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The six-factor asset pricing model in Paris

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ABSTRACT:

This study employs the Fama-French (2018) six-factor model on a unique dataset of country-specific data from the French stock market, spanning from July 2010 to December 2023, during the low interest rate environment following the 2008 financial crisis. Left-hand side regressions indicate that the six-factor model significantly outperforms the three- and five-factor models in explaining stock returns on the Paris Stock Exchange, substantially reducing portfolio alphas. The GRS test rejects all models; however, the six-factor model demonstrates considerably better performance than the competing models. Spanning regressions reveal that the market, value, investment, and momentum factors expand the mean-variance frontier, while size and profitability factors are redundant. Moreover, the momentum factor contributes significant monthly returns of 0.79% that are not captured by the five-factor model. Throughout the period, the value premium averages a strongly negative -0.88%, suggesting that factor premiums are prone to significant variations across different periods, particularly under varying macroeconomic conditions. The results also show that the value, momentum, and profitability factors can enhance risk adjusted portfolio returns. A one unit increase in the momentum factor loading increases average monthly portfolio returns by 0.78%. A similar one unit increase in the value loading decreases monthly returns by 0.71%, while a one unit increase in the profitability factor loading delivers a 0.37% increase in average monthly returns. This implies that smart beta styled factor investing strategies are viable in France.

KEYWORDS: Factor models, Asset pricing models, Fama-French, Three-factor model, Five-factor model, Six-factor model, Factor premiums, Risk premiums, French stock market

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Tiivistelmä:

Tässä tutkimuksessa käytetään Fama-French (2018) kuuden faktorin mallia Ranskan osakemarkkinoiden maakohtaisessa tutkimuksessa, joka kattaa heinäkuusta 2010 joulukuuhun 2023 ulottuvan jakson, joka ajoittuu vuoden 2008 finanssikriisiä seuranneeseen matalan korkotason ympäristöön. Left-hand side regressiot osoittavat, että kuuden faktorin malli selittää Pariisin pörssin osaketuottoja huomattavasti paremmin kuin kolmen ja viiden faktorin mallit, vähentämällä merkittävästi salkkujen alfoja. GRS-testi hylkää kaikki mallit, mutta kuuden faktorin malli osoittaa huomattavasti parempaa suorituskykyä kuin kilpailevat mallit. Spanning-regressiot paljastavat, että markkina-, arvo-, sijoitus- ja momentumfaktorit laajentavat tuotto-riskirintamaa, kun taas koko- ja kannattavuusfaktorit ovat tarpeettomia Ranskan osakeuniversumissa. Lisäksi momentum faktori tarjoaa jopa 0.79 prosentin kuukausituottoja, mitä viiden faktorin malli ei kykene selittämään. Arvopreemio on koko ajanjakson aikana keskimäärin merkittävästi negatiivinen -0.88 prosenttia, mikä viittaa siihen, että faktoripreemiot ovat alttiita merkittäville vaihteluille eri ajanjaksoilla, erityisesti vaihtelevissa makrotaloudellisissa olosuhteissa. Tulokset osoittavat myös, että arvo-, momentum- ja kannattavuusfaktorit voivat parantaa riskikorjattuaja osaketuottoja. Yhden yksikön nousu momentum-faktorin betassa lisää salkun keskimääräistä kuukausituottoa 0.78 prosenttia. Samanlainen yhden yksikön nousu arvofaktorin betassa vähentää kuukausituottoja 0.71 prosenttia, kun taas kannattavuusfaktorin betan yhden yksikön nousu kasvattaa keskimääräisiä kuukausituottoja 0.37 prosenttia. Tämä tarkoittaa, että näitä faktoreita hyödyntävät smart beta -tyyppiset faktoristrategiat ovat toteuttamiskelpoisia Ranskassa.

AVAINSANAT: Faktorimallit, osakehinnoittelumallit, Fama-French, Kolmen faktorin malli, Viiden faktorin malli, Kuuden faktorin malli, Faktoripreemio, Riskipreemio, Ranskan osakemarkkinat

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1 Introduction

In recent years, factor investing has become a thriving trend. Following the 2008 financial crisis, investors started exploring new investment strategies to generate excess returns in unstable markets. One of the strategies that gained prominence was factor investing. Though it was already well-known in academia and hedge funds prior to the crisis, it was only after 2008 that the approach became popular with retail investors through factor-based mutual and exchange-traded funds (ETFs). One of the early adopters was AQR Capital Management, bringing momentum-based funds to the market in 2009. Since inception, their US-based momentum funds have delivered annualized average returns up to 14.90%, significantly outperforming the market. The success of these investing products encouraged AQR and other asset management firms to launch a growing number of factor-based funds, employing strategies ranging from betting-against-beta to multi-factor approaches combining value, size, momentum, and quality in one portfolio. While demonstrating the benefits of factor-based investment products, early adopters of this approach have contributed significantly to the rising traction of factor investing. Their success has been acknowledged by investors and fund managers alike, as numerous factor ETFs are launched annually by various investment management firms, with around 260 ETFs utilizing multi-factor strategies traded on US markets by the end of 2023. Nowadays, all the major players in the investment industry, such as BlackRock, Invesco, and Goldman Sachs, offer a range of factor-based products. Factor investing has unquestionably reached the mainstream, and as a result, asset pricing needs to evolve to capture the changing risk-return dynamics in modern markets.

Estimating expected returns for factor-based portfolios using conventional asset pricing models, such as CAPM, is ineffective as factor strategies leverage additional sources of risk over the market factor. Therefore, advanced models are needed for explaining the returns of such portfolios. Multi-factor models have emerged as popular equilibrium asset pricing approaches since the introduction of the Fama and French (1993) three-factor model. Over the years, the model has undergone revisions and augmentations, incorporating various additional factors, such as the Carhart (1997) four-factor model and the Fama-French (2015) five-factor model. Harvey et al. (2016) observe that the range of new factors found in asset pricing research is overly extensive, complicating the evaluation of competing factors. As a result, Fama and French's (2015) approach considers only those factors in their multi-factor model, which have robust support from empirical research and are grounded in solid theoretical frameworks. For an extended period, the academic community advocated for the inclusion of the Momentum factor in their models. Fama and French initially opposed this due to the absence of a theoretical foundation for the momentum phenomenon. Even so, Fama and French (2018) introduce a six-factor model, reluctantly augmenting their five-factor model with a momentum factor.

In their initial study, the Fama and French (2018) six-factor model offers promising results when applied in North America, advocating for the significance of the momentum factor in explaining stock returns. Hou et al. (2018) argue that the performance of factor models must be examined by testing the factors with different sample data across various time periods to evaluate their robustness. Consequently, Grobys and Kolari (2022) replicate the study by analyzing the performance of the six-factor model using a novel sample period in Europe, Japan, broader Asia, and North America. While predominantly indicating that the six-factor model outperforms models omitting the momentum factor, their results also find significant variations in model efficacy, including disparities in factor premiums and factor loadings across regions. Fama and French (2017) similarly observe regional variations in performance when testing their five-factor model internationally. These results undermine the robustness of the model from a global perspective.

The international studies of factor models are primarily conducted in broad markets, such as the entire European market or the aggregate stock markets of emerging nations. Country-specific studies, while less common, provide more significant insights into the performance of factor models across nations with unique economic, governmental, and legal characteristics that warrant attention. Country-specific studies also offer plenty of new sample data to replicate studies assessing factor model robustness. If the models are robust, they should be applicable in any stock market, though with potentially differing levels of risk premiums and factor loadings. Country-specific studies, including Chiah et al. (2016), Guo et al. (2017), and Foye (2018a), provide valuable insights into the varying performance of factor models across stock universes of specific countries. Outside academia, these studies can also be helpful for portfolio managers, as factor strategies are increasingly employed in investment portfolios. Country-specific studies of factor performance can assist managers in determining which risk factors to follow within a specific investment universe.

In response to Hou et al. (2018) advocating for examining factor models using novel sets of sample data, this study tests the performance of the Fama-French six-factor model in the French stock market, where no significant studies have previously examined asset pricing models. The French stock market is highly concentrated and heavily weighted towards the luxury goods and energy sectors, with a growing presence of the technology sector, making it unique compared to other major European countries, such as the UK and Germany. Luxury goods firms dominate the CAC 40 index, with LVMH accounting for approximately 10% of the total market capitalization of the index as of the end of 2024. Overall, the top ten firms in the index account for over 60% of the total index. Macroeconomic conditions may also have a more significant impact on French stock returns due to the concentrated nature of the market. For instance, luxury goods sales may experience a significant decline in the event of poor economic conditions, while energy companies are prone to the risk of rising global energy prices, and banks can encounter poor profitability under low interest rates. This can have a profound effect on the overall market and factor returns. Along with these market-specific factors, cultural disparities in customer and investor behavior may also influence stock returns in France relative to other regions in Europe and North America.

The behavioral, cultural and legal dimensions of France considerably differ from those of other major European nations and the United States, which foster more liberal economies. The French market has suffered from a lack of participation from retail investors, with only about 6.2% of the population holding direct equity investments according to the OECD (2023) report. Furthermore, the French can be considered generally risk-averse investors, with only about one-quarter of their financial assets invested in risky assets. This lack of market participation and conservative risk-taking behavior distinguishes the French market from the rest of Europe and the United States, where retail investors tend to favor higher financial risk in pursuit of higher returns. France also stands out due to the government's significant impact on commerce. France's strategic industries, such as energy, transportation, and defense, frequently receive substantial subsidies from the government, demonstrating the state's substantial control over the economy. The nation also possesses an extensive welfare state. France finances its substantial government expenditure through one of the highest tax-to-GDP ratios in the European Union. Alongside high taxation, France has comprehensive labor protection legislation, with labor unions playing a significant influence on the economy. This combination has resulted in reduced comparative growth due to decreased productivity relative to the US, UK, and other liberal economies. The unique legal and cultural aspects of the French economy, combined with the distinct structural features of its stock market, can significantly influence the behavior of stock returns, making the French stock market a significant and original subject for study.

To evaluate the robustness of factor performance, Hou et al. (2018) advocate for conducting asset pricing model tests using novel time periods. Factor model studies are generally conducted over a sample period that encompasses several decades. A restricted number of studies have assessed the performance of the Fama-French models over shorter sample periods. Understanding the time-varying performance of factor models is crucial for short-term investment horizons. As a result, this study analyzes stock returns in the French stock market from 2010 to 2023 during the historically low interest rate period. The study serves as a case study examining factor model performance in France during the post-2008 financial crisis, to analyze stock behavior during a period characterized by unusual macroeconomic conditions in Europe. Given that the ECB maintained low and occasionally negative interest rates throughout the study period, the study examines the behavior of stock returns in France during a period when capital is unusually inexpensive. This thesis contributes to the existing literature by providing a novel, country-specific sample for testing the efficacy of the six-factor model and by examining stock return behavior during extraordinary macroeconomic conditions, specifically in the post-financial crisis recovery phase marked by historically low interest rates.

1.1 Research question and hypotheses

This thesis analyzes the performance of the six-factor model within the French stock market. The objective is to examine the efficacy of the model in explaining French stock returns. This study also analyzes the model's performance relative to the Fama-French three- and five-factor models. Fama and French (2018), as well as Grobys and Kolari (2022), identify the six-factor model as the winner in their studies. Therefore, this study equally presumes that the six-factor model is the superior model in France. As a result, the research question of the thesis is: How effective is the six-factor model in explaining the cross-section of stock returns in the French stock market during the post-2008 financial crisis low interest rate period, and how does it compare to the three- and five-factor models?

Three hypotheses are defined to guide this study. This thesis examines the six-factor model as the primary equilibrium asset pricing model. Hence, the first hypothesis proposes that the model correctly explains the cross-section of stock returns within the French stock market. Secondly, this thesis argues that the six-factor model should outperform the three- and five-factor models in explaining stock returns. Thus, the second hypothesis concerns the comparative performance of the models. Finally, each factor in the optimal asset pricing model should pull its weight in explaining stock returns and provide supplementary explanatory power not captured by other factors. As a result, the third hypothesis concerns the complementary performance that each individual factor should add to the model. The hypotheses are defined as follows:

H_1: The six-factor model explains the cross-section of average stock returns within the French stock market.

H_2: The six-factor model significantly outperforms the three- and five-factor models in explaining the cross-section of average stock returns within the French stock market.

H_3: The incorporation of all individual factors in the six-factor model significantly expands the mean-variance frontier and provides increased explanatory power for the model.

1.2 Structure of the study

The study is organized as follows: The second chapter examines the fundamental theories of finance that underlie the six-factor model in the following order: The modern portfolio theory, efficient market hypothesis, Capital Asset Pricing Model (CAPM), arbitrage pricing theory, and the Fama-French three-, five-, and six-factor models. Chapter 3 provides a literature review on the empirical performance of the independent factors within the six-factor model, along with a review of the model's empirical performance and its limitations. Chapter 4 presents the sample data and methodology employed in this study. Following that, the empirical analysis will be delivered in Chapter 5, followed by a discussion section in Chapter 6. The study closes with conclusions in Chapter 7.

2 Theoretical framework

Asset pricing models aim to explain the relationship between equity risk and stock returns. Single index models treat market risk as the only risk factor. In contrast, multi-factor models assume that multiple underlying systematic risk factors influence equity risk. As these more complex models consider more factors, they show potential to significantly reduce the number of stock market anomalies that are problematic for simpler models. Multi-factor models are based on existing literature and theories, therefore comprehending the concepts that underlie them is crucial. Next, this thesis will look at modern portfolio theory, which serves as the theoretical foundation for equilibrium asset pricing models.

2.1 Modern portfolio theory

Before the 1950s, investors had a limited number of inconsistent quantitative approaches to determining the optimal portfolio composition. Markowitz (1952) establishes the principles of modern portfolio theory (MPT). MPT provides tangible quantitative tools for investors to construct mean-variance efficient portfolios. The theory argues that all investors are intrinsically averse to risk. Hence, a rational investor constructs an optimal portfolio that minimizes risk while achieving the desired level of expected return. Furthermore, an investor chooses the less risky portfolio when presented with two portfolios with identical expected returns. Portfolio risk is effectively reduced through diversification and the deliberate application of a stock weighting scheme, which exploits the inherent covariances between the stocks within the portfolio. Portfolio risk can be minimized by finding the optimal weights for each asset within the portfolio.

The expected return for a portfolio is calculated by the weighted average expected returns of all assets included. The expected return for a portfolio utilizing its total capital is written as,

$$E(r_p) = \sum_{i=1}^{N} X_i E(R_i), \quad \text{where } \sum_{i=1}^{N} X_i = 1, \qquad (1)$$

where X_i is the weight and $E(R_i)$ is the expected return of asset i. Since all capital is employed, the sum of the individual stock weights equals 1. The variance and standard deviation of a portfolio is estimated using the following formula,

$$\sigma_P^2 = w^T \Sigma w, \text{ and } \sigma_p = \sqrt{\sigma_P^2}. \quad (2)$$

The portfolio variance is represented by σ_P^2. The transpose of the weight vector, which includes the weights of all stocks in the portfolio, is represented as w^T. To calculate the expected portfolio variance (σ_P^2), the transposed weight vector is multiplied by the co-variance matrix Σ, which is then multiplied by the weight vector w. The matrix contains the covariances of each stock pair within the portfolio as well as the variances of each individual stock. The portfolio standard deviation σ_p is the square root of the variance.

The covariance of two stocks is associated with Pearson's correlation coefficient ρ, expressed as $\sigma_{ij} = \rho\sigma_i\sigma_j$, where σ_{ij} represents the covariance and σ_i and σ_j denote the standard deviations of the respective stocks. A correlation coefficient of 1 indicates a perfect linear relationship between assets. A coefficient of -1 signifies a perfect negative correlation. A coefficient value of 0 indicates the absence of correlation between the assets. Choosing assets with low or negative correlations can significantly reduce the overall risk of a portfolio. Consequently, investors analyze correlations between different asset classes to determine optimal allocations that minimize portfolio risk.

Markowitz (1952) argues that investors can find their optimal portfolio composition from all available options through mean-variance analysis. Portfolios that offer the highest expected return for a specified level of risk are known as efficient portfolios. Portfolios that yield lower returns at an equivalent risk level are considered inefficient. Figure 1 illustrates potential portfolios with varying asset allocations. Portfolios located above the global minimum-variance portfolio on the blue line, called the efficient frontier, provide the highest expected returns for a given risk level. Under the theory of Markowitz, investors are mean-variance optimizers; thus, rational investors will choose their portfolios from the efficient frontier according to their risk tolerance.

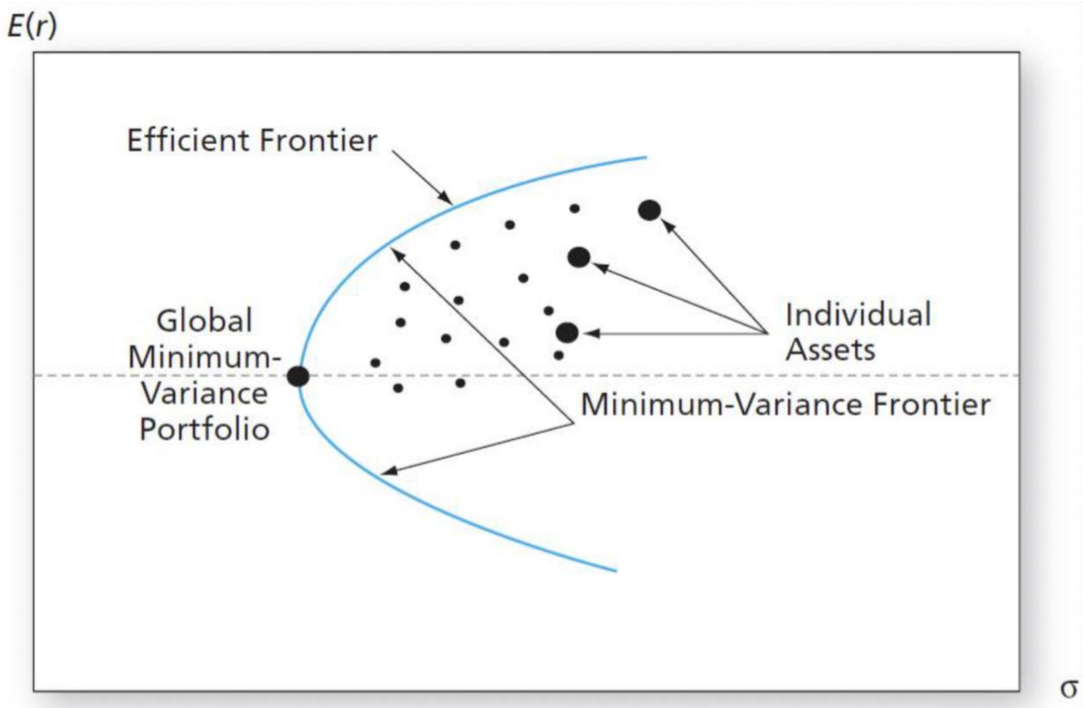

Figure 1. The efficient frontier (Bodie, Kane & Marcus, 2011).

The MPT framework encounters practical and theoretical obstacles. As the number of possible stock combinations within a given investment universe rises, the necessary amount of approximated coefficients for the MPT framework climbs significantly, as the covariance between each stock must be estimated separately. For instance, in a stock universe of 500 stocks, it is required to estimate 125,700 unique coefficients to apply the MPT framework. Portfolio optimization involving an ever greater quantity of stocks becomes considerably resource intensive. In addition, the methodology can display significant sensitivity to variations in its inputs. Since the methodology generally relies on estimates derived from historical stock returns rather than a forward-looking approximation, the optimization results may be chronically inaccurate. As a result, the MPT framework is not the ultimate solution for portfolio optimization but rather a useful tool when applied with discretion.

Nonetheless, Markowitz's contributions have profoundly fostered the development of modern approaches for evaluating expected stock returns and portfolio performance. The Capital Asset Pricing Model (CAPM) is one of the most significant adaptations of the Modern Portfolio Theory, augmenting the model by incorporating an option for borrowing and lending at a risk-free rate. Before this paper discusses the CAPM, it is essential to first review the efficient market hypothesis, as asset pricing models are frequently assessed together with it in the context of the joint hypothesis problem.

2.2 Efficient market hypothesis

Like the Modern Portfolio Theory, the Efficient Market Hypothesis (EMH) is a fundamental concept within the domain of investments. EMH proposes that stock prices always reflect all available information. Consequently, mispricing does not occur in the market. Therefore, price fluctuations occur only due to new information, as the market readjusts in response to new data. The concept of efficient capital markets is a common topic in financial research, as testing an asset pricing model simultaneously acts as a test of the Efficient Market Hypothesis. This is because all attempts to evaluate market efficiency necessitate testing the returns of an asset pricing model against actual returns. Fama (1965) and Samuelson (1965) establish the theoretical foundation for the Efficient Market Hypothesis, while Fama's (1970) publication has popularized the concept. In the paper, Fama establishes the commonly recognized levels of market efficiency.

Fama (1970) presents the forms of market efficiency as follows: Weak efficiency implies that, across the entire market, the price of a stock reflects all historical stock price information. Therefore, achieving excess returns using technical analysis is impossible. Semi-strong efficiency suggests that all publicly available information regarding a company is fully reflected in its market price. Thus, fundamental stock analysis and technical analysis cannot produce excess returns. Lastly, strong market efficiency proposes that all available data, including insider information, is completely incorporated into a stock’s price. According to Fama (1991), the strong version of EMH is unattainable since the preconditions for strong efficiency, including no information and trade costs, are practically impossible.

If the EMH holds, then market price movements follow a random walk process, implying that movements in stock prices are random and cannot be forecasted (Malkiel, 2003). A random walk is a process in which each subsequent movement is unpredictable and uncorrelated with prior movements. Within the framework of the stock market, a random walk can be understood in the following manner. Initially, stock prices contain all available information as of the present day. Only new information can impact stock prices. Consequently, in the light of tomorrow's news, stock prices will adjust accordingly. Price movements are random due to the randomness of new information. As a result, stock prices display a random walk behavior, provided the markets are efficient. The random walk process can be mathematically described as follows:

$$X_t = X_{t-1}\epsilon_t, \tag{3}$$

where X_t is the price of a stock at time t, which is influenced by the stock price at $t-1$ and the random effect of new information ϵ_t**,** which stands for white noise process with zero mean. According to the formula, the prices for tomorrow are exclusively determined by today's stock prices and randomness.

Due to the relatively high cost of information, Grossman and Stiglitz (1980), as well as Tirole (1982), argue that the market cannot achieve complete efficiency. Considering that the Efficient Market Hypothesis claims that the market "fully" reflects all available information, the hypothesis cannot be entirely confirmed. Malkiel (2003) argues that the market is more efficient than previously believed. Also, Schwert (2003) finds evidence supporting a strengthening of market efficiency. He argues that when research papers addressing anomalies are published, the anomalies tend to disappear rapidly. Consistent with increasing market efficiency, Altinkiliç et al. (2016) note that in aggregate, stock analysts fail to generate significant excess returns for their clients. Despite the rejection of the EMH in practically all asset pricing model studies, empirical evidence from the post-2000s suggests that markets have become progressively more efficient in recent years.

