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Author(s): Chowdury, Md. Abdul Malek; Nath, Rahul; Shukla, Amit K.; Rauniyar, Amit; Muhuri, Pranab K.

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Year: 2025

Version: Accepted manuscript

Please cite the original version:

Chowdury, M. A.M., Nath, R., Shukla, A., Rauniyar, A. & Muhuri, P. K. (2025). Multi-objective Multi-Factorial Evolutionary Algorithm for Fuzzy Reliability Redundancy Allocation. In: 2025 IEEE International Conference on Fuzzy Systems (FUZZ). https://doi.org/10.1109/FUZZ62266.2025.11152082

Multi-objective Multi-Factorial Evolutionary Algorithm for Fuzzy Reliability Redundancy Allocation

Md. Abdul Malek Chowdury[1], Rahul Nath[2], Amit K. Shukla[3], Amit Rauniyar[1], Pranab K. Muhuri[1]
[1]*Department of Computer Science and Engineering*, *South Asian University*, Rajpur Road, Maidan Garhi, New Delhi-110068, India
[2]*Department of Informatics, University of Bergen*, Thormøhlens Gate 55, 5008 Bergen, Norway
[3]*School of Technology and Innovations, University of Vaasa*, Wolffintie 34, FI-65200 Vaasa, Finland
mdamalekchy@gmail.com; rahul.nath@uib.no; amit.shukla@uwasa.fi; amitrauniyar90@gmail.com; pranabmuhuri@cs.sau.ac.in

Abstract— **The Reliability Redundancy Allocation Problem (RRAP) seeks to optimize system reliability by enhancing component reliability and reducing redundancy under constraints such as cost, weight, and volume. Given dynamic manufacturing uncertainties, fuzzy modeling with the meta-heuristic-based solution approach is often preferred. While evolutionary multi-task optimization (EMTO) has been used to solve two single-objective RRAPs simultaneously, the recent multi-objective RRAPs (MO-RRAP) formulation incorporate both reliability and cost or weight for greater real-world applicability. Simultaneously optimizing multiple MO-RRAPs with similar system structures enables effective knowledge transfer, such as sharing genetic material between tasks leading to faster convergence, improved solutions, and reduced memory consumption. In this paper, we propose a multi-objective multi-factorial evolutionary algorithm (MO-MFEA) to simultaneously solve two fuzzy multi-objective RRAPs, one for a series system and one for a complex bridge system, across two test sets (reliability with cost and reliability with weight). Our results indicate that proposed MO-MFEA based approach provides superior solution quality and reduced computation time compared to the non-dominated sorting genetic algorithm (NSGA-II) when solving each problem independently.**

Keywords—Fuzzy modeling, Multi-task Optimization, Multi-factorial optimization, Multi-objective optimization, Reliability-redundancy allocation problem, Series, Complex bridge system

I. INTRODUCTION

System reliability is a probability, which specifies a system will continue its functioning without any failure for a certain amount of time under defined conditions. Higher system reliability is an important requirement in many sectors including transportations, communication, electrical power systems [1] and many engineering applications such as pharmaceutical plants, safety critical systems, electrical power plants, Hydraulic Systems etc. [2]. To maximize the overall system reliability, one may either improve the components' reliability or add redundant components (called redundancy allocation problem) [3]. When these strategies are combined, the problem becomes the reliability-redundancy allocation problem (RRAP) [4]. Optimizing RRAP is particularly challenging because it involves continuous decision variables for component reliability alongside integer variables for redundancy allocation, and the overall system reliability calculation is non-linear. Moreover, increasing system reliability often results in higher costs, weight, and volume, making RRAP a complex NP-hard problem [5]. Various techniques, including dynamic programing [6], branch and bound, Lagrange multiplier etc. [7], have been employed to tackle RRAP; however traditional approaches tend to fall short due to the problem's inherent complexity.

Although the RRAP has traditionally been studied as a single-objective optimization problem, focusing solely on maximizing overall system reliability [8]–[11], recent years have seen a growing interest in Multi-Objective RRAP (MO-RRAP). In MO-RRAP, the goal of maximizing system reliability is paired with a conflicting objective, such as minimizing total cost or weight [12]–[14]. This approach provides decision-makers with a set of trade-off solutions. Various heuristic-based methods, including the widely adopted NSGA-II [15], have been used to tackle the problem. More recently, the multi-factorial evolutionary algorithm (MFEA) has been utilized to simultaneously solve multiple single-objective RRAP problems [16].

