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Ville Vainioranta

AN ADAPTIVE DIFFERENTIAL EVOLUTION ALGORITHM

Master's thesis in Technology for the degree of Master of Science in Technology
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Instructor Professor Jouni Lampinen

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VAASAN YLIOPISTO
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TIIVISTELMÄ

Differentiaalievoluutio on evoluutioalgoritmi, joka on suunniteltu globaaliin optimointiin. Sen tärkeimmät edut ovat yksinkertaisuus, helppokäyttöisyys ja tehokkuus, mikä on osoitettu eri tutkimuksissa ja evoluutiolaskennassa järjestetyissä kilpailuissa. Differentiaalievoluutiossa on kolme säätöparametria, ja jokaisella niistä on merkittävä vaikutus algoritmin suorituskykyyn.

Tämä tutkielma esittelee kolme muunneltua algoritmia, jotka mukauttavat säätöparametreja optimointiprosessin aikana. Ensimmäinen modifikaatio VDE-1 mukauttaa mutaatiovakiota, toinen modifikaatio VDE-2 mukauttaa risteytysvakiota ja kolmas modifikaatio molempia vakioita. Ajatuksena näissä algoritmeissa on käyttää aiemmin onnistuneita parametriarvoja ja laskea niistä eksponentiaalinen liukuva keskiarvo. Tätä keskiarvoa sovelletaan uusien parametriarvojen luomisessa. Lisäksi modifikaatioissa sovelletaan Daniela Zaharien teoriaa populaation monimuotoisuuden vaikutuksesta differentiaalievoluutioon ja siitä, miten säätöparametrit vaikuttavat algoritmin suppenemisominaisuuksiin.

Modifikaatioiden suorituskykyä verrataan algoritmin alkuperäiseen versioon kokeessa, jossa ajetaan ”IEEE Congress on Evolutionary Computation 2005” -evoluutiolaskentakilpailun ongelmafunktioita. Nämä 25 funktiota ajetaan kahdessa eri ulottuvuudessa ja tulokset kerätään kaikista algoritmeista. Tuloksia analysoidaan ja niitä verrataan toisiinsa. Analyysissä keskitytään pääasiassa siihen, miten modifikaatioiden säätöparametrit käyttäytyvät.

Kokeelliset tulokset osoittavat, että muunnellut algoritmit ovat lupaavan tehokkaita verrattuna alkuperäiseen differentiaalievoluutioon. Erityisesti suppenemisnopeudet VDE-1 ja VDE-3 algoritmeilla ovat paljon suurempia kuin alkuperäisellä versiolla. Tietyissä funktioissa erot suorituskyvyssä ovat erityisen isoja.

AVAINSANAT: globaalioptimointi, evoluutioalgoritmi, differentiaalievoluutio, adaptiivinen, säätöparametrit

UNIVERSITY OF VAASA	
Faculty of technology	
Author:	Ville Vainioranta
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ABSTRACT

Differential Evolution is an evolutionary algorithm designed for global optimization. Its main assets are simplicity, ease of use, and effectiveness, which has been demonstrated in different research papers and evolution computation competitions. Differential Evolution has three control parameters and each of them has a significant impact on the performance of the algorithm.

This paper introduces three modified algorithms which adapt the control parameters during the optimization process. The first modification VDE-1 adapts the mutation factor, the second modification VDE-2 adapts the crossover factor and the third modification adapts both factors. The idea in these modified algorithms is to use previously successful factor parameter values and calculate an exponential moving average value of these parameter values. This average value is then utilized to generate new control parameter values. In addition, the modifications apply Daniela Zaharie's theory on population diversity in Differential evolution and on how control parameters affect the convergence properties.

The performances of the modified algorithms are compared with the original algorithm version with results on a benchmark of 25 functions provided by the "IEEE Congress on Evolutionary Computation 2005" evolution computation contest. The 25 functions are run on two different dimensions, and the results are gathered from all algorithms. The results are then analyzed and compared with each other. The main focus in the analysis is on the behavior of the control parameters for the modified algorithms.

The experimental results indicate that the modified algorithms are promisingly more effective and efficient when compared to the original Differential Evolution. Especially the convergence speeds of VDE-1 and VDE-3 are much faster compared to the original algorithm. The differences in performance are remarkably high for certain functions.

KEYWORDS: global optimization, evolutionary algorithm, Differential evolution, adaptation, control parameters

1. INTRODUCTION

Differential evolution, often referred to as DE, is an evolutionary algorithm designed for solving numerical optimization problems. It is a population based stochastic algorithm. After the initial introduction of the original version, researchers have been trying to develop new and more efficient versions. The original DE has three user defined control parameters and a common approach to recent research has been to somehow adapt the parameters during the optimization process, thus eliminating the sometimes problematic selection of these control parameters. This thesis is about adapting two control parameters (mutation factor and crossover factor) during the optimization process, as well as about analyzing how the adaptation affects the results. Adaptation of these control parameter values will be based on a theory by Daniela Zaharie (2002).

In her article "Critical Values for the Control Parameters of Differential Evolution Algorithms" Daniela Zaharie (2002) introduces a method for choosing control parameters so that the algorithm does not prematurely converge. Zaharie's objective is to find a relationship between the three control parameters and the population diversity which is measured by statistical variances, and also to identify a relationship between population variance and convergence properties. To prevent premature convergence and stagnation, the control parameters chosen should keep the population diversity on a reasonable level. Increasing the diversity in the population is achieved by using control parameters which induce an increase in the population variance.

In general, optimization seeks to find the best solution for a given problem. Optimization is, for example, minimizing or maximizing the result of an objective function which defines the cost of the problem. A problem has a certain number of parameters which have their own constraints and thus, optimization aims to find the values of the active parameters which then give the best cost (the global optima) or, and as in most cases, a satisfactory solution with solution near the global optima.

1.1. Aim and scope of research

The aim of this research is to conduct an experiment on four different real parameter optimization algorithm versions and to find out whether adapting one or two of the control parameters (mutation factor *F*, crossover factor *CR*) is successful in Differential Evolution (DE). Four different versions of DE will be tested on 25 real parameter optimization problems that were given in CEC05 contest (IEEE Congress on Evolutionary Computation, 2005) for real parameter optimization (see Suganthan, Hansen, Liang, Chen, Auger & Tiwari 2005). DE algorithm was tested in the contest and the results for the 25 optimization problems were reported by Rönkkönen, Kukkonen and Price (2005).

There are different versions of DE algorithm and the report by Rönkkönen *et al*. (2005) states that DE/rand/1/bin was chosen to be used in the contest. DE/rand/1/bin is called the classic version of DE, and several studies have showed its usefulness for global optimization. DE/rand/1/bin is the first algorithm to be tested on the test functions. The report (Rönkkönen *et al*. 2005) also reveals the control parameter settings for the functions and thus, the reproduced test will use the same settings for the DE/rand/1/bin. The second algorithm is a similar DE/rand/1/bin algorithm but, instead of predetermined user-chosen control parameters, new mutation factor values are generated separately as the optimization is running. New parameter values are generated by utilizing an exponential moving average (EMA) of previously successful values. The third algorithm is a DE/rand/1/bin algorithm and it generates new crossover factor values during the optimization process, and also utilizes EMA value of successful values. Finally, the fourth algorithm is also a DE/rand/1/bin algorithm which generates values for both of the control parameters: the mutation and the crossover factor. Both factors are generated from the EMA values of previously successful values.

For all algorithms the control parameters, the generation of new values will apply Zaharie’s theory which provides the boundaries for the generated values. The aim is to test and find effective values for the competition functions and see if the parameters can automatically adapt to good values depending on the function at hand.

Many researches in the field of DE have made conclusions in their studies about control parameters and given guidelines on how to choose the control parameters. Furthermore, if the control parameters in question can be automated and are able to provide satisfactory results, then the algorithms could be developed as a black box system which could be used to solve various different problems or functions. If the algorithms prove to have good performance, they will also provide useful information about effective control parameters. The CEC05 contest provides some of the performance criteria for this study, which an experimental benchmark will record. Also the values of the control parameters are gathered throughout the optimization. The results from every algorithm version are then represented, analyzed and compared with each other.

1.2. Outline of the thesis

Chapter 2 introduces Differential Evolution (DE) in detail. The original DE algorithm is described and also other DE strategies are briefly introduced. The control parameter selection is critical for DE, and some basic advice is presented. Daniela Zaharie’s work is significant for this thesis and hence, described in more detail in section 2.4. Researchers across the world have developed similar adaptive DE variants and the most relevant ones are introduced. DE is applicable for solving many different problems and some of the recent applications are also presented in this thesis.

Chapter 3 introduces the modifications developed for the thesis. Modifications will apply an exponential moving average (EMA) in generating new mutation and crossover factor values. The basic principle of EMA is presented and discussed as well as Zaharie’s variance factor in relation to the mutation and crossover factor values. The operations of the modification named VDE-1, VDE-2 and VDE-3 are then presented in detail.

Chapter 4 introduces the experimental arrangements. The control parameters used for each algorithm are listed. The benchmark and the performance criteria are introduced. Chapter 5 presents the results. The results include error value tables and tables which show the number of function evaluations (FES) it took the algorithm to solve the function in question**.** The detailed analysis on convergence properties and behavior of the control parameters are presented in graphs and figures for selected functions. Chapter 6 includes the conclusions of the thesis and suggestions for further study.

2. DIFFERENTIAL EVOLUTION

Differential evolution (DE), introduced by Kenneth Price and Rainer Storn in 1995 (Storn & Price 1995), was developed for global optimization over continuous spaces. DE promises to be a simple algorithm as well as fast and robust numerical optimization algorithm accessible to everyone (Price & Storn 1997: 18). DE is a population based algorithm which has characteristics of biological evolution.

Individuals (vectors) of the initial population (first generation) undergo mutation and crossover processes, and the resulting trial vectors (candidate solutions) are compared to the initial population vectors. Comparison is done by fitness (cost) of an objective function and the vector with a better cost is selected to the next generation. This process continues until an acceptable solution is found or other termination conditions are met.

In the following chapter, the classic version DE/rand/1/bin is described in detail, and the objective is to find a solution which minimizes the objective function in question. All the optimization problems in this thesis are minimization problems which can be expressed as follows:

Find vector

$X = \{x_1, x_2, \dots, x_D\}$

to minimize

$f(X)$

where each parameter (1)

$x_i \in R$

and can be subject to boundary constraints

$x_i^{low} \leq x_i \leq x_i^{high} \qquad i = 1, \dots, D$

2.1. Description of the DE/rand/1/bin algorithm

DE operates on a population of D-dimensional vectors. First, a constant population size *NP* is chosen. *NP* must be greater or equal to four for the algorithm to work. Often there is no knowledge of the global optimum of the objective function and therefore, the initial population of the parameters of the *NP* vectors is generated randomly within the given parameter boundaries. The basic process of the algorithm is shown in Figure 1.

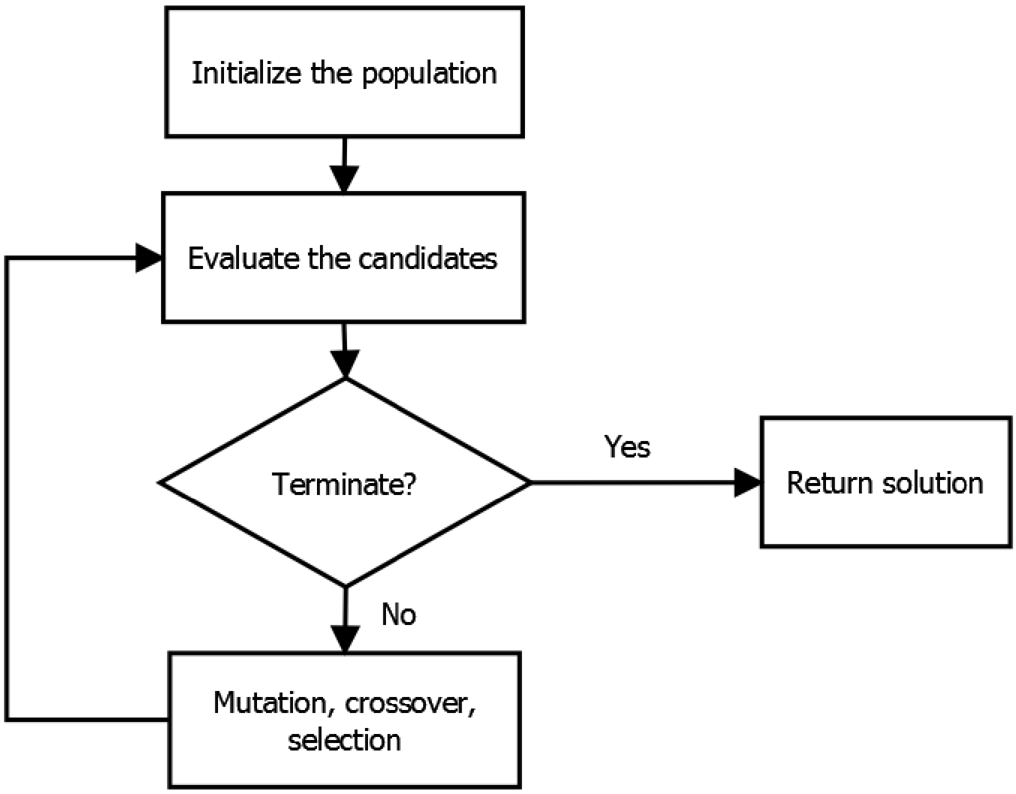

Figure 1. Basic process of DE algorithm.

Vectors in the first generation are evaluated by an objective function. If an acceptable solution is not found, then the current population vectors undergo mutation and crossover operations which produce new candidates (trial vectors). Every vector in the current population is compared to a trial vector, and if the trial vector gives a better cost it is selected and it replaces the current population vector in the next generation.

Operations are then iteratively applied to the vectors until an acceptable cost is found or other termination criteria are met. Mutation, crossover and selection operations are explained in detail next.

Mutation is done by adding a weighted difference of two other random population vectors to a third random population vector resulting in a new parameter vector (mutant vector). Weighted difference is defined by a mutation factor *F*. Mutation process is described as follows:

For each target vector $x_{i,G},\ i = 1, 2, 3, \ldots, NP$, a mutant vector (is also called noisy vector in DE literature) v_i is formed with a mutation scheme described by Storn and Price (1997) as follows:

$$v_{i,G+1} = x_{r1,G} + F \cdot (x_{r2,G} - x_{r3,G}) \tag{2}$$

with random indexes $\{r_1,\ r_2,\ r_3 \in \{1, 2, \ldots,\ NP\}$, integer, mutually different and $F > 0$. The randomly chosen integers $r_1,\ r_2$ and r_3 are also chosen to be different from the running index *i*, so that *NP* must be greater or equal to four to allow for this condition. *F* is a real and constant factor usually in range [0, 2] which controls the amplification of the differential variation $(x_{r2,G} - x_{r3,G})$. (Storn & Price 1997: 344.)

where *i* is the index of the current target vector and x_{r1}, x_{r2} and x_{r3} are randomly selected and mutually different vectors in the population. The mutation constant *F* is a constant chosen in range [0, 2]. Figure 2 illustrates a two-dimensional example of the mutation process. (Storn & Price 1997: 344.)

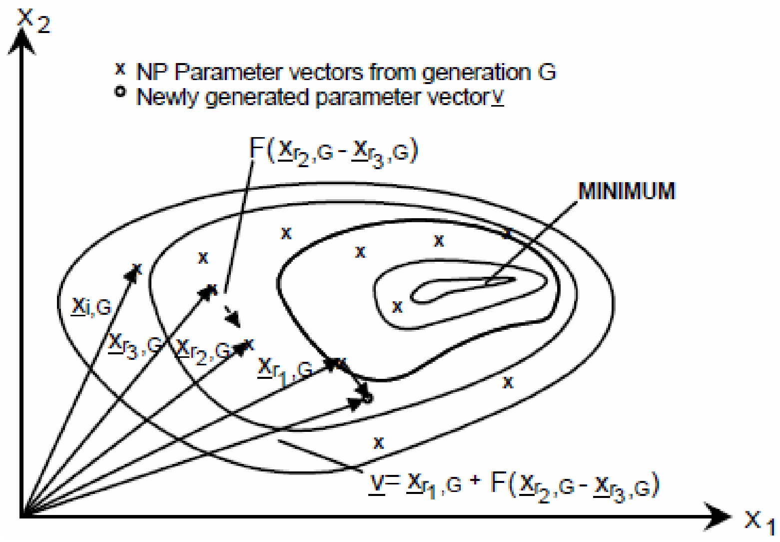

Figure 2. An example of a two-dimensional cost function showing its contour lines and the process for generating $v_{ji,G+1}$. (Storn & Price 1997: 344.)

Crossover is done by mixing the parameters, index *j*, of the newly found mutant vector $v_{ji,G+1}$ with parameters of the target vector $x_{ji,G}$ in a crossover operation scheme forming a trial vector $u_{ji,G+1}$:

$$u_{\mathrm{ji,G+1}} = (u_{\mathrm{1i,G+1}}, u_{\mathrm{2i,G+1}}, \dots, u_{\mathrm{Di,G+1}}) \tag{3}$$

The trial vector is formed with the following scheme:

$$u_{ji,G+1} = \begin{cases} v_{ji,G+1} & \text{if } (randb(j) \leq CR) \text{or } j = rnbr(i) \\ x_{ji,G} & \text{if } (randb(j) > CR) \text{ and } j \neq rnbr(i) \end{cases} \tag{4}$$

$$j = 1, 2, \dots, D.$$

randb(j) is the jth evaluation of a uniform random number generator with outcome ∈ [0; 1]. *CR* is the crossover constant ∈ [0; 1] which has to be determined by the user. *rnbr(i)* is a randomly chosen index ∈ 1, 2,…,*D* which ensures that $u_{ji,G+1}$ gets at least one parameter from $v_{ji,G+1}$. Example of the crossover process is illustrated in Figure 3. (Storn & Price 1997: 344-345.)

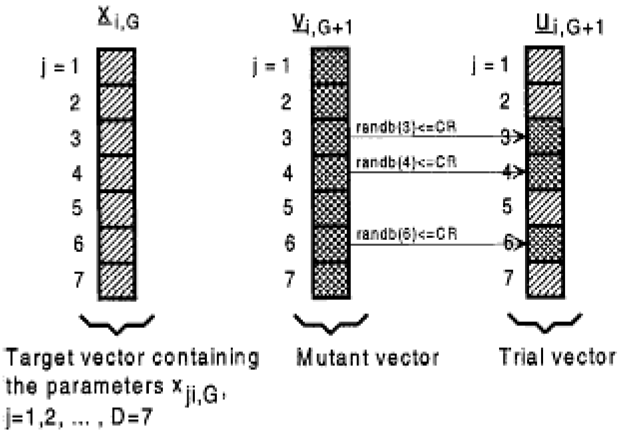

Figure 3. Illustration of the crossover process for D = 7 parameters. (Storn & Price 1997: 344.)

Selection is done by evaluating the trial vector's cost against the target vector's cost. To decide whether or not it should become a member of generation G+1, the trial vector $u_{i,G+1}$ is compared to the target vector $x_{i,G}$ using the greedy criterion. If vector $u_{i,G+1}$ yields a smaller cost function value than $x_{i,G}$, then $x_{i,G+1}$ is set to $u_{i,G+1}$, otherwise, the old value $x_{i,G}$ is retained. (Storn & Price 1997: 345.)

Other variant of the selection process is to also select the trial vector if it yields an equal cost function score. In this thesis the algorithms will use this selection variant. This variant was chosen because Rönkkönen *et al.* (2005) have used it also.

2.2. Other DE strategies

DE/rand/1/bin is not the only variant of DE. Other strategies, especially for the mutation process, have been developed. Other variants can be expressed with the following notation:

DE/x/y/z

where

x specifies the vector to be mutated which can be “rand” (a randomly chosen population vector) or “best” (the vector of lowest cost from the current population). Also “current-to-best” and “current-to-rand” have been introduced, which add the current target vector to the mutation. Some of the mutation schemes are explained in Table 1.
y is the number of difference vectors used.
z denotes the crossover scheme. Mostly used is “bin”, which means that crossover of the trial vectors parameters is done by independent binomial experiments. Other variant is the exponential crossover “exp”, where the trial vector gets a sequence of parameters from the mutant vector.

Table 1. Different mutation schemes.

Strategy	**Mutation scheme**
DE/best/1	$v_{i,G+1} = x_{best,G} + F \cdot (x_{r1,G} - x_{r2,G})$
DE/best/2	$v_{i,G+1} = x_{best,G} + F \cdot (x_{r1,G} - x_{r2,G}) + F \cdot (x_{r3,G} - x_{r4,G})$
DE/rand/2	$v_{i,G+1} = x_{r1,G} + F \cdot (x_{r2,G} - x_{r3,G}) + F \cdot (x_{r4,G} - x_{r5,G})$
DE/current-to-rand/1	$v_{i,G+1} = x_{i,G} + F \cdot (x_{r1,G} - x_{i,G}) + F \cdot (x_{r2,G} - x_{r3,G})$
DE/current-to-best/1	$v_{i,G+1} = x_{i,G} + F \cdot (x_{best,G} - x_{i,G}) + F \cdot (x_{r1,G} - x_{r2,G})$

2.3. Control parameter selection (DE/rand/1/bin)

Differential Evolution control parameters are a popular research subject. While the parameters are reasonably easy to choose, they are quite problem specific. Choosing the parameters with a trial-and-error method can be very time-consuming. Some sets of parameters work well for some problems but not for other problems. Storn and Price (Storn & Price 1997: 356) offer some rules of thumb:

> According to our experience a reasonable choice for *NP* is between 5·*D* and 10·*D* but *NP* must be at least 4 to ensure that DE will have enough mutually different vectors with which to work. As for *F*, *F* = 0.5 is usually a good initial choice. If the population converges prematurely, then F and/or *NP* should be increased. Values of *F* smaller than 0.4, like those

> greater than 1, are only occasionally effective. A good first choice for *CR* is 0.1, but since a large *CR* often speeds convergence, to first try *CR* = 0.9 or *CR* = 1.0 is appropriate in order to see if a quick solution is possible. (Storn & Price 1997: 356.)

Rönkkönen *et al.* (2005) suggest that good values of *F* are typically in range [0.4, 0.95] and for *CR* the range is [0, 0.2] for separable functions and [0.9, 1] for the non-separable functions. *NP* size should be chosen from the range 2·D to 40·D.

2.4. Daniela Zaharie's work

In the article "Critical Values for the Control Parameters of Differential Evolution Algorithms", Daniela Zaharie (2002) introduces a method for choosing control parameters of Differential evolution algorithm. Zaharie points out that only a few theoretical results have been published on Differential evolution behavior. The problem with DE is that it has a risk of prematurely converging to local optima and thus, a risk of losing population diversity (population consisting of identical vectors). If this happens the DE mutation process cannot generate new different trial vectors as there are no differences between the current population vectors. To prevent this, the control parameters should be chosen so that the diversity, which is measured by statistical variances in the population vectors, is kept on a reasonable level. Zaharie's objective is to find a relationship between the three control parameters and population variance evolution, and to identify a relationship between population variance evolution and convergence properties.

In DE the variance operations (mutation and crossover) usually try to increase the variance in the population. The effect of these operations on population variance can be expressed as follows:

$$Var(Y) = c \cdot Var(X) \tag{5}$$

where Y is the trial population after the mutation and crossover operations. Factor c is a variance factor representing the effect of the control parameters on the trial population variance. X is the current population before the variance operations.

After the variance operations, the selection operation usually tries to reduce the variance in the population thus causing convergence. Selection can be expressed as follows:

$$Var(Z) = a \cdot Var(Y) \tag{6}$$

where Z is the selected population for the next generation. Factor a represents the effect of the selection operation on the population variance. Y is the trial population after the variance operations.

When these two equations (5) and (6) are combined:

$$Var(Z) = ac \cdot Var(X) \tag{7}$$

From this is seen that ac >1 the population variance increases and the algorithm diverges. If ac <1 the population variance decreases and the algorithm converges. The value of a varies with the function properties and the selection operation of the algorithm. In most cases the user has no control over the value of a. However, the user can control the value of c with the control parameters (*F*, *CR*, *NP)*. Depending on the function, if the c value is too low then the algorithm might converge too fast towards local optima since the problem domain is not searched very efficiently. Then again, if c is too high, the convergence speed will slow down. The aim in this thesis is to find out if the modified algorithms can adapt the control parameters to good values effectively and efficiently, and thus find a good c value automatically. The variance factor c is discussed in more detail in section 3.2.

2.5. Adaptive DE variants

Recent advances in DE research have introduced many adaptive variants which use multiple strategies for mutation, and/or update the control parameter values during the optimization process. Some variants encode the parameters within the vectors. The population size, *NP*, is constant in these strategies. Some of the adaptive variants and their basic methods are introduced next.

Fuzzy Adaptive Differential Evolution (FADE), introduced by Liu and Lampinen (2005), adapts *F* and *CR* values of rand/1/bin during the optimization using fuzzy logic controllers. Controllers have information on the magnitude of change in vectors' parameters and the function cost in the last two successive generations. Based on the information stored in the controllers, good *F* and *CR* values are set for the next generation.

Selfadaptive Differential Evolution algorithm (SaDE), introduced by Qin and Suganthan (2005) uses two strategies (rand/1/bin, current-to-best/2/bin) for the mutation and crossover. The strategies are first used equally, but later in the optimization process a more successful strategy is given a higher probability. Values for *F* are random for each individual in the population from range (0, 2]. This range allows the optimization process to have good local (small *F* values) and global (large *F* values) search abilities. *CR* is also generated randomly but successful values are recorded. After a prespecified generation period, successful values influence new *CR* values for the next generation period. SaDE also employes a Quasi-Newton local search method for the best individuals after 200 generations have passed.

Self-Adapting Control Parameters in Differential Evolution (jDE), introduced by Brest, Greiner, Boskovic, Mernik and Zumer (2006) is also based on the rand/1/bin scheme. The idea is to encode the *F* and *CR* values to the vectors with the vector parameters. Succeeding vectors retain the applied control parameters to the next generation. Algorithm introduces two new control parameters τ_1 and τ_2 for the probabilities that *F* and/or *CR* will mutate before the mutation and crossover of the trial vector. Probabilities are set to 0.1 and new *F* and *CR* values are randomly generated from ranges [0.1, 1.0] and [0, 1].

Adaptive Differential Evolution With Optional External Archive (JADE), developed by Jingqiao and Sanderson (2009), introduces a new mutation strategy current-to-pbest where pbest is chosen randomly from certain percentage (input parameter *p*) of top individuals in the current population. Percentage *p* is tested best in the range of 5–20% by Jingqiao and Sanderson. JADE also introduces an optional external archive that stores a certain amount of vectors which fail in the selection process. The second random vector in the mutation phase has a chance to be chosen from the current population or from the archive.

JADE also adapts *F* and *CR* values during the optimization. In each generation *F* and *CR* values are individually generated. A successful parameter value updates the mean valued for the distributions (*F* - Cauchy distribution, *CR* - normal distribution). A new control parameter $c \in [0, 1]$ is introduced and it affects the rate of parameter adaption.

Differential Evolution Algorithm with Ensemble of Parameters and Mutation and Crossover Strategies (EPSDE), introduced in a report by Mallipeddi, Suganthan, Pan and Tasgetiren (2011), uses pools for mutation and crossover strategies and pools of values for *F* and *CR*. These values are encoded to the vector in a similar way as in jDE. In the report by Mallipeddi *et al.* (2011), the pools are for mutation: {JADE, DE/current-to-rand/1}, for crossover {bin, exp}, and for the control parameters *F* {0.5, 0.9}, *CR* {0.1, 0.5, 0.9}.

First, the strategies and control parameters are randomly assigned to the initial population. If a trial vector is found better than the target vector, then the strategies and control parameters are inherited to the next generation. A combination of successful strategies and parameter values is stored in memory. If the trial vector is worse than the target vector, then the target vector goes through to the next generation and the strategies and control parameters of the target vector are randomly regenerated or taken from a store of successful combinations.

Differential Evolution with Composite Trial Vector Generation Strategies and Control Parameters (CoDE), presented by Yong, Zixing and Qingfu (2011), has a different strategy for producing the trial vector. The algorithm produces three trial vectors for each target vector and the best one goes through to the selection phase. The trial vectors are mutated by three different strategies (rand/1/bin, rand/2/bin, current-to-rand/1) and the control parameters are chosen randomly from predefined pools.

In general, results on all these adaptive versions outperformed the basic rand/1/bin version. This suggests that adaptive DE variants are effective and here to stay. Criticism could be made towards introducing new control parameters and losing the simplicity of the original version.

2.6. Recent real-world problems solved by DE

The most common aim in real-world optimization, for example in engineering, is to design an optimized system, so, maximizing the system's beneficial qualities and minimizing the undesired qualities. The following are summaries of scientific papers written on applications which have solved real-world problems applying the DE algorithm. The papers have been published after the year 2010, and they are:

1. Gunda, Acharjee and S. Biswas (2011) proposed a method for solving combined economic and emission dispatch (CEED) problem. The problem in CEED is to optimize a power system so that it fulfills an economic objective (providing enough electricity while minimizing the fuel costs) and an environmental objective (minimizing the emissions produced). The researchers used a hybrid optimization algorithm which was created by combining a DE algorithm and a particle swarm optimization (PSO) algorithm.

2. Chiha, Ghabi and Liouane (2012) demonstrated that it is possible to find optimal parameters using DE for a proportional–integral–derivative (PID) controller. PID controllers are widely used in industrial control systems and PID controllers have three control parameters which define the control system's behaviour. Optimization of these parameters was done by the original DE algorithm.

3. Lezama, Castañón and Sarmiento (2012) introduced a method on optimal placement of wavelength converters in nodes of an optical network. In the article, extensive experiments were made on different network topologies and DE was proven more effective than other optimization algorithms. The DE version was a classic DE but it had been altered to a binary format.

4. Qu, Lianhai and Juan (2010) applied DE for estimating future water demand in the Yellow River Basin area in China. DE was used to optimize a smoothing factor in a general regression neural network (GRNN) learning model. The DE version in the paper is the original DE but mutation factor F values have been adjusted automatically during the process by adding chaotic variance to the values.

5. Myeong-Chun and Sung-Bae (2012) developed an image enhancement tool for smart phones that automatically creates new and enhanced images from existing images based on user preferences. The original version of DE was used but the cost-based selection was replaced with user-based selection. The users were shown a parent image and a trial image which the DE mutated from the parent image (brightness, color, contrast *etc*.), and they were asked to select the one they preferred. This method proved to be effective in comparison to manual image enhancement done by expert tools.

6. Wei, Lingbo and Xingsheng (2010) created a new algorithm which they applied to an application whose aim was to maximize the profit of butane alkylation process. The article introduces a new hybrid algorithm combining a cultural algorithm and the DE algorithm. DE was only used for calculating and mutating

> a “center individual”, an arithmetic average value of all candidates, which influenced the creation of the new population individuals. Furthermore, only the mutation scheme of the original DE was used in this hybrid algorithm.

These papers are only a randomly gathered, and interest-based sample and do not represent the entire publication collection since 2010. They illustrate the possibilities of using DE to solve various different problems. What is noteworthy is that if the solutions can be represented and evaluated somehow (usually by an objective function) then DE can be utilized to generate and better the potential solutions.

3. MODIFICATIONS (VDE)

The proposed modifications (named here VDE-1, VDE-2 and VDE-3) to the DE/rand/1/bin scheme are made to adapt the mutation factor *F* or/and the crossover factor *CR* during the optimization process. The population size *NP* remains fixed. Adaptation of the control parameters is done by calculating an exponential moving average (EMA) of successful values. New values are generated for each generation by using the current EMA values and by adding random variance to it. Section 3.1 explains the EMA more specifically.

For each VDE-algorithm the minimum and maximum values of the crossover and mutation factors are controlled by the variance factor *c*. Section 3.2 explains the variance factor more specifically.

3.1. Exponential moving average (EMA)

EMA calculates an average value emphasizing on newer values and exponentially discounting older values. Calculating the EMA can be expressed as follows:

$$EMA_{new} = EMA_{old} + \alpha \cdot (newvalue - EMA_{old}) \tag{8}$$

where EMA_{new} is the resulting average value, EMA_{old} is the previous average value, $newvalue$ is the value to be added to the average, and coefficient α is the degree of weight decrease, a constant $\in [0,1]$. Higher α values discount older values faster. Constant α can also be expressed as follows:

$$\alpha = 2/(N+1) \tag{9}$$

where N is the length of the period of values that user wants to have significant weighted effect on the average. An example of difference in weights is shown in Figure 4.

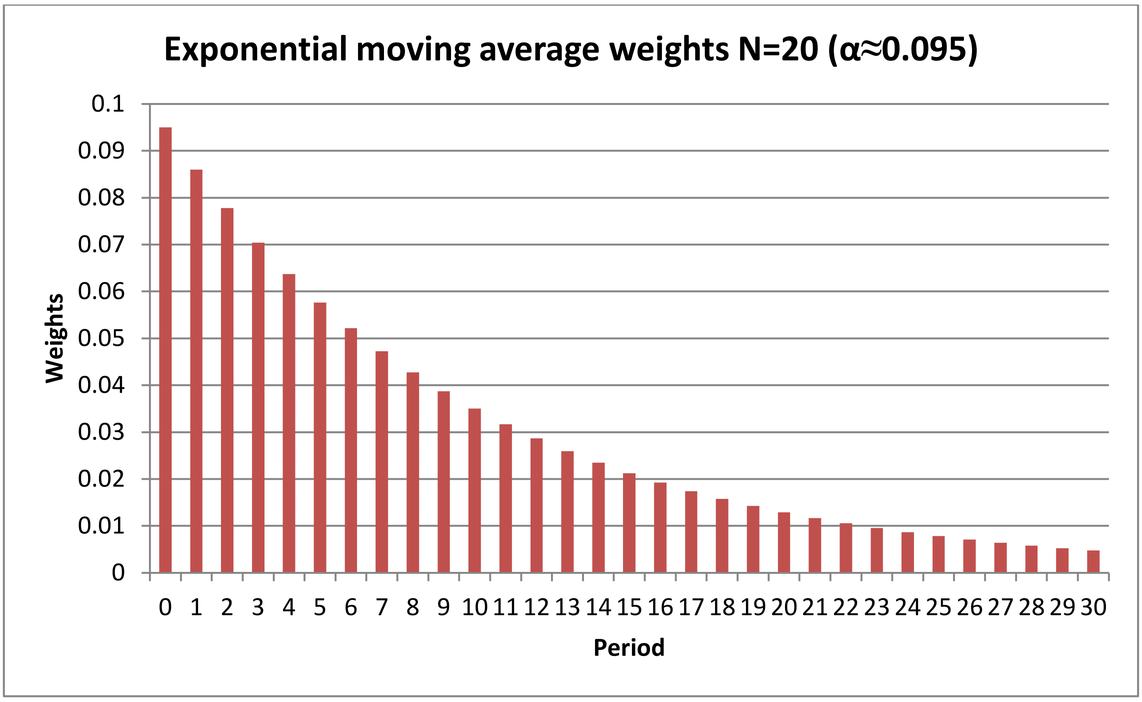

Figure 4. Example of EMA weights.

3.2. Mutation and crossover factor values applying variance factor *c*

The modifications also apply Zaharie's variance factor *c* (2002) to determine the limits or the range of which the mutation factor and crossover factor get their values. Variance factor *c* from equation (5) can be calculated as follows:

$$c = \sqrt{2F^2CR - \frac{2CR}{NP} + \frac{CR^2}{NP} + 1} \tag{10}$$

Zaharie suggests that *c* values which are a little greater than 1 provide good convergence properties, and previously used and reported parameter values (Storn & Price 1995) are in range [1.0, 1.5]. To illustrate the resulting values of factor *c*, two tables are provided for two different NP sizes, and values *c* which result in previously suggested range are in red (values are rounded down).

Table 2. Variance factor *c* values for different control parameter values (NP20).

Value of variance factor *c* for *NP*=20											
F	*CR*										
	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.1	1.00	0.99	0.99	0.99	0.98	0.98	0.98	0.98	0.98	0.98	0.98
0.3	1.00	1.00	1.00	1.01	1.01	1.02	1.03	1.03	1.04	1.05	1.06
0.5	1.00	1.02	1.04	1.06	1.08	1.10	1.12	1.14	1.16	1.18	1.20
0.7	1.00	1.04	1.08	1.12	1.16	1.20	1.24	1.28	1.31	1.35	1.38
0.9	1.00	1.07	1.14	1.20	1.27	1.33	1.38	1.44	1.49	1.55	1.60
1.1	1.00	1.11	1.21	1.30	1.39	1.47	1.55	1.62	1.69	1.76	1.83
1.3	1.00	1.15	1.28	1.41	1.52	1.62	1.72	1.82	1.91	1.99	2.08
1.5	1.00	1.20	1.37	1.52	1.66	1.79	1.91	2.02	2.13	2.23	2.33

Table 3. Variance factor *c* values for different control parameter values (NP100).

Value of variance factor *c* for *NP*=100											
F	*CR*										
	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.1	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
0.3	1.00	1.00	1.01	1.02	1.03	1.04	1.04	1.05	1.06	1.07	1.08
0.5	1.00	1.02	1.04	1.07	1.09	1.11	1.13	1.15	1.17	1.20	1.22
0.7	1.00	1.04	1.09	1.13	1.17	1.21	1.25	1.29	1.33	1.36	1.40
0.9	1.00	1.07	1.14	1.21	1.28	1.34	1.40	1.45	1.51	1.56	1.61
1.1	1.00	1.11	1.21	1.31	1.40	1.48	1.56	1.63	1.71	1.77	1.84
1.3	1.00	1.15	1.29	1.41	1.53	1.63	1.73	1.83	1.92	2.00	2.09
1.5	1.00	1.20	1.37	1.53	1.67	1.80	1.92	2.03	2.14	2.24	2.34

From these tables it is evident that the population size (*NP*) has no major effect on the *c* values. Of course the population size has a major effect in relation to the problem difficulty. Multimodal problems would need a larger population size for the algorithm to find the global optima.

3.3. VDE-1

The first modification VDE-1 adapts the mutation factor *F* during the optimization. New *F* values are generated for each generation after the first generation. *F* values are controlled by the variance factor *c* which provides the minimum and maximum values, and ensures that the values stay in effective range. *F* can be solved from equation (10) as follows:

$$F = \sqrt{\frac{c^2 - 1}{2CR} + \frac{2 - CR}{2NP}} \tag{11}$$

Table 4 illustrates values of *F* in relation to *c* and *CR*. Reported effective values from the range [0.4, 0.95] (Rönkkönen *et al.* 2005) are in red.

Table 4. Example of *F* values in relation to *CR* and *c* (*NP*=50).

