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Markov Chain Cost/Life-Cycle Model

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ABSTRACT:
In maintenance management, predictive and preventive strategies are considered essential for the purpose of improving the overall efficiency of the systems, and for extending the operational lifespan of equipment. Stochastic behavior of a system's maintenance states can be effectively modeled with Markov chains, a mathematical framework providing the needed capability. With this approach for calculation of the system state probabilities for specific time intervals, maintenance planners can determine future conditions and plan maintenance schedules better.

The following paper looks into the Markov chain deployment in maintenance optimization. The first step is to determine a set of discrete states that would represent the condition of a piece of equipment at any given time. Such conditions cover a wide spectrum, from sustainable to completely dysfunctional ones. Then, probabilities of the transition between these states are determined using historical maintenance data and equipment condition. Through implementing these probabilities, we model behavior of the system with time and find the most profitable maintenance policies.

Our approach involves the construction of a state-transition matrix and its solution for steady – state probabilities to assess the long-term behavior of the equipment under various condition based maintenance strategies. Another cost function that includes the costs of distinct maintenance actions and the implications of equipment failure is presented as well. Through this optimization process, we determine the ideal maintenance plan which leads to the minimization of total operational expenditures as well as maximum equipment reliability.

The results show that the use of the Markov chain optimization maintenance is a remarkable tool for improving decision-making processes in maintenance management thus resulting to an analytical technique used for increasing equipment uptime and reducing maintenance-related costs. This method turns out to be very effective in industries where the cost of equipment failure is very high and efficiency of production is the main trend. The paper is concluded with case studies and suggestions for building the Markov chain models into the current maintenance management systems.

KEYWORDS: Markov Chain, Maintenance Optimization, Probabilistic Modeling, Cost Analysis, Life-cost Modelling

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1 Introduction

The Markov Chain Cost/Life-cycle Model, which involves a combination of the concepts of Markov chains, cost modeling, and life cycle modeling, altogether forms the most powerful and comprehensive approach of modeling. The model has undergone significant changes in the area of project management, product development, and risk analysis, which is a crucial aspect of how modern companies approach existing systems (Rad, et al.2021).

The primary competence of the Markov Chain Cost/Life-cycle Model becomes evident in the fact that it allows for the mathematical analysis of the system in terms of lifecycle and can therefore very well predict how operational this system is going to be. This cycle will study in detail all steps of life and fill the gaps such as long term costs and resource use instead of just the upfront expenses. The adoption of this holistic perspective provides project managers with better information for decisions to be made plus resources to be allocated proportionally, which are all factors that result in project success.(Hao et al.2022)

Particularly, the Markov Chain Cost/Life-cycle Model has a core function as it belongs to the industry of product development. Through an in-depth analysis of the cost impact both on the product process itself and on the product life-time, organizations acquire a deeper financial awareness of how the financial processes work. This way, they can arrive at the best strategies to develop economically, and also take into account environmental factors, with the eventual creation profitability, and sustainable competitive advantage.(Rad, et al.2021)

However, it also represents a fundamental instrument for programmed maintenance cross-checking. Causing more sophisticated preventive maintenance strategies to emerge in companies, Markov Chains become a tool to help with the information-based leadership strategies raising reliability values and avoiding costly downtime incidents. Since there remain uncertainties that can be included in the analysis, including fluctuations in transition rules between different stages of the life cycle, the model is even more dependable and reliable.