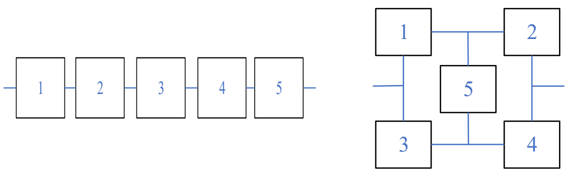
In real-world applications, system parameters like reliability, cost, and weight are often uncertain due to factors such as design variability, environment, and manufacturing [15]. To handle this, fuzzy modeling has been used in RRAP to create more realistic and robust optimization models. Unlike traditional crisp methods, fuzzy approaches better capture uncertainty, enhancing solution applicability. While much of the research on RRAP has focused on addressing the problem as a single-objective optimization problem, considering different objectives independently, except in [17], proposed solving multiple fuzzy RRAPs simultaneously using an MFEA-based evolutionary multi-tasking framework. However, their work is limited to single-objective problems, lacking support for multiple conflicting objectives in a fuzzy context.

Therefore, this study proposes a novel approach within the evolutionary multi-tasking framework called multi-objective multi-factorial evolutionary algorithm (MO-MFEA) for simultaneously solving multiple fuzzy MO-RRAPs. To validate its effectiveness, we also implemented the NSGA-II algorithm, where each MO-RRAP problem is solved independently, and the parameters are modeled using fuzzy sets (FSs).

The remainder of the paper is organized as follows: Section II outlines the fuzzy MO-RRAP formulation for both series and complex bridge systems. Section III details the fuzzy MO-MFEA approach for simultaneously solving two fuzzy MO-RRAP problems. Section IV presents experimental results, parameter settings, and input data alongside a comparative analysis. Finally, Section V concludes with limitations and directions for future work.

II. Problem Formulation

The formulation by Kuo and Prasad [1] is widely used in reliability optimization. In our study, we employ two well-known fuzzy RRAPs namely series system and complex bridge system and termed them as *Problem-1 & Problem-2* [17]. Each problem is then formulated into two different multi-objective optimization problem. In *Formulation-I,* the objectives are to maximize total system reliability (R_s) and minimize total system cost (C_s), while *Formulation-II* includes, maximize (R_s) and minimize total system weight (W_s). The following sections elaborate on these problems and their corresponding fuzzy MO-RRAP formulations.



(a) (b)

Fig. 1: Structure of (a) series system; and (b) Complex Bridge system

A. Problem-1: Fuzzy Series system

Series system is represented by sequentially connected five sub-systems, depicted in Fig.1 (a). The mathematical formulation for single objective RRAPs for such system is follows:

$$Maximize\ R_s(r,n) = \prod_{i=1}^{m}[1-(1-r_i)^{n_i}] \quad (1)$$

subject to,

$$C_s(r,n) = \sum_{i=1}^{m} \alpha_i * \left(-\frac{1000}{\ln(r_i)}\right)^{\beta_i} * \left[n_i + \exp\left(\frac{n_i}{4}\right)\right] - \tilde{C} \leq 0$$

$$V_s(r,n) = \sum_{i=1}^{m} v_i * n_i^2 - \tilde{V} \leq 0$$

$$W_S(r,n) = \sum_{i=1}^{m} w_i * n_i * \exp\left(\frac{n_i}{4}\right) - \widetilde{W} \leq 0$$

$$0.5 \leq r_i \leq 1, r_i \in [0,1] \subset \mathbb{R}^+, 1 \leq n_i \leq 5, n_i \in \mathbb{Z}^+; i = 1,2,3, \dots 5$$

1) *Case study-1: Fuzzy Multi-objective (R_s & C_s) series system based on Formulation-I*

$$Max\ R_s(r,n) \quad Min\ C_s(r,n) \quad (2)$$
$$Subject\ to \quad V_s(r,n), W_s(r,n)$$

2) *Case study-2: Fuzzy Multi-objective (R_s & W_s) series system based on Formulation-II.*

$$Max\ R_s(r,n) \quad Min\ W_s(r,n) \quad (3)$$
$$Subject\ to \quad V_s(r,n), C_s(r,n)$$