2.3 CAPM

William Sharpe (1964) contributes to Modern Portfolio Theory by examining the implications of universal portfolio optimization based on Markowitz's methodology. He proposes that, in theory, all individuals hold a combination of the identical market portfolio and a risk-free asset, depending on their risk appetite. Lintner (1965) and Treynor (1961) adopt equivalent theories. The model they establish is the capital asset pricing model (CAPM). CAPM uses systematic asset risk and a risk-free rate to estimate asset returns.

Sharpe (1964) and Lintner (1965) provide two fundamental assumptions for the theory. First, investors can lend and borrow at an identical risk-free rate. Secondly, investors possess homogenous information regarding expected returns, standard deviations, and correlations within the market. Furthermore, CAPM assumes that investors are rational mean-variance optimizers and seek to hold a mean-variance efficient portfolio at all times. Figure 2 illustrates a set of various alternative portfolios. The vertical axis represents the expected return, and the horizontal axis depicts risk as standard deviation. The abc Curve depicts the minimum variance frontier, with b representing the minimum variance portfolio. Portfolios located on the frontier above b are called efficient, as they provide the highest expected return for a given degree of risk. Portfolios on the frontier are constructed from a mix of risky assets. When risk-free (R_f) lending and borrowing is allowed, the frontier transforms into a linear representation of investors' capital allocation options, referred to as the capital allocation line (CAL), as illustrated in figure 2 by the line extending from the intercept R_f trough portfolio g. Investors of portfolio g can adjust their allocation between the risky portfolio and risk-free asset depending on their desired level of risk by lending and borrowing at the risk-free rate, moving left and right along the CAL. As depicted in Figure 2, portfolio g is inefficient, as it does not reside on the efficient frontier.

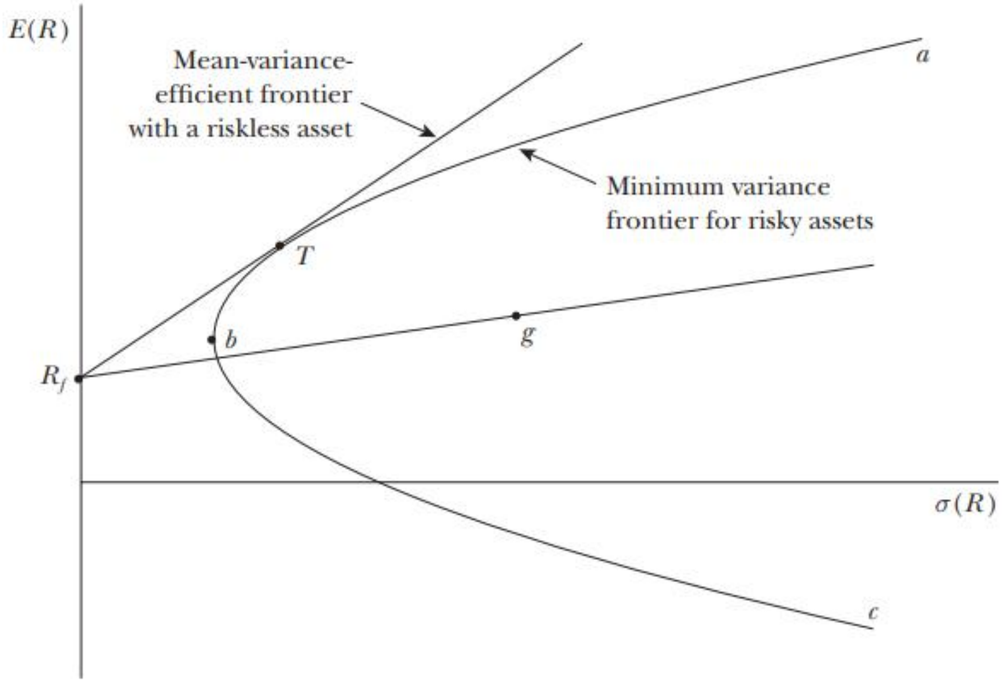

Figure 2. The capital allocation line and the efficient frontier (Fama & French, 2004).

Assuming that investors are rational mean-variance optimizers with homogeneous information and can lend and borrow at the equivalent risk-free rate, they will eventually reach the same efficient risky portfolio, which offers the highest possible risk-to-return ratio of all portfolios (Sharpe, 1964). This portfolio, when combined with the risk-free asset, forms a tangent line to the efficient frontier. In other words, the portfolio has the highest Sharpe ratio out of all alternatives. When combined with a risk-free asset, no other portfolio can provide a higher return for the same level of risk. Since each investor holds the same portfolio T, it can be inferred that it represents the value-weighted market portfolio for all risky assets (Fama & French, 2004). The CAL, where the risky portfolio is the market portfolio, is called the Capital market line (CML).

Let us consider an investor who exclusively invests in the market portfolio and has the ability to borrow and lend at a risk-free rate. If he chooses to allocate all his funds to the risk-free asset, his portfolio will be R_f. If he allocates all his funds in the market portfolio, he is at point T. If he borrows at the risk-free fate R_f and invests in portfolio T with leverage, he is positioned on the CML, to the right of point T.

A crucial component in the CAPM formula is the measure of market risk of an asset, denoted as beta (β). It is estimated through a single index model regression, where the returns of an asset are regressed against the returns of a market index. Beta is the resulting slope coefficient of that regression. CAPM assumes a positive linear relationship between beta and expected return. Consequently, two portfolios possessing identical betas must display equivalent expected returns. The CAPM formula is written as follows,

$$E(R_i) = R_F + \beta_i \cdot \left(R_m - R_f\right), \tag{4}$$

where $E(R_i)$ denotes the expected return of asset i, R_F represents the risk-free rate of return and β_i is the beta coefficient of asset i. The market risk premium is derived by subtracting the risk-free rate R_F from the market return R_m within the parentheses. Expected return for asset i is calculated as the product of beta and the market risk premium plus the risk-free rate. The linear representation of the relationship between beta and expected return is called the security market line (SML). CAPM proposes that all securities should be positioned on the SML relative to their beta coefficients. Nonetheless, this may not consistently hold in reality. According to CAPM, a stock located above the Security Market Line (SML) is considered undervalued, whereas a stock located below the line is regarded as overvalued.

CAPM faces several empirical challenges in its applications. Since the 1970s, numerous researchers have challenged the assumption of a linear relationship between beta and expected return. While Fama and MacBeth (1973) reaffirm the validity of the Capital Asset Pricing Model (CAPM) in the US stock market, the evidence for a consistent positive linear relationship is weak. Reinganum (1981) challenges them by showing that the CAPM fails to account for the seemingly constant excess returns produced by portfolios constructed with size and P/E ratios. Fama and French (1992) show that the

CAPM inaccurately represents market equilibrium. They find that the value and book-to-market risk factors explain stock returns that the CAPM does not effectively capture. Also, Fama and French (2004) demonstrate the behavior of various Book-to-Market (B/M) ratio portfolios in relation to CAPM predictions, as shown in Figure 3. They categorized stocks from NYSE, AMEX, and NASDAQ into ten value-weight portfolios based on their book-to-market ratio. Figure 3 illustrates the weakness of the positive linear correlation assumed by CAPM. The beta-return relationship is negative, and all but one of the portfolios represented are deemed undervalued, while none are positioned on the SML.

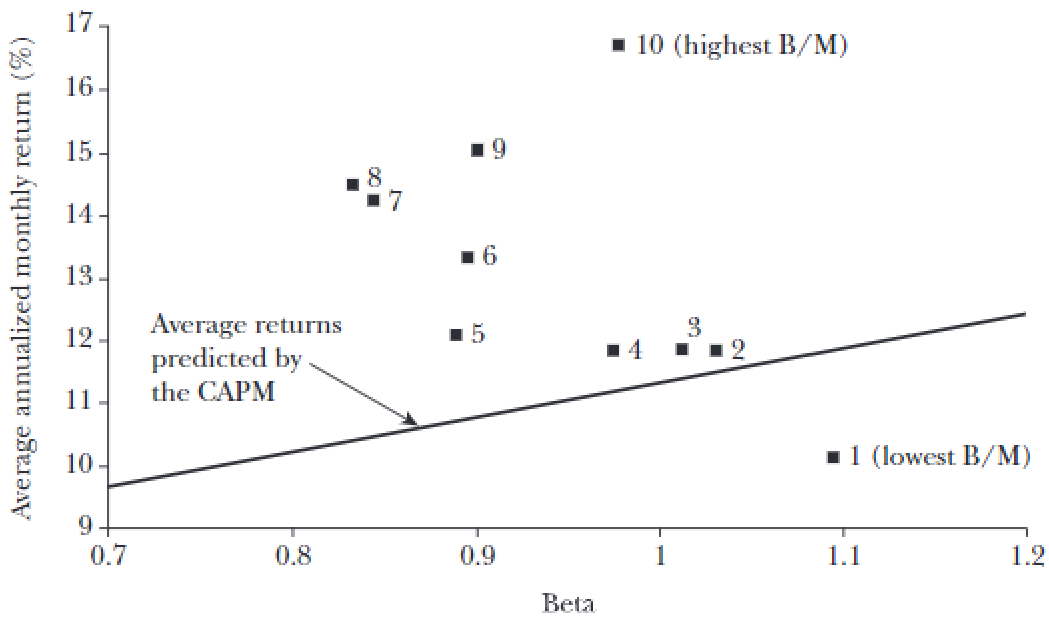

Figure 3. Return to beta for portfolios constructed on B/M (Fama & French, 2004).

The main advantage of the CAPM is simultaneously its weakness. The model fundamentally represents a basic market equilibrium structure utilizing only market risk to estimate expected returns. Although the model is easy to use, it lacks the complexity needed to accurately represent the cross-section of stock returns across different types of portfolios. CAPM solely considers market risk, ignoring other sources of risk that might be priced in the market. Fama and French (2004) claim that using the CAPM to estimate expected stock returns is impractical because alternative models incorporating additional risk factors, such as size and value, provide more accurate stock return estimations.

2.4 Multi-Factor Models

Basu (1977) shows that price-to-earnings ratios significantly impact stock returns, contradicting CAPM predictions. The P/E effect, along with other anomalies such as the B/M anomaly identified by Fama and French (1993), undermines the efficacy of the CAPM. This thesis will next analyze asset pricing models developed to address issues that remain problematic to the CAPM. These multi-factor asset pricing models explain asset returns through multiple risk factors, aiming to exploit multiple sources of risk instead of just one. Prior to discussing the Fama-French models, this thesis will address the arbitrage pricing theory proposed by Stephen Ross (1976).

2.4.1 Arbitrage pricing theory

Stephen Ross (1976) offers the arbitrage pricing theory (APT) as an alternative to the CAPM. Unlike the CAPM, the APT estimates expected returns based on multiple macroeconomic factors instead of market risk exposure. These factors may include, among other parameters, changes in inflation, GDP, and the yield curve. Moreover, the Arbitrage Pricing Theory (APT) does not claim that markets are always efficient; instead, it suggests that markets can occasionally be mispriced, allowing arbitrageurs to capitalize on transitory pricing discrepancies through APT modeling. The model assumes a linear relationship between the expected return and the asset's sensitivity to various macroeconomic factors, expressed mathematically as follows:

$$E(R_i) = R_F + \beta_{i1}\lambda_1 + \beta_{i2}\lambda_2 + \ \dots\ \beta_{in}\lambda_n. \tag{5}$$

In the formula, $E(R_i)$ represents the expected return of asset i, R_F is the risk-free rate, and β_{in} is the asset’s beta (factor loading) to factor n. λ_n represents the risk premium of factor n. Therefore, the expected excess return of asset i is the sum of factor loadings multiplied by the respective factor risk premiums.

The CAPM, being a one-factor model, is significantly more straightforward to use than the APT. However, its simplicity makes it prone to empirical constraints. The CAPM requires the input of expected market return, while the APT model estimates stock returns by utilizing risk premiums associated with macroeconomic factors. In addition, the more complex APT offers a more flexible asset pricing model despite possessing notable drawbacks. The primary challenge of implementing the APT is choosing the appropriate variables, as the theory does not specify which factors or even the number of factors to include. This allows significant room for user interpretation, resulting in varied estimations for APT model users who select to utilize different factors. Moreover, due to the non-generalizable nature of the APT factors, the model's results lack comparability.

2.4.2 Three-factor model

Fama and French (1992, 1993) discover that historically small-cap stocks have consistently outperformed large-cap stocks and that value stocks have similarly outperformed growth stocks. They proposed a three-factor model incorporating size and value factors alongside the CAPM market factor to account for these anomalies. They argue that this model offers greater explanatory power of the cross-section of stock returns than CAPM alone. The model comprises of the size factor (small minus big, SMB), the value factor (high minus low, HML), and the market factor (MRF). The SMB factor represents the spread between the returns of a diversified portfolio of small-cap stocks and a diversified portfolio of large-cap stocks. The HML factor, consequently, is the return spread between the portfolios with high and low book-to-market ratio stocks. The sensitivity of a particular portfolio to these factors can be estimated through multiple regression analysis. The three-factor model is written as follows,

$$R_{it} - R_{Ft} = a_i + b_i\left(R_{Mt} - R_{ft}\right) + s_i SMB_t + h_i HML_t + e_{it}, \tag{6}$$

where $R_{it} - R_{Ft}$ denotes the excess return of asset i and $(R_{Mt} - R_{Ft})$ represents the market factor, SMB is the size factor, and HML is the value factor. Corresponding factor loadings are denominated by b_i, s_i and h_i respectively. Unexplained excess returns are denoted as a_i, meanwhile e_{it} is a zero-mean error term. Fama and French (1992) argue that HML, SMB, and the market factor are significantly uncorrelated, thus each significantly contributes to return variability and the cross-section of average stock returns.

Fama and French (1993) identify the effectiveness of the three-factor model in explaining the returns of portfolios formed on size and book-to-market ratio. Moreover, they argue that the model can also account for returns linked to sales growth, cash flow/price, and earnings/price variables, as these patterns are explained by their exposure to value, size, and market factors (Fama & French, 1996). They suggest that, because of the significant return explanatory power provided by test results, the three-factor model is clearly superior to CAPM in explaining the cross-section of average stock returns in the US. However, the explanatory power of the model might vary based on the structure of the test portfolios. Nonetheless, the model has certain shortcomings, as noted by Asness (1995), Jegadeesh & Titman (1993), Fama & French (1996), and Titman et al. (2004), who show that the three-factor model fails to account for returns associated with anomalies such as low investments, momentum, and short-term stock returns.

2.4.3 Five-factor model

The three-factor model fails to account for significant average returns related to investments and profitability, as demonstrated by Novy-Marx (2013), Cohen et al. (2002), and Fama & French (2006, 2008). Fama and French (2015) add two new factors to the model and base their justification for incorporating them on the Miller-Modigliani (1961) valuation model and a modified dividend discount model. The dividend discount model proposes that a company's market value is the present value of its projected dividends. The formula for the company market value is as follows,

$$M_t = \sum_{\tau=1}^{\infty} E(d_t - \tau)/(1+r)^{\tau}, \tag{7}$$

where M_t denotes the market value at time t. The average long-term return is represented by r, and $E(d_{t+\tau})$ is the projected dividend for the period $t + \tau$. The model can then be augmented with Miller and Modigliani's (1961) relationship between expected return, investment, profitability, and book-to-market ratio as follows,

$$M_t = \sum_{\tau=1}^{\infty} E(Y_{t+\tau} - dB_{t+\tau})/(1+r)^{\tau}. \tag{8}$$

Here, $Y_{t+\tau}$ represents total equity earnings and $dB_{t+\tau}$ denotes the change in book equity for the period $t+\tau$. Dividing by the book value of equity at time t provides,

$$\frac{M_t}{B_t} = \frac{\sum_{\tau=1}^{\infty} E(Y_{t+\tau} - dB_{t+\tau})/(1+r)^{\tau}}{B_t}. \tag{9}$$

Equation 9 indicates that holding all else constant, a lower stock value or a higher book-to-market ratio results in increased expected returns. Also stronger earnings provide higher expected stock returns. An increase in investments, resulting in higher book equity B_t, leads to decreased expected returns. Consequently, profitability and investments influence the expected stock returns.

Motivated by academic research and the implications of the modified dividend discount model, Fama & French (2015) incorporate investment (conservative minus aggressive, CMA) and profitability (robust minus weak, RMW) factors into their model. The authors argue that the revised Fama & French five-factor model captures a broader range of average stock returns compared to its three-factor counterpart. Fama and French (2015) define the model as follows,

$$R_{it} - R_{Ft} = a_i + b_i\left(R_{Mt} - R_{ft}\right) + s_i SMB_t + h_i HML_t + r_i RMW_t + c_i CMA_t + e_{it} \,. \tag{10}$$

The five-factor model adds RMW and CMA factors into the three-factor model. In the formula, RMW represents the spread between the return of a diversified portfolio of stocks exhibiting strong operating profitability and the return of a diversified portfolio with weak operating profitability. CMA denotes the return spread between a well-diversified portfolio comprising stocks with conservative investing characteristics and a diversified portfolio with aggressive investment characteristics. If the factor loadings: b_i, s_i, h_i, r_i, and c_i explain all variability in the returns, the intercept (a_i) is zero.

The Fama-French models are often augmented with additional factors. A common variation is the incorporation of a momentum factor, as displayed in Carhart's (1997) four-factor model. Conversely, Fama and French (2015) exclude it from their five-factor model. They argue that incorporating momentum would yield only negligible improvements to the model's performance. Nonetheless, the momentum factor plays an important role when explaining returns for portfolios explicitly built on momentum.

2.4.4 Six-factor model

To address the issues encountered by the Fama-French three- and five-factor models concerning momentum-based portfolios, Fama and French (2018) expand the model by incorporating a momentum factor. They are hesitant about doing so, as the theoretical justification for momentum in stock returns is lacking despite empirical evidence from, among others, Jegadeesh & Titman (1993), Asness (1995), and Asness et al. (2013) demonstrating the persistence of the momentum anomaly in practice. The momentum factor UMD (up minus down), established by Fama and French (2018), resembles other factors in the model, consisting of a long-short portfolio of high and low momentum sub-portfolios. The long portfolio consists of a well-diversified set of stocks with high trailing 12-month returns, while the short portfolio comprises a well-diversified set of stocks with low trailing 12-month returns. The equation for the six-factor model is as follows,

$$R_{it} - R_{Ft} = a_i + b_i\left(R_{Mt} - R_{ft}\right) + s_i SMB_t + h_i HML_t + r_i RMW_t + c_i CMA_t + u_i UMD + e_{it} \,. \tag{11}$$

UMD represents the momentum factor premium, while u_i denotes the UMD factor loading or factor beta. In contrast to other factor portfolios, the composition of the UMD factor is rebalanced monthly instead of annually to correspond with empirical evidence regarding the behavior of short-term momentum returns.

Fama and French (2018) demonstrate that the augmented model incorporating the momentum factor greatly enhances the model's ability to explain stock returns for portfolios that are utilizing momentum. Nonetheless, as previously stated, the root causes of the momentum anomaly remain obscure. Conventional financial theory suggests that factor exposure should increase expected stock returns only if the factor represents a genuine source of systematic risk. The authors argue that a conclusive reason for the momentum anomaly is missing. Therefore, the momentum factor explains market behavior that cannot be attributable to exposure to fundamental risk characteristics of the market. Nonetheless, the momentum anomaly has several explanations.

Among other studies, Daniel et al. (2002) suggest that the anomaly can be explained behaviorally, linking market over- and undercorrections to investors' overconfidence and biased self-attribution. This results in stock prices diverging from their intrinsic values, creating both positive and negative momentum. Scholars also challenge the behavioral justification for momentum, proposing structural explanations instead. Korajczyk and Sadka (2004) argue that economic frictions, such as trading costs, may influence stock momentum. The persistent nature of the momentum effect is firmly confirmed in empirical research, supporting the rationale behind the incorporation of the momentum factor in the six-factor model.

3 Literature review

Measuring the performance of any asset pricing model addresses two fundamental questions. Does the model correctly explain stock returns, and are the markets efficient? This is known as the joint hypothesis problem, which arises because any test of an asset pricing model simultaneously assesses market efficiency. This complicates the interpretation of empirical results from asset pricing model tests, as observed shortcomings may be interpreted either as flaws in the model or as market inefficiency. Therefore, it is crucial to ensure that the asset pricing models are carefully defined. Only empirically and theoretically robust factors that consistently reflect real-life market behavior should be incorporated in the models. Factors should also exhibit significant risk premiums to be useful in asset pricing. This section of the thesis will first analyze the literature related to the theoretical and empirical justification for each factor of the six-factor model. This paper will then analyze research on the performance of the Fama-French multi-factor models as applied in various markets.

3.1 Standalone factors

Financial literature has found hundreds of individual factors, creating a “factor zoo” as expressed by Feng et al. (2020). The wide range of competing factors complicates the selection of the right mix of factors for the optimal model. This section of this thesis visits the factors identified by Fama and French (1993, 2015, 2018) included in the six-factor model, analyzing the robustness of their factor selection beyond the market factor.

The size premium is a widely documented phenomenon in financial literature. Intuitively, small firms with limited access to financial markets are more adversely affected by economic downturns and liquidity restrictions. On the other hand, small companies typically exhibit greater growth opportunities compared to their large-cap counterparts, resulting in a greater focus on distant future cash flows in stock valuation, which are prone to higher variability. Perez-Quiros and Timmermann (2000) validate this by showing that the expected returns of small-caps are more significantly affected by prevailing economic conditions compared to large-caps. Holding all other factors constant, small-caps are expected to offer higher returns than high-caps assuming rational asset pricing.

The size factor provides significant explanatory power for US stock returns, as Fama and French (1995) demonstrate that a factor model incorporating a size factor outperforms the CAPM. The results are aligned with Fama and French (1992), where the authors show that the size factor possesses independent explanatory power of expected stock returns. The persistence of the size effect is also validated by notable studies such as Banz (1981) and Reinganum (1981). However, the size effect seems to have significantly weakened post 1980s in the US, but it remains significant when controlling for quality factors (Asness et al., 2018). Despite the size effect being significant in the US, small-caps do not consistently outperform large-caps globally (Alquist et al., 2018). In addition to the lack of an international size effect, Alquist et al. (2018) observe that the size factor possesses limited efficacy in isolation. Overall, existing research indicates that the size factor remains significant and relevant in the United States, but inconsistent globally.

The fundamental rationale behind the value premium is typically explained in the literature in two ways. Some scholars argue that the book-to-market ratio serves as an indicator of risk. Chen and Zhang (1998) suggest that value stocks are inherently riskier than growth stocks due to their tendency of being distressed companies with elevated financial leverage, and highly unpredictable future returns. The overall risk associated with value stocks disproportionately increases during bad economic conditions compared to non-value stocks (Zhang, 2005). Hence, investors who hold high book-market-ratio stocks should receive compensation for their exposure to increased risk. The second explanation for the value effect is that value stocks become undervalued due to the higher general interest towards growth companies and subsequent market overvaluation, leaving value stocks neglected and undervalued, as suggested in Piotroski & So (2012).