Value of *F* for *NP*=50										
c	*CR*									
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
1	0.14	0.13	0.13	0.13	0.12	0.12	0.11	0.11	0.10	0.10
1.05	0.73	0.52	0.43	0.38	0.34	0.32	0.29	0.28	0.26	0.25
1.1	1.03	0.74	0.61	0.53	0.47	0.43	0.40	0.38	0.36	0.34
1.15	1.28	0.91	0.74	0.65	0.58	0.53	0.49	0.46	0.44	0.41
1.2	1.49	1.06	0.87	0.75	0.67	0.62	0.57	0.54	0.51	0.48
1.25	1.68	1.19	0.98	0.85	0.76	0.69	0.64	0.60	0.57	0.54
1.3	1.86	1.32	1.08	0.94	0.84	0.77	0.71	0.67	0.63	0.60
1.35	2.03	1.44	1.18	1.02	0.92	0.84	0.77	0.73	0.68	0.65
1.4	2.20	1.55	1.27	1.10	0.99	0.90	0.84	0.78	0.74	0.70
1.45	2.35	1.67	1.36	1.18	1.06	0.97	0.89	0.84	0.79	0.75
1.5	2.50	1.77	1.45	1.26	1.12	1.03	0.95	0.89	0.84	0.80
1.55	2.65	1.88	1.53	1.33	1.19	1.09	1.01	0.94	0.89	0.84
1.6	2.80	1.98	1.62	1.40	1.25	1.15	1.06	0.99	0.94	0.89
1.65	2.94	2.08	1.70	1.47	1.32	1.20	1.12	1.04	0.98	0.93
1.7	3.08	2.18	1.78	1.54	1.38	1.26	1.17	1.09	1.03	0.98
1.75	3.21	2.27	1.86	1.61	1.44	1.32	1.22	1.14	1.08	1.02
1.8	3.35	2.37	1.94	1.68	1.50	1.37	1.27	1.19	1.12	1.06

Detailed description of the VDE-1 applying the EMA and *c*:

1. Before optimization the user has to initialize the population and control parameters *NP*, *CR*, *F* and the average value F_{EMA} for the first generation. Next, the user has to choose the EMA degree of weight coefficient $\alpha \in [0, 1]$.
2. Every time a trial vector is successful (better or equal cost compared to the target vector), the current *F* value updates the F_{EMA} value.
3. After the first generation, in each successive generation a new value of F_{new} is formed:

$$F_{new} = F_{EMA} + rand(-x, x) \quad (12)$$

   where F_{EMA} is the current EMA value and $rand(-x, x)$ is a uniformly distributed random value in range $[-x, x]$. Value of x is chosen by the user and suggested range is [0.01, 0.2].

4. Resulting F_{new} is bounded by the variance factor *c* limits c_{min} and c_{max}. As seen in Table 4, the value of *CR* has a major effect on value of *c*. Boundaries should be chosen so that the value of *F* stays in reasonable range. If the value F_{new} violates these boundaries, the value of *F* should be set to F_{EMA}. Different boundary settings for separable and non-separable functions should be applied.

3.4. VDE-2

The second modification VDE-2 adapts the crossover factor *CR* during the optimization. New *CR* values are also generated for each generation after the first generation. *CR* values are again controlled by the variance factor *c* which provides the minimum and maximum values and ensures that the values stay in effective range. *CR* can be solved from equation (10) as follows:

$$CR = \frac{-(2F^2 - \frac{2}{NP}) \pm \sqrt{(2F^2 - \frac{2}{NP})^2 - 4(\frac{1}{NP} * (1 - c^2))}}{2/NP} \qquad (13)$$

From the equation (9) it is possible only to use positive values and values which are in range [0, 1]. Table 5 shows an example how the *CR* value behaves in relation to *F* and *c*.

Table 5. Example of *CR* values in relation to *F* and *c* (*NP*=50).

Value of *CR* for *NP*=50										
c	*F*									
	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1	1.1	1.2
1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
1.05	0.67	0.36	0.22	0.15	0.11	0.08	0.06	0.05	0.04	0.04
1.1	1.27	0.71	0.45	0.31	0.22	0.17	0.13	0.11	0.09	0.07
1.15	1.83	1.07	0.68	0.47	0.34	0.26	0.20	0.16	0.14	0.11
1.2	2.35	1.43	0.92	0.64	0.46	0.35	0.28	0.22	0.18	0.15
1.25	2.85	1.78	1.16	0.81	0.59	0.45	0.35	0.29	0.24	0.20
1.3	3.34	2.14	1.41	0.99	0.72	0.55	0.43	0.35	0.29	0.24
1.35	3.81	2.49	1.67	1.17	0.86	0.66	0.52	0.42	0.34	0.29
1.4	4.26	2.85	1.93	1.36	1.00	0.76	0.60	0.49	0.40	0.34
1.45	4.71	3.20	2.19	1.55	1.14	0.88	0.69	0.56	0.46	0.39
1.5	5.15	3.56	2.46	1.75	1.29	0.99	0.78	0.63	0.52	0.44
1.55	5.58	3.91	2.73	1.95	1.45	1.11	0.88	0.71	0.59	0.49
1.6	6.00	4.27	3.00	2.16	1.60	1.23	0.98	0.79	0.65	0.55
1.65	6.42	4.62	3.28	2.37	1.77	1.36	1.08	0.87	0.72	0.60
1.7	6.83	4.98	3.56	2.58	1.93	1.49	1.18	0.95	0.79	0.66
1.75	7.24	5.33	3.84	2.80	2.10	1.62	1.28	1.04	0.86	0.72
1.8	7.65	5.69	4.13	3.02	2.27	1.76	1.39	1.13	0.93	0.78

VDE-2 also calculates an EMA value CR_{EMA} of successful *CR* values. The logic in the algorithm is the same as in VDE-1 in section 3.3. but in addition, the value of *CR* is also limited to range [0, 1]. Also the DE literature suggest that *CR* should be kept fairly high for non-separable [0.8, 1.0] and low for separable functions [0.01, 0.2]. Because the effective *CR* ranges are smaller compared to *F* ranges, the random variance added to the EMA value CR_{EMA} should also be smaller.

3.5. VDE-3

The third modification VDE-3 adapts both the mutation factor *F* and the crossover factor *CR* during the optimization. Both factors get new values for each generation, and minimum and maximum values controlled by the variance factor limits c_{min} and c_{max}. EMA values F_{EMA} and CR_{EMA} are calculated from successful values for both factors.

Because the *CR* value can more easily go out of bounds, the new value of *CR* is generated first. Like in VDE-2, the value of *CR* should have limits which ensure effective range. If the generated value goes over or under the *CR* limit, the value is set to CR_{EMA}.

The value of *F* is generated next, and the resulting value is checked for boundary violations. If the resulting value violates the variance factor limits then the value of *F* is set to F_{EMA}. If also the F_{EMA} value violates the limits, then *F* is calculated with the equation (8) with *c* set to the violated boundary value. The value of *c* is set to c_{min} if *F* violates the lower bound and c_{max} if it violates the upper bound.

4. EXPERIMENTAL ARRANGMENTS

The comparison between the basic DE and the modifications VDE-1, VDE-2 and VDE-3 is done by extensive test functions provided by the CEC05 (IEEE Congress on Evolutionary Computation 2005) contest (Suganthan *et al.* 2005). The test functions consist of 5 unimodal ($f_1 - f_5$) and 20 multimodal functions ($f_6 - f_{25}$). The functions are designed to test the optimizer in different environments. In addition to modality, characteristics of the functions include: rotation, high condition, noise in fitness, non-continuity, optimum shifted from origin, optimum on bounds, and optimum out of initial bounds.

Results of the functions are recorded for the basic DE and for the modifications – the VDE algorithms. In order to get more accurate results in each dimension and function, all of the algorithms are given the same initial population but only if the population size set for the algorithms is the same. The same initial population is achieved with a fixed random seed. After the initialization, the random seed is reset.

4.1. Algorithm control parameter settings

The basic DE uses the same control parameter settings as Rönkkönen *et al.* (2005) have used. They adjusted the *NP* values for different functions and the values can be seen in Table 6 which includes the *NP* settings for DE and VDE algorithms. For VDE-algorithms I will experiment with also *NP* size 200. For DE, *F* is set to 0.9 for all functions and *CR* is set to 0.1 for separable functions (f_1 and f_9) and 0.9 for all other functions. These control parameter settings for DE means that the variance factor *c* value is ≈1.07 for separable functions and ≈1.56 for non-separable functions.

Table 6. *NP* values for the test functions in 10- and 30 dimensions

Function	10D				30D			
	DE	VDE-1	VDE-2	VDE-3	DE	VDE-1	VDE-2	VDE-3
1	20	20	20	20	20	20	20	20
2	20	20	20	20	20	20	20	20
3	50	50	50	50	20	20	20	200
4	20	20	20	20	20	20	20	100
5	20	20	20	20	20	50	20	100
6	20	20	20	20	20	50	20	50
7	20	50	20	50	50	50	50	50
8	20	20	20	20	100	100	100	100
9	20	20	20	20	50	50	50	50
10	100	100	100	100	20	20	20	20
11	50	50	50	50	20	50	20	200
12	100	100	100	100	50	100	50	100
13	50	50	50	50	20	20	20	20
14	50	50	50	50	20	20	20	20
15	100	100	100	200	100	200	100	200
16	100	100	100	200	100	200	100	200
17	50	100	50	100	100	100	100	200
18	100	100	100	200	100	100	100	200
19	100	100	100	200	100	100	100	200
20	100	100	100	200	100	100	100	200
21	100	100	100	200	100	100	100	200
22	100	100	100	200	100	100	100	200
23	100	100	100	200	100	100	100	200
24	100	100	100	200	100	100	100	200
25	100	100	100	200	100	100	100	200

VDE-algorithms have a few more settings for the user to choose. Most of the test functions in the CEC05 competition are very difficult to solve and the settings are not optimal. The relatively low maximum allowed function evaluation value forces to make a compromise especially with population size. In preliminary testing VDE-3 showed good performance even with larger population sizes while the other algorithms did not provide better results with larger populations.

The adaptation of *F* (VDE-1) should presumably give better results than adapting the *CR* (VDE-2) for non-separable functions. To compare the performance of VDE-2 with the basic DE, the population sizes for these algorithms will be set to the same value.

When adapting both of the control parameters (VDE-3) I have decided to limit the minimum value that the *CR* can have. This is also due to the fact that if *CR* is very low then the value of *F* has to compensate for the loss of variance. Preliminary testing of VDE-3 showed risks of *F* value compensating too much (resulting value *F*>1.5) if the value of *CR* falls too low.

Figure 5 shows examples for functions f_7 and f_{12} what may happen if *CR* values are not limited. In the figure the variance factor limit *c* is also presented and it is limited to the range 1.2-1.6. *CR* is limited to the range 0.01-1.0 and *F*>0. All factors are in the same graph so the scale has been adjusted.

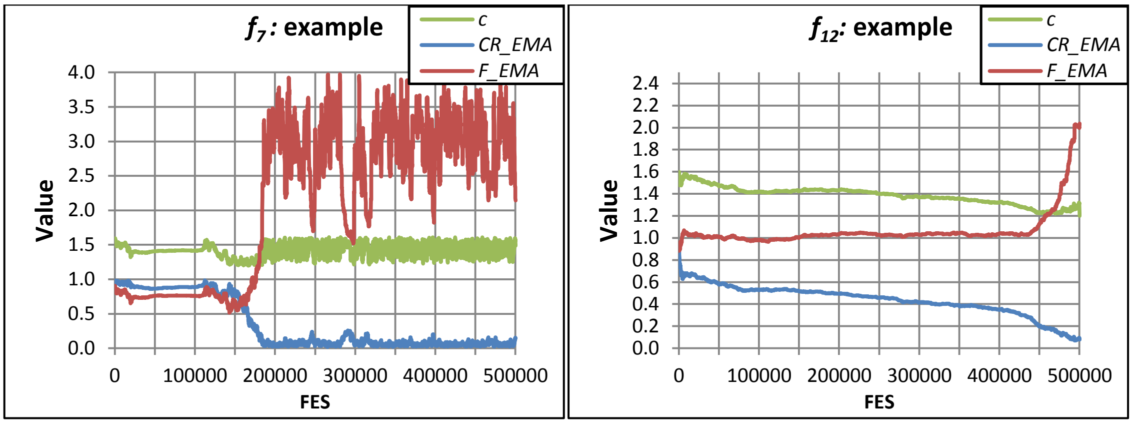

Figure 5. Examples on how F_{EMA} can behave if CR_{EMA} is not limited. The variance factor *c* boundaries in these examples are set to 1.2-1.6.

Settings for the VDE-algorithms are as follows:

VDE-1:

- *F* and the average value F_{EMA} initialized to 0.9
- *CR* same as DE
- F_{EMA} weight coefficient α set to 0.06 (period N≈33)
- random variance x set to 0.1
- Variance factor boundaries c_{min} and c_{max} set to

  - Separable functions: 1.01 and 1.15 respectively
  - All other functions: 1.25 and 1.65 respectively

VDE-2:

- CR and the average value CR_{EMA} initialized to 0.1 for separable functions and 0.9 for all others
- F same as DE
- CR_{EMA}weight coefficient α set to 0.05 (period N=41)
- random variance x set to 0.05
- Variance factor boundaries c_{min} and c_{max} set to
  - Separable functions: 1.01 and 1.35 respectively
  - All other functions: 1.4 and 1.6 respectively

VDE-3:

- CR and the average value CR_{EMA} initialized to 0.1 for separable functions and 0.9 for all others
- CR_{EMA} weight coefficient α set to 0.04 (period N=49)
- random variance x set to 0.05 for CR
- CR limited to range 0.7-1.0
- F and the average value F_{EMA} initialized to 0.9
- F_{EMA} weight coefficient α set to 0.06 (period N≈33)
- random variance x set to 0.1 for F
- Variance factor boundaries $c_{min}$and c_{max} set to
  - Separable functions: 1.01 and 1.15 respectively
  - All other functions: 1.2 and 1.6 respectively

Most of the test functions have boundary constraints for the parameter values and the settings for the benchmark are the same as Rönkkönen *et al.*(2005) have used. If the trial parameters violate the boundary constraints of the function in question, the parameters are reflected back as follows:

$$u_{ji,G} = \begin{cases} 2 \cdot x_{j,low} - u_{ji,G} & \text{if } u_{ji,G} < x_{j,low} \\ 2 \cdot x_{j,high} - u_{ji,G} & \text{if } u_{ji,G} > x_{j,high} \end{cases} \quad (14)$$

4.2. Benchmark

Functions are tested on 10- and 30-dimensions. List of functions can be seen in Table 8. Each function and dimension is run 25 times each. Performance criteria in this thesis are, in short:

- The maximum number of function evaluations (FES) for the algorithm is defined as MAXFES = 10·dimension (100000 for 10D and 300000 for 30D).
- Best function error value achieved on checkpoints at 1000(1E+03), 10000(1E+04), 100000(1E+05) and MAXFES function evaluations.
- Report the 1st (best), 7th, 13th (median), 18th and 25th (worst), mean and standard deviation error values at each checkpoint.
- FES needed to achieve a fixed accuracy level error level (see Table 7).
- Report the 1st (best), 7th, 13th (median), 18th and 25th (worst), mean and standard deviation value of FES at termination.
- Report the success rate = (# of successful runs) / total runs performance.
- Report the success performance = mean(FES for successful runs) · (# total runs) / (# of successful runs)

Table 7. Accuracy level for each function (Suganthan *et al* (2005: 40-41)).

Function	Accuracy	Function	Accuracy
1	1E-06	14	1E-02
2	1E-06	15	1E-02
3	1E-06	16	1E-02
4	1E-06	17	1E-01
5	1E-06	18	1E-01
6	1E-02	19	1E-01
7	1E-02	20	1E-01
8	1E-02	21	1E-01
9	1E-02	22	1E-01
10	1E-02	23	1E-01
11	1E-02	24	1E-01
12	1E-02	25	1E-01
13	1E-02		

Table 8. CEC05 Test Functions.

TEST FUNCTIONS
f_1: Shifted Sphere Function
f_2: Shifted Schwefel's Problem 1.2
f_3: Shifted Rotated High Conditioned Elliptic Function
f_4: Shifted Schwefel's Problem 1.2 with Noise in Fitness
f_5: Schwefel's Problem 2.6 with Global Optimum on Bounds
f_6: Shifted Rosenbrock's Function
f_7: Shifted Rotated Griewank's Function without Bounds
f_8: Shifted Rotated Ackley's Function with Global Optimum on Bounds
f_9: Shifted Rastrigin's Function
f_{10}: Shifted Rotated Rastrigin's Function
f_{11}: Shifted Rotated Weierstrass Function
f_{12}: Schwefel's Problem 2.13
f_{13}: Expanded Extended Griewank's plus Rosenbrock's Function (F8F2)
f_{14}: Shifted Rotated Expanded Scaffer's F6
f_{15}: Hybrid Composition Function
f_{16}: Rotated Hybrid Composition Function
f_{17}: Rotated Hybrid Composition Function with Noise in Fitness
f_{18}: Rotated Hybrid Composition Function
f_{19}: Rotated Hybrid Composition Function with a Narrow Basin for the Global Optimum
f_{20}: Rotated Hybrid Composition Function with the Global Optimum on the Bounds
f_{21}: Rotated Hybrid Composition Function
f_{22}: Rotated Hybrid Composition Function with High Condition Number Matrix
f_{23}: Non-Continuous Rotated Hybrid Composition Function
f_{24}: Rotated Hybrid Composition Function
f_{25}: Rotated Hybrid Composition Function without Bounds

5. RESULTS

All results for the two algorithms (DE and VDE-algorithms) are summarized in the following chapters. In addition to the benchmark checkpoints, the results also include extended performance for 500000 (5.0e5) function evaluations. Tables 9-14 summarize the function error values for 10-dimensional functions, and Tables 18-23 summarize the function error values for 30-dimensional functions. Tables 15-16 summarize successful runs which reach the fixed accuracy level and success performance for 10-dimensional functions, and Tables 24-25 summarize successful runs which reach the fixed accuracy level and success performance for 30-dimensional functions. Best values are marked in **bold.** If all algorithms have reached the global optimum or have achieved the same value at given point, the values are not bolded.

5.1. Results for 10-dimensional functions

Table 9. Error values achieved for f_1-f_4 (10D).

10D		f_1 DE	f_1 VDE-1	f_1 VDE-2	f_1 VDE-3	f_2 DE	f_2 VDE-1	f_2 VDE-2	f_2 VDE-3	f_3 DE	f_3 VDE-1	f_3 VDE-2	f_3 VDE-3	f_4 DE	f_4 VDE-1	f_4 VDE-2	f_4 VDE-3
1E+03	1st	1.57E+02	1.96E+02	**1.41E+02**	1.55E+02	1.40E+03	7.99E+02	7.59E+02	**4.55E+02**	1.02E+07	5.74E+06	6.92E+06	**3.79E+06**	1.84E+03	1.91E+03	2.13E+03	**8.84E+02**
	7th	**3.27E+02**	3.32E+02	3.37E+02	3.36E+02	2.62E+03	**1.78E+03**	2.89E+03	2.31E+03	2.40E+07	**2.25E+07**	2.37E+07	2.56E+07	3.57E+03	3.23E+03	3.82E+03	**1.99E+03**
	13th	4.59E+02	**4.16E+02**	4.38E+02	4.37E+02	3.28E+03	**2.31E+03**	3.73E+03	2.80E+03	3.05E+07	**3.01E+07**	3.69E+07	3.67E+07	5.70E+03	4.30E+03	4.68E+03	**3.14E+03**
	19th	5.87E+02	5.99E+02	5.60E+02	**5.29E+02**	4.59E+03	**3.27E+03**	5.21E+03	4.14E+03	**4.31E+07**	4.38E+07	5.64E+07	4.53E+07	7.63E+03	6.09E+03	5.99E+03	**4.44E+03**
	25th	7.90E+02	1.04E+03	1.41E+03	**7.71E+02**	6.78E+03	7.24E+03	8.68E+03	**6.36E+03**	**5.96E+07**	7.19E+07	1.38E+08	9.09E+07	1.09E+04	8.33E+03	8.93E+03	**7.00E+03**
	Mean	4.54E+02	4.71E+02	4.64E+02	**4.19E+02**	3.71E+03	**2.94E+03**	3.99E+03	3.25E+03	**3.39E+07**	**3.39E+07**	4.20E+07	4.10E+07	5.78E+03	4.57E+03	5.09E+03	**3.41E+03**
	Std	1.74E+02	2.05E+02	2.38E+02	1.58E+02	1.50E+03	1.67E+03	1.87E+03	1.57E+03	1.34E+07	1.71E+07	2.78E+07	2.28E+07	2.49E+03	1.71E+03	1.97E+03	1.65E+03
1E+04	1st	2.68E-11	**0.00E+00**	3.30E-11	**0.00E+00**	4.92E-02	**1.41E-06**	1.42E-02	1.32E-05	9.75E+05	3.56E+04	1.00E+06	**1.16E+03**	1.76E-01	8.21E-05	9.84E-03	**5.57E-05**
	7th	1.08E-10	**0.00E+00**	1.54E-10	**0.00E+00**	1.09E-01	9.38E-05	8.95E-02	**4.24E-05**	1.57E+06	1.21E+05	1.53E+06	**7.12E+03**	6.27E-01	**6.57E-04**	1.76E-01	3.42E-03
	13th	1.50E-10	**1.48E-12**	2.99E-10	1.59E-12	2.81E-01	**3.17E-04**	3.55E-01	4.94E-04	2.05E+06	1.94E+05	1.91E+06	**2.14E+04**	1.58E+00	**2.27E-03**	4.60E-01	1.00E-02
	19th	3.20E-10	2.24E-10	7.06E-10	**3.30E-11**	6.46E-01	**8.44E-04**	1.31E+00	5.97E-03	2.45E+06	5.34E+05	2.35E+06	**1.36E+05**	2.19E+00	**6.72E-03**	1.61E+00	5.98E-02
	25th	**7.99E-10**	7.31E-08	3.62E-09	3.61E-09	1.25E+00	**2.21E-02**	8.82E+00	1.10E-01	3.77E+06	**1.56E+06**	3.92E+06	2.21E+06	9.32E+00	**2.77E-02**	2.30E+01	2.60E+00
	Mean	2.32E-10	3.23E-09	6.49E-10	**1.89E-10**	4.19E-01	**2.80E-03**	1.29E+00	1.42E-02	2.11E+06	3.80E+05	2.05E+06	**2.16E+05**	2.08E+00	**5.22E-03**	2.98E+00	2.04E-01
	Std	1.98E-10	1.43E-08	8.76E-10	7.05E-10	3.54E-01	5.71E-03	2.11E+00	3.06E-02	7.11E+05	4.17E+05	7.82E+05	4.64E+05	2.32E+00	7.36E-03	6.13E+00	5.63E-01
1E+05	1st	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	1.28E+00	**0.00E+00**	1.22E-02	**0.00E+00**	0.00E+00	0.00E+00	0.00E+00	0.00E+00
	7th	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	3.32E+00	**0.00E+00**	4.03E-02	**0.00E+00**	0.00E+00	0.00E+00	0.00E+00	0.00E+00
	13th	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	5.01E+00	**0.00E+00**	6.55E-02	**0.00E+00**	0.00E+00	0.00E+00	0.00E+00	0.00E+00
	19th	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	1.01E+01	3.41E-12	1.58E-01	**0.00E+00**	0.00E+00	0.00E+00	0.00E+00	0.00E+00
	25th	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	3.11E+01	8.69E-11	5.23E-01	**0.00E+00**	0.00E+00	0.00E+00	0.00E+00	0.00E+00
	Mean	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	7.35E+00	5.98E-12	1.21E-01	**0.00E+00**	0.00E+00	0.00E+00	0.00E+00	0.00E+00
	Std	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	6.55E+00	1.70E-11	1.24E-01	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
5E+05	1st	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
	7th	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
	13th	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
	19th	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
	25th	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
	Mean	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
	Std	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00

Table 10. Error values achieved for f_5-f_8 (10D).

10D		f_5				f_6				f_7				f_8			
		DE	VDE-1	VDE-2	VDE-3	DE	VDE-1	VDE-2	VDE-3	DE	VDE-1	VDE-2	VDE-3	DE	VDE-1	VDE-2	VDE-3
1E+03	1st	3.00E+03	2.97E+03	3.49E+03	**1.97E+03**	1.21E+06	8.50E+05	3.30E+06	**5.47E+05**	**1.19E+02**	3.53E+02	1.28E+02	2.55E+02	2.04E+01	2.05E+01	2.03E+01	**2.02E+01**
	7th	4.51E+03	4.08E+03	4.31E+03	**3.80E+03**	9.77E+06	3.29E+06	8.60E+06	**3.15E+06**	**2.60E+02**	5.49E+02	2.86E+02	5.29E+02	**2.06E+01**	2.07E+01	2.07E+01	**2.06E+01**
	13th	5.12E+03	4.61E+03	5.09E+03	**4.50E+03**	1.58E+07	1.18E+07	2.25E+07	**5.48E+06**	**3.57E+02**	6.80E+02	4.09E+02	6.41E+02	2.08E+01	**2.07E+01**	**2.07E+01**	2.08E+01
	19th	6.03E+03	5.52E+03	5.84E+03	**5.49E+03**	2.50E+07	2.01E+07	4.01E+07	**1.34E+07**	**4.96E+02**	8.02E+02	5.17E+02	7.23E+02	**2.08E+01**	**2.08E+01**	**2.08E+01**	**2.08E+01**
	25th	9.88E+03	**7.01E+03**	7.81E+03	7.47E+03	9.03E+07	1.16E+08	1.27E+08	**5.88E+07**	8.51E+02	1.30E+03	**7.72E+02**	1.08E+03	**2.09E+01**	**2.09E+01**	**2.09E+01**	**2.09E+01**
	Mean	5.32E+03	4.85E+03	5.15E+03	**4.47E+03**	2.27E+07	1.89E+07	3.12E+07	**9.91E+06**	4.17E+02	7.00E+02	**4.14E+02**	6.28E+02	**2.07E+01**	**2.07E+01**	**2.07E+01**	**2.07E+01**
	Std	1.43E+03	1.11E+03	1.07E+03	1.33E+03	2.07E+07	2.59E+07	3.26E+07	1.17E+07	1.93E+02	2.39E+02	1.66E+02	2.06E+02	1.37E-01	1.17E-01	1.34E-01	1.72E-01
1E+04	1st	2.51E-01	1.77E-03	2.90E-01	**3.56E-04**	6.55E+00	2.91E-01	4.85E+00	**2.74E-02**	5.56E-01	9.85E-01	**9.65E-02**	5.78E-01	2.04E+01	2.03E+01	2.03E+01	**2.02E+01**
	7th	1.47E+00	4.67E-03	6.87E-01	**1.23E-03**	1.10E+01	**3.85E+00**	1.60E+01	4.78E+00	**7.14E-01**	1.20E+00	7.72E-01	1.12E+00	**2.05E+01**	**2.05E+01**	**2.05E+01**	**2.05E+01**
	13th	2.18E+00	1.14E-02	1.11E+00	**6.40E-03**	1.74E+01	**5.02E+00**	2.35E+01	5.37E+00	**8.01E-01**	1.55E+00	8.36E-01	1.25E+00	2.06E+01	**2.05E+01**	2.06E+01	2.06E+01
	19th	3.75E+00	3.09E-02	2.49E+00	**1.57E-02**	5.36E+01	**6.37E+00**	9.13E+01	6.56E+00	**8.60E-01**	1.95E+00	8.95E-01	2.28E+00	**2.06E+01**	**2.06E+01**	**2.06E+01**	**2.06E+01**
	25th	1.28E+01	**1.76E-01**	1.37E+01	2.21E+00	2.11E+02	**2.29E+01**	2.34E+02	2.84E+02	**9.32E-01**	5.87E+00	1.11E+00	6.32E+00	2.07E+01	**2.06E+01**	2.08E+01	2.07E+01
	Mean	3.00E+00	**3.23E-02**	2.23E+00	1.05E-01	4.34E+01	**6.27E+00**	5.91E+01	2.97E+01	**7.87E-01**	2.10E+00	8.22E-01	1.73E+00	**2.05E+01**	**2.05E+01**	2.06E+01	**2.05E+01**
	Std	2.61E+00	4.52E-02	2.84E+00	4.31E-01	5.30E+01	4.91E+00	6.47E+01	7.33E+01	9.57E-02	1.43E+00	1.81E-01	1.20E+00	8.79E-02	8.20E-02	1.03E-01	1.07E-01
1E+05	1st	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	**7.40E-03**	9.86E-03	9.86E-03	1.23E-02	2.03E+01	**2.02E+01**	2.03E+01	**2.02E+01**
	7th	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	**5.16E-02**	5.17E-02	6.40E-02	7.39E-02	2.04E+01	**2.03E+01**	2.04E+01	**2.03E+01**
	13th	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	9.10E-02	**6.64E-02**	9.61E-02	1.40E-01	2.05E+01	**2.03E+01**	2.05E+01	2.04E+01
	19th	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	1.35E-01	3.84E-01	**1.30E-01**	5.03E-01	2.05E+01	**2.04E+01**	2.06E+01	**2.04E+01**
	25th	3.64E-12	**0.00E+00**	**0.00E+00**	**0.00E+00**	**0.00E+00**	3.99E+00	3.99E+00	3.99E+00	**4.61E-01**	5.92E-01	5.09E-01	6.56E-01	2.06E+01	**2.05E+01**	2.07E+01	**2.05E+01**
	Mean	2.18E-13	**0.00E+00**	**0.00E+00**	**0.00E+00**	**0.00E+00**	7.97E-01	7.97E-01	4.79E-01	**1.10E-01**	1.88E-01	1.27E-01	2.65E-01	2.05E+01	**2.03E+01**	2.05E+01	2.04E+01
	Std	7.84E-13	0.00E+00	0.00E+00	0.00E+00	0.00E+00	1.59E+00	1.59E+00	1.30E+00	9.41E-02	2.07E-01	1.06E-01	2.18E-01	8.38E-02	7.19E-02	1.20E-01	7.66E-02
5E+05	1st	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	**7.40E-03**	9.86E-03	9.86E-03	1.23E-02	2.03E+01	**2.02E+01**	2.03E+01	**2.02E+01**
	7th	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	5.16E-02	**4.19E-02**	6.40E-02	5.66E-02	2.04E+01	**2.02E+01**	2.04E+01	**2.02E+01**
	13th	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	9.10E-02	**5.41E-02**	9.61E-02	1.03E-01	2.05E+01	**2.03E+01**	2.05E+01	**2.03E+01**
	19th	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	1.35E-01	**7.38E-02**	1.30E-01	1.50E-01	2.05E+01	**2.03E+01**	2.06E+01	**2.03E+01**
	25th	0.00E+00	0.00E+00	0.00E+00	0.00E+00	**0.00E+00**	3.99E+00	3.99E+00	3.99E+00	4.61E-01	**1.45E-01**	5.09E-01	5.26E-01	2.06E+01	**2.04E+01**	2.07E+01	**2.04E+01**
	Mean	0.00E+00	0.00E+00	0.00E+00	0.00E+00	**0.00E+00**	7.97E-01	7.97E-01	4.79E-01	1.10E-01	**6.04E-02**	1.27E-01	1.47E-01	2.05E+01	**2.03E+01**	2.05E+01	**2.03E+01**
	Std	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	1.59E+00	1.59E+00	1.30E+00	9.41E-02	3.43E-02	1.06E-01	1.34E-01	8.59E-02	5.49E-02	1.20E-01	5.23E-02

Table 11. Error values achieved for f_9-f_{12} (10D).

10D		f_9				f_{10}				f_{11}				f_{12}			
		DE	VDE-1	VDE-2	VDE-3	DE	VDE-1	VDE-2	VDE-3	DE	VDE-1	VDE-2	VDE-3	DE	VDE-1	VDE-2	VDE-3
1E+03	1st	1.90E+01	1.80E+01	**1.79E+01**	2.13E+01	**7.13E+01**	8.52E+01	7.75E+01	7.92E+01	9.47E+00	9.70E+00	**8.82E+00**	9.49E+00	1.82E+04	**1.70E+04**	2.85E+04	3.17E+04
	7th	2.95E+01	2.61E+01	2.60E+01	**2.52E+01**	9.55E+01	9.37E+01	**9.19E+01**	1.01E+02	1.16E+01	1.14E+01	**1.10E+01**	**1.10E+01**	6.53E+04	5.08E+04	5.92E+04	**4.94E+04**
	13th	3.13E+01	**2.98E+01**	3.12E+01	3.03E+01	**1.04E+02**	1.05E+02	1.05E+02	1.07E+02	1.21E+01	**1.18E+01**	1.20E+01	1.20E+01	7.39E+04	6.76E+04	**6.49E+04**	6.62E+04
	19th	3.38E+01	3.36E+01	3.40E+01	**3.26E+01**	**1.13E+02**	**1.13E+02**	1.19E+02	1.17E+02	1.24E+01	**1.23E+01**	**1.23E+01**	**1.23E+01**	9.14E+04	8.27E+04	8.46E+04	**7.35E+04**
	25th	3.97E+01	**3.91E+01**	4.39E+01	4.20E+01	1.42E+02	1.30E+02	1.34E+02	**1.28E+02**	1.33E+01	1.38E+01	**1.29E+01**	1.30E+01	1.17E+05	1.07E+05	1.16E+05	**8.76E+04**
	Mean	3.15E+01	**2.97E+01**	3.06E+01	3.04E+01	**1.05E+02**	**1.05E+02**	**1.05E+02**	1.08E+02	1.19E+01	1.18E+01	**1.16E+01**	1.17E+01	7.20E+04	6.57E+04	6.91E+04	**6.28E+04**
	Std	4.01E+00	5.49E+00	6.50E+00	5.33E+00	1.43E+01	1.26E+01	1.52E+01	1.27E+01	8.89E-01	8.20E-01	1.04E+00	9.42E-01	2.63E+04	2.30E+04	1.90E+04	1.53E+04
1E+04	1st	3.47E-10	**0.00E+00**	6.67E-11	**0.00E+00**	4.36E+01	4.04E+01	4.44E+01	**2.89E+01**	8.88E+00	8.87E+00	8.82E+00	**8.60E+00**	7.92E+03	4.68E+03	8.01E+03	**1.14E+03**
	7th	2.26E-09	**3.33E-11**	6.44E-10	2.10E-10	5.54E+01	4.87E+01	5.29E+01	**4.43E+01**	9.74E+00	9.53E+00	9.98E+00	**9.49E+00**	1.30E+04	7.64E+03	1.11E+04	**4.63E+03**
	13th	4.18E-09	1.62E-09	1.77E-09	**4.30E-10**	5.99E+01	5.50E+01	5.60E+01	**4.91E+01**	1.00E+01	1.02E+01	1.03E+01	**9.99E+00**	1.59E+04	9.12E+03	1.50E+04	**8.32E+03**
	19th	1.21E-08	3.58E-08	3.18E-09	**1.65E-09**	6.30E+01	5.71E+01	6.06E+01	**5.42E+01**	**1.05E+01**	1.07E+01	1.07E+01	1.08E+01	1.83E+04	1.31E+04	1.76E+04	**1.02E+04**
	25th	9.95E-01	2.33E-07	**7.78E-08**	9.95E-01	6.75E+01	**6.32E+01**	7.01E+01	6.71E+01	1.14E+01	1.15E+01	**1.11E+01**	1.14E+01	2.09E+04	2.33E+04	2.06E+04	**1.85E+04**
	Mean	1.19E-01	2.90E-08	**8.78E-09**	3.98E-02	5.87E+01	5.29E+01	5.62E+01	**4.97E+01**	1.01E+01	1.01E+01	1.02E+01	**1.00E+01**	1.51E+04	1.08E+04	1.43E+04	**8.23E+03**
	Std	3.23E-01	5.27E-08	1.96E-08	1.95E-01	6.21E+00	6.15E+00	6.56E+00	7.63E+00	5.86E-01	7.80E-01	6.25E-01	7.56E-01	3.90E+03	4.30E+03	3.71E+03	4.58E+03
1E+05	1st	0.00E+00	0.00E+00	0.00E+00	0.00E+00	2.36E+01	**2.00E+01**	2.92E+01	2.26E+01	3.00E-01	**6.02E-04**	2.54E-01	1.07E-03	1.00E+02	8.61E-06	1.88E+02	**9.15E-12**
	7th	0.00E+00	0.00E+00	0.00E+00	0.00E+00	3.47E+01	**2.73E+01**	3.53E+01	2.91E+01	3.37E+00	**9.93E-02**	6.37E-01	1.50E+00	3.88E+02	6.43E-05	7.08E+02	**3.22E-10**
	13th	0.00E+00	0.00E+00	0.00E+00	0.00E+00	3.86E+01	3.21E+01	3.78E+01	**3.13E+01**	6.42E+00	3.95E+00	**3.23E+00**	7.98E+00	7.34E+02	2.41E-04	1.38E+03	**1.90E-08**
	19th	0.00E+00	0.00E+00	0.00E+00	0.00E+00	4.05E+01	3.54E+01	4.12E+01	**3.30E+01**	8.57E+00	8.73E+00	**8.50E+00**	8.85E+00	9.60E+02	4.15E-03	2.03E+03	**3.27E-07**
	25th	9.95E-01	**0.00E+00**	**0.00E+00**	9.95E-01	4.38E+01	3.91E+01	4.64E+01	**3.79E+01**	**9.26E+00**	9.77E+00	9.47E+00	9.53E+00	2.37E+03	1.06E+01	3.97E+03	**1.02E+01**
	Mean	1.19E-01	**0.00E+00**	**0.00E+00**	3.98E-02	3.74E+01	3.13E+01	3.80E+01	**3.06E+01**	5.93E+00	**4.00E+00**	4.20E+00	5.32E+00	7.73E+02	8.30E-01	1.61E+03	**8.15E-01**
	Std	3.23E-01	0.00E+00	0.00E+00	1.95E-01	4.78E+00	4.83E+00	4.33E+00	3.59E+00	2.72E+00	3.79E+00	3.69E+00	3.97E+00	5.25E+02	2.79E+00	1.12E+03	2.74E+00
5E+05	1st	0.00E+00	0.00E+00	0.00E+00	0.00E+00	**1.99E+00**	2.98E+00	**1.99E+00**	3.98E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	1.48E-11	**0.00E+00**	**0.00E+00**	**0.00E+00**
	7th	0.00E+00	0.00E+00	0.00E+00	0.00E+00	**4.97E+00**	**4.97E+00**	5.07E+00	8.95E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	3.27E-10	**0.00E+00**	6.67E-11	**0.00E+00**
	13th	0.00E+00	0.00E+00	0.00E+00	0.00E+00	**6.96E+00**	9.95E+00	8.95E+00	1.99E+01	**0.00E+00**	**0.00E+00**	1.74E-10	9.84E-01	3.11E-09	**0.00E+00**	1.53E-10	**0.00E+00**
	19th	0.00E+00	0.00E+00	0.00E+00	0.00E+00	**8.95E+00**	2.26E+01	2.35E+01	2.38E+01	1.00E+00	**0.00E+00**	9.78E-01	1.58E+00	4.03E-08	**0.00E+00**	9.06E-10	**0.00E+00**
	25th	9.95E-01	**0.00E+00**	**0.00E+00**	9.95E-01	2.68E+01	**2.59E+01**	3.19E+01	2.84E+01	6.42E+00	4.06E+00	4.78E+00	**3.52E+00**	**5.96E-07**	1.00E+01	9.46E-01	1.00E+01
	Mean	1.19E-01	**0.00E+00**	**0.00E+00**	3.98E-02	**8.75E+00**	1.36E+01	1.35E+01	1.66E+01	1.01E+00	**4.06E-01**	7.03E-01	9.51E-01	**6.17E-08**	8.00E-01	3.78E-02	8.00E-01
	Std	3.23E-01	0.00E+00	0.00E+00	1.95E-01	6.02E+00	8.74E+00	1.01E+01	8.07E+00	1.87E+00	9.97E-01	1.16E+00	1.00E+00	1.37E-07	2.71E+00	1.85E-01	2.71E+00

Table 12. Error values achieved for f_{13}-f_{16} (10D).