B. Problem 2: Fuzzy Complex Bridge System

The structure of complex bridge system consisting five sub-system is depicted in Fig.1 (b). The mathematical formulation for single objective RRAPs for complex bridge system is as follows:

$$Maximize\ R_s(r,n) = R_1R_2 + R_3R_4 + R_1R_4R_5 + R_2R_3R_5 - R_1R_2R_3R_4 - R_1R_2R_3R_5 - R_1R_2R_4R_5 - R_1R_3R_4R_5 - R_2R_3R_4R_5 + 2R_1R_2R_3R_4R_5 \quad (4)$$

subject to,

$$C_s(r,n) = \sum_{i=1}^{m} \alpha_i * \left(-\frac{1000}{\ln(r_i)}\right)^{\beta_i} * \left[n_i + \exp\left(\frac{n_i}{4}\right)\right] - \tilde{C}$$

$$V_s(r,n) = \sum_{i=1}^{m} v_i * n_i^2 - \tilde{V} \leq 0$$

$$W_s(r,n) = \sum_{i=1}^{m} w_i * n_i * \exp\left(\frac{n_i}{4}\right) - \widetilde{W} \leq 0$$

$$0.5 \leq r_i \leq 1, r_i \in [0,1] \subset \mathbb{R}^+, 1 \leq n_i \leq 5, n_i \in \mathbb{Z}^+; i = 1,2,3, \dots 5$$

1) *Case study-3: Fuzzy Multi-objective (R_s & C_s) complex bridge system (based on Formulation-I)*

$$Max \quad R_s(r,n) \quad Min\ C_s(r,n) \quad (5)$$
$$Subject\ to, \quad V_s(r,n), W_s(r,n)$$

2) *Case study-4: Fuzzy Multi-objective (R_s & W_s) complex bridge system (based on Formulation -II)*

$$Max \quad R_s(r,n) \quad Min\ W_s(r,n) \quad (6)$$
$$Subject\ to, \quad V_s(r,n), C_s(r,n)$$

In all the cases, "R_s" indicates the total system reliability which need to be maximized. The value of R_s has to be within the range: $0.5 \leq R_s \leq 1$. "m" is the total number of sub-systems in the system, $r_i(r_1, r_2, \dots r_m)$ and $n_i(n_1, n_2, \dots n_m)$ are the components reliability and number of redundant components in the i^{th} sub-system, respectively. The values of r_i takes real number and its ranges between 0 to 1. The value of n_i is an integer number and ranges between 1 to 5.

In case studies 1 & 3, "C_s" indicates the total system costs to be minimized, constrained by a fuzzified upper limit $\tilde{C}$. The "V_s" and "W_s" denotes the system's total volume and weight, respectively.

Similarly, in case studies 2 & 4, "W_s" indicates the total weight of the system needs to be minimized and need to be less or equal to its fuzzified upper limit $\widetilde{W}$. The "V_s" and "C_s" are the system's total volume and costs constraints, respectively. In all the cases, "$\widetilde{W}$", "$\tilde{V}$", and "$\tilde{C}$" indicates the fuzzified upper bound for weight, volume and cost constraints, respectively. These constraints are modeled using type-1 fuzzy sets (T1 FSs). Every member of the FS is assigned membership values ranges from 0 to 1. Each parameter quantifies the membership level of its respective attribute within the fuzzy set. We have utilized Triangular membership function for the fuzzification. According to the definition, T1 FS denoted by 'let's say $\tilde{A}$' (in our case, $\tilde{C}, \widetilde{W}\ and\ \tilde{V}$) and can be described using the following mathematical expression:

$$\tilde{A} = \{(x, \mu_A(x)) | \forall_x \in X\} \quad (7)$$

For the defuzzification popular centroid defuzzification is used. The "w_i" and "v_i" indicates the i^{th} sub-systems weight and volume, respectively. Further, "α_i" and "β_i" indicates i^{th} sub systems shaping and scaling factors, respectively.

Table I presents the test sets used to evaluate the proposed MO-MFEA approach for solving MO-RRAPs. As the approach is multi-tasking, it solves two tasks simultaneously. Two test sets are used: Test Set-1 (T1: Case Study-1, T2: Case Study-3) and Test Set-2 (T1: Case Study-2, T2: Case Study-4).

Table I: Description of Test sets based on MO-RRAP for MO-EMTO setup.