Historically value stocks have provided a notable risk premium in the US stock market. Portfolios with high exposure to the value factor have provided increased portfolio returns compared to portfolios with lower exposures, even after controlling for market beta (Fama and French, 1992). In other words, HML has provided a significantly positive risk premium from 1963 to 1990. Due to the tendency of anomalies disappearing post-publication, Fama & French (2021) reexamine the value premium in the US market. They observe that in the latter half of the 1963–2019 examination period, the value premium is significantly reduced and statistically insignificant, although still detectable. They suggest that conclusions regarding the potential disappearance of the value premium cannot be drawn due to the significant monthly return volatility observed in the latter half of the study period, which is also noted by Ali et al. (2003), showing that value stocks' returns exhibit greater volatility than value stocks.

The HML factor has consistently underperformed the market since 2007, resulting in an approximate –55% depreciation through 2020, increasing criticism towards the relevance of the value premium (Arnott et al., 2021). The significant underperformance of value stocks has led to the idea of "value's death". However, Arnott et al. (2021) argue that this perception can be considerably exaggerated. The underperformance of value can be ascribed to many factors. First, the standard HML factor definition fails to account for intangible assets in the book value of equity. In addition, the valuations of value stocks, unlike those of growth stocks, experience substantial declines due to cyclical mispricing, thereby increasing their potential returns. Arnott et al. (2021) highlight both attributes as opposing the idea of value's death, claiming that value is in “hibernation”. In fact, Asness et al. (2015) demonstrate that the integration of value in portfolio construction along with additional factors can significantly improve the performance of factor strategies. In conclusion, the value factor finds more consistent support from the literature than the size factor. Utilizing a value factor in portfolio selection can expand the mean-variance frontier, but its efficacy is significantly improved when combined with additional factors. Furthermore, the value factor also exhibits significance on an international level (Chiah et al., 2016; Fama & French, 2012, 2017; Kubota & Takehara, 2017).

Asset growth is a rational predictor for stock returns as demonstrated in equation 9. Assuming everything else remains unchanged, an increase in company investments yields higher book equity, resulting in decreased expected returns. Furthermore, economic theory argues that firms undertaking significant investments may encounter diminishing marginal returns on invested capital. Academic research also suggests that firms encountering corporate events that increase their assets, such as acquisitions and debt or equity offerings, experience extended periods of decreased stock returns (Cooper et al., 2008). Cooper et al. (2008) also argue that firms that reduce their assets, such as through dividend payments or share repurchases, typically experience prolonged periods of increased stock returns.

Cooper et al. (2008) suggest that the change in firm assets may be a reliable indicator of future stock returns, especially in the US stock markets. Their results indicate that companies with low asset growth rates yield an average risk-adjusted return of 9.1%, whereas those with high asset growth rates suffer a risk-adjusted returns of -10.4% annually from 1963 to 2003. The findings indicate that a company's annual asset growth rate is a significant predictor of stock returns. Titman et al. (2004) support the notion that company investments, measured by the change in total assets, negatively affect future stock returns. The correlation between investments and stock returns is even higher for companies with low debt-to-equity ratios and higher cash flows (Titman et al., 2004). On average, companies that increase their investments experience a five-year period of decreased returns. The existing research highlights a significant correlation between company investments and stock returns. As a result, the CMA factor provides significant power for predicting stock returns. Nonetheless, the investment factor does not consistently reflect stock returns internationally (Chiah et al., 2016; Dirkx & Peter, 2018; Fama & French, 2017).

The profitability factor is likely the most intuitive driver of expected stock returns. In equation 9, while holding everything else constant, higher profitability increases stock returns, as profitable firms are expected to generate higher returns than unprofitable ones. One might argue that there is no fundamental risk characteristic driving the profitability factor, as rational asset pricing suggests that higher profitability should be reflected in higher market prices. Nonetheless, high-profitability firms may be regarded as riskier due to the potential difficulty of maintaining strong returns over prolonged periods, as their profitability may diminish more drastically, compared to firms with lower profitability, during unfavorable economic conditions. Still, evidence suggests that firms exhibiting higher profitability, as measured by the ratio of gross profits to assets, yield higher returns than their less profitable counterparts (Novy-Marx, 2013). Moreover, Ball et al. (2015) argue that operating profitability as an alternative measure for Novy-Marx's gross profits-to-assets ratio, exhibits a more robust correlation with expected returns.

The profitability premium is strongly supported by empirical evidence as operating profitability can forecast returns up to ten years ahead in the US (Ball et al., 2015). The utility of profitability is further highlighted in combined profitability-value strategies, where the factor significantly expands the mean-variance frontier (Novy-Marx, 2013). Nonetheless, the RMW factor produces inconsistent outcomes in the international context, although it exhibits greater consistency than CMA (Chiah et al., 2016; Fama & French, 2017).

Fama and French (2018) extend their five-factor model by introducing a momentum factor. However, they are reluctant to do so, arguing that the momentum effect is not attributable to systematic risk. And therefore, momentum is not a risk premium; instead, it is a market anomaly. However, the persistence of momentum in stock returns is extensively documented in financial literature. Jegadeesh and Titman (1993) demonstrate that purchasing well-performing stocks while shorting poorly performing stocks based on their historical performance generates significant positive abnormal returns in the short term of 3 to 12 months. The authors find that the momentum effect of previous winners and losers is positive during the initial 12 months but dissipates over the following years.

Jegadeesh and Titman (2001) expand upon their prior research and verify that the momentum effect persists following their initial 1993 study. Consequently, they demonstrate that the momentum effect captured through purchasing past winners and shorting past losers is not a result of data snooping. Furthermore, Hurst et al. (2017) demonstrate that momentum has remained relevant and effective for every century following the 1880s while displaying low correlations with traditional asset classes, indicating that momentum investing has been consistently profitable for over a century. Positive momentum returns are also prevalent in most global markets, and these returns continue to be positive in the post-2000 period (Jegadeesh and Titman, 2023). However, after the 2000s, momentum appears to lack consistency in specific individual markets worldwide and has also diminished in significance in the United States. In contrast, Daniel and Moskowitz (2016) observe that although momentum is statistically significant and generally yields strong returns, momentum portfolios are prone to crashing during market turmoil, resulting in subsequent negative returns.

Extensive empirical evidence indicates that momentum is a prevalent phenomenon in global financial markets, enduring rather than vanishing over decades. Furthermore, momentum is not considered a proxy for systematic risk. Instead, momentum seems to arise from behavioral factors. The primary behavioral reasons for momentum are commonly regarded as the stock market over- and underreactions and investor cognitive biases (De Bondt & Thaler, 1985; Daniel et al., 1998; Daniel et al., 2001). Nonetheless, as defined by Fama and French (2018), the momentum factor exhibits strong empirical support, but it lacks a theoretical justification. However, the substantial body of empirical evidence supports its incorporation into asset pricing models.

The individual components of the six-factor model can offer considerable utility in predicting stock returns. Although the size factor has limited value in isolation, it achieves efficacy when integrated with other factors, particularly in combined value strategies. Value has recently experienced subpar performance but has the potential to yield significant returns in combination with other factors. Investment behavior of firms has been shown to significantly influence future stock returns. Profitability has exhibited significant explanatory power of returns despite the absence of definitive consensus on the theoretical rationale for the profitability premium. Finally, momentum is supported by a substantial body of empirical research. While individual factors can extend the mean-variance frontier, their combination is considerably more effective. Nonetheless, the literature does not provide definitive conclusions regarding whether the performance of the factors is due to market efficiency or inefficiency. Overall, the existing literature validates the theoretical and empirical justifications for the FF6F factors.

3.2 Empirical performance

This section of the literature review examines the performance of the Fama-French factor models. Since the six-factor model is not an established model, the majority of researchers examining the Fama-French models employ the three- and five-factor models in their tests of factor performance. Given the scarcity of studies employing the six-factor model, this literature review will primarily focus on research papers utilizing the five-factor model.

Fama and French (2015) set up the standard definitions for test portfolios and the five-factor model factor construction. They construct test portfolios by categorizing the excess monthly returns of NYSE, AMEX, and NASDAQ stocks into value-weighted portfolios. The timeframe spans from July 1963 to December 2013. They construct three sets of 5 x 5 double-sorted portfolios categorized by Size-B/M, Size-Operating Profitability (OP), and Size-Investment (INV). The portfolios are sorted into five size groups and five groups for the other characteristics using NYSE percentile breakpoints, resulting in a total of 25 value-weighted portfolios. These left-hand-side portfolios (LHS) are then used in the study to evaluate model performance. Fama and French also test alternative test portfolio construction schemes, and the testing indicates that the five-factor model appears to be resilient to the way the test portfolios are constructed. The 5 x 5 test portfolio composition is regarded as the industry standard with the LHS regression testing method.

Fama and French (2015) also demonstrate that altering the factor definitions has minimal impact on the model performance. They independently categorize individual equities into two sorts based on size and three sorts based on stock characteristics, which include operating profitability, investment, and B/M ratios. The initial definition utilizes an independent 2 x 3 sort, comprising two size groups and three stock characteristic groups. The second factor definition is given by 2 x 2 sort of two size groups and two stock characteristic groups. The final factor definition further isolates the factor results using a 2 x 2 x 2 x 2 structure. Upon examination, the authors conclude that the model's performance remains resilient across variations in the composition of its factors. The 2 x 3 sort is currently regarded as the standard for factor definitions.

The Fama-French five-factor model captures around 71% to 94% of the average expected stock returns (Fama and French, 2015). Despite this, the model is easily rejected with the Gibbons, Ross, and Shanken (1989) test (GRS). This is a noticeable improvement over the three-factor model, as the RMW and CMA factors boost the model's performance. Fama and French (2015) therefore validate the profitability premium proposed by Novy-Marx (2013) and the widely observed relationship between investments and stock returns in the US.

The improved power of the five-factor mode is also evident by its improved ability to explain anomalies that remain unexplained with the three-factor model. In contrast to the three-factor model, the five-factor model reduces the number of observed anomalies, as the additional factors improve the model's capacity to account for returns associated with value and profitability-related market returns (Fama and French, 2016). After risk adjustment, the number of unexplained anomalies decreases, by significantly reducing the observed alphas of anomaly portfolios when utilizing the five-factor model over the three-factor model (Huynh, 2018). Furthermore, other anomalies, such as the abnormal returns of US vice stocks, which appear anomalous when explained with CAPM or the three-factor model, effectively disappear when the returns are explained with the five-factor model (Richey, 2017). The five-factor model also seems to explain the returns of low-beta stocks significantly more effectively. The widely recognized weak correlation between low market beta portfolios and expected returns (the low-beta anomaly) is explained by the RMW and CMA factors, where positive exposure to these factors increases the expected returns of low-beta stocks. Meanwhile, stocks with high betas generally display negative CMA and RMW factor loadings, thus lowering their expected returns.

Although the five-factor model enhances the explanatory power of the three-factor model, it continues to encounter numerous shortcomings. The five-factor model performs poorly with momentum-based portfolios (Fama and French, 2015). The model's explanatory power would increase with the addition of a momentum factor. The five-factor model also fails to capture returns linked to some anomalies, including the accruals anomaly and returns linked to net share issues in microcap stocks (Fama and French, 2016). In addition, Fama and French (2015) highlight more issues regarding the model. A four-factor model omitting the HML factor provides results comparable to the five-factor model, thereby rendering value factor redundant. This is due to the other factors absorbing value returns. The model also fails to explain the low average returns of small-cap firms, whose returns behave similarly to stocks that invest heavily despite poor profitability.

The robustness of the five-factor model, or any asset pricing model, is best assessed with out-of-sample validation. The model should also work outside of the original data sample. The robustness of the five-factor model in the US stock market finds support in studies utilizing fresh sample data and new approaches, yet these studies reveal certain shortcomings. While the five-factor model provides more consistent results than its three-factor predecessor, the model does not sufficiently explain long-term abnormal performance of companies in the midst of significant corporate events, especially in the long run, shedding doubt on long-term model robustness (Dutta, 2019). Dutta also supports the findings of Fama and French (2015) regarding the 4-factor model, which omits the HML factor, delivering equivalent power to the five-factor model, affirming the redundancy of the value factor. Dhaoui and Bensalah's (2016) tackle the model's incapability to account for the returns of small and unprofitable firms, augmenting the model with a momentum factor and a factor that measures variations in investor sentiment. This augmented model accounts for portfolio returns linked to invested capital, profitability, and size, demonstrating that the incorporation of investor sentiment and other behavioral factors into asset pricing models improves their explanatory power. Furthermore, the study highlights the importance of the momentum factor.

The US stock market analyses support the five-factor model's robustness, with explanatory power for the cross-section of stock returns ranging from 71% to 94%. A major weakness of the model is its failure to account for the low returns of small-cap stocks that exhibit behavior similar to companies that invest heavily despite poor profitability. Moreover, momentum-based portfolio returns are not properly captured by the model. The five-factor model demonstrates significantly superior performance compared to the three-factor model in the US market.

Research indicates that the five-factor model effectively captures a significant portion of the cross-section of stock returns in the US markets. However, regional disparities in stock markets and economies raise questions about the model's international applicability. Fama & French (2012) identify consistencies and discrepancies in global factor premiums by analyzing size, value, and momentum factors across markets in North America, Europe, Japan, and Asia Pacific. A global value premium that diminishes with size is identified, alongside momentum premiums that consistently decline as size increases. Nonetheless, momentum appears to be nonexistent in Japanese market. The authors also demonstrate that global stock returns are significantly lower for larger stocks. Thus, the SMB factor possesses considerable explanatory power globally. Moreover, their experiments with global asset pricing models applied to forecast local stock returns are unsuccessful, indicating that global factor definitions should not be applied locally, which is also demonstrated by Griffin (2002). However, the research demonstrates global persistence of global size and value premiums. Moreover, Asness et al. (2013) provide further evidence of global value and momentum premiums.

Fama and French (2017) find more mixed results of the international applicability of their five-factor model. Only in North America, Europe, and Asia Pacific, the HML and RMW factors expand the mean-variance frontier. In all regions, the smallest High INV portfolios yield significantly lower returns. The CMA factor yields systematic results exclusively for larger stocks and only in the US. In Japan, stock returns display minimal factor loadings to profitability and investment factors. Aligned with the evidence from the United States, the model's primary global shortcoming is its failure to account for the low average returns of small-cap stocks, which behave similarly to firms that invest heavily despite low profitability. The investment factor appears to diminish in power globally, whereas the size, profitability, and value factors serve as reliable predictors of stock returns worldwide.

The five-factor model demonstrates significant variability in overall performance across various markets, demonstrated in the discrepancies in the results of regional studies. Chiah et al. (2016) demonstrate that the five-factor model is superior to the three-factor model in Australia. The five-factor model in the Chinese stock market outperforms its competitors by passing the GRS test for most portfolios analyzed by Guo et al. (2017). Furthermore, the five-factor model demonstrates statistical superiority in Eastern Europe and Latin America, as evidenced by Foye (2018b) and Berggrun et al. (2020). Nonetheless, the three-factor model exhibits performance nearly identical to that of the five-factor model within the Japanese investment universe from 1972 to 2014 (Kubota and Takehara, 2017). Meanwhile, Jiao and Lilti (2017) contend that the five-factor model provides negligible supplementary explanatory power relative to the three-factor model in the Chinese A-share stock market. Furthermore, Foye (2018a) argues that neither the five-factor nor the three-factor models can consistently explain UK equity returns, thereby offering an unreliable estimation of financial risk. Foye (2018a) contends that the models are inadequate asset pricing models in the UK. Overall, the five-factor model's performance varies globally, indicating that regional economic and market dynamics significantly influence the applicability of factor models.

A recurring finding in regional studies is the persistent significance of the value factor, which contradicts the findings of Fama and French (2015) concerning the US stock market, where the inclusion of RMW and CMA factors renders HML redundant. Chiah et al. (2016) provide evidence that in the Australian market, the value factor retains its significance alongside the investment and profitability factors. Foye (2018b) analyzes 18 emerging markets and discovers that all regions examined exhibit significant value premiums. Guo et al. (2017) identify unique value patterns in the Chinese market. Conversely, Kubota and Takahera (2017) contend that the HML factor does not provide significant predictive power for expected returns in Japan. In a global context, these findings collectively contradict the idea that the value factor is redundant in the five-factor model due to the presence of RMW and CMA factors.

The two new factors, RMW and CMA, demonstrate varying degrees of significance across the studies. The factors, while providing explanatory power in various regions, demonstrate reduced or negligible power in others. Chiah et al. (2016) demonstrate that the model using the two additional factors slightly outperforms the three-factor model across all metrics in Australia. However, in Japan, the CMA factor does not provide any additional explanatory power (Kubota and Takehara, 2017). Foye (2018b) notes that, unlike in other emerging markets, there is no incremental power from CMA and RMW in Asia, aligning with the findings of Chiah et al. (2016), Fama & French (2012), and Quo et al. (2020). Furthermore, in the UK, additional factors fail to improve the model's performance (Foye, 2018a). The variability in results indicates that the RMW and CMA factors do not share global relevance and tend to be insignificant.

In summary, international examination of the five-factor model yields inconsistent results regarding its efficacy. The model's performance is significantly affected by the market in which it tested in, indicating that stock market and economy-specific factors greatly impact stock returns and, consequently, the applicability of factor models. The five-factor model appears to offer additional power over the three-factor model in North America, Australia, Europe, and Asia-Pacific. The investment factor, however, seems to be consistently significant only in North America. The model explains the cross-section of stock returns fairly well around the world, although with less consistency than in North America. Inconsistencies in the performance of individual factors highlight how important it is to test the model in different markets to ensure its global robustness.

3.3 Limitations and the extended model

Despite the relative success of the five-factor model, it still has significant flaws. Blitz et al. (2018) underscore some of the issues concerning the model. They note that the model continues to employ the CAPM market factor despite empirical studies indicating that the relationship between beta and expected return is flatter than what CAPM estimates. In addition, the model neglects the widely recognized momentum phenomenon. Also, concerns regarding model robustness have emerged due to the investment and profitability factors, as the theoretical and economic rationale behind these two factors, compared to the others in the model, remains ambiguous.

Some augmentations of the model, addressing its shortcomings, have achieved noteworthy success; however, the complexity of these augmentations tends to render them inferior alternatives. A notable limitation of the five-factor model is its ineffectiveness in predicting returns for small-cap stocks with low profitability (Fama and French, 2015; Fama and French, 2017). Dhaoui and Bensalah (2016) address this issue by proposing the integration of momentum and investor sentiment factors. Their revised model can efficiently capture portfolio returns related to capital investment, profitability, and size. They illustrate that incorporating investor sentiment and other behavioral factors into the asset pricing model significantly improves the explanatory power of the original model, especially for the problematic small-cap. Anderson et al. (2005) approach the issues of the model by utilizing a proxy for investor heterogeneity to the three-factor model. Integrating the range of investor beliefs into conventional asset pricing improves the accuracy of empirical models, improving the explanatory power of the model. Nevertheless, such modifications to the five-factor model are impractical and difficult to replicate, making them insufficient solutions to the model's shortcomings.

One major problem of the model is that the HML factor is largely ineffective in explaining expected returns in US equities within the five-factor model (Fama and French, 2015, 2016; Dutta, 2019). If other factors account for HML returns, what justifies utilizing the factor in the model? Nonetheless, the value factor has both theoretical and empirical support. Although the factor appears redundant in the United States, it holds significance in various other markets, such as Australia (Chiah et al., 2016), China (Guo et al., 2017), and in all of the 18 emerging markets studied by Foye (2018b). The HML factor can also achieve varying levels of significance when analyzed over shorter time periods. The performance of the value factor seems to be affected by regional and time period-specific conditions.

Overall, the five-factor model consistently faces the same challenges. The HML factor is redundant in the US market, and the new RMW and CMA factors show varying levels of success globally. In addition, the model struggles with small-cap growth stocks displaying returns like companies that invest heavily despite poor profitability. The HML factor's lack of significance in North America is counterbalanced by its prominence in other markets. Academics also criticize Fama and French's (2015) decisions to exclude momentum from their five-factor model despite the momentum anomaly being extensively documented in the literature and found to be one of the most persistent anomalies in financial research (Jegadeesh & Titman, 1993; Lee & Swaminathan, 2000; Zakamulin & Giner, 2022). In response to the demand from academia, Fama & French (2018) include a momentum factor in their six-factor model.

The momentum factor has significant explanatory power for stock returns as the anomaly occurs in most markets around the world. The Fama-French six-factor model outperforms the five-factor model, with the momentum factor providing the highest individual explanatory power after the market factor in the US (Fama & French, 2018). Likewise, Grobys and Kolari (2022) find momentum significant when explaining stock returns in Europe, North America and Asia (excluding Japan). Similarly, Dirkx and Peter (2020) show that the momentum factor delivers significant explanatory power in the German market. However, Dirkx and Peter argue that the momentum factor is primarily useful in explaining the returns of Size-Momentum sorted portfolios. The explanatory power of the factor is negligible when explaining returns for portfolios not formed on momentum. The presence of the momentum effect in various stock universes around the world warrants the use of the factor, despite the absence of robust theoretical justification for it.

The six-factor model generally outshines models that exclude the momentum factor; however, in specific regional markets, the momentum factor's additional power does not significantly improve the overall power of the six-factor model compared to the five- or even the three-factor model. Fama and French (2018) and Barillas and Shanken (2018) demonstrate that the model incorporating a momentum factor convincingly outperforms competing models in the United States. Meanwhile, Grobys and Kolari (2022) observe that the six-factor model tends to outperform the five-factor model in Europe, Japan, and North America, though not consistently in Asia (excluding Japan). In contrast, Dirkx and Peter (2020) conclude that the six-factor model does not provide significant additional explanatory power compared to the three-factor model in Germany. Once again, the relative performance of factor models is shown to be highly dependent on the given stock universe examined.

Table 1 at the end of the literature review summarizes the contributions of key studies discussed in this thesis. Overall, empirical research on the Fama-French asset pricing models suggests the additional factors in the extended models generally contribute to significantly increased explanatory power. However, analysis of various regional stock universes reveals that the explanatory power of these factors varies significantly across different markets and time periods. Only the market factor seems to be universally significant in global markets. However, most markets also feature significant momentum, value, and size premiums. Research also shows that defining risk factors unique to each market, rather than global ones, is the best practice for model design. As a result, market- and time-specific asset pricing model tests are required to determine how the relationships between stock returns and risk factors behave in each investment universe.

3.4 Research gap

Most studies on asset pricing models focus on the United States or large regional investment universes such as Europe or Asia. However, these broad analyses fail to explain risk factor behavior in smaller, country-specific stock markets. Fama & French (2017) and Grobys & Kolari (2022) identify similarities in factor performance within Europe, with the market, HML, and RMW factors demonstrating significant explanatory power in both studies, alongside momentum, which is highlighted in Grobys & Kolari (2022). Although both studies produce comparable results for Europe as a whole, Dirkx and Peter (2020) show that only the market and momentum factors hold significant explanatory power in the German market. These contradictory findings indicate that county-specific markets can exhibit significantly different stock return behavior relative to the broader market, rendering some factors irrelevant within that stock market. Consequently, insights from the studies of the broader European market cannot be directly utilized for investment decision-making in country-specific stock universes, such as France. This emphasizes the necessity of evaluating asset pricing models at the individual country level.