10D		f_{13} DE	f_{13} VDE-1	f_{13} VDE-2	f_{13} VDE-3	f_{14} DE	f_{14} VDE-1	f_{14} VDE-2	f_{14} VDE-3	f_{15} DE	f_{15} VDE-1	f_{15} VDE-2	f_{15} VDE-3	f_{16} DE	f_{16} VDE-1	f_{16} VDE-2	f_{16} VDE-3
1E+03	1st	8.07E+00	8.44E+00	8.52E+00	**7.06E+00**	4.12E+00	**3.87E+00**	4.07E+00	4.09E+00	**5.79E+02**	6.30E+02	6.00E+02	6.74E+02	3.17E+02	3.13E+02	3.19E+02	**2.95E+02**
	7th	1.02E+01	**9.73E+00**	1.03E+01	1.02E+01	4.27E+00	**4.19E+00**	4.30E+00	4.28E+00	7.20E+02	6.99E+02	**6.85E+02**	7.56E+02	3.71E+02	3.72E+02	**3.61E+02**	3.94E+02
	13th	1.15E+01	1.19E+01	1.32E+01	**1.14E+01**	4.31E+00	**4.29E+00**	4.35E+00	4.34E+00	7.42E+02	7.41E+02	**7.31E+02**	7.90E+02	3.93E+02	**3.87E+02**	**3.87E+02**	4.21E+02
	19th	1.32E+01	1.32E+01	1.49E+01	**1.22E+01**	**4.37E+00**	4.38E+00	4.43E+00	4.42E+00	7.84E+02	7.63E+02	**7.61E+02**	8.14E+02	4.13E+02	**4.06E+02**	4.22E+02	4.49E+02
	25th	**1.45E+01**	1.86E+01	1.83E+01	1.78E+01	**4.50E+00**	4.51E+00	4.54E+00	4.52E+00	8.41E+02	**8.16E+02**	8.96E+02	8.87E+02	4.80E+02	**4.69E+02**	5.06E+02	5.02E+02
	Mean	1.16E+01	1.18E+01	1.31E+01	**1.12E+01**	4.32E+00	**4.29E+00**	4.34E+00	4.34E+00	7.38E+02	7.32E+02	**7.23E+02**	7.82E+02	3.95E+02	**3.87E+02**	3.95E+02	4.20E+02
	Std	1.68E+00	2.63E+00	2.79E+00	2.24E+00	8.92E-02	1.53E-01	1.12E-01	1.10E-01	6.30E+01	4.80E+01	6.27E+01	5.14E+01	3.87E+01	3.78E+01	4.99E+01	4.05E+01
1E+04	1st	**2.43E+00**	2.74E+00	3.87E+00	2.65E+00	3.71E+00	3.47E+00	**3.29E+00**	3.42E+00	3.99E+02	4.22E+02	**3.85E+02**	4.50E+02	2.08E+02	**1.63E+02**	1.73E+02	2.01E+02
	7th	4.12E+00	3.68E+00	4.46E+00	**3.37E+00**	3.91E+00	**3.77E+00**	3.89E+00	3.96E+00	**4.63E+02**	5.39E+02	4.80E+02	5.06E+02	2.27E+02	**1.93E+02**	2.14E+02	2.21E+02
	13th	4.94E+00	3.96E+00	5.04E+00	**3.72E+00**	**4.01E+00**	4.05E+00	**4.01E+00**	**4.01E+00**	5.45E+02	5.51E+02	5.50E+02	**5.29E+02**	2.36E+02	**2.05E+02**	2.31E+02	2.32E+02
	19th	5.51E+00	4.75E+00	5.53E+00	**4.12E+00**	4.15E+00	**4.09E+00**	4.11E+00	4.12E+00	5.79E+02	5.71E+02	5.81E+02	**5.66E+02**	2.49E+02	**2.18E+02**	2.41E+02	2.45E+02
	25th	6.06E+00	**5.47E+00**	6.20E+00	5.53E+00	4.22E+00	4.23E+00	**4.18E+00**	4.21E+00	6.23E+02	**5.87E+02**	6.10E+02	6.22E+02	2.67E+02	**2.34E+02**	2.58E+02	2.69E+02
	Mean	4.73E+00	4.11E+00	5.01E+00	**3.79E+00**	4.01E+00	**3.94E+00**	3.97E+00	3.99E+00	**5.23E+02**	5.39E+02	5.29E+02	5.33E+02	2.37E+02	**2.03E+02**	2.27E+02	2.33E+02
	Std	9.70E-01	6.89E-01	6.40E-01	6.61E-01	1.40E-01	2.04E-01	1.88E-01	1.65E-01	6.59E+01	4.57E+01	5.89E+01	4.57E+01	1.52E+01	1.89E+01	2.15E+01	1.59E+01
1E+05	1st	**3.22E-01**	4.75E-01	4.29E-01	5.55E-01	2.15E+00	2.32E+00	**1.55E+00**	2.66E+00	1.57E+02	1.11E+02	**1.05E+02**	1.36E+02	1.66E+02	1.49E+02	1.37E+02	**1.23E+02**
	7th	**7.41E-01**	1.22E+00	1.34E+00	1.68E+00	3.30E+00	3.46E+00	**3.25E+00**	3.27E+00	1.96E+02	2.07E+02	**1.53E+02**	1.99E+02	1.66E+02	**1.59E+02**	1.69E+02	1.60E+02
	13th	**1.62E+00**	2.28E+00	2.26E+00	2.09E+00	3.59E+00	3.62E+00	3.63E+00	**3.44E+00**	2.24E+02	4.36E+02	**1.93E+02**	2.31E+02	1.73E+02	**1.65E+02**	1.76E+02	1.69E+02
	19th	2.70E+00	2.61E+00	2.87E+00	**2.34E+00**	3.78E+00	3.70E+00	3.70E+00	**3.55E+00**	4.61E+02	4.69E+02	4.57E+02	**2.56E+02**	1.84E+02	**1.72E+02**	1.83E+02	**1.72E+02**
	25th	3.63E+00	3.12E+00	3.33E+00	**2.73E+00**	3.93E+00	3.83E+00	3.92E+00	**3.69E+00**	5.26E+02	4.97E+02	5.15E+02	**4.63E+02**	1.95E+02	1.87E+02	1.94E+02	**1.78E+02**
	Mean	**1.71E+00**	1.98E+00	2.09E+00	2.00E+00	3.47E+00	3.52E+00	3.42E+00	**3.35E+00**	2.90E+02	3.39E+02	2.59E+02	**2.50E+02**	1.74E+02	**1.65E+02**	1.74E+02	**1.65E+02**
	Std	1.02E+00	8.34E-01	9.03E-01	4.63E-01	4.23E-01	3.01E-01	5.02E-01	2.89E-01	1.33E+02	1.42E+02	1.50E+02	8.56E+01	1.34E+01	1.00E+01	1.35E+01	1.16E+01
5E+05	1st	3.13E-01	**9.87E-03**	4.10E-01	2.76E-01	6.69E-01	**1.52E-01**	2.07E-01	1.19E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	9.16E+01	**5.09E+01**	9.09E+01	1.21E+02
	7th	5.36E-01	**4.11E-01**	5.51E-01	5.90E-01	2.70E+00	**1.22E+00**	3.18E+00	1.75E+00	4.59E-05	4.04E+01	**0.00E+00**	2.33E-09	**9.69E+01**	9.80E+01	9.70E+01	1.36E+02
	13th	7.41E-01	**4.90E-01**	8.67E-01	1.31E+00	3.17E+00	**2.12E+00**	3.27E+00	2.17E+00	3.00E-03	4.00E+02	**0.00E+00**	2.85E-03	**1.02E+02**	1.05E+02	**1.02E+02**	1.46E+02
	19th	9.55E-01	**6.45E-01**	1.60E+00	1.47E+00	3.52E+00	2.69E+00	3.42E+00	**2.62E+00**	4.04E+02	4.00E+02	4.00E+02	**8.05E+01**	**1.05E+02**	1.10E+02	1.35E+02	1.48E+02
	25th	**1.62E+00**	2.13E+00	2.41E+00	2.09E+00	3.93E+00	**3.19E+00**	3.92E+00	3.29E+00	4.71E+02	4.07E+02	**4.01E+02**	4.14E+02	1.51E+02	**1.46E+02**	1.63E+02	1.64E+02
	Mean	7.51E-01	**6.05E-01**	1.13E+00	1.11E+00	2.86E+00	**2.01E+00**	3.08E+00	2.21E+00	1.25E+02	2.44E+02	1.12E+02	**6.34E+01**	**1.06E+02**	1.09E+02	1.15E+02	1.43E+02
	Std	2.86E-01	4.26E-01	6.36E-01	5.04E-01	9.41E-01	8.71E-01	8.48E-01	5.92E-01	1.96E+02	1.82E+02	1.80E+02	1.16E+02	1.49E+01	2.21E+01	2.37E+01	1.06E+01

Table 13. Error values achieved for f_{17}-f_{20} (10D).

10D		f_{17}				f_{18}				f_{19}				f_{20}			
		DE	VDE-1	VDE-2	VDE-3	DE	VDE-1	VDE-2	VDE-3	DE	VDE-1	VDE-2	VDE-3	DE	VDE-1	VDE-2	VDE-3
1E+03	1st	**2.95E+02**	3.33E+02	3.22E+02	3.56E+02	**1.06E+03**	**1.06E+03**	**1.06E+03**	**1.06E+03**	1.09E+03	**1.07E+03**	1.11E+03	1.10E+03	1.11E+03	1.09E+03	1.09E+03	**1.07E+03**
	7th	**3.62E+02**	4.05E+02	3.63E+02	4.13E+02	1.16E+03	**1.14E+03**	1.15E+03	1.16E+03	**1.13E+03**	**1.13E+03**	1.15E+03	1.16E+03	**1.13E+03**	1.14E+03	**1.13E+03**	1.17E+03
	13th	**3.79E+02**	4.21E+02	3.87E+02	4.41E+02	1.17E+03	**1.16E+03**	1.17E+03	1.19E+03	**1.15E+03**	**1.15E+03**	1.16E+03	1.19E+03	**1.16E+03**	1.17E+03	**1.16E+03**	1.18E+03
	19th	4.17E+02	4.46E+02	**4.05E+02**	4.83E+02	1.19E+03	**1.18E+03**	1.20E+03	1.22E+03	1.19E+03	**1.16E+03**	1.18E+03	1.20E+03	1.20E+03	1.19E+03	**1.18E+03**	1.20E+03
	25th	4.95E+02	5.37E+02	**4.87E+02**	5.27E+02	**1.22E+03**	1.24E+03	1.28E+03	1.27E+03	1.22E+03	**1.19E+03**	1.21E+03	1.30E+03	1.26E+03	**1.21E+03**	1.22E+03	1.27E+03
	Mean	**3.88E+02**	4.25E+02	3.89E+02	4.45E+02	1.17E+03	**1.16E+03**	1.17E+03	1.19E+03	1.16E+03	**1.14E+03**	1.17E+03	1.18E+03	1.17E+03	**1.16E+03**	**1.16E+03**	1.18E+03
	Std	4.64E+01	3.94E+01	3.78E+01	4.32E+01	3.30E+01	3.73E+01	4.81E+01	4.90E+01	3.60E+01	3.05E+01	2.52E+01	4.32E+01	4.12E+01	3.32E+01	3.00E+01	3.98E+01
1E+04	1st	2.01E+02	2.14E+02	**1.97E+02**	2.03E+02	6.27E+02	**4.70E+02**	6.72E+02	6.50E+02	6.38E+02	**5.01E+02**	6.40E+02	6.43E+02	6.46E+02	**4.70E+02**	6.31E+02	6.49E+02
	7th	**2.29E+02**	2.39E+02	2.47E+02	2.32E+02	6.97E+02	**5.85E+02**	7.40E+02	8.31E+02	7.25E+02	**5.93E+02**	7.35E+02	8.42E+02	7.30E+02	**5.35E+02**	7.79E+02	9.15E+02
	13th	2.52E+02	**2.48E+02**	2.61E+02	2.51E+02	7.67E+02	**6.43E+02**	8.75E+02	9.18E+02	8.22E+02	**6.69E+02**	7.95E+02	8.94E+02	7.77E+02	**6.27E+02**	8.02E+02	9.71E+02
	19th	**2.58E+02**	2.62E+02	2.66E+02	2.62E+02	**8.29E+02**	8.36E+02	9.36E+02	9.64E+02	8.93E+02	**8.51E+02**	9.12E+02	9.41E+02	8.43E+02	**7.15E+02**	9.06E+02	9.94E+02
	25th	**2.76E+02**	2.85E+02	2.82E+02	2.83E+02	**9.90E+02**	1.00E+03	1.01E+03	1.01E+03	9.97E+02	**9.82E+02**	9.98E+02	1.01E+03	1.01E+03	**8.97E+02**	1.01E+03	1.05E+03
	Mean	**2.45E+02**	2.50E+02	2.56E+02	2.47E+02	7.74E+02	**6.91E+02**	8.44E+02	8.89E+02	8.20E+02	**7.00E+02**	8.10E+02	8.82E+02	7.95E+02	**6.36E+02**	8.37E+02	9.43E+02
	Std	2.02E+01	1.86E+01	1.93E+01	2.29E+01	9.93E+01	1.48E+02	1.07E+02	9.57E+01	1.09E+02	1.52E+02	1.07E+02	9.19E+01	9.53E+01	1.12E+02	1.01E+02	8.64E+01
1E+05	1st	1.29E+02	1.74E+02	**1.24E+02**	1.66E+02	3.00E+02	3.00E+02	3.00E+02	3.00E+02	3.00E+02	3.00E+02	3.00E+02	3.00E+02	3.00E+02	3.00E+02	3.00E+02	3.00E+02
	7th	**1.64E+02**	1.84E+02	1.80E+02	1.77E+02	3.00E+02	3.00E+02	3.00E+02	3.00E+02	3.00E+02	3.00E+02	3.00E+02	3.00E+02	3.00E+02	3.00E+02	3.00E+02	3.00E+02
	13th	**1.83E+02**	1.92E+02	1.89E+02	1.86E+02	3.00E+02	3.00E+02	3.00E+02	3.00E+02	3.00E+02	3.00E+02	3.00E+02	3.00E+02	3.00E+02	3.00E+02	3.00E+02	3.00E+02
	19th	2.01E+02	1.97E+02	1.97E+02	**1.94E+02**	3.00E+02	3.00E+02	3.00E+02	3.00E+02	3.00E+02	3.00E+02	3.00E+02	3.00E+02	3.00E+02	3.00E+02	3.00E+02	3.00E+02
	25th	2.18E+02	2.12E+02	**2.07E+02**	**2.07E+02**	**3.00E+02**	8.00E+02	8.00E+02	8.00E+02	8.00E+02	8.00E+02	**3.00E+02**	**3.00E+02**	3.00E+02	3.00E+02	3.00E+02	3.00E+02
	Mean	**1.80E+02**	1.92E+02	1.83E+02	1.86E+02	**3.00E+02**	3.40E+02	3.20E+02	3.20E+02	3.20E+02	3.40E+02	**3.00E+02**	**3.00E+02**	3.00E+02	3.00E+02	3.00E+02	3.00E+02
	Std	2.43E+01	1.07E+01	2.16E+01	1.25E+01	5.09E-02	1.36E+02	9.80E+01	9.80E+01	9.80E+01	1.36E+02	4.14E-02	6.37E-03	6.27E-02	7.34E-09	2.59E-02	6.33E-03
5E+05	1st	**9.11E+01**	1.03E+02	9.28E+01	9.52E+01	3.00E+02	3.00E+02	3.00E+02	3.00E+02	3.00E+02	3.00E+02	3.00E+02	3.00E+02	3.00E+02	3.00E+02	3.00E+02	3.00E+02
	7th	**1.01E+02**	1.53E+02	**1.01E+02**	1.47E+02	3.00E+02	3.00E+02	3.00E+02	3.00E+02	3.00E+02	3.00E+02	3.00E+02	3.00E+02	3.00E+02	3.00E+02	3.00E+02	3.00E+02
	13th	**1.06E+02**	1.59E+02	1.11E+02	1.56E+02	3.00E+02	3.00E+02	3.00E+02	3.00E+02	3.00E+02	3.00E+02	3.00E+02	3.00E+02	3.00E+02	3.00E+02	3.00E+02	3.00E+02
	19th	**1.15E+02**	1.66E+02	1.19E+02	1.66E+02	3.00E+02	3.00E+02	3.00E+02	3.00E+02	3.00E+02	3.00E+02	3.00E+02	3.00E+02	3.00E+02	3.00E+02	3.00E+02	3.00E+02
	25th	**1.27E+02**	1.76E+02	1.75E+02	1.84E+02	**3.00E+02**	8.00E+02	8.00E+02	8.00E+02	8.00E+02	8.00E+02	**3.00E+02**	**3.00E+02**	3.00E+02	3.00E+02	3.00E+02	3.00E+02
	Mean	**1.07E+02**	1.56E+02	1.14E+02	1.52E+02	**3.00E+02**	3.40E+02	3.20E+02	3.20E+02	3.20E+02	3.40E+02	**3.00E+02**	**3.00E+02**	3.00E+02	3.00E+02	3.00E+02	3.00E+02
	Std	1.01E+01	1.76E+01	1.91E+01	2.18E+01	0.00E+00	1.36E+02	9.80E+01	9.80E+01	9.80E+01	1.36E+02	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00

Table 14. Error values achieved for f_{21}-f_{25} (10D).

10D		f_{21}				f_{22}				f_{23}				f_{24}				f_{25}			
		DE	VDE-1	VDE-2	VDE-3	DE	VDE-1	VDE-2	VDE-3	DE	VDE-1	VDE-2	VDE-3	DE	VDE-1	VDE-2	VDE-3	DE	VDE-1	VDE-2	VDE-3
1E+03	1st	1.23E+03	1.28E+03	1.22E+03	**1.17E+03**	9.21E+02	**8.82E+02**	9.36E+02	9.73E+02	**1.19E+03**	1.31E+03	1.22E+03	1.31E+03	1.10E+03	**1.01E+03**	1.15E+03	1.21E+03	6.92E+02	**5.93E+02**	6.17E+02	1.37E+03
	7th	**1.34E+03**	**1.34E+03**	**1.34E+03**	1.37E+03	**1.01E+03**	1.04E+03	1.02E+03	1.07E+03	**1.34E+03**	1.35E+03	1.35E+03	1.36E+03	**1.19E+03**	1.23E+03	1.26E+03	1.29E+03	**9.06E+02**	1.07E+03	9.15E+02	1.55E+03
	13th	1.37E+03	**1.36E+03**	**1.36E+03**	1.40E+03	**1.05E+03**	1.06E+03	1.07E+03	1.12E+03	**1.36E+03**	**1.36E+03**	1.37E+03	1.39E+03	**1.26E+03**	1.27E+03	1.27E+03	1.32E+03	**1.01E+03**	1.27E+03	1.08E+03	1.60E+03
	19th	1.39E+03	**1.38E+03**	**1.38E+03**	1.42E+03	1.11E+03	**1.09E+03**	1.10E+03	1.15E+03	1.39E+03	**1.38E+03**	1.39E+03	1.40E+03	1.31E+03	**1.30E+03**	1.32E+03	1.34E+03	**1.15E+03**	1.42E+03	1.25E+03	1.64E+03
	25th	1.43E+03	**1.41E+03**	**1.41E+03**	1.46E+03	1.18E+03	1.17E+03	**1.15E+03**	1.21E+03	**1.40E+03**	**1.40E+03**	1.41E+03	1.45E+03	1.38E+03	**1.34E+03**	1.40E+03	1.38E+03	**1.45E+03**	1.53E+03	1.49E+03	1.73E+03
	Mean	1.37E+03	1.36E+03	**1.35E+03**	1.39E+03	**1.06E+03**	**1.06E+03**	**1.06E+03**	1.11E+03	**1.35E+03**	1.36E+03	1.36E+03	1.38E+03	1.25E+03	**1.24E+03**	1.28E+03	1.31E+03	**1.02E+03**	1.18E+03	1.06E+03	1.59E+03
	Std	4.19E+01	3.06E+01	5.26E+01	5.28E+01	6.54E+01	5.52E+01	5.32E+01	5.98E+01	4.59E+01	2.51E+01	3.90E+01	3.45E+01	7.78E+01	8.09E+01	5.76E+01	4.31E+01	2.03E+02	2.78E+02	2.33E+02	8.58E+01
1E+04	1st	7.07E+02	**5.19E+02**	7.00E+02	7.28E+02	7.99E+02	**7.89E+02**	8.03E+02	8.16E+02	7.53E+02	**5.95E+02**	7.17E+02	7.33E+02	3.06E+02	**2.10E+02**	2.44E+02	3.09E+02	4.24E+02	**4.08E+02**	4.36E+02	4.24E+02
	7th	8.10E+02	**5.74E+02**	8.18E+02	9.59E+02	8.25E+02	**7.99E+02**	8.29E+02	8.37E+02	8.37E+02	**6.44E+02**	8.43E+02	8.47E+02	3.75E+02	**2.42E+02**	3.21E+02	3.59E+02	4.61E+02	**4.15E+02**	4.52E+02	4.49E+02
	13th	8.48E+02	**6.09E+02**	8.68E+02	9.91E+02	8.29E+02	**8.06E+02**	8.34E+02	8.48E+02	8.74E+02	**6.82E+02**	9.03E+02	9.42E+02	4.03E+02	**2.56E+02**	3.55E+02	4.17E+02	4.71E+02	**4.27E+02**	4.64E+02	4.54E+02
	19th	8.76E+02	**6.69E+02**	9.24E+02	1.05E+03	8.35E+02	**8.16E+02**	8.42E+02	8.56E+02	9.39E+02	**7.42E+02**	9.73E+02	1.02E+03	4.44E+02	**2.84E+02**	4.04E+02	5.66E+02	4.77E+02	**4.38E+02**	4.86E+02	4.68E+02
	25th	1.01E+03	**7.93E+02**	1.07E+03	1.13E+03	**8.50E+02**	8.54E+02	8.53E+02	8.85E+02	1.04E+03	**9.41E+02**	1.03E+03	1.18E+03	4.71E+02	**4.29E+02**	4.93E+02	7.12E+02	4.98E+02	**4.60E+02**	5.05E+02	5.25E+02
	Mean	8.43E+02	**6.28E+02**	8.71E+02	9.93E+02	8.29E+02	**8.09E+02**	8.34E+02	8.49E+02	8.83E+02	**7.03E+02**	9.02E+02	9.34E+02	4.04E+02	**2.74E+02**	3.64E+02	4.74E+02	4.69E+02	**4.28E+02**	4.69E+02	4.58E+02
	Std	6.68E+01	7.35E+01	9.50E+01	9.16E+01	1.06E+01	1.38E+01	1.20E+01	1.70E+01	6.80E+01	8.04E+01	8.27E+01	1.26E+02	4.56E+01	5.63E+01	6.36E+01	1.27E+02	1.57E+01	1.42E+01	1.87E+01	2.02E+01
1E+05	1st	5.00E+02	5.00E+02	5.00E+02	5.00E+02	**3.00E+02**	7.63E+02	3.03E+02	7.64E+02	5.59E+02	5.59E+02	5.59E+02	5.59E+02	2.00E+02	2.00E+02	2.00E+02	2.00E+02	3.97E+02	3.94E+02	**3.92E+02**	3.94E+02
	7th	5.00E+02	5.00E+02	5.00E+02	5.00E+02	**7.71E+02**	7.72E+02	7.75E+02	**7.71E+02**	5.59E+02	5.59E+02	5.59E+02	5.59E+02	2.00E+02	2.00E+02	2.00E+02	2.00E+02	4.02E+02	**4.00E+02**	4.03E+02	4.03E+02
	13th	5.00E+02	5.00E+02	5.00E+02	5.00E+02	7.79E+02	**7.73E+02**	7.78E+02	7.74E+02	5.59E+02	5.59E+02	5.59E+02	5.59E+02	2.00E+02	2.00E+02	2.00E+02	2.00E+02	4.05E+02	**4.04E+02**	4.06E+02	4.06E+02
	19th	5.00E+02	5.00E+02	5.00E+02	5.00E+02	7.81E+02	**7.75E+02**	7.82E+02	7.76E+02	5.59E+02	5.59E+02	5.59E+02	5.59E+02	2.00E+02	2.00E+02	2.00E+02	2.00E+02	4.08E+02	**4.06E+02**	4.07E+02	4.07E+02
	25th	5.00E+02	5.00E+02	5.00E+02	5.00E+02	7.85E+02	**7.80E+02**	7.85E+02	7.82E+02	5.59E+02	5.59E+02	5.59E+02	5.59E+02	2.00E+02	2.00E+02	2.00E+02	2.00E+02	4.11E+02	**4.09E+02**	4.11E+02	4.10E+02
	Mean	5.00E+02	5.00E+02	5.00E+02	5.00E+02	**6.84E+02**	7.73E+02	7.60E+02	7.73E+02	5.59E+02	5.59E+02	5.59E+02	5.59E+02	2.00E+02	2.00E+02	2.00E+02	2.00E+02	4.05E+02	**4.03E+02**	4.05E+02	4.05E+02
	Std	2.60E-05	3.27E-13	7.80E-06	1.62E-05	1.90E+02	4.03E+00	9.32E+01	3.63E+00	3.41E-13	6.11E-13	3.41E-13	3.41E-13	1.26E-05	0.00E+00	3.60E-07	2.17E-06	4.16E+00	4.00E+00	4.58E+00	4.08E+00
5E+05	1st	5.00E+02	5.00E+02	5.00E+02	5.00E+02	**3.00E+02**	7.15E+02	**3.00E+02**	7.45E+02	5.59E+02	5.59E+02	5.59E+02	5.59E+02	2.00E+02	2.00E+02	2.00E+02	2.00E+02	3.94E+02	3.93E+02	**3.91E+02**	3.93E+02
	7th	5.00E+02	5.00E+02	5.00E+02	5.00E+02	**7.25E+02**	7.58E+02	7.66E+02	7.63E+02	5.59E+02	5.59E+02	5.59E+02	5.59E+02	2.00E+02	2.00E+02	2.00E+02	2.00E+02	4.00E+02	**3.99E+02**	4.01E+02	4.02E+02
	13th	5.00E+02	5.00E+02	5.00E+02	5.00E+02	7.68E+02	**7.64E+02**	7.71E+02	7.65E+02	5.59E+02	5.59E+02	5.59E+02	5.59E+02	2.00E+02	2.00E+02	2.00E+02	2.00E+02	**4.03E+02**	**4.03E+02**	**4.03E+02**	4.05E+02
	19th	5.00E+02	5.00E+02	5.00E+02	5.00E+02	7.73E+02	7.68E+02	7.73E+02	**7.66E+02**	5.59E+02	5.59E+02	5.59E+02	5.59E+02	2.00E+02	2.00E+02	2.00E+02	2.00E+02	4.06E+02	**4.05E+02**	**4.05E+02**	4.06E+02
	25th	5.00E+02	5.00E+02	5.00E+02	5.00E+02	7.76E+02	7.70E+02	7.77E+02	**7.69E+02**	5.59E+02	5.59E+02	5.59E+02	5.59E+02	2.00E+02	2.00E+02	2.00E+02	2.00E+02	4.09E+02	**4.08E+02**	4.09E+02	**4.08E+02**
	Mean	5.00E+02	5.00E+02	5.00E+02	5.00E+02	**6.54E+02**	7.59E+02	7.50E+02	7.63E+02	5.59E+02	5.59E+02	5.59E+02	5.59E+02	2.00E+02	2.00E+02	2.00E+02	2.00E+02	4.03E+02	**4.02E+02**	4.03E+02	4.04E+02
	Std	3.41E-13	3.27E-13	3.41E-13	3.41E-13	1.99E+02	1.35E+01	9.21E+01	5.67E+00	3.41E-13	6.11E-13	3.41E-13	3.41E-13	0.00E+00	0.00E+00	0.00E+00	0.00E+00	4.17E+00	4.08E+00	4.50E+00	4.00E+00

Table 15. FES to reach the specified accuracy and success rate and performance for f_1-f_{13} (10D).

Func	Strategy	1st	7th	13th	19th	25th	Mean	Std	Successrate		Success Performance	
									1.0e5	5.0e5	1.0e5	5.0e5
f_1	DE	6700	7057	7320	7391	**7611**	7.23E+03	2.52E+02	**100 %**	**100 %**	7225	7225
	VDE-1	5284	**5948**	6619	7216	9108	6.75E+03	9.47E+02	**100 %**	**100 %**	6753	6753
	VDE-2	6713	7227	7388	7577	8421	7.43E+03	3.54E+02	**100 %**	**100 %**	7425	7425
	VDE-3	**4863**	5988	**6379**	**6901**	7885	**6.42E+03**	7.43E+02	**100 %**	**100 %**	**6423**	**6423**
f_2	DE	19653	21367	22012	22817	24883	2.21E+04	1.25E+03	**100 %**	**100 %**	22077	22077
	VDE-1	**10374**	**12470**	**13158**	**14299**	**15156**	**1.31E+04**	1.41E+03	**100 %**	**100 %**	**13082**	**13082**
	VDE-2	17317	18804	20620	21844	28806	2.10E+04	2.84E+03	**100 %**	**100 %**	20973	20973
	VDE-3	11118	12513	14578	15732	18032	1.44E+04	2.00E+03	**100 %**	**100 %**	14407	14407
f_3	DE	198379	210434	214462	221219	228329	2.15E+05	7.80E+03	0 %	**100 %**	-	214981
	VDE-1	61742	66000	68280	70015	83189	6.90E+04	4.46E+03	**100 %**	**100 %**	69047	69047
	VDE-2	148189	154231	157669	162679	169465	1.58E+05	6.09E+03	0 %	**100 %**	-	158404
	VDE-3	**27771**	**34235**	**37874**	**41353**	**53307**	**3.87E+04**	6.35E+03	**100 %**	**100 %**	**38705**	**38705**
f_4	DE	22615	23851	24842	26284	28467	2.52E+04	1.59E+03	**100 %**	**100 %**	25150	25150
	VDE-1	11908	**13366**	**14252**	**14991**	**17233**	**1.43E+04**	1.30E+03	**100 %**	**100 %**	**14269**	**14269**
	VDE-2	19195	20814	23428	26538	29249	2.37E+04	3.12E+03	**100 %**	**100 %**	23718	23718
	VDE-3	**11634**	16449	19682	23142	35650	2.02E+04	5.24E+03	**100 %**	**100 %**	20181	20181
f_5	DE	24110	24713	25871	26576	29054	2.59E+04	1.38E+03	**100 %**	**100 %**	25941	25941
	VDE-1	14994	15858	16419	17028	**18591**	1.65E+04	9.95E+02	**100 %**	**100 %**	16521	16521
	VDE-2	22239	23328	24396	26084	30038	2.50E+04	2.06E+03	**100 %**	**100 %**	24966	24966
	VDE-3	**13265**	**14127**	**15799**	**16918**	20600	**1.58E+04**	1.86E+03	**100 %**	**100 %**	**15784**	**15784**
f_6	DE	20830	23359	25762	**27061**	**32193**	**2.57E+04**	2.97E+03	**100 %**	**100 %**	**25695**	**25695**
	VDE-1	**13214**	**21535**	**24948**	31530	-	1.19E+05	1.91E+05	80 %	80 %	148598	148598
	VDE-2	21596	23967	27646	34331	-	1.22E+05	1.89E+05	80 %	80 %	151964	151964
	VDE-3	19707	25724	37291	44755	-	9.05E+04	1.52E+05	88 %	88 %	102892	102892
f_7	DE	**17053**	-	-	-	-	**4.81E+05**	9.46E+04	**4 %**	**4 %**	**12017053**	**12017053**
	VDE-1	92495	-	-	-	-	4.84E+05	7.99E+04	**4 %**	**4 %**	12092495	12092495
	VDE-2	25086	-	-	-	-	4.81E+05	9.31E+04	**4 %**	**4 %**	12025086	12025086
	VDE-3	-	-	-	-	-	-	-	0 %	0 %	-	-
f_8	DE	-	-	-	-	-	-	-	0 %	0 %	-	-
	VDE-1	-	-	-	-	-	-	-	0 %	0 %	-	-
	VDE-2	-	-	-	-	-	-	-	0 %	0 %	-	-
	VDE-3	-	-	-	-	-	-	-	0 %	0 %	-	-
f_9	DE	4864	5059	5273	5722	-	6.46E+04	1.61E+05	88 %	88 %	73431	73431
	VDE-1	**4286**	5202	5488	5781	**6193**	5.47E+03	4.67E+02	**100 %**	**100 %**	5471	5471
	VDE-2	4358	**4761**	**5023**	**5352**	6363	**5.13E+03**	4.65E+02	**100 %**	**100 %**	**5131**	**5131**
	VDE-3	4433	4819	5036	5393	-	2.49E+04	9.70E+04	96 %	96 %	25987	25987
f_{10}	DE	-	-	-	-	-	-	-	0 %	0 %	-	-
	VDE-1	-	-	-	-	-	-	-	0 %	0 %	-	-
	VDE-2	-	-	-	-	-	-	-	0 %	0 %	-	-
	VDE-3	-	-	-	-	-	-	-	0 %	0 %	-	-
f_{11}	DE	166607	241520	281317	-	-	3.47E+05	1.28E+05	0 %	60 %	-	577880
	VDE-1	**81502**	**114160**	**146990**	**309156**	-	**2.36E+05**	1.57E+05	**8 %**	**76 %**	**2943979**	**309892**
	VDE-2	163091	190560	324064	-	-	3.49E+05	1.46E+05	0 %	56 %	-	623373
	VDE-3	91127	168894	-	-	-	3.54E+05	1.65E+05	4 %	48 %	8851540	737628
f_{12}	DE	246805	279548	304142	320004	**357217**	3.02E+05	2.90E+04	0 %	**100 %**	-	301774
	VDE-1	72749	76204	86604	96289	-	1.19E+05	1.13E+05	84 %	92 %	141862	129526
	VDE-2	233437	270650	290990	311889	-	3.07E+05	6.31E+04	0 %	96 %	-	319799
	VDE-3	**37570**	**56162**	**63623**	**72037**	-	**9.81E+04**	1.19E+05	**88 %**	92 %	**111446**	**106601**
f_{13}	DE	-	-	-	-	-	-	-	0 %	0 %	-	-
	VDE-1	**470739**	-	-	-	-	**4.99E+05**	5.73E+03	0 %	**4 %**	-	**12470739**
	VDE-2	-	-	-	-	-	-	-	0 %	0 %	-	-
	VDE-3	-	-	-	-	-	-	-	0 %	0 %	-	-

Table 16. FES to reach the specified accuracy and success rate and performance for f_{14}-f_{25} (10D).

Func	Strategy	1st	7th	13th	19th	25th	Mean	Std	Successrate 1.0e5	Successrate 5.0e5	Success Performance 1.0e5	Success Performance 5.0e5
f_{14}	DE	-	-	-	-	-	-	-	0 %	0 %	-	-
	VDE-1	-	-	-	-	-	-	-	0 %	0 %	-	-
	VDE-2	-	-	-	-	-	-	-	0 %	0 %	-	-
	VDE-3	-	-	-	-	-	-	-	0 %	0 %	-	-
f_{15}	DE	322368	467460	498281	-	-	4.73E+05	4.49E+04	0 %	56 %	-	843918
	VDE-1	271448	-	-	-	-	4.79E+05	5.27E+04	0 %	20 %	-	2394940
	VDE-2	**165024**	**192324**	**238663**	-	-	**2.95E+05**	1.32E+05	0 %	**72 %**	-	**409138**
	VDE-3	304732	444091	494302	-	-	4.55E+05	6.00E+04	0 %	60 %	-	758861
f_{16}	DE	-	-	-	-	-	-	-	0 %	0 %	-	-
	VDE-1	-	-	-	-	-	-	-	0 %	0 %	-	-
	VDE-2	-	-	-	-	-	-	-	0 %	0 %	-	-
	VDE-3	-	-	-	-	-	-	-	0 %	0 %	-	-
f_{17}	DE	-	-	-	-	-	-	-	0 %	0 %	-	-
	VDE-1	-	-	-	-	-	-	-	0 %	0 %	-	-
	VDE-2	-	-	-	-	-	-	-	0 %	0 %	-	-
	VDE-3	-	-	-	-	-	-	-	0 %	0 %	-	-
f_{18}	DE	-	-	-	-	-	-	-	0 %	0 %	-	-
	VDE-1	-	-	-	-	-	-	-	0 %	0 %	-	-
	VDE-2	-	-	-	-	-	-	-	0 %	0 %	-	-
	VDE-3	-	-	-	-	-	-	-	0 %	0 %	-	-
f_{19}	DE	-	-	-	-	-	-	-	0 %	0 %	-	-
	VDE-1	-	-	-	-	-	-	-	0 %	0 %	-	-
	VDE-2	-	-	-	-	-	-	-	0 %	0 %	-	-
	VDE-3	-	-	-	-	-	-	-	0 %	0 %	-	-
f_{20}	DE	-	-	-	-	-	-	-	0 %	0 %	-	-
	VDE-1	-	-	-	-	-	-	-	0 %	0 %	-	-
	VDE-2	-	-	-	-	-	-	-	0 %	0 %	-	-
	VDE-3	-	-	-	-	-	-	-	0 %	0 %	-	-
f_{21}	DE	-	-	-	-	-	-	-	0 %	0 %	-	-
	VDE-1	-	-	-	-	-	-	-	0 %	0 %	-	-
	VDE-2	-	-	-	-	-	-	-	0 %	0 %	-	-
	VDE-3	-	-	-	-	-	-	-	0 %	0 %	-	-
f_{22}	DE	-	-	-	-	-	-	-	0 %	0 %	-	-
	VDE-1	-	-	-	-	-	-	-	0 %	0 %	-	-
	VDE-2	-	-	-	-	-	-	-	0 %	0 %	-	-
	VDE-3	-	-	-	-	-	-	-	0 %	0 %	-	-
f_{23}	DE	-	-	-	-	-	-	-	0 %	0 %	-	-
	VDE-1	-	-	-	-	-	-	-	0 %	0 %	-	-
	VDE-2	-	-	-	-	-	-	-	0 %	0 %	-	-
	VDE-3	-	-	-	-	-	-	-	0 %	0 %	-	-
f_{24}	DE	-	-	-	-	-	-	-	0 %	0 %	-	-
	VDE-1	-	-	-	-	-	-	-	0 %	0 %	-	-
	VDE-2	-	-	-	-	-	-	-	0 %	0 %	-	-
	VDE-3	-	-	-	-	-	-	-	0 %	0 %	-	-
f_{25}	DE	-	-	-	-	-	-	-	0 %	0 %	-	-
	VDE-1	-	-	-	-	-	-	-	0 %	0 %	-	-
	VDE-2	-	-	-	-	-	-	-	0 %	0 %	-	-
	VDE-3	-	-	-	-	-	-	-	0 %	0 %	-	-

The ranking of the algorithms is sorted according to the results from the functions that were reliably solved by an algorithm (extended success performance >50%). 10 functions were solved by the algorithms (f_1, f_2, f_3, f_4, f_5, f_6, f_9, f_{11}, f_{12} and f_{15}). Table 17 shows how many times the algorithms ranked first, second, third, and fourth on the basis of their extended success performance (5.0e5)~~,~~ in addition to the average ranking. On average VDE-3 is the best but very even with VDE-1. VDE-3 is the third best and DE performs worst. Noteworthy are the good performances of DE for f_6, VDE-1 for f_{11}, VDE-2 for f_{15} and VDE-3 for f_3 and f_{12}. In section 5.3 these functions are examined more in detail.

Table 17. Ranking based on success performance of the algorithms for solved 10D functions. Table illustrates how many times the algorithm ranked $1^{st}/2^{nd}/3^{rd}/4^{th}$.

Strategy	1^{st}	2^{nd}	3^{rd}	4^{th}	Avg. Rank
DE	1	1	3	5	3.2
VDE-1	3	5	1	1	2
VDE-2	2	0	5	3	2.9
VDE-3	4	4	1	1	1.9

5.2. Results for 30-dimensional functions

Table 18. Error values achieved for f_1-f_4 (30D).