The Markov Chain Cost/Life-cycle Model comprises three key components: Markov Chain, Cost Model, and Life-cycle Model for instance. The treatment is the Markov Chain theory followed by the subsequent procedures as the starting point. This framework combines the Alan Turing machine and its evolutionary keys. The economic analysis section is further expanded to tackle the specifics of setting up a cost model that includes all essential cost factors. In this regard, the model provides the supplies of a fully systematic life cycle oriented approach, delivering sophisticated techniques to accurately determine and forecast a system's path.(Hao et al.2022)

The fact is that the Markov Chain Cost/Life-cycle Model is a must-have resource for any organization or sector involved in the development of projects, products, or systems. This model produces efficiency, streamlines decision-making and provides unique insights through the continuous dissection of cost dynamics and system life all over operation. The application of this model guarantees that the decisions of project managers and authorities are always successful. What is more, the model greatly reduces the uncertainties, thus, organizations gain a possibility of cost flexibility, increased efficiency of the system, and reliability.

In the final analysis, the Markov Chain Cost/Life-cycle Model remains an essential tool for the organizations that are future-oriented. It’s thorough and strong modeling approach assures cost, life-cycle, and decision-making accurate analysis. Utilization of this model will help organizations gain heightened levels of efficiency, cost flexibility and system reliability, thus making them successful in the fast-changing business world.(Hao et al.2022)

1.1 Definition of Markov Chain

This model's analysis is based on a principle that the entire system consists of a preset amount of fixed states of things. Their reality unfolds sequentially as they start from one to get to the other. The condition of the present is the core idea of the Markov property, therefore, enables us to bring the probability of the future state being in a specific value, considering the value of previous states.

Markov chains are the reason for modelling cost/life-Cycle generally because of the fact that they exhibit the overall dynamics of a system in addition to those transitions in the system from one state to another with fact timing. Policy options are reviewed and their respective outcomes and costs get calculated. Such coincides with the Markov principle that in a future event set, movement happens solely based on the current state of the system and not with regard to its past movement.

The Markov chain is often used to model all possible system states while running cost/life-cycle modeling in projects. The variety of possible arrangements keep one mind busy while observing its continued existence for a specific time interval. Forms with certain states transition gradually into other ones in the course of time. Through the modeling of the system's life-cycle as a finite set of states and making each transition move from one state to another in a Markov chain, we will represent the whole system dynamics accurately. The Markov chain is one of the most useful tools for the comprehensive representation of the system's transitions with the aid of calculating the potential outcomes.

A Markov chain has mathematically designed a framework of transitioning from one state to the other, which is based on particular rules of probabilistic values. The sample definition is the one that keeps in that no matter how this came to be it is fixed that the future prospects given the present state are definite. By transition from state to state is accompanied by certain probability values which have been already discussed hence, likely denotes the Markov process.(Stewart, 2021)

1.2 Significance of Cost/Life-cycle Modeling

The Markov Chain Cost/Life-cycle Model is a very robust and versatile modeling technique that integrates the basic principles of Markov chains, cost modeling, and life-cycle modeling. What makes this model special is the fact that it can include all stages of the life of the system and offers an unrivaled level of understanding and forecasting in many domains, such as project management, product development, and risk analysis. Unlikely to numerous other models, the Markov Chain Cost/Life-cycle Model does not only focus on short term, which means that the decision making process will be well-informed and will include the entire life of the system. (Rad et al.2021)

The dynamic character of this model is therefore, seen as its main benefit, as it allows a general view of system changes over the time. Through successful exploitation of developments in computing power, the Markov Chain Cost/Life-cycle Model makes it possible to run thousands of iterations thus ensuring accurate forecasts of long-run behavior. This property has been key in the practical use of the model to different hardware and software systems.

The output, which is generated from the Markov Chain Cost/Life-cycle Model, has been instrumental in multiple ways, as per the specifics of what the system being analyzed needs and what questions are being asked. In product development, for instance, the model is used to identify the design alternative that has the lowest “average” or “long term expected” cost, which implies an efficient allocation of resources. In terms of risk analysis, the model has ability to assess the probability of the system to reach a certain state in a given time. This essential information is further used to recognize the critical components of a system and develop the required risk factors mitigation measures.(Hao et al.2022)

As emphasized in this paper, one of the significant merits of Markov Chain Cost/Life-cycle Model relative to simpler models is its ability to perform thorough probabilistic life-cycle cost analysis. Offering a better and complex analysis of the decision options, the outcomes from this model can be providing valuable assistance in all the acquisition stages. This aspect is very important in the case of complicated defense systems in which a very large number of interrelated and big design decisions have to be made over extended periods of time. The Markov Chain Cost/Life-cycle Model is one of the methodologies that contribute to a successful navigation of the complexity that is characteristic for such environments, so that decisions are aligned with the ultimate objectives and resources are optimally distributed.

