unconstrained

portfolios, excluding Min-CVaR, yielded higher risk-adjusted returns with cryptocurrencies. Additionally, under GVBC, all of the portfolios with cryptocurrencies outperformed. Cryptocurrencies improved the diversification of the Max-Sharpe and Max-OR portfolios, as measured by both metrics. However, for the Min-CVaR, the benefits were zero in the unconstrained case and negative under GVBC. The Min-Var results were negative across all portfolios. These outcomes reflect the 2021 cryptocurrency bull market conditions, where they reached all-time highs and maintained low or negative correlations with traditional assets for most of the year. This drives the outperformance and diversification improvements of cryptocurrency-inclusive portfolios, particularly for return-seeking strategies that allocate more heavily to cryptocurrencies.

In 2022, cryptocurrency weights were on average the lowest for the entire sample period, ranging from 0.1% (Min-CVaR for both portfolios) to 2.5% (Max-Sharpe under GVBC). Both Min-Var portfolios were the only ones to outperform their traditional counterparts in terms of risk-adjusted returns. Nevertheless, the diversification benefits were positive for almost all portfolios, except for Min-CVaR, which had only a positive Diversification Ratio in the unconstrained portfolio. The cryptocurrency bear market in 2022, marked by events such as the Terra-Luna collapse and the FTX bankruptcy, resulted in extreme losses for cryptocurrencies, which in turn negatively impacted portfolios with higher cryptocurrency exposure, as reflected in the risk-adjusted metrics. However, cryptocurrency weights quickly fell to zero or near-zero, controlling portfolio downside risk and preserving the early-year diversification benefits.

In 2023, the weights for cryptocurrencies were from 0.5% (both Min-Var portfolios) to 33.6% (unconstrained Max-Sharpe portfolio). The Max-Ret and Min-Var portfolios show consistent outperformance in risk-adjusted returns across all metrics. Otherwise, the rest of the portfolios underperformed, except for the unconstrained Min-CVaR, as measured by the Omega ratio, and under GVBC, as measured by both the Sharpe and Omega ratios. Cryptocurrencies improved diversification for all portfolios, except for the unconstrained Min-CVaR, as measured by the Effective Number of Bets. The recovery of

cryptocurrency prices in 2023, partly driven by renewed optimism surrounding crypto regulation and Bitcoin ETF filings, led to higher cumulative returns and reduced correlations with traditional assets, which explains the improved diversification. However, due to the trailing nature of the optimization process, the early-year cryptocurrency rally was only partially captured, while the mid-year downward trend was fully reflected. This explains why only some of the strategies achieved improved risk-adjusted returns.

Overall, the time-varying results of cryptocurrency inclusion are consistent with earlier findings in the literature. The positive diversification effects observed during normal or bullish market conditions (e.g., 2019, 2020, and 2021) are consistent with the findings of Bouri et al. (2017), who concluded that Bitcoin typically acts as a diversifier but not as a safe haven during crises. Similarly, the expansion of efficient frontiers during periods of cryptocurrency outperformance, particularly during the 2020-2021 bull run, supports the findings of Brière et al. (2015) and Petukhina et al. (2021), who highlighted the ability of cryptocurrencies to enhance portfolio efficiency. In contrast, the diminished risk-adjusted returns and diversification benefits during bear phases, such as the 2022 downturn, align with the observations of Dofleitner and Lung (2018) and Allen (2022), who noted the regime-dependency of cryptocurrencies' portfolio benefits. These findings reinforce the notion that cryptocurrencies enhance performance and diversification during regular or bull market periods, but fade or reverse during market stress. This highlights the importance of dynamic risk management when incorporating cryptocurrencies into portfolios.

From the time-varying findings, it is evident that the effect of cryptocurrency inclusion fluctuates with market conditions, confirming Hypothesis 3. Digging deeper, the results show apparent differences across the regimes. During bull markets (e.g., 2020-2021), cryptocurrencies enhance risk-adjusted returns and diversification. Conversely, during bear markets (e.g., 2022), the improvements mostly disappear, and portfolios with cryptocurrencies generally underperformed. Across the regimes, the most consistent performance was observed for risk-oriented strategies, which partly challenges the findings of

Petukhina et al. (2021), which emphasized the benefits of return-seeking strategies when including cryptocurrencies.

7.3 Optimization Method Effects

The Mean-Variance optimization strategies (Min-Var and Max-Sharpe) were compared to the higher-moment optimization strategies (Min-CVaR and Max-OR) based on economic significance, by comparing the average levels of rolling risk-adjusted return metrics (Sharpe, Sortino, and Omega Ratios) over the entire 2019-2023 period. The results show mixed outcomes across strategy types. For risk-focused portfolios, Min-Var outperformed Min-CVaR (Sharpe: +0.05 daily avg, Sortino: +0.06 daily avg, and Omega: +0.08 daily avg), indicating that minimizing variance was more efficient than minimizing conditional value-at-risk in cryptocurrency-inclusive portfolios. Conversely, for return-seeking portfolios, Max-OR outperformed Max-Sharpe (Sharpe: +0.02 daily avg, Sortino: +0.05 daily avg, and Omega: +0.07 daily avg), suggesting that optimizing for skewness and tail-return asymmetry provided a modest advantage over traditional mean-variance optimization.

These findings partially support Hypothesis 4, indicating that optimizations with higher moments deliver better out-of-sample risk-adjusted returns on average for return-seeking strategies, but not when considering risk alone. The Global Variance-Based Constraints narrowed the differences between the optimization methods while still aligning with the findings of the unconstrained portfolios. GVBC appeared to reduce the relative advantage of the Max-OR portfolio by preventing extreme cryptocurrency allocations and distributing the allocations more uniformly. The comparison between unconstrained and constrained optimizations, using cryptocurrencies, Max-Sharpe, and Min-CVaR portfolios, revealed consistent improvements in risk-adjusted metrics when GVBC was introduced. At the same time, the Max-OR strategy encountered a slight decline in Sharpe and Omega ratios.

7.4 Implications for Portfolio Management

The empirical findings bring forth essential implications for portfolio management regarding the role of cryptocurrencies in European portfolios. In particular, they emphasize that time-varying, regime-dependent effects and strategy choices are critical factors in determining whether cryptocurrency allocations enhance or weaken the portfolio performance.

First, the study confirms that the selective inclusion of cryptocurrencies, rather than large-scale allocations, yields the most consistent results. Even modest allocations of cryptocurrencies were sufficient to produce statistically significant increases in the performance metrics. For instance, the Min-Var portfolio, which limited the cryptocurrency allocation to an average of 4%, saw a clear improvement in the risk-adjusted return measures. This suggests that small positions can capture the upside when the cryptocurrency market rises, but the limited allocation prevents them from overwhelming the portfolio during downturns.

Second, the results reinforce that performance is regime dependent and call for a momentum-based allocation framework. For instance, cryptocurrency inclusion was highly beneficial during the 2020-2021 bull run; however, the benefits disappeared or turned negative during the downturns. Therefore, this volatility clustering suggests that giving more weight to recent performance than to long-term performance can enhance outcomes. This way, the portfolio's cryptocurrency exposure can be increased in clear uptrends and decreased in downtrends more efficiently. This is associated with the third: frequency of rebalancing. The rapidly evolving nature of the cryptocurrency market necessitates lower-frequency rebalancing, as evidenced by the superior results of weekly rebalancing. However, the differences were mostly insignificant, and monthly rebalancing was sufficient to capture most of the benefits, suggesting that it is a more feasible solution as transaction costs are considered.

Fourth, the choice of strategy and specific cryptocurrency asset matters. Return-seeking strategies that account for higher moments (Max-OR) consistently outperformed variance-only strategies (Max-Sharpe), indicating that skewness and kurtosis should be considered when including cryptocurrencies. Regarding the cryptocurrency selection, the most consistent inclusion is seen in Bitcoin. However, Ethereum is often the next choice or sometimes even preferred to Bitcoin, and altcoins occasionally hold a small weight in the portfolios. In practice, this suggests that Bitcoin serves as the core holding, with Ethereum and other altcoins serving as complementary allocations under specific market conditions.

Fifth, the results indicate that implementing constraints, such as the GVBC, plays a critical role in shaping portfolio outcomes when cryptocurrencies are included. The GVBC helped mitigate extreme allocation swings, particularly in return-maximizing strategies, by capping risk concentrations and enforcing diversification across the assets. This led to more stable and predictable outcomes, with reduced volatility and improved risk-adjusted returns for specific strategies. From a practical perspective, this suggests that upper bounds or variance-based constraints should be considered when incorporating cryptocurrencies, especially to balance the upside potential with downside protection.

Ultimately, based on the findings, cryptocurrencies should be viewed as dynamic, highly volatile investments that necessitate ongoing monitoring and periodic adjustments, rather than being treated as static long-term allocations.

8 Limitations

Although the study offers a comprehensive view of the effects of incorporating cryptocurrencies into traditional portfolios, several limitations should be considered. First, diversified products are used for traditional assets and Altcoins, which mitigates concerns about insufficient diversification within each category. However, the fixed and simplified structure can limit the generalizability of the results to more complex, multi-asset portfolio structures or portfolios that include a broader set of alternative assets. Second, the analysis relies on historical data, introducing estimation error. While the use of GVBC helps mitigate this issue, it cannot entirely eliminate it. Third, the results are time-dependent due to the unique market characteristics of the 2019-2023 sample period, which may limit the applicability of the findings to future periods. Fourth, because the risk and diversification metrics are calculated using overlapping monthly rolling windows, applying paired sample t-tests introduces autocorrelation, which may affect the validity of the statistical significance results.

Additionally, the study assumes frictionless markets with perfect rebalancing at specified intervals. In reality, transaction costs, slippage, and liquidity constraints can significantly affect the performance of the strategies. Cryptocurrency markets, particularly smaller altcoins, often suffer from limited liquidity, resulting in wide bid-ask spreads and significant market impact during rebalancing. Furthermore, slippage resulting from rapid price movements and exchange fees can significantly impact returns, particularly in lower-frequency rebalancing. These costs are not considered, which can overstate the portfolios' performance.

9 Conclusions

This study examined the time-varying benefits of incorporating cryptocurrencies into traditional European portfolios, which include equities, bonds, and commodities, from 2019 to 2023. Employing a dynamic rolling-window approach, portfolios that include Bitcoin, Ethereum, and a value-weighted altcoin portfolio were systematically compared to those containing only traditional assets across different rebalancing intervals (weekly, monthly, quarterly) and optimization methods (mean-variance, CVaR, and Omega-ratio). The empirical findings demonstrate that incorporating cryptocurrencies can significantly enhance portfolio performance, although these benefits depend heavily on the allocation strategy and market regime.

Cryptocurrencies generally increased absolute returns considerably. For instance, the unconstrained Max-Sharpe portfolio achieved a cumulative log return of 213.7% when cryptocurrencies were included, compared to 30.5% without them over the entire analysis period. However, the higher return was accompanied by substantially higher volatility (e.g., Max-Sharpe daily volatility of 2.43% vs. 0.81% without cryptocurrencies), resulting in varied outcomes in risk-adjusted returns. Specifically, return-seeking strategies (Max-Sharpe and Max-Ret) with cryptocurrency inclusion underperformed their traditional counterparts on risk-adjusted measures. In contrast, portfolios optimized for risk reduction (Min-Var and Min-CVaR) and higher-moment considerations (Max-OR) generally experienced significant out-of-sample improvements in risk-adjusted performance through cryptocurrency exposure.

Moreover, diversification metrics improved with cryptocurrency exposure across nearly all optimization methods (except the constrained Min-CVaR method), but the effect was adverse during certain market conditions. Combined findings from both risk-adjusted return and diversification metrics indicate significant regime-dependent behavior. During bullish and regular market periods, particularly from 2020 to 2021, cryptocurrency-inclusive portfolios consistently outperformed traditional portfolios on risk-adjusted metrics, and more often on diversification metrics. Conversely, during market downturns

such as the 2022 cryptocurrency bear market, these benefits either diminished or became adverse. Therefore, the observed benefits of cryptocurrency allocation demonstrate recognizable regime dependency, highlighting the necessity for dynamic and momentum-based strategies when integrating cryptocurrencies into traditional portfolios.

Regarding portfolio construction, the optimization method had a moderate impact on the outcomes of cryptocurrency inclusion. Portfolios focusing on risk minimization experienced the most consistent statistically significant benefits, with the Min-Var outperforming the Min-CVaR portfolio. Among return-seeking strategies, only the Max-OR portfolio sustained improvement from the inclusion of cryptocurrency throughout the entire period. This suggests that portfolios with modest allocations in cryptocurrencies experience the most benefits. Although both Max-OR and Max-Sharpe portfolios benefited during the same periods (2020-2021), the Max-OR strategy demonstrated greater resilience during downturns, underscoring the importance of incorporating higher-order risk considerations when implementing aggressive return strategies involving cryptocurrencies. Furthermore, the importance of constraining the portfolios is emphasized to achieve more stable allocations and predictable outcomes.

The empirical findings of this study largely align with the previous literature, particularly regarding the benefits of cryptocurrencies and their regime-dependent behavior. Consistent with earlier studies such as Brière et al. (2015), Petukhina et al. (2021), and Bouri et al. (2017), this thesis confirms that cryptocurrencies typically enhance diversification and risk-adjusted returns during normal or bullish market conditions, expanding the efficient frontier. Additionally, the observed disappearance of these benefits during bearish market conditions, such as in 2022, aligns with the findings of Dofleitner and Lung (2018) and Allen (2022), reinforcing the importance of market regime considerations in cryptocurrency portfolio management. Furthermore, by examining a more recent and extended period (2019-2023), this study adds value by validating these earlier findings in modern market conditions, which reflect a notably more mature cryptocurrency

environment with broader institutional adoption, particularly in European markets, with the out-of-sample framework.

However, there are also unique findings: contrary to Petukhina et al. (2021) and Huang et al. (2023), who highlighted substantial benefits primarily for return-seeking strategies, this study found the most consistent cryptocurrency-related improvements in portfolios focused explicitly on risk minimization, such as the Min-Var strategy. Moreover, the clear advantage of higher-order risk consideration (Max-OR strategy) over mean-variance optimization (Max-Sharpe) in terms of downturn resilience is a distinctive insight from this analysis, emphasizing the practical value of incorporating skewness and kurtosis into portfolio strategies involving cryptocurrencies.

While these findings offer clear insights into the benefits of cryptocurrency in portfolio management, the study is subject to certain limitations, including the absence of transaction costs and the assumption of perfect rebalancing, as discussed in Chapter 9. However, recognizing these limitations opens up possibilities for future research, which can be conducted in more realistic settings, including different market frictions, as well as focusing on finding the best portfolio management strategies for incorporating cryptocurrencies into traditional portfolios.