No study has previously examined the six-factor model within the French stock market, despite the Paris stock exchange being one of the largest equity markets in Europe. The studies by Fama & French (2017) and Krobys and Kolari (2022) targeting Europe neglect the unique stock return dynamics of the French market. In the spirit of Dirkx and Peter (2020), this gap necessitates a market-specific analysis of the French stock market, contrasting the performance of the six-factor model against previous studies, and gaining deeper insights into the international efficacy of the model while examining this underexplored stock universe. Furthermore, as Fama and French (2018) challenge the theoretical justification of the momentum factor, robust empirical evidence is required to justify its application, which has yet to be examined in the French market. This thesis will address these gaps by building upon prior research offering new evidence regarding the behavior of Fama-French factor models in a country-specific context, in response to Hou et al. (2018). This thesis refines and expands on the work of Fama and French (2017) and Krobys and Kolari (2022) in broader European market, by conducting a deeper examination of the French market, and evaluating the model's performance in contrast to the broader market. This approach considers whether the factors of the broader market can be used to explain stock returns in regional markets.

Table 1. Summary table of key studies in asset pricing.

This table summarizes the contributions from key studies examined in the literature review and theoretical framework sections of this thesis. The table presents the model tested, the methodology used and highlights the contribution to the asset pricing literature. They are split into studies introducing new models and methods, additional studies from the US markets, studies covering broader markets, and studies covering country-specific markets.

#	Reference	Model	Methodology	Contribution
	New models			
1	Markowitz (1952)	MPT	Quadratic programming (Historical returns, variances and covariances)	Development of the Modern Portfolio Theory: The well-known approach to mean-variance optimization.
2	Treynor (1961)	CAPM	Mathematical reasoning and applying statistical assumptions	Building on top of MPT contributed to the creation of CAPM.
3	Sharpe (1964)	CAPM	Mathematical reasoning and applying statistical assumptions	Building on top of MPT contributed to the creation of CAPM.
4	Lintner (1965)	CAPM	Mathematical reasoning and applying statistical assumptions	Building on top of MPT contributed to the creation of CAPM.
5	Ross (1976)	APT	LHS regression, Mathematical reasoning	The introduction of the Arbitrage Pricing Theory. An asset pricing model utilizing macroeconomic factors in modeling stock returns.

6	Gibbons, Ross & Shanken (1989)	GRS	GRS test	A new methodology utilizing the F-distribution for the hypothesis that all alphas are zero. Used in testing the performance of asset-pricing models against test portfolios.
7	Fama & French (1992)	FF3F	LHS regression	A new asset pricing model extended on CAPM, utilizing three factors: market, value, and size.
8	Carhart (1997)	C4FM	LHS regression	Showing the persistent nature of the momentum anomaly and augmenting the FF3F model with a momentum factor
9	Fama & French (2015)	FF5F	LHS regression, Spanning regression, GRS	A new asset pricing model extending the FF3F, utilizing two additional factors: investment and profitability
10	Fama & French (2018)	FF6F	LHS regression, spanning regression, GRS, Maxed squared Sharpe Ratio	A new asset pricing model introducing a momentum to the FF5F
	Additional tests in the US			
11	Fama & French (2016)	FF5F	LHS regression, GRS	The list of anomalies significantly decreases when examined with the five-factor model in the US. Momentum only explains momentum-based portfolio returns.
12	Richey (2017)	FF5F	LHS regression	The seemingly excessive returns of US vice stocks, when explained with FF3F and C4FM, disappear when explained with FF5F, providing support for the model's efficacy.
13	Dhaoui and Bensalah (2016)	Augmented FF5F	LHS regression	The FF5F predictive power increases when augmented with additional investor sentiment and momentum factors.
	Broad market analyses			

14	Fama & French (2012)	Size, Value and Momentum	LHS regression, GRS	Value premiums tend to persist in most global markets. Furthermore, momentum is strong in North America, Europe, and Asia Pacific but not in Japan. The global four-factor model (C4FM) is efficient in explaining global portfolio returns.
15	Fama & French (2017)	FF5F	LHS regression, GRS, Spanning regression	Applying the FF5F to the same regions studied in Fama & French (2012) (North America, Europe, Asia Pacific, and Japan). Global versions of FF3F and FF5F fail, but local ones, while more effective, show vast variation of factor explanatory power between regions.
16	Foye (2018b)	FF5F	LHS regression, GRS, Spanning regression	Testing the FF5F model in emerging markets (18 countries from Eastern Europe, Latin America, and Asia). FF5F is found to consistently beat FF34F in Europe and Latin America, but not consistently in Asia.
17	Grobys & Kolari (2022)	FF6F	LHS regression, GRS, spanning regression, Pairs bootstrap	Applying the FF6F to North America, Europe, Asia, and Japan exclusively. The six-factor model achieves the highest squared Sharpe ratio in most regions. They also discover that the size factor is insignificant globally.
	Country-specific tests			
18	Chiah et al. (2016)	FF5F	LHS regression, GRS	Testing the performance of the FF5F in the Australian stock market. Five-factor model outperforms the FF3F, while HML remains significant.
19	Guo et al. (2017)	FF5F	LHS regression, GRS	Testing the FF5F in the Chinese stock market. Interestingly, the FF5F passes the GRS test, while CMA is found redundant.
20	Kubota & Takehara (2017)	FF5F	LHS regression, GRS, GMM test	Testing the FF5F in Japan. The model is found not to be the best model in Japan, while CMA and RMW are not statistically significant.

21	Foye (2018a)	FF5F	LHS regression, GRS	The tests in the UK reveal that the FF3F is unable to improve on FF3F, which is also deemed an insufficient asset pricing model in the UK.
22	Dirkx & Peter (2020)	FF6F	LHS regression, GRS, Spanning regression	Implementing the FF5F in the German stock universe. Only the market and momentum deliver explanatory power in Germany, with only marginal added explanatory power of the FF6F over the FF3F.
23	Huynh (2018)	FF5F	LHS regression, GRS	Extends on Chiah et al.'s (2016) analysis of the FF5F in Australia to explain returns of anomaly portfolios. The five-factor model improves FF3F efficacy, but some anomalies remain unexplained.

4 Data and methodology

This chapter establishes the sample and empirical methodology used in this study and presents the test portfolio and factor definitions.

4.1 Sample

The data sample consists of monthly stock returns for 162 months, spanning from July 2010 to December 2023, from French companies listed on the Paris Stock Exchange. The data is derived from the LSEG database. The final data sample excludes companies with market capitalization under €50 million. Moreover, financial stocks are excluded due to their fundamentally different balance sheets, which may skew the financial ratios and consequently distort the results. Additionally, consistent with Fama & French (2015), firms displaying negative book-to-market ratios are excluded from the final sample. Stock returns are calculated using a return index incorporating the distribution of profits influencing stock prices, such as dividends.

The sample includes accounting values from financial statements required to compute financial ratios for factor and test portfolio construction. The total number of stocks in the sample varies over the period as companies enter and exit the stock market. Companies must be listed and offer valid financial statement data at the time of portfolio reconstruction at the end of each June to be included in the portfolios for the subsequent period. Companies that file for bankruptcy or delist from the stock exchange through a buyout or equivalent arrangement may distort true portfolio returns. Consequently, stocks that enter or exit the stock exchange between portfolio rebalancing dates are excluded. The sample averages 352 stocks annually. Due to the stock count being considerably lower than that in the Fama & French (2015) study conducted in the US stock market, this study employs 4 x 4 double-sorted test portfolios instead of the 5 x 5 sort. This guarantees that each test portfolio contains an ample quantity of stocks for sufficient diversification.

4.2 Methodology

This thesis utilizes the Fama and French (2015) methodology for evaluating the performance of factors and to compare factor models. This thesis will initially adopt a left-hand side (LHS) approach to measure the performance of the FF3F, FF5F, and FF6F models against a set of test portfolios with varying levels of P/E, OP, INV, and MOM ratios. Test portfolios are regressed against the models, with the analysis focusing on the differences in portfolio alphas and the behavior of factor loadings. This thesis will then employ a right-hand side (RHS) approach, conducting spanning regressions to identify the individual factor contributions to the overall performance of the six-factor model. In the spanning regressions, each factor is regressed independently against the rest of the model to examine whether the individual factors yield significant intercepts, indicating significant additional explanatory power. Finally, a comparative analysis is conducted, where the models are tested against each other in base versus extended model regressions.

4.3 Construction of factors and test portfolios

This study utilizes the risk factor and test portfolio definitions established by Fama & French (2015) and Fama & French (2018). The primary model of this thesis is the FF6F, which includes the market factor ($R_m - R_f$), size factor (SMB), value factor (HML), profitability factor (RMW), investment factor (CMA), and momentum factor (UMD). Next, this thesis discusses how each factor is constructed.

The main French equity index, the CAC 40, contains only the 40 largest stocks on the Paris stock exchange. Sharpe (1964) proposes that the market factor represents a value-weighted index of all securities within an investment universe. Consequently, the CAC 40 is an inadequate proxy for the French equity market due to its limited number of stocks. Therefore, a comprehensive market index is constructed to proxy the market factor. The market factor ($R_m - R_f$) represents value-weighted returns of all stocks in the sample, updated monthly, subtracted by the monthly risk-free rate. The risk-free rate is the 1-

year French government bond yield, matching the portfolio rebalancing interval, averaging 0.01% monthly throughout the study period.

Fama and French (2015) demonstrate that their factor model remains robust regardless of the way its factors are defined. Therefore, this study applies their standardized 2 x 3 factor design. Risk factors are formed at the end of each June as a 2 x 3 size-characteristic sort, with the exception of the momentum factor, which is adjusted monthly. The size sort breakpoint is the sample median market capitalization at the time of portfolio rebalancing. The stock characteristic coefficient breakpoints are the sample 30th and 70th percentiles. The characteristic coefficients are the stock-specific B/M, OP, INV, and MOM ratios, which are derived from the most recent available company annual statement and market data.

The value factor (HML) is calculated using the B/M ratio, which is the book value of equity from the latest annual financial statement divided by the market capitalization at the end of June. The profitability factor (RMW) utilizes the OP ratio, which is defined as operating profit divided by book equity from the most recent financial statement. The investment factor (CMA) utilizes the INV ratio, expressing the percentage change in the company's total assets over the previous full accounting period. The momentum factor's characteristic coefficient, MOM is the trailing 12-month stock return, excluding the most recent month, accounting for short-term reversals. The coefficient is therefore calculated as the percentage change between the return index at t –1 month and t –13 month. Equations 12 to 15 present the equations for the characteristic coefficients.

The factors are defined by independent 2 x 3 size-characteristic sorts. In HML, stocks are split into two size groups: Big (B) and Small (S) and further subdivided into three Book-to-Market (B/M) groups: High (H), Neutral (N), and Low (L). These sorts generate six distinct portfolios, which are then weighted by market capitalization. The RMW factor is generated by dividing the sample into two size groups and three groups based on the OP ratio, resulting in six portfolios that include large- and small-cap variants of Robust (R), Neutral (N), and Weak (W) portfolios. The CMA portfolios are similarly constructed by sorting the sample into six value-weighted portfolios of small and large Conservative (C), Neutral (N), and Aggressive (A) based on their INV ratio. The identical procedure is applied to the UMD factor, utilizing the trailing 12-month returns, resulting in classifications for Up (U), Neutral (N), and Down (D) portfolios across both size categories. The SMB factor is computed utilizing the returns from the big and small HML, RMW, and CMA portfolios. Each SMB sub-factor portfolio is constructed by subtracting the average returns of the big portfolios from those of the small portfolios, as illustrated in equations 16 to 18. The final SMB factor is derived by averaging these three SMB portfolios, as shown in equation 19.

The value factor (HML) is the spread between the mean of the two high book-to-market ratio portfolios (B_H & S_H) and the mean of the two low book-to-market ratio portfolios (B_L& S_L), as illustrated in equation 20. The profitability factor is the spread between the two robust profitability portfolios (B_R & S_R) and the two weak profitability portfolios (B_W & S_W), as presented in equation 21. The investment factor is the spread between the average of the two conservative portfolios (B_C & S_C) and the average of the aggressive portfolios (B_A & S_A), shown in equation 22. Similarly, the momentum factor is the difference between the averages of the two high momentum portfolios (B_U & S_U) and the two low momentum portfolios (B_D& S_D), as illustrated in equation 23. Averaging the returns of large and small portfolios aims to isolate the effect of size.

$$B/M_t = \frac{Book\ value\ of\ equity_t}{Market\ value\ of\ equity_t} \tag{12}$$

$$OP_t = \frac{Operating\ profit_t}{Book\ value\ of\ equity_t} \tag{13}$$

$$INV_t = \frac{Total\ assets_t - Total\ assets\ _{t-1}}{Total\ assets\ _{t-1}} \tag{14}$$

$$MOM_t = \frac{Return\ index_{t-1} - Return\ index\ assets\ _{t-13}}{Return\ index\ _{t-13}} \tag{15}$$

$$SMB_{HML} = \frac{S_H + S_N + S_L}{3} - \frac{B_H + B_N + B_L}{3} \tag{16}$$

$$SMB_{RMW} = \frac{S_R + S_N + S_W}{3} - \frac{B_R + B_N + B_W}{3} \tag{17}$$

$$SMB_{CMA} = \frac{S_C + S_N + S_A}{3} - \frac{B_C + B_N + B_A}{3} \tag{18}$$

$$SMB = \frac{SMB_{HML} + SMB_{RMW} + SMB_{CMA}}{3} \tag{19}$$

$$HML = \frac{B_H + S_H}{2} - \frac{B_L + S_L}{2} \tag{20}$$

$$RMW = \frac{B_R + S_R}{2} - \frac{B_W + S_W}{2} \tag{21}$$

$$CMA = \frac{B_C + S_C}{2} - \frac{B_A + S_A}{2} \tag{22}$$

$$UMD = \frac{B_U + S_U}{2} - \frac{B_D + S_D}{2} \tag{23}$$

Table 2 displays descriptive statistics and correlations of the factors. Panel A displays the average monthly factor returns, standard deviations, and t-statistics. The market, value, and momentum are the only factors exhibiting high, statistically significant factor premiums. Conversely, size, profitability, and investment factors do not provide significant premiums. The market factor yields a solid return, delivering an average monthly return of 0.91 percent during the period. Momentum appears strong in the sample, as the factor generates an average monthly premium of 0.98 percent. Strangely, the value premium is strongly negative, averaging -0.88 percent per month, indicating that growth stocks substantially outperformed value stocks during the period. This may have occurred due to historically low interest rates, as more accessible and affordable capital potentially benefits growth companies more than value firms.

Table 2. Factor descriptive statistics and correlations.

This table displays the mean, standard deviation, and sample mean T-statistic for each of the FF6F factors. Panel B displays the correlation matrix of the factors. The values are expressed as percentages, and the mean denotes the average monthly risk premium associated with the factor. The critical values and notations for the T-statistic are as follows: At 1% significance level, the value is ±2.61 (***); at 5% level, it is ±1.98 (**); and at 10% level, it is ±1.65 (*).

Panel A: Factor means, standard deviations, and t-statistics

	MRF	*SMB*	*HML*	*RMW*	*CMA*	*UMD*
Mean	0.91	-0.06	-0.88	0.26	0.09	0.98
STD	2.81	2.81	2.90	3.04	2.10	4.03
T-Stat	4.14***	-0.27	-3.87***	1.07	0.56	3.11***

Panel B: Factor Correlations

	MRF	*SMB*	*HML*	*RMW*	*CMA*	*UMD*
MRF	1	-0.03	0.19	-0.10	-0.03	-0.39
SMB	-0.03	1	0.07	-0.21	0.04	-0.36
HML	0.19	0.07	1	-0.26	0.44	-0.51
RMW	-0.10	-0.21	-0.26	1	-0.38	0.32
CMA	-0.03	0.04	0.44	-0.38	1	-0.24
MOM	-0.39	-0.36	-0.51	0.32	-0.24	1

Compared to the factor premiums of North America and Europe from the same time period, displayed in Appendix 1, the factors exhibit relatively comparable premiums. The market factor in France yields higher average returns than in the broader European market, yet lower than in North America. The size and investment factors offer insignificant premiums across all regions. Momentum is pronounced in all markets, with the highest premium in France. The profitability factor is significant only in broader Europe. Notably, the value factor is negative across all regions. However, the factor premium is significant only in France. It is worth noting that the factor premiums diverge between the broad European and the specific French stock universes.

The sample factor correlations vary greatly, as presented in panel B of Table 2. Most factors have weak correlations with the market. The value factor has a slight positive correlation with the market and a relatively strong negative correlation with momentum. This suggests that negative value exposure may absorb some momentum returns. The size factor has weak correlations across the board, except for the significant negative correlations with profitability and momentum. The profitability factor displays weak to moderate correlations, with the strongest being a positive correlation with momentum and a negative correlation with the investment factor. The momentum factor is negatively correlated with the market, implying that the momentum effect is independent of the market. The degree of low and negative correlations between factors signifies that they capture distinct sources of systematic risk. This reduces the probability of multicollinearity, enhancing the reliability of the model.

In left-hand side (LHS) regressions, the factor models are evaluated with test portfolios constructed based on size and book-to-market (B/M), operating profitability (OP), investment (INV), or momentum (MOM) to evaluate their ability to explain the monthly excess returns of these portfolios. Test portfolios are well-diversified portfolios structured in a 4 x 4 size-characteristic structure. Each June, stocks are independently divided into quartiles according to their market capitalization and quartiles according to their book-to-market ratio. The intersection of these two sorts produces 16 Size-B/M test portfolios, which are subsequently value-weighted based on the market capitalization of the companies in each portfolio. The identical process is used to create 16 Size-OP and 16 Size-INV portfolios. The 16 Size-MOM portfolios are constructed identically but are rebalanced on a monthly basis.

Table 3 presents the average excess returns and coefficients of the test portfolios. The average excess returns are displayed on the left, and the average coefficients of the portfolios are on the right. The portfolios sorted by size and B/M exhibit a distinct negative value premium, where the portfolios with the highest B/M ratios offer substantially lower returns compared to those with lower B/M ratios. The B/M ratios are consistent across the size sorts, with the exception of the largest size quartile, which exhibits substantial variation in B/M ratios. No significant size effect is observed. The second B/M quartile portfolio in the third size quartile marginally outperforms the lowest B/M sort, while the second B/M sort portfolio significantly outperforms the lowest B/M sort portfolio in the smallest size quartile. Overall, growth stocks outperform value stocks.

Table 3. Average excess returns and coefficients of the 4 x 4 test portfolios.

This table displays the average excess monthly returns for each test portfolio. The average returns are displayed on the left side, with the portfolios organized in a 4 x 4 matrix based on their market capitalization and characteristic coefficient. The average characteristic coefficients of the respective test portfolios are displayed on the right-hand side.

	Average Excess returns				Average coefficients			
	Low	2	3	High	Low	2	3	High
4 x 4 Portfolios formed on size and B/M								
	Size x Book-to-market				B/M			
Small	0.98	1.46	0.91	0.13	0.22	0.51	0.88	2.09
2	1.56	1.31	0.66	0.26	0.21	0.49	0.85	3.00
3	1.21	1.31	0.77	0.20	0.22	0.49	0.83	2.58
Big	1.17	0.81	0.47	0.28	0.23	0.48	0.83	1.65
4 x 4 Portfolios formed on size and OP								
	Size x Operating Profitability				OP			
Small	0.44	0.76	0.71	1.08	-0.89	0.03	0.15	1.02
2	0.46	0.73	0.93	1.10	-1.00	0.04	0.15	0.57
3	0.73	0.63	1.00	0.87	-0.87	0.05	0.15	0.46
Big	1.19	0.79	0.80	0.91	-0.26	0.04	0.15	0.34
4 x 4 Portfolios formed on size and INV								
	Size x Investment				INV			
Small	0.53	1.43	1.31	0.42	-0.15	0.00	0.08	0.62
2	1.01	0.98	0.92	0.76	-0.14	0.00	0.08	0.57
3	0.62	1.01	0.94	1.10	-0.10	0.01	0.08	0.37
Big	1.21	0.67	0.90	0.94	-0.10	0.01	0.08	0.33
4 x 4 Portfolios formed on size and MOM								
	Size x Momentum				MOM			
Small	0.75	0.16	0.97	1.46	-0.35	-0.07	0.14	0.70
2	0.49	0.74	1.09	1.80	-0.35	-0.07	0.14	0.74
3	0.64	0.98	0.78	2.03	-0.32	-0.07	0.15	0.76
Big	0.74	1.11	1.23	1.42	-0.29	-0.06	0.15	0.53

The portfolios sorted by size and operating profitability exhibit a noticeable profitability premium. The average OP ratios of the portfolios are consistent in the two middle OP quartiles but vary more in the highest and lowest OP quartiles, where the OP ratio typically decreases as market capitalization increases, indicating a wider range of OP ratios among small-cap portfolios. High OP stocks consistently offer superior returns in the two smallest size quartiles. The effect decreases in the third size quartile and is undetectable in the largest quartile. Table 4 displays the average number of stocks in each 4 x 4 test portfolio sort. The largest size and lowest OP quartile portfolio averaged just four stocks during the study period, rendering its returns potentially unreliable. The returns indicate that the profitability premium is stronger in small-cap stocks, although it is also evident to a smaller degree in large-cap stocks.

The portfolios sorted by size and investment do not display a noticeable investment premium. The INV coefficients across size quartiles remain consistent, exhibiting greater variability in the highest INV quartile, where investment ratios are higher in small-cap portfolios. Returns exhibit significant variability across the OP quartiles, and a clear relationship between the OP coefficient and portfolio returns is nonexistent. Moreover, no observable size effect is evident in the returns of the test portfolio. The inconsistency of returns among the portfolios and the insignificant investment premium indicate that the CMA factor is likely to provide inconsistent explanatory power.

The portfolios sorted by size and MOM display a high momentum premium. The MOM coefficients are negative in the first two quartiles and positive in the two highest MOM quartiles. In both the lowest and highest MOM quartiles, the coefficients are significantly lower in high-cap portfolios, suggesting that large shifts in stock prices are typically greater in small-caps. The portfolio returns demonstrate a consistent increase in average returns as MOM increases. The effect is evident across all size quartiles, with the exception of the second MOM quartile within the smallest size quartile, where the returns are significantly lower than those of the lowest MOM sort. Size has virtually no impact on the returns on portfolios formed on momentum. During the study period, the portfolio in the largest size and lowest momentum quartile averaged seven stocks, as only a small number of large-cap stocks exhibit poor momentum returns. The high momentum portfolios consistently yield the highest returns among all tested portfolios, indicating a significant momentum premium throughout the study period.

Table 4. Average number of stocks in the 4 x 4 test portfolios.

This table displays the average quantity of stocks included in a specified portfolio throughout the study period. The Size and Characteristic sorts are independent, resulting in significant variation in the number of stocks across the different sets of portfolios.