30D		f_1				f_2				f_3				f_4			
		DE	VDE-1	VDE-2	VDE-3	DE	VDE-1	VDE-2	VDE-3	DE	VDE-1	VDE-2	VDE-3	DE	VDE-1	VDE-2	VDE-3
1E+03	1st	1.51E+04	1.39E+04	1.36E+04	**1.03E+04**	5.28E+04	**4.76E+04**	4.99E+04	5.17E+04	3.84E+08	**2.80E+08**	3.03E+08	5.45E+08	7.09E+04	6.04E+04	**5.53E+04**	6.11E+04
	7th	1.91E+04	1.66E+04	1.98E+04	**1.63E+04**	6.83E+04	6.46E+04	7.21E+04	**5.87E+04**	5.07E+08	**4.55E+08**	4.56E+08	1.09E+09	9.40E+04	8.43E+04	**8.12E+04**	9.14E+04
	13th	2.10E+04	2.12E+04	2.13E+04	**1.96E+04**	7.58E+04	8.30E+04	8.68E+04	**6.89E+04**	**6.12E+08**	6.39E+08	7.00E+08	1.36E+09	1.00E+05	9.13E+04	**9.11E+04**	1.05E+05
	19th	2.33E+04	**2.32E+04**	2.37E+04	2.36E+04	8.43E+04	9.14E+04	9.22E+04	**7.72E+04**	7.69E+08	**7.35E+08**	8.08E+08	1.46E+09	1.13E+05	**1.05E+05**	**1.05E+05**	1.21E+05
	25th	**2.66E+04**	3.22E+04	3.24E+04	3.59E+04	1.07E+05	9.71E+04	1.12E+05	**8.21E+04**	**9.25E+08**	1.05E+09	1.02E+09	1.73E+09	1.31E+05	1.31E+05	**1.27E+05**	1.54E+05
	Mean	2.10E+04	**2.02E+04**	2.18E+04	2.05E+04	7.68E+04	7.82E+04	8.33E+04	**6.75E+04**	6.30E+08	**6.12E+08**	6.51E+08	1.28E+09	1.02E+05	**9.33E+04**	9.36E+04	1.07E+05
	Std	2.95E+03	4.06E+03	4.51E+03	6.36E+03	1.34E+04	1.44E+04	1.54E+04	9.41E+03	1.68E+08	2.08E+08	2.02E+08	2.81E+08	1.56E+04	1.60E+04	1.71E+04	2.26E+04
1E+04	1st	2.61E+00	1.73E-03	1.32E+00	**1.37E-03**	1.08E+04	2.35E+03	5.31E+03	**2.30E+03**	4.62E+07	**5.13E+06**	8.02E+06	2.39E+08	1.78E+04	**7.34E+03**	1.14E+04	3.53E+04
	7th	4.13E+00	**1.86E-02**	2.55E+00	4.12E-01	1.60E+04	6.37E+03	1.03E+04	**4.00E+03**	6.95E+07	**1.34E+07**	1.55E+07	3.10E+08	3.28E+04	**1.45E+04**	1.69E+04	4.66E+04
	13th	5.26E+00	**3.97E-02**	3.12E+00	1.28E+00	2.09E+04	1.05E+04	1.45E+04	**6.21E+03**	8.88E+07	**1.93E+07**	2.13E+07	3.63E+08	3.69E+04	**1.72E+04**	2.43E+04	5.80E+04
	19th	5.79E+00	**2.55E-01**	4.46E+00	3.77E+00	2.85E+04	1.30E+04	2.12E+04	**7.16E+03**	1.15E+08	**3.89E+07**	6.42E+07	4.33E+08	4.59E+04	**2.73E+04**	3.65E+04	6.58E+04
	25th	7.15E+00	8.62E+01	**6.87E+00**	8.23E+00	3.84E+04	3.37E+04	4.84E+04	**2.18E+04**	2.06E+08	**9.25E+07**	2.73E+08	6.63E+08	5.12E+04	**4.50E+04**	5.81E+04	8.00E+04
	Mean	5.05E+00	5.92E+00	3.45E+00	**2.22E+00**	2.22E+04	1.06E+04	1.78E+04	**6.45E+03**	9.67E+07	**3.03E+07**	5.50E+07	3.81E+08	3.74E+04	**2.06E+04**	2.82E+04	5.71E+04
	Std	1.05E+00	1.81E+01	1.32E+00	2.29E+00	7.95E+03	6.45E+03	1.18E+04	3.86E+03	3.75E+07	2.41E+07	6.15E+07	1.06E+08	8.79E+03	9.05E+03	1.28E+04	1.84E+01
1E+05	1st	0.00E+00	0.00E+00	0.00E+00	0.00E+00	1.22E-01	**3.30E-06**	3.83E-03	6.21E-03	3.65E+05	1.80E+05	**1.32E+05**	3.59E+06	1.57E+01	**2.72E+00**	1.67E+02	1.66E+02
	7th	0.00E+00	0.00E+00	0.00E+00	0.00E+00	9.22E-01	**8.39E-05**	2.14E-02	2.15E-02	7.38E+05	**3.03E+05**	3.46E+05	8.67E+06	1.59E+02	**1.87E+01**	2.53E+02	2.59E+02
	13th	0.00E+00	0.00E+00	0.00E+00	0.00E+00	2.10E+00	**2.56E-04**	5.09E-02	3.34E-02	9.58E+05	**4.34E+05**	4.37E+05	9.96E+06	2.47E+02	**4.95E+01**	4.02E+02	1.24E+04
	19th	0.00E+00	0.00E+00	0.00E+00	0.00E+00	2.66E+00	**1.18E-03**	1.04E-01	9.19E-02	1.72E+06	**6.13E+05**	**6.13E+05**	1.68E+07	5.17E+02	**1.19E+02**	6.54E+02	7.21E+02
	25th	0.00E+00	0.00E+00	0.00E+00	0.00E+00	7.50E+00	**3.44E-02**	7.02E+00	6.43E-01	2.61E+06	1.22E+06	**9.54E+05**	1.12E+08	1.85E+03	**1.08E+03**	3.78E+03	2.99E+04
	Mean	0.00E+00	0.00E+00	0.00E+00	0.00E+00	2.16E+00	**2.18E-03**	4.49E-01	1.13E-01	1.19E+06	**4.77E+05**	4.81E+05	2.06E+07	4.26E+02	**1.52E+02**	6.68E+02	3.10E+03
	Std	0.00E+00	0.00E+00	0.00E+00	0.00E+00	1.73E+00	6.73E-03	1.41E+00	1.74E-01	6.11E+05	2.35E+05	2.22E+05	2.63E+07	4.16E+02	2.56E+02	7.95E+02	7.56E+03
3E+05	1st	0.00E+00	0.00E+00	0.00E+00	0.00E+00	7.82E-11	**0.00E+00**	**0.00E+00**	**0.00E+00**	7.03E+04	4.25E+04	3.07E+03	**1.63E+03**	3.19E-04	**8.75E-06**	1.74E-02	1.60E-04
	7th	0.00E+00	0.00E+00	0.00E+00	0.00E+00	6.07E-10	**0.00E+00**	**0.00E+00**	**0.00E+00**	1.71E+05	6.81E+04	5.76E+04	**1.13E+04**	5.87E-02	**2.11E-04**	1.33E-01	4.65E-03
	13th	0.00E+00	0.00E+00	0.00E+00	0.00E+00	1.93E-09	**0.00E+00**	**0.00E+00**	**0.00E+00**	2.14E+05	1.02E+05	6.86E+04	**2.55E+04**	1.77E-01	**2.50E-03**	2.53E+00	1.46E-02
	19th	0.00E+00	0.00E+00	0.00E+00	0.00E+00	9.90E-09	**0.00E+00**	**0.00E+00**	1.99E-12	2.76E+05	1.23E+05	9.98E+04	**4.92E+04**	3.64E-01	**2.44E-02**	1.23E+01	8.82E-02
	25th	0.00E+00	0.00E+00	0.00E+00	0.00E+00	7.98E-08	**0.00E+00**	3.71E-10	4.27E-09	8.08E+05	1.90E+05	1.44E+05	**7.07E+04**	1.20E+01	**3.85E-01**	8.07E+02	3.21E+01
	Mean	0.00E+00	0.00E+00	0.00E+00	0.00E+00	1.07E-08	**0.00E+00**	1.55E-11	1.75E-10	2.51E+05	1.01E+05	7.45E+04	**2.89E+04**	1.27E+00	**3.38E-02**	4.36E+01	1.47E+00
	Std	0.00E+00	0.00E+00	0.00E+00	0.00E+00	2.03E-08	0.00E+00	7.26E-11	8.36E-10	1.50E+05	3.83E+04	3.44E+04	2.05E+04	2.82E+00	7.98E-02	1.58E+02	6.29E+00
5E+05	1st	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	3.69E+04	1.59E+04	1.30E+03	**3.46E+00**	1.01E-07	**1.38E-10**	2.05E-05	1.10E-08
	7th	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	7.15E+04	2.99E+04	2.02E+04	**5.59E+01**	1.60E-05	**5.18E-09**	9.45E-04	9.68E-08
	13th	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	9.90E+04	4.18E+04	3.02E+04	**1.07E+02**	6.20E-05	**9.35E-08**	3.92E-03	2.07E-06
	19th	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	1.53E+05	5.27E+04	3.42E+04	**2.95E+02**	2.61E-04	**8.13E-06**	5.44E-02	1.73E-05
	25th	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	3.95E+05	1.21E+05	4.57E+04	**2.82E+03**	3.30E-02	1.26E-03	3.54E+02	**1.02E-03**
	Mean	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	1.20E+05	4.72E+04	2.84E+04	**3.08E+02**	2.05E-03	7.14E-05	1.44E+01	**5.80E-05**
	Std	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	7.43E+04	2.50E+04	1.17E+04	5.54E+02	6.75E-03	2.53E-04	6.93E+01	1.99E-04

Table 19. Error values achieved for f_5-f_8 (30D).

30D		f_5				f_6				f_7				f_8			
		DE	VDE-1	VDE-2	VDE-3	DE	VDE-1	VDE-2	VDE-3	DE	VDE-1	VDE-2	VDE-3	DE	VDE-1	VDE-2	VDE-3
1E+03	1st	2.00E+04	2.76E+04	**1.85E+04**	2.98E+04	**3.54E+09**	1.34E+10	6.80E+09	1.08E+10	**4.85E+03**	5.11E+03	5.00E+03	5.20E+03	2.11E+01	**2.10E+01**	2.11E+01	2.11E+01
	7th	**2.44E+04**	3.02E+04	2.56E+04	3.26E+04	1.08E+10	2.46E+10	**1.02E+10**	1.78E+10	**5.71E+03**	6.40E+03	6.57E+03	6.07E+03	2.12E+01	2.12E+01	2.12E+01	**2.11E+01**
	13th	**2.73E+04**	3.11E+04	2.78E+04	3.46E+04	**1.36E+10**	2.65E+10	1.47E+10	2.41E+10	7.03E+03	7.10E+03	7.30E+03	**6.35E+03**	2.12E+01	2.12E+01	2.12E+01	2.12E+01
	19th	**2.84E+04**	3.18E+04	2.90E+04	3.62E+04	2.04E+10	3.36E+10	**1.68E+10**	2.67E+10	7.92E+03	7.50E+03	7.75E+03	**7.12E+03**	2.13E+01	2.13E+01	2.13E+01	**2.12E+01**
	25th	**3.24E+04**	3.74E+04	3.30E+04	3.92E+04	3.37E+10	5.89E+10	**2.34E+10**	4.06E+10	9.48E+03	8.72E+03	1.03E+04	**8.41E+03**	2.13E+01	2.13E+01	2.13E+01	2.13E+01
	Mean	**2.65E+04**	3.13E+04	2.71E+04	3.43E+04	1.61E+10	2.84E+10	**1.44E+10**	2.40E+10	6.97E+03	7.02E+03	7.22E+03	**6.57E+03**	2.12E+01	2.12E+01	2.12E+01	2.12E+01
	Std	2.97E+03	2.18E+03	3.35E+03	2.40E+03	7.63E+09	9.76E+09	4.40E+09	7.58E+09	1.27E+03	8.59E+02	1.18E+03	7.92E+02	6.25E-02	6.06E-02	5.25E-02	6.07E-02
1E+04	1st	5.74E+03	8.70E+03	**5.39E+03**	1.17E+04	2.46E+06	2.15E+07	**1.83E+06**	4.86E+06	2.02E+03	4.23E+02	2.79E+03	**1.07E+02**	**2.09E+01**	**2.09E+01**	2.10E+01	2.10E+01
	7th	7.42E+03	9.86E+03	**6.78E+03**	1.39E+04	**7.20E+06**	5.28E+07	1.15E+07	1.68E+07	3.60E+03	7.72E+02	3.74E+03	**3.31E+02**	2.11E+01	2.11E+01	2.11E+01	2.11E+01
	13th	8.50E+03	1.13E+04	**7.16E+03**	1.54E+04	**1.03E+07**	9.87E+07	2.38E+07	3.26E+07	3.92E+03	1.01E+03	4.13E+03	**5.57E+02**	2.11E+01	2.11E+01	2.11E+01	2.11E+01
	19th	9.55E+03	1.26E+04	**8.23E+03**	1.72E+04	**2.13E+07**	1.78E+08	4.58E+07	9.34E+07	4.76E+03	1.35E+03	4.61E+03	**1.10E+03**	2.11E+01	2.11E+01	2.11E+01	2.11E+01
	25th	**1.20E+04**	1.76E+04	**1.20E+04**	2.09E+04	1.56E+08	6.22E+08	**1.45E+08**	1.03E+09	5.14E+03	2.94E+03	5.73E+03	**2.21E+03**	2.12E+01	2.12E+01	2.12E+01	2.12E+01
	Mean	8.49E+03	1.17E+04	**7.92E+03**	1.58E+04	**2.24E+07**	1.46E+08	3.40E+07	1.31E+08	4.01E+03	1.18E+03	4.14E+03	**7.54E+02**	2.11E+01	2.11E+01	2.11E+01	2.11E+01
	Std	1.53E+03	2.39E+03	1.86E+03	2.51E+03	3.20E+07	1.46E+08	3.35E+07	2.30E+08	7.72E+02	5.99E+02	6.79E+02	5.47E+02	6.10E-02	6.67E-02	4.44E-02	4.68E-02
1E+05	1st	**1.24E+02**	1.57E+02	4.31E+02	3.47E+02	**1.56E+01**	1.97E+01	2.09E+01	1.87E+01	3.59E+01	4.83E-04	1.13E+01	**1.74E-07**	2.09E+01	2.09E+01	2.09E+01	**2.08E+01**
	7th	7.37E+02	**3.10E+02**	7.60E+02	6.25E+02	2.32E+01	2.21E+01	2.79E+01	**2.12E+01**	6.75E+01	2.92E-03	6.09E+01	**4.72E-06**	2.10E+01	2.10E+01	2.10E+01	**2.09E+01**
	13th	1.11E+03	**4.66E+02**	1.25E+03	8.29E+02	2.61E+01	2.29E+01	2.94E+01	**2.28E+01**	9.69E+01	4.80E-03	3.77E+02	**4.06E-05**	2.10E+01	2.10E+01	2.10E+01	2.10E+01
	19th	1.42E+03	**6.74E+02**	1.60E+03	9.54E+02	1.13E+02	**2.40E+01**	1.10E+02	2.44E+01	1.30E+02	1.16E-02	9.50E+02	**1.17E-03**	2.10E+01	2.10E+01	2.10E+01	2.10E+01
	25th	2.19E+03	**1.43E+03**	3.55E+03	1.45E+03	2.08E+02	1.52E+02	1.56E+03	**9.98E+01**	2.04E+02	**3.57E-02**	1.87E+03	1.09E-01	2.11E+01	2.11E+01	2.11E+01	2.11E+01
	Mean	1.08E+03	**5.82E+02**	1.30E+03	8.46E+02	7.44E+01	3.88E+01	1.50E+02	**2.99E+01**	1.04E+02	**8.04E-03**	5.97E+02	8.82E-03	2.10E+01	2.10E+01	2.10E+01	2.10E+01
	Std	5.16E+02	3.58E+02	6.57E+02	2.89E+02	6.56E+01	3.49E+01	3.27E+02	2.10E+01	4.66E+01	7.96E-03	6.04E+02	2.23E-02	4.08E-02	5.25E-02	4.56E-02	6.23E-02
3E+05	1st	2.78E+00	**3.82E-02**	6.61E+01	3.82E-01	2.69E-04	**2.18E-07**	4.90E-04	9.00E-07	5.61E-01	**0.00E+00**	3.64E-01	**0.00E+00**	2.09E+01	**2.08E+01**	2.09E+01	**2.08E+01**
	7th	1.24E+02	**2.60E-01**	3.74E+02	6.70E-01	2.38E-01	**6.64E-05**	3.99E+00	2.11E-04	9.79E-01	**0.00E+00**	1.33E+00	**0.00E+00**	2.09E+01	2.09E+01	2.09E+01	2.09E+01
	13th	2.19E+02	**4.64E-01**	6.50E+02	1.03E+00	2.73E+00	**7.98E-04**	7.93E+00	1.94E-03	1.01E+00	**0.00E+00**	3.96E+00	**0.00E+00**	**2.09E+01**	2.10E+01	2.10E+01	**2.09E+01**
	19th	4.97E+02	**9.29E-01**	1.11E+03	2.44E+00	6.49E+00	**3.89E-03**	1.09E+01	1.21E-02	1.06E+00	**0.00E+00**	5.19E+01	**0.00E+00**	2.10E+01	2.10E+01	2.10E+01	2.10E+01
	25th	1.45E+03	7.44E+00	1.41E+03	**5.48E+00**	1.90E+01	2.95E+00	7.33E+01	**7.52E-01**	1.10E+00	**7.40E-03**	3.15E+02	2.95E-02	2.10E+01	2.10E+01	2.10E+01	2.10E+01
	Mean	3.49E+02	**1.16E+00**	6.88E+02	1.59E+00	4.85E+00	1.23E-01	1.24E+01	**3.89E-02**	9.74E-01	**8.88E-04**	4.88E+01	4.33E-03	2.10E+01	2.10E+01	2.10E+01	**2.09E+01**
	Std	3.36E+02	1.71E+00	4.12E+02	1.29E+00	5.40E+00	5.78E-01	1.83E+01	1.47E-01	1.32E-01	2.40E-03	7.83E+01	9.24E-03	3.60E-02	5.27E-02	3.61E-02	5.49E-02
5E+05	1st	2.51E-01	**2.21E-06**	9.72E+00	2.44E-04	6.42E-11	**0.00E+00**	5.91E-12	**0.00E+00**	5.09E-04	**0.00E+00**	5.52E-05	**0.00E+00**	2.08E+01	**2.07E+01**	2.08E+01	2.08E+01
	7th	3.05E+01	**2.05E-04**	1.08E+02	7.86E-04	8.47E-09	**0.00E+00**	1.96E-06	**0.00E+00**	2.29E-03	**0.00E+00**	1.43E-01	**0.00E+00**	2.09E+01	2.09E+01	2.09E+01	2.09E+01
	13th	8.11E+01	**5.41E-04**	3.34E+02	1.12E-03	1.42E-05	**0.00E+00**	5.93E-03	**0.00E+00**	5.23E-03	**0.00E+00**	1.35E+00	**0.00E+00**	**2.09E+01**	**2.09E+01**	2.10E+01	**2.09E+01**
	19th	2.30E+02	**1.47E-03**	7.46E+02	3.80E-03	7.32E-05	**0.00E+00**	1.58E-01	**0.00E+00**	1.40E-02	**0.00E+00**	9.42E+00	**0.00E+00**	**2.09E+01**	2.10E+01	2.10E+01	**2.09E+01**
	25th	1.38E+03	**1.55E-02**	1.11E+03	1.83E-02	3.99E+00	**0.00E+00**	4.01E+00	**0.00E+00**	3.49E-02	**7.40E-03**	4.54E+01	2.95E-02	2.10E+01	2.10E+01	2.10E+01	2.10E+01
	Mean	1.73E+02	**2.23E-03**	4.33E+02	3.11E-03	6.38E-01	**0.00E+00**	6.86E-01	**0.00E+00**	9.78E-03	**8.88E-04**	7.49E+00	4.33E-03	2.09E+01	2.09E+01	2.09E+01	2.09E+01
	Std	2.73E+02	3.95E-03	3.40E+02	4.14E-03	1.46E+00	0.00E+00	1.45E+00	0.00E+00	9.11E-03	2.40E-03	1.14E+01	9.24E-03	4.11E-02	5.87E-02	4.11E-02	4.73E-02

Table 20. Error values achieved for f_9-f_{12} (30D).

30D		f_9				f_{10}				f_{11}				f_{12}			
		DE	VDE-1	VDE-2	VDE-3	DE	VDE-1	VDE-2	VDE-3	DE	VDE-1	VDE-2	VDE-3	DE	VDE-1	VDE-2	VDE-3
1E+03	1st	**2.84E+02**	3.11E+02	3.33E+02	3.18E+02	4.03E+02	3.49E+02	4.05E+02	**3.46E+02**	4.07E+01	4.23E+01	**3.97E+01**	4.20E+01	1.29E+06	**9.36E+05**	1.19E+06	1.30E+06
	7th	3.55E+02	3.65E+02	3.67E+02	**3.52E+02**	4.55E+02	4.35E+02	4.57E+02	**4.02E+02**	**4.40E+01**	4.41E+01	4.47E+01	4.50E+01	**1.44E+06**	1.56E+06	1.45E+06	1.50E+06
	13th	3.71E+02	3.79E+02	3.82E+02	**3.68E+02**	4.99E+02	4.60E+02	4.83E+02	**4.50E+02**	4.55E+01	**4.50E+01**	4.54E+01	4.58E+01	1.61E+06	1.67E+06	**1.56E+06**	1.69E+06
	19th	**3.88E+02**	3.98E+02	4.01E+02	3.95E+02	5.22E+02	4.91E+02	5.41E+02	**4.71E+02**	4.65E+01	**4.58E+01**	4.63E+01	4.72E+01	1.74E+06	1.82E+06	**1.66E+06**	1.75E+06
	25th	4.29E+02	**4.20E+02**	4.26E+02	4.61E+02	5.60E+02	5.78E+02	5.97E+02	**5.57E+02**	4.83E+01	**4.71E+01**	4.87E+01	4.81E+01	2.02E+06	2.14E+06	**1.88E+06**	2.03E+06
	Mean	**3.69E+02**	3.76E+02	3.82E+02	3.72E+02	4.87E+02	4.61E+02	4.96E+02	**4.46E+02**	4.52E+01	**4.48E+01**	4.54E+01	4.58E+01	1.60E+06	1.67E+06	**1.55E+06**	1.66E+06
	Std	3.30E+01	2.71E+01	2.35E+01	3.47E+01	4.32E+01	5.41E+01	5.70E+01	5.01E+01	1.66E+00	1.20E+00	1.82E+00	1.56E+00	1.77E+05	2.22E+05	1.65E+05	1.91E+05
1E+04	1st	8.79E+01	8.76E+01	6.84E+01	**6.45E+01**	2.31E+02	1.94E+02	2.31E+02	**1.15E+02**	3.89E+01	4.07E+01	**3.13E+01**	4.07E+01	7.55E+05	6.44E+05	6.52E+05	**6.21E+05**
	7th	9.56E+01	9.57E+01	8.60E+01	**8.44E+01**	2.64E+02	**2.29E+02**	2.63E+02	2.36E+02	4.17E+01	4.23E+01	**4.16E+01**	4.20E+01	9.01E+05	8.71E+05	9.54E+05	**8.36E+05**
	13th	1.02E+02	1.01E+02	9.10E+01	**8.86E+01**	2.78E+02	**2.45E+02**	2.77E+02	2.51E+02	4.31E+01	4.29E+01	**4.21E+01**	4.27E+01	9.81E+05	9.10E+05	9.81E+05	**9.03E+05**
	19th	1.05E+02	1.11E+02	1.03E+02	**1.01E+02**	2.93E+02	**2.56E+02**	2.95E+02	2.59E+02	4.38E+01	4.39E+01	4.36E+01	**4.35E+01**	1.09E+06	1.04E+06	1.04E+06	**9.91E+05**
	25th	**1.15E+02**	1.19E+02	1.66E+02	1.41E+02	3.32E+02	**2.76E+02**	3.19E+02	2.80E+02	4.50E+01	**4.49E+01**	4.53E+01	4.52E+01	1.26E+06	1.35E+06	**1.15E+06**	**1.15E+06**
	Mean	1.01E+02	1.03E+02	9.83E+01	**9.36E+01**	2.78E+02	**2.42E+02**	2.77E+02	**2.42E+02**	4.26E+01	4.30E+01	**4.13E+01**	4.27E+01	9.87E+05	9.60E+05	9.81E+05	**9.04E+05**
	Std	6.86E+00	8.98E+00	2.29E+01	1.79E+01	2.13E+01	1.97E+01	2.26E+01	3.12E+01	1.55E+00	1.12E+00	3.70E+00	1.02E+00	1.23E+05	1.68E+05	1.02E+05	1.28E+05
1E+05	1st	1.78E-08	**0.00E+00**	1.00E-10	**0.00E+00**	2.50E+01	2.39E+01	3.38E+01	**2.31E+01**	1.42E+01	**1.10E+01**	1.42E+01	3.77E+01	1.38E+05	1.08E+05	4.74E+04	**2.11E+03**
	7th	4.11E-08	**0.00E+00**	2.82E-10	**0.00E+00**	4.77E+01	**3.68E+01**	5.67E+01	5.87E+01	3.91E+01	3.90E+01	**2.01E+01**	4.01E+01	2.20E+05	2.29E+05	2.66E+05	**7.69E+03**
	13th	7.93E-08	4.83E-12	4.40E-10	**1.14E-12**	6.82E+01	**5.49E+01**	2.05E+02	1.81E+02	4.05E+01	4.07E+01	**3.05E+01**	4.09E+01	2.64E+05	3.37E+05	4.90E+05	**3.62E+04**
	19th	1.04E-07	6.51E-11	1.23E-09	**1.03E-11**	2.01E+02	1.92E+02	2.18E+02	**1.91E+02**	4.12E+01	4.10E+01	**3.97E+01**	4.12E+01	**3.38E+05**	4.41E+05	5.78E+05	3.69E+05
	25th	6.20E-07	3.06E-08	7.52E-09	**2.21E-09**	2.28E+02	2.24E+02	2.37E+02	**2.03E+02**	4.25E+01	**4.18E+01**	4.21E+01	4.19E+01	6.32E+05	6.08E+05	6.80E+05	**5.05E+05**
	Mean	1.09E-07	2.83E-09	1.25E-09	**1.47E-10**	1.22E+02	**1.13E+02**	1.55E+02	1.33E+02	3.90E+01	3.87E+01	**2.91E+01**	4.06E+01	2.94E+05	3.25E+05	4.31E+05	**1.52E+05**
	Std	1.23E-07	8.05E-09	1.71E-09	4.48E-10	7.98E+01	8.06E+01	8.05E+01	6.92E+01	5.64E+00	6.04E+00	1.03E+01	9.44E-01	1.17E+05	1.37E+05	1.94E+05	1.80E+05
3E+05	1st	0.00E+00	0.00E+00	0.00E+00	0.00E+00	2.29E+01	2.39E+01	2.69E+01	**2.19E+01**	1.17E+01	9.48E+00	**6.94E+00**	3.60E+01	1.42E+03	1.44E+03	2.28E+02	**1.90E-02**
	7th	0.00E+00	0.00E+00	0.00E+00	0.00E+00	**3.08E+01**	3.28E+01	4.48E+01	4.08E+01	2.28E+01	**1.58E+01**	1.84E+01	3.88E+01	4.84E+03	4.36E+03	4.26E+03	**1.21E+02**
	13th	0.00E+00	0.00E+00	0.00E+00	0.00E+00	**4.08E+01**	4.28E+01	9.09E+01	6.37E+01	3.89E+01	3.85E+01	**2.23E+01**	4.03E+01	7.50E+03	8.20E+03	2.37E+05	**1.19E+03**
	19th	0.00E+00	0.00E+00	0.00E+00	0.00E+00	**5.37E+01**	1.69E+02	2.00E+02	1.73E+02	3.99E+01	4.00E+01	**3.79E+01**	4.09E+01	1.16E+04	2.87E+04	3.79E+05	**3.26E+03**
	25th	0.00E+00	0.00E+00	0.00E+00	0.00E+00	**9.89E+01**	1.91E+02	2.17E+02	1.91E+02	4.11E+01	4.10E+01	**4.07E+01**	4.12E+01	**3.71E+04**	1.79E+05	5.50E+05	3.36E+05
	Mean	0.00E+00	0.00E+00	0.00E+00	0.00E+00	**4.38E+01**	8.19E+01	1.18E+02	1.03E+02	3.23E+01	3.09E+01	**2.58E+01**	3.96E+01	**9.11E+03**	2.44E+04	2.12E+05	3.54E+04
	Std	0.00E+00	0.00E+00	0.00E+00	0.00E+00	1.77E+01	6.81E+01	7.68E+01	6.75E+01	1.07E+01	1.24E+01	1.10E+01	1.44E+00	7.10E+03	3.79E+04	1.95E+05	9.28E+04
5E+05	1st	0.00E+00	0.00E+00	0.00E+00	0.00E+00	2.29E+01	2.39E+01	2.69E+01	**2.09E+01**	1.17E+01	9.48E+00	**6.94E+00**	3.60E+01	2.92E+01	2.21E+01	2.98E+01	**2.22E-09**
	7th	0.00E+00	0.00E+00	0.00E+00	0.00E+00	3.08E+01	**2.89E+01**	4.28E+01	3.65E+01	2.13E+01	**1.26E+01**	1.84E+01	3.78E+01	7.27E+02	4.95E+02	1.28E+03	**3.11E+00**
	13th	0.00E+00	0.00E+00	0.00E+00	0.00E+00	4.08E+01	**3.68E+01**	7.50E+01	5.87E+01	3.69E+01	3.81E+01	**2.23E+01**	3.92E+01	2.24E+03	1.13E+03	7.60E+03	**4.31E+02**
	19th	0.00E+00	0.00E+00	0.00E+00	0.00E+00	5.37E+01	**4.28E+01**	1.89E+02	1.73E+02	3.90E+01	3.95E+01	**3.60E+01**	4.02E+01	3.33E+03	2.28E+03	1.88E+05	**2.20E+03**
	25th	0.00E+00	0.00E+00	0.00E+00	0.00E+00	**9.89E+01**	1.81E+02	2.17E+02	1.91E+02	4.06E+01	4.10E+01	**4.04E+01**	4.08E+01	**7.71E+03**	1.03E+04	4.60E+05	3.36E+05
	Mean	0.00E+00	0.00E+00	0.00E+00	0.00E+00	**4.38E+01**	5.02E+01	1.08E+02	9.13E+01	3.02E+01	2.79E+01	**2.51E+01**	3.89E+01	2.47E+03	**2.03E+03**	1.18E+05	1.45E+04
	Std	0.00E+00	0.00E+00	0.00E+00	0.00E+00	1.77E+01	4.32E+01	7.33E+01	6.71E+01	1.09E+01	1.29E+01	1.04E+01	1.35E+00	2.06E+03	2.38E+03	1.71E+05	6.56E+04

Table 21. Error values achieved for f_{13}-f_{16} (30D).

30D		f_{13}				f_{14}				f_{15}				f_{16}			
		DE	VDE-1	VDE-2	VDE-3	DE	VDE-1	VDE-2	VDE-3	DE	VDE-1	VDE-2	VDE-3	DE	VDE-1	VDE-2	VDE-3
1E+03	1st	6.99E+01	5.48E+01	6.25E+01	**4.60E+01**	**1.38E+01**	**1.38E+01**	1.39E+01	**1.38E+01**	8.93E+02	9.31E+02	**8.36E+02**	8.92E+02	6.23E+02	**6.11E+02**	6.30E+02	7.69E+02
	7th	1.54E+02	1.00E+02	1.63E+02	**9.83E+01**	1.41E+01	1.41E+01	1.41E+01	**1.40E+01**	**1.03E+03**	1.08E+03	1.05E+03	1.12E+03	**7.48E+02**	8.11E+02	7.64E+02	8.41E+02
	13th	2.23E+02	**1.60E+02**	2.31E+02	1.62E+02	**1.42E+01**	**1.42E+01**	**1.42E+01**	**1.42E+01**	**1.05E+03**	1.13E+03	1.08E+03	1.13E+03	**7.95E+02**	9.17E+02	8.12E+02	9.10E+02
	19th	3.34E+02	**2.16E+02**	2.73E+02	2.25E+02	1.43E+01	**1.42E+01**	**1.42E+01**	1.43E+01	**1.10E+03**	1.15E+03	1.11E+03	1.16E+03	**8.40E+02**	9.63E+02	8.63E+02	9.54E+02
	25th	5.42E+02	4.35E+02	4.16E+02	**3.21E+02**	1.44E+01	1.44E+01	**1.43E+01**	1.44E+01	1.21E+03	1.23E+03	**1.16E+03**	1.22E+03	**9.54E+02**	1.02E+03	1.02E+03	1.09E+03
	Mean	2.47E+02	1.74E+02	2.25E+02	**1.72E+02**	**1.42E+01**	**1.42E+01**	**1.42E+01**	**1.42E+01**	**1.05E+03**	1.11E+03	1.07E+03	1.12E+03	**7.93E+02**	8.84E+02	8.18E+02	9.10E+02
	Std	1.22E+02	9.36E+01	9.16E+01	7.83E+01	1.28E-01	1.40E-01	1.19E-01	1.84E-01	7.26E+01	8.00E+01	6.61E+01	6.50E+01	7.75E+01	1.04E+02	8.53E+01	7.71E+01
1E+04	1st	**1.51E+01**	1.65E+01	1.84E+01	1.73E+01	1.35E+01	1.37E+01	1.36E+01	**1.34E+01**	5.97E+02	6.10E+02	6.08E+02	**5.66E+02**	3.90E+02	3.61E+02	4.02E+02	**3.29E+02**
	7th	2.16E+01	**1.82E+01**	2.19E+01	1.91E+01	**1.38E+01**	**1.38E+01**	**1.38E+01**	**1.38E+01**	6.73E+02	6.51E+02	**6.34E+02**	6.37E+02	4.17E+02	4.23E+02	4.38E+02	**3.86E+02**
	13th	2.30E+01	**1.93E+01**	2.23E+01	1.96E+01	1.39E+01	1.39E+01	1.39E+01	**1.38E+01**	6.89E+02	**6.78E+02**	6.80E+02	6.97E+02	4.54E+02	4.43E+02	4.59E+02	**4.02E+02**
	19th	2.43E+01	2.11E+01	2.39E+01	**2.03E+01**	**1.39E+01**	**1.39E+01**	1.40E+01	**1.39E+01**	7.47E+02	7.47E+02	**7.06E+02**	7.45E+02	4.84E+02	4.79E+02	4.74E+02	**4.45E+02**
	25th	2.86E+01	2.32E+01	3.61E+01	**2.19E+01**	**1.41E+01**	**1.41E+01**	**1.41E+01**	**1.41E+01**	8.54E+02	8.12E+02	8.54E+02	**8.10E+02**	5.52E+02	5.42E+02	5.36E+02	**5.15E+02**
	Mean	2.27E+01	**1.95E+01**	2.38E+01	**1.95E+01**	1.39E+01	1.39E+01	1.39E+01	**1.38E+01**	7.02E+02	6.96E+02	**6.83E+02**	6.89E+02	4.53E+02	4.51E+02	4.58E+02	**4.13E+02**
	Std	2.75E+00	1.76E+00	4.37E+00	1.01E+00	1.14E-01	1.16E-01	1.25E-01	1.37E-01	6.46E+01	5.87E+01	5.82E+01	6.69E+01	4.10E+01	4.39E+01	3.54E+01	4.84E+01
1E+05	1st	1.83E+00	1.93E+00	1.88E+00	**1.54E+00**	1.31E+01	1.34E+01	**1.26E+01**	1.29E+01	5.05E+02	4.77E+02	4.85E+02	**4.54E+02**	2.90E+02	**2.24E+02**	2.58E+02	2.29E+02
	7th	2.59E+00	2.81E+00	3.17E+00	**2.36E+00**	1.35E+01	1.34E+01	**1.33E+01**	1.34E+01	5.12E+02	4.81E+02	4.89E+02	**4.70E+02**	3.06E+02	2.64E+02	2.74E+02	**2.45E+02**
	13th	3.22E+00	2.96E+00	4.62E+00	**2.72E+00**	1.36E+01	**1.35E+01**	**1.35E+01**	**1.35E+01**	5.17E+02	4.82E+02	4.94E+02	**4.74E+02**	3.17E+02	2.70E+02	2.85E+02	**2.54E+02**
	19th	4.50E+00	**3.83E+00**	7.58E+00	3.92E+00	1.37E+01	**1.36E+01**	**1.36E+01**	**1.36E+01**	5.22E+02	4.84E+02	5.00E+02	**4.77E+02**	3.24E+02	2.78E+02	3.07E+02	**2.60E+02**
	25th	**7.18E+00**	1.56E+01	1.72E+01	1.62E+01	**1.37E+01**	1.38E+01	1.38E+01	**1.37E+01**	6.35E+02	**4.90E+02**	5.65E+02	5.66E+02	3.70E+02	2.96E+02	3.19E+02	**2.75E+02**
	Mean	**3.59E+00**	3.61E+00	6.02E+00	5.14E+00	1.36E+01	**1.35E+01**	**1.35E+01**	**1.35E+01**	5.38E+02	4.83E+02	4.98E+02	**4.79E+02**	3.17E+02	2.68E+02	2.88E+02	**2.53E+02**
	Std	1.38E+00	2.53E+00	4.25E+00	4.96E+00	1.59E-01	1.24E-01	2.71E-01	1.91E-01	4.61E+01	2.83E+00	1.58E+01	2.34E+01	1.71E+01	1.45E+01	1.91E+01	1.18E+01
3E+05	1st	1.80E+00	1.92E+00	1.84E+00	**1.48E+00**	1.29E+01	1.29E+01	1.26E+01	**1.18E+01**	4.63E+02	4.02E+02	4.53E+02	**4.00E+02**	2.56E+02	**2.10E+02**	2.48E+02	**2.10E+02**
	7th	2.53E+00	2.66E+00	2.89E+00	**2.29E+00**	1.33E+01	1.32E+01	1.31E+01	**1.30E+01**	4.78E+02	4.03E+02	4.59E+02	**4.00E+02**	2.79E+02	2.29E+02	2.58E+02	**2.26E+02**
	13th	3.22E+00	2.91E+00	3.22E+00	**2.56E+00**	1.34E+01	1.33E+01	1.33E+01	**1.32E+01**	4.80E+02	4.03E+02	4.62E+02	**4.00E+02**	2.86E+02	2.39E+02	2.61E+02	**2.32E+02**
	19th	4.50E+00	**3.44E+00**	4.70E+00	3.92E+00	1.35E+01	**1.34E+01**	1.35E+01	**1.34E+01**	4.83E+02	4.05E+02	4.68E+02	**4.00E+02**	2.89E+02	2.47E+02	2.73E+02	**2.37E+02**
	25th	7.18E+00	**4.09E+00**	1.50E+01	1.51E+01	**1.36E+01**	1.37E+01	1.38E+01	1.37E+01	5.92E+02	**4.06E+02**	4.88E+02	5.00E+02	3.53E+02	2.61E+02	2.89E+02	**2.44E+02**
	Mean	3.54E+00	**2.98E+00**	4.36E+00	4.73E+00	1.34E+01	1.33E+01	1.32E+01	**1.31E+01**	4.97E+02	**4.04E+02**	4.64E+02	4.08E+02	2.85E+02	2.38E+02	2.66E+02	**2.31E+02**
	Std	1.39E+00	6.18E-01	2.76E+00	4.29E+00	1.60E-01	1.67E-01	2.98E-01	4.19E-01	3.99E+01	1.16E+00	7.14E+00	2.71E+01	1.74E+01	1.25E+01	1.24E+01	7.85E+00
5E+05	1st	1.80E+00	1.91E+00	1.84E+00	**1.47E+00**	1.26E+01	1.29E+01	1.26E+01	**1.11E+01**	**3.51E+02**	4.00E+02	3.98E+02	4.00E+02	2.41E+02	2.04E+02	2.04E+02	**1.94E+02**
	7th	2.53E+00	2.66E+00	2.89E+00	**2.25E+00**	1.32E+01	1.31E+01	1.31E+01	**1.25E+01**	4.47E+02	**4.00E+02**	4.22E+02	**4.00E+02**	2.60E+02	2.21E+02	2.49E+02	**2.14E+02**
	13th	3.22E+00	2.91E+00	3.22E+00	**2.54E+00**	1.33E+01	1.33E+01	1.33E+01	**1.30E+01**	4.53E+02	**4.00E+02**	4.31E+02	**4.00E+02**	2.70E+02	2.29E+02	2.54E+02	**2.23E+02**
	19th	4.50E+00	3.44E+00	4.70E+00	**3.23E+00**	1.34E+01	1.34E+01	1.34E+01	**1.33E+01**	4.61E+02	**4.00E+02**	4.36E+02	**4.00E+02**	2.81E+02	2.36E+02	2.60E+02	**2.27E+02**
	25th	7.18E+00	**4.09E+00**	8.46E+00	1.50E+01	1.36E+01	**1.35E+01**	1.38E+01	**1.35E+01**	5.73E+02	**4.00E+02**	4.42E+02	5.00E+02	3.49E+02	2.50E+02	2.79E+02	**2.41E+02**
	Mean	3.54E+00	**2.96E+00**	4.05E+00	3.69E+00	1.33E+01	1.32E+01	1.32E+01	**1.28E+01**	4.66E+02	**4.00E+02**	4.28E+02	4.08E+02	2.72E+02	2.29E+02	2.54E+02	**2.20E+02**
	Std	1.39E+00	6.38E-01	1.88E+00	3.16E+00	2.41E-01	1.63E-01	2.84E-01	6.61E-01	4.90E+01	5.39E-03	1.12E+01	2.71E+01	2.06E+01	1.11E+01	1.51E+01	1.17E+01

Table 22. Error values achieved for f_{17}-f_{20} (30D).