Test sets	Tasks	Dimension(D)	Objectives
Test set-1	T-1: Case study-1	10	obj-1: Reliability
			obj-2: Cost
	T-2: Case study-3	10	obj-1: Reliability
			obj-2: Cost
Test set-2	T-1: Case study-2	10	obj-1: Reliability
			obj-2: Weight
	T-2: Case study-4	10	obj-1: Reliability
			obj-2: Weight

III. Solution Approach

Evolutionary Algorithms (EAs), as population-based metaheuristics, effectively solve single-, multi-, and many-objective optimization problems. Recently, Gupta et al. [18], introduced the Multi-Factorial Evolutionary Algorithm (MFEA), which allows simultaneous optimization of multiple problems with knowledge sharing. MFEA has been successfully applied in machine learning and engineering domains [19]–[21]. However, it was originally limited to single-objective problems. To overcome this, the same authors later proposed the Multi-Objective Multi-Factorial Evolutionary Algorithm (MO-MFEA), capable of handling multiple multi-objective problems simultaneously [21].

A. *Multi-objective Multi-factorial Evolutionary Algorithm*

In MO-MFEA, the main ingredients are (i) unified representation of solution, and (ii) knowledge sharing among the tasks. Let's assume K number of optimization tasks (each having multiple objective function) need to be optimized within MO-MFEA. The search space of j^{th} task (T_j) is denoted as X_j and its objective function is defined as $F_j: X_j \rightarrow \mathbb{R}^{M_j}$, where M_j indicates the total number of elements in the objective function vector. Since all the problems are by default considers are minimization problem, so aim is to minimize the $\{F_1(x), F_2(x), F_3(x), \dots \dots . F_k(x)\}$. In MO-MFEA, each F_j represents an added factor and can be termed as $K-factorial$ environment. The unified representation of search space for all $K-factorial$ problem is defined as $Y(X_1, X_2, X_3, \dots . X_k)$ and it can be translated into task specific solution. In MO-MFEA each tasks have its own dimension and T_k has dimension of d_k. The unified search space will take maximum dimension ($D = \max\{d_1, d_2, d_3, d_k\}$) among all the tasks. All the solutions are encoded into real values ranges between 0-1. The individual solution (P_i) of the population (P) has some basic properties namely skill factor, factorial costs, factorial rank and scalar fitness; for interacting in MO-MFEA environments. *Skill factor* (τ)*:* indicates the belongingness of specific task (T_j) for an individual (P_i). It simply takes the index of the T_j task. During the fitness evaluation P_i will be evaluated based on skill factor only. *Factorial cost* (ψ_j^i)*:* indicates the objective values of tasks T_j for individual P_i. On the other hand, *Factorial rank* (r_j^i)*:* indicates task specific ranking, which is the rank of an individual (P_i) among all the solution of task T_j The calculation is based on multiple objective values. *Scalar fitness* (φ_i): indicates the of an individual (P_i) among all solution across all the tasks. The calculation is given by $\varphi_i = 1/r_{\tau_i}^i$.

B. *Working of MO-MFEA for solving fuzzy MO-RRAPs*

This subsection outlines the working of MO-MFEA based solution approach for solving multiple MO-RRAPs simultaneously. MO-MFEA treats each MO-RRAP (as discussed in Section II) as a separate task e.g., in TS-1, T1 corresponds to case study-1 and T2 to case study-3. Leveraging shared problem characteristics, MO-MFEA enables knowledge transfer (via genetic material exchange), enhancing convergence speed and reducing memory consumption.

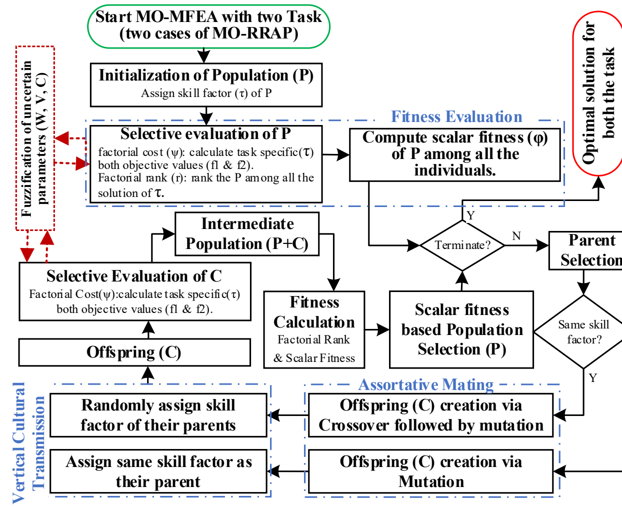


Fig. 2: Working of MO-MFEA for solving multiple MO-RRAP

The process begins with population initialization, followed by task assignment through skill factors (τ). Each individual is evaluated for task-specific objectives (f1 & f2), ranked within and across tasks using factorial rank (r) and scalar fitness (φ). Selection is based on scalar fitness, and genetic operations (crossover and mutation) generate offspring. These undergo fitness evaluation and are combined with the parent population to form the next generation. This evolutionary cycle continues until a termination condition is met. Figure 2 illustrates the proposed MO-MFEA framework for solving fuzzy MO-RRAPs. The key steps are presented below.