All in all, the Markov Chain Cost/Life-cycle Model is a state-of-the-art approach that elegantly combines Markov chains, cost modeling, and life-cycle modeling. Through covering the whole life-cycle of a system, both short-term and long-term costs, and dynamic analysis capabilities, this model ensures accurate long-term behavior prediction and informed decision making. Finally, the outcomes generated by the model form the basis of the best resource distribution, risk evaluation, and strategic planning across different industries and areas.(Gavrikova et al., 2020)

2 Markov Chain Basics

The Markov chain sequence steps from one state to another at discrete time intervals. The term step is used to define each move or transition, and the time between transitions is of no importance. Such one-step transition probability, P {X (t+s) = j | X(t) = i, X(s) = k for 0<=s<=t} is conditional on the state k during the current time t and not on past states. The set of all states is called the state space of the chain, and the whole matrix of transition probabilities is the transition matrix P of the Markov chain. The fundamental characteristics of any Markov chain are as follows: the state space and the transitions between the state in the state space. The space of state is all possible system states. The majority of practical problems of engineering have a state space discretely, and it can be characterized via the analysis of the system's behavior, and transitions between states are easy to represent by state transition diagram. The system changes from state to state in discrete time points. The system transition probabilities define all transitions of the system and they are time-invariant. These attributes include Markov property and memoryless. That is, the Markov property can be formulated as "the future is independent of the past given the present" (Dizaj et al.2021).

The idea of the Markov chain is very popular in different areas like stochastic processes, physics, economics, and computer science. The importance of the Markov process in this case is to represent systems that undergo random changes. Understanding the dynamics of the Markov chain enables one to predict what the future behavior of the system would be.

In application purposes, the state space of a Markov chain can be of significantly different sizes. It can stretch from a finite set of distinct states to an infinite set of continuous states. Also, the transition probabilities between states may vary depending on what the system is that is being modeled.

As an example of a Markov chain, let us take a simple case. Consider a predictive weather system that represents weather as a Markov chain. The weather state space includes sunny, cloudy and rainy. The probabilities of transition signify the chance for the weather to switch from one state to another over a specific time period. With the help of the historical data, the probabilities of the transitions can be estimated and thus, the transition matrix is created. This matrix enables us to compute the occurrence probability of different weather conditions in future time periods. We can irrepeatedly apply the transition matrix to simulate the weather forecast for a period.(Bolhuis and Swenson2021)

Modeling of complex systems and their behavior prediction is an area in which Markov chains are highly useful. Starting with the analysis of stock markets and ending with studies of the spread of diseases, the framework of Markov chains allows not only to understand but also to make informed decisions.

In brief, Markov chain is a mathematical concept that has been applied in many areas. The fact that it can capture the probabilistic characteristic of the system transitions renders it a helpful analysis and predictive tool. Through the knowledge of the Markov property and the state space, one can understand the dynamics of the dynamic systems. In economics, physics, or computer science the Markov chain is at the core of our understanding of randomness and uncertainty.(Bolhuis and Swenson2021)

2.1 Transition Matrix

A transition matrix is used to define the transition probabilities among states in a Markov Chain. Each row of the matrix represents the probabilities of moving from a particular state to all the other states, and the matrix is defined such that each element is a number between 0 and 1 and each row sums to 1. Let's take the example of our software system again. Suppose we have 3 possible states for the software system, state 1: working properly, state 2: small error, and state 3: shutdown. Then, the transition matrix T will be like the following:

$$\mathrm{T} = [\ \mathrm{P00\ P01\ P02} \\ \mathrm{P10\ P11\ P12} \\ \mathrm{P20\ P21\ P22}] \qquad (1)$$