	Low	2	3	High	Low	2	3	High
	Size x Book-to-market				Size x Operating profitability			
Small	19	17	17	31	21	24	18	14
2	26	26	23	29	18	25	28	26
3	20	28	25	20	8	19	35	29
Big	30	26	25	14	4	15	35	38
	Size x Investment				Size x Momentum			
Small	17	17	17	20	17	19	18	18
2	20	23	25	27	16	22	26	27
3	13	26	26	23	12	23	26	24
Big	10	28	34	20	7	21	28	22

The average returns for factors and test portfolios tell a similar story. Risk premiums, evident in average factor returns, are also visible in test portfolios. The market and momentum display significant risk premiums. Furthermore, the value premium is significantly negative. In contrast, risk premiums associated with size, profitability and investment appear insignificant. Table 4 shows that all but three of the test portfolios held a minimum of ten stocks over the span of the study. This is caused by the fact that only a limited number of the sample's high-cap companies have poor profitability and momentum. Ultimately, the highest factor premium identified is momentum, whereas the lowest was the significantly negative value premium. This suggests that from 2010 to 2023, during the low interest rate regime and post-financial crisis period, growth stocks and stocks exhibiting high momentum returns consistently performed the best. Factor correlations indicate that value and momentum factors have a low correlation with the market, but they have a relatively strong negative correlation with each other. The weak and negative correlations between the FF6F factors indicate a low likelihood of multicollinearity.

5 Empirical analysis

Test portfolios are independently regressed against the FF3F, FF5F, and FF6F models. The regression results for each set of 16 test portfolios are displayed in this chapter. The results of the LHS regressions for the Size-Value, Size-Profitability, Size-Investment, and Size-Momentum test portfolios are displayed in Tables 5 through 8, respectively. Each regression table presents the intercepts for the FF3F, FF5F, and FF6F models, along with the FF6F factor loadings. In the regression tables, α_i denotes the intercept term and β_i signifies the market beta. The coefficients s_i, h_i, r_i c_i, and u_i represent the factor loadings for SMB, HML, RMW, CMA and UMD, respectively. Full FF3F and FF5F regression results are presented in the Appendices. The Size-Value regressions are displayed in Appendix 2 and Appendix 3. Size-Profitability in Appendices 4 and 5. Size-Investment in Appendices 6 and 7. Finally, the Size-Momentum FF3F and FF5F regressions are presented in Appendices 8 and 9.

The three-factor model identifies significant alphas for two portfolios as presented in Table 5. The model effectively explains the returns for the lowest and third value quartiles, but it overestimates the returns for the portfolios in the highest quartile while underestimating those in the second value quartile. The five-factor model displays problems with the same portfolios as the three-factor model. The model appears to moderately reduce the alphas, but not significantly. The average alphas in the six-factor model exhibit a slight reduction in significance, while the number of non-zero alphas decreases to a single portfolio. The alphas decrease especially in the highest B/M quartile, but the smallest portfolio in the lowest B/M quartile, although not statistically significant, experiences an increased overestimation with an alpha of -0.23 (0.59). On average, the FF3F leaves 0.23 percent of absolute monthly returns unexplained, whereas the FF6F only reduces that number only by 0.01 to 0.22 percent; however with less significant intercepts.

The market betas in the six-factor model regressions range from 0.73 to 1.21 and are all statistically significant. No apparent relationship exists between B/M or size and the market beta. The factor loadings for SMB are consistently higher for small-caps and weak or slightly negative for large-cap portfolios, indicating a consistent relationship between portfolio size sorts and SMB factor loadings. The factor loadings for the value factor remain consistent. Portfolios with low B/M ratios exhibit negative factor loadings, while portfolios with high book-to-market ratios display positive loadings. The h_i t-statistics are significant in all quartiles except the second, which provides the biggest challenges for the model. The factor betas for RMW are generally slightly negative and are significant for only three portfolios, one of which is the lowest value quartile and smallest size quartile portfolio, exhibiting a r_i coefficient of -0.41 (-3.23), potentially contributing to increased mispricing in the six-factor model. The investment factor loadings are fairly low, showing no obvious relationship between the various size and value quartiles. Only the second size quartile portfolio in the highest B/M sort has a significant CMA factor loading of -0.20 (-1.91). The factor betas for UMD (u_i) range from 0.19 to -0.23, displaying varying levels of significance. Six portfolios exhibit significant u_i loadings. The momentum factor could explain some of the lowered t-statistics of intercepts, especially within the large-cap high B/M portfolios. The adjusted R-squared values for the portfolios vary from 0.54 to 0.87, indicating a decent model fit. The R-squared tends to increase with larger size and higher book-to-market ratios. The additional factors in extended models slightly improve the performance of the FF3F with the Size-Value portfolios.

Table 5. LHS regressions with 16 Size-Value sorted portfolios.

This table presents the regression results from the 16 Size-B/M portfolios. Panel A displays the alphas and t-statistics for the test portfolios regressed using the three-factor model. Panel B displays the alphas and t-statistics for the test portfolios regressed using the five-factor model. Panel C presents the alphas, factor loadings and modified R-squared values of the test portfolios regressed with the six-factor model. The critical values for the t-statistic are as follows: At 1% significance level, the critical value is ±2.61; at 5% level, it is ±1.98; and at 10% level, it is ±1.65. t-statistics exceeding the 5% significance threshold are highlighted in bold.

B/M →	Low	2	3	High	Low	2	3	High
Panel A: FF3F regressions for 4 x 4 test portfolios formed on size and B/M								
	α_i				t (α_i)			
Small	-0.06	0.66	0.36	-0.32	-0.14	**2.43**	1.59	-1.49
2	0.07	0.37	0.05	-0.05	0.29	1.74	0.26	-0.26
3	-0.03	0.56	-0.12	-0.49	-0.14	**2.78**	-0.59	-1.62
Big	-0.06	0.04	0.08	-0.44	-0.52	0.33	0.51	-1.95

Panel B: FF5F regressions for 4 x 4 test portfolios formed on size and B/M

	α_i				$t(\alpha_i)$			
Small	-0.17	0.67	0.31	-0.25	-0.43	**2.41**	1.35	-1.17
2	-0.01	0.40	0.02	0.04	-0.03	1.83	0.09	0.20
3	-0.02	0.64	-0.08	-0.51	-0.06	**3.12**	-0.39	-1.68
Big	-0.01	0.00	0.04	-0.42	-0.09	-0.04	0.24	-1.80

Panel C: FF6F regressions for 4 x 4 test portfolios formed on size and B/M

	α_i				$t(\alpha_i)$			
Small	-0.23	0.52	0.22	-0.36	-0.59	1.86	0.92	-1.66
2	0.17	0.39	-0.02	0.08	0.66	1.74	-0.09	0.38
3	0.00	0.58	-0.10	-0.43	0.00	**2.76**	-0.46	-1.38
Big	-0.07	-0.01	0.05	-0.31	-0.60	-0.05	0.31	-1.34
	β_i				$t(\beta_i)$			
Small	1.07	0.94	0.94	0.88	**9.55**	**11.77**	**14.02**	**14.08**
2	0.94	1.05	0.89	0.90	**12.72**	**16.63**	**17.02**	**15.13**
3	1.02	0.93	1.21	1.21	**13.30**	**15.57**	**20.14**	**13.71**
Big	1.06	0.89	0.73	1.16	**31.51**	**25.43**	**16.46**	**17.42**
	s_i				$t(s_i)$			
Small	1.23	0.68	1.05	1.07	**9.13**	**7.08**	**12.98**	**14.23**
2	0.96	1.11	0.87	0.88	**10.72**	**14.46**	**13.83**	**12.31**
3	0.43	0.47	0.61	0.53	**4.64**	**6.56**	**8.39**	**5.03**
Big	-0.03	-0.06	-0.12	0.09	-0.79	-1.50	**-2.30**	1.09
	h_i				$t(h_i)$			
Small	-0.34	0.05	0.28	0.38	**-2.29**	0.48	**3.16**	**4.62**
2	-0.93	-0.04	0.15	0.58	**-9.40**	-0.43	**2.18**	**7.38**
3	-0.41	0.16	0.25	0.31	**-4.02**	**2.03**	**3.12**	**2.68**
Big	-0.22	0.00	0.27	0.35	**-4.95**	-0.09	**4.49**	**3.95**
	r_i				$t(r_i)$			
Small	-0.41	-0.11	0.05	-0.12	**-3.23**	-1.18	0.69	-1.73
2	-0.11	-0.05	-0.02	-0.05	-1.35	-0.73	-0.29	-0.81
3	-0.10	-0.04	0.00	-0.30	-1.19	-0.62	-0.01	**-2.97**
Big	0.05	-0.06	-0.08	-0.08	1.31	-1.49	-1.53	-1.13
	c_i				$t(c_i)$			
Small	0.31	0.00	0.10	-0.11	1.62	-0.02	0.84	-1.05
2	0.19	-0.05	0.07	-0.20	1.50	-0.49	0.76	-1.91
3	-0.02	-0.17	-0.08	0.08	-0.18	-1.65	-0.79	0.54
Big	-0.12	0.11	0.10	-0.04	**-2.04**	1.77	1.35	-0.35
	u_i				$t(u_i)$			
Small	0.08	0.19	0.12	0.14	0.72	**2.25**	1.75	**2.19**
2	-0.23	0.01	0.04	-0.05	**-2.98**	0.17	0.77	-0.79
3	-0.02	0.08	0.02	-0.10	-0.25	1.26	0.35	-1.12
Big	0.08	0.00	-0.02	-0.13	**2.23**	0.06	-0.33	-1.85

	Adj. R^2			
Small	0.57	0.54	0.72	0.75
2	0.72	0.78	0.78	0.80
3	0.59	0.67	0.79	0.68
Big	0.87	0.83	0.73	0.75

In Table 6, only the five- and six-factor model find significant intercepts for a portfolio at 5 % significance level. However, at 10 % level the three-factor model finds significant non-zero intercepts for three portfolios from the lowest and highest profitability quartiles in the Size-Profitability sorted portfolios. FF5F faces challenges with low profitability portfolios, overestimating returns for the three smallest size quartiles in the low OP sort while underestimating returns for the largest portfolio in the same size quartile. The six-factor model decreases the number of significant non-zero alphas at the 10 % level to a single portfolio, while lowering overall intercept significance. The remaining significant intercept comes from the lowest OP and highest size quartile portfolio, exhibiting an average unexplained monthly return of 0.86% (2.71). However, the average returns of this portfolio are not as robust due to the low stock count of the portfolio. The six-factor model reduces average absolute alphas universally from 0.29 with the FF3F to 0.26. Still, low OP portfolios remain problematic for all models.

The market betas in the six-factor model range from 0.77 to 1.68. A distinct relationship between market beta and size or OP is absent. The SMB factor loadings consistently decrease with increasing market capitalization in the Size-Profitability portfolios. The size factor betas are significant for all portfolios except the largest size quartile. The h_i coefficients are mixed. About half of the portfolios exhibit significant value loadings, with the lowest found in the low OP portfolios, which is logical given that growth stocks frequently experience prolonged periods of low or negative profitability. All small quartile portfolios exhibit significant h_i values. For the remaining portfolios, HML loadings are comparatively weak. Low OP and big size portfolios have the most consistently significant RMW loadings. Nonetheless, the profitability factor loadings are mainly negative across all quartiles yet increasing from low to high OP sorts. The profitability factor does not consistently capture the various OP exposures. Meanwhile, no apparent relationship exists between the various portfolio sorts and CMA loadings. The same is true for the UMD factor, as the factor loadings are inconsistent and only four portfolios exhibit significant coefficients. The results suggest that, while RMW loadings are inconsistent, the incorporation of the momentum factor marginally improves the model's explanatory power. The adjusted R-squared coefficients are mostly high, except in the low OP quartile, suggesting that the model performs well overall but experiences most problems with low OP portfolios.

Table 6. LHS regressions with 16 Size-Profitability sorted portfolios.

This table presents the regression results from the 16 Size-OP portfolios. Panel A displays the alphas and t-statistics for the test portfolios regressed using the three-factor model. Panel B displays the alphas and t-statistics for the test portfolios regressed using the five-factor model. Panel C presents the alphas, factor loadings and modified R-squared values of the test portfolios regressed with the six-factor model. The critical values for the t-statistic are as follows: At 1% significance level, the critical value is ±2.61; at 5% level, it is ±1.98; and at 10% level, it is ±1.65. t-statistics exceeding the 5% significance threshold are highlighted in bold.

OP →	Low	2	3	High	Low	2	3	High
Panel A: FF3F regressions for 4 x 4 test portfolios formed on size and OP								
	α_i				t (α_i)			
Small	-0.45	0.26	0.10	0.38	-1.10	1.20	0.43	1.28
2	-0.75	0.21	0.24	-0.05	-1.89	1.30	1.38	-0.22
3	-0.80	-0.13	0.18	-0.09	-1.33	-0.58	1.05	-0.49
Big	0.62	0.17	0.02	-0.22	1.76	1.04	0.16	-1.88
	α_i				t (α_i)			
Small	-0.55	0.23	0.11	0.35	-1.36	1.05	0.49	1.13
2	-0.57	0.15	0.23	-0.10	**-2.12**	0.91	1.29	-0.48
3	-0.80	-0.07	0.20	-0.04	-1.35	-0.31	1.12	-0.21
Big	0.70	0.18	-0.04	-0.15	**2.23**	1.07	-0.37	-1.33
Panel C: FF6F regressions for 4 x 4 test portfolios formed on size and OP								
	α_i				t (α_i)			
Small	-0.60	0.16	0.06	0.17	-1.44	0.73	0.26	0.56
2	-0.44	0.21	0.18	0.05	-1.60	1.24	0.97	0.23
3	-0.69	-0.01	0.19	-0.05	-1.14	-0.05	1.03	-0.26
Big	0.86	0.24	-0.08	-0.14	**2.71**	1.39	-0.71	-1.22
	β_i				t (β_i)			
Small	1.04	0.77	0.94	1.06	**8.82**	**12.02**	**14.11**	**12.03**
2	1.22	0.82	0.98	1.02	**15.79**	**17.40**	**18.82**	**16.48**
3	1.68	0.91	1.05	1.18	**9.77**	**13.54**	**20.04**	**20.87**
Big	0.83	0.88	0.89	1.05	**9.17**	**17.83**	**28.77**	**33.12**

	s_i				t (s_i)			
Small	1.22	0.81	0.91	1.27	**8.62**	**10.57**	**11.24**	**11.95**
2	1.05	0.76	1.04	1.01	**11.24**	**13.35**	**16.56**	**13.59**
3	0.97	0.48	0.54	0.52	**4.67**	**5.85**	**8.54**	**7.64**
Big	-0.27	-0.05	-0.07	-0.03	**-2.46**	-0.78	-1.83	-0.67
	h_i				t (h_i)			
Small	-0.22	0.17	0.27	0.22	-1.41	**2.05**	**3.02**	1.89
2	-0.39	0.11	0.18	-0.25	**-3.79**	1.82	**2.58**	**-3.06**
3	-0.24	0.08	0.14	0.13	-1.06	0.85	**1.98**	1.69
Big	0.07	0.17	-0.02	-0.09	0.60	**2.66**	-0.57	**-2.22**
	r_i				t (r_i)			
Small	-0.46	0.05	0.07	-0.09	**-3.43**	0.64	0.96	-0.93
2	-1.23	-0.08	0.05	0.58	**-14.02**	-1.49	0.77	**8.32**
3	-0.60	-0.06	-0.02	-0.10	**-3.09**	-0.76	-0.33	-1.57
Big	-0.70	-0.12	-0.07	0.10	**-6.81**	**-2.12**	**-2.07**	**2.87**
	c_i				t (c_i)			
Small	0.29	0.06	-0.04	0.11	1.42	0.51	-0.39	0.70
2	-0.19	0.15	0.02	0.01	-1.44	1.89	0.17	0.05
3	0.10	-0.13	-0.04	-0.10	0.34	-1.08	-0.42	-1.02
Big	-0.07	0.00	0.14	-0.17	-0.44	0.04	**2.65**	**-3.16**
	u_i				t (u_i)			
Small	0.06	0.08	0.06	0.22	0.52	1.25	0.94	**2.42**
2	-0.17	-0.07	0.07	-0.20	**-2.13**	-1.51	1.29	**-3.07**
3	-0.14	-0.08	0.02	0.01	-0.75	-1.11	0.28	0.23
Big	-0.20	-0.08	0.05	-0.01	**-2.18**	-1.47	1.50	-0.33
	Adj. R^2							
Small	0.55	0.63	0.70	0.65				
2	0.84	0.81	0.81	0.79				
3	0.51	0.64	0.78	0.79				
Big	0.57	0.75	0.86	0.89				

In Table 7, the three-factor model identifies significant alphas for three Size-Investment sorted test portfolios. The alphas range from 0.74 to -0.39, with the highest values being in the two middle INV quartiles. The five-factor model reduces the number of significantly non-zero alphas to two. The model also reduces average absolute intercepts noticeably. The six-factor model is better at explaining the two middle INV quartile portfolios than the previous models, while not finding any significant alphas at the 5 % level. Overall, the average t-statistics of intercepts are reduced with FF6F. The six-factor model slightly increases the explanatory power over the other models, while leaving an average of 0.25 percent of absolute monthly returns unexplained.

The market betas for the portfolios are all statistically significant, ranging from 0.87 to 1.34. No relationship between market beta and size or investments is evident. The factor loadings for SMB are high for small-cap stocks and consistently decrease with increasing size, becoming weak or slightly negative for large-cap portfolios. The HML loadings exhibit an inconsistent distribution, ranging from 0.53 to -0.42, indicating a weak relationship between value and Size-INV sorted portfolios. The profitability factor loadings are inconsistent and generally weak or negative. The CMA loadings accurately capture the varying investment characteristics of the portfolios. Conservative portfolios have positive c_i values, while aggressive portfolios display negative values consistently. The CMA loadings vary from 0.75 to -0.56, which are significant for eight portfolios, with the highest t-statistics observed in the large and small size quartiles, indicating that investment risk is best captured in the small and large market capitalization portfolios. The momentum factor shows significant loadings in four portfolios. However, the distribution of UMD factor betas displays no distinct relationship between Size-INV portfolios and momentum. The adjusted R-squared coefficients range from 0.56 to 0.82, indicating a decent fit of model. The coefficients are evenly distributed, displaying only minor disparity in R-squared values in the small size quartile that consistently displays the lowest values, suggesting that the model struggles to explain small-cap investment related returns. The six-factor model marginally beats the five-factor model, which in turn significantly outperforms the three-factor model with the 16 Size-Investment portfolios.

Table 7. LHS regressions with 16 Size-Investment sorted portfolios.

This table presents the regression results from the 16 Size-INV portfolios. Panel A displays the alphas and t-statistics for the test portfolios regressed using the three-factor model. Panel B displays the alphas and t-statistics for the test portfolios regressed using the five-factor model. Panel C presents the alphas, factor loadings and modified R-squared values of the test portfolios regressed with the six-factor model. The critical values for the t-statistic are as follows: At 1% significance level, the critical value is ±2.61; at 5% level, it is ±1.98; and at 10% level, it is ±1.65. t-statistics exceeding the 5% significance threshold are highlighted in bold.

INV →	Low	2	3	High	Low	2	3	High
Panel A: FF3F regression for 4 x 4 test portfolios formed on size and INV								
	α_i				t (α_i)			
Small	-0.08	0.74	0.74	-0.39	-0.24	**2.81**	**2.61**	-1.47
2	0.18	0.37	0.24	-0.19	0.62	1.70	1.31	-1.03
3	-0.14	0.24	0.15	-0.20	-0.38	1.24	0.84	-0.67
Big	0.62	-0.14	-0.23	-0.33	**2.09**	-1.04	-1.75	-1.59
Panel B: FF5F regression for 4 x 4 test portfolios formed on size and INV								
	α_i				t (α_i)			
Small	-0.27	0.63	0.72	-0.18	-0.85	**2.36**	**2.50**	-0.70
2	0.20	0.34	0.26	-0.18	0.81	1.51	1.40	-0.95
3	-0.16	0.23	0.16	0.00	-0.43	1.15	0.84	-0.01
Big	0.29	-0.21	-0.17	-0.08	1.04	-1.55	-1.26	-0.42
Panel C: FF6F regression for 4 x 4 test portfolios formed on size and INV								
	α_i				t (α_i)			
Small	-0.42	0.53	0.51	-0.24	-1.27	1.95	1.77	-0.90
2	0.29	0.32	0.17	-0.13	1.13	1.39	0.88	-0.66
3	-0.22	0.23	0.11	0.09	-0.58	1.14	0.58	0.30
Big	0.27	-0.26	-0.10	-0.04	0.96	-1.85	-0.77	-0.23
	β_i				t (β_i)			
Small	1.01	0.91	0.97	0.98	**10.93**	**11.73**	**11.86**	**12.96**
2	1.08	0.87	1.00	1.04	**15.13**	**13.40**	**18.39**	**18.68**
3	1.34	1.09	1.03	1.14	**12.35**	**18.83**	**19.21**	**13.50**
Big	1.10	1.00	0.97	1.05	**13.73**	**25.04**	**25.08**	**19.13**
	s_i				t (s_i)			
Small	1.28	0.77	0.84	1.00	**11.39**	**8.23**	**8.52**	**11.01**
2	1.04	0.86	0.91	0.93	**12.06**	**11.03**	**13.83**	**13.90**
3	0.76	0.39	0.56	0.62	**5.77**	**5.61**	**8.72**	**6.10**
Big	-0.08	-0.03	-0.06	0.08	-0.78	-0.71	-1.37	1.19
	h_i				t (h_i)			
Small	0.10	0.07	0.32	0.16	0.78	0.66	**2.94**	1.60
2	-0.06	0.11	0.27	-0.08	-0.60	1.25	**3.75**	-1.03
3	0.41	0.19	0.13	-0.18	**2.86**	**2.54**	1.85	-1.63
Big	0.16	0.05	-0.23	-0.12	1.50	1.02	**-4.47**	-1.66
	r_i				t (r_i)			
Small	-0.19	0.09	-0.11	-0.29	-1.78	1.04	-1.22	**-3.37**
2	-0.60	-0.08	0.04	-0.02	**-7.42**	-1.11	0.65	-0.31
3	-0.25	-0.08	-0.06	-0.01	**-2.07**	-1.21	-1.05	-0.13
Big	-0.07	-0.04	0.01	0.04	-0.74	-0.90	0.25	0.59

	c_i				t (c_i)			
Small	0.46	0.23	0.06	-0.40	**2.91**	1.70	0.40	**-3.10**
2	0.05	0.09	-0.05	-0.02	0.43	0.82	-0.57	-0.21
3	0.09	0.04	0.00	-0.43	0.47	0.40	0.01	**-2.92**
Big	0.75	0.16	-0.14	-0.56	**5.42**	**2.39**	**-2.10**	**-5.86**
	u_i				t (u_i)			
Small	0.18	0.13	0.27	0.08	1.89	1.55	**3.20**	0.97
2	-0.11	0.02	0.12	-0.06	-1.48	0.31	**2.14**	-1.12
3	0.08	0.00	0.06	-0.12	0.70	-0.08	1.07	-1.34
Big	0.02	0.06	-0.08	-0.04	0.21	1.45	**-2.07**	-0.75
	Adj. R^2							
Small	0.64	0.56	0.58	0.69				
2	0.79	0.69	0.79	0.80				
3	0.61	0.76	0.76	0.64				
Big	0.64	0.82	0.83	0.76				

Table 8 shows that with the Size-Momentum test portfolios, the three-factor model struggles to explain the returns of high and medium momentum portfolios. Consequently, the model finds significant non-zero alphas for three portfolios. Low momentum portfolios are overvalued, while high momentum portfolios are undervalued. The exception is the big size, high MOM portfolio with an intercept of 0.08 (0.35), which is explained by its high market and value factor loadings. The five-factor model, which displays slightly worse performance compared to the three-factor model, struggles with the same portfolios. The six-factor model still finds significant alphas for two portfolios but vastly decreases their significance. The six-factor model significantly reduces average absolute alphas in comparison to the other models. The model reduces the average absolute monthly alphas from 0.35 (FF3F) to 0.29.