1) *Stage-1: Initialization of Population (P)*

In this step, the unified chromosome is created based on the highest task dimension. Since both case studies share the same decision variables, i.e. five component's reliability(r_i) and five redundant components (n_i), the chromosome dimension is 10. Chromosomes are encoded as real values between 0 and 1. An example of encoded chromosome structure is provided in Fig. 3. Each chromosome holds properties like factorial costs, rank, scalar fitness, and skill factor, as described in Section III(A). The task specific population conversion is made using continuous decoding scheme [18]. Task-specific populations are assigned skill factors, ensuring proper fitness evaluation during the process.

0.7849	0.8734	0.9053	0.7138	0.7973	3	2	2	3	3
Component's Reliability (r_i)					Number of Redundant Components(n_i)				

Fig. 3: Sample chromosome structure

2) *Stage-2: Fitness evaluation and parent selection*

Fitness evaluation consists of two parts. First, factorial costs are computed using task-specific objectives—here, total system reliability and cost—incorporating fuzzy parameters (as discussed in Section II). Individuals are then ranked within each task using non-dominated sorting and crowding distance [23]. It ensures the ranking of the task specific best solution to ensure convergence. In the second part, scalar fitness is calculated to rank individuals across both tasks.

Tournament selection is used to choose parents for the next generation [22].

3) Offspring creation (C) and new population selection (P)

To create offspring (C), selected parents undergo assortative mating and vertical cultural transmission. Assortative mating enables knowledge transfer between tasks using a random mating probability (rmp). If two parents share the same skill factor or meet the rmp condition, they undergo simulated binary crossover (SBX) followed by polynomial mutation to produce offspring [21]. Otherwise, each parent undergoes slight mutation to generate offspring individually.

If offspring are created via crossover, they inherit a randomly chosen parent's skill factor; if created by mutation alone, they retain their own parent's skill factor. This vertical cultural transmission ensures each individual is evaluated for only one task. All offspring are then evaluated task-specifically and their factorial costs updated. The offspring and parent populations are merged into an intermediate pool (P+C), which undergoes task specific fitness evaluation stage followed by factorial rank and scalar fitness calculation. The final population selection is based on scalar fitness, ensuring survival of the fittest individuals.

4) Stage-4 Termination criteria

This optimization process continues from Stage-2 until the termination criterion, defined as the maximum number of generations, is met.

IV. Experimental Results and Comparison Analysis

For the simulation, two test sets of multi-objective RRAPs were used to evaluate the proposed MO-MFEA, with each case study in a set treated as a separate task. For comparison, the well-known NSGA-II was applied to solve each case study independently. Experiments were conducted on a system with 64 GB RAM, Intel(R) Xeon(R) E5-2650 @ 2.00 GHz CPU, using MATLAB 2021b. Each algorithm was run independently 20 times per problem, and the results were used for performance analysis.

Table II. Data used in all case study-1 to case study-4.

i	$10^5\alpha_i$	β_i	v_i	w_i	$\widetilde{V}$	$\widetilde{C}$	$\widetilde{W}$
1	2.330	1.5	1	7			
2	1.450	1.5	2	8			
3	0.541	1.5	3	8	111.11	181.6	204.37
4	8.050	1.5	4	6			
5	1.950	1.5	2	9			

Table III: Average HV and IGD values for TS-1 and TS-2.

Test Sets	Task	Approach	IGD	HV
TS-1	T-1	NSGA-II	40.0199	0.8564
		MO-MFEA	38.3671	0.8607
	T-2	NSGA-II	31.4811	0.9035
		MO-MFEA	22.5837	0.9294
TS-2	T-1	NSGA-II	142.9551	0.5647
		MO-MFEA	95.8071	0.6996
	T-2	NSGA-II	142.0741	0.5694
		MO-MFEA	95.8063	0.7095

A. Parameters settings and Input data

For both approaches, initial population is set to 50 individuals and maximum number of generations are set to 1000. The crossover and mutation index are set to $\eta_c = \eta_m = 10$. The crossover probability in NSGA-II is set as $P_c = 0.8$, mutation probability, $P_m = \frac{1}{D} = \frac{1}{10} = 0.1$. For MO-MFEA, the random mating probability, rmp = 0.8 [16], [17]. The performance indicator namely, inverted generalized distance (IGD) and hyper volume (HV) is considered for showing the convergence and divergence of our MO-MFEA approach and NSGA-II. The input data for all the case studies are provided in Table II. Five set of input data are provided including values of shaping factor, scaling factor, components volume, weights, fuzzy upper bound for weight, volume and costs are provided.