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Appendix 2. Altcoin Portfolio Weight Statistics

	Mean	Maximum	Minimum
ZRX	0,05 %	0,89 %	0,00 %
AAVE	0,17 %	4,14 %	0,00 %
AE	0,04 %	0,84 %	0,00 %
ALGO	0,28 %	2,53 %	0,00 %
APT	0,02 %	1,48 %	0,00 %
REP	0,01 %	0,54 %	0,00 %
AVAX	1,40 %	6,63 %	0,00 %
AXS	0,06 %	1,76 %	0,00 %
BAT	0,06 %	1,19 %	0,00 %
BCH	7,28 %	20,11 %	1,12 %
BTG	0,22 %	2,01 %	0,00 %
BTS	0,02 %	0,76 %	0,00 %
BTT	0,02 %	3,15 %	0,00 %
BNB	12,95 %	31,25 %	0,00 %
BTM	0,01 %	1,01 %	0,00 %
ADA	7,00 %	20,31 %	2,54 %
CEL	0,02 %	1,92 %	0,00 %
LINK	2,38 %	10,65 %	0,00 %
COMP	0,03 %	1,62 %	0,00 %
ATOM	1,23 %	3,06 %	0,00 %
CRO	1,22 %	6,21 %	0,00 %
DASH	1,04 %	3,70 %	0,00 %
MANA	0,06 %	1,61 %	0,00 %
DCR	0,06 %	0,80 %	0,00 %
DCN	0,00 %	0,63 %	0,00 %
DGD	0,02 %	0,84 %	0,00 %
DOGE	3,23 %	16,59 %	0,00 %
EOS	3,87 %	12,02 %	0,00 %
ETC	1,36 %	3,26 %	0,00 %
FTM	0,02 %	1,81 %	0,00 %
FIL	0,29 %	3,66 %	0,00 %
FLOW	0,02 %	1,55 %	0,00 %
FTT	0,34 %	2,39 %	0,00 %
HBAR	0,06 %	1,41 %	0,00 %
HEDG	0,14 %	1,88 %	0,00 %
HEX	0,13 %	3,36 %	0,00 %
HOT	0,01 %	1,41 %	0,00 %
HT	0,43 %	2,91 %	0,00 %
ICX	0,10 %	1,89 %	0,00 %
ICP	0,20 %	3,78 %	0,00 %
MIOTA	1,18 %	5,16 %	0,00 %
KAS	0,01 %	1,45 %	0,00 %
KLAY	0,04 %	3,07 %	0,00 %

LDO	0,03 %	1,43 %	0,00 %
LSK	0,15 %	2,30 %	0,00 %
LTC	5,59 %	13,66 %	1,94 %
MKR	0,41 %	2,03 %	0,00 %
XMR	2,04 %	4,41 %	0,00 %
EGLD	0,01 %	1,85 %	0,00 %
NANO	0,08 %	1,65 %	0,00 %
NEAR	0,29 %	3,12 %	0,00 %
XEM	0,87 %	5,22 %	0,00 %
NEO	1,16 %	6,48 %	0,00 %
OKB	0,26 %	1,84 %	0,00 %
OMG	0,18 %	1,59 %	0,00 %
ONT	0,28 %	1,73 %	0,00 %
CAKE	0,01 %	1,38 %	0,00 %
DOT	3,14 %	16,00 %	0,00 %
MATIC	1,56 %	7,06 %	0,00 %
PZM	0,01 %	0,81 %	0,00 %
QTUM	0,21 %	2,11 %	0,00 %
QNT	0,01 %	0,99 %	0,00 %
REV	0,02 %	1,58 %	0,00 %
SHIB	1,23 %	6,52 %	0,00 %
SC	0,01 %	0,90 %	0,00 %
SOL	3,12 %	18,74 %	0,00 %
STEEM	0,02 %	0,81 %	0,00 %
XLM	3,22 %	9,08 %	0,00 %
STRAT	0,01 %	0,69 %	0,00 %
SNX	0,02 %	1,78 %	0,00 %
LUNA	0,79 %	10,99 %	0,00 %
XTZ	1,07 %	5,62 %	0,00 %
THETA	0,29 %	5,49 %	0,00 %
RUNE	0,01 %	1,10 %	0,00 %
TON	0,47 %	5,69 %	0,00 %
TRX	2,99 %	6,43 %	1,21 %
UMA	0,02 %	2,00 %	0,00 %
UNI	1,35 %	7,34 %	0,00 %
LEO	1,31 %	4,20 %	0,00 %
VSYS	0,01 %	0,85 %	0,00 %
XVG	0,05 %	1,64 %	0,00 %
WAVES	0,04 %	1,04 %	0,00 %
XRP	20,07 %	46,65 %	8,00 %
YFI	0,02 %	2,21 %	0,00 %
ZEC	0,40 %	1,64 %	0,00 %
ZIL	0,05 %	1,03 %	0,00 %

Appendix 3. Yearly Correlation Matrices

	Bitcoin	Ethereum	Altcoins	Stocks	Bonds	Commodities
Bitcoin	1,00					
Ethereum	0,77	1,00				
Altcoins	0,85	0,86	1,00			
Stocks	0,06	0,10	0,07	1,00		
Bonds	-0,01	0,03	0,02	0,07	1,00	
Commodities	0,03	0,05	0,05	0,24	-0,03	1,00

Table 9. 2018 Correlation Matrix

	Bitcoin	Ethereum	Altcoins	Stocks	Bonds	Commodities
Bitcoin	1,00					
Ethereum	0,80	1,00				
Altcoins	0,80	0,88	1,00			
Stocks	-0,07	-0,01	0,03	1,00		
Bonds	-0,10	-0,09	-0,08	-0,05	1,00	
Commodities	0,05	0,12	0,08	0,40	-0,03	1,00

Table 10. 2019 Correlation Matrix

	Bitcoin	Ethereum	Altcoins	Stocks	Bonds	Commodities
Bitcoin	1,00					
Ethereum	0,88	1,00				
Altcoins	0,82	0,92	1,00			
Stocks	0,48	0,47	0,44	1,00		
Bonds	0,40	0,36	0,34	0,37	1,00	
Commodities	0,35	0,32	0,29	0,60	0,30	1,00

Table 11. 2020 Correlation Matrix

	Bitcoin	Ethereum	Altcoins	Stocks	Bonds	Commodities
Bitcoin	1,00					
Ethereum	0,75	1,00				
Altcoins	0,77	0,79	1,00			
Stocks	0,21	0,22	0,24	1,00		
Bonds	0,07	0,11	0,04	- 0,03	1,00	
Commodities	0,10	0,14	0,12	0,35	- 0,13	1,00

Table 12. 2021 Correlation Matrix

	Bitcoin	Ethereum	Altcoins	Stocks	Bonds	Commodities
Bitcoin	1,00					
Ethereum	0,90	1,00				
Altcoins	0,88	0,89	1,00			
Stocks	0,33	0,37	0,32	1,00		
Bonds	0,12	0,08	0,08	0,19	1,00	
Commodities	0,13	0,12	0,10	-0,06	-0,03	1,00

Table 13. 2022 Correlation Matrix

	Bitcoin	Ethereum	Altcoins	Stocks	Bonds	Commodities
Bitcoin	1,00					
Ethereum	0,84	1,00				
Altcoins	0,72	0,78	1,00			
Stocks	-0,01	0,06	0,10	1,00		
Bonds	0,02	0,03	-0,01	0,08	1,00	
Commodities	-0,03	-0,02	-0,02	0,08	-0,10	1,00