When we write down the elements of T:

$$
\begin{gathered}
T[0][0] = P00 = P\ (\text{state } 1 \rightarrow \text{state } 1) \\
T[0][1] = P01 = P\ (\text{state } 1 \rightarrow \text{state } 2) \\
T[0][2] = P02 = P\ (\text{state } 1 \rightarrow \text{state } 3) \\
T[1][0] = P10 = P\ (\text{state } 2 \rightarrow \text{state } 1) \\
T[1][1] = P11 = P\ (\text{state } 2 \rightarrow \text{state } 2) \qquad (2) \\
T[1][2] = P12 = P\ (\text{state } 2 \rightarrow \text{state } 3) \\
T[2][0] = P20 = P\ (\text{state } 3 \rightarrow \text{state } 1) \\
T[2][1] = P21 = P\ (\text{state } 3 \rightarrow \text{state } 2) \\
T[2][2] = P22 = P\ (\text{state } 3 \rightarrow \text{state } 3)
\end{gathered}
$$

Where Pij is the probability to move from state i to state j and it satisfies the following conditions: $0 \leq Pij \leq 1$, $\Sigma j\ Pij = 1$. The elements in the first row of the matrix define the probability to move from state 1 to other states. In our example, P00 = P (state 1 → state 1) is the probability to remain the software in working properly condition after one month; P01 = P (state 1 → state 2) is the probability to have small error after one month given that the current state is working properly; P02 = P (state 1 → state 3) is the probability the software may shutdown after one month given that current status is working properly. And similarly, other elements can be defined. (Stewart, 2021)

A transition matrix identifies the possible transitions of a system or entity between a number of states. The two-dimensional matrix provides row information of the current state and column information of the probability of occurring other states in the future. Although all transition matrices adhere in general to the same rules and mathematical definitions, each of them has a different scale and specific transitions and states. The transition matrix is the crucial component to analyze and simulate the Markov Chains and Markov processes.

So, in essence, our life-cycle application p provides you with a stage-to-stage transition probability, if you square your transition matrix n times and then examine the aggregated transition matrix, T to the power of n, n times, this allows you to get an overview of what the probabilities for being in each state would be after n stage transitions have occurred. This is a highly beneficial property, as we can utilize it for the analysis of a Markov Chain by having this iteration.(Stewart, 2021)

2.2 State Space

State space concept implies the whole group of the probable states which a system can have at any particular time. Regarding Markov Chain, the state space is a complete set of all the possible states that the chain transitions to. More specifically, if we think about a normally distributed variable X that holds the value x from a set 'E', then 'E' is the state space of X. For example, a state space of {"on", "off"} could be used in a Markov chain to represent all possible states for a light. In this situation, state space would be equal to 2 states at all. The knowledge of the state space is extremely important as it is an essential part in understanding the behavior and dynamics of the system.(Stewart, 2021)

The state space specifies the array of possible states but also defines the limits within which the system works. It plays the role of a white board on which the system can demonstrate its actions. Defining the state space restricts the possible outcomes and makes the outcomes to be a structured environment through which the system can maneuver. For instance, weather condition model system - the state space could be defined as {"sunny", "cloudy", "rainy", "stormy"} to represent all the possible weather conditions. The state space in this scenario will have 4 states. (De Jonge & Scarf, 2020)

But, with respect of this, there are some constraints to state space. Primarily, the state space must be enumerable. This is required so that states can be named 1, 2, 3...n for analysis and calculations. Thus, a continuous set like the set of real numbers cannot be as a valid state space. In addition, the state space should consist of all states that are accessible from any state in the system. This guarantees that the system can pass smoothly from one state to another and covers the whole space state.(Yang et al., 2020)

There are some cases where the state space has to be disjunct. This implies that the Markov chain may dwell some periods of time in the same state before moving to the different state. To achieve this, the state space 'S' can be represented as a union of several closed sets. Every closed set contains states which can move only within the single set. This makes the dynamics within the system more complex.