The market betas in the six-factor model are statistically weaker compared to the other models, especially for the portfolios in the biggest size quartile. This indicates that the momentum factor absorbs returns that are accounted for by the market factor in the other models. The beta coefficients range from 1.28 to 0.85, with higher values for portfolios in the low and high momentum quartiles, indicating that portfolios with significant positive or negative momentum demonstrate greater market risk compared to those in the middle momentum quartiles. The SMB factor loadings show consistency, with s_i

coefficients decreasing by size. The value factor shows inconsistency, as only four portfolios have significant HML loadings at the 5 % significance level. No noticeable relationship exists between HML loadings and Size-MOM sorted portfolios. The RMW betas are predominantly negative or weakly positive. High and low momentum quartile portfolios exhibit the highest significance for r_i. The investment factor loadings are scattered and generally weak, with only two portfolios presenting statistically significant CMA coefficients. The momentum factor effectively captures the returns of Size-MOM portfolios. UMD loadings are negative for low momentum portfolios and increase with higher MOM quartiles. The UMD loadings are generally significant in all sorts, except for the second MOM quartile. The adjusted R-squared values vary significantly from 0.83 to 0.41, suggesting that the model fails to account for the variance in the returns of low momentum portfolios. The six-factor model significantly enhances the explanatory power of momentum portfolio returns in comparison to the three- and five-factor models.

Table 8. LHS regressions with 16 Size-Momentum sorted portfolios.

This table presents the regression results from the 16 Size-MOM portfolios. Panel A displays the alphas and t-statistics for the test portfolios regressed using the three-factor model. Panel B displays the alphas and t-statistics for the test portfolios regressed using the five-factor model. Panel C presents the alphas, factor loadings and modified R-squared values of the test portfolios regressed with the six-factor model. The critical values for the t-statistic are as follows: At 1% significance level, the critical value is ±2.61; at 5% level, it is ±1.98; and at 10% level, it is ±1.65. t-statistics exceeding the 5% significance threshold are highlighted in bold.

MOM →	Low	2	3	High	Low	2	3	High
Panel A: FF3F regression for 4 x 4 test portfolios formed on size and MOM								
	α_i				t (α_i)			
Small	-0.16	-0.40	0.38	0.86	-0.37	-1.66	1.80	**2.23**
2	-0.42	-0.10	0.34	0.81	-1.12	-0.39	1.81	**3.49**
3	-0.41	0.18	0.09	0.87	-0.76	0.85	0.47	**3.32**
Big	-0.29	0.19	0.05	0.08	-0.65	0.88	0.39	0.35
Panel B: FF5F regression for 4 x 4 test portfolios formed on size and MOM								
	α_i				t (α_i)			
Small	-0.22	-0.48	0.36	0.93	-0.49	-1.95	1.67	**2.37**
2	-0.50	-0.06	0.32	0.81	-1.35	-0.24	1.70	**3.44**
3	-0.17	0.17	0.13	0.91	-0.32	0.77	0.67	**3.40**
Big	-0.30	0.19	0.01	0.19	-0.65	0.82	0.05	0.87

Panel C: FF6F regression for 4 x 4 test portfolios formed on size and MOM

	α_i				t (α_i)			
Small	0.08	-0.49	0.22	0.68	0.17	-1.96	1.00	1.73
2	-0.03	-0.03	0.20	0.52	-0.08	-0.12	1.02	**2.34**
3	0.55	0.21	0.04	0.72	1.12	0.97	0.18	**2.69**
Big	0.47	0.34	-0.03	-0.08	1.17	1.50	-0.21	-0.37
	β_i				t (β_i)			
Small	1.06	0.88	0.85	0.97	**10.08**	**9.82**	**11.91**	**7.77**
2	1.28	1.00	0.90	1.22	**9.94**	**10.95**	**12.20**	**15.92**
3	1.26	1.10	0.90	1.23	**2.38**	**8.96**	**7.46**	**8.23**
Big	1.38	1.12	1.24	1.28	**-2.73**	-1.87	-1.47	1.60
	s_i				t (s_i)			
Small	3.18	1.57	1.50	1.74	**9.16**	**8.62**	**9.43**	**6.29**
2	2.79	1.83	1.40	2.04	**9.73**	**9.68**	**10.19**	**11.54**
3	1.53	1.32	0.89	1.32	**4.04**	**8.56**	**6.55**	**7.64**
Big	-0.02	-0.28	-0.32	0.03	-0.05	-1.86	**-2.73**	0.26
	h_i				t (h_i)			
Small	-0.16	0.14	0.17	0.30	-0.93	1.52	**2.03**	**2.04**
2	-0.05	0.01	0.07	0.08	-0.37	0.09	1.03	1.01
3	0.00	0.15	0.17	-0.04	0.00	1.78	**2.26**	-0.44
Big	0.04	0.08	-0.05	-0.09	0.29	0.93	-0.95	-1.11
	r_i				t (r_i)			
Small	0.07	-0.05	-0.03	-0.31	0.47	-0.68	-0.47	**-2.48**
2	-0.25	-0.16	0.06	-0.23	**-2.22**	-1.92	1.01	**-3.26**
3	-0.39	-0.05	-0.04	-0.17	**-2.47**	-0.69	-0.54	**-1.98**
Big	-0.05	0.00	0.06	-0.20	-0.37	0.04	1.30	**-3.07**
	c_i				t (c_i)			
Small	0.10	0.18	0.05	-0.09	0.46	1.49	0.50	-0.48
2	0.20	-0.06	0.02	0.06	1.21	-0.46	0.25	0.60
3	-0.48	0.03	-0.08	-0.05	-2.01	0.32	-0.82	-0.39
Big	0.00	0.01	0.10	-0.21	-0.01	0.12	1.35	**-2.06**
	u_i				t (u_i)			
Small	-0.38	0.02	0.18	0.32	**-2.81**	0.27	**2.87**	**2.72**
2	-0.60	-0.04	0.16	0.37	**-5.89**	-0.49	**2.89**	**5.68**
3	-0.92	-0.06	0.12	0.24	**-6.34**	-0.92	**2.05**	**3.08**
Big	-0.98	-0.20	0.05	0.34	**-8.33**	**-2.96**	1.13	**5.58**
	Adj. R^2							
Small	0.49	0.59	0.61	0.41				
2	0.65	0.64	0.68	0.70				
3	0.51	0.74	0.67	0.69				
Big	0.59	0.73	0.80	0.83				

The results from the FF3F, FF5F, and FF6F regressions indicate that the six-factor model generally offers superior explanatory power within the French stock universe compared to the other models. Certain portfolios, particularly those exhibiting high momentum, continue to present difficulties for the model, suggesting that the momentum factor may require recalibration in order to better capture the returns of extreme momentum portfolios. The six-factor model beats the five-factor model across all sets of test portfolios, demonstrating that the momentum factor accounts for variations in stock returns, even in portfolios not explicitly constructed on momentum. Only in the Size-Value portfolios, the additional factors of FF6F provide minimal additional explanatory power, as the alpha reduction from the FF3F to the FF5F and FF6F models is negligible.

Table 9 displays the summary results from the LHS regressions with the three asset pricing models against each set of the 16 test portfolios. The table displays the average absolute alphas ($A|\alpha|$), average absolute alpha t-statistics ($A|t(\alpha)|$), and GRS test results utilizing chi-squared (χ^2) as the test statistic in place of the F distribution, along with the associated p-values. The GRS test is a specific application of the Wald test, where the null hypothesis states that all intercepts of the test portfolio set are jointly equal to zero, signifying that the model fully explains the variation in average portfolio returns. The critical value for the GRS chi-squared (χ^2) test statistic, with 16 degrees of freedom, is 26.30 at the 5% significance level. For the average absolute alpha, t-statistic, and chi-squared, a lower value indicates a better fit of model.

The chi-square test statistics are above the critical value for all but the six-factor model in the Size-Profitability (24.87) and Size-Momentum (23.46) portfolios, indicating that the model correctly prices both sets of test portfolios. However, their p-values are above 0.05. As a result, the null hypothesis is rejected in all sets of test portfolios for all models. The GRS test shows that for all sets of test portfolios, the six-factor model consistently provides the best explanatory power. The average absolute alphas consistently decrease with the addition of more risk factors. For the Size-Value, Size-Profitability, and Size-Momentum portfolios, the alpha reduction from the FF3F to FF5F is modest but distinct in

Size-Investment portfolios. The six-factor model significantly decreases average absolute alphas from the five-factor model in Size-OP and drastically in Size-MOM portfolios. The six-factor model demonstrates superior performance across all models, as indicated by the significantly lower GRS test scores and average absolute alphas.

Table 9. LHS regression result summary and Wald test.

This table displays the summary statistics from the left-hand side regressions. The results from each model regression are summarized by the sets of test portfolios. The table presents the average absolute alphas (A|α|) and the corresponding average absolute t-statistics (A|t(α)|). The table also presents the Wald test (GRS) chi-squared (χ^2) test statistics, and the corresponding p-values. The Wald test null hypothesis states that all intercepts of the 16 test portfolios are jointly equal to zero when explained with the given model. The critical value for the χ^2 statistic is 26.30 at the 5% significance level, where the null hypothesis is rejected if the χ^2 statistic is higher than the critical value. If the null hypothesis is accepted, it signifies that the model correctly explains the returns of a given set of test portfolios.

	A|α|	A|t(α)|	χ^2	p-value (χ^2)
16 test portfolios formed on size and value				
3-factor model	0.23	1.04	44.88	0.00
5-factor model	0.22	0.93	44.52	0.00
6-factor model	0.22	0.93	37.71	0.00
16 test portfolios formed on size and operating profitability				
3-factor model	0.29	1.07	35.47	0.00
5-factor model	0.28	1.05	28.14	0.03
6-factor model	0.26	0.97	24.87	0.07
16 test portfolios formed on size and investment				
3-factor model	0.31	1.34	52.06	0.00
5-factor model	0.25	1.11	44.94	0.00
6-factor model	0.25	1.02	35.68	0.00
16 test portfolios formed on size and momentum				
3-factor model	0.35	1.28	32.79	0.01
5-factor model	0.36	1.30	32.81	0.01
6-factor model	0.29	1.04	23.46	0.10

Spanning regression analysis is conducted to assess the degree to which each factor in the six-factor model provides additional explanatory power not accounted for by the other factors. Table 10 shows the six-factor model spanning regressions, where five factors are used to explain the returns of the sixth. The table displays the intercepts, factor betas, and adjusted R-squared statistics for each regression. If the intercept is non-zero, the factor expands the mean-variance frontier and adds significant explanatory power to the model.

Table 10. Spanning regressions using five factors to explain the returns of the sixth.

This table displays the results from the spanning regressions, where the returns of one of the FF6F factors are explained by the rest of the five. The table presents the intercepts, factor loadings and R-squared coefficients of the regressions. If a factor has a significant intercept, that factor holds significant explanatory power not captured by the other factors and expands the mean-variance frontier. The critical values for the t-statistic are as follows: At 1% significance level, the critical value is ±2.61; at 5% level, it is ±1.98; and at 10% level, it is ±1.65. t-statistics exceeding the 5% significance threshold are highlighted in bold.

LHS	*Int*	*MRF*	*SMB*	*HML*	*RMW*	*CMA*	*UMD*	*R^2*
MRF								
Coefficient	1.36		-0.24	0.03	-0.06	-0.27	-0.40	0.18
t-Statistic	**5.22**		**-2.54**	0.31	-0.61	**-2.00**	**-5.17**	
SMB								
Coefficient	0.34	-0.17		-0.14	-0.14	-0.10	-0.33	0.17
t-Statistic	1.47	**-2.54**		-1.57	-1.82	-0.84	**-5.23**	
HML								
Coefficient	-0.62	0.02	-0.11		-0.01	0.46	-0.33	0.36
t-Statistic	**-3.01**	0.31	-1.57		-0.10	**4.69**	**-5.79**	
RMW								
Coefficient	0.20	-0.04	-0.15	-0.01		-0.48	0.12	0.19
t-Statistic	0.82	-0.61	-1.82	-0.10		**-4.17**	1.65	
CMA								
Coefficient	0.48	-0.09	-0.05	0.27	-0.21		-0.02	0.27
t-Statistic	**3.05**	**-2.00**	-0.84	**4.69**	**-4.17**		-0.41	
UMD								
Coefficient	0.79	-0.37	-0.45	-0.54	0.14	-0.05		0.45
t-Statistic	**2.98**	**-5.17**	**-5.23**	**-5.79**	1.65	-0.41		

The intercept for the market factor is 1.36, indicating that 1.36% of monthly returns remain unaccounted for by the other factors. Omitting the value factor, -0.62% of returns remain unexplained, while other factors cannot explain 0.79% of the UMD returns. The regression results show that the intercepts for MRF, HML, CMA, and UMD are non-zero. The intercepts for CMA and SMB are equivalent to zero at the 5% significance level.

The HML intercept is negative as the value premium is negative, yet interestingly, HML has a significant positive factor loading with CMA despite the investment premium being positive. This indicates that value stocks are generally associated with companies that exhibit conservative investment strategies. The investment factor has a significant positive loading with HML and a negative loading with RMW, indicating that conservatively investing stocks are value stocks that exhibit low profitability. At the same time, the factor loadings of UMD suggest that winner stocks are large-cap growth stocks. the findings demonstrate that with the exception of size and profitability, all factors provide significant independent explanatory power.

A base model versus extended model test is conducted to examine the additional explanatory power additional factors provide when comparing two asset pricing models, where the second model is an extended version of the other. Table 11 presents the regressions, where the base model is used to explain the returns of each additional factor in the extended model. The market factor model is used to explain the returns of the (FF3F) size and value factors. The three-factor model is used to explain the returns of the (FF5F) RMW and CMA factors, and the five-factor model is used to explain the returns of the (FF6F) UMD factor.

The regression results show that all extended models provide increased explanatory power compared to their base model counterparts. Only the value factor at -1.03 (4.47) provides significant explanatory power over the market factor model, while the SMB intercept is insignificant at -0.04 (-0.17). In the five-factor model, CMA is the only factor that offers significant explanatory power compared to the three-factor model, with an intercept of 0.45 (2.98). The momentum factor's intercept is 0.79 (2.98), as the five-factor model cannot explain momentum returns. The greatest improvement in model performance happens from the CAPM to the three-factor model. The model-versus-model tests verify the significance of the value, investment, and momentum factors on top of the market factor. Moreover, the size and profitability factors do not contribute significant explanatory power to the models, as evidenced by the model versus model as well as the spanning regression results.

Table 11. Base model versus extended model.

This table presents model vs model tests, where the returns of the additional factors of the extended model are explained with the base model. The table displays the intercepts of the factors as well as their factor loadings. If the intercept of a factor is significant, that factor possesses additional return explanatory power not captured with the base model, and as a result, the extended model outperforms the base model significantly. The critical values for the t-statistic are as follows: At 1% significance level, the critical value is ±2.61; at 5% level, it is ±1.98; and at 10% level, it is ±1.65. t-statistics exceeding the 5% significance threshold are highlighted in bold.

LHS	*Int*	*MRF*	*SMB*	*HML*	*RMW*	*CMA*
MRF VS FF3F						
SMB	-0.04	-0.02				
	(-0.17)	(-0.34)				
HML	-1.03	0.16				
	(-4.47)	**(2.49)**				
FF3F VS FF5F						
RMW	0.08	-0.05	-0.22	-0.24		
	(0.32)	(-0.78)	**(-2.68)**	**(-3.04)**		
CMA	0.45	-0.07	0.01	0.33		
	(2.82)	(-1.67)	(0.14)	**(6.44)**		
FF5F VS FF6F						
UMD	0.79	-0.37	-0.45	-0.54	0.14	-0.05
	(2.98)	**(-5.17)**	**(-5.23)**	**(-5.79)**	(1.65)	(-0.41)

The empirical evidence confirms that the FF6F is the superior model in the French stock universe. However, not all factors contribute equally. While RMW and SMB appear redundant, momentum and value, alongside the market factor, are the most important. But how can these insights be applied in practice? Results from a cross-sectional regression of average monthly returns against the six-factor model loadings are presented in table 12. The regression results represent the percentage of additional returns that result from a one unit increase in each factor loading, holding all else constant. On average, a one unit increase in MOM loading increases monthly portfolio returns by approximately 0.78%. A one unit increase in HML loading decreases monthly returns by 0.71%. Meanwhile, one unit increase in market beta provides a 0.48% boost in average monthly returns, but this relationship is relatively weak. For CMA and SMB, the relationship between average returns and factor loadings is both economically and statistically insignificant. Remarkably, a one unit increase in RMW loading results in 0.37% higher average monthly returns. In summary, portfolios with higher exposure to the market, momentum, and profitability factors, and lower exposure to the value factor, can achieve economically significant excess monthly returns. The Momentum and Value factors exhibit the strongest loading-to-return relationships. As a result, all factors, apart from CMA and SMB, are useful in expanding the mean-variance frontier in portfolio selection in the French stock universe. These findings encourage the use of these factors in rule-based smart beta investment strategies, as the independent factor loadings significantly affect average stock returns, expanding the mean-variance frontier.

Table 12. Impact of factor loadings on average monthly portfolio returns.

This table presents the factor coefficients and T-statistics from a regression, where the average monthly returns of the test portfolios are regressed against the respective six-factor loadings. The regression results represent the percentage of extra returns that result from a one unit increase of a given factor loading. The critical values for the t-statistic with 57 degrees of freedom are as follows: at 1% significance level, the critical value is ±2.67 (***); at 5% level, it is ±2.00 (**); and at 10% level, it is ±1.67 (*).

Factor	*Intercept*	*MKT*	SMB	HML	RMW	CMA	MOM
Coefficient	0.50	0.48	-0.01	-0.71	0.37	-0.09	0.78
t Stat	1.75*	1.73*	-0.17	-3.84***	2.00**	-0.42	4.04***

5.1 Factor robustness tests

To assess the robustness of the six-factor model regression results, this study implements a subsample rolling window regression analysis to test how stable the factor loadings remain throughout the period. 36-month and 60-month rolling windows are used to independently regress each test portfolio against the six-factor model. The regression is repeated each month, with the window updating each time to include the most recent 36 or 60 months of data. From the monthly rolling window regression results, the rolling factor loadings are then subtracted from the full-sample factor loadings to calculate the absolute difference between the two values. This “absolute difference” variable is used to analyze how far the rolling factor loadings stray from their full-sample values. If the factor loadings are stable throughout the period, the absolute differences should remain near zero.

Figures 4 and 5 display the group average deviations of the rolling window factor betas from their full sample coefficients for each factor. The rolling window extends over 36 months in Figure 4 and 60 months in Figure 5. The coefficients displayed in the graphs for each group represent the averages of the 16 test portfolios within each set. With the 36-month window, the average rolling factor loadings fluctuate around the full-sample mean, deviating by as much as ±0.3 units from the full-sample beta. Distinct trends with changes in the average loading deviations are evident across all factors. The uniformity of average deviation trends indicates that the deviations are probably due to the factor portfolios rather than alterations in individual portfolio characteristics over time. The chart indicates that, in general, the average factor loadings exhibit a lack of stability in the short term. However, extending the window to five years (60 months) significantly reduces the variability in the coefficients, resulting in the rolling coefficients consistently aligning more closely with the full sample ones and exhibiting relative stability over time.

The summary statistics from the rolling window regressions, as described in Appendix 10, demonstrate that the 60-month rolling factor loadings exhibit relative stability throughout the study period in contrast to the 36-month window. The mean absolute difference between the rolling factor loadings and their full-sample values is approximately 0.12, with a standard deviation of roughly 0.15, while the 36-month window averages 0.17 with a standard deviation of around 0.23. The findings indicate that although the six-factor loadings demonstrate instability over a three-year span, they become noticeably more stable when assessed over a five-year period.

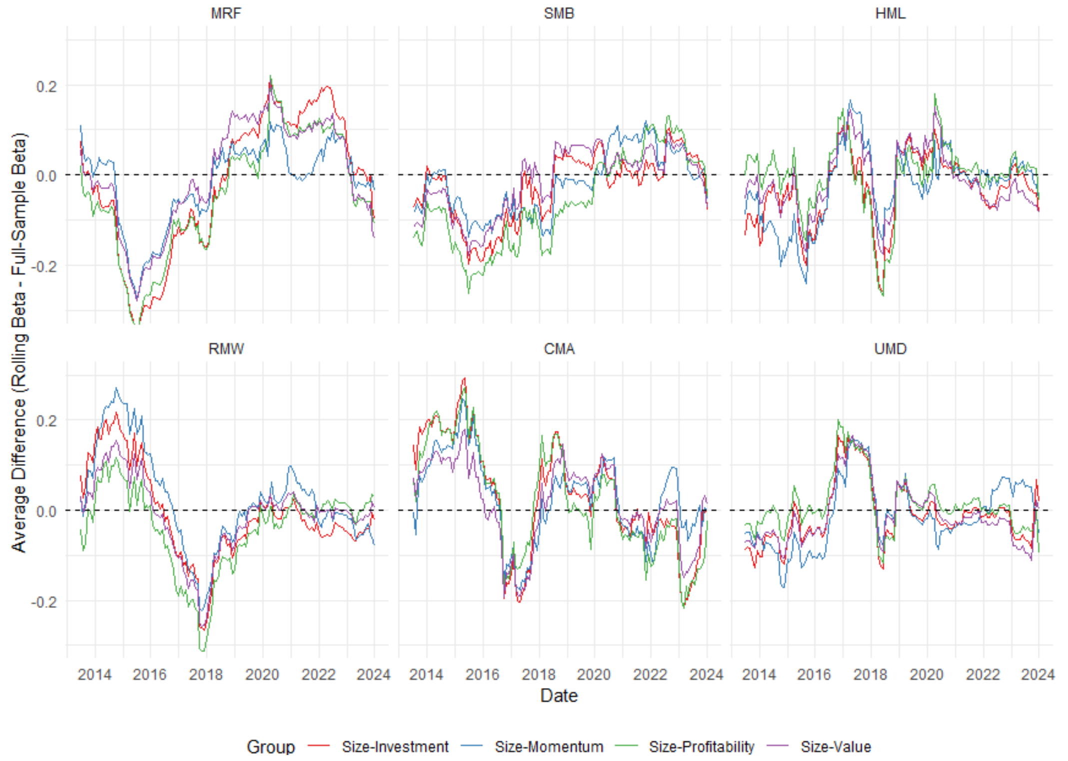

Figure 4. Average absolute differences of 36-month rolling factor loadings from full-sample coefficients.