Table 14. 2023 Correlation Matrix

Appendix 4. Weights of Monthly and Quarterly Rebalancing Intervals

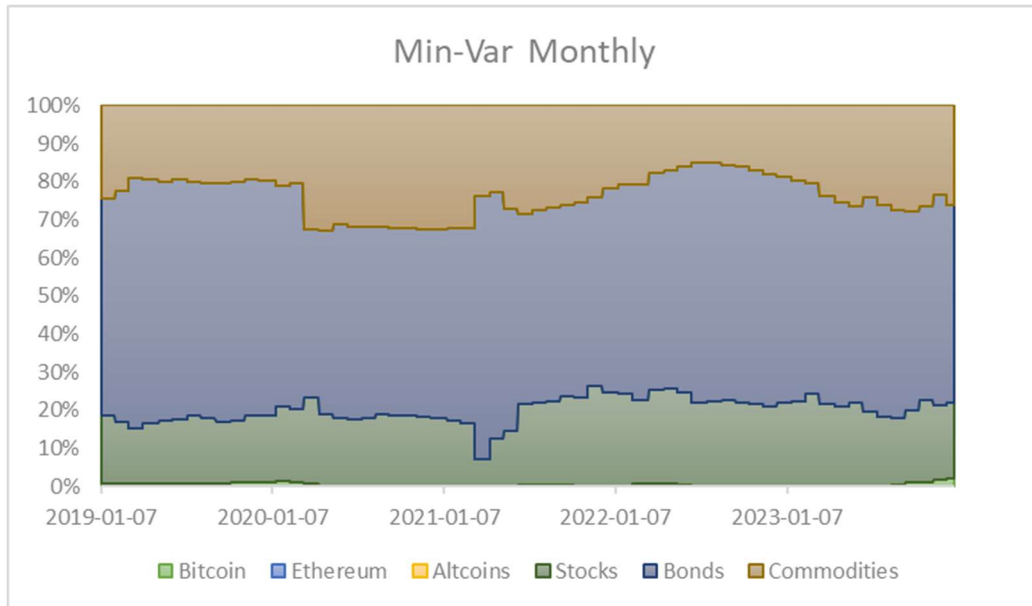


Figure 30. Min-Var Monthly Rebalancing

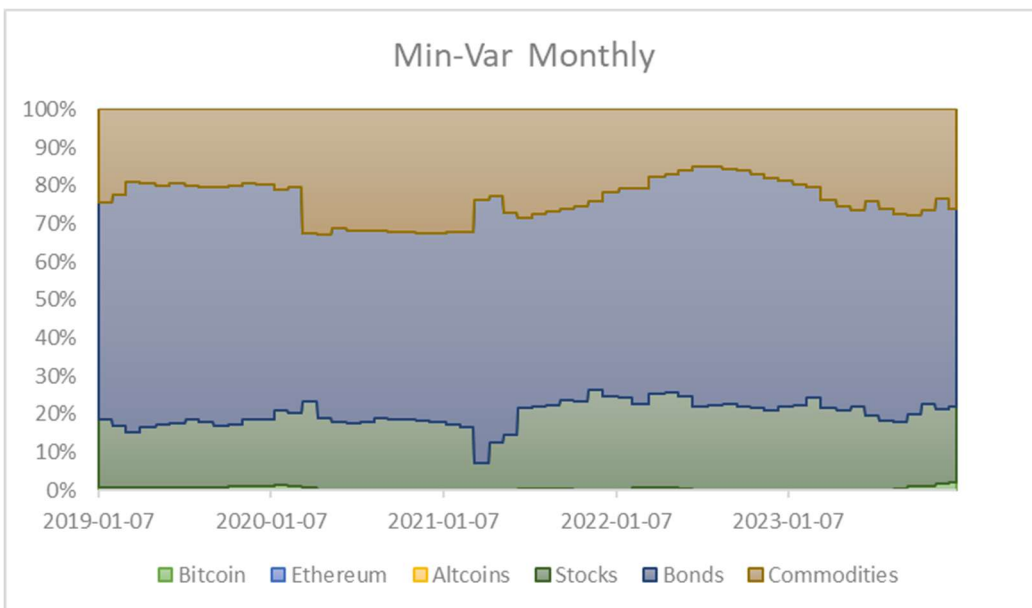


Figure 31. Min-Var Monthly Rebalancing with GVBC

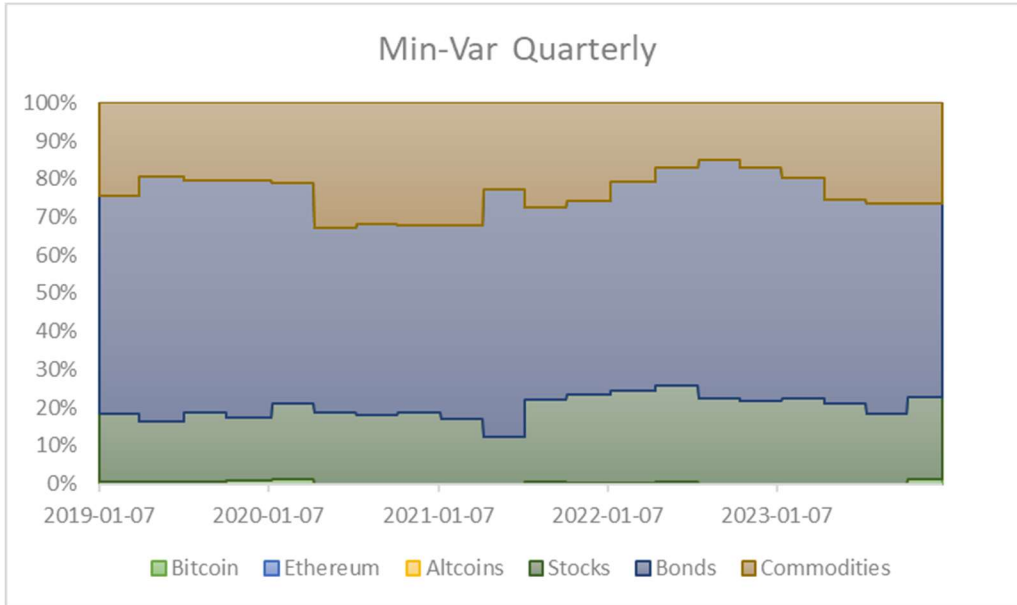


Figure 32. Min-Var Quarterly Rebalancing

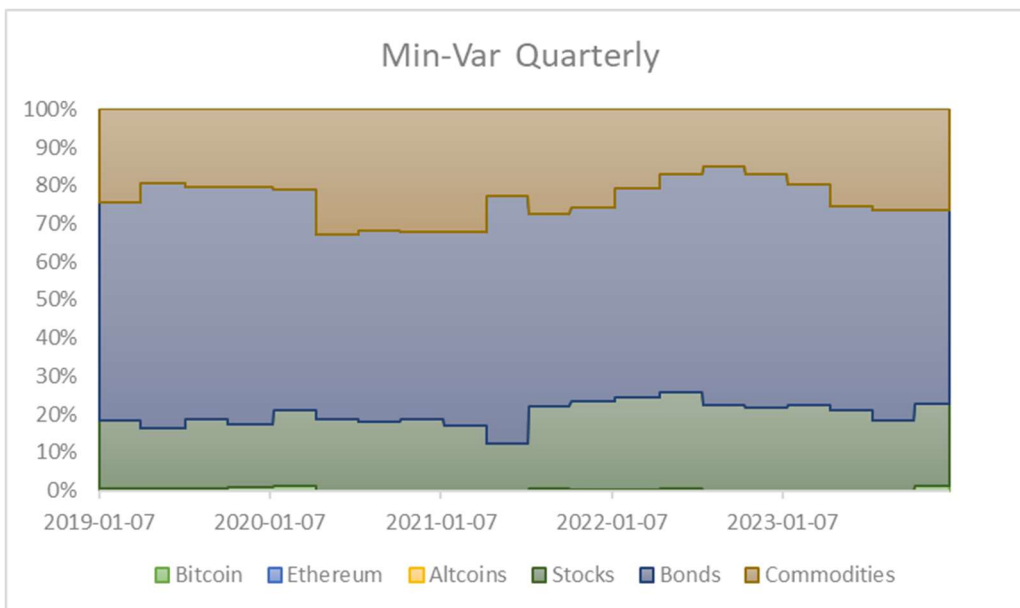


Figure 33. Min-Var Quarterly Rebalancing with GVBC

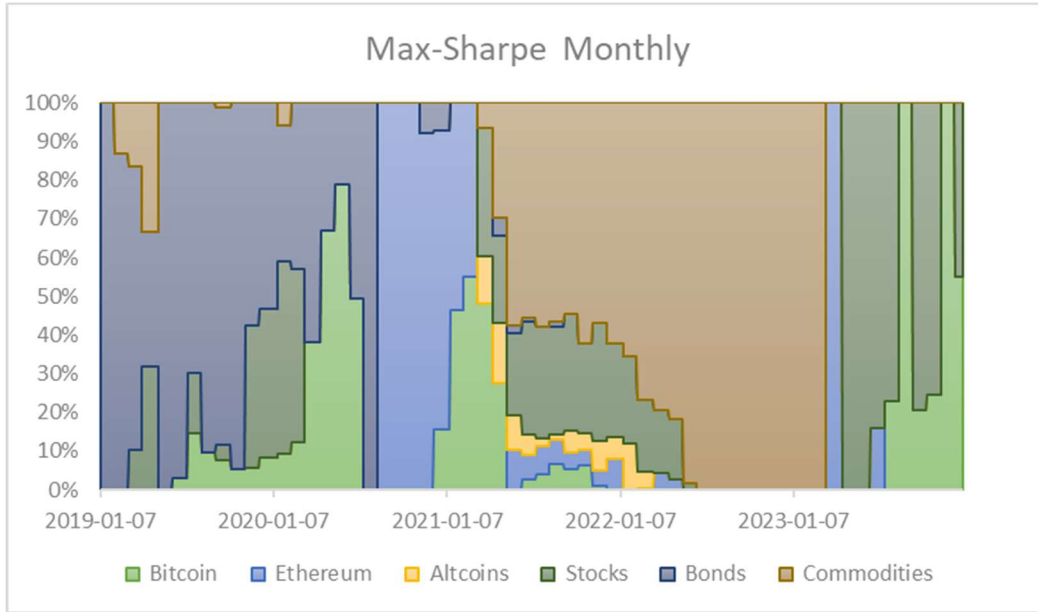


Figure 34. Max-Sharpe Monthly Rebalancing

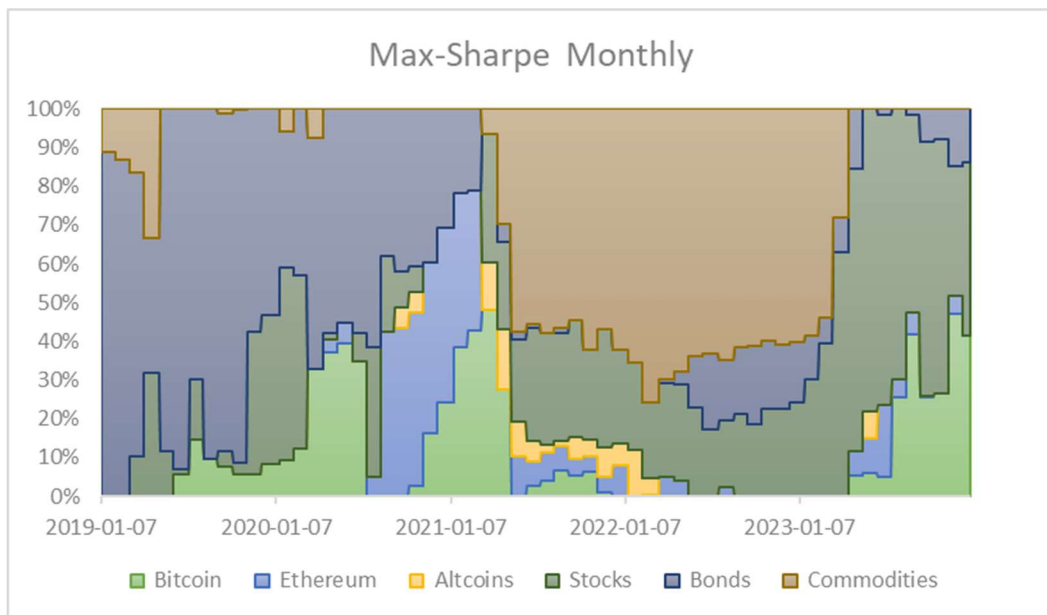


Figure 35. Max-Sharpe Monthly Rebalancing with GVBC

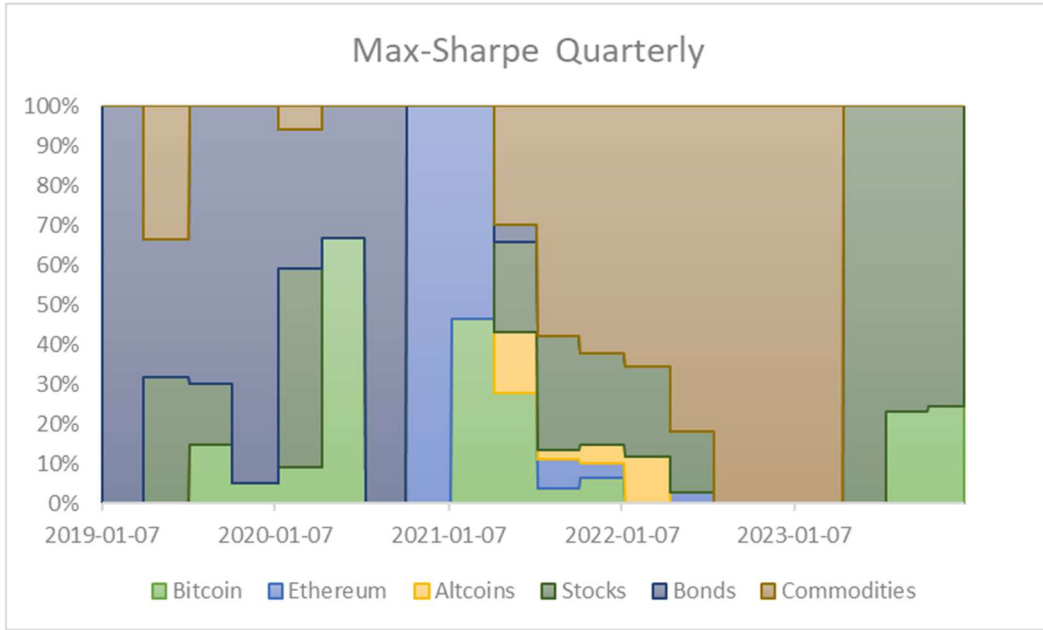


Figure 36. Max-Sharpe Quarterly Rebalancing