30D		f_{17}				f_{18}				f_{19}				f_{20}			
		DE	VDE-1	VDE-2	VDE-3	DE	VDE-1	VDE-2	VDE-3	DE	VDE-1	VDE-2	VDE-3	DE	VDE-1	VDE-2	VDE-3
1E+03	1st	7.44E+02	5.99E+02	**5.39E+02**	7.63E+02	1.24E+03	**1.17E+03**	**1.17E+03**	1.18E+03	1.20E+03	**1.08E+03**	1.21E+03	1.17E+03	**1.19E+03**	1.20E+03	1.20E+03	1.20E+03
	7th	8.39E+02	**8.36E+02**	8.60E+02	9.55E+02	1.28E+03	**1.24E+03**	1.27E+03	1.31E+03	1.27E+03	**1.26E+03**	**1.26E+03**	1.31E+03	**1.25E+03**	1.26E+03	1.27E+03	1.29E+03
	13th	**8.88E+02**	9.44E+02	9.24E+02	1.01E+03	1.30E+03	**1.27E+03**	1.30E+03	1.33E+03	**1.28E+03**	**1.28E+03**	1.29E+03	1.32E+03	**1.28E+03**	1.29E+03	1.29E+03	1.31E+03
	19th	9.69E+02	1.02E+03	**9.64E+02**	1.08E+03	1.33E+03	**1.30E+03**	1.33E+03	1.34E+03	1.31E+03	**1.30E+03**	1.31E+03	1.34E+03	**1.30E+03**	**1.30E+03**	1.31E+03	1.34E+03
	25th	**1.06E+03**	1.19E+03	**1.06E+03**	1.15E+03	1.37E+03	**1.34E+03**	1.36E+03	1.39E+03	1.34E+03	**1.33E+03**	1.35E+03	1.38E+03	1.36E+03	**1.33E+03**	1.35E+03	1.35E+03
	Mean	9.05E+02	9.21E+02	**8.97E+02**	1.00E+03	1.30E+03	**1.27E+03**	1.29E+03	1.31E+03	1.29E+03	**1.27E+03**	1.29E+03	1.32E+03	**1.27E+03**	1.28E+03	1.28E+03	1.30E+03
	Std	8.84E+01	1.33E+02	1.07E+02	9.31E+01	3.35E+01	3.99E+01	5.10E+01	4.29E+01	3.32E+01	5.04E+01	3.34E+01	4.13E+01	4.15E+01	3.43E+01	4.15E+01	4.30E+01
1E+04	1st	4.32E+02	**3.54E+02**	4.16E+02	4.10E+02	1.07E+03	**9.95E+02**	1.04E+03	1.05E+03	1.04E+03	**9.68E+02**	1.02E+03	1.06E+03	1.04E+03	**9.69E+02**	1.04E+03	1.03E+03
	7th	4.86E+02	**3.86E+02**	4.75E+02	4.68E+02	1.09E+03	**1.01E+03**	1.08E+03	1.08E+03	1.07E+03	**1.01E+03**	1.07E+03	1.07E+03	1.09E+03	**1.01E+03**	1.07E+03	1.06E+03
	13th	5.21E+02	**4.12E+02**	5.27E+02	4.92E+02	1.11E+03	**1.03E+03**	1.09E+03	1.09E+03	1.10E+03	**1.03E+03**	1.08E+03	1.09E+03	1.10E+03	**1.04E+03**	1.09E+03	1.09E+03
	19th	5.37E+02	**4.22E+02**	5.48E+02	5.36E+02	1.12E+03	**1.05E+03**	1.11E+03	1.12E+03	1.12E+03	**1.05E+03**	1.10E+03	1.11E+03	1.11E+03	**1.07E+03**	1.10E+03	1.11E+03
	25th	5.89E+02	**5.14E+02**	6.70E+02	5.95E+02	1.14E+03	**1.10E+03**	1.13E+03	1.17E+03	1.14E+03	1.13E+03	**1.12E+03**	1.14E+03	1.14E+03	**1.11E+03**	1.12E+03	1.15E+03
	Mean	5.15E+02	**4.14E+02**	5.17E+02	5.01E+02	1.11E+03	**1.03E+03**	1.09E+03	1.10E+03	1.10E+03	**1.04E+03**	1.09E+03	1.09E+03	1.10E+03	**1.04E+03**	1.08E+03	1.09E+03
	Std	3.84E+01	3.82E+01	5.87E+01	5.46E+01	1.99E+01	2.83E+01	2.51E+01	3.33E+01	2.74E+01	3.74E+01	2.23E+01	2.68E+01	2.01E+01	3.73E+01	2.07E+01	3.02E+01
1E+05	1st	3.35E+02	**2.50E+02**	3.02E+02	2.58E+02	9.30E+02	**9.07E+02**	9.17E+02	9.10E+02	9.29E+02	**9.07E+02**	9.18E+02	9.11E+02	9.28E+02	**9.07E+02**	9.18E+02	9.10E+02
	7th	3.46E+02	**2.67E+02**	3.24E+02	2.75E+02	9.35E+02	**9.07E+02**	9.20E+02	9.11E+02	9.33E+02	**9.07E+02**	9.22E+02	9.12E+02	9.33E+02	**9.07E+02**	9.20E+02	9.12E+02
	13th	3.59E+02	**2.81E+02**	3.35E+02	2.92E+02	9.38E+02	**9.08E+02**	9.24E+02	9.12E+02	9.37E+02	**9.07E+02**	9.26E+02	9.14E+02	9.35E+02	**9.07E+02**	9.24E+02	9.13E+02
	19th	3.69E+02	**2.89E+02**	3.49E+02	3.01E+02	9.45E+02	**9.08E+02**	9.32E+02	9.14E+02	9.39E+02	**9.08E+02**	9.35E+02	9.15E+02	9.38E+02	**9.08E+02**	9.28E+02	9.14E+02
	25th	3.89E+02	**3.05E+02**	4.05E+02	3.15E+02	9.61E+02	**9.08E+02**	9.48E+02	9.17E+02	9.52E+02	**9.08E+02**	9.44E+02	9.22E+02	9.48E+02	**9.08E+02**	9.42E+02	9.25E+02
	Mean	3.59E+02	**2.78E+02**	3.37E+02	2.88E+02	9.40E+02	**9.08E+02**	9.27E+02	9.13E+02	9.37E+02	**9.07E+02**	9.27E+02	9.14E+02	9.36E+02	**9.07E+02**	9.25E+02	9.14E+02
	Std	1.60E+01	1.41E+01	2.15E+01	1.59E+01	7.24E+00	3.83E-01	7.67E+00	1.94E+00	5.31E+00	2.44E-01	7.23E+00	2.77E+00	4.65E+00	2.48E-01	6.42E+00	2.87E+00
3E+05	1st	2.93E+02	**2.22E+02**	2.72E+02	2.45E+02	9.11E+02	**9.06E+02**	9.07E+02	**9.06E+02**	9.11E+02	**9.05E+02**	9.08E+02	9.06E+02	9.10E+02	**9.05E+02**	9.08E+02	9.06E+02
	7th	3.04E+02	**2.45E+02**	2.92E+02	2.53E+02	9.12E+02	**9.06E+02**	9.08E+02	**9.06E+02**	9.12E+02	**9.06E+02**	9.08E+02	**9.06E+02**	9.12E+02	**9.06E+02**	9.08E+02	**9.06E+02**
	13th	3.14E+02	**2.59E+02**	3.08E+02	**2.59E+02**	9.13E+02	**9.06E+02**	9.09E+02	**9.06E+02**	9.13E+02	**9.06E+02**	9.08E+02	**9.06E+02**	9.13E+02	**9.06E+02**	9.09E+02	**9.06E+02**
	19th	3.32E+02	**2.63E+02**	3.24E+02	2.64E+02	9.13E+02	**9.06E+02**	9.09E+02	**9.06E+02**	9.14E+02	**9.06E+02**	9.10E+02	9.07E+02	9.14E+02	**9.06E+02**	9.09E+02	9.07E+02
	25th	3.41E+02	**2.74E+02**	4.04E+02	2.80E+02	9.15E+02	**9.07E+02**	9.12E+02	**9.07E+02**	9.16E+02	**9.06E+02**	9.12E+02	9.07E+02	9.16E+02	**9.07E+02**	9.13E+02	**9.07E+02**
	Mean	3.17E+02	**2.54E+02**	3.11E+02	2.60E+02	9.13E+02	**9.06E+02**	9.09E+02	**9.06E+02**	9.13E+02	**9.06E+02**	9.09E+02	**9.06E+02**	9.13E+02	**9.06E+02**	9.09E+02	**9.06E+02**
	Std	1.48E+01	1.30E+01	2.55E+01	8.07E+00	1.29E+00	2.10E-01	1.24E+00	2.32E-01	1.45E+00	3.28E-01	1.31E+00	2.87E-01	1.42E+00	2.28E-01	1.45E+00	2.59E-01
5E+05	1st	2.53E+02	2.21E+02	2.67E+02	**2.14E+02**	9.08E+02	**9.03E+02**	9.07E+02	9.04E+02	9.08E+02	**9.03E+02**	9.07E+02	9.04E+02	9.08E+02	**9.03E+02**	9.07E+02	9.04E+02
	7th	2.93E+02	2.42E+02	2.90E+02	**2.40E+02**	9.08E+02	**9.05E+02**	9.07E+02	**9.05E+02**	9.08E+02	**9.04E+02**	9.07E+02	9.05E+02	9.08E+02	**9.04E+02**	9.07E+02	9.05E+02
	13th	3.00E+02	**2.43E+02**	2.96E+02	2.48E+02	9.09E+02	**9.06E+02**	9.07E+02	**9.06E+02**	9.08E+02	**9.04E+02**	9.07E+02	9.06E+02	9.09E+02	**9.05E+02**	9.07E+02	9.06E+02
	19th	3.08E+02	**2.53E+02**	3.05E+02	2.54E+02	9.09E+02	**9.06E+02**	9.07E+02	**9.06E+02**	9.09E+02	**9.06E+02**	9.07E+02	**9.06E+02**	9.09E+02	**9.06E+02**	9.07E+02	**9.06E+02**
	25th	3.34E+02	**2.63E+02**	3.89E+02	2.66E+02	9.10E+02	**9.06E+02**	9.08E+02	**9.06E+02**	9.10E+02	**9.06E+02**	9.08E+02	**9.06E+02**	9.09E+02	**9.06E+02**	9.08E+02	9.07E+02
	Mean	2.99E+02	**2.44E+02**	2.98E+02	2.47E+02	9.08E+02	**9.05E+02**	9.07E+02	9.06E+02	9.09E+02	**9.05E+02**	9.07E+02	9.06E+02	9.09E+02	**9.05E+02**	9.07E+02	9.06E+02
	Std	1.63E+01	1.14E+01	2.21E+01	1.18E+01	5.40E-01	8.88E-01	2.19E-01	7.88E-01	4.94E-01	1.07E+00	3.17E-01	5.39E-01	5.06E-01	1.03E+00	2.63E-01	6.46E-01

Table 23. Error values achieved for f_{21}-f_{25} (30D).

30D		f_{21} DE	f_{21} VDE-1	f_{21} VDE-2	f_{21} VDE-3	f_{22} DE	f_{22} VDE-1	f_{22} VDE-2	f_{22} VDE-3	f_{23} DE	f_{23} VDE-1	f_{23} VDE-2	f_{23} VDE-3	f_{24} DE	f_{24} VDE-1	f_{24} VDE-2	f_{24} VDE-3	f_{25} DE	f_{25} VDE-1	f_{25} VDE-2	f_{25} VDE-3
1E+03	1st	1.38E+03	1.38E+03	1.38E+03	**1.34E+03**	1.40E+03	1.37E+03	**1.32E+03**	1.44E+03	1.35E+03	1.38E+03	1.37E+03	**1.34E+03**	1.42E+03	**1.41E+03**	1.42E+03	1.43E+03	1.61E+03	1.60E+03	**1.59E+03**	1.68E+03
	7th	1.41E+03	**1.40E+03**	1.43E+03	1.44E+03	**1.51E+03**	1.52E+03	**1.51E+03**	1.55E+03	**1.40E+03**	1.41E+03	1.41E+03	1.41E+03	**1.45E+03**	**1.45E+03**	**1.45E+03**	1.48E+03	**1.67E+03**	1.68E+03	**1.67E+03**	1.76E+03
	13th	**1.43E+03**	**1.43E+03**	1.44E+03	1.46E+03	**1.56E+03**	**1.56E+03**	1.57E+03	1.59E+03	**1.43E+03**	**1.43E+03**	1.44E+03	1.45E+03	1.48E+03	1.47E+03	**1.46E+03**	1.51E+03	1.72E+03	1.73E+03	**1.70E+03**	1.79E+03
	19th	**1.44E+03**	1.45E+03	1.47E+03	1.49E+03	**1.60E+03**	1.62E+03	1.61E+03	1.66E+03	**1.45E+03**	**1.45E+03**	1.46E+03	1.46E+03	1.49E+03	**1.48E+03**	**1.48E+03**	1.52E+03	1.78E+03	1.74E+03	**1.73E+03**	1.82E+03
	25th	1.49E+03	**1.48E+03**	1.49E+03	1.52E+03	**1.64E+03**	1.74E+03	1.80E+03	1.77E+03	1.51E+03	**1.48E+03**	1.50E+03	1.52E+03	1.54E+03	**1.52E+03**	**1.52E+03**	1.53E+03	1.83E+03	1.83E+03	**1.79E+03**	1.87E+03
	Mean	**1.43E+03**	**1.43E+03**	1.45E+03	1.46E+03	**1.55E+03**	1.56E+03	**1.55E+03**	1.60E+03	**1.43E+03**	**1.43E+03**	**1.43E+03**	1.45E+03	**1.47E+03**	**1.47E+03**	**1.47E+03**	1.50E+03	1.73E+03	1.72E+03	**1.70E+03**	1.79E+03
	Std	2.55E+01	2.95E+01	2.91E+01	3.93E+01	6.96E+01	8.29E+01	1.12E+02	8.89E+01	4.36E+01	2.63E+01	3.39E+01	4.48E+01	3.07E+01	2.56E+01	2.57E+01	3.06E+01	6.32E+01	5.68E+01	5.29E+01	4.81E+01
1E+04	1st	1.22E+03	**9.94E+02**	1.20E+03	1.19E+03	1.13E+03	**1.09E+03**	1.16E+03	1.16E+03	1.22E+03	**1.10E+03**	1.18E+03	1.16E+03	1.18E+03	**1.01E+03**	1.17E+03	1.16E+03	4.57E+02	**3.22E+02**	4.64E+02	5.09E+02
	7th	1.24E+03	**1.13E+03**	1.23E+03	1.22E+03	1.25E+03	**1.15E+03**	1.21E+03	1.21E+03	1.23E+03	**1.15E+03**	1.22E+03	1.24E+03	1.22E+03	**1.10E+03**	1.20E+03	1.20E+03	6.36E+02	**4.24E+02**	5.20E+02	5.50E+02
	13th	1.25E+03	**1.17E+03**	1.23E+03	1.26E+03	1.26E+03	**1.17E+03**	1.27E+03	1.24E+03	1.23E+03	**1.18E+03**	1.24E+03	1.27E+03	1.24E+03	**1.12E+03**	1.21E+03	1.26E+03	7.18E+02	**4.77E+02**	6.77E+02	6.77E+02
	19th	1.26E+03	**1.21E+03**	1.27E+03	1.26E+03	1.29E+03	**1.19E+03**	1.29E+03	1.27E+03	1.27E+03	**1.22E+03**	1.25E+03	1.28E+03	1.26E+03	**1.16E+03**	1.23E+03	1.27E+03	8.22E+02	**5.15E+02**	7.36E+02	7.83E+02
	25th	1.29E+03	**1.25E+03**	1.28E+03	1.28E+03	1.36E+03	**1.29E+03**	1.33E+03	1.35E+03	1.28E+03	**1.25E+03**	1.28E+03	1.31E+03	1.31E+03	**1.19E+03**	1.26E+03	1.31E+03	1.19E+03	**6.55E+02**	1.07E+03	1.04E+03
	Mean	1.25E+03	**1.16E+03**	1.24E+03	1.24E+03	1.26E+03	**1.17E+03**	1.25E+03	1.24E+03	1.25E+03	**1.18E+03**	1.24E+03	1.26E+03	1.24E+03	**1.12E+03**	1.21E+03	1.24E+03	7.41E+02	**4.80E+02**	6.71E+02	6.90E+02
	Std	1.53E+01	6.30E+01	2.32E+01	2.76E+01	4.81E+01	4.09E+01	4.86E+01	4.35E+01	2.18E+01	4.42E+01	2.24E+01	3.48E+01	2.97E+01	4.78E+01	2.48E+01	4.20E+01	1.55E+02	8.73E+01	1.55E+02	1.53E+02
1E+05	1st	8.21E+02	**5.00E+02**	6.28E+02	5.11E+02	9.90E+02	**8.92E+02**	9.78E+02	9.02E+02	8.94E+02	**5.67E+02**	7.13E+02	6.11E+02	6.19E+02	**2.00E+02**	4.51E+02	2.19E+02	2.82E+02	2.20E+02	2.67E+02	**2.12E+02**
	7th	8.63E+02	**5.00E+02**	6.79E+02	5.21E+02	1.02E+03	**9.06E+02**	1.02E+03	9.17E+02	9.72E+02	**5.89E+02**	7.91E+02	6.37E+02	7.14E+02	**2.01E+02**	4.72E+02	2.55E+02	3.45E+02	2.24E+02	3.18E+02	**2.16E+02**
	13th	8.81E+02	**5.01E+02**	7.11E+02	5.31E+02	1.04E+03	**9.17E+02**	1.03E+03	9.22E+02	9.98E+02	**5.97E+02**	8.17E+02	6.51E+02	7.67E+02	**2.01E+02**	5.24E+02	2.59E+02	3.96E+02	2.26E+02	3.66E+02	**2.18E+02**
	19th	9.15E+02	**5.01E+02**	7.62E+02	5.37E+02	1.05E+03	**9.24E+02**	1.05E+03	9.42E+02	1.00E+03	**6.08E+02**	8.40E+02	6.60E+02	7.88E+02	**2.02E+02**	5.91E+02	2.79E+02	4.29E+02	2.30E+02	3.94E+02	**2.21E+02**
	25th	9.51E+02	**5.02E+02**	9.16E+02	6.06E+02	1.07E+03	**9.50E+02**	1.09E+03	9.58E+02	1.03E+03	**6.37E+02**	9.42E+02	7.24E+02	8.30E+02	**2.03E+02**	7.76E+02	3.87E+02	4.50E+02	**2.42E+02**	4.69E+02	2.46E+02
	Mean	8.88E+02	**5.01E+02**	7.29E+02	5.36E+02	1.03E+03	**9.16E+02**	1.03E+03	9.27E+02	9.85E+02	**5.97E+02**	8.15E+02	6.51E+02	7.55E+02	**2.01E+02**	5.52E+02	2.69E+02	3.84E+02	2.27E+02	3.61E+02	**2.20E+02**
	Std	3.32E+01	3.70E-01	6.78E+01	2.29E+01	2.04E+01	1.30E+01	2.27E+01	1.49E+01	3.17E+01	1.57E+01	4.80E+01	2.23E+01	5.04E+01	5.76E-01	9.25E+01	3.36E+01	4.78E+01	5.40E+00	5.46E+01	6.98E+00
3E+05	1st	5.41E+02	**5.00E+02**	5.08E+02	**5.00E+02**	9.44E+02	8.75E+02	9.29E+02	**8.70E+02**	6.46E+02	**5.34E+02**	5.50E+02	**5.34E+02**	2.60E+02	**2.00E+02**	2.14E+02	**2.00E+02**	2.39E+02	2.10E+02	2.25E+02	**2.09E+02**
	7th	5.56E+02	**5.00E+02**	5.10E+02	**5.00E+02**	9.56E+02	8.80E+02	9.52E+02	**8.78E+02**	6.80E+02	**5.34E+02**	5.60E+02	5.49E+02	2.84E+02	**2.00E+02**	2.20E+02	**2.00E+02**	2.58E+02	2.10E+02	2.30E+02	**2.09E+02**
	13th	5.72E+02	**5.00E+02**	5.13E+02	**5.00E+02**	9.64E+02	8.82E+02	9.62E+02	**8.81E+02**	7.12E+02	**5.34E+02**	5.66E+02	5.60E+02	2.97E+02	**2.00E+02**	2.26E+02	**2.00E+02**	2.78E+02	2.11E+02	2.38E+02	**2.09E+02**
	19th	5.86E+02	**5.00E+02**	5.17E+02	**5.00E+02**	9.78E+02	**8.84E+02**	9.68E+02	8.85E+02	7.39E+02	**5.34E+02**	5.74E+02	5.61E+02	3.35E+02	**2.00E+02**	2.36E+02	**2.00E+02**	2.93E+02	2.11E+02	2.95E+02	**2.09E+02**
	25th	6.22E+02	**5.00E+02**	5.33E+02	**5.00E+02**	9.91E+02	8.98E+02	9.81E+02	**8.88E+02**	7.76E+02	**5.47E+02**	6.19E+02	5.73E+02	4.14E+02	**2.00E+02**	2.73E+02	**2.00E+02**	3.26E+02	2.12E+02	3.66E+02	**2.09E+02**
	Mean	5.75E+02	**5.00E+02**	5.15E+02	**5.00E+02**	9.66E+02	8.82E+02	9.59E+02	**8.81E+02**	7.12E+02	**5.35E+02**	5.69E+02	5.56E+02	3.11E+02	**2.00E+02**	2.29E+02	**2.00E+02**	2.78E+02	2.11E+02	2.59E+02	**2.09E+02**
	Std	2.36E+01	2.47E-08	5.82E+00	7.28E-03	1.17E+01	5.50E+00	1.35E+01	4.84E+00	3.66E+01	2.51E+00	1.50E+01	9.91E+00	3.39E+01	4.93E-08	1.26E+01	8.47E-03	2.33E+01	6.25E-01	4.26E+01	5.42E-02
5E+05	1st	5.05E+02	**5.00E+02**	**5.00E+02**	**5.00E+02**	9.09E+02	8.67E+02	9.05E+02	**8.60E+02**	5.34E+02	5.34E+02	5.34E+02	5.34E+02	2.05E+02	**2.00E+02**	2.01E+02	**2.00E+02**	2.25E+02	**2.09E+02**	2.13E+02	**2.09E+02**
	7th	5.07E+02	**5.00E+02**	5.01E+02	**5.00E+02**	9.32E+02	**8.72E+02**	9.19E+02	**8.72E+02**	5.50E+02	**5.34E+02**	**5.34E+02**	**5.34E+02**	2.10E+02	**2.00E+02**	2.01E+02	**2.00E+02**	2.30E+02	**2.09E+02**	2.16E+02	**2.09E+02**
	13th	5.10E+02	**5.00E+02**	5.01E+02	**5.00E+02**	9.39E+02	8.76E+02	9.24E+02	**8.75E+02**	5.70E+02	**5.34E+02**	**5.34E+02**	**5.34E+02**	2.14E+02	**2.00E+02**	2.02E+02	**2.00E+02**	2.34E+02	**2.09E+02**	2.19E+02	**2.09E+02**
	19th	5.14E+02	**5.00E+02**	5.01E+02	**5.00E+02**	9.46E+02	8.79E+02	9.37E+02	**8.77E+02**	5.88E+02	**5.34E+02**	**5.34E+02**	**5.34E+02**	2.18E+02	**2.00E+02**	2.02E+02	**2.00E+02**	2.38E+02	**2.09E+02**	2.20E+02	**2.09E+02**
	25th	5.23E+02	**5.00E+02**	5.02E+02	**5.00E+02**	9.57E+02	8.84E+02	9.59E+02	**8.79E+02**	6.13E+02	**5.34E+02**	**5.34E+02**	**5.34E+02**	2.43E+02	**2.00E+02**	2.04E+02	**2.00E+02**	2.78E+02	2.10E+02	3.18E+02	**2.09E+02**
	Mean	5.11E+02	**5.00E+02**	5.01E+02	**5.00E+02**	9.38E+02	8.76E+02	9.28E+02	**8.73E+02**	5.69E+02	**5.34E+02**	**5.34E+02**	**5.34E+02**	2.16E+02	**2.00E+02**	2.02E+02	**2.00E+02**	2.37E+02	**2.09E+02**	2.31E+02	**2.09E+02**
	Std	4.86E+00	1.14E-13	4.00E-01	1.70E-06	1.19E+01	4.83E+00	1.24E+01	5.00E+00	2.23E+01	4.55E-13	4.22E-13	8.59E-05	1.01E+01	1.19E-12	8.11E-01	1.83E-06	1.17E+01	1.74E-01	3.00E+01	4.50E-02

Table 24. FES to reach the specified accuracy and success rate and performance for f_1-f_{13} (30D).

Func	Strategy	1st	7th	13th	19th	25th	Mean	Std	Successrate		Success Performance	
									3.0e5	5.0e5	3.0e5	5.0e5
f_1	DE	26472	27452	27616	28154	**28714**	2.78E+04	4.99E+02	**100 %**	**100 %**	27781	27781
	VDE-1	**13955**	**16192**	**17311**	**20032**	35161	**1.92E+04**	4.91E+03	**100 %**	**100 %**	**19236**	**19236**
	VDE-2	24739	25529	26405	26773	30767	2.64E+04	1.19E+03	**100 %**	**100 %**	26424	26424
	VDE-3	15786	18601	22398	25107	29367	2.22E+04	3.98E+03	**100 %**	**100 %**	22243	22243
f_2	DE	216416	229360	233489	251649	269462	2.40E+05	1.48E+04	**100 %**	**100 %**	239684	239684
	VDE-1	**108084**	**123154**	**134039**	**137428**	**157964**	**1.32E+05**	1.16E+04	**100 %**	**100 %**	**132137**	**132137**
	VDE-2	148451	172322	176471	190957	218857	1.81E+05	1.60E+04	**100 %**	**100 %**	181486	181486
	VDE-3	157207	172891	181850	192061	227741	1.84E+05	1.77E+04	**100 %**	**100 %**	184253	184253
f_3	DE	-	-	-	-	-	-	-	0 %	0 %	-	-
	VDE-1	-	-	-	-	-	-	-	0 %	0 %	-	-
	VDE-2	-	-	-	-	-	-	-	0 %	0 %	-	-
	VDE-3	-	-	-	-	-	-	-	0 %	0 %	-	-
f_4	DE	457973	-	-	-	-	4.98E+05	8.24E+03	0 %	4 %	-	12457973
	VDE-1	**333315**	**403615**	**449553**	-	-	**4.47E+05**	5.15E+04	0 %	**68 %**	-	**657309**
	VDE-2	-	-	-	-	-	-	-	0 %	0 %	-	-
	VDE-3	394724	464806	-	-	-	4.79E+05	2.96E+04	0 %	44 %	-	1088397
f_5	DE	-	-	-	-	-	-	-	0 %	0 %	-	-
	VDE-1	-	-	-	-	-	-	-	0 %	0 %	-	-
	VDE-2	-	-	-	-	-	-	-	0 %	0 %	-	-
	VDE-3	-	-	-	-	-	-	-	0 %	0 %	-	-
f_6	DE	273385	348740	398937	438005	-	3.94E+05	6.96E+04	12 %	84 %	3280646	468663
	VDE-1	**229497**	**272059**	**285691**	**293913**	353038	**2.83E+05**	2.36E+04	**84 %**	**100 %**	**336829**	**282936**
	VDE-2	258379	407828	491068	-	-	4.41E+05	7.09E+04	4 %	52 %	11035796	848907
	VDE-3	244551	275131	290412	301025	**330673**	2.88E+05	1.78E+04	72 %	**100 %**	399946	287961
f_7	DE	433097	466096	487504	-	-	4.80E+05	2.10E+04	0 %	56 %	-	857736
	VDE-1	83739	92824	96234	100762	**113870**	**9.69E+04**	6.40E+03	**100 %**	**100 %**	**96861**	**96861**
	VDE-2	385601	-	-	-	-	4.80E+05	3.65E+04	0 %	24 %	-	2001147
	VDE-3	**61926**	**71044**	**76162**	**93306**	-	1.61E+05	1.70E+05	80 %	80 %	201871	201871
f_8	DE	-	-	-	-	-	-	-	0 %	0 %	-	-
	VDE-1	-	-	-	-	-	-	-	0 %	0 %	-	-
	VDE-2	-	-	-	-	-	-	-	0 %	0 %	-	-
	VDE-3	-	-	-	-	-	-	-	0 %	0 %	-	-
f_9	DE	61591	64253	65296	66444	70831	6.54E+04	2.09E+03	**100 %**	**100 %**	65397	65397
	VDE-1	54100	58425	60787	63971	76712	6.14E+04	4.88E+03	**100 %**	**100 %**	61377	61377
	VDE-2	46969	50719	51694	53679	**62044**	5.23E+04	3.29E+03	**100 %**	**100 %**	52285	52285
	VDE-3	**43652**	**45961**	**49230**	**51330**	64702	**4.94E+04**	4.21E+03	**100 %**	**100 %**	**49358**	**49358**
f_{10}	DE	-	-	-	-	-	-	-	0 %	0 %	-	-
	VDE-1	-	-	-	-	-	-	-	0 %	0 %	-	-
	VDE-2	-	-	-	-	-	-	-	0 %	0 %	-	-
	VDE-3	-	-	-	-	-	-	-	0 %	0 %	-	-
f_{11}	DE	-	-	-	-	-	-	-	0 %	0 %	-	-
	VDE-1	-	-	-	-	-	-	-	0 %	0 %	-	-
	VDE-2	-	-	-	-	-	-	-	0 %	0 %	-	-
	VDE-3	-	-	-	-	-	-	-	0 %	0 %	-	-
f_{12}	DE	-	-	-	-	-	-	-	0 %	0 %	-	-
	VDE-1	-	-	-	-	-	-	-	0 %	0 %	-	-
	VDE-2	-	-	-	-	-	-	-	0 %	0 %	-	-
	VDE-3	**321143**	-	-	-	-	**4.78E+05**	5.52E+04	0 %	**16 %**	-	**2989702**
f_{13}	DE	-	-	-	-	-	-	-	0 %	0 %	-	-
	VDE-1	-	-	-	-	-	-	-	0 %	0 %	-	-
	VDE-2	-	-	-	-	-	-	-	0 %	0 %	-	-
	VDE-3	-	-	-	-	-	-	-	0 %	0 %	-	-

Table 25. FES to reach the specified accuracy and success rate and performance for f_{14}-f_{25} (30D).

Func	Strategy	1st	7th	13th	19th	25th	Mean	Std	Successrate		Success Performance	
									3.0e5	5.0e5	3.0e5	5.0e5
f_{14}	DE	-	-	-	-	-	-	-	0 %	0 %	-	-
	VDE-1	-	-	-	-	-	-	-	0 %	0 %	-	-
	VDE-2	-	-	-	-	-	-	-	0 %	0 %	-	-
	VDE-3	-	-	-	-	-	-	-	0 %	0 %	-	-
f_{15}	DE	-	-	-	-	-	-	-	0 %	0 %	-	-
	VDE-1	-	-	-	-	-	-	-	0 %	0 %	-	-
	VDE-2	-	-	-	-	-	-	-	0 %	0 %	-	-
	VDE-3	-	-	-	-	-	-	-	0 %	0 %	-	-
f_{16}	DE	-	-	-	-	-	-	-	0 %	0 %	-	-
	VDE-1	-	-	-	-	-	-	-	0 %	0 %	-	-
	VDE-2	-	-	-	-	-	-	-	0 %	0 %	-	-
	VDE-3	-	-	-	-	-	-	-	0 %	0 %	-	-
f_{17}	DE	-	-	-	-	-	-	-	0 %	0 %	-	-
	VDE-1	-	-	-	-	-	-	-	0 %	0 %	-	-
	VDE-2	-	-	-	-	-	-	-	0 %	0 %	-	-
	VDE-3	-	-	-	-	-	-	-	0 %	0 %	-	-
f_{18}	DE	-	-	-	-	-	-	-	0 %	0 %	-	-
	VDE-1	-	-	-	-	-	-	-	0 %	0 %	-	-
	VDE-2	-	-	-	-	-	-	-	0 %	0 %	-	-
	VDE-3	-	-	-	-	-	-	-	0 %	0 %	-	-
f_{19}	DE	-	-	-	-	-	-	-	0 %	0 %	-	-
	VDE-1	-	-	-	-	-	-	-	0 %	0 %	-	-
	VDE-2	-	-	-	-	-	-	-	0 %	0 %	-	-
	VDE-3	-	-	-	-	-	-	-	0 %	0 %	-	-
f_{20}	DE	-	-	-	-	-	-	-	0 %	0 %	-	-
	VDE-1	-	-	-	-	-	-	-	0 %	0 %	-	-
	VDE-2	-	-	-	-	-	-	-	0 %	0 %	-	-
	VDE-3	-	-	-	-	-	-	-	0 %	0 %	-	-
f_{21}	DE	-	-	-	-	-	-	-	0 %	0 %	-	-
	VDE-1	-	-	-	-	-	-	-	0 %	0 %	-	-
	VDE-2	-	-	-	-	-	-	-	0 %	0 %	-	-
	VDE-3	-	-	-	-	-	-	-	0 %	0 %	-	-
f_{22}	DE	-	-	-	-	-	-	-	0 %	0 %	-	-
	VDE-1	-	-	-	-	-	-	-	0 %	0 %	-	-
	VDE-2	-	-	-	-	-	-	-	0 %	0 %	-	-
	VDE-3	-	-	-	-	-	-	-	0 %	0 %	-	-
f_{23}	DE	-	-	-	-	-	-	-	0 %	0 %	-	-
	VDE-1	-	-	-	-	-	-	-	0 %	0 %	-	-
	VDE-2	-	-	-	-	-	-	-	0 %	0 %	-	-
	VDE-3	-	-	-	-	-	-	-	0 %	0 %	-	-
f_{24}	DE	-	-	-	-	-	-	-	0 %	0 %	-	-
	VDE-1	-	-	-	-	-	-	-	0 %	0 %	-	-
	VDE-2	-	-	-	-	-	-	-	0 %	0 %	-	-
	VDE-3	-	-	-	-	-	-	-	0 %	0 %	-	-
f_{25}	DE	-	-	-	-	-	-	-	0 %	0 %	-	-
	VDE-1	-	-	-	-	-	-	-	0 %	0 %	-	-
	VDE-2	-	-	-	-	-	-	-	0 %	0 %	-	-
	VDE-3	-	-	-	-	-	-	-	0 %	0 %	-	-

In 30D only 6 functions were reliably solved by the algorithms (f_1, f_2, f_3, f_4, f_6, f_7 and f_9). Table 26 shows how the algorithms ranked. In 10D, VDE-1 and VDE-3 rank very close but in 30D VDE-1 performs better having the best performance for 5/6 functions.

Table 26. Ranking based on success performance of the algorithms for solved 30D functions. Table illustrates how many times the algorithm ranked $1^{st}/2^{nd}/3^{rd}/4^{th}$.

Strategy	1^{st}	2^{nd}	3^{rd}	4^{th}	Avg. Rank
DE	0	0	3	3	3.5
VDE-1	5	0	1	0	1.33
VDE-2	0	2	1	3	3.17
VDE-3	1	4	1	0	2

In order to see the overall performance of the algorithms, Table 27 combines the 10D and 30D results. The table clearly shows that VDE-1 was the overall best with VDE-3 a close second. VDE-3 ranked third while DE performed the worst.

Table 27. Ranking based on success performance of the algorithms for all solved functions (10D and 30D). Table illustrates how many times the algorithm ranked $1^{st}/2^{nd}/3^{rd}/4^{th}$.

Strategy	1^{st}	2^{nd}	3^{rd}	4^{th}	Avg. Rank
DE	1	1	6	8	3.31
VDE-1	8	5	2	1	1.75
VDE-2	2	2	6	6	3.00
VDE-3	5	8	2	1	1.94

5.3. Convergence and value (F_{EMA}, CR_{EMA}, c) distribution graphs

In this section more detailed figures and graphs are provided for the convergence properties of all algorithms for selected functions. The semi-log convergence graphs show the error value achieved against the number of function evaluations (FES). Convergence graphs include the error values of the best, median and the worst run for each algorithm. The runs are sorted by the number of function evaluations to reach the specified accuracy or the error value at MAXFES if the specified accuracy is not reached. Because the functions show great variation, the scale of FES shown in the figures has been adjusted accordingly to present the data in a suitable scale. The data in the graphs has been gathered from every 100th function evaluations.

In addition to the convergence graphs, figures of the control parameter behavior for the best, median and the worst run are provided for the VDE-algorithms. VDE-1 graphs show the value of F_{EMA} for each run and VDE-2 graphs show the value of CR_{EMA}. Also provided is the maximum, mean and the minimum value of F_{EMA} and CR_{EMA} during the optimization. The max, mean and min values have been gathered until the median run reaches an accuracy of 1.0E-08 or the maximum function evaluation (MAXFES) value.

For VDE-3, the graphs show both the F_{EMA} and CR_{EMA} values in addition to the value of the variance factor c which is calculated by these EMA values. Also the max, mean and min value of c during the optimization is provided for VDE-3. For other VDE algorithms the value of c is not provided in the graphs since its behavior is linear with the control parameter which is being adapted.

The functions have been selected according to the achieved results. The performance of one algorithm has been exceedingly better, or the function in question is otherwise interesting. The selected functions are f_2(10D), f_2(30D), f_3(10D), f_6(10D), f_6(30D), f_7(30D), f_9(30D), f_{11}(10D), f_{12}(10D) and f_{15}(10D).

First to be analyzed is f_2 which is a unimodal and non-separable function. A 3D landscape illustration of the 2D version of the function is shown in Figure 6.