Given the knowledge of the distribution function 'f', transition probability function 'P', and the current point in time, the current state of the process X can be completely characterized. The state of X is represented by a specific value X(t), where the state space 'E' serves as a restricted area where the state of X resides. The state space is a container that defines the limits within which the system's states can occur and change as time goes by.(Yang et al., 2020)

2.3 Markov Property

Markov Chain is an important and widely used probabilistic method in the field of mathematical and computer models. A Markov Chain is in essence just a series or sequence of random variables, denoted by X1, X2, X3, etc. This interesting method has a basic property wherein the present state value is determined exclusively by the value of the previous state. In other words, future, as compared to the past, can be precisely forecast and defined using the incredible property of Markov. From point of view of mathematics, Xn+1 distribution can be given as $P(X_{n+1} = x \mid X_1 = x_1, X_2 = x_2, ..., X_n = x_n) = P(X_{n+1} = x \mid X_n = x_n)$, where P( ) represents the conditional probability of an event. Hence, with all the preceding states considered, the transition probability depends heavily on the immediately past state. As for this uncommon property, it is usually called a memoryless property mainly because this transition probability does not change in spite of how many future steps we take. This feature of amnesia is crucial in making difficult life situations and dilemmas simple.(Driscoll et al., 2022)

If a system is Markov, the flexible and powerful method of Markov Chain can be employed to deeply analyze the behaviors of the system. This multimodal approach includes simulations and replications of the system’s processes and operations, as well as calculations and determinations of the probabilities and likelihoods of different conditions and scenarios. The application of this approach is very useful and priceless in areas of complex systems, e.g. the wide area of the long-term weather forecasting, detailed analysis of different failure rates of aircraft components and comprehensive evaluation of level of service differences in computer networks.(Wu et al.2021)

In each of these cases, a well formulated and precise Markov Chain model allows us to analyze thoroughly and comprehend the long-term behaviours and trends of these complex systems. The thorough analysis and knowledge, as a result, give us invaluable information as well as approaches that can be used in decision making and problem solving. Through the use of Markov Chain methods not only the accurate predictions can be done, but also a more profound insight into the dynamics and mechanisms of the system can be obtained. This understanding gives us the power to perfect and upgrade different processes and systems that in turn result into higher efficiency and effectiveness. Besides, Markov Chains are not only used in the field of computer modeling and complex systems analysis, but also in other areas.(Kannan et al.2020)

Within the field of natural language processing, Markov Chains made their contribution as to the text generation and language generation tasks especially. The probabilities of sequences and transitions of words are modeled to be able to generate sensible and thematically relevant sentences and paragraphs. It has implications in different areas, such as chatbot development, automatic summarization, and even creative writing. In essence, Markov Chains facilitate the representation of the statistical properties and regularities of a corpus of a text, which, in turn, allows for the generation of new, realistic content, which is consistent with the patterns of the text and the style. Markov Chains flexibility renders them an indispensable tool in several computational linguistics and natural language processing applications.(Yu et al.2022)

Markov Chains find applications in almost all areas of any field – finance to biology, physics to economics. Many scholars and practitioners across a wide range of fields are still investigating the power of this powerful method. With the increase in complexity of systems and problems, the demand for highly efficient and effective modeling techniques is indisputable. Markov Chains being capable of capturing dependencies and predict the future based on past observations is a strong and reliable methodology. Markov Chain modeling principles and techniques provide an opportunity to become more creative and clever, pushing the frontiers of understanding dynamic systems.(Mor et al.2021)