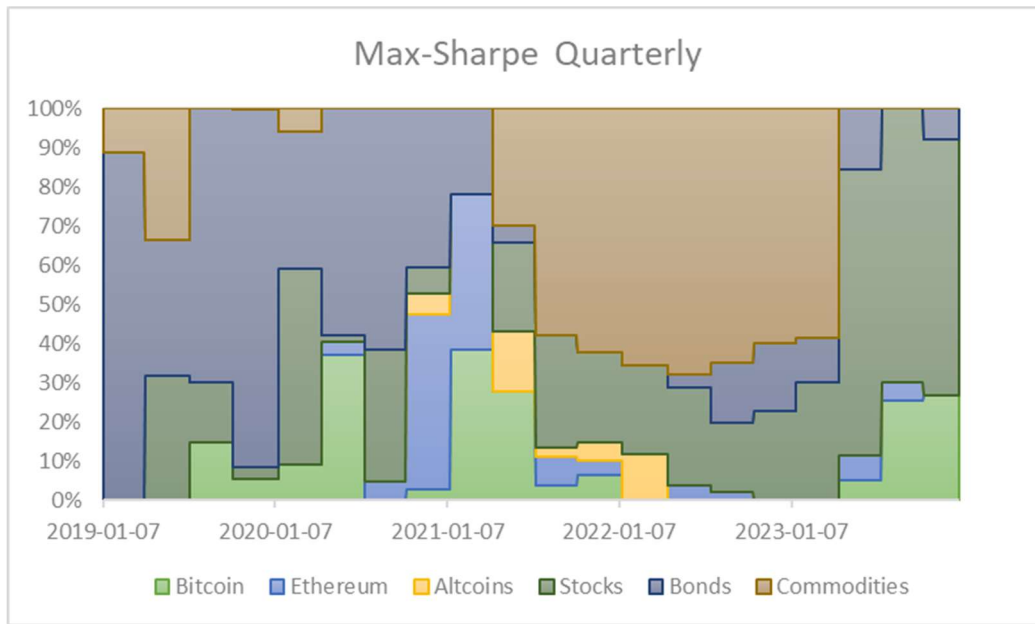


Figure 37. Max-Sharpe Quarterly Rebalancing with GVBC

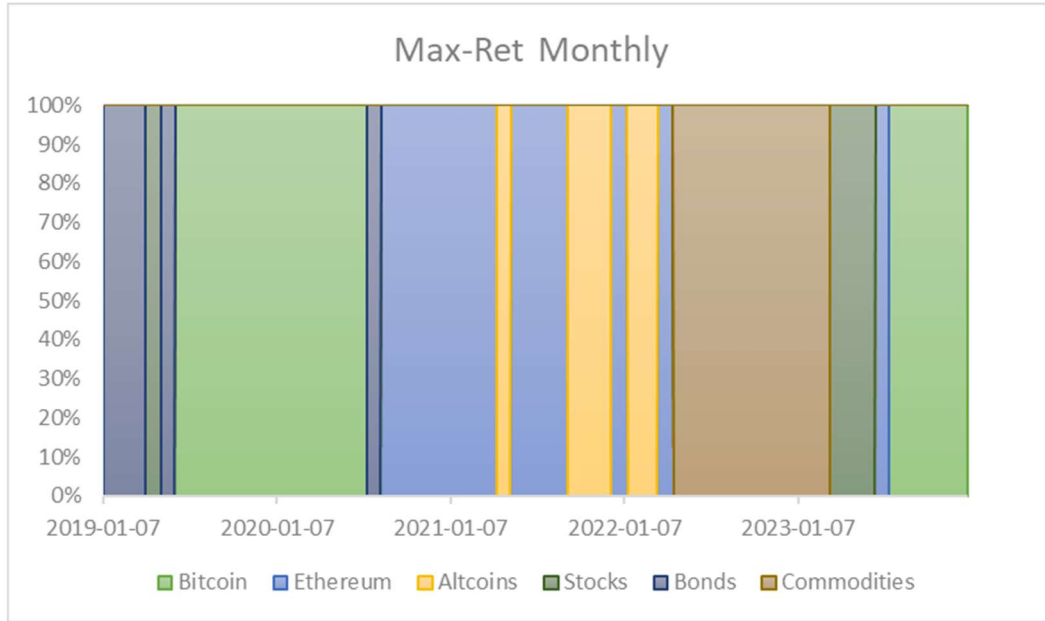


Figure 38. Max-Ret Monthly Rebalancing

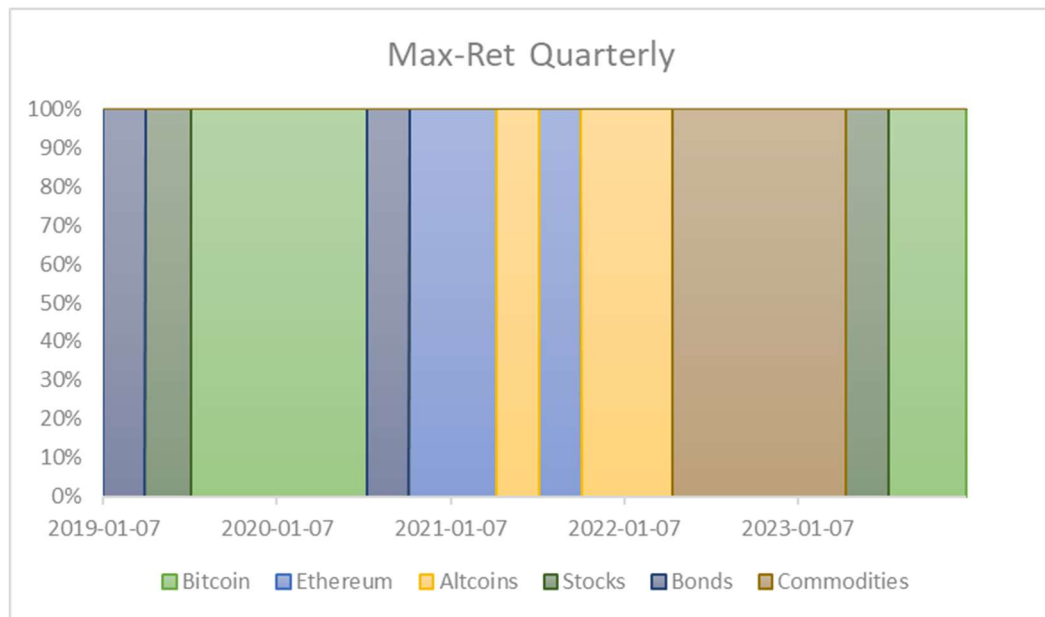


Figure 39. Max-Ret Quarterly Rebalancing

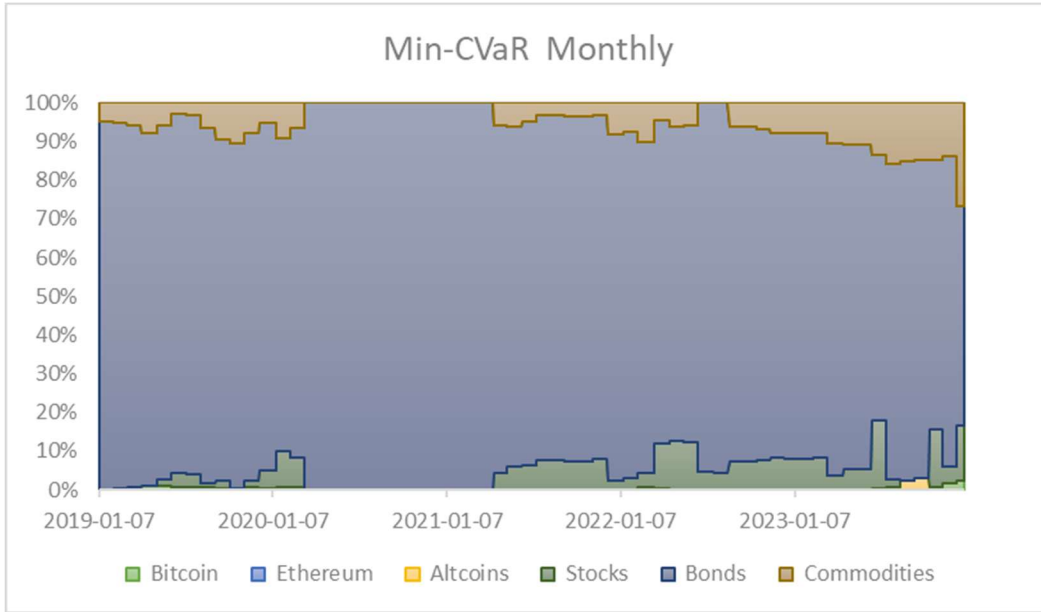


Figure 40. Min-CVaR Monthly Rebalancing

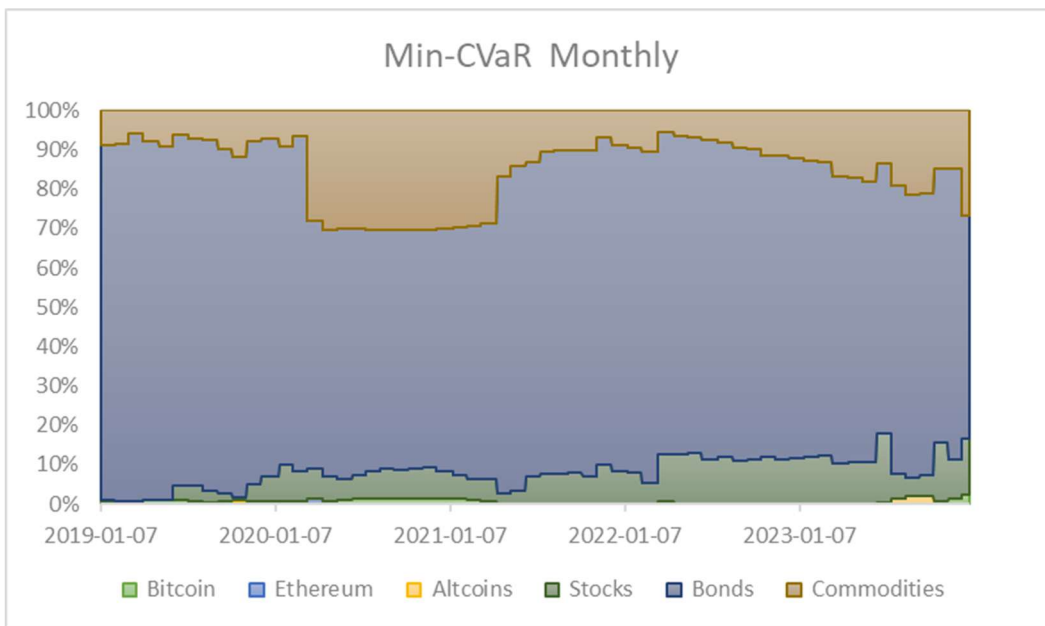


Figure 41. Min-CVaR Monthly Rebalancing with GVBC

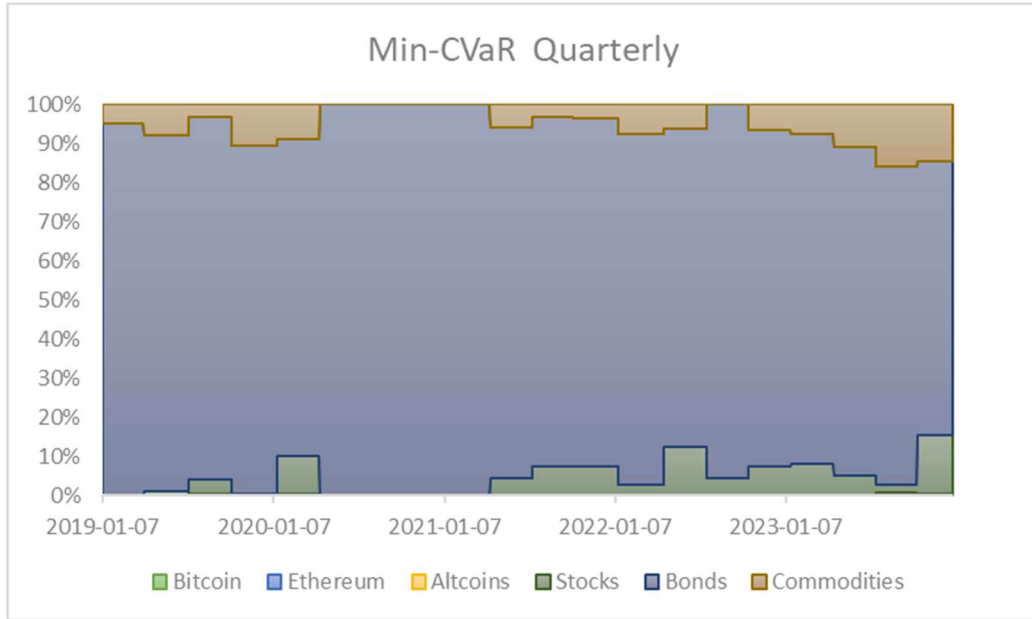


Figure 42. Min-CVaR Quarterly Rebalancing

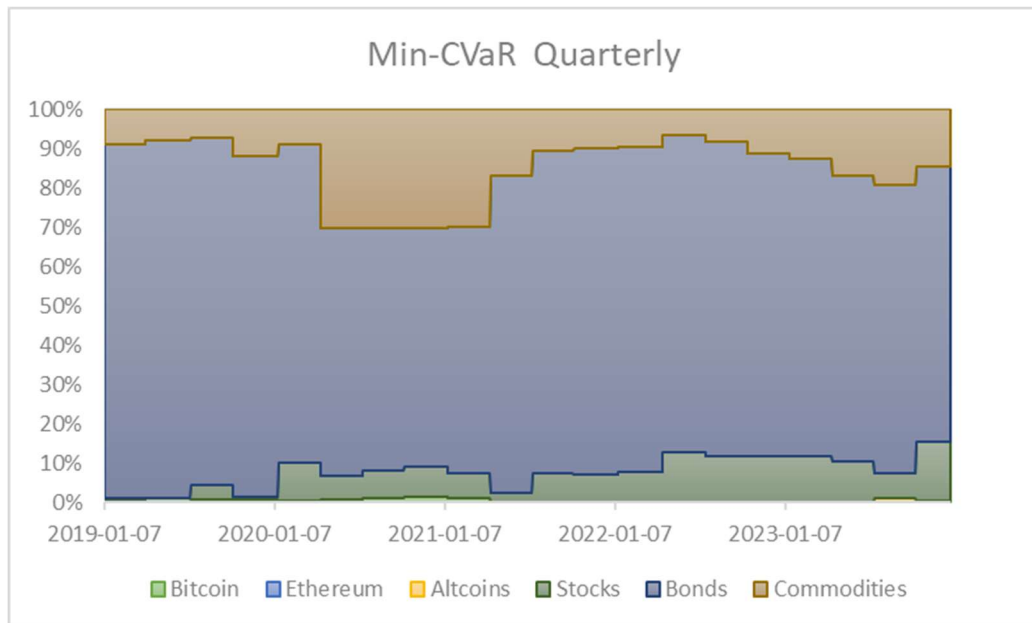


Figure 43. Min-CVaR Quarterly Rebalancing with GVBC

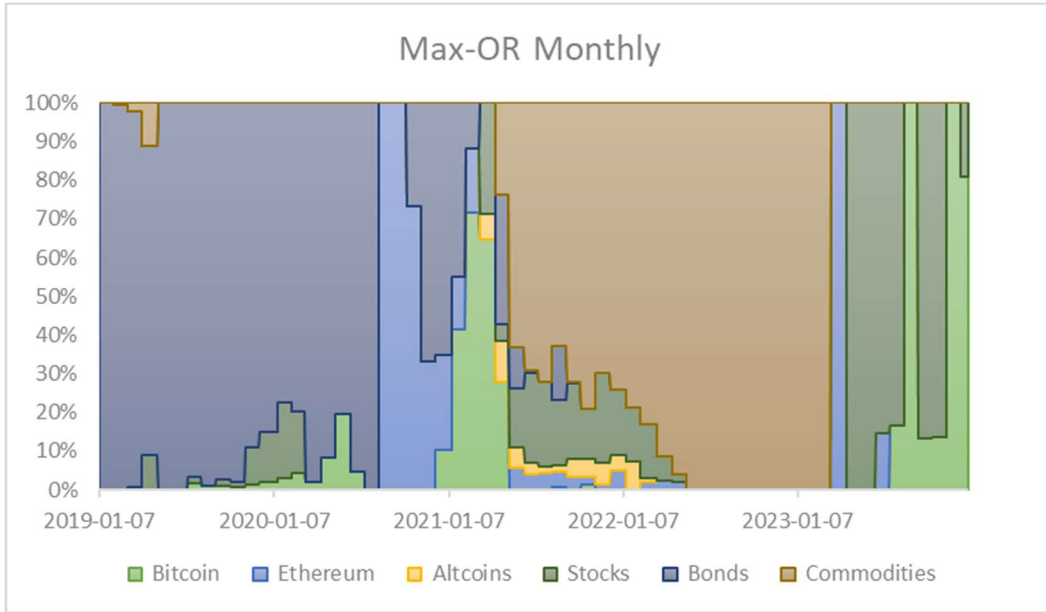


Figure 44. Max-OR Monthly Rebalancing

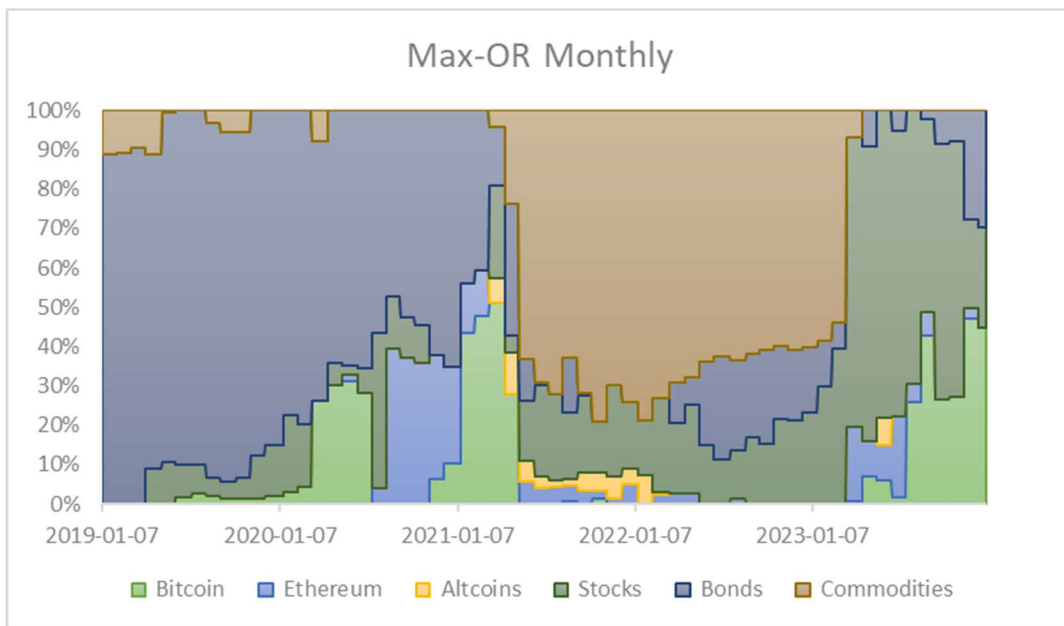