B. Results of TS-1and TS-2 in terms of IGD & HV values.

Table III shows the average IGD and HV values over 20 runs for each task in TS-1 and TS-2. MO-MFEA consistently outperforms NSGA-II, yielding lower IGD and higher HV, indicating better convergence and diversity. Fig. 4(a)-(d) illustrates IGD trends over generations, highlighting MO-MFEA's superior convergence behavior across both test sets.

In T-1 of TS-1 (Fig. 4a), both MO-MFEA and NSGA-II show similar convergence trends, converged at around 150 generations, but MO-MFEA yields a lower final IGD. In T-2 of TS-1 (Fig. 4b), MO-MFEA again outperforms NSGA-II, with a wider margin. For T-1 of TS-2 (Fig. 4c), NSGA-II converges quickly (by 50 generations), while MO-MFEA takes longer (around 800 generations) but achieves significantly better IGD. A similar pattern is seen in T-2 of TS-2 (Fig. 4d), where MO-MFEA delivers superior IGD performance.

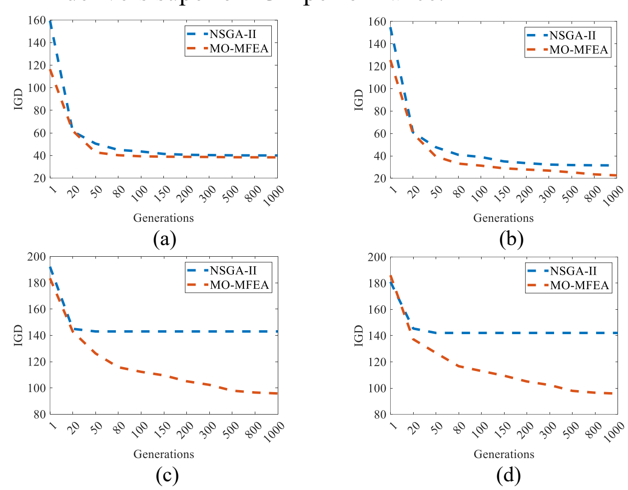


Fig. 4: NSGA-II vs MO-MFEA comparison in average IGD values of (a) T-1 of TS-1, (b) T-2 of TS-1, (c) T-1 of TS-2 and (d) T-2 of TS-2.

Similarly, Fig. 5(a)–(d) shows HV convergence trends over 20 runs for both algorithms. In T-1 of TS-1 (Fig. 5a), MO-MFEA and NSGA-II converge around 200 generations, but MO-MFEA yields higher final HV. In T-2 of TS-1 (Fig. 5b), MO-MFEA again performs better with a larger margin. For T-1 of TS-2 (Fig. 5c), NSGA-II converges early (50 gens), but MO-MFEA, though slower (500 gens), achieves a significantly higher HV. Similar results are observed in T-2 of TS-2 (Fig. 5d), with MO-MFEA consistently outperforming NSGA-II.

C. Results of TS-1and TS-2 in terms of quality of solution

1) Sample pareto solution

Figures 6 (a)-(d) depicts the comparative pareto front produced by proposed MO-MFEA and NSGA-II for both conflicting

Table IV. Maximum and Minimum objective values obtained by MO-MFEA and NSGA-II for TS-1 and TS-2.

Task	Approach	Test sets							
		TS-1				TS-2			
		Max R_s	Max C_s	Min R_s	Min C_s	Max R_s	Max W_s	Min R_s	Min W_s
T1	NSGA-II	0.934952	181.60	0.500004	37.89	0.932412	204.23	0.773582	125.30
	MO-MFEA	0.935372	181.60	0.500001	37.79	0.935387	192.48	0.574752	66.91
T2	NSGA-II	0.999881	181.59	0.837196	28.79	0.999868	201.64	0.998640	125.30
	MO-MFEA	0.999914	181.60	0.625000	19.79	0.999915	202.96	0.975625	66.91

Table V. NSGA-II vs MO-MFEA on task specific best solution in terms of maximum reliability among all the solutions obtained in 20 independent runs for TS-1