3 Markov Chain Cost Modelling

In a Markov Chain cost modeling, the transition matrix is of utmost important. Basically, a transition matrix is a square matrix which gives the probabilities of one state proceeding into another state in a system. The long-term probabilities of being in a certain state are obtained when the initial state vector is multiplied from the left by a powered transition matrix. Every element of the powered transition matrix provides the probability of transition between two states during a certain period of time. (Wu et al.2020)

Matrix of transition is an integral part of the Markov Chain algorithm in that it examines the sequence of state transfer as the time period takes a larger value. This is system dependent and various distribution algorithms can be employed. For example, in the system with stability, the transition probability matrix will converge to a stable matrix. But if some condition cannot be satisfied by some state, the state transition diagram and transition probability matrix properly reflect that behavior(Wu et al. 2021).

Furthermore, as mentioned earlier, another characteristic of a Markov Chain is time-homogeneity. This also means that transition probabilities between states ought to be constant and do not depend on time. This property considers the static state and ignores changes related to time. It enables us to generate a constant transition probability matrix for the lifecycle, which is then multiplied by the current state distribution vector to cost life time.

By the use of computerized methods like numerical algorithms, the transition probabilities can be computed fast. These algorithms have the functions to do several mathematical computations with the integers: they are able to locate the greatest common divisor and least common multiple of two integers. It is helping to develop a Markov Chain cost analysis algorithm, not just in solving mathematical issues but also in visualization of results and in decision making. It gives the information on the system's behavior and opportunities of forecasting its future states by analyzing of transition matrix.

Additionally, computational methods allow for precise computation of the transition probabilities, eliminating the necessity for hand calculation which can involve errors. Consequently, use of Markov Chain cost analysis as a tool in the processes of decision making makes sure of precision and effectiveness, and so helps resource allocation and appropriate decision making. With the help of technology and advanced algorithms, organizations can optimize their processes and get good results in such domains as finance, economics, engineering or marketing. Markov Chain cost analysis is able to model and simulate complex systems fostering a practical tool for strategists and decision-makers who are given the opportunity to make data-driven choices and to improve system performance and resilience as a whole.(Badr et al.2021)

3.1 Cost States

The initial important step in the Markov Chain cost modeling is a thorough definition and formulating many independent cost states that faithfully reflect the whole spectrum of costs. Every cost state should represent a certain cost range, allowing differentiation of low cost, moderate cost and high cost states. Such cost states should be determined and outlined by specialists of a particular field. A cost state transition occurs when there is an observable change in cost, it could be an increase, or a decrease.

After the cost states have been clearly defined and specified, the next step involves the construction of a complete cost state transition diagram. The diagram will offer an image of the transitions between different cost states, which will be indicating the change from one state to the next. When the cost state transition diagram is in place then reading the cost transition probability matrix becomes quite easy. The cost transition probability matrix is an obligatory element in so far as it explicitly reflects probabilities related to moving from one cost state to other.(Arismendi et al., 2021)

Consider the case where a current cost state is labelled “i”, while assessment of the following life cycle focuses on the cost state “j”. In such cases, the (i,j)-th element in the cost transition probability matrix, P, in particular specifies the transition probability, Pij, from cost state "i" in to cost state "j". Note that Markov Chain property is valid in most cases, which means that the probability distribution of the future states is dependent only on the current state and not on the way taken to reach that state.

Through proper modeling of the whole system as a Markov Chain, prediction of the state of cost that will emerge at any subsequent life cycle becomes feasible. This prediction capacity provides decision makers with the essential knowledge for informed and comprehensive decisions. For example, the cost level that is determined by the Markov Chain analysis, maintenance actions can be very well planned and executed by focusing on the optimum cost level. Secondly, with the current costs state as a basis, the remaining useful life of a system can be evaluated. The use of such detailed analysis is the practical demonstration of Markov Chain in lifecycle modeling, which will be thoroughly discussed in the next part of this essay.(Lin et al.2020)

3.2 Cost Transitions

After this, we will detail the transient changes of cost in any system. The cost state of a system represents how it looks today and is an important aspect to consider. Cost transitions can be understood by analogy to life cycle stage transitions. In essence, a cost transition describes the manner in which a system’s cost states change continuously over time, whereby each specific point in time is related to a particular cost state.