Figure 45. Max-OR Monthly Rebalancing with GVBC

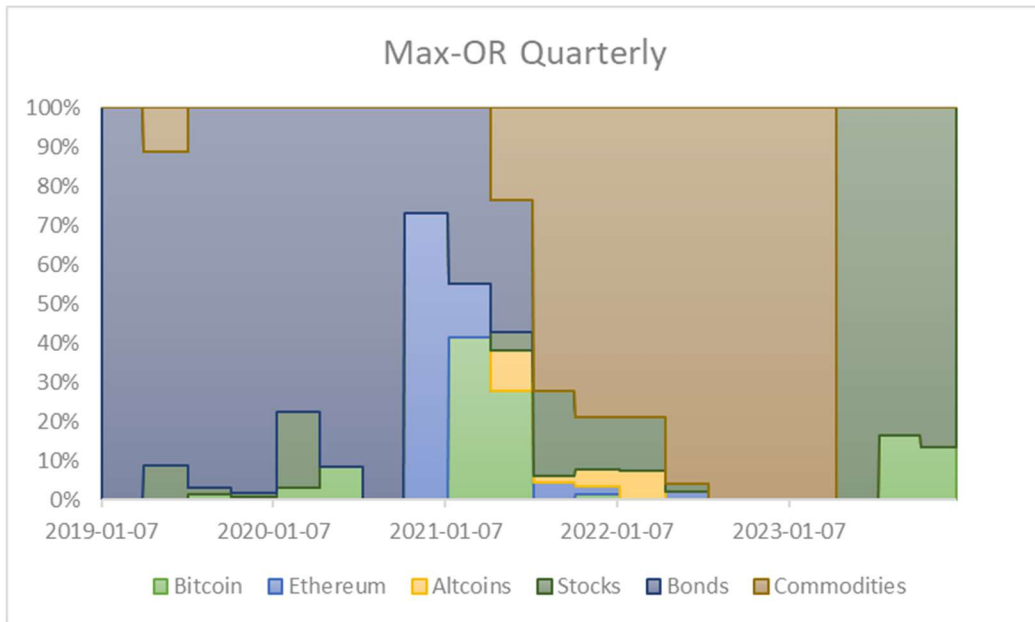


Figure 46. Max-OR Quarterly Rebalancing

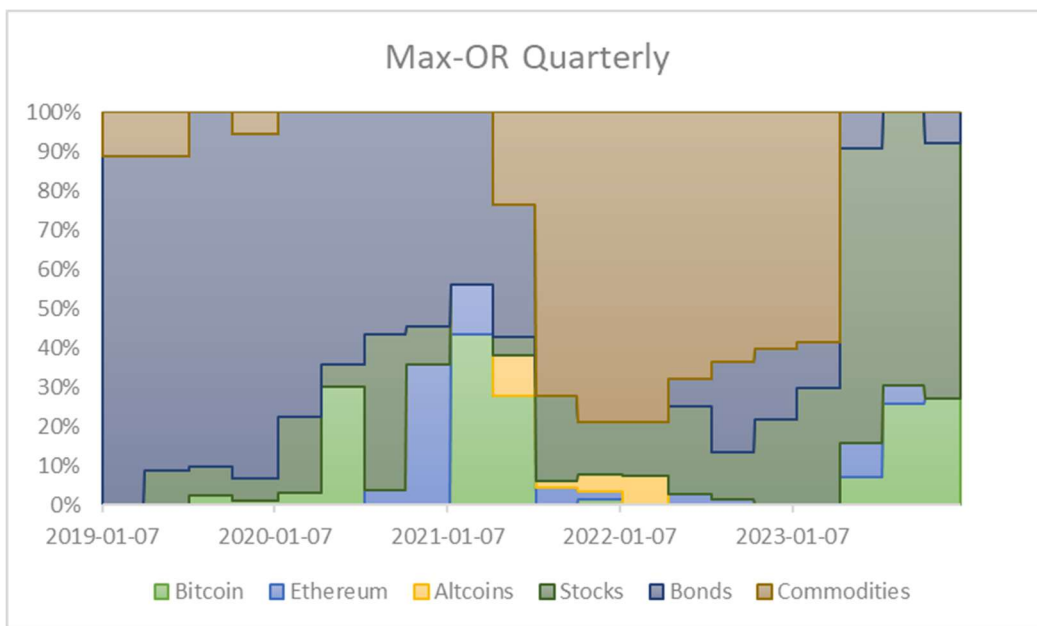


Figure 47. Max-OR Quarterly Rebalancing with GVBC

Appendix 5. Portfolio Weights without Cryptocurrencies

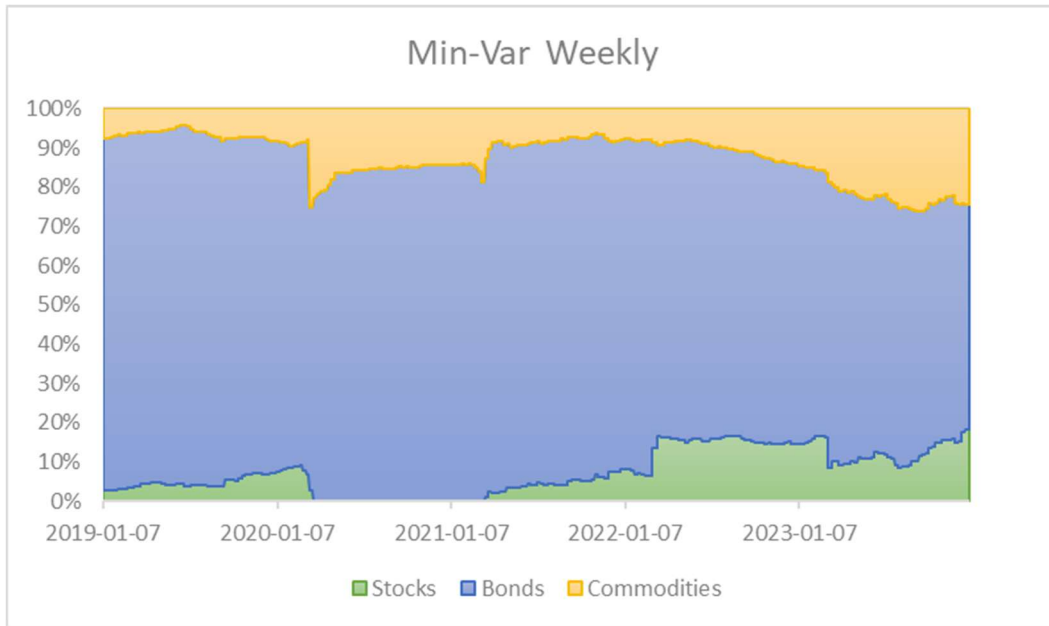


Figure 48. Min-Var Weekly without Cryptocurrencies

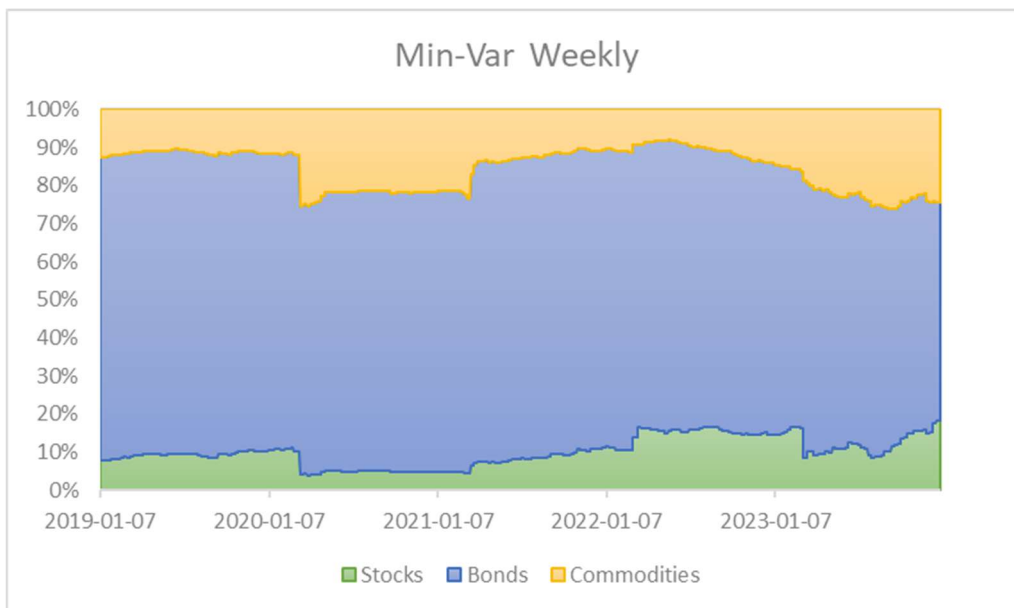


Figure 49. Min-Var Weekly without Cryptocurrencies with GVBC

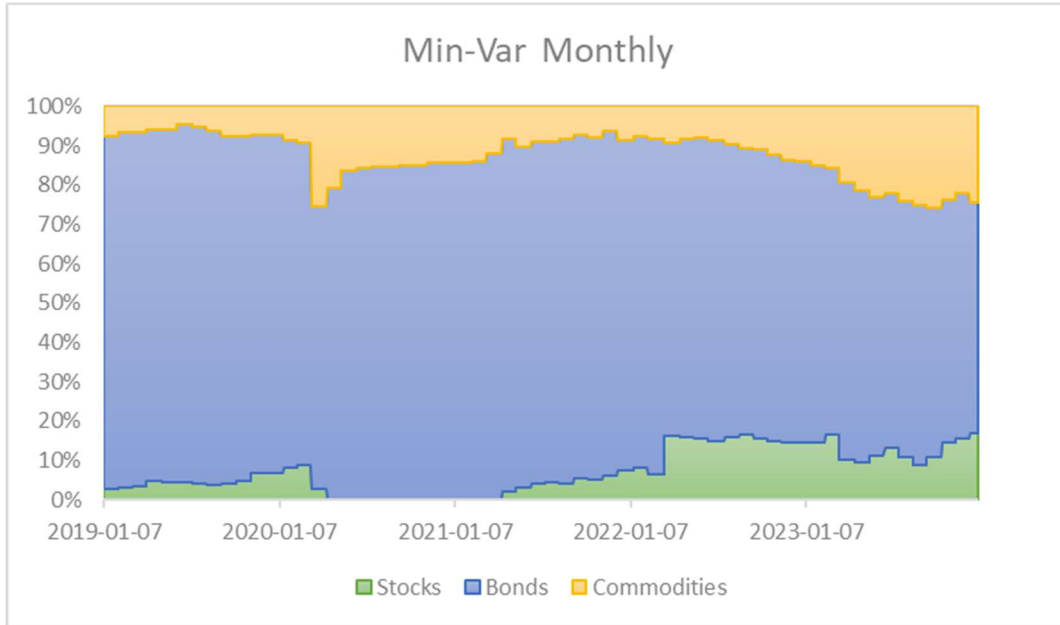


Figure 50. Min-Var Monthly without Cryptocurrencies

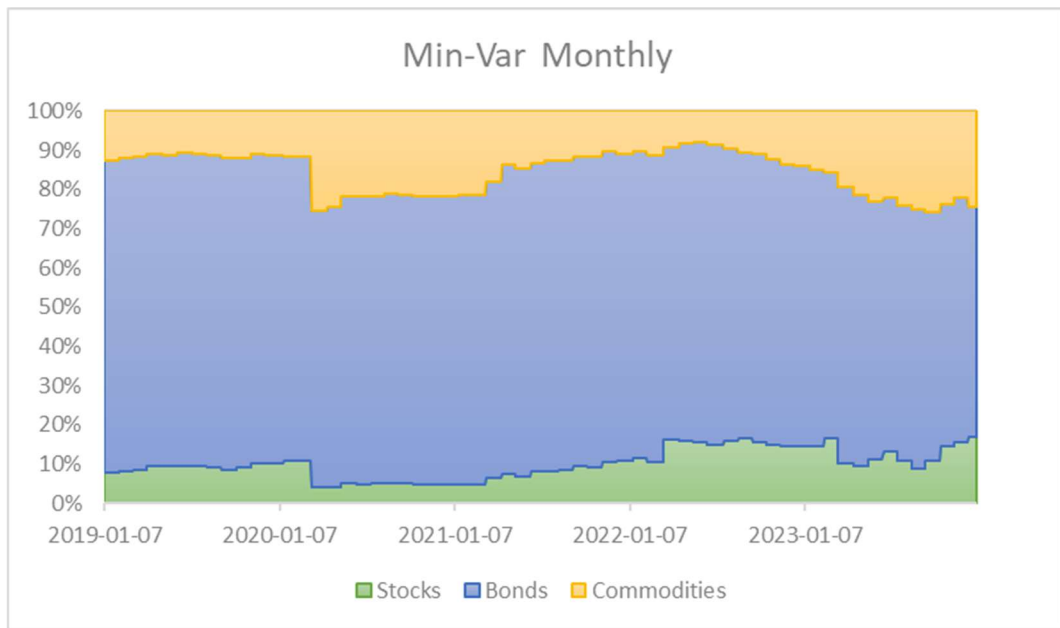


Figure 51. Min-Var Monthly without Cryptocurrencies with GVBC

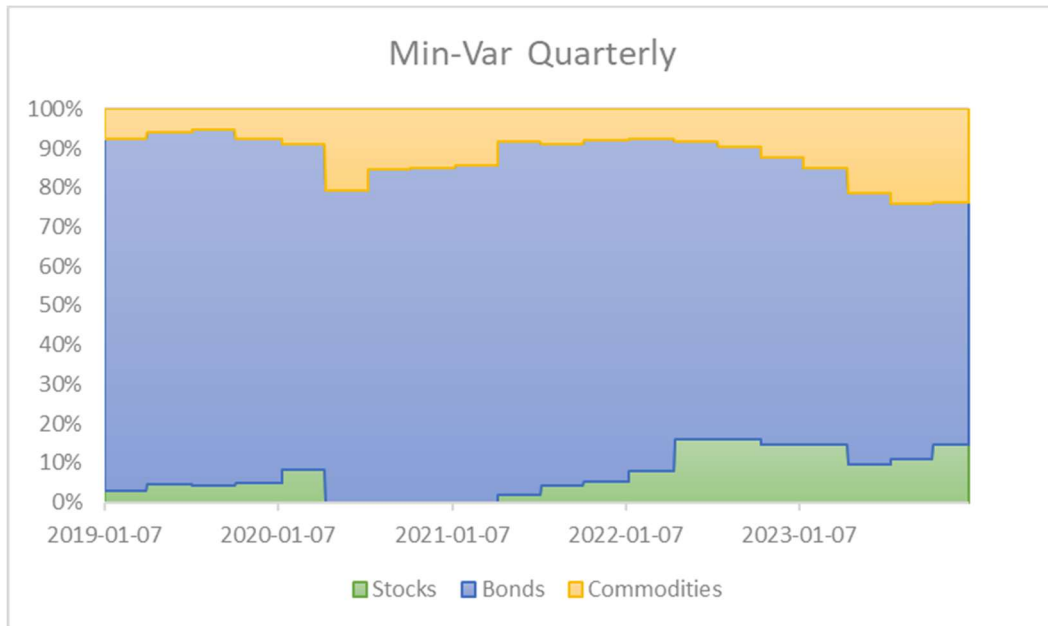


Figure 52. Min-Var Quarterly without Cryptocurrencies