Task	Approach	Decision Variable		Objectives		Constraints	
		Component's reliability (r_i)	Redundant components (n_i)	R_s	C_s	V_s	W_s
T-1	NSGA-II	[0.7849, 0.8734, 0.9053, 0.7138, 0.7973]	[3,2,2,3,3]	0.934952	180.9104	83.00	192.48
	MO-MFEA	[0.7849, 0.8767, 0.9055, 0.7156, 0.7905]	[3,2,2,3,3]	0.935372	181.5931	83.00	192.48
T-2	NSGA-II	[0.8185, 0.8796, 0.9139, 0.6823, 0.6946]	[3,3,2,3,2]	0.999881	181.5358	83.00	189.43
	MO-MFEA	[0.8016, 0.8879, 0.8796, 0.6858, 0.7028]	[4,3,2,3,2]	0.999914	181.3811	84.00	202.96

Table VI. NSGA-II vs MO-MFEA on task specific best solution in terms of maximum reliability among all the solutions obtained in 20 independent runs for TS-2

Task	Approach	Decision Variable		Objectives		Constraints	
		Component's reliability (r_i)	Redundant components (n_i)	R_s	W_s	V_s	C_s
T-1	NSGA-II	[0.7956, 0.8101, 0.8886, 0.7126, 0.8664]	[3,3,2,3,2]	0.932412	189.4275	83.00	181.59
	MO-MFEA	[0.7831, 0.8744, 0.9055, 0.7178, 0.7926]	[3,2,2,3,3]	0.935387	192.4811	83.00	181.59
T-2	NSGA-II	[0.8433, 0.8654, 0.8908, 0.7040, 0.7610]	[2,3,3,3,2]	0.999868	192.4811	93.00	181.59
	MO-MFEA	[0.7906, 0.8807, 0.9031, 0.6982, 0.7007]	[4, 3,2,3,2]	0.999915	202.9617	84.00	181.58

objectives (F1: Max R_s & F2: Min C_s or Min W_s) of every task of both the test sets. For T-1 of TS-1 (Fig. 6a), both MO-MFEA and NSGA-II produce diverse solutions with reliability ranging from 0.5 to 0.93. However, MO-MFEA delivers superior solution quality compared to NSGA-II.

Similarly, for T-2 of TS-1 (Fig. 6b), proposed MO-MFEA produces solutions with reliability ranging from 0.8 to 0.99, while NSGA-II only covers from 0.9 onward, indicating limited diversity. In both tasks of TS-2, MO-MFEA consistently offers more diverse and higher-quality solutions. As shown in Fig. 6(c) & (d), MO-MFEA outperforms NSGA-II in solution quality. Notably, in T-2 of TS-2 (Fig. 6c), MO-MFEA covers reliability from 0.991 to 0.9999, while NSGA-II starts from 0.996, again showing less diversity.

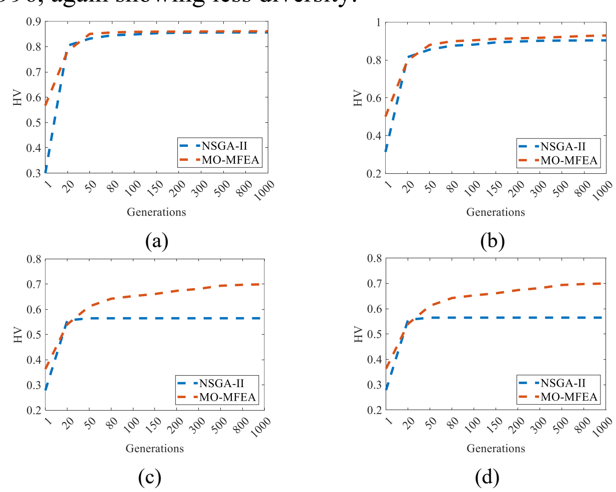


Fig. 5: NSGA-II vs MO-MFEA comparison in average HV values of (a) T-1 of TS-1, (b) T-2 of TS-1, (c) T-1 of TS-2 and (d) T-2 of TS-2.