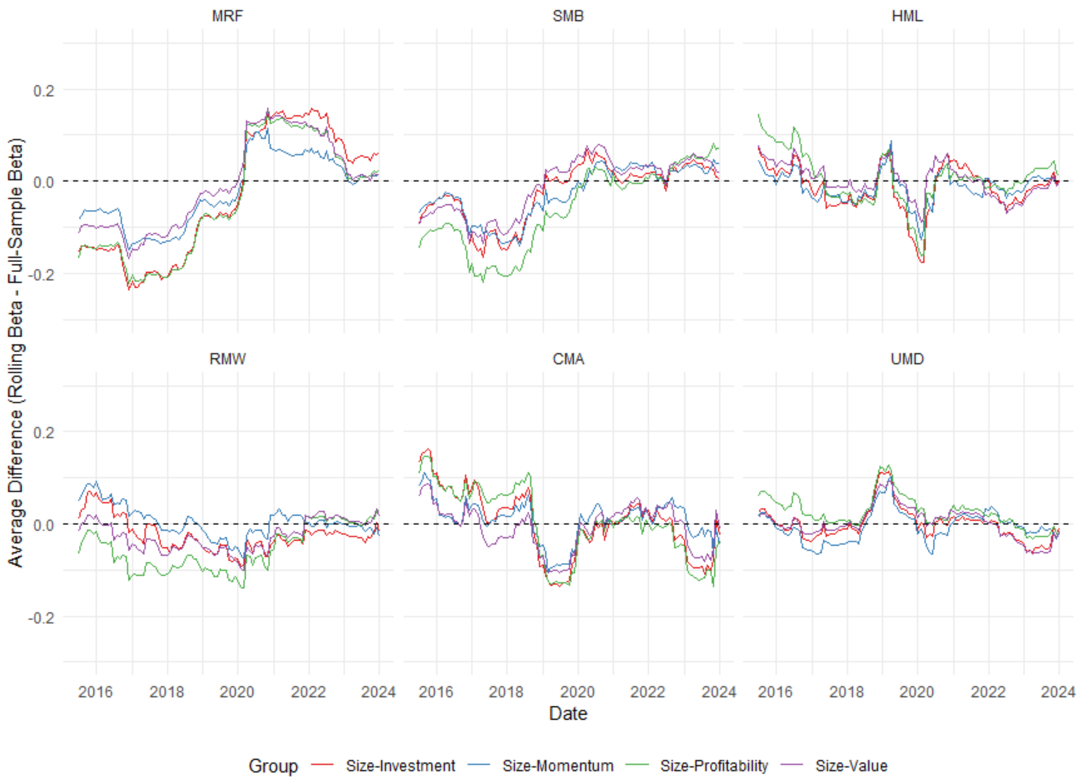

Figure 5. Average absolute differences of 60-month rolling factor loadings from full-sample coefficients.

The rolling window regressions alone do not provide sufficient evidence to make conclusions about the stability factor of factor loadings. The Wald test is utilized to assess the stability of the factor loadings. The sample is divided into two 81-month periods, and the six-factor model is estimated for both intervals. The Wald test is then utilized to evaluate whether the factor loadings remain consistent across each portfolio between the two time periods. The test is applied for each portfolio independently, with the null hypothesis stating that the factor loadings for each individual factor within each portfolio are identical across the two sub-periods. For instance, portfolio one incorporates a total of seven tests, with the initial test checking that the intercept of the portfolio in period one is equivalent to the intercept in period two. Table 13 displays the summary statistics derived from the Wald tests. If the coefficients differ significantly between the periods, the Wald test p-value is below 0.05. As the average p-values increase, the statistical stability of the factor loadings between the two periods also strengthens. The analysis reveals that the factor coefficients generally exhibit statistically significant stability across the two periods, with the exception of the market factor, which is identified as the most unstable coefficient, as 11 portfolios demonstrate statistically significant variations in market betas. However, for the remaining factors, the p-values are high, and only a small percentage of portfolios exhibit significantly different factor loadings across the periods, indicating that the loadings and the overall model are mostly stable during the study period.

The rolling window regressions indicate that the six-factor model loadings display noticeable instability within a 3-year rolling sample, while exhibiting relative stability over a 5-year window, highlighting the significant short-term variability of the factor loadings. Nevertheless, over a moderately extended time frame, the factor betas demonstrate statistically significant stability when examining the disparity between the factor loadings of the two subsample periods. The rolling window and half-sample regressions therefore support the robustness of the six-factor model in the French stock market during the study period by validating time-varying factor stability.

Table 13. Factor loading stability Wald test summary.

This table presents the results of the Wald test, which assesses whether the regression coefficients differ significantly between the two half-periods of the sample for each test portfolio. For each factor, the table reports the number of portfolios with Wald test p-values below 0.05, the median p-value across all portfolios, and the median coefficient estimates for both periods. A Wald test p-value below 0.05 for a given portfolio indicates a statistically significant difference in the coefficient estimates between the two periods. Conversely, a higher median p-value suggests greater similarity between the sub-sample coefficients on average, indicating higher factor stability.

Coefficient	Portfolios with p < 0.05	Median p-value	Median Period 1 Estimate	Median Period 2 Estimate
Intercept	6	0.36	0.30	-0.05
MRF	11	0.22	0.92	1.08
SMB	4	0.45	0.62	0.79
HML	2	0.48	0.16	0.07
RMW	5	0.50	-0.08	-0.07
CMA	4	0.43	0.04	0.01
UMD	3	0.49	0.06	0.01

6 Discussion

In the French investment universe, portfolios constructed on OP and INV show incoherent patterns in average returns. Also, no significant size effect is evident in the portfolio returns. This is also evident in the factor premiums, as the average monthly SMB, RMW, and CMA returns are all insignificant. Conversely, throughout the study period, average returns increase with higher momentum but decline with B/M ratio, signifying a persistent negative value premium. The presence of a negative value premium is relatively uncommon, as the majority of studies, including those by Grobys and Kolari (2022) and Fama and French (2012, 2018), identify positive value premiums in the EU and North America. However, an analysis of the HML returns in the EU and North America during the same timeframe as this study, displayed in Appendix 1, reveals that the value premium is also negative in both the US and the broader EU markets. Nonetheless, the negative value premium is significant only in the French market. The unprecedentedly low interest rates during this period, along with the recovery from the 2008 crisis, are likely the factors contributing to the negative value premium in France, as growth stocks benefitted from the low costs of capital more than value stocks. However, what remains puzzling is why the negative value premium is so prominent in France compared to the rest of the European market.

This study supports the findings of Grobys and Kolari (2022) and Fama and French (2018), indicating that the six-factor model is superior among the models examined. The additional momentum factor significantly enhances the predictive power of the FF3F and FF5F models. The LHS results and the spanning regressions largely reflect the same picture. The increase in explanatory power from the FF3F to FF5F is especially apparent in Size-INV test portfolios, as also demonstrated by the significant intercept of the CMA factor in the spanning regression. On the contrary, the FF6F appears to explain portfolio returns significantly better than other models across all test portfolio sets. This effect is evident in the spanning regression, as UMD exhibits significant factor loadings with MRF, SMB, and HML, indicating that during the study period, winner stocks are predominantly high-cap growth stocks. This reinforces the theory that the prolonged low interest rate environment highly benefited growth stocks, concurrently driving market momentum.

The study indicates that only the market, value, investment, and momentum factors significantly explain the variation in stock returns in the French stock market. This is mostly aligned with Grobys and Kolari (2022), who also show that the size factor is redundant in Europe, while detecting significance for RMW and insignificance for CMA, contrary to this study. Dirkx and Peter (2020) discover that in the German stock market, the six-factor model only slightly improves on the FF3F. They also find that only the market and momentum factors expand the mean-variance frontier in the context of the six-factor model, rendering the remaining factors practically redundant. Their results do not support the application of the six-factor model in Germany, which contradicts the findings of this study in France. These studies concerning European stock markets indicate that the size premium is nonexistent across the region, while the momentum premium remains persistent. This disputes the findings from North America, where Fama and French (2018) identify significance for all factors except for value.

The varying results in asset pricing model research internationally indicate that the efficacy of factor models is inconsistent between samples. The results are dependent upon the examined region and timeframe, suggesting that the optimal factors for asset pricing models must be analyzed in relation to the specific investment universe. This study demonstrates that, despite robust in-sample results, factor premiums can display considerable regional and temporal variation when contrasted with the findings of Grobys and Kolari (2022), Dirkx and Peter (2020), and Fama and French (2018). As the variation in factor premiums is apparent over time, ex-post estimation of factor premiums and loadings is not particularly useful for practitioners in portfolio selection. This problem could be tackled by examining if factor premiums can be forecasted given different macroeconomic factors, for example, by examining if lower interest rates and GDP growth can explain the negative value premium. This would make an interesting domain for future research.

6.1 The negative value premium

This study reveals a critical finding: the value factor consistently demonstrates a negative premium during the observed period, indicating the significant outperformance of growth stocks relative to value stocks on the Paris Stock Exchange. In financial literature, the value premium is typically positive, as value is often regarded as an underlying systematic risk factor, and hence value stocks should yield a risk premium. Furthermore, most studies covering a longer study period report positive value premiums in Europe (Fama and French, 2017; Grobys and Kolari, 2022). Therefore, the negative value premium is peculiar. This is analytically justified by the consistently low interest rates during the period, which have historically benefited rapidly growing companies, as accessible and inexpensive capital promotes growth. To understand this phenomenon, it is vital to diagnose which industries contribute to the negative value premium.

Two regressions are conducted to analyze this, utilizing sector portfolios to explain HML returns. The first regression utilizes a value-weighted approach, while the second one uses equal-weighted sector portfolios. The results presented in Appendix 11 demonstrate a consistent trend: The poor relative performance of the high B/M energy and utilities sectors contributes to the negative value premium, as evidenced by their positive loadings with HML. Simultaneously, the overperformance of low B/M and high growth technology and the pharmaceuticals sectors, coupled with their negative loadings with HML, compounds the disparity between low and high value stock returns. Despite the financial industry demonstrating significant explanatory power of HML, its high performance does not significantly improve HML returns due to the relatively small aggregate market capitalization of these non-bank financial firms. Also, the consumer goods industry significantly explains HML returns in the value-weighted sort, but not in the equal-weighted one, as one low B/M firm, LVMH, represents nearly half of the sector's market capitalization and had high stock returns during the period.

Additionally, the composition of the HML factor is analyzed. Appendix 12 also displays the composition of the low and high B/M sorts utilized in the formulation of the HML factor. The number of stocks from different sectors in both groups is analyzed to determine whether the performance of high or low value portfolios can be attributed to over- or underexposure to a particular industry. In accordance with the regression results, the low B/M sort is overweighted with stocks from the pharmaceutical and technology sectors, averaging 17 and 18 more stocks than the high B/M portfolio respectively.

In conclusion, the negative value premium in the French stock market during the study period is primarily attributable to the exceptional performance of the high-growth technology, pharmaceutical, and healthcare sectors, as well as LVMH, contrasted with the comparatively weak returns of high B/M industrials and the energy & utilities sectors. This indicates that low interest rates benefit companies with significant growth potential by enabling them to access inexpensive capital to drive their growth. Alternatively, the low interest rates might boost demand for high-tech and healthcare. While value stocks generate positive returns, these remain significantly lower than those of growth stocks, suggesting that negative value premiums persist during low interest rate environments.

6.2 Limitations

This study examines the post-2008 low interest rate environment in the French stock market, though it is constrained by the duration of the analysis period. The results of this study are therefore influenced by the prevailing economic conditions of the period. The insights might be specific to low interest rate environments, and the short sample period can limit the generalizability of the findings to other macroeconomic conditions. As the study only covers a period of low interest rates, it does not investigate potential structural breaks in factor performance across different macroeconomic environments. Historically, factor premiums have fluctuated cyclically. As a result, factors can behave differently during periods of high interest rates and inflation. Under such conditions, value and investment premiums may become highly positive as capital becomes less abundant, while growth and heavily investing companies suffer. The size factor may suffer as funding conditions diminish. Momentum may slip if volatility increases significantly. Overall, changing macroeconomic regimes can have a significant impact on factor performance.

7 Conclusion

This study examines the performance of the Fama-French six-factor model in the French stock universe during the period between July 2010 and December 2023. Comparing the results to the three- and five-factor models, the six-factor model delivers significantly improved power in explaining the cross-section of stock returns, underscoring the importance of the momentum factor in the French equity market.

The study period is established to analyze stock returns during the historically low interest rate environment preceding the 2008 financial crisis. During this period, consistent patterns in average stock returns are observed in portfolios sorted by B/M as well as in portfolios sorted by momentum. Stock returns increase with momentum, but with B/M sorted portfolios, average returns decrease as B/M increases, displaying a negative value premium. The results also indicate that while the SMB loadings shows consistency in left-hand side regressions, the size premium is virtually nonexistent in France. The LHS regressions show that the six-factor model beats the competing models across all four sets of test portfolios, evidenced by significantly reduced average absolute alphas. The improvement of the model is most significant for the Size-OP and Size-MOM portfolios. However, the model is still rejected by the GRS test for all test portfolios at 5% significance level. Spanning regressions reveal that in the French stock universe, the market, value, investment, and momentum factors expand the mean-variance frontier. Meanwhile, size and profitability factors are redundant in the French equity market, as their risk premiums are captured by other factors.

The results of this study demonstrate that the performance of factor models and their associated risk premiums are subject to variability across different investment universes and time periods. Under extended low interest rate environments, factor risk premiums, particularly the value premium, can become negative, contrary to the expectation of a positive risk premium. The significance of factors, as well as their risk premiums, varies between the sub-markets of the broader European equity universe. As a result, choosing risk factors for the optimal factor model should be region-specific.

7.1 Practical implications

This study underscores the superior efficacy of the Fama-French six-factor model when compared against the competing asset pricing models in France. However, not all of the factors have relevant predictive power for stock returns. This means that some of the factors are irrelevant in the context of the French stock market. The primary applications for asset pricing models are asset pricing, such as estimating expected returns, and portfolio management, such as portfolio selection and benchmarking. For asset pricing, this study suggests that when estimating expected returns for French equities, implementing value, investment, and momentum factor exposures in conjunction with the CAPM is necessary to more precisely account for the systematic risks captured by these factors, as they independently capture French stock returns. The rest of the factors of the six-factor model are virtually obsolete in this purpose, as most of their explanatory power is captured by other factors.

Moreover, for portfolio management applications, the results of this study imply that smart beta styled rule-based investment strategies are feasible in the French stock market. Especially strategies utilizing factor exposures with the momentum and value factors are efficient in France. Holding all else fixed, a one unit increase in momentum loading increases average monthly portfolios returns by 0.78% and a one unit increase in value loadings decreases monthly returns by 0.71%. In addition, conversely to LHS and spanning regression results, a one unit increase in the profitability factor loading offers a 0.37% increase in average monthly returns. As a result, these factors can be implemented in smart beta strategies to expand the mean variance frontier. In essence, this can be achieved by constructing a portfolio whose exposure to momentum, profitability, and value factor loadings is dynamically adjusted based on expected future factor premiums. On the other hand, this thesis also suggests that traditional pure value strategies are inefficient in France, at least during the low interest rate period examined.

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Appendices

Appendix 1. Factor returns from NA and Europe: 6/2010–12/2023.

This table displays the mean, standard deviation, and sample mean T-statistic for each of the FF6F factors from the North American and European broader aggregate stock markets. The factor returns are derived from the Kenneth French Data library. The values are expressed as percentages, and the means denotes the average monthly risk premium associated with the factor. The critical values and notations for the t-statistic are as follows: At 1% significance level, the value is ±2.607 (***); at 5% level, it is ±1.98 (**); and at 10% level, it is ±1.65 (*).

Panel A: Descriptive statistics of factors from **North America** sourced from the Eugene Fama data library.

	Rm-Rf	*SMB*	*HML*	*RMW*	*CMA*	*UMD*
Mean	1.07	-0.15	-0.23	0.30	0.10	0.36
STD	4.42	2.38	3.23	1.72	2.20	3.10
T-Stat	3.09***	-0.81	-0.89	2.20**	0.59	1.47

Panel B: Descriptive statistics of factors from **Europe** sourced from the Eugene Fama data library.

	Rm-Rf	*SMB*	*HML*	*RMW*	*CMA*	*UMD*
Mean	0.68	0.06	-0.07	0.32	-0.06	0.76
STD	5.11	1.67	2.90	1.66	1.49	3.24
T-Stat	1.68*	0.48	-0.30	2.45**	-0.49	3.00***

Appendix 2. LHS regression with FF3F and Size-Value sorted portfolios.

This table presents the regression results from the 16 Size-B/M portfolios regressed against the FF3F. The critical values for the t-statistic are as follows: At 1% significance level, the critical value is ±2.61; at 5% level, it is **±1.98 (bolded in the table)**; and at 10% level, it is ±1.65.

B/M→	Low	2	3	High	Low	2	3	High
FF3F regression for 4 x 4 test portfolios formed on size and B/M								
	α_i				t (α_i)			
Small	-0.06	0.66	0.36	-0.32	-0.14	**2.43**	1.59	-1.49
2	0.07	0.37	0.05	-0.05	0.29	1.74	0.26	-0.26
3	-0.03	0.56	-0.12	-0.49	-0.14	**2.78**	-0.59	-1.62
Big	-0.06	0.04	0.08	-0.44	-0.52	0.33	0.51	-1.95
	β_i				t (β_i)			
Small	1.04	0.88	0.89	0.84	**9.64**	**11.84**	**14.30**	**14.47**
2	1.02	1.06	0.87	0.93	**14.37**	**18.27**	**18.20**	**17.05**
3	1.03	0.92	1.21	1.25	**14.74**	**16.61**	**22.02**	**15.02**
Big	1.03	0.89	0.73	1.21	**32.35**	**27.10**	**17.77**	**19.71**
	s_i				t (s_i)			
Small	1.28	0.62	0.98	1.03	**10.00**	**6.96**	**13.29**	**14.83**
2	1.10	1.11	0.86	0.92	**12.92**	**16.09**	**15.01**	**14.00**
3	0.46	0.44	0.60	0.65	**5.50**	**6.73**	**9.12**	**6.50**
Big	-0.08	-0.05	-0.10	0.17	**-2.12**	-1.31	**-2.00**	**2.28**
	h_i				t (h_i)			
Small	-0.19	-0.04	0.23	0.29	-1.48	-0.40	**3.14**	**4.25**
2	-0.70	-0.05	0.15	0.56	**-8.35**	-0.70	**2.73**	**8.67**
3	-0.38	0.07	0.21	0.47	**-4.60**	1.05	**3.24**	**4.80**
Big	-0.32	0.04	0.33	0.43	**-8.44**	1.15	**6.73**	**5.95**
	Adj. R^2							
Small	0.53	0.53	0.71	0.74				
2	0.70	0.78	0.78	0.79				
3	0.59	0.67	0.79	0.66				
Big	0.87	0.82	0.72	0.75				

Appendix 3. LHS regression with FF5F and Size-Value sorted portfolios.

This table presents the regression results from the 16 Size-B/M portfolios regressed with the FF5F. The critical values for the t-statistic are as follows: At 1% significance level, the critical value is ±2.61; at 5% level, it is **±1.98 (bolded in the table)**; and at 10% level, it is ±1.65.

B/M→	Low	2	3	High	Low	2	3	High
FF5F regression for 4 x 4 test portfolios formed on size and B/M								
	α_i				t (α_i)			
Small	-0.17	0.67	0.31	-0.25	-0.43	**2.41**	1.35	-1.17
2	-0.01	0.40	0.02	0.04	-0.03	1.83	0.09	0.20
3	-0.02	0.64	-0.08	-0.51	-0.06	**3.12**	-0.39	-1.68
Big	-0.01	0.00	0.04	-0.42	-0.09	-0.04	0.24	-1.80
	β_i				t (β_i)			
Small	1.04	0.87	0.90	0.82	**10.02**	**11.63**	**14.31**	**14.13**
2	1.03	1.05	0.88	0.92	**14.61**	**17.93**	**18.07**	**16.67**
3	1.02	0.90	1.21	1.24	**14.49**	**16.25**	**21.65**	**15.24**
Big	1.03	0.89	0.74	1.21	**32.68**	**27.50**	**17.94**	**19.41**
	s_i				t (s_i)			
Small	1.20	0.60	1.00	1.01	**9.58**	**6.63**	**13.22**	**14.30**
2	1.06	1.10	0.86	0.91	**12.53**	**15.60**	**14.64**	**13.65**
3	0.44	0.44	0.60	0.58	**5.13**	**6.55**	**8.94**	**5.90**
Big	-0.07	-0.06	-0.12	0.15	-1.77	-1.66	**-2.36**	1.94
	h_i				t (h_i)			
Small	-0.29	-0.05	0.22	0.31	**-2.86**	-0.51	**2.64**	**4.02**
2	-0.39	-0.04	0.13	0.61	**-8.74**	-0.55	**2.04**	**8.49**
3	-0.40	0.12	0.24	0.37	**-4.31**	1.65	**3.28**	**3.46**
Big	-0.26	-0.01	0.27	0.42	**-6.40**	-0.13	**5.10**	**5.16**
	r_i				t (r_i)			
Small	-0.40	-0.08	0.07	-0.10	**-3.16**	-0.88	0.92	-1.43
2	-0.15	-0.05	-0.01	-0.06	-1.71	-0.71	-0.19	-0.92
3	-0.11	-0.03	0.00	-0.31	-1.23	-0.45	0.04	**-3.13**
Big	0.06	-0.06	-0.08	-0.10	1.59	-1.49	-1.59	-1.37
	c_i				t (c_i)			
Small	0.31	-0.01	0.09	-0.12	1.60	-0.10	0.77	-1.11
2	0.21	-0.05	0.07	-0.19	1.56	-0.50	0.73	-1.88
3	-0.02	-0.17	-0.08	0.09	-0.17	-1.69	-0.80	0.57
Big	-0.12	0.11	0.10	-0.03	**-2.08**	1.76	1.36	-0.29
	Adj. R^2							
Small	0.57	0.52	0.71	0.74				
2	0.71	0.78	0.78	0.80				
3	0.59	0.67	0.79	0.68				
Big	0.87	0.83	0.73	0.75				

Appendix 4. LHS regression with FF3F and Size-OP sorted portfolios.

This table presents the regression results from the 16 Size-OP portfolios regressed with the FF3F. The critical values for the t-statistic are as follows: At 1% significance level, the critical value is ±2.61; at 5% level, it is **±1.98 (bolded in the table)**; and at 10% level, it is ±1.65.