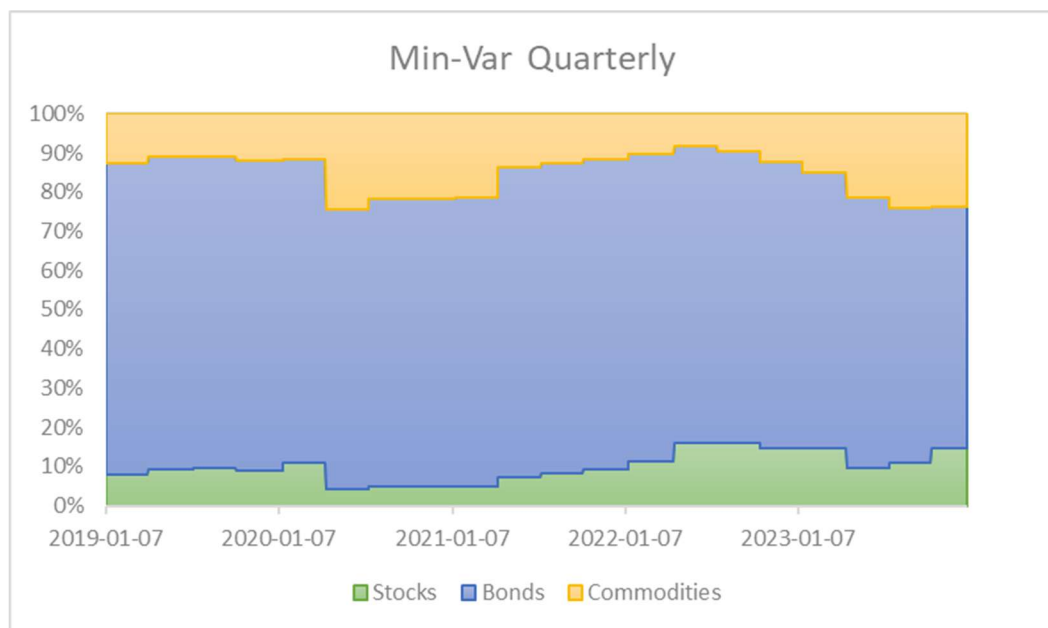


Figure 53. Min-Var Quarterly without Cryptocurrencies with GVBC

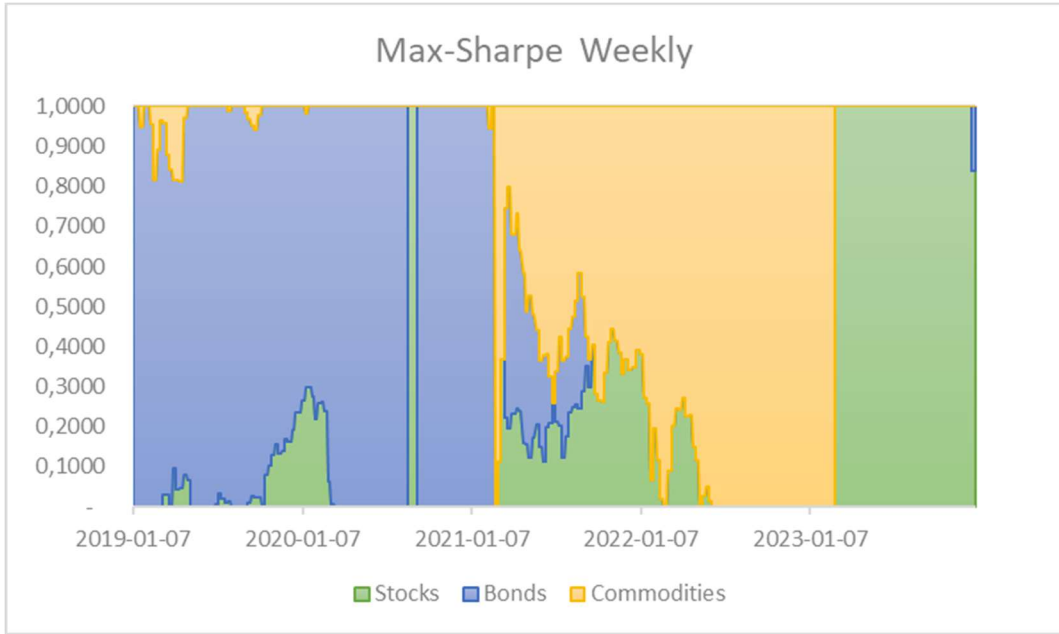


Figure 54. Max-Sharpe Weekly without Cryptocurrencies

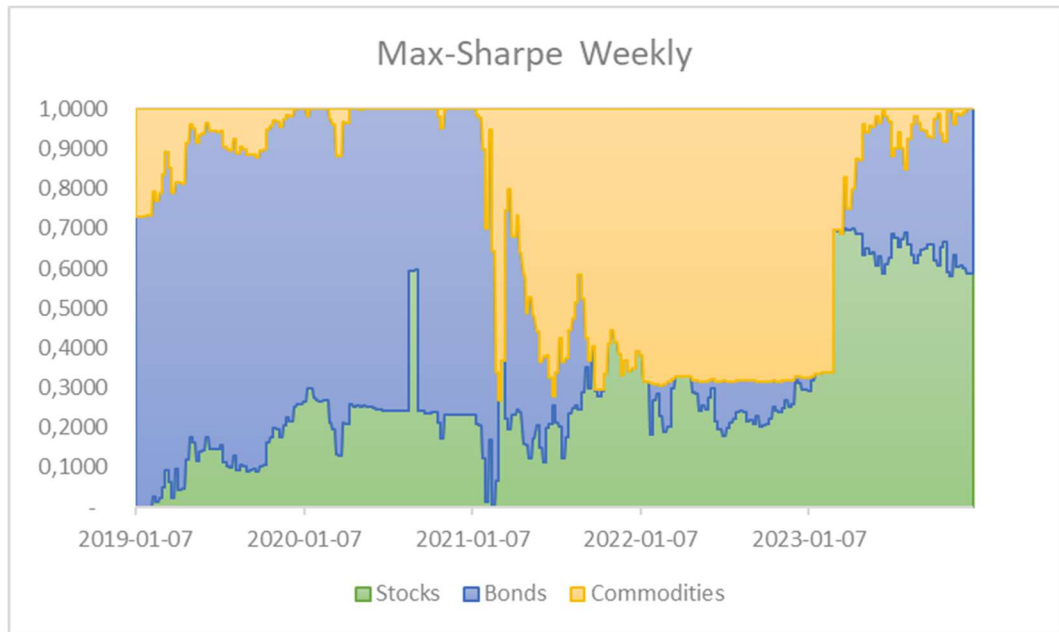


Figure 55. Max-Sharpe Weekly without Cryptocurrencies with GVBC

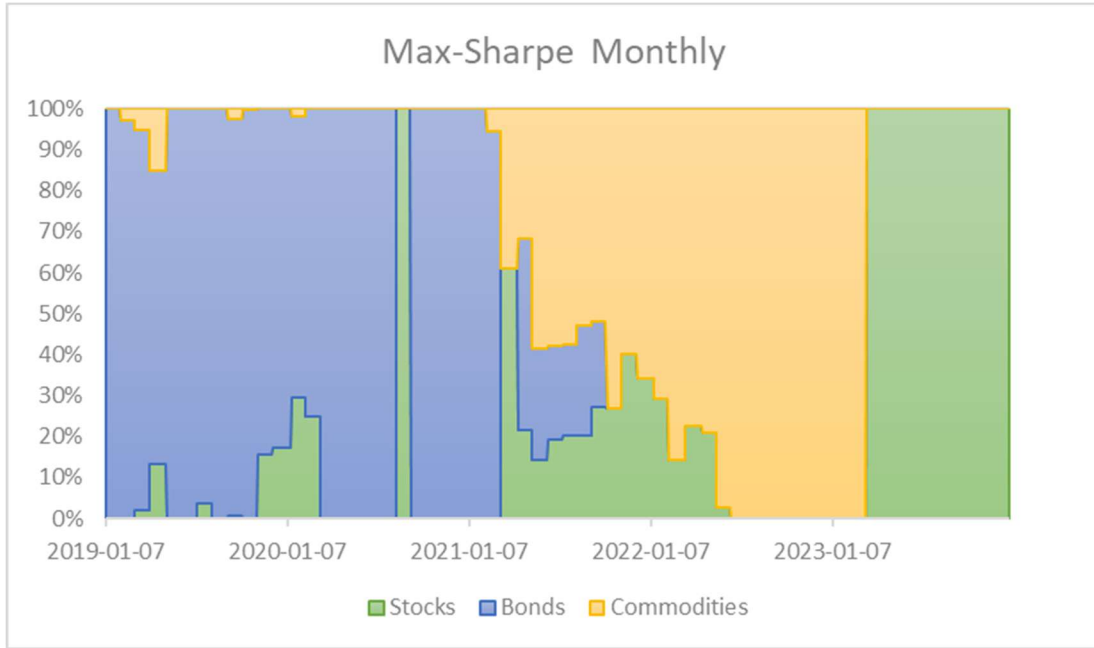


Figure 56. Max-Sharpe Monthly without Cryptocurrencies

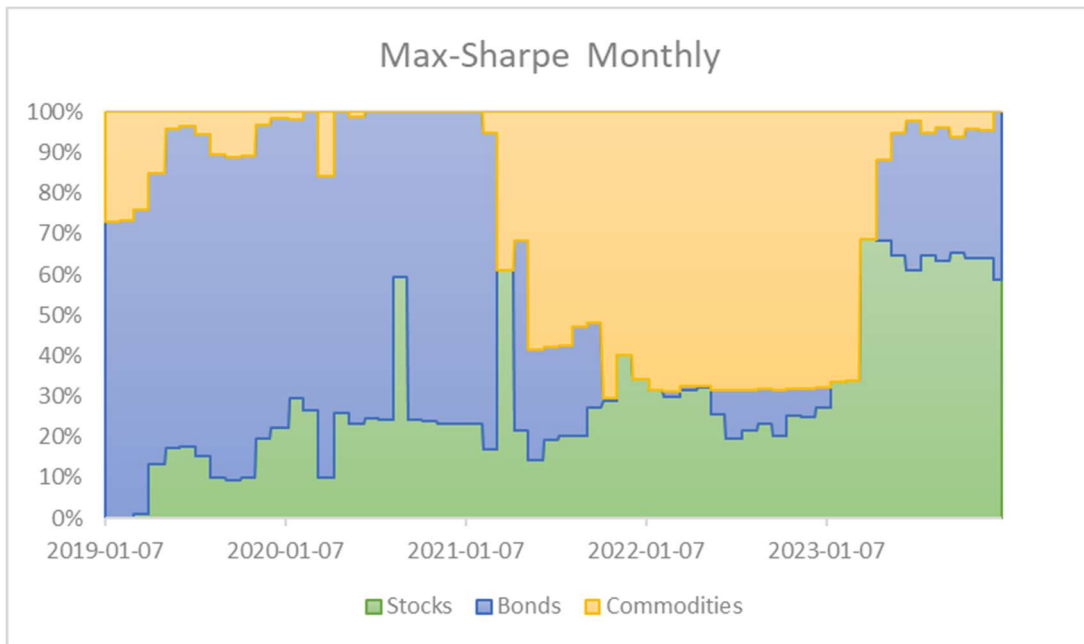


Figure 57. Max-Sharpe Monthly without Cryptocurrencies with GVBC

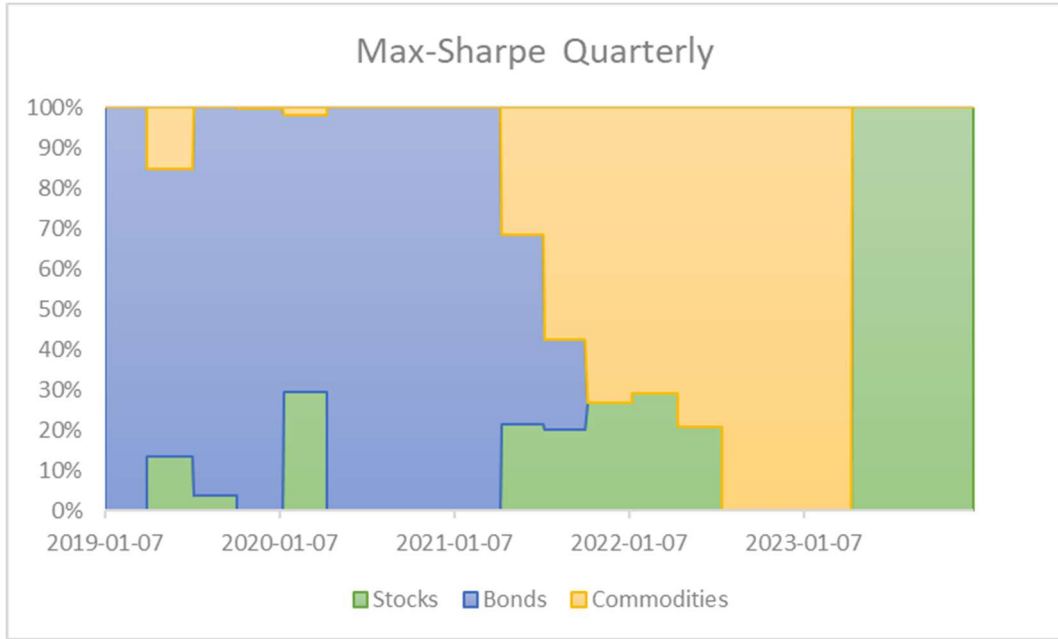


Figure 58. Max-Sharpe Quarterly without Cryptocurrencies

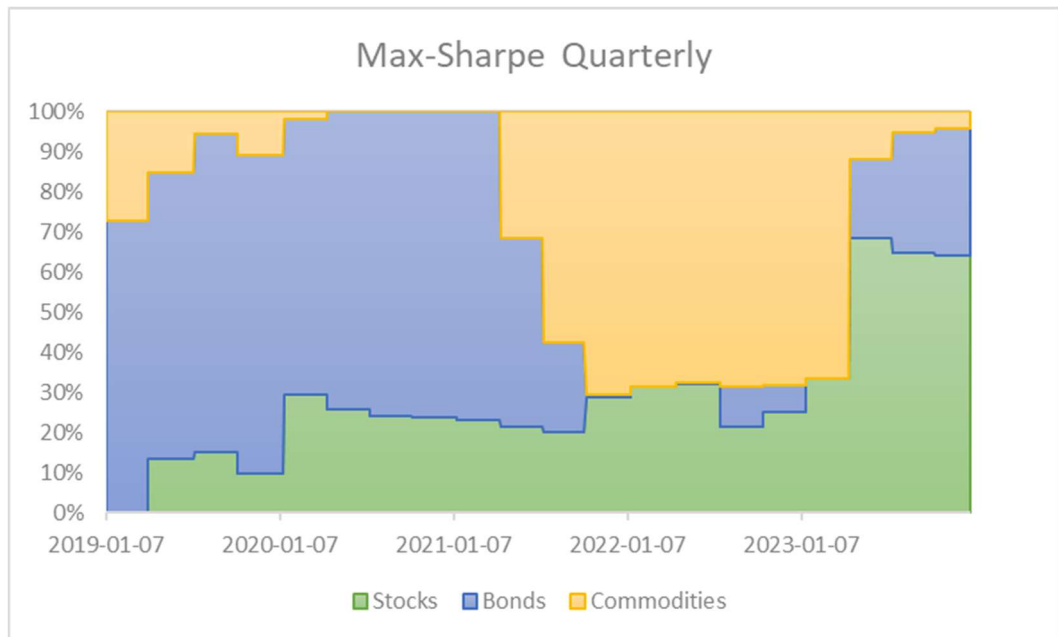


Figure 59. Max-Sharpe Quarterly without Cryptocurrencies with GVBC