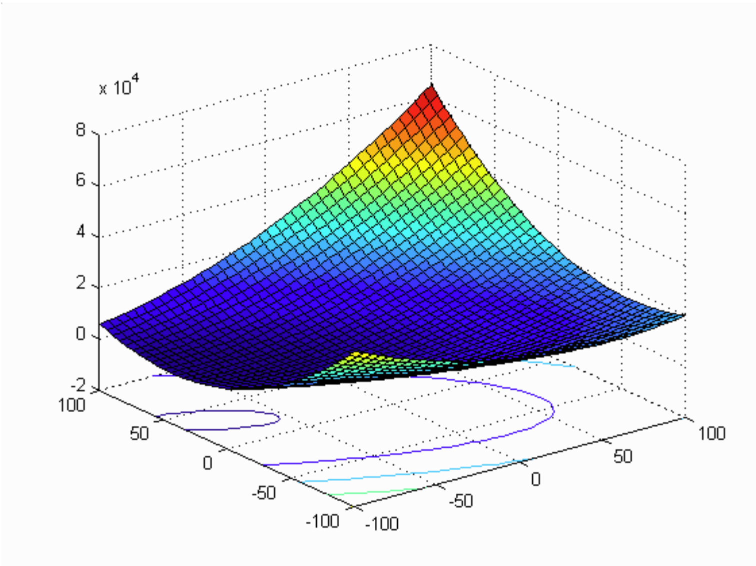

Figure 6. f_2:*Shifted Schwefel's Problem 1.2*. 3-D map for 2-D function. **(**Suganthan *et al.* 2005)

Convergence of the algorithms for f_2 is illustrated in Figure 7. Each algorithm converges reasonably steady towards the global optimum. The best run is achieved by VDE-1 and the worst by VDE-2. Overall, VDE-2 was the best. In VDE-3 it is interesting to see that the worst run actually has the best error value in the early part of the optimization but its convergence slows down later.

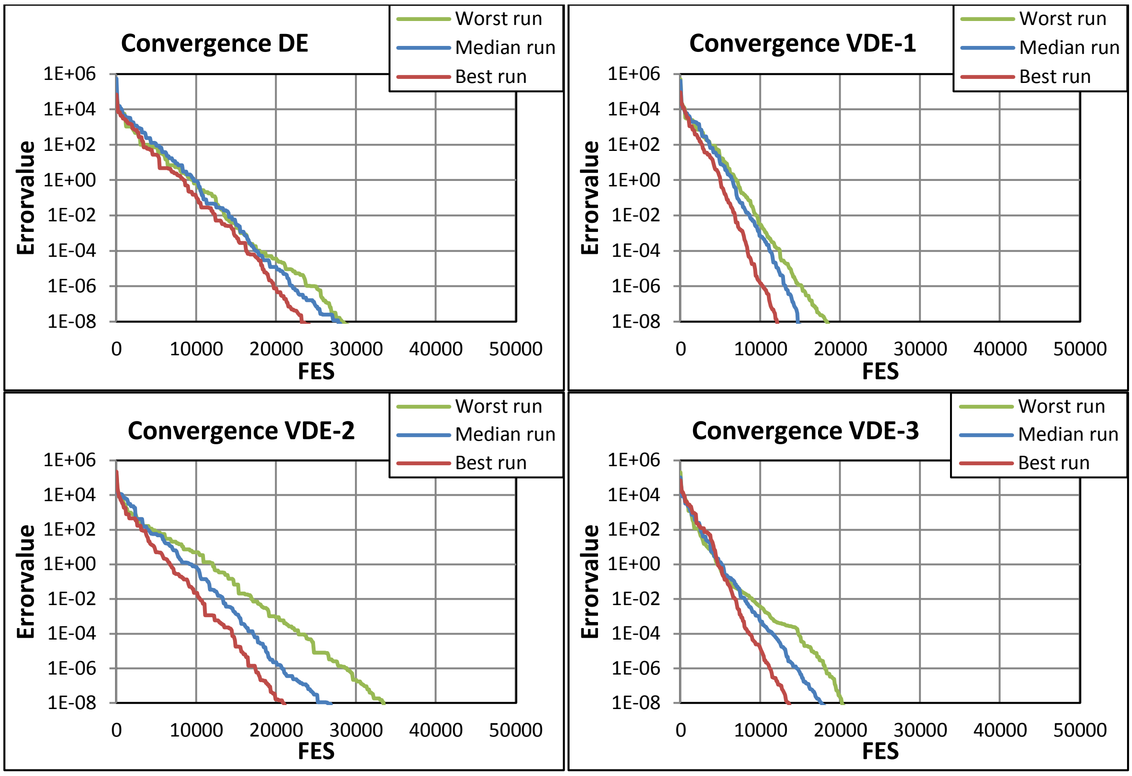

Figure 7. Convergence graphs for f_2(10D).

As a reminder, the scale of the following graphs for the VDE-algorithms is the same as in the convergence graphs so that the reader can compare them as need be. The EMA value distribution graphs show the maximum, mean and the minimum value for the corresponding parameter of all runs during the optimization. This data has been gathered until the median run has reached an error value of 1.0E-08 or the maximum has allowed function evaluations.

For VDE-1 (Figure 8) in the best run the F_{EMA} value falls very fast to ≈0.7 while in the median and in the worst run the value stays relatively near the starting point of 0.9 and falls later to the ≈0.7 values. Distribution graph also shows the same behaviour and the F_{EMA} values are steadily falling towards the lower bound of the value range which is ≈0.58 with the settings used for the test.

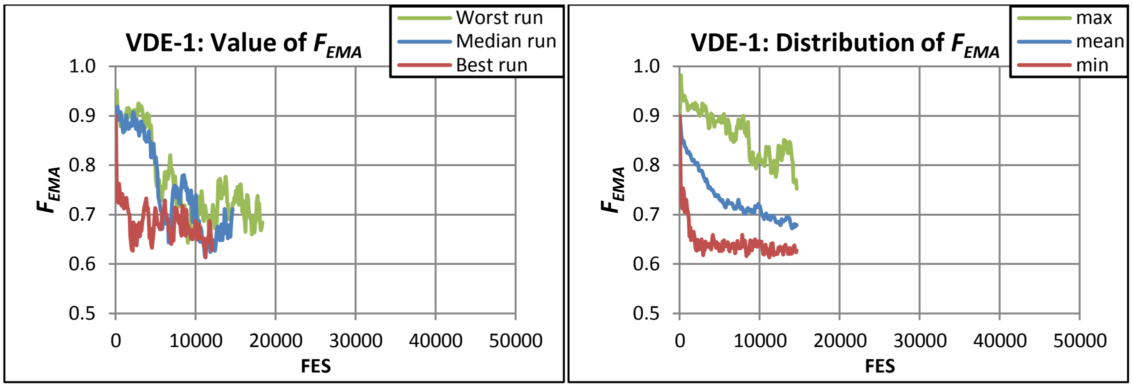

Figure 8. VDE-1: F_{EMA} values for best, median and worst runs, and F_{EMA} max, mean and min value during optimization for f_2(10D).

For VDE-2 (Figure 9) the most interesting run is the worst run where the CR_{EMA} value stays below 0.9 for the most part of the optimization while the best and median runs CR_{EMA} values are above 0.9 and also result in better convergence speed compared to DE. Noteworthy is also the fact that the minimum value of CR_{EMA} does not fall near the lower bound of the value range which was ≈0.62 during the optimization.

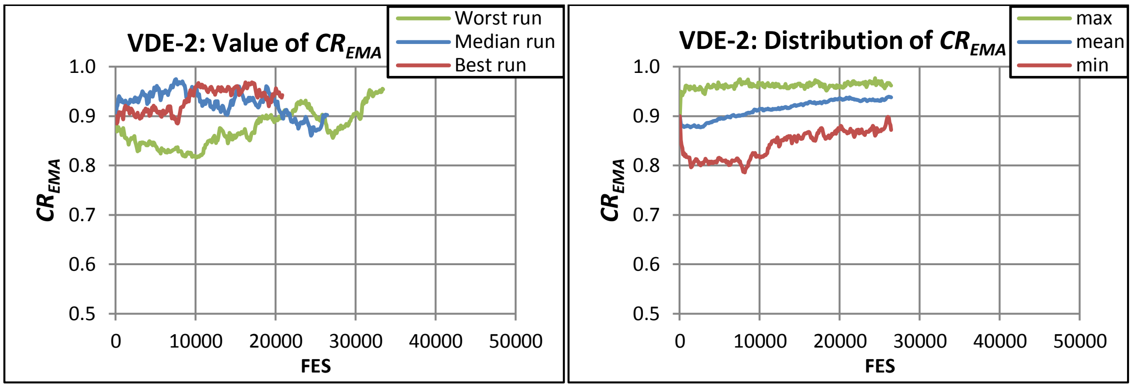

Figure 9. VDE-2: CR_{EMA} values for best, median and worst runs, and CR_{EMA} max, mean and min value during optimization for f_2(10D).

For VDE-3 (Figure 10) the parameter values of the best, median and the worst are quite identical. F_{EMA} falls to range of 0.6-0.8 and CR_{EMA} varies in range 0.8-1.0. Convergence of the worst run seems to slow down at approximately 10000 FES as the CR_{EMA} value falls and F_{EMA} rises. Variance factor c mean values are little greater than 1.3 so VDE-3 parameter values tend to be nearer to the lower bound than the upper bound. Overall, the behavior of the control parameters is individually similar in VDE-1 and VDE-2.

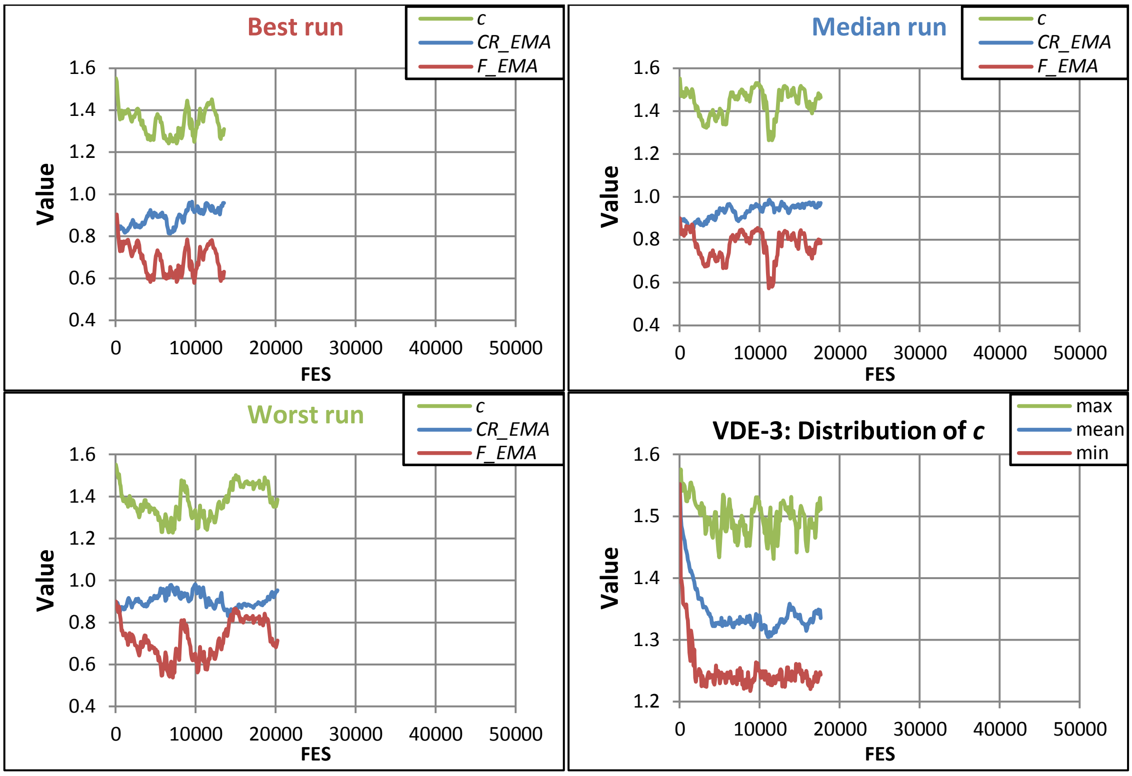

Figure 10. VDE-3: F_{EMA}, CR_{EMA} and c values for best, median and worst runs, and c max, mean and min value during optimization for f_2(10D).

Next are shown the convergence graphs for the 30D version of f_2. Again VDE-1 has clearly the best convergence speed compared to the other algorithms. But also VDE-2 is converging faster and its performance is much better than that of DE. VDE-2 shows similar performance with VDE-3.

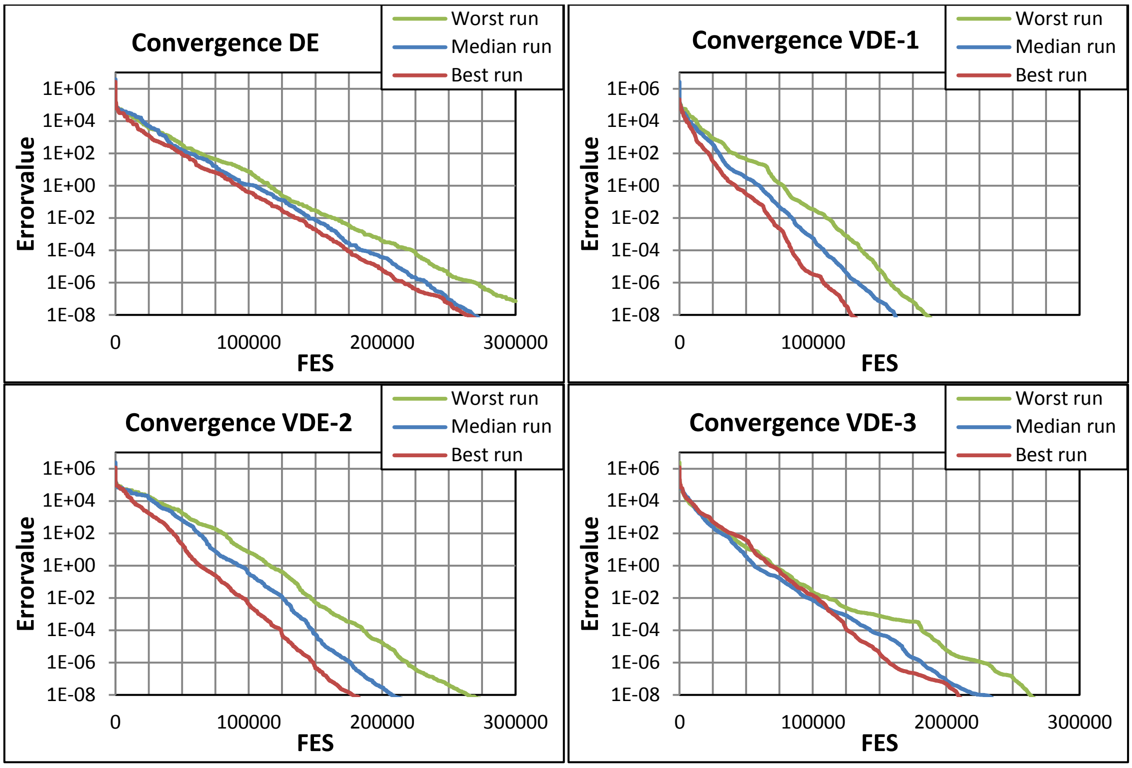

Figure 11. Convergence graphs for f_2(30D).

Values of F_{EMA} for VDE-1 are again near 0.7, as in 10D, and consistent with each run. The high fluctuation of the value is explained by the fact that the population is small (NP=20) and the new F values are generated at each generation.

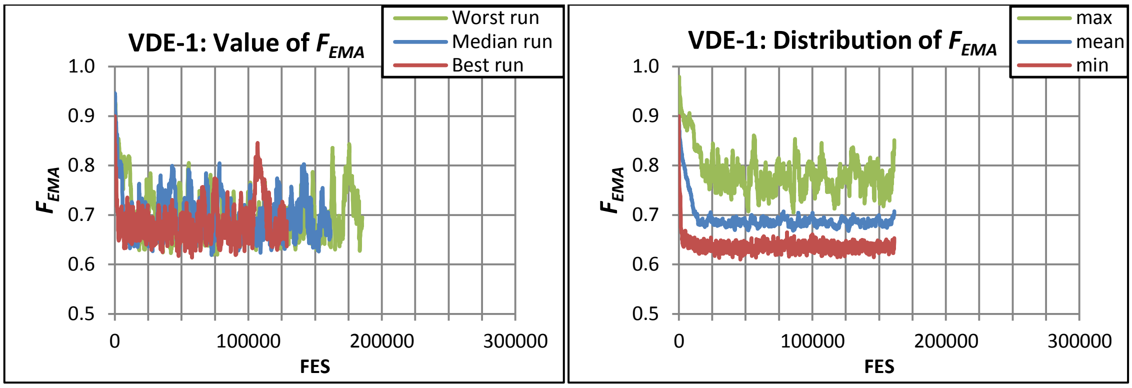

Figure 12. VDE-1: F_{EMA} values for best, median and worst runs, and F_{EMA} max, mean and min value during optimization for f_2(30D).

For VDE-2 the behaviour of CR_{EMA} is very consistent for each run. In the best run the value rises to ≈0.95 the fastest and in the worst run the slowest. The distribution graph shows that for every run the value of CR_{EMA} has stabilized >0.9. VDE-2 performed better than DE in this function so a minor change in the *CR* can be quite effective. Maximum *CR* value with variance factor limit 1.6 and *NP*=20 is ≈0.99.

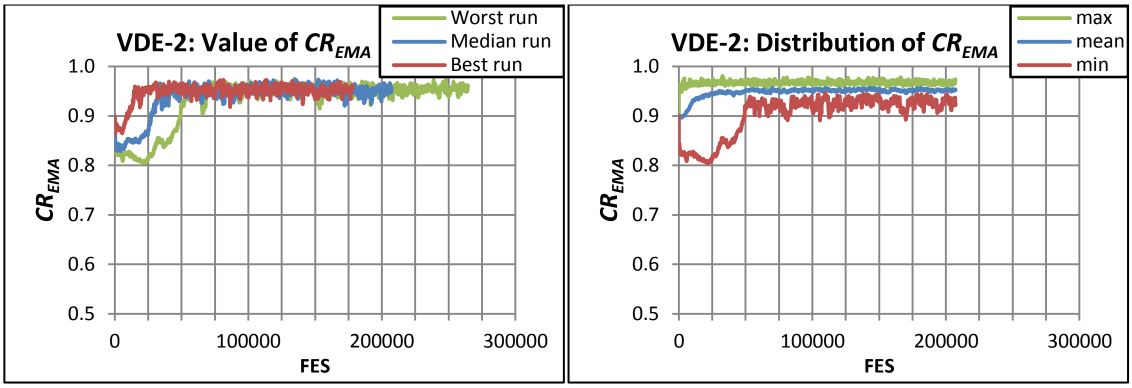

Figure 13. VDE-2: CR_{EMA} values for best, median and worst runs, and CR_{EMA} max, mean and min value during optimization for f_2(30D).

While the performance of VDE-1 and VDE-3 was alike in 10D, in 30D VDE-3 no longer performs that well. In all the runs the control parameter values are close together. CR_{EMA} is mostly >0.9 and F_{EMA} <0.8. VDE-3 was set up with a lower variance factor boundary, 1.2, than VDE-1, 1.25, and this allows lower F_{EMA} values which seem to slow down convergence. The *c* distribution graph shows that the average value of the variance factor *c* was a little greater than 1.3. Again the values fluctuate greatly due to the small *NP* which is set to 20.

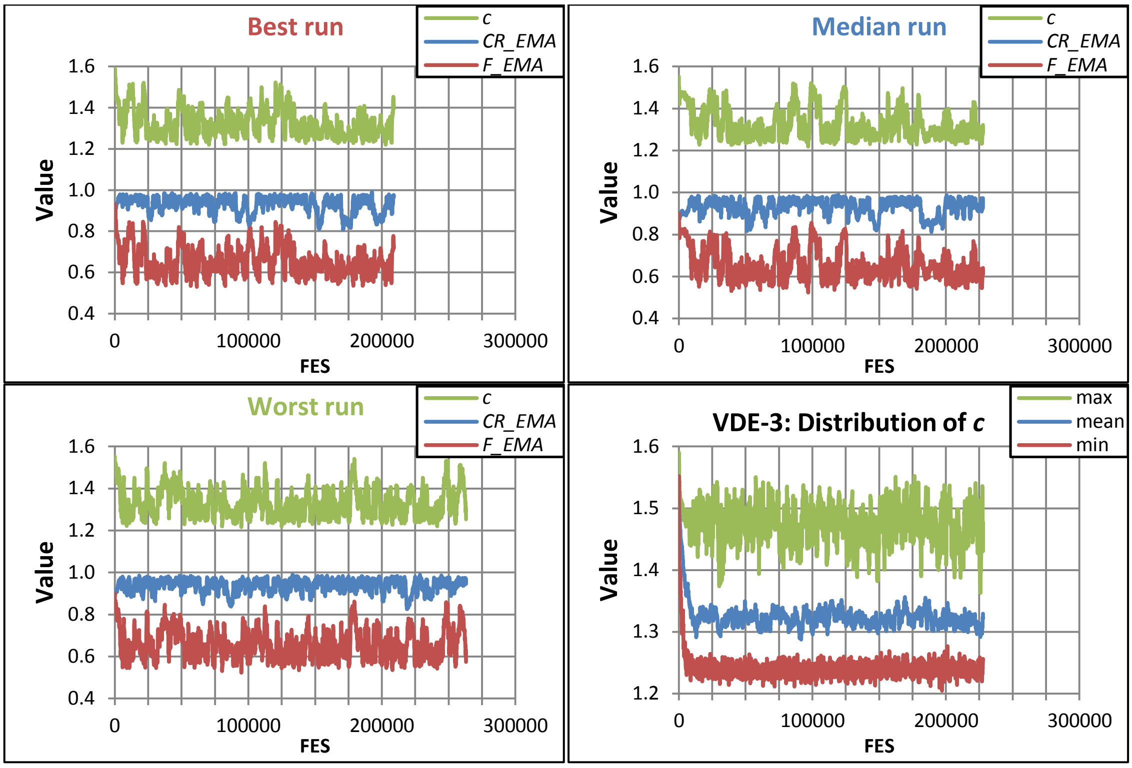

Figure 14. VDE-3: F_{EMA}, CR_{EMA} and c values for best, median and worst runs, and c max, mean and min value during optimization for f_2(30D).

Now for the graphs of f_3(10D) (Figure 15**)** which is also a unimodal non-separable function. The difference between the other unimodal 10D functions in the benchmark was that f_3 was run with a larger population: NP=50. The convergence graphs in Figure 16 show great differences in convergence speeds. VDE-3 solves the function in approximately 1/5 of the number of function evaluations when compared to DE and approximately 1/2 compared to VDE-1 which also performs well. VDE-2 performs third best.

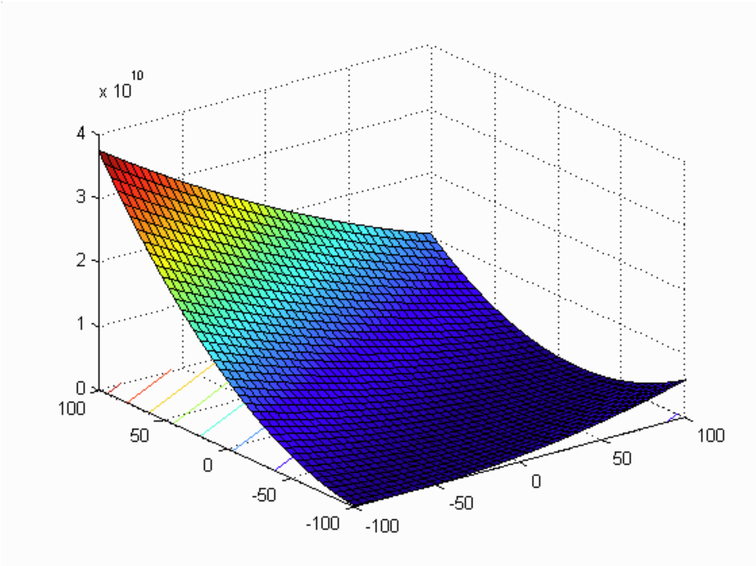

Figure 15. f_3: *Shifted Rotated High Conditioned Elliptic Function*. 3-D map for 2-D function. **(**Suganthan *et al.* 2005)

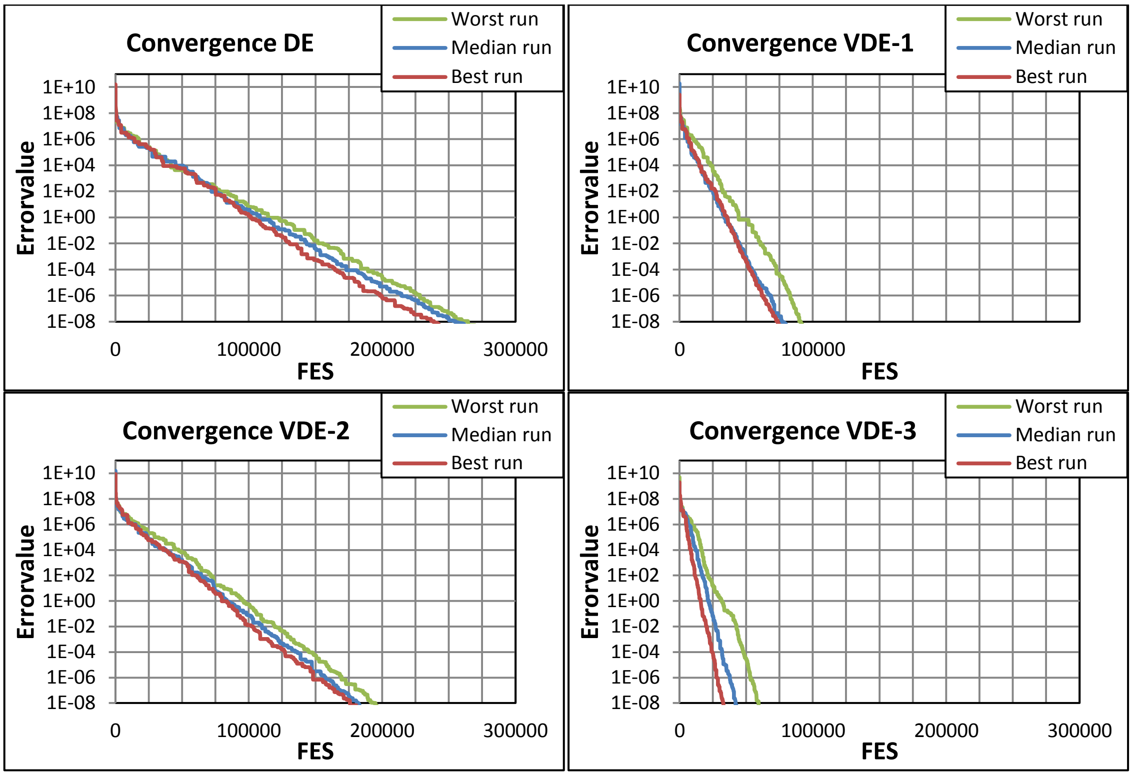

Figure 16. Convergence graphs for f_3(10D).

As in the previous function, VDE-1 F_{EMA} values in Figure x again fall very fast to <0.7 with the exception of the worst run in which the value is higher until ≈50000 FES. In the convergence graph it is also shown that the convergence was a bit slower until that point. The larger *NP* size allows a minimum value of ≈0.57 for *F* and the value distribution is slightly lower compared to the values for f_2.

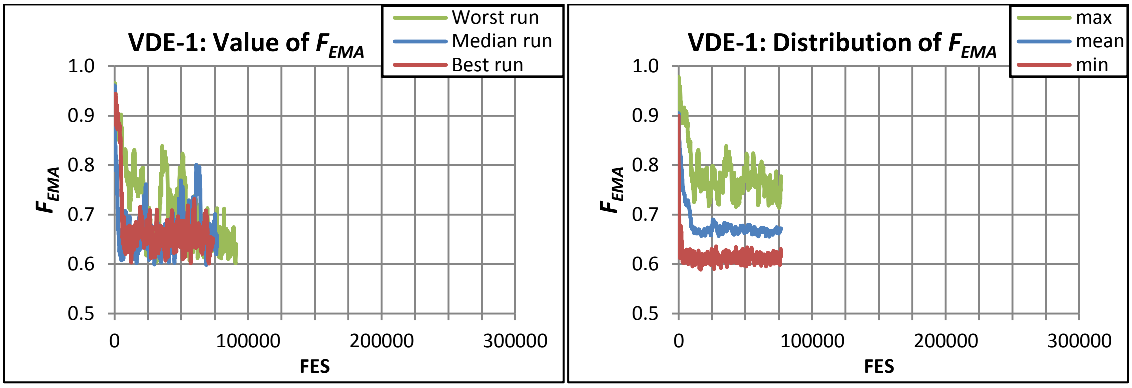

Figure 17. VDE-1: F_{EMA} values for best, median and worst runs, and F_{EMA} max, mean and min value during optimization for f_3(10D).

VDE-2 CR_{EMA} values again are consistently >0.9 which provide better performance compared to DE. The allowed maximum *CR* value with variance factor limit 1.6 and *NP*=50 is ≈0.98.

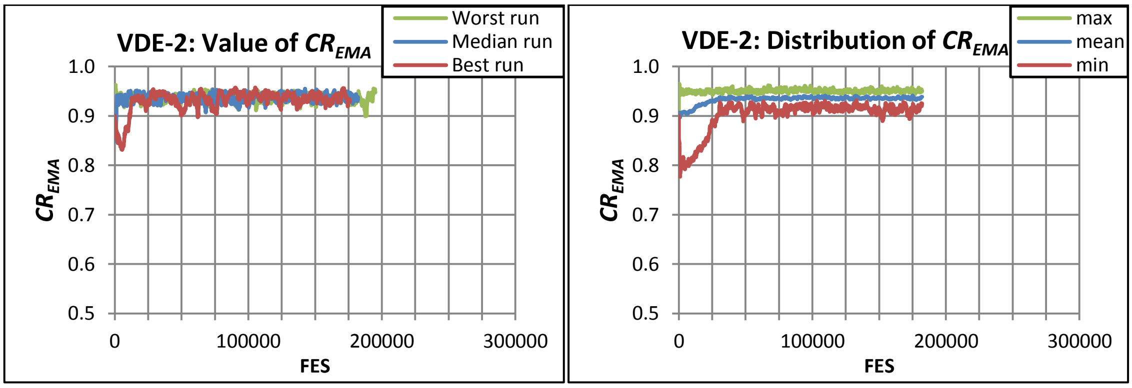

Figure 18. VDE-2: CR_{EMA} values for best, median and worst runs, and CR_{EMA} max, mean and min value during optimization for f_3(10D).

The reason for the good VDE-3 performance seems to be that lower *F* values are allowed. F_{EMA} values again fall rapidly near 0.6 while CR_{EMA} values stay in range 0.9-1.0. Compared to f_2 , lower *F* values are more effective for f_3 and confirm the fact that the optimal control parameter values vary with the function in question. High *CR* values are better at least for the unimodal functions.

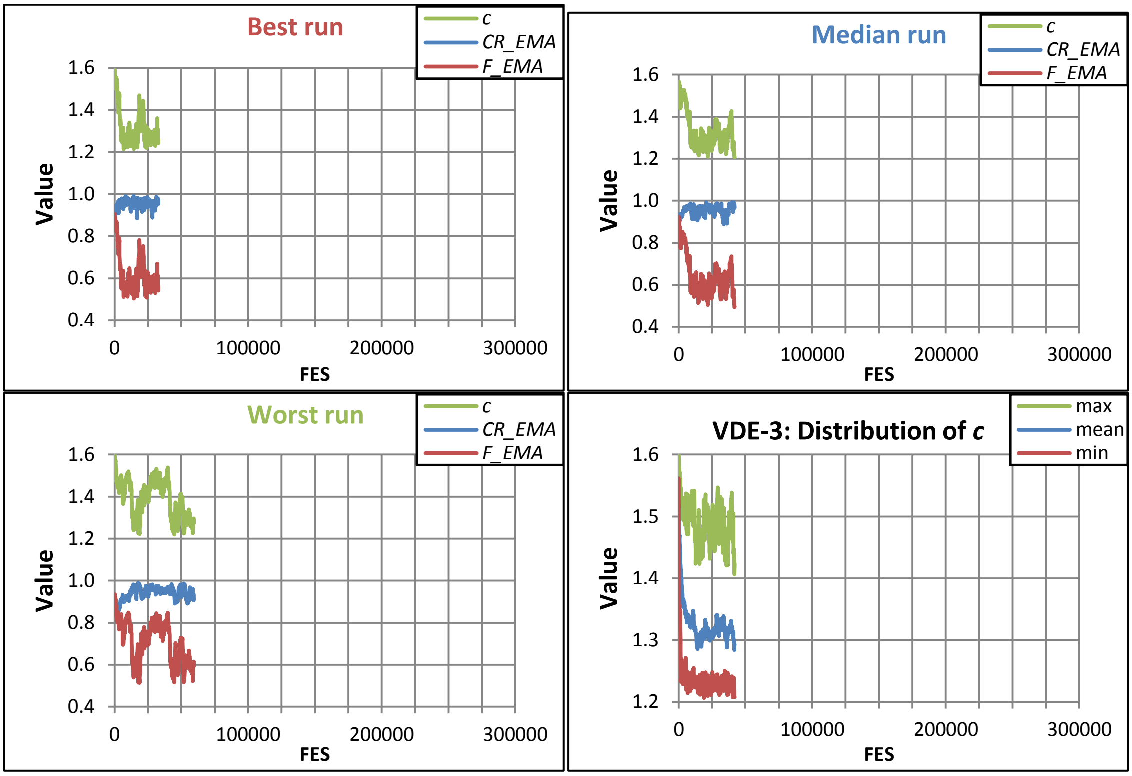

Figure 19. VDE-3: F_{EMA}, CR_{EMA} and *c* values for best, median and worst runs, and *c* max, mean and min value during optimization for f_3(10D).

Overall, in unimodal non-separable functions in VDE-1 and VDE-3 values of *F* systematically approached the lower boundary of the allowed value range and thus speeded the convergence. In VDE-2 and VDE-3 the *CR* values, in contrast, approached the higher boundary of the value range. Next some of the multimodal functions are analyzed.

First multimodal function is the easiest unimodal function f_6 (Figure 20) which has a narrow valley from the local optimum to the global optimum. Population size *NP*=20 was used for this function which caused some failed runs. DE had 100% success rate in 10D for f_6 and it was the only function in which DE had the best performance. For the VDE-algorithms a larger population size fixed the success rate in the expense of speed but *NP* was kept low for the actual test firstly for comparison and secondly in order to analyze what might be the problem that causes the premature convergence or stagnation.

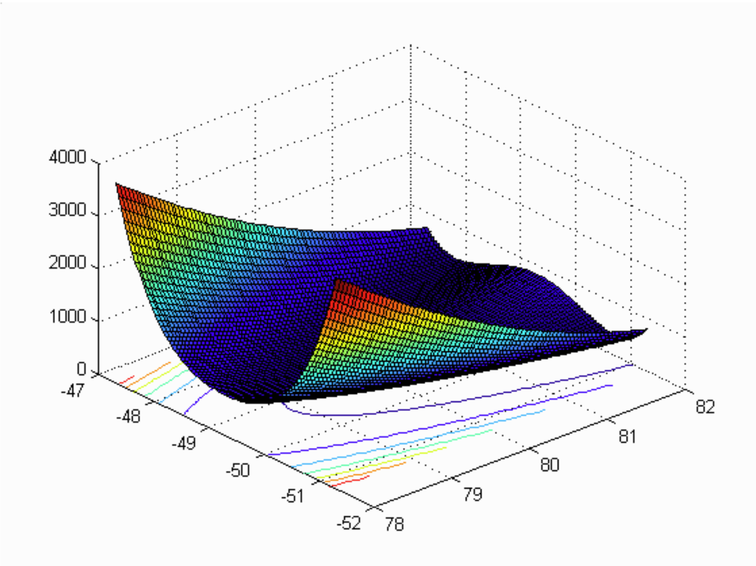

Figure 20. f_6: *Shifted Rosenbrock's Function*. 3-D map for 2-D function. **(**Suganthan *et al.* 2005)

The convergence graphs (Figure 21) show that VDE-1 and VDE-3 converge much faster in the beginning and this seems to cause premature convergence.

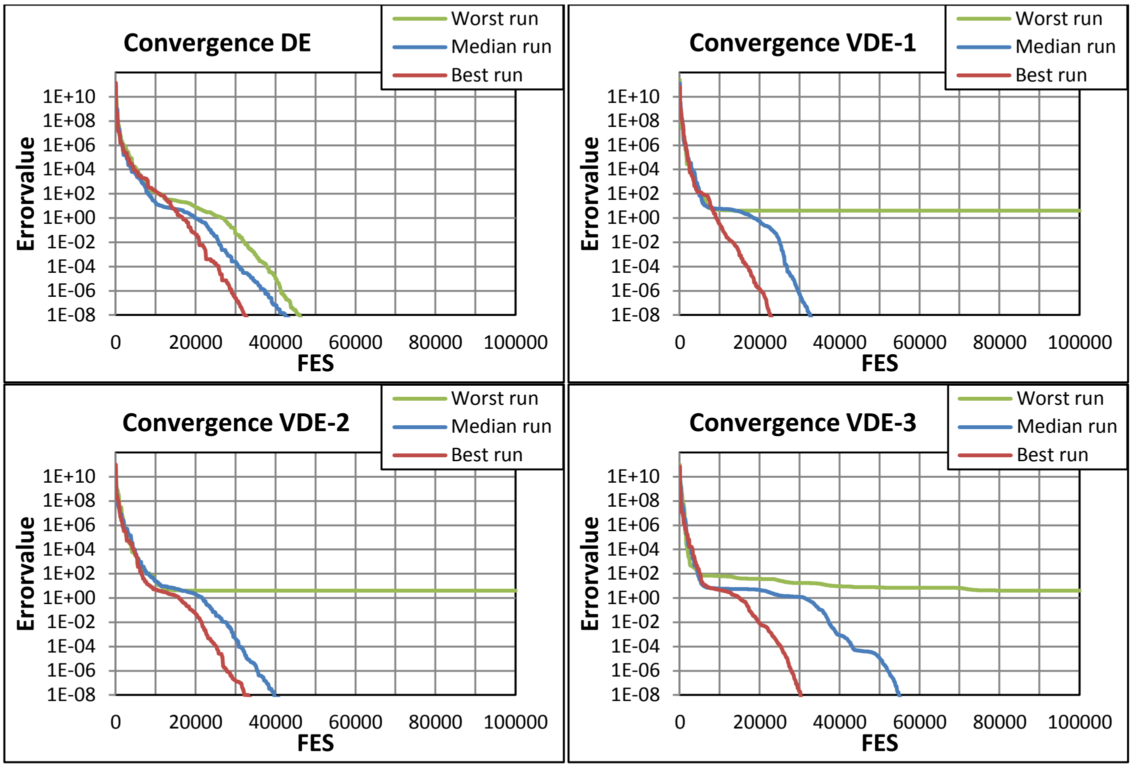

Figure 21. Convergence graphs for f_6(10D).

For VDE-1 the behavior of F_{EMA} is again similar with the unimodal functions and the average approaches value of ≈0.7. There is no major difference with the best, median and the worst runs. The high variance in the worst run indicates that the population is clustering in the local optimum. As the selection operation selects a trial vector of an equal error value to the next generation, the current F value updates the F_{EMA} value for the majority of the function evaluations.