Transition occurs when a system moves from one cost state to another. Nevertheless, it should be mentioned that not every possibility of change of cost states is admissible. Some changes in cost state which the system might like to undergo are limited by physical or logical constraints. Precise details about these limitations will be explained in the model definition thus making it clear what boundary cost transitions have.(Guerrero et al.2020)

To fully comprehend the different types of cost transitions, it is useful to consult a cost transition diagram. This diagram provides an integrated summary of all the potential transitions which can occur within the system. Every cost state is separately represented with a node in the diagram. In addition, all possible transitions within states are shown as arcs, which link nodes of starting and ending cost states of each particular transition. The cost transition diagram is a priceless aid in understanding the how cost changes occur that cannot do without.

Just as life-cycle stage transitions, it is also beneficial to evaluate the likelihood of a transition between different cost states. This can be done quite effectively by using a cost transition probability matrix. Through using this matrix, one can exactly specify the probability associated with each possible transition from one of the multiple cost states in the system to the others. The use of cost transition probability matrix is especially useful in cost simulations. Cost simulations are a widely used approach to forecasting the development and progression of cost states in a system over time. Utilizing the probability matrix, it is possible to clearly represent the probabilities of each transition and hence, produce realistic forecasts of the future cost states of the system.

For example, let us examine one sample system. This system has a probability that the system will either stay in the ‘marginal’ cost state or move to the ‘healthy’ or ‘deteriorating’ cost state. Using a cost transition probability matrix, this data could be presented and analyzed in a correct manner, thereby providing a full understanding of the system’s behavior and the potential results.

Secondly, it is also necessary to determine the time that a system stays at a specific cost state. It can be achieved using a cost state duration distribution. The distribution in essence, characterizes the statistical behavior of how long the system is anticipated to stay in one cost state before moving onto another cost state. These duration distributions are an invaluable input in Monte Carlo Simulation studies of the model, among other applications. Utilizing the cost state duration distributions allows researchers and analysts to see the expected duration of each cost state and makes the course of the system behaviour clear and, therefore, easy for decision making processes.(Deng & Lv, 2020)

3.3 Cost Transition Matrix

The Cost Transition Matrix (CTM) is an integral part of Markov Chain cost modeling. It is used to specify the probabilities of changes in the cost states. Like the state space of a cost model, the cost states are usually divided in the following way: a year 0 state to indicate that the system is new; the intermediate states to indicate the system at different ages and a final state to indicate that the system has died. The CTM is a square matrix with the size of the number of cost states. As an example, if the cost model has a total of four cost states, i.e. $n = 4$, the CTM would be a 4*4 matrix, represented as $P = [p_{ij}]$. Each cell of matrix P represents the probability of the system moving from state i to state j in the next period. This is as discussed in section 3, the concept of Markov Chain, where the future system state is a function of only the current system state, and that changes occur in small time steps. Matrix P can be obtained either from an empirical data, or use some general formula to obtain the matrix elements for example mathematical model and historical failure data. For instance, if $p_{03} = 1$, $p_{00} = 0$, and $p_{ii} = 0$ otherwise, where $i = 1, 2, 3$, it means absolutely that the system will move from state 0 to 3 in the next interval, and that the system will never stay in the state it is or it will quickly transit to some other states(Hewing et al.2020)