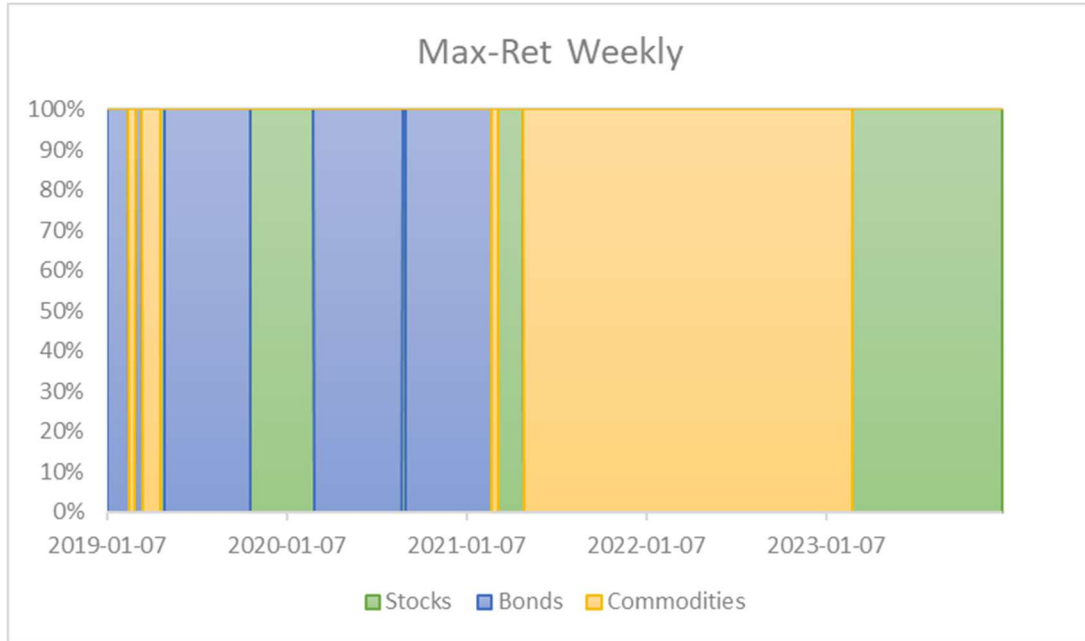


Figure 60. Max-Ret Weekly without Cryptocurrencies

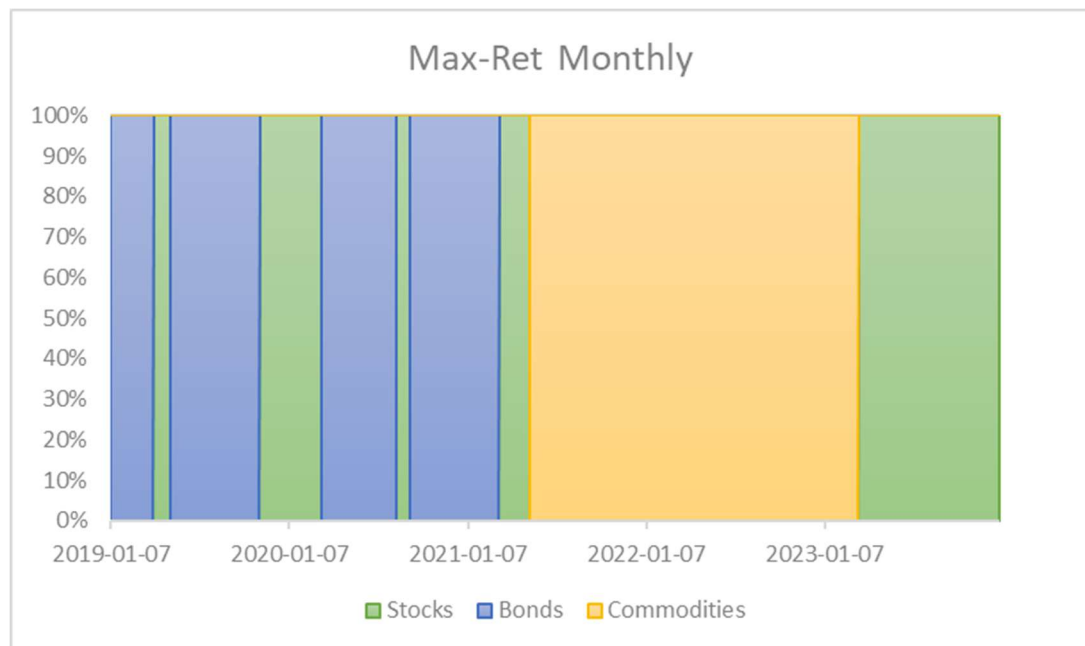


Figure 61. Max-Ret Monthly without Cryptocurrencies

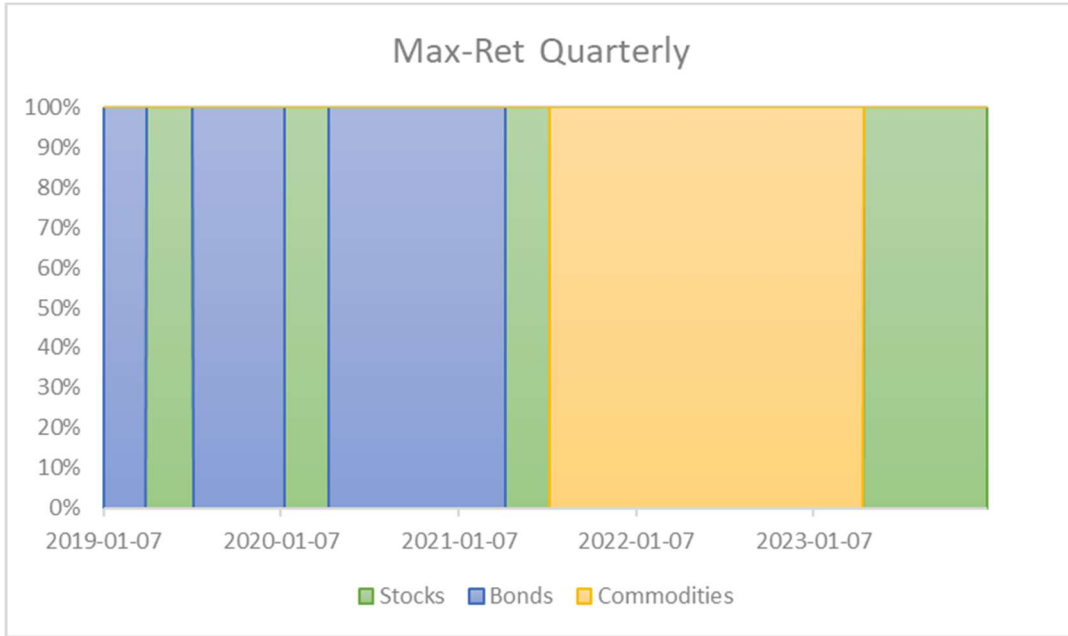


Figure 62. Max-Ret Quarterly without Cryptocurrencies

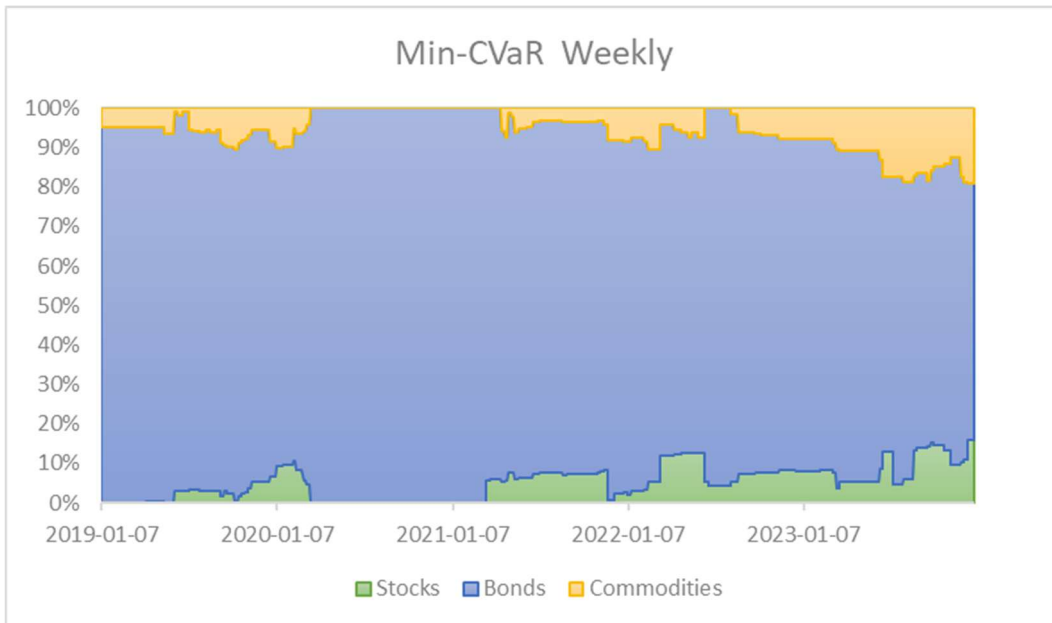


Figure 63. Min-CVaR Weekly without Cryptocurrencies

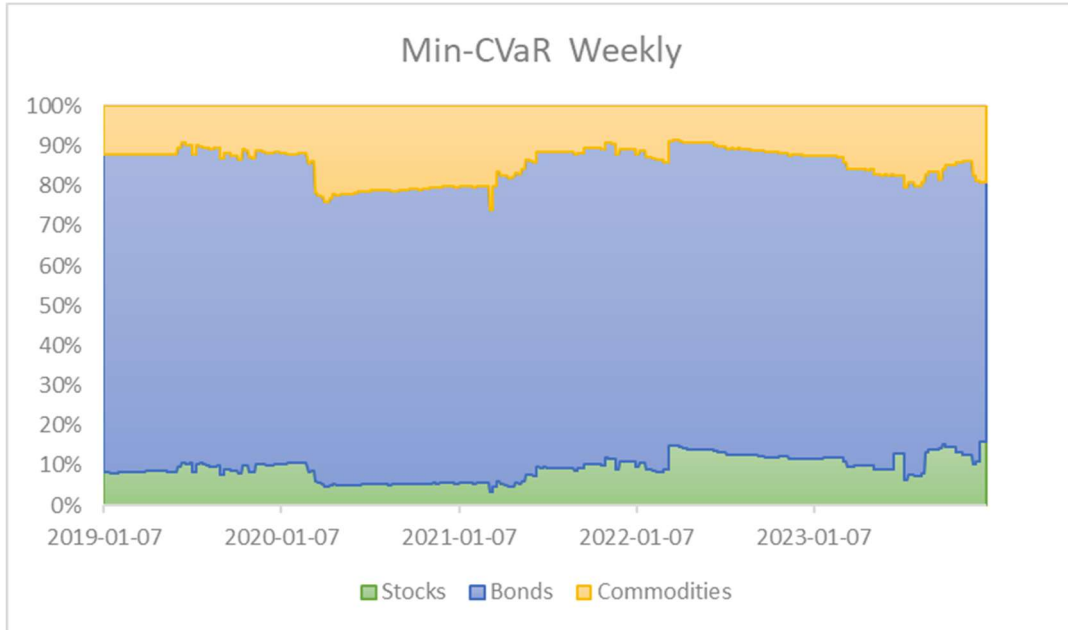


Figure 64. Min-CVaR Weekly without Cryptocurrencies with GVBC

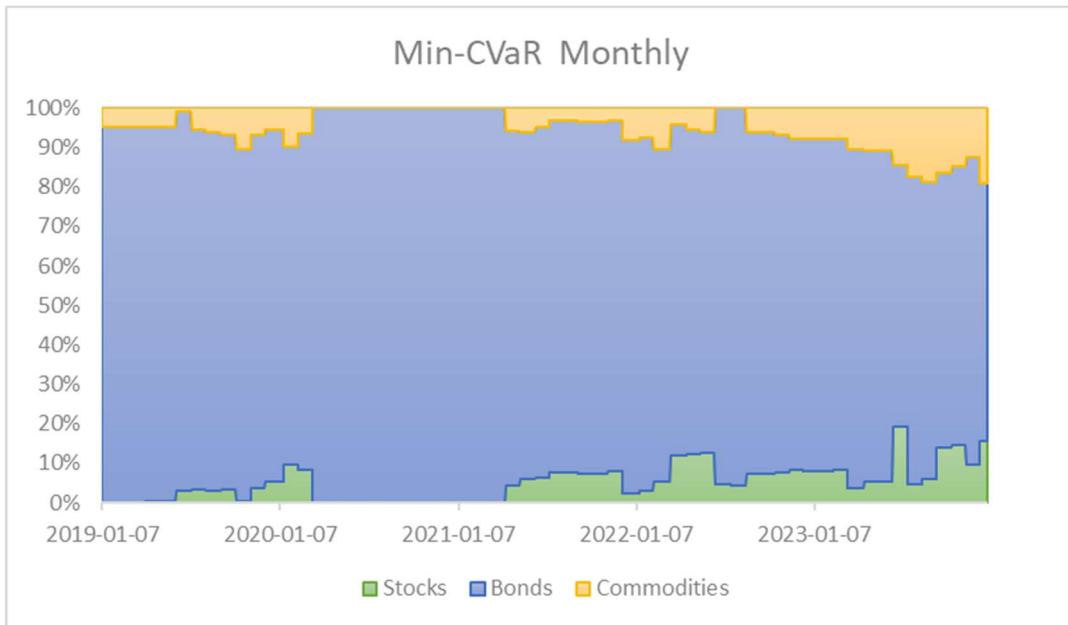


Figure 65. Min-CVaR Monthly without Cryptocurrencies

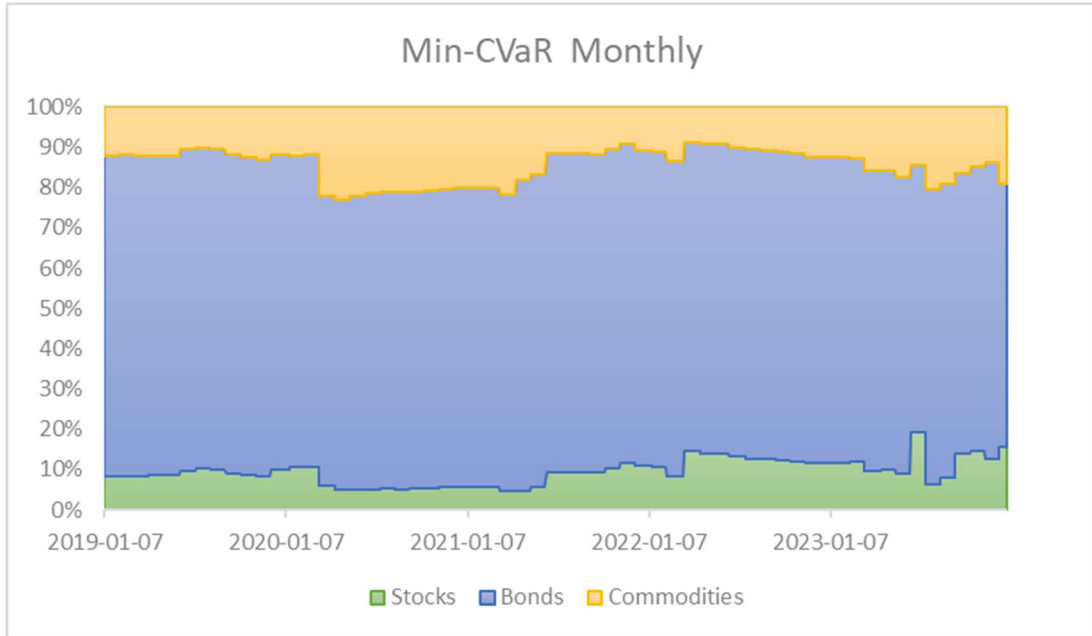


Figure 66. Min-CVaR Monthly without Cryptocurrencies with GVBC

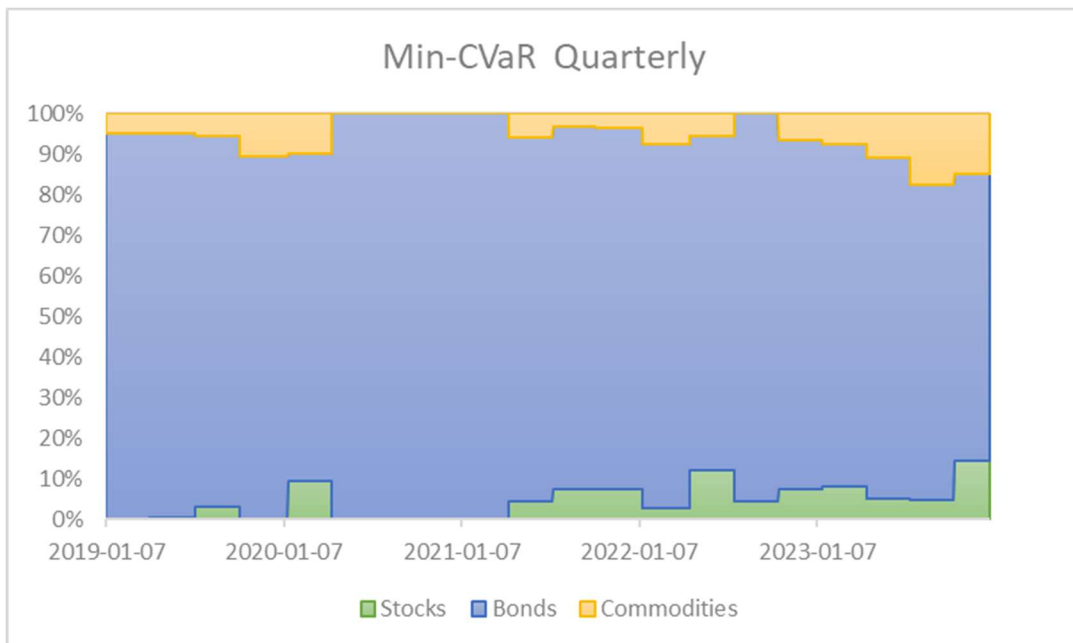