OP →	Low	2	3	High	Low	2	3	High
FF3F regression for 4 x 4 test portfolios formed on size and OP								
	α_i				t (α_i)			
Small	-0.45	0.26	0.10	0.38	-1.10	1.20	0.43	1.28
2	-0.75	0.21	0.24	-0.05	-1.89	1.30	1.38	-0.22
3	-0.80	-0.13	0.18	-0.09	-1.33	-0.58	1.05	-0.49
Big	0.62	0.17	0.02	-0.22	1.76	1.04	0.16	-1.88
	β_i				t (β_i)			
Small	1.01	0.73	0.92	0.97	**8.99**	**12.47**	**14.97**	**11.89**
2	1.36	0.84	0.95	1.06	**12.53**	**18.89**	**19.89**	**15.43**
3	1.76	0.96	1.04	1.19	**10.77**	**15.41**	**21.95**	**22.87**
Big	0.94	0.91	0.86	1.06	**9.82**	**19.83**	**29.20**	**34.05**
	s_i				t (s_i)			
Small	1.29	0.76	0.86	1.18	**9.58**	**10.94**	**11.72**	**12.11**
2	1.39	0.81	0.99	0.98	**10.73**	**15.30**	**17.48**	**11.97**
3	1.17	0.53	0.54	0.54	**5.99**	**7.10**	**9.41**	**8.64**
Big	-0.02	0.02	-0.07	-0.04	-0.18	0.28	**-2.11**	-1.18
	h_i				t (h_i)			
Small	-0.05	0.13	0.20	0.15	-0.38	1.92	**2.73**	1.53
2	-0.06	0.23	0.13	-0.27	-0.44	**4.34**	**2.34**	**-3.38**
3	0.02	0.09	0.12	0.11	0.08	1.29	**2.15**	1.80
Big	0.34	0.25	0.01	-0.17	**2.98**	**4.58**	0.37	**-4.62**
	Adj. R^2							
Small	0.50	0.64	0.70	0.64				
2	0.62	0.80	0.81	0.69				
3	0.48	0.64	0.78	0.79				
Big	0.42	0.74	0.84	0.88				

Appendix 5. LHS regression with FF5F and Size-OP sorted portfolios.

This table presents the regression results from the 16 Size-OP portfolios regressed with the FF5F. The critical values for the t-statistic are as follows: At 1% significance level, the critical value is ±2.61; at 5% level, it is **±1.98 (bolded in the table)**; and at 10% level, it is ±1.65.

OP →	Low	2	3	High	Low	2	3	High
FF5F regression for 4 x 4 test portfolios formed on size and OP								
	α_i				t (α_i)			
Small	-0.55	0.23	0.11	0.35	-1.36	1.05	0.49	1.13
2	-0.57	0.15	0.23	-0.10	**-2.12**	0.91	1.29	-0.48
3	-0.80	-0.07	0.20	-0.04	-1.35	-0.31	1.12	-0.21
Big	0.70	0.18	-0.04	-0.15	**2.23**	1.07	-0.37	-1.33
	β_i				t (β_i)			
Small	1.01	0.74	0.92	0.98	**9.33**	**12.44**	**14.85**	**11.80**
2	1.28	0.85	0.95	1.09	**17.72**	**19.33**	**19.74**	**18.57**
3	1.73	0.94	1.04	1.18	**10.86**	**15.06**	**21.57**	**22.49**
Big	0.90	0.90	0.87	1.06	**10.67**	**19.77**	**30.31**	**35.97**
	s_i				t (s_i)			
Small	1.19	0.78	0.88	1.17	**9.12**	**10.88**	**11.76**	**11.73**
2	1.13	0.79	1.01	1.10	**12.89**	**15.00**	**17.32**	**15.57**
3	1.03	0.51	0.53	0.52	**5.37**	**6.78**	**9.14**	**8.19**
Big	-0.18	-0.01	-0.09	-0.02	-1.73	-0.23	**-2.59**	-0.59
	h_i				t (h_i)			
Small	-0.25	0.13	0.23	0.10	-1.79	1.67	**2.89**	0.94
2	-0.30	0.15	0.14	-0.15	**-3.15**	**2.68**	**2.24**	-1.90
3	-0.17	0.12	0.13	0.12	-0.82	1.45	**2.05**	1.75
Big	0.18	0.21	-0.05	-0.09	1.65	**3.59**	-1.31	**-2.29**
	r_i				t (r_i)			
Small	-0.45	0.06	0.08	-0.06	**-3.39**	0.81	1.09	-0.61
2	-1.25	-0.09	0.06	0.55	**-14.23**	-1.69	0.94	**7.76**
3	-0.62	-0.07	-0.02	-0.10	**-3.21**	-0.91	-0.30	-1.55
Big	-0.73	-0.13	-0.07	0.10	**-7.06**	**-2.31**	-1.87	**2.85**
	c_i				t (c_i)			
Small	0.28	0.05	-0.05	0.09	1.40	0.47	-0.42	0.61
2	-0.18	0.16	0.01	0.02	-1.36	1.93	0.13	0.15
3	0.11	-0.12	-0.04	-0.10	0.36	-1.04	-0.43	-1.03
Big	-0.06	0.01	0.14	-0.17	-0.36	0.09	**2.58**	**-3.15**
	Adj. R^2							
Small	0.55	0.63	0.70	0.64				
2	0.83	0.81	0.81	0.78				
3	0.51	0.64	0.78	0.79				
Big	0.56	0.74	0.86	0.89				

Appendix 6. LHS regression with FF3F and Size-INV sorted portfolios.

This table presents the regression results from the 16 Size-INV portfolios regressed with the FF3F. The critical values for the t-statistic are as follows: At 1% significance level, the critical value is ±2.61; at 5% level, it is **±1.98 (bolded in the table)**; and at 10% level, it is ±1.65.

INV →	Low	2	3	High	Low	2	3	High
FF3F regression for 4 x 4 test portfolios formed on size and INV								
	α_i				t (α_i)			
Small	-0.08	0.74	0.74	-0.39	-0.24	**2.81**	**2.61**	-1.47
2	0.18	0.37	0.24	-0.19	0.62	1.70	1.31	-1.03
3	-0.14	0.24	0.15	-0.20	-0.38	1.24	0.84	-0.67
Big	0.62	-0.14	-0.23	-0.33	2.09	-1.04	-1.75	-1.59
	β_i				t (β_i)			
Small	0.92	0.84	0.87	0.99	**10.34**	**11.71**	**11.30**	**13.78**
2	1.15	0.86	0.96	1.07	**14.73**	**14.42**	**18.95**	**20.94**
3	1.32	1.09	1.01	1.21	**13.10**	**20.59**	**20.58**	**15.32**
Big	1.04	0.96	1.01	1.11	**12.92**	**25.82**	**27.94**	**19.74**
	s_i				t (s_i)			
Small	1.23	0.69	0.74	1.02	**11.56**	**8.06**	**7.99**	**11.91**
2	1.22	0.87	0.84	0.97	**13.12**	**12.25**	**13.94**	**15.93**
3	0.78	0.41	0.55	0.68	**6.44**	**6.49**	**9.37**	**7.15**
Big	-0.06	-0.05	-0.03	0.09	-0.67	-1.19	-0.63	1.30
	h_i				t (h_i)			
Small	0.19	0.05	0.20	0.05	1.79	0.57	**2.25**	0.59
2	0.17	0.15	0.17	-0.04	1.85	**2.07**	**2.89**	-0.67
3	0.46	0.23	0.11	-0.25	**3.85**	**3.69**	1.94	**-2.70**
Big	0.42	0.08	-0.23	-0.29	**4.36**	1.89	**-5.40**	**-4.39**
	Adj. R^2							
Small	0.60	0.55	0.56	0.67				
2	0.71	0.69	0.78	0.81				
3	0.60	0.76	0.77	0.62				
Big	0.56	0.81	0.83	0.70				

Appendix 7. LHS regression with FF5F and Size-INV sorted portfolios.

This table presents the regression results from the 16 Size-INV portfolios regressed with the FF5F. The critical values for the t-statistic are as follows: At 1% significance level, the critical value is ±2.61; at 5% level, it is **±1.98 (bolded in the table)**; and at 10% level, it is ±1.65.

INV →	Low	2	3	High	Low	2	3	High
FF5F regression for 4 x 4 test portfolios formed on size and INV								
	α_i				$t(\alpha_i)$			
Small	-0.27	0.63	0.72	-0.18	-0.85	**2.36**	**2.50**	-0.70
2	0.20	0.34	0.26	-0.18	0.81	1.51	1.40	-0.95
3	-0.16	0.23	0.16	0.00	-0.43	1.15	0.84	-0.01
Big	0.29	-0.21	-0.17	-0.08	1.04	-1.55	-1.26	-0.42
	β_i				$t(\beta_i)$			
Small	0.95	0.86	0.87	0.95	**10.93**	**11.97**	**11.16**	**13.58**
2	1.12	0.86	0.96	1.06	**16.88**	**14.37**	**18.76**	**20.60**
3	1.31	1.09	1.01	1.18	**13.05**	**20.41**	**20.28**	**15.09**
Big	1.10	0.97	1.00	1.07	**14.77**	**26.33**	**27.64**	**20.98**
	s_i				$t(s_i)$			
Small	1.19	0.72	0.72	0.97	**11.43**	**8.21**	**7.66**	**11.50**
2	1.09	0.85	0.85	0.96	**13.60**	**11.82**	**13.90**	**15.48**
3	0.72	0.39	0.54	0.67	**5.95**	**6.11**	**8.98**	**7.14**
Big	-0.08	-0.06	-0.03	0.10	-0.94	-1.37	-0.61	1.60
	h_i				$t(h_i)$			
Small	0.00	0.00	0.17	0.12	-0.02	0.01	1.70	1.30
2	0.00	0.10	0.21	-0.04	0.02	1.24	**3.10**	-0.61
3	0.37	0.20	0.10	-0.12	**2.82**	**2.84**	1.54	-1.17
Big	0.15	0.02	-0.19	-0.10	1.55	0.45	**-3.92**	-1.48
	r_i				$t(r_i)$			
Small	-0.16	0.11	-0.07	-0.28	-1.53	1.25	-0.79	**-3.26**
2	-0.62	-0.08	0.06	-0.03	**-7.63**	-1.08	0.93	-0.46
3	-0.24	-0.08	-0.06	-0.03	**-1.99**	-1.23	-0.91	-0.31
Big	-0.06	-0.03	0.00	0.03	-0.72	-0.72	-0.02	0.49
	c_i				$t(c_i)$			
Small	0.45	0.22	0.04	-0.41	**2.81**	1.64	0.28	**-3.13**
2	0.06	0.09	-0.06	-0.02	0.47	0.81	-0.63	-0.17
3	0.08	0.04	0.00	-0.42	0.45	0.40	-0.02	**-2.87**
Big	0.75	0.16	-0.14	-0.55	**5.41**	**2.33**	**-2.01**	**-5.83**
	Adj. R^2							
Small	0.63	0.56	0.55	0.69				
2	0.79	0.70	0.78	0.80				
3	0.61	0.76	0.76	0.64				
Big	0.64	0.82	0.83	0.76				

Appendix 8. LHS regression with FF3F and Size-MOM sorted portfolios.

This table presents the regression results from the 16 Size-MOM portfolios regressed with the FF3F. The critical values for the t-statistic are as follows: At 1% significance level, the critical value is ±2.61; at 5% level, it is **±1.98 (bolded in the table)**; and at 10% level, it is ±1.65.

MOM→	Low	2	3	High	Low	2	3	High
FF3F regression for 4 x 4 test portfolios formed on size and MOM								
	α_i				t (α_i)			
Small	-0.16	-0.40	0.38	0.86	-0.37	-1.66	1.80	**2.23**
2	-0.42	-0.10	0.34	0.81	-1.12	-0.39	1.81	**3.49**
3	-0.41	0.18	0.09	0.87	-0.76	0.85	0.47	**3.32**
Big	-0.29	0.19	0.05	0.08	-0.65	0.88	0.39	0.35
	β_i				t (β_i)			
Small	1.19	0.86	0.78	0.88	**9.92**	**13.16**	**13.49**	**8.30**
2	1.50	1.03	0.84	1.09	**14.73**	**15.50**	**16.38**	**17.21**
3	1.66	1.13	0.87	1.16	**11.37**	**19.62**	**16.15**	**16.18**
Big	1.74	1.19	1.21	1.18	**14.12**	**19.79**	**31.81**	**20.00**
	s_i				t (s_i)			
Small	1.73	0.85	0.81	0.96	**12.07**	**10.82**	**11.71**	**7.65**
2	1.52	1.00	0.71	1.07	**12.44**	**12.63**	**11.62**	**14.13**
3	0.92	0.72	0.47	0.67	**5.28**	**12.44**	**12.63**	**7.87**
Big	0.11	-0.05	-0.11	-0.01	0.73	-0.71	**-2.40**	-0.11
	h_i				t (h_i)			
Small	0.08	0.21	0.08	0.16	0.57	**2.67**	1.23	1.31
2	0.43	0.05	-0.03	-0.06	**3.61**	0.63	-0.49	-0.76
3	0.48	0.21	0.08	-0.16	**2.77**	**3.07**	1.31	-1.95
Big	0.63	0.20	-0.06	-0.31	**4.33**	**2.81**	-1.41	**-4.39**
	Adj. R^2							
Small	0.60	0.65	0.66	0.44				
2	0.71	0.71	0.71	0.75				
3	0.52	0.76	0.66	0.66				
Big	0.60	0.73	0.86	0.71				

Appendix 9. LHS regression with FF5F and Size-MOM sorted portfolios.

This table presents the regression results from the 16 Size-INV portfolios regressed with the FF5F. The critical values for the t-statistic are as follows: At 1% significance level, the critical value is ±2.61; at 5% level, it is **±1.98 (bolded in the table)**; and at 10% level, it is ±1.65.

MOM→	Low	2	3	High	Low	2	3	High
FF5F regression for 4 x 4 test portfolios formed on size and MOM								
	α_i				t (α_i)			
Small	-0.22	-0.48	0.36	0.93	-0.49	-1.95	1.67	**2.37**
2	-0.50	-0.06	0.32	0.81	-1.35	-0.24	1.70	**3.44**
3	-0.17	0.17	0.13	0.91	-0.32	0.77	0.67	**3.40**
Big	-0.30	0.19	0.01	0.19	-0.65	0.82	0.05	0.87
	β_i				t (β_i)			
Small	1.20	0.87	0.78	0.85	**9.86**	**13.27**	**13.35**	**8.09**
2	1.50	1.01	0.84	1.09	**15.08**	**15.28**	**16.35**	**17.21**
3	1.60	1.13	0.86	1.14	**11.12**	**19.40**	**15.82**	**15.90**
Big	1.74	1.19	1.22	1.15	**13.95**	**19.53**	**31.91**	**19.70**
	s_i				t (s_i)			
Small	1.73	0.83	0.81	0.91	**11.82**	**10.53**	**11.42**	**7.13**
2	1.44	0.96	0.73	1.04	**12.01**	**12.06**	**11.72**	**13.59**
3	0.81	0.70	0.47	0.64	**4.68**	**10.08**	**7.14**	**7.42**
Big	0.07	-0.06	-0.10	-0.04	0.44	-0.77	**-2.06**	-0.55
	h_i				t (h_i)			
Small	0.04	0.13	0.07	0.13	0.27	1.55	0.89	0.97
2	0.27	0.03	-0.01	-0.11	**2.11**	0.33	-0.20	-1.39
3	0.50	0.18	0.11	-0.18	**2.63**	**2.39**	1.53	-1.86
Big	0.57	0.19	-0.08	-0.27	**3.50**	**2.33**	-1.56	**-3.49**
	r_i				t (r_i)			
Small	0.01	-0.05	-0.01	-0.27	0.10	-0.65	-0.09	**-2.09**
2	-0.33	-0.16	0.09	-0.18	**-2.74**	**-2.00**	1.37	**-2.31**
3	-0.52	-0.06	-0.02	-0.14	**-2.98**	-0.82	-0.27	-1.55
Big	-0.19	-0.03	0.07	-0.15	-1.23	-0.34	1.46	**-2.16**
	c_i				t (c_i)			
Small	0.12	0.18	0.04	-0.11	0.54	1.48	0.40	-0.55
2	0.24	-0.06	0.01	0.04	1.28	-0.44	0.15	0.38
3	-0.43	0.04	-0.09	-0.06	-1.61	0.35	-0.88	-0.48
Big	0.05	0.02	0.09	-0.23	0.22	0.21	1.31	**-2.06**
	Adj. R^2							
Small	0.59	0.66	0.66	0.45				
2	0.73	0.71	0.71	0.75				
3	0.54	0.76	0.66	0.66				
Big	0.60	0.72	0.86	0.71				

Appendix 10. Rolling window regression summary statistics.

This table displays the summary statistics for the rolling window regressions utilizing both 36-month and 60-month windows. The rolling window regression uses a rolling sample to estimate the six-factor model on a monthly basis, adjusting the sample to represent the preceding 36 or 60 months. The table presents the mean absolute deviation from the full-sample coefficient, the average deviation from the full-sample coefficient, and the STDEV of the average deviation for the four groups of test portfolios. The mean absolute difference measures the average deviation of the rolling factor beta from its full-sample coefficient in terms of absolute values. Lower coefficients correspond to higher stability in loadings. Likewise, Avg. Diff measures the average deviation from full sample values while allowing negative values. The mean absolute difference provides a better representation of coefficient stability, as it shows how far from zero (full sample estimate) the coefficients deviate on average.

	36-month window			60-month window		
Intercept	Mean abs. Diff.	Avg. Diff.	STDEV	Mean abs. Diff.	Avg. Diff.	STDEV
Size-Investment	0.42	0.05	0.54	0.31	0.09	0.38
Size-Momentum	0.50	0.09	0.64	0.36	0.12	0.45
Size-Profitability	0.41	0.08	0.54	0.27	0.09	0.36
Size-Value	0.39	0.05	0.50	0.28	0.08	0.36
MRF	Mean abs. Diff.	Avg. Diff.	STDEV	Mean abs. Diff.	Avg. Diff.	STDEV
Size-Investment	0.18	-0.02	0.23	0.15	-0.03	0.18
Size-Momentum	0.16	-0.02	0.21	0.11	-0.02	0.13
Size-Profitability	0.18	-0.04	0.24	0.14	-0.04	0.19
Size-Value	0.17	0.00	0.22	0.14	-0.01	0.18
SMB	Mean abs. Diff.	Avg. Diff.	STDEV	Mean abs. Diff.	Avg. Diff.	STDEV
Size-Investment	0.15	-0.03	0.20	0.11	-0.02	0.15
Size-Momentum	0.17	-0.02	0.22	0.12	-0.03	0.16
Size-Profitability	0.18	-0.06	0.27	0.13	-0.06	0.20
Size-Value	0.15	-0.01	0.19	0.10	-0.01	0.13
HML	Mean abs. Diff.	Avg. Diff.	STDEV	Mean abs. Diff.	Avg. Diff.	STDEV
Size-Investment	0.18	-0.03	0.26	0.11	-0.01	0.16
Size-Momentum	0.21	-0.03	0.27	0.14	-0.01	0.18
Size-Profitability	0.17	0.00	0.24	0.11	0.01	0.15
Size-Value	0.18	-0.01	0.25	0.12	0.00	0.17
RMW	Mean abs. Diff.	Avg. Diff.	STDEV	Mean abs. Diff.	Avg. Diff.	STDEV
Size-Investment	0.13	-0.01	0.17	0.08	-0.02	0.11
Size-Momentum	0.16	0.02	0.22	0.10	0.00	0.14
Size-Profitability	0.17	-0.04	0.24	0.12	-0.05	0.16
Size-Value	0.15	-0.01	0.19	0.10	-0.03	0.13
CMA	Mean abs. Diff.	Avg. Diff.	STDEV	Mean abs. Diff.	Avg. Diff.	STDEV
Size-Investment	0.22	0.03	0.28	0.13	0.00	0.16
Size-Momentum	0.22	0.03	0.30	0.14	0.01	0.18
Size-Profitability	0.22	0.02	0.30	0.15	0.00	0.19
Size-Value	0.19	0.02	0.26	0.13	-0.01	0.18
UMD	Mean abs. Diff.	Avg. Diff.	STDEV	Mean abs. Diff.	Avg. Diff.	STDEV
Size-Investment	0.12	-0.01	0.16	0.08	0.00	0.11
Size-Momentum	0.15	-0.01	0.20	0.11	-0.01	0.14
Size-Profitability	0.12	0.01	0.16	0.08	0.03	0.11
Size-Value	0.12	-0.01	0.15	0.08	0.00	0.10
Average ignoring the intercept	0.17	0.00	0.23	0.12	0.00	0.15

Appendix 11. Regressing HML with industry portfolios and HML factor industry composition.

This table displays the investigative methods on the factors that contribute to the negative value premium. Regressions, where the returns of the HML factor are explained with portfolios categorized by industry, are conducted with value-weighted portfolios in panel A and equal-weighted portfolios in panel B. The industry composition of the HML factor is examined in Panel C by analyzing the disparity in total stock quantity by industry between the low and high B/M sub-portfolios of the HML factor. The critical values and notations for the t-statistic are as follows: At 1% significance level, the critical value is ±2.61 (***); at 5% level, it is ±1.98 (**); and at 10% level, it is ±1.65 (*).

Panel A: Explaining HML returns with value weighted sector portfolios

Coefficient	Coefficients	t-Stat	p-value	Significance	AVG. return
Intercept	-0.86	-4.22	0.00	***	
Consumer Goods	-0.25	-2.49	0.01	**	0.98
Energy and Utilities	0.12	1.74	0.08	*	0.58
Health and Pharmaceuticals	-0.06	-0.63	0.53		1.03
Industrials and Manufacturing	0.12	1.39	0.17		0.90
Technology and Communication	-0.27	-3.28	0.00	***	1.27
Travel and Leisure	0.05	0.78	0.44		0.92
Financials (non-banks)	0.30	4.36	0.00	***	1.34
R-squared	0.57				

Panel B: Explaining HML returns with equal weighted sector portfolios

Coefficient	Coefficients	t-Stat	p-value	Significance
Intercept	-1.17	-5.21	0.00	***
Consumer Goods	0.00	0.03	0.98	
Energy and Utilities	0.09	1.91	0.06	*
Health and Pharmaceuticals	-0.14	-2.08	0.04	**
Industrials and Manufacturing	0.10	1.86	0.07	*
Technology and Communication	-0.06	-0.94	0.35	
Travel and Leisure	0.06	1.34	0.18	
Financials (non-banks)	0.12	1.91	0.06	*
R-squared	0.20			

Panel C: Average quantity of stocks in the Value factor sub-portfolios

Industry	High B/M	Low B/M	Difference
Consumer Goods	19.2	15.3	3.9
Energy and Utilities	7.8	4.7	3.1
Health and Pharmaceuticals	4.6	21.8	17.2
Industrials and Manufacturing	14.5	18.2	3.7
Technology and Communication	9.4	27.6	18.2
Travel and Leisure	5.0	4.4	0.6
Financials	4.9	1.7	3.2