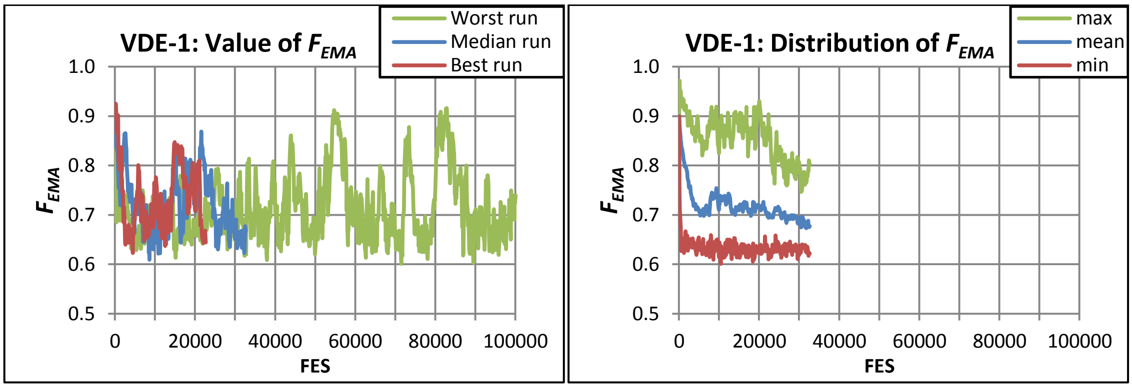

Figure 22. VDE-1: F_{EMA} values for best, median and worst runs, and F_{EMA} max, mean and min value during optimization for f_6(10D).

CR_{EMA} values for VDE-2 had more variance compared to the unimodal functions. In the beginning successful trial vectors are found very quickly and the some of the CR values are approaching the lower bound. After the algorithm has found a local optimum (≈10000 FES), the values start to rise again and average values are above 0.9. The performance of the best and median run was very even with DE.

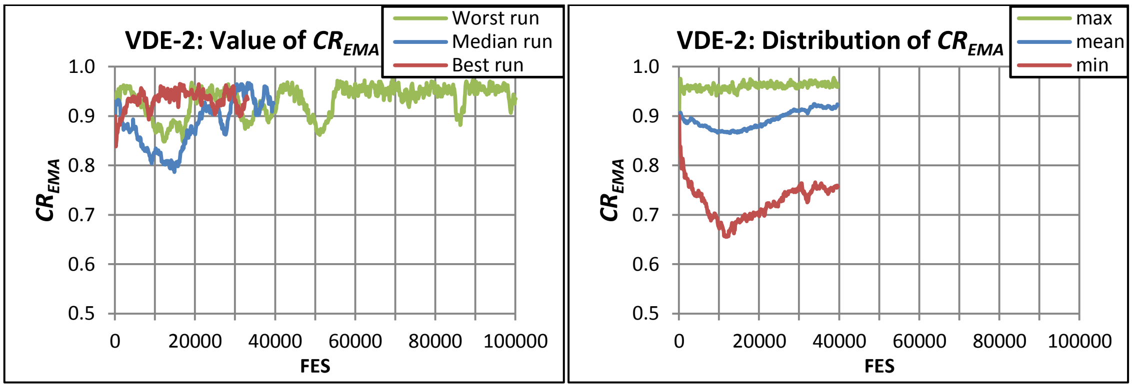

Figure 23. VDE-2: CR_{EMA} values for best, median and worst runs, and CR_{EMA} max, mean and min value during optimization for f_6(10D).

VDE-3 had the most unstable convergence of the algorithms. VDE-3 has very fast convergence in the beginning but slow in the end. The worst run actually converges the fastest from early on. Most of the population vectors converge to local optimums very fast and the algorithm struggles to find better solutions after that. In the best run CR_{EMA} value falls early to 0.8 and this might slow the convergence down enough.

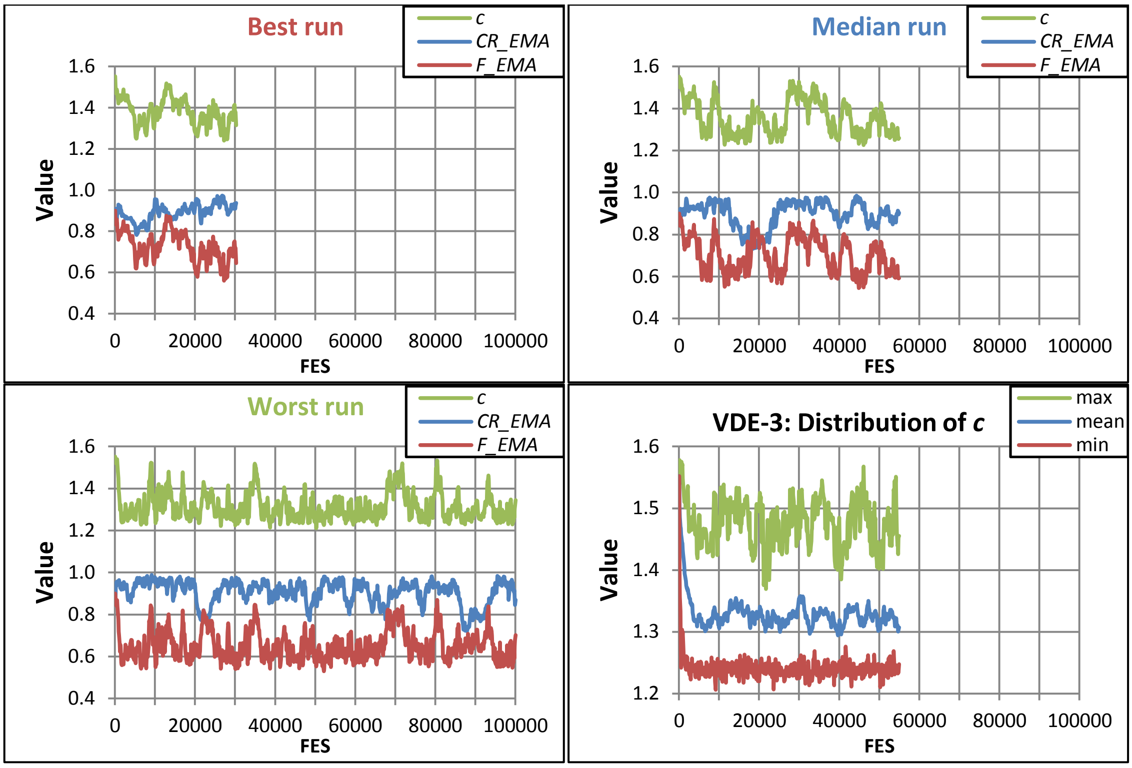

Figure 24. VDE-3: F_{EMA}, CR_{EMA} and c values for best, median and worst runs, and c max, mean and min value during optimization for f_6(10D).

In 10D the *NP* value was too small for reliable convergence. In 30D VDE-1 and VDE-3 was run with a larger population, *NP*=50, and this lead to a good effect on the performance. DE and VDE-2 was run with *NP*=20. In Figure 25 it is clearly shown that VDE-3 has the best and the most stable performance in 30D and, despite the larger population, the convergence rate at the beginning is similar with DE. VDE-1 is converging more slowly early, but when the global optimum is found the convergence rate is good and very stable.

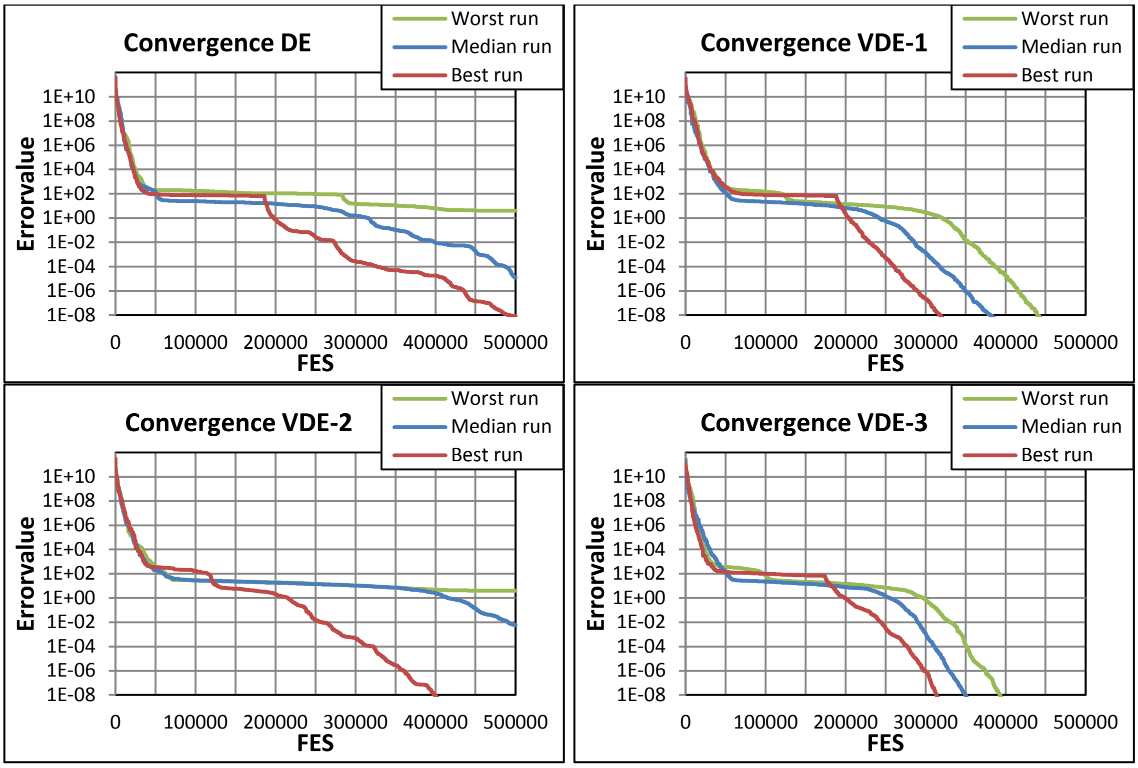

Figure 25. Convergence graphs for f_6(30D).

A closer look to VDE-1 F_{EMA}behaviour shows that the values are stable in range ≈0.6-0.8 averaging 0.65. Interestingly the F_{EMA}value of the best run has a spike right at the point when the algorithm starts to converge towards the global optimum. Other than that, the values are consistent.

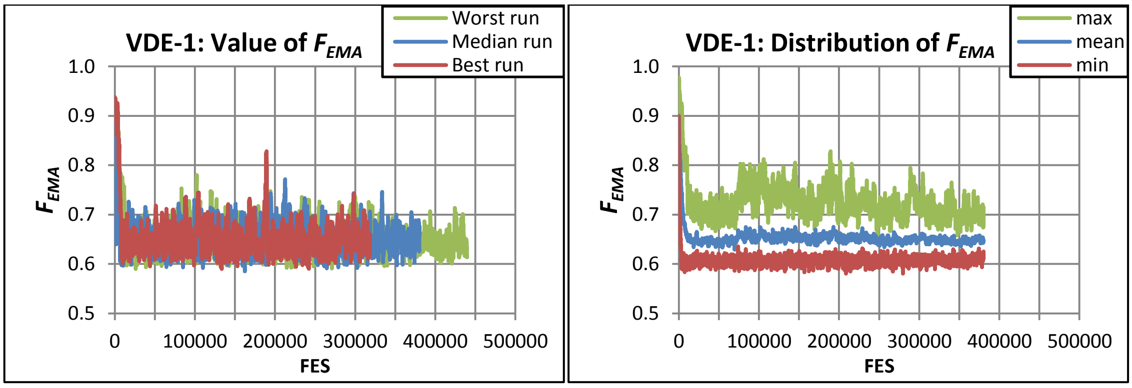

Figure 26. VDE-1: F_{EMA} values for best, median and worst runs, and F_{EMA} max, mean and min value during optimization for f_6(30D).

VDE-2 has the worst performance of the algorithms overall. The best run is good and for that run the CR_{EMA} value stays fairly high. For other runs CR_{EMA} falls more rapidly towards the lower bound. Distribution graph confirms that the values are lower compared to unimodal functions and compared with 10D results of f_6. Again the high variance is explained by the low *NP*=20 which allows many more *CR* value generations, and in the early part of the optimization better solutions are found more regularly. Interestingly in the best and median runs the CR_{EMA} starts to rise when the convergence slows down. The algorithm clearly adjusts itself. The same kind of behaviour was evident also in 10D.

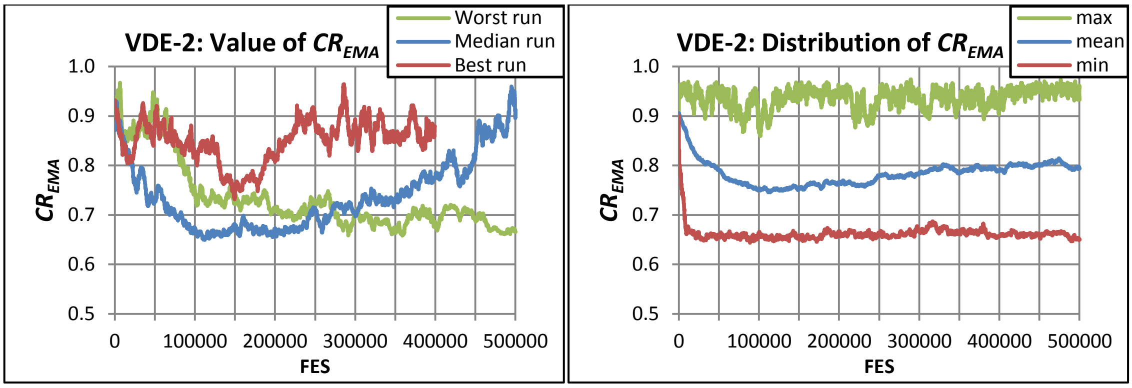

Figure 27. VDE-2: CR_{EMA} values for best, median and worst runs, and CR_{EMA} max, mean and min value during optimization for f_6(30D).

For VDE-3 F_{EMA} values fall rapidly as in VDE-1. VDE-3 settings allow lower *F* values and values are on average around 0.6. CR_{EMA} values again vary more compared with unimodal functions but the lower bound limit of 0.7 ensures that values stay in effective range. The *c* distribution graph illustrates that the variance factor is very low early when the convergence is the fastest and that it slowly rises when the local optimums are found at about 50000 FES.

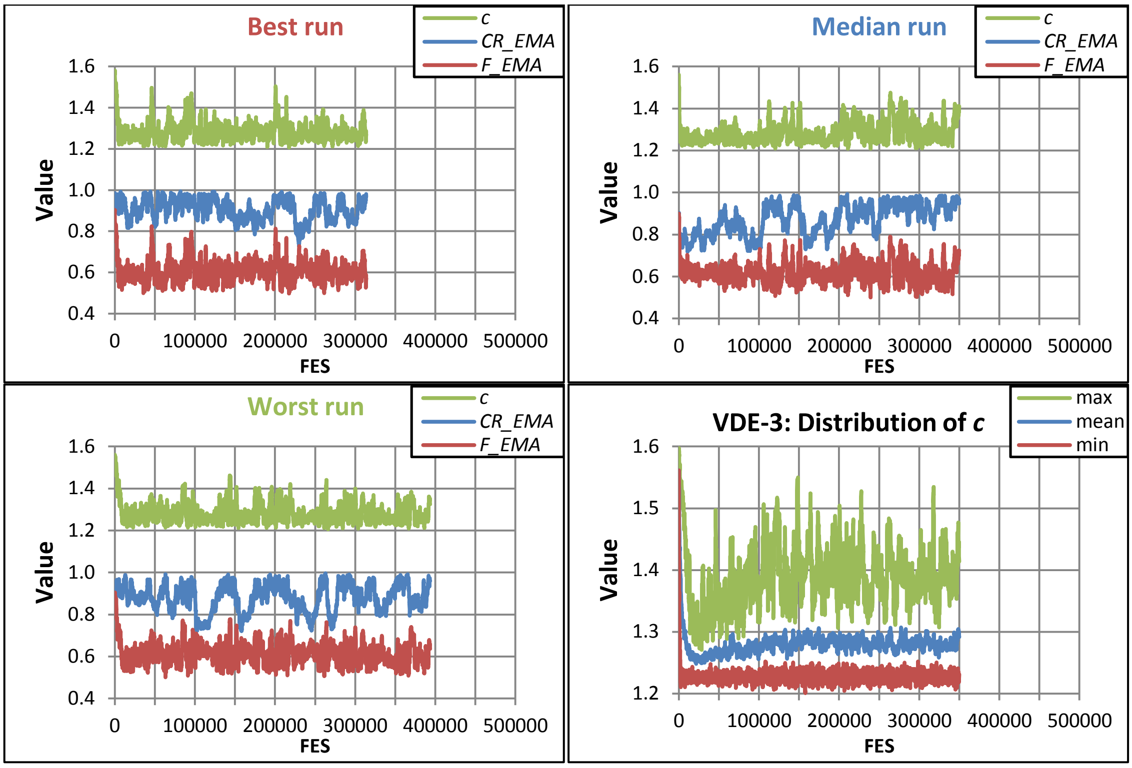

Figure 28. VDE-3: F_{EMA}, CR_{EMA} and c values for best, median and worst runs, and c max, mean and min value during optimization for f_6(30D).

Next, are provided the graphs for the second multimodal function f_7(Figure 29) in 30D. It is a rotated non-separable multimodal function in which the global optimum is out of the initial population range. For this function VDE-1 had the best overall performance but VDE-3 had the best individual runs but with poorer reliability. Interestingly the worst run for VDE-1 does not find the global optimum but it is successful because the error value for the local optimum is enough to reach the specified accuracy level (1E-02). Every algorithm used NP=50 for f_7.

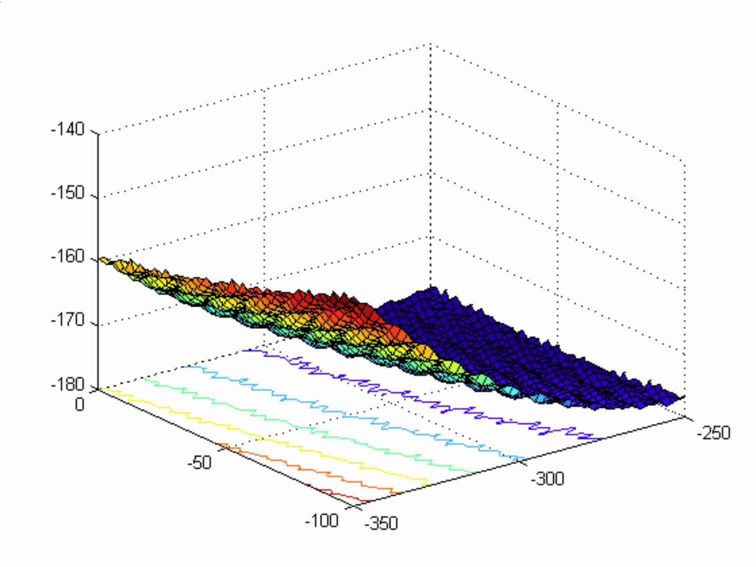

Figure 29. f_7: *Shifted Rotated Griewank's Function without Bounds*. 3-D map for 2-D function. **(**Suganthan *et al.* 2005)

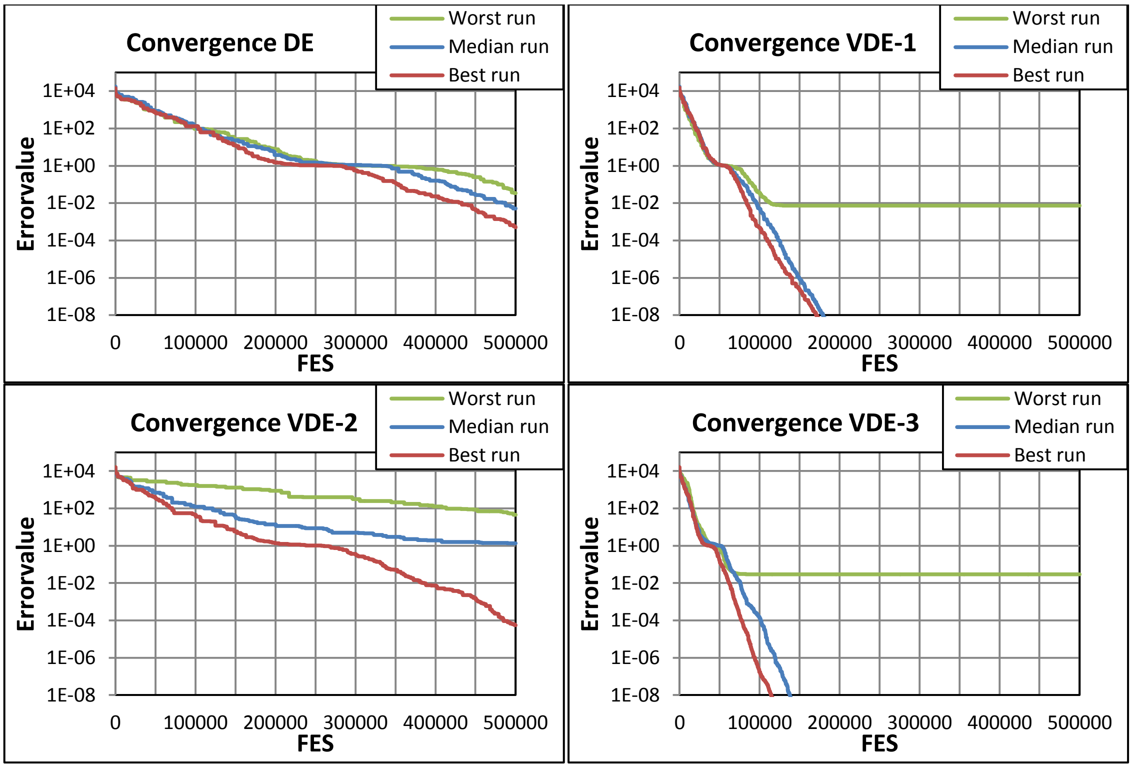

Figure 30. Convergence graphs for f_7(30D).

The F_{EMA}value behavior for VDE-1 is exactly the same as for f_6.Again values average at 0.65 and with these values the convergence rate is major compared to DE. Values of the worst run again vary greatly as the algorithm has prematurely converged to local optimum.

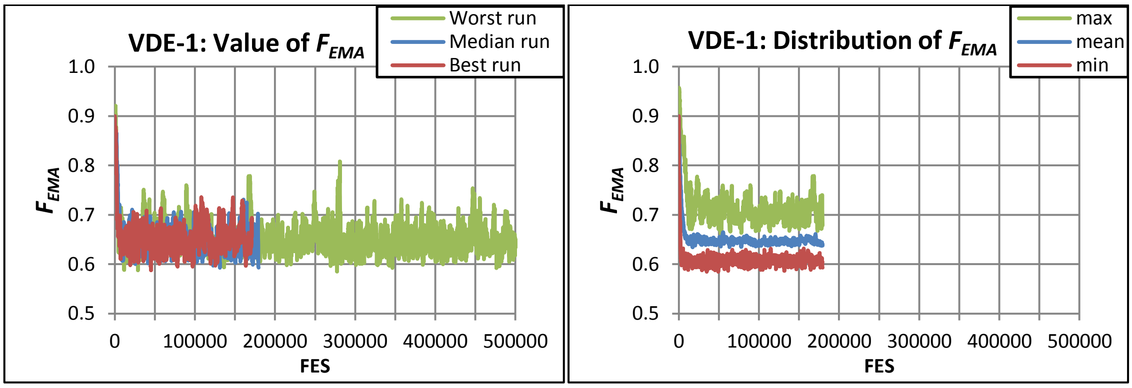

Figure 31. VDE-1: F_{EMA} values for best, median and worst runs, and F_{EMA} max, mean and min value during optimization for f_7(30D).

For VDE-2 the best run is better than the best run for DE but otherwise it performs worse than DE. In the best run the CR_{EMA} averages a little greater than 0.9 which explains the faster convergence. In the median run the value slowly falls near the lower bound and in the worst run the value falls more quickly. Distribution graph shows the steady fall. Lower CR values clearly slow down the convergence of VDE-3 for this function.

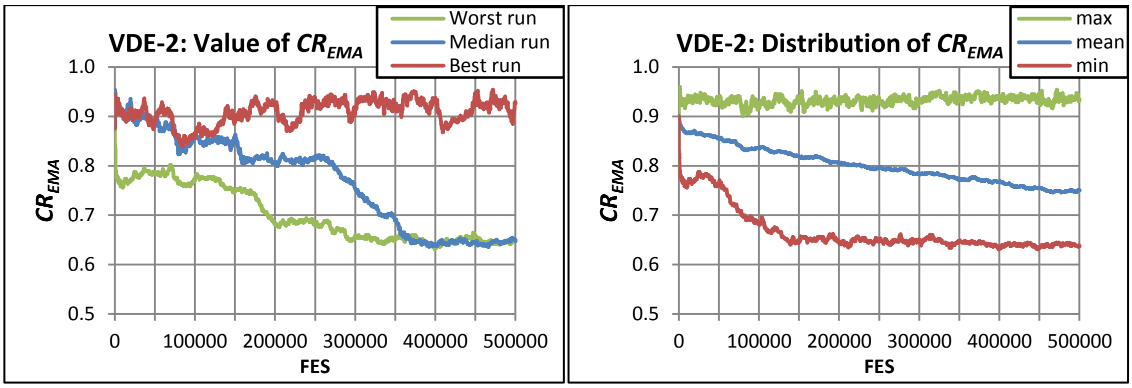

Figure 32. VDE-2: CR_{EMA} values for best, median and worst runs, and CR_{EMA} max, mean and min value during optimization for f_7(30D).

VDE-3 control parameter behavior in Figure x is also similar with f_6. F_{EMA} values are near the 0.6 and CR_{EMA} between 0.8-1.0. Problem with reliability (80% success rate) would have to be that the lower limit of c=1.2 allows too low F values which causes premature convergence. But this could be avoided with a larger population as for f_6(30D).

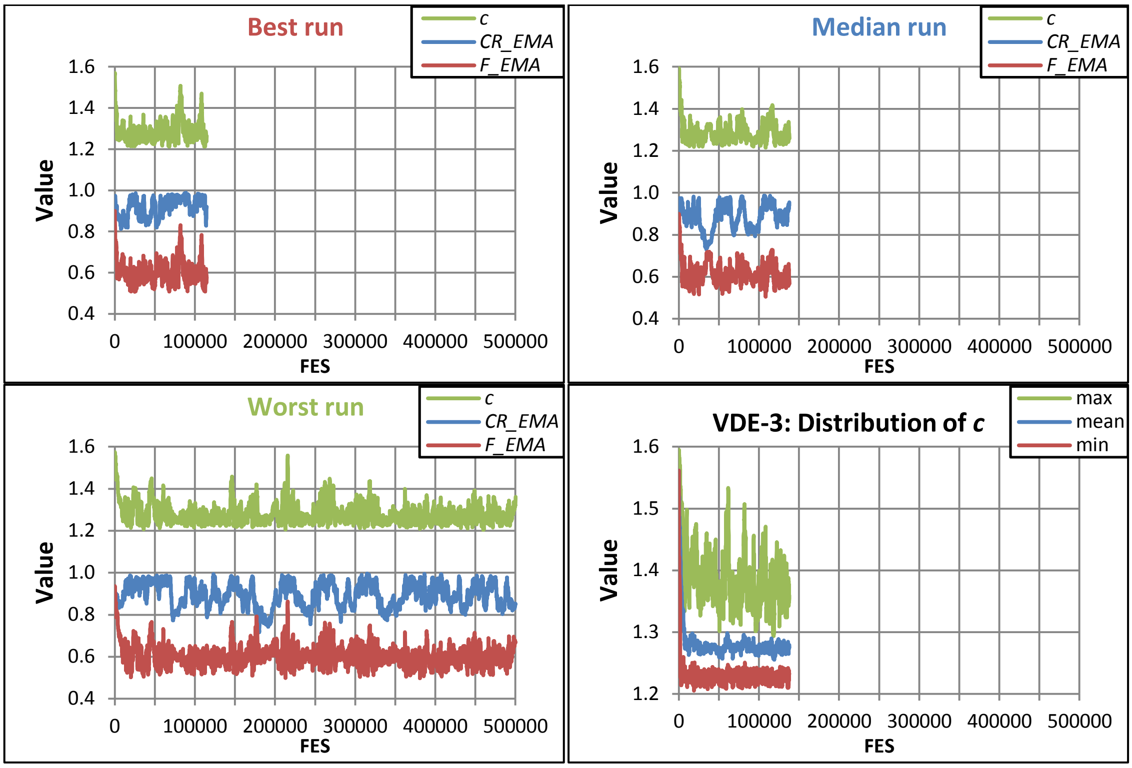

Figure 33. VDE-3: F_{EMA}, CR_{EMA} and c values for best, median and worst runs, and c max, mean and min value during optimization for f_7(30D).

Next illustrated are the results for f_9 (Figure 34) which is a separable multimodal function in 30D. For separable functions the CR value has the greater effect on performance. The c limits for the VDE-algorithms were also set to more aggressive settings which allowed more variance especially to F values. Convergence of DE and VDE-2 was the most consistent and the best performance was achieved by VDE-3.

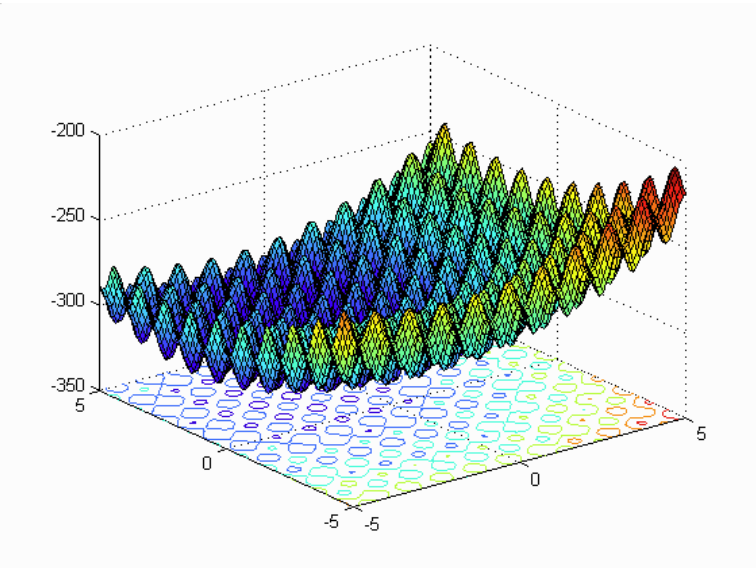

Figure 34. f_9: *Shifted Rastrigin's Function*. 3-D map for 2-D function. **(**Suganthan *et al.* 2005)

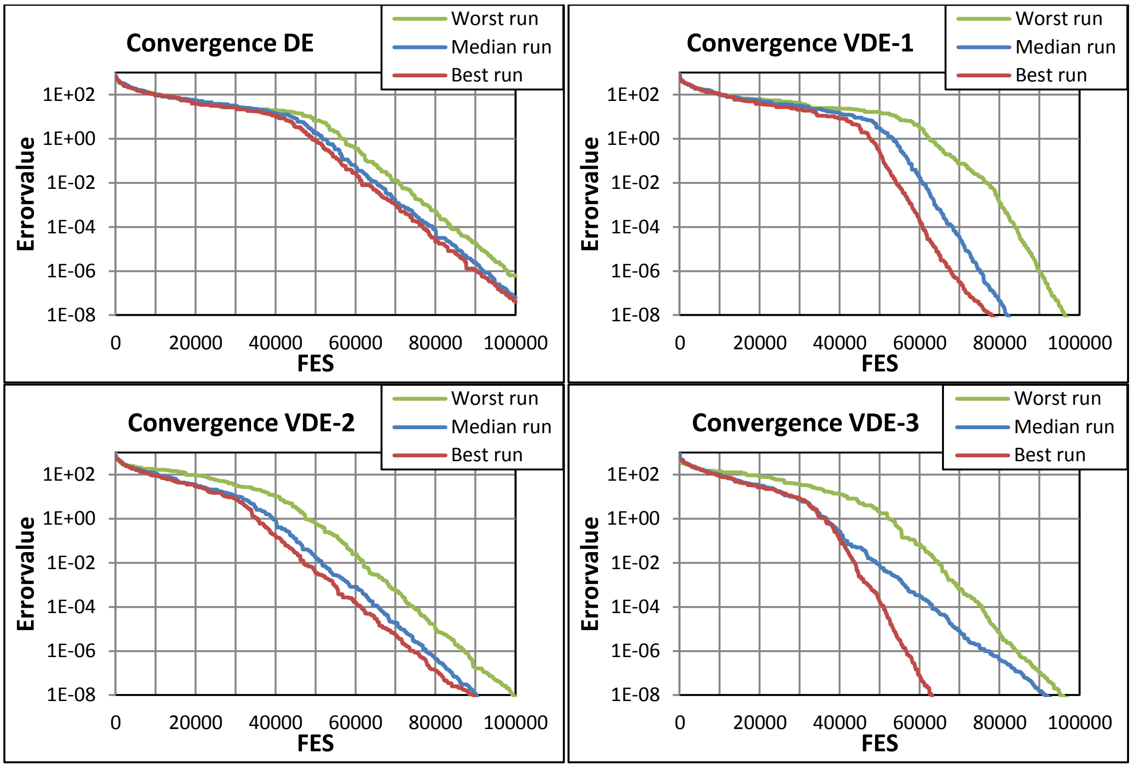

Figure 35. Convergence graphs for f_9(30D).

For VDE-1 the best and median performance were rather close with each other, and the behavior of F_{EMA} value is also similar staying high in the beginning and falling when the region of the global optimum is found. In the worst run the value falls quickly but it does not have a great impact on the result. Eventually the value rises and again falls towards the lower bound. The distribution graph also shows that the average values start to fall towards the end. Compared to DE and VDE-2 the low F values speed up the convergence when the global optimum is found.

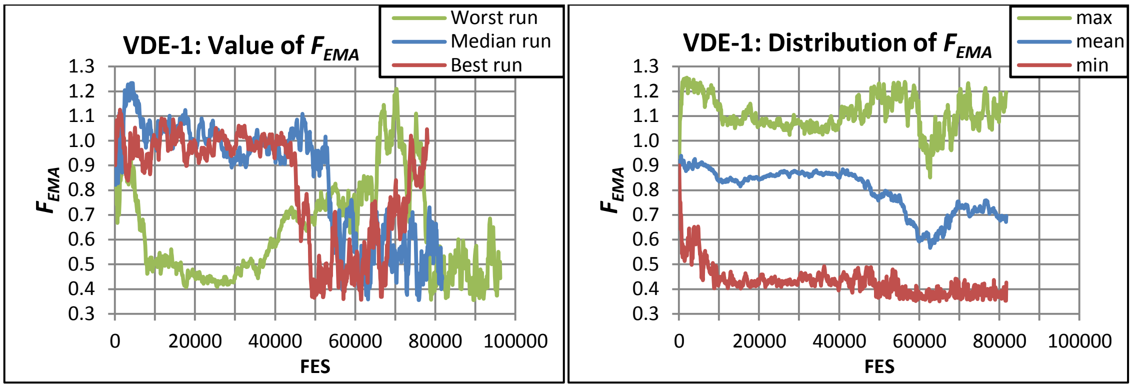

Figure 36. VDE-1: F_{EMA} values for best, median and worst runs, and F_{EMA} max, mean and min value during optimization for f_9(30D).

VDE-2 had better performance than DE and this is because VDE-2 allows CR values below 0.1. In the best and median run faster convergence is achieved by CR_{EMA} values averaging around 0.05. In the worst run CR_{EMA} rises slowing down the convergence. When the algorithm starts to converge towards the global optimum the CR_{EMA} values seem to rise a little.

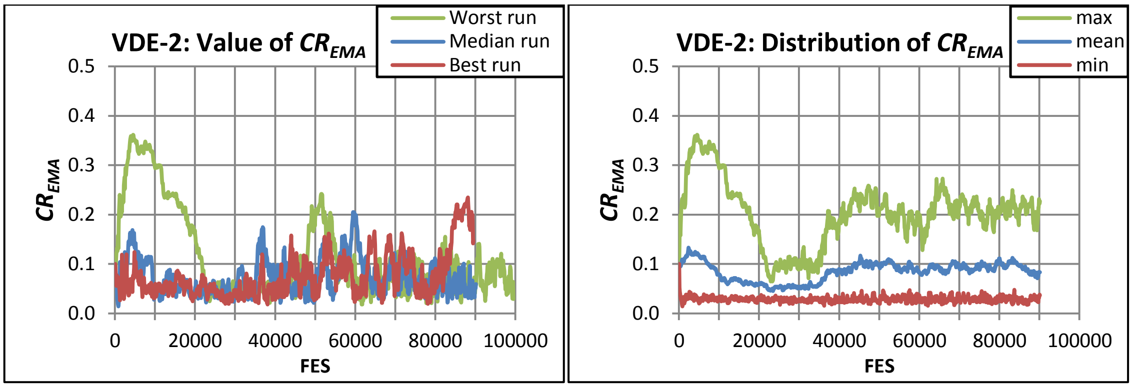

Figure 37. VDE-2: CR_{EMA} values for best, median and worst runs, and CR_{EMA} max, mean and min value during optimization for f_9(30D).

The best run is accomplished by VDE-3 and the behaviour of the control parameters in it is really a combination of best runs for VDE-1 and VDE-2. F_{EMA}stays high while CR_{EMA} falls very low and when the algorithm starts to converge to the global optimum, F_{EMA}falls and CR_{EMA} rises near the higher bound of 0.3. In the median run CR_{EMA} is low for the whole run and it slows the convergence towards the end. In the worst run F_{EMA} value has a spike >1.4 but does not make a major difference.

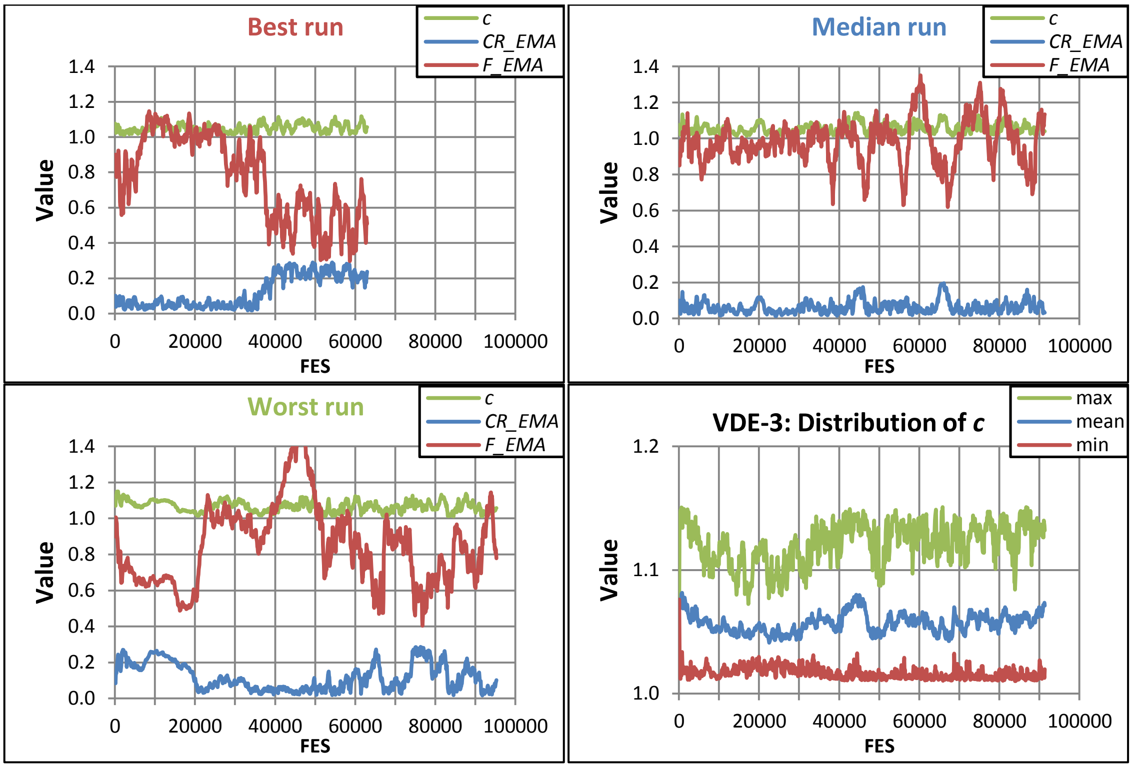

Figure 38. VDE-3: F_{EMA}, CR_{EMA} and c values for best, median and worst runs, and c max, mean and min value during optimization for f_9(30D).