Figure 67. Min-CVaR Quarterly without Cryptocurrencies

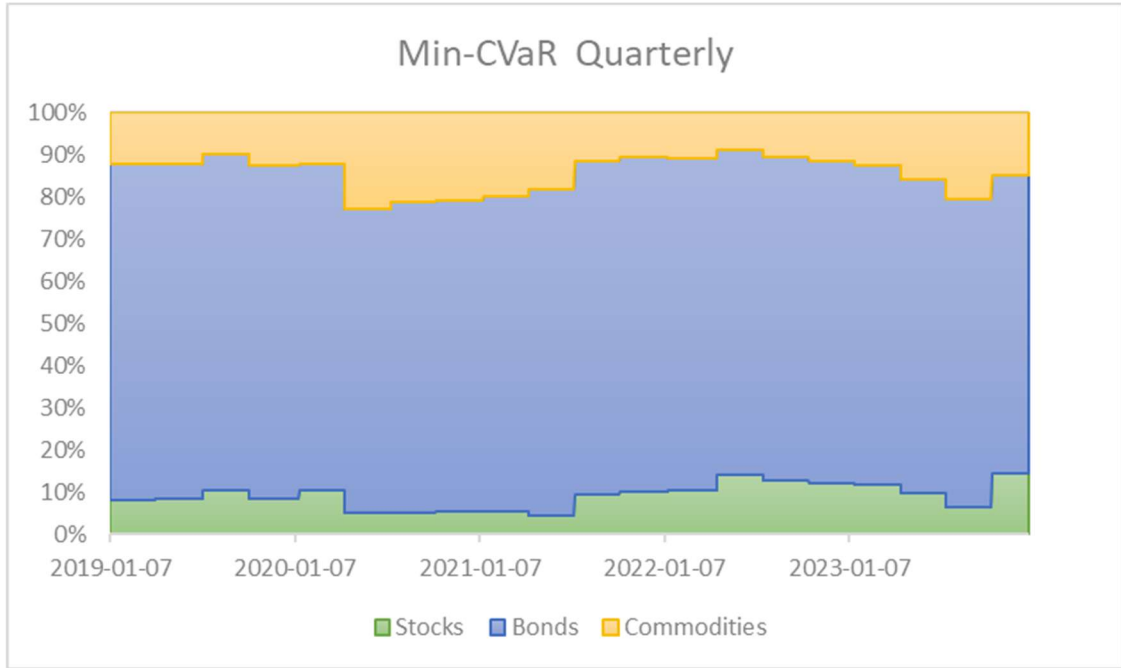


Figure 68. Min-CVaR Quarterly without Cryptocurrencies with GVBC

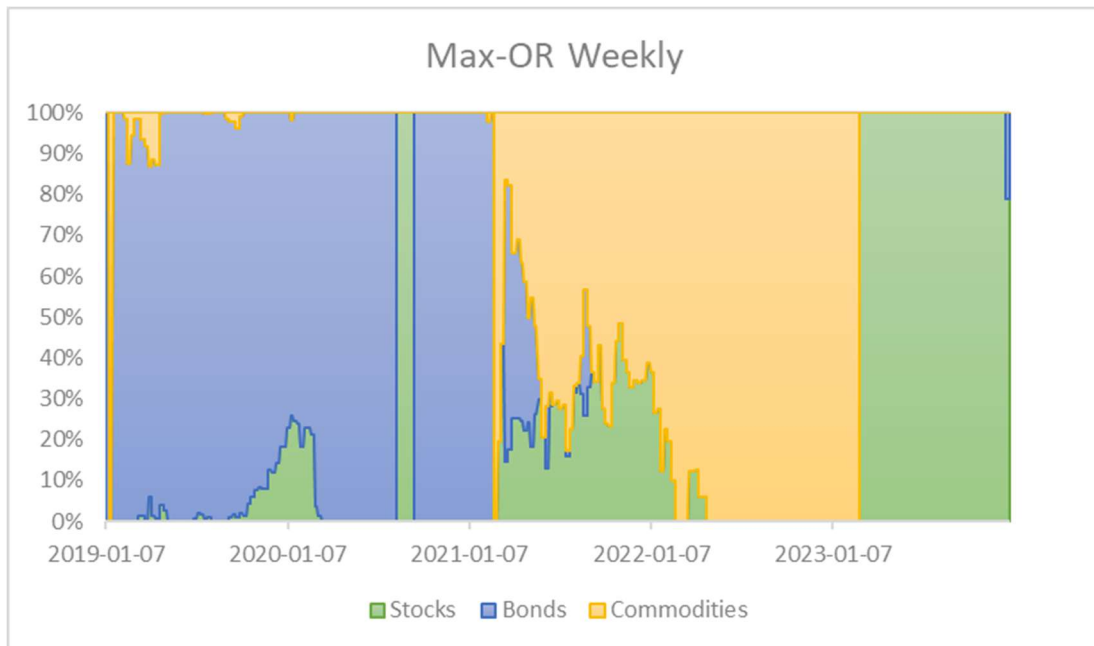


Figure 69. Max-OR Weekly without Cryptocurrencies

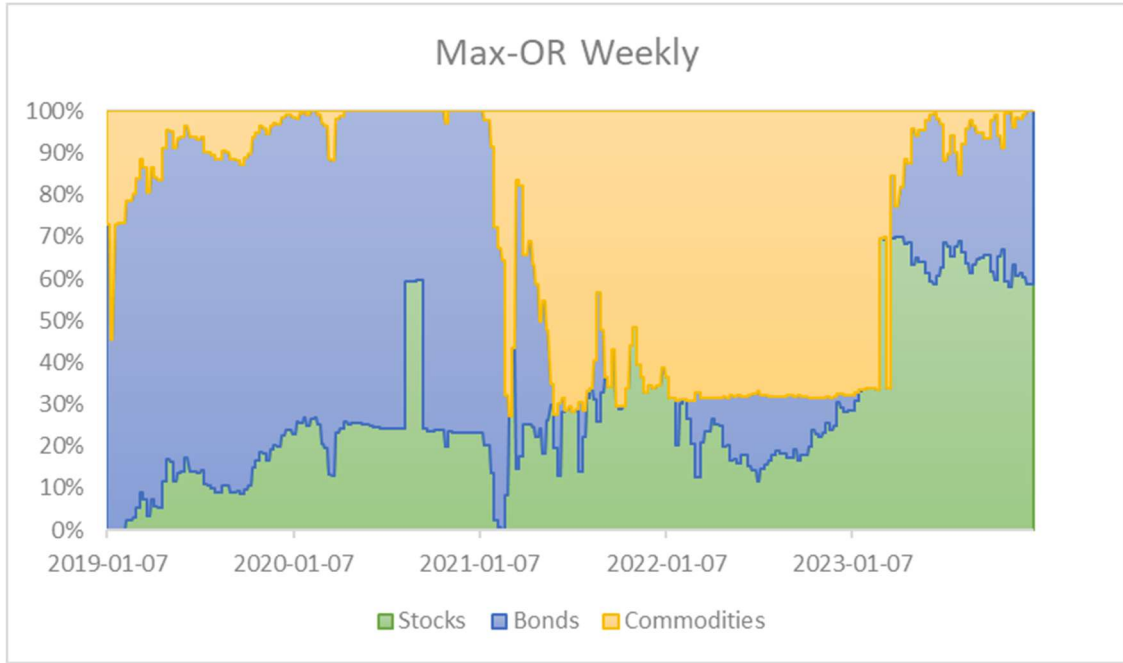


Figure 70. Max-OR Weekly without Cryptocurrencies with GVBC

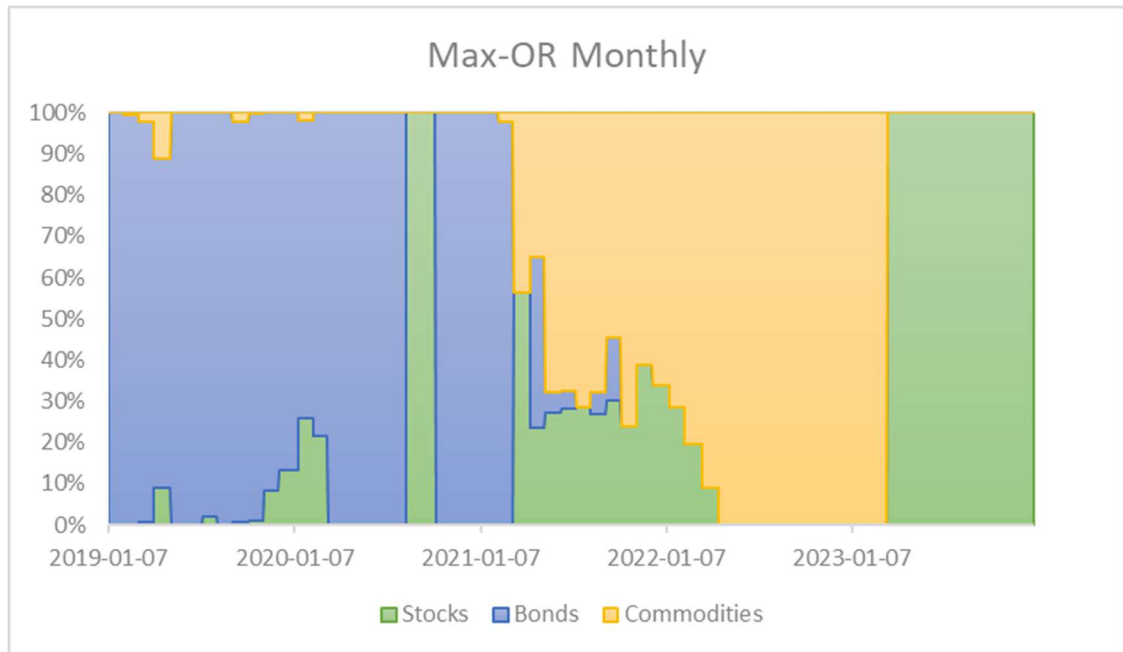


Figure 71. Max-OR Monthly without Cryptocurrencies

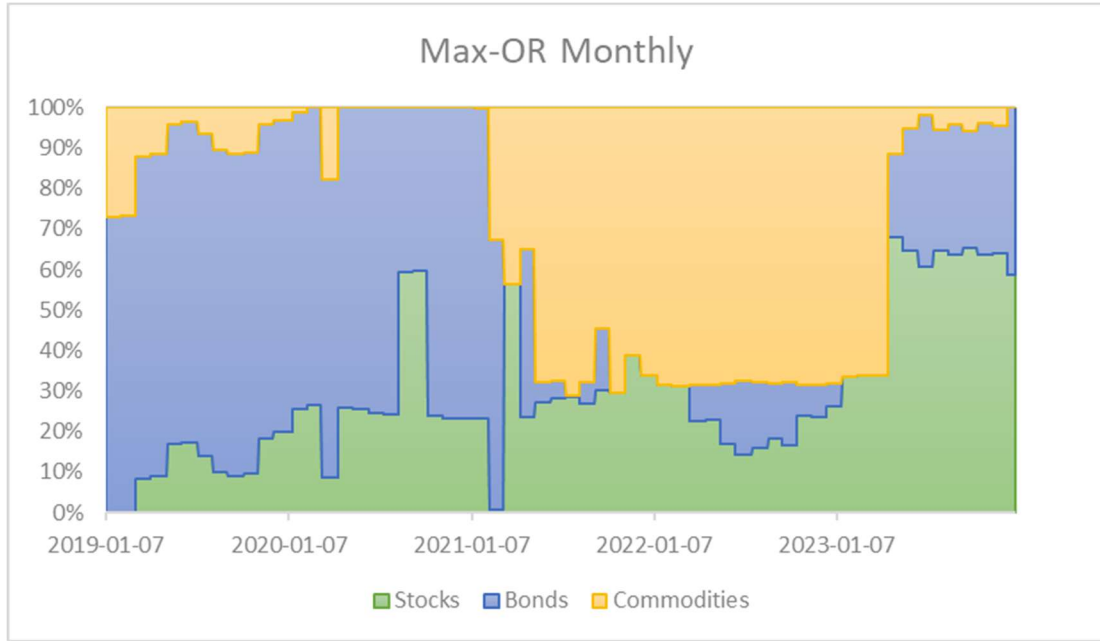


Figure 72. Max-OR Monthly without Cryptocurrencies with GVBC

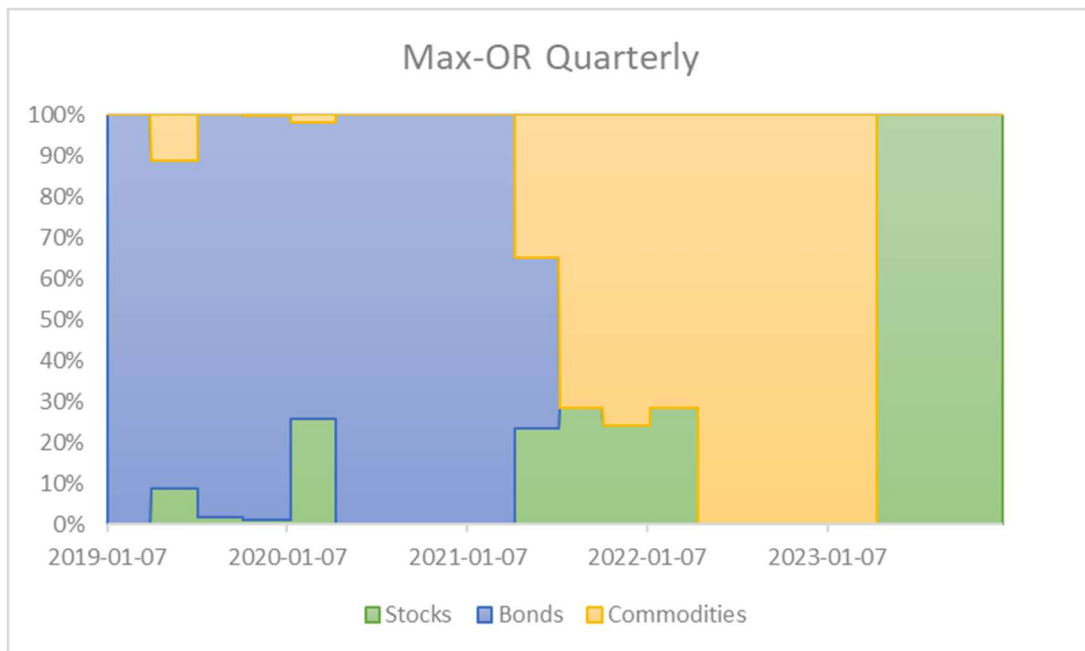


Figure 73. Max-OR Quarterly without Cryptocurrencies

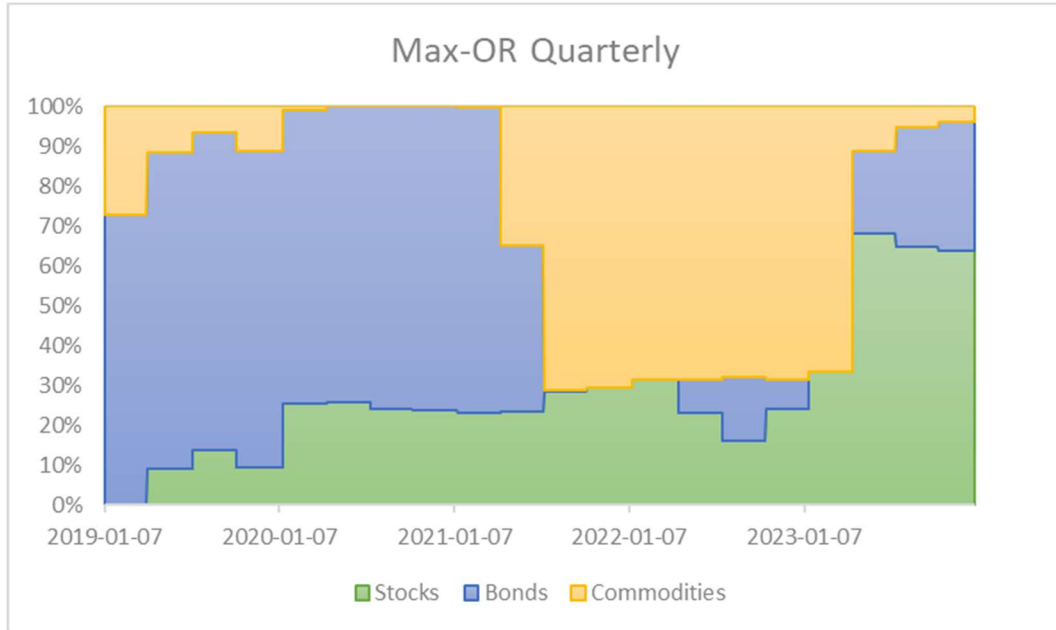


Figure 74. Max-OR Quarterly without Cryptocurrencies with GVBC