2) *Task specific Maximum and Minimum objective values for TS-1 and TS-2*

To understand the quality of solution (in terms of both of the conflicting objectives) generated by proposed MO-MFEA and its competitor NSGA-II for the task specific, the maximum and minimum objective values for TS-1 & TS-2 are reported in Table IV. For the analysis of maximum and minimum objective values we have utilized all the solutions generated by MO-MFEA and NSGA-II at the end of 20 independent runs. For T-1 of TS-1, the proposed MO-MFEA has generated better values of maximum reliability (0.935372), minimum reliability (0.500001) and minimum cost (37.79) compared to NSGA-II. However, in case of generating maximum costs both the MO-MFEA and NSGA-II approach provides equal value which is 181.60. In case of T-2 of TS-1, proposed MFEA outperformed in generating maximum and minimum values of reliability and costs compared to NSGA-II. For T-1 of TS-2, the proposed MO-MFEA has generated better values of maximum reliability (0.935387), minimum reliability (0.574752), and minimum weight (66.91) compared to NSGA-II. However, in case of generating maximum weight (204.23) NSGA-II approach provides better value compared to MO-MFEA. In case of T-2 of TS-2, proposed MO-MFEA outperformed in generating maximum and minimum values of reliability and weights compared to NSGA-II.

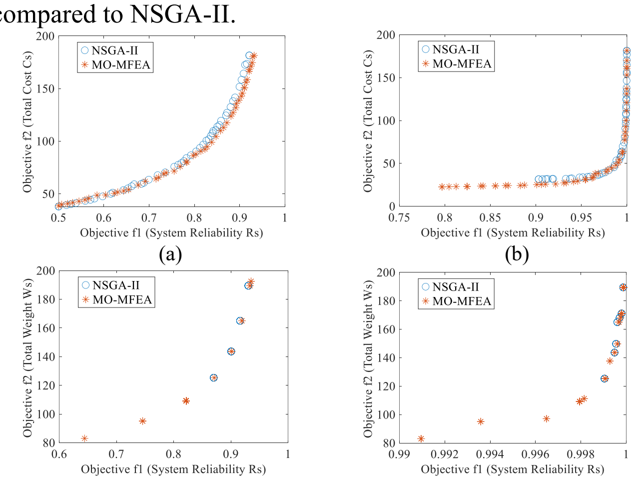


Fig. 6: NSGA-II vs MO-MFEA comparitive sample pareto front (aprox) for (a) T-1 of TS-1, (b) T-2 of TS-1, (c) T-1 of TS-2, (d) T-2 of TS-2.

3) Best solution of TS-1 and TS-2 with respect to maximum reliability

In real scenario, finding better system reliability is the main concern with considerable costs. By keeping it in mind, in Table V and VI, we have provided a sample of best solution (in terms of maximum reliability) and its associated parameters values among 20 independent runs obtained by proposed MO-MFEA based approach and its competitor NSGA-II based solution approach for TS-1 and TS-2, respectively.

Table VII. NSGA vs MO-MFEA in terms of average computation time

Approach	Total computation time		Improvements in terms of computation time of the MO-MFEA	
	TS-1	TS-2	TS-1	TS-2
NSGA-II	500.39	550.95	46.78%	48.18%
MO-MFEA	266.31	285.49	-	-

D. Computation time for both TS-1 & TS-2

The average computation time for optimizing both the tasks of TS-1 and TS-2 for 20 independent run using NSGA-II and proposed MO-MFEA is provided in Table VII. It also provides percentage of improvements of proposed MO-MFEA for both the tasks compared when compared to NSGA-II. As we can see, proposed MO-MFEA took less average computation time compared to NSGA-II which is 46.78% & 48.18% faster for solving TS-1 & TS-2, respectively.

V. Conclusion and Future Work

This paper has proposed a MO-MFEA-based approach to simultaneously solve two fuzzy MO-RRAPs: a series system and a complex bridge system. Two multi-objective formulations were used, where the formulation-1 includes maximize system reliability and minimize cost, and, formulation-2 includes maximize system reliability and minimize weight. We have considered two multi-tasking test sets for our study where the first set includes first formulation of series and complex bridge system. And the second set includes second formulation of both the problems. Each problem in a test set was treated as a separate is task within the MO-MFEA framework. For comparison, NSGA-II has been used to solve each problem independently. Performances are evaluated using average IGD and HV metrics, along with comparisons on pareto fronts, best solutions, and computation time. Results have established that MO-MFEA outperformed NSGA-II in all metrics for both test sets, except in Task-1 of Test Set-1 where both yielded the same system cost. In terms of computation time, MO-MFEA has been found 46.78% faster for Test Set 1 and 48.18% faster for Test Set 2.

However, this study has focused only on small-scale MO-RRAPs. In the future, we plan to extend our work to large-scale systems, incorporate additional metaheuristic algorithms for broader validation, and include fuzzification of component reliabilities to enhance the modeling of MO-RRAPs

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