Next presented are the graphs for f_{11}(10D). The function is a non-separable rotated multimodal function (Figure 39). The best performance for this function was achieved by VDE-1. DE and VDE-2 performed very evenly and VDE-3 the worst because it had the worst success rate although the best run was on par with VDE-1. From the convergence graphs (Figure x) it is seen that contrary to the previous multimodal functions, algorithms are not converging very fast in the beginning. *NP* was set to 50 for all algorithms.

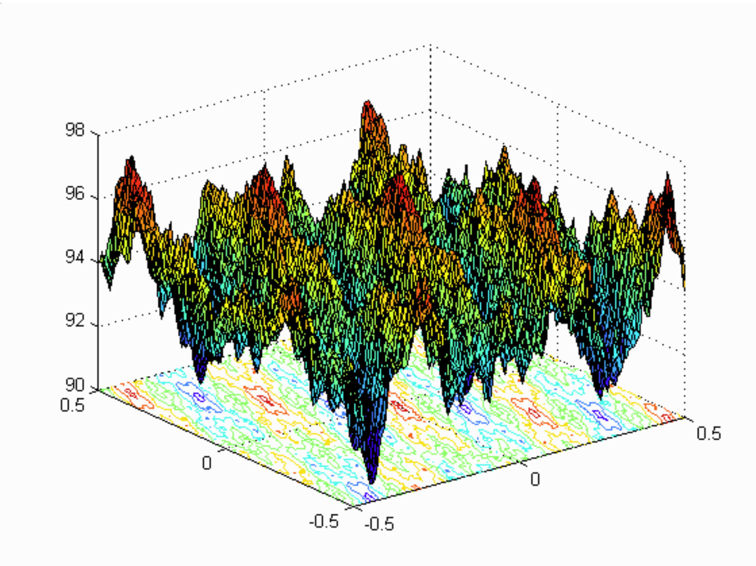

Figure 39. f_{11}: *Shifted Rotated Weierstrass Function*. 3-D map for 2-D function. **(**Suganthan *et al.* 2005)

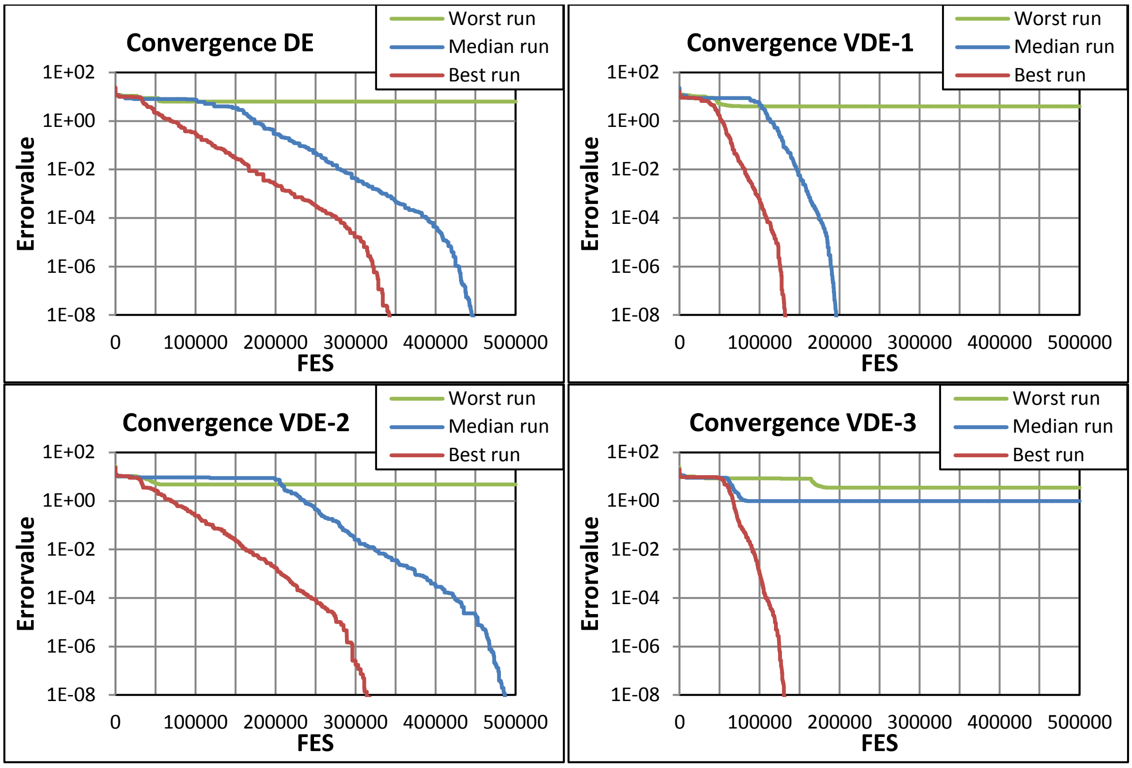

Figure 40. Convergence graphs for f_{11}(10D).

For VDE-1 F_{EMA} values are a little different compared to earlier multimodal functions. The value in the best and median run stays high until the algorithm finds better region in the search space and F_{EMA}values fall very quickly. The distribution graph confirms this behavior. In the worst run F_{EMA}falls instantly and the high fluctuation shows that the algorithm is finding better or equal trial vectors but in local optimums. Towards the end the fluctuation reduces which indicates that the population is scattered in local optimums and algorithm is stagnating.

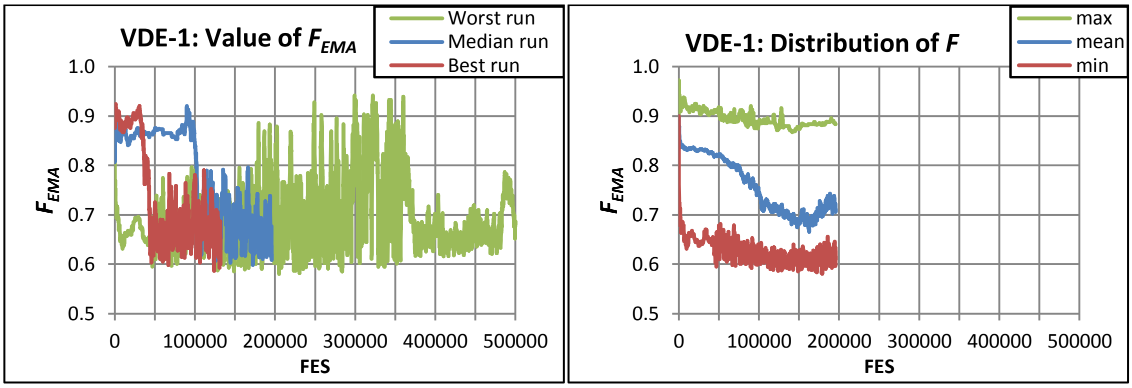

Figure 41. VDE-1: F_{EMA} values for best, median and worst runs, and F_{EMA} max, mean and min value during optimization for f_{11}(10D).

VDE-2 convergence speed was very close to that of DE. In the best and median run CR_{EMA} value stays high and in the worst run falls towards the lower bound and rises right at the end when the algorithm has presumably converged to a local optimum. On average the CR_{EMA} value was ≈0.9.

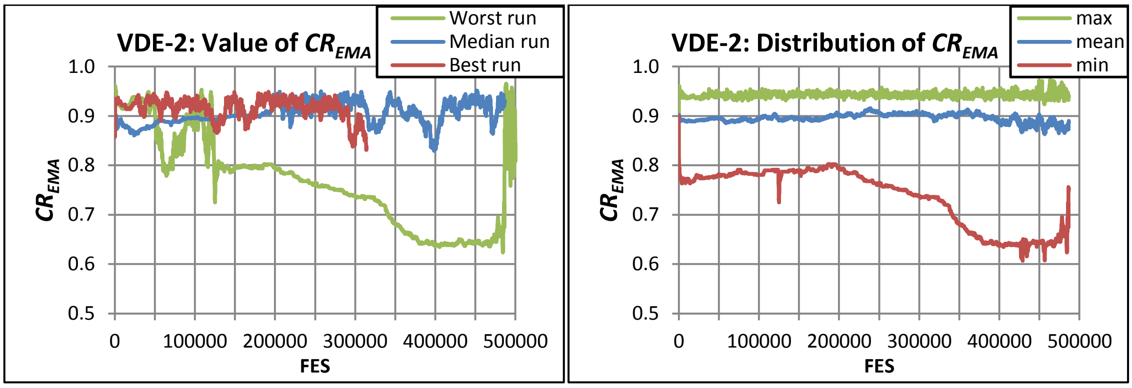

Figure 42. VDE-2: CR_{EMA} values for best, median and worst runs, and CR_{EMA} max, mean and min value during optimization for f_{11}(10D).

VDE-3 behaves similarly with VDE-1 and when comparing the convergence of the best runs for these algorithms, VDE-3 has a small advantage in pure speed. This is again the result of lower F values but at the same time the lower values are the problem with the success rate. CR_{EMA} values stay fairly high. VDE-3 converges prematurely more easily and this could be fixed with a larger population or raising the variance factor boundaries.

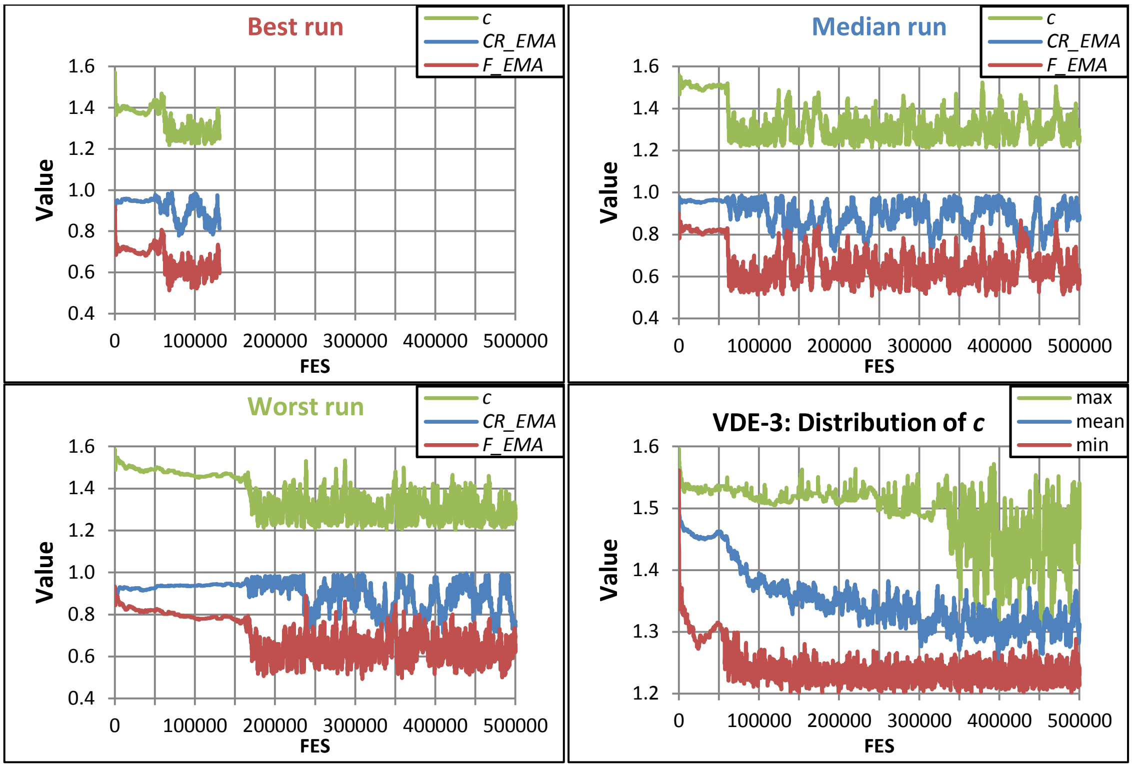

Figure 43. VDE-3: F_{EMA}, CR_{EMA} and c values for best, median and worst runs, and c max, mean and min value during optimization for f_{11}(10D).

Next are the graphs for f_{12}(10D). This function is a non-separable multimodal function (Figure 44). VDE-3 performed the best with VDE-1 a close second. VDE-2 was slightly better than DE apart from the worst run. For VDE-1 and VDE-3 two runs failed to solve the function. Overall, most runs in the convergence graphs had stable convergence rate towards the global optimum. *NP* was set to 100 for all algorithms.

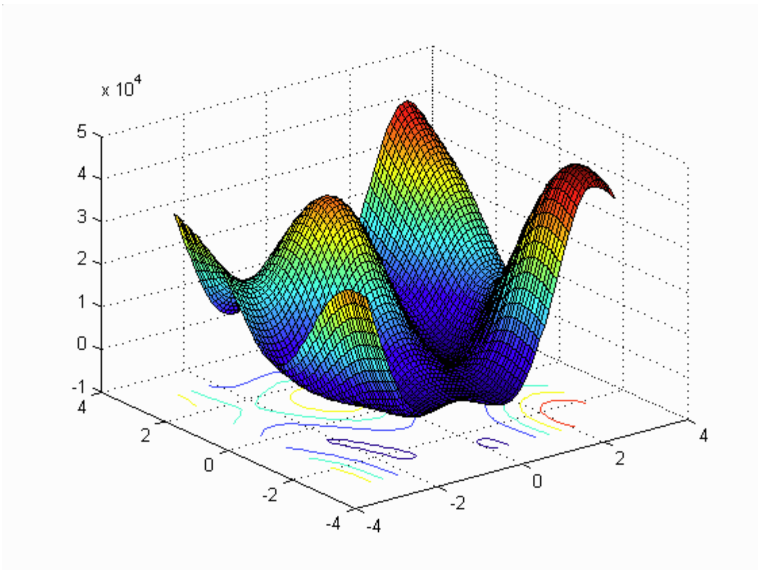

Figure 44. f_{12}: *Schwefel's Problem 2.13*. 3-D map for 2-D function. (Suganthan *et al.* 2005)

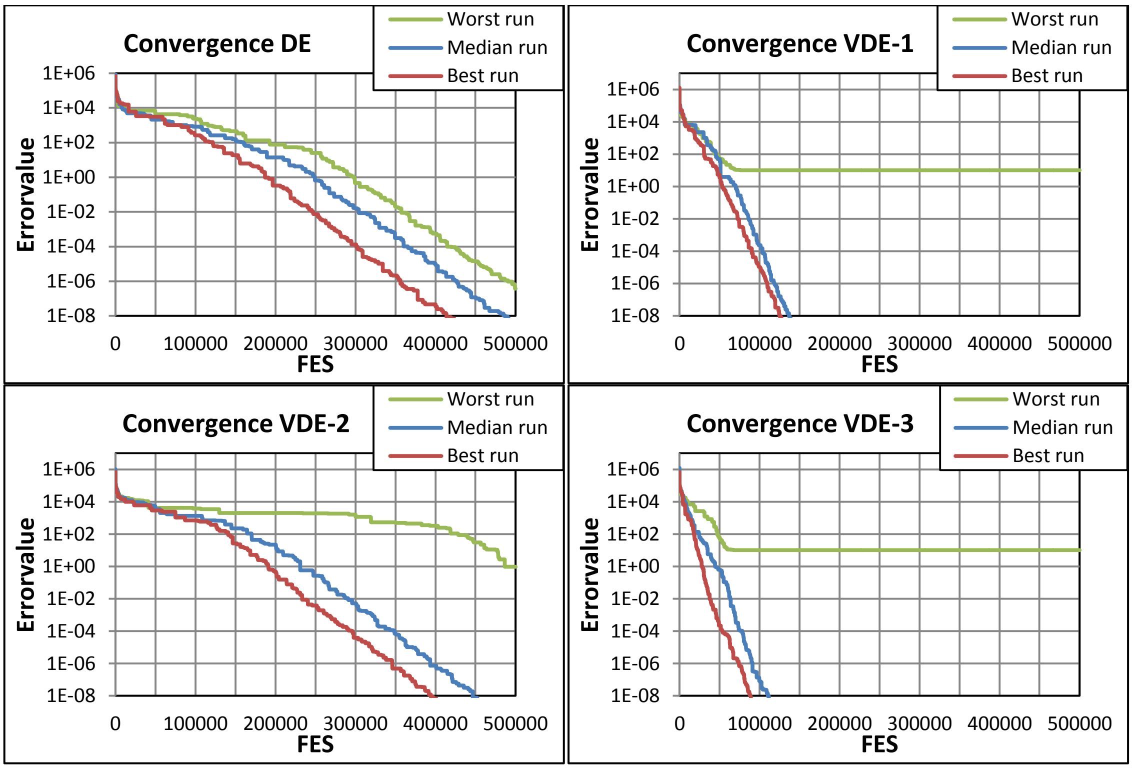

Figure 45. Convergence graphs for f_{12}(10D).

VDE-1 F_{EMA} values average ≈0.7 for most runs. This again provides much faster convergence when compared to DE and VDE-2. In the worst run there are signs of stagnation towards the end.

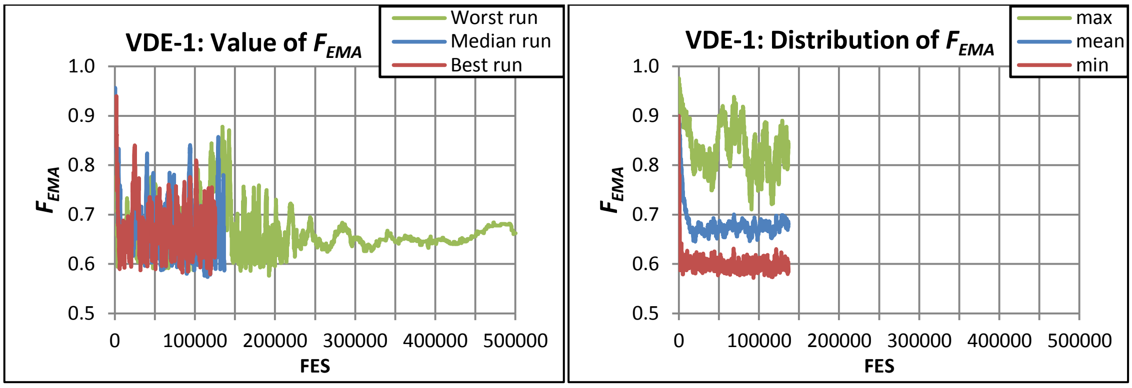

Figure 46. VDE-1: F_{EMA} values for best, median and worst runs, and F_{EMA} max, mean and min value during optimization for f_{12}(10D).

VDE-2 performance was a little better when compared to DE apart from the worst run. The problem with the worst run is the lower CR_{EMA} value during the most part of the run. In the end the value rises and from the convergence graph it is seen that the algorithm starts to converge faster and more likely towards the global optimum.

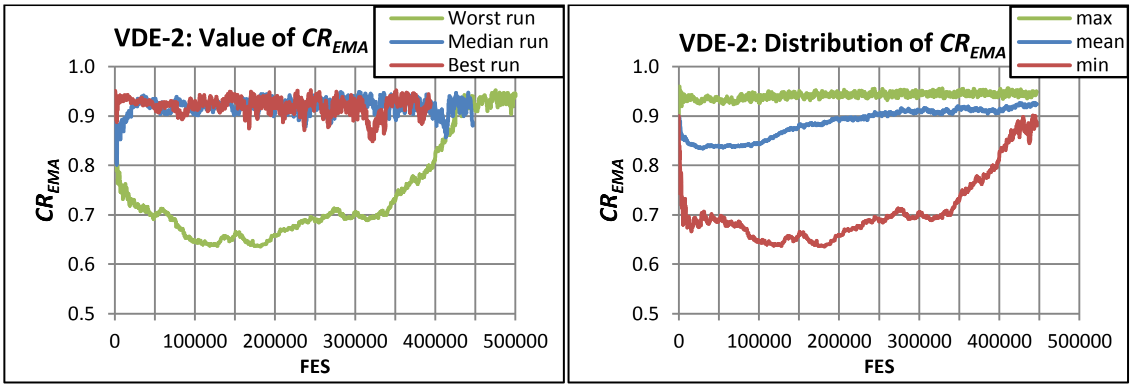

Figure 47. VDE-2: CR_{EMA} values for best, median and worst runs, and CR_{EMA} max, mean and min value during optimization for f_{12}(10D).

VDE-3 had the best convergence speed and the behaviour is similar to that with the previous functions. If good search region is found, the F_{EMA} values fall quickly towards the low bound while CR_{EMA} values remain fairly high. In the worst run the algorithm eventually stagnates and does not find better trial vectors.

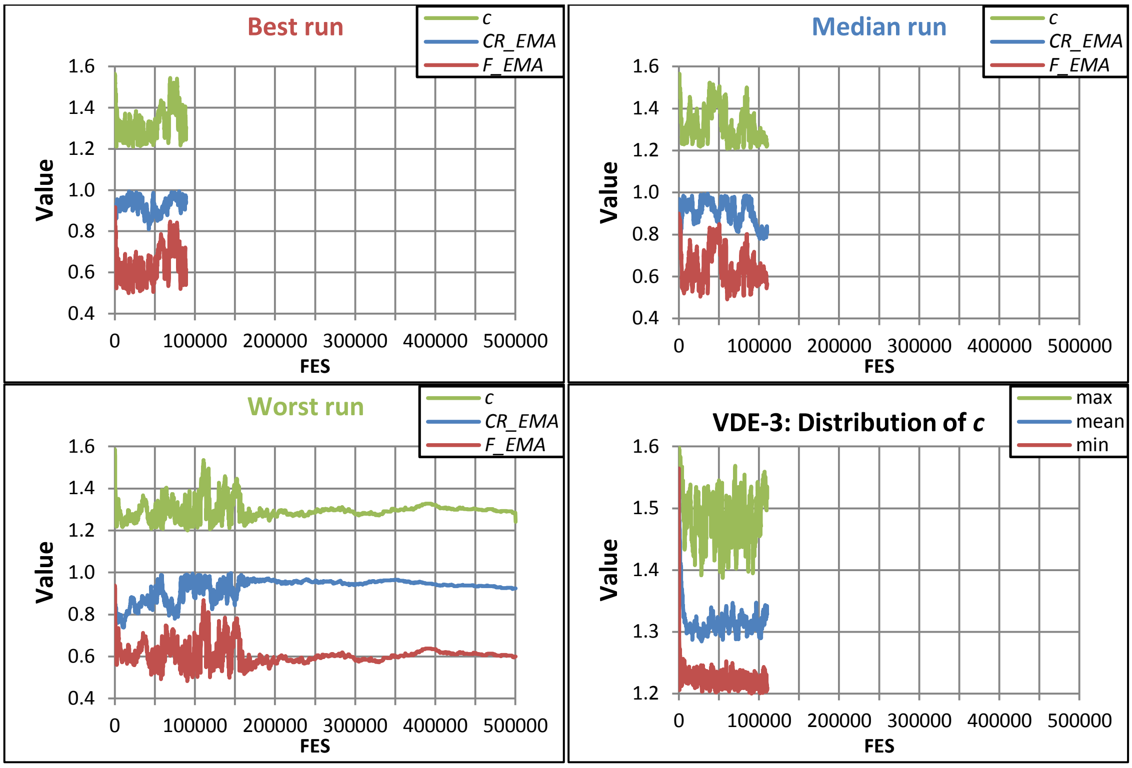

Figure 48. VDE-3: F_{EMA}, CR_{EMA} and *c* values for best, median and worst runs, and *c* max, mean and min value during optimization for f_{12}(10D).

The last function I have decided to report here is f_{15}(10D) (Figure 49). This function is a multimodal composition function where different function properties are mixed together. This function is a mix of Rastrigin, Weierstrass, Griewank, Ackley and Sphere functions. The function has a huge number of local optima but the Rastrigin function is the function with the global optimum. This makes the function separable near the global optimum. This seems to be the reason why VDE-2 had the best performance and VDE-3 the second best. VDE-3 was run with *NP*=200 and others with *NP*=100.

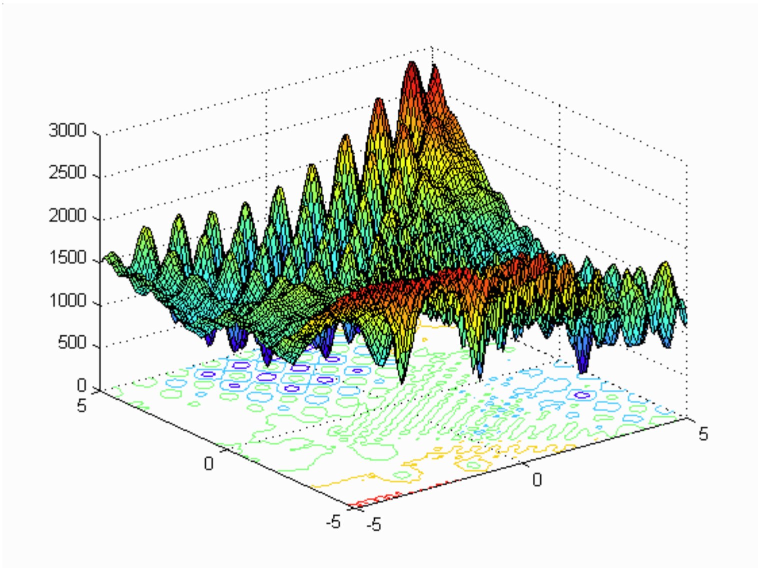

Figure 49. f_{15}: *Hybrid Composition Function*. 3-D map for 2-D function. **(**Suganthan *et al*. 2005)

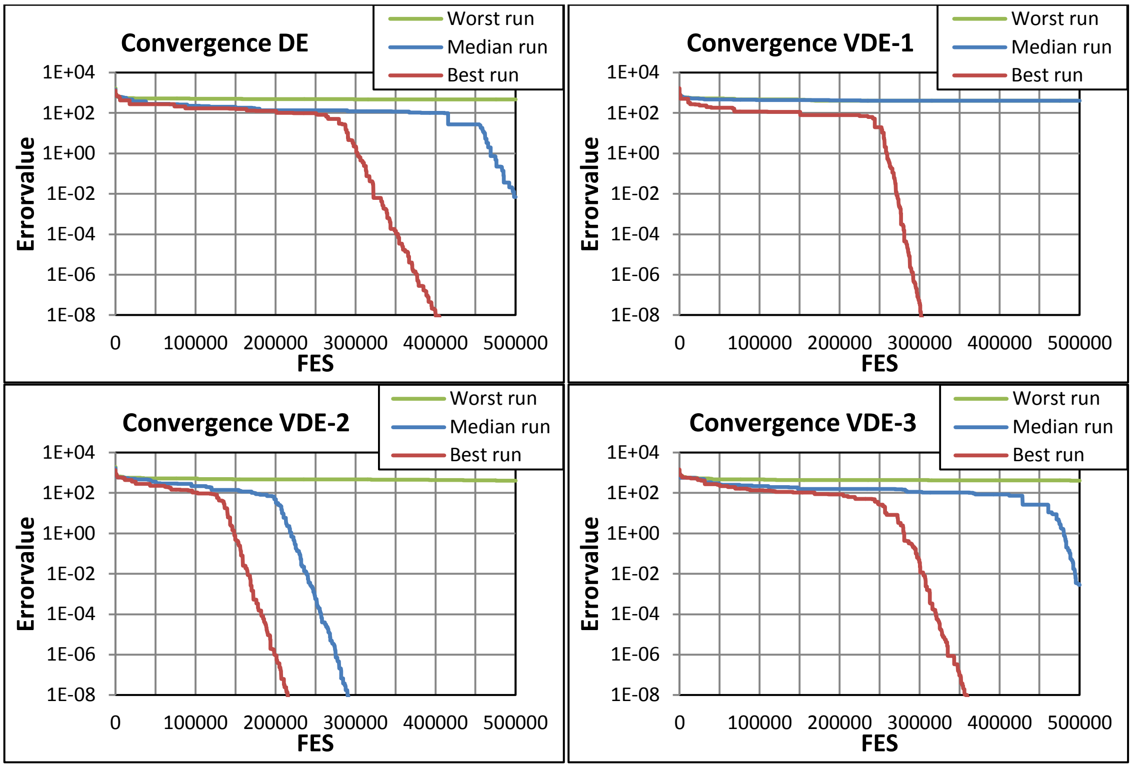

Figure 50. Convergence graphs for f_{15}(10D).

VDE-1 had the worst performance of the algorithms for this function. Interestingly in the best and median run the F_{EMA} value behavior is almost identical. After the initialization, F_{EMA} falls fast but then rises as the convergence slows down and falls again when a good search region is found. The only difference is that in the best run the algorithm manages to find the global optimum and converges very fast towards it. Also the range of effective F values is wider compared to other multimodal functions.

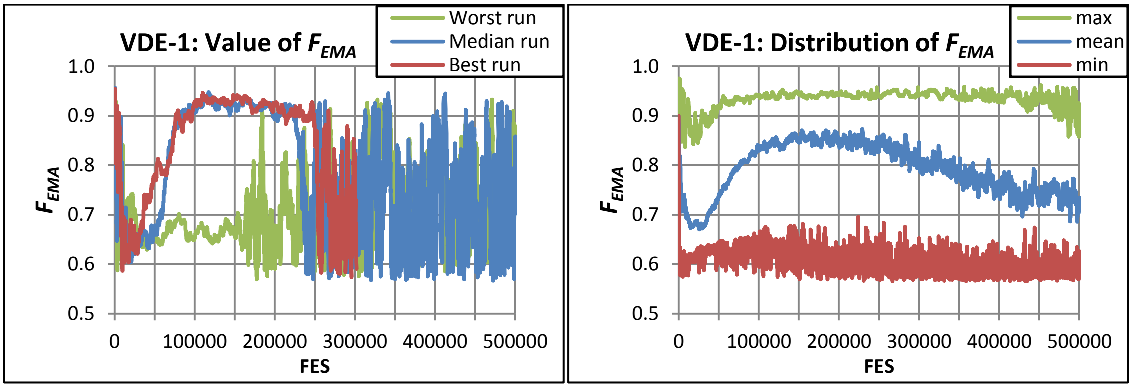

Figure 51. VDE-1: F_{EMA} values for best, median and worst runs, and F_{EMA} max, mean and min value during optimization for f_{15}(10D).

VDE-2 had the best performance. In the best and median runs CR_{EMA} values fall quickly and in the worst the value stays high in the beginning but is slowly falling. When the algorithm finds the region of the global optimum, CR_{EMA} values start to rise. In the worst run there are also signs that the global optimum is found as the value of CR_{EMA} is rising fast. It is clear that VDE-2 benefits from this partly separable function with low CR_{EMA} values.

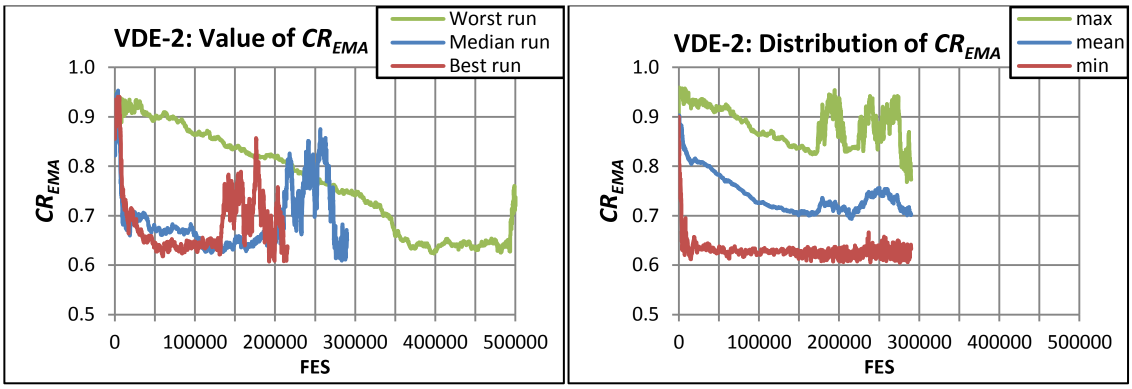

Figure 52. VDE-2: CR_{EMA} values for best, median and worst runs, and CR_{EMA} max, mean and min value during optimization for f_{15}(10D).

VDE-3 behaves similarly to VDE-1 and VDE-2. In the beginning F_{EMA} values drop only to rise again after the convergence slows down. And as in VDE-1, F_{EMA} value falls when the region of global optimum is found. In addition, CR_{EMA} also acts like in VDE-2, falling towards the low bound of 0.7. The c values in the graphs clearly show the behavior of the algorithm. If a good search region is found, the algorithm tries to lower the variance, and thus tries to converge as fast as possible. If the algorithm does not find good trial vectors the algorithm raises the variance.

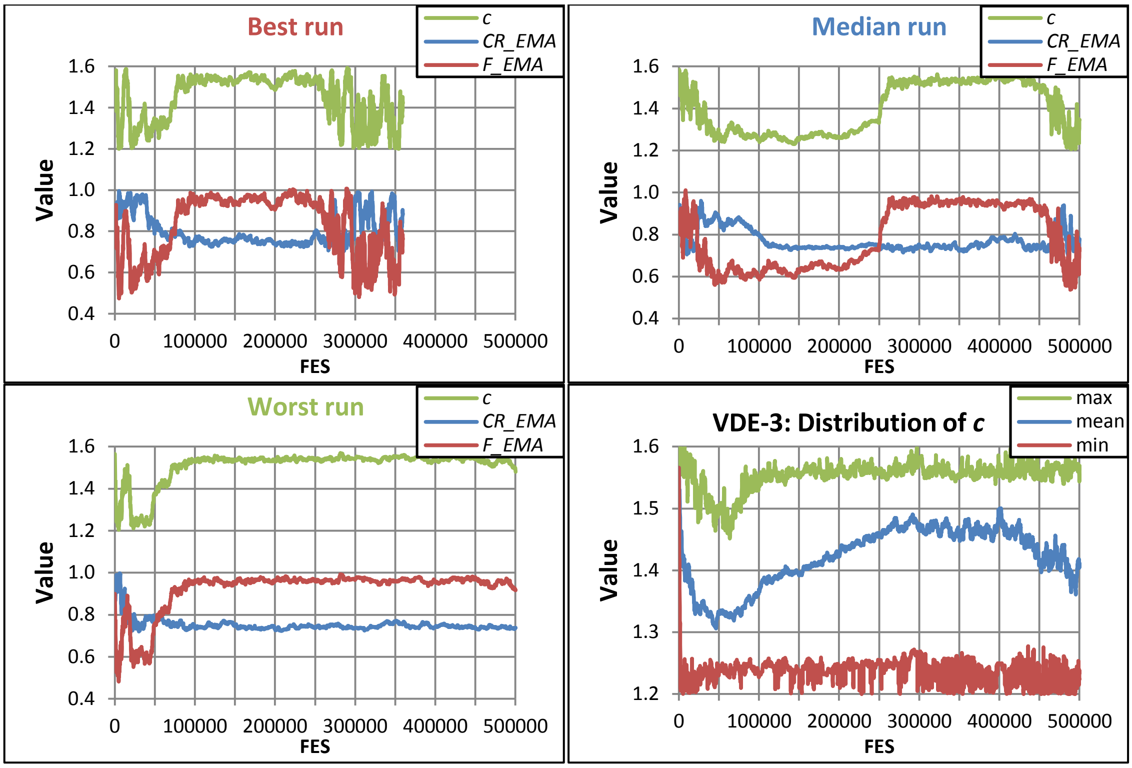

Figure 53. VDE-3: F_{EMA}, CR_{EMA} and c values for best, median and worst runs, and c max, mean and min value during optimization for f_{15}(10D).

6. CONCLUSIONS

The results achieved in chapter 5 (Table 27) shows that on average VDE-1 and VDE-3 had the best results. Error value tables in sections 5.1 and 5.2 show that the best error values at given checkpoints were achieved mainly by these algorithms. Fast convergence rates were illustrated in section 5.3. Performance was superior compared to original DE especially in the non-separable and unimodal functions. That said, these algorithms also had a larger risk of prematurely converging to a local optimum in the multimodal functions. The reasons, which were analyzed in section 5.3, for the premature convergence seemed to be that lower mutation factor *F* values caused the algorithms to converge too fast towards local optima.

The aim of the thesis was to find out if the original DE algorithm could be developed into a black box system in which the control parameters could be automatically adapted to good values. The modifications clearly adapt the parameters successfully. The black box could be applied to all VDE-algorithms. Also the effective control parameter values were analyzed in the thesis.

Overall, the effective value range for *F* was around 0.6-0.8 and VDE-1 and VDE-3 adapted *F* to this range when successful trial vectors were found regularly. When successful trial vectors were not found, the *F* values started to rise and the algorithms started to search for better regions in the search space. Hence, the adaptation especially for *F* evidently worked well. For future use of the DE/rand/1/bin scheme I would suggest an initial value of 0.7 for *F*.

DE and VDE-2 which had the mutation factor constant at 0.9 had slower convergence rates yet this also lead to better success rates, especially for DE. Generally VDE-2 had better performance for most functions when compared to DE, and especially in 30D functions the reason was that the crossover factor *CR* was allowed to go over 0.9. But VDE-2 also had worse worst runs for most functions in comparison with DE. In the worst runs the convergence was slowed down by *CR* values lower than 0.9. However, the lower *CR* values benefited VDE-2 in the partly separable f_{15}. For non-separable functions I would suggest a value of *CR* between 0.85-1.0 and for separable functions a low value between 0.05-0.2. The higher the dimensionality of a non-separable problem is, higher *CR* values seem to work better.

The optimal variance factor *c* value varied with the function in question. For VDE-1 and VDE-3 good convergence properties were achieved when the variance factor was around 1.3. Mutation factor *F* tried to reduce the variance, and crossover factor *CR* tried to increase it. For DE the variance factor was constant at around 1.55 and in VDE-2 the best performance was achieved when the variance factor was near 1.6.

Most functions in the competition were too difficult to solve in the given boundaries. Most of the multimodal functions would require a very large population size due to the large amount of local optima. The population sizes used in the benchmark were mostly very small which lead to lower success rates but also less function evaluations. For VDE-3 I used a larger population size for some functions and it allowed the algorithm to use lower *F* values more effectively. It allowed the algorithm to utilize its fast convergence rate while preventing the algorithm to prematurely converge. Good initial minimum *NP* value of 5·D for unimodal problems and 10·D for multimodal problems should be a.

The VDE settings used for the benchmark were also set to adapt fairly fast to changing conditions. The random variance and the exponential moving average factor could be set to smaller values to give more stable behavior to the algorithms. Slower adaptation could also add more variance in the beginning and thus go through the search space more effectively. The settings used in the benchmark were a compromise between speed and reliability and also chosen so that they work well on multiple different functions.

For further study I would suggest testing the modifications on other functions and benchmarks. Most of the functions in CEC05 benchmark were too difficult to solve reliably and they require much computing time. Testing different settings on the hybrid composition functions proved very time-consuming and impractical. Also testing the modifications on solving real-world problems would be an interesting experiment.

Zaharie's theory does not directly apply to other DE mutation schemes but it would be interesting to experiment similar adaptation with other DE mutation schemes.

Finding the balance between speed and reliability is challenging and there are no universal control parameter settings which work well for every problem. But the results were promising since speed and reliability is most easily improved by raising the population size. Raising the population size also increases the value range of effective control parameters. In order to find good universal settings which could be applied to a black box system, the modifications would require more testing with problems that can be realistically solved.

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