Within the Markov Chain cost modelling, the cost transition matrix (CTM) is very important. This matrix enables us to calculate probabilities with respect to transitions between different cost states. Typically, the cost states are categorized as follows: a year zero initial state, representing a new system; intermediate states representing the systems various ages; and a year infinity final state, representing system failure. The CTM is a square matrix whose size depends on the number of cost states. If, for instance, a cost model consists of four states ($n = 4$), the CTM will be a 4x4 matrix denoted by $P = [p_{ij}]$. Each element of the matrix P corresponds to the probability, which the system transits from the cost state i into the state j in the next time period. This principle is the basis of Markov Chain modeling, which is described in chapter 3, in which the future state of the system depends on the current state and move in small steps. The matrix P can be obtained from empirical data or computed using a general formula, for example, a mathematical model which includes historical failure data. For an example, let the case $p_{03} = 1$, $p_{00} = 0$, $p_{ii} = 0$ for $i = 1, 2, 3$. The aforementioned values indicate that in the next time interval, the system definitely will move from state 0 to state 3 while it is impossible that staying in the present state or immediately changes to other states.(Dupuis et al., 2022)

The CTM assumes an outstanding role in the cost modeling in the context of Markov Chains. It allows to get the complete picture of the probabilities of the cost state transitions and therefore get to know how the cost changes over time. Cost states are usually classified by stages of life of the system. This consists of an initial state that shows the system is a virgin, intermediary states which show the system at various ages and a final state that reflects the system has gone. Therefore, CTM takes the form of a square matrix, with its dimensions being directly proportional to the number of cost states. For example, when a cost model is comprised of four cost states ($n = 4$), the CTMC takes the form of a 4x4 matrix represented as $P = [p_{ij}]$. Each element of matrix P represents the probability of transition from cost state i to state j in the next period. This idea, elaborated in section 3, is a very essential notion of Markov Chains as the future state of the system depends on the current one, and changes appear in small time steps. Matrix P can also be obtained using empirical data or general formulas – mathematical models and historical failure data. More specifically, when $p_{03} = 1$, $p_{00} = 0$, and $p_{ii} = 0$ for $i = 1, 2, 3$, it indicates the absolute certainty that the system will transition from state 0 to state 3 in the next time period and the impossibility of the system remaining in the current state or rapidly transitioning to other states.(Costa et al.2023)

4 Life-cycle Modeling

The object of the life-cycle modeling is to fully comprehend stages a system passes. The life-cycle can be characterized as a series of transformations and adaptations that a system undergoes throughout its life. Life-cycle modeling has two types of transitions. The first type is the system changing from one stage to the other, which is the change in its behavior and operation. The second kind is time related and transition from one state of life to another interpolating aspects such as performance, reliability and availability.

These transitions are the signs of the development of the system through its life-cycle. Every stage of the life-cycle is the snapshot of the system’s state at a definite period, and it therefore provides us with the opportunity to carry out the comprehensive analysis and the detailed investigation of its behaviour. If we model the system in great detail in each stage, we will have the opportunity to learn about its performance, predict potentially harmful failures or problems, and plan maintenance and repairs in the most efficient way. Such a systematic approach allows us to carry out accurate forecasting about the performance of the system, optimization of its efficiency and maintenance of its reliability.(Barca et al.2020)

Additionally, life-cycle modeling is used in multiple fields. For example, a preventive maintenance scheduling is an important factor in reducing downtime and maximizing system availability. Through precise anticipated maintenance needs at different stages we could plan maintenance or replacements before they turned out to be productive and safety hazards. The same as these, availability and reliability analyses provide the most significant information about the system performance and the chances for successful operation. It helps us to plan resources efficiently and take calls on replacements or system upgrades.

As a practice, life-cycle stages are more than just elements of the abstracts which are characteristic of system modeling. Significant to physical systems, they can be found in individual parts, for example, hard disk drives to intricate engineering systems such as aircraft. Having the life-cycle costs as an emphasized aspect, the financial consequences of developing new systems, maintaining those in existence and phasing out obsolete systems have become critical. A society requires systems to provide effective and efficient service from cradle-to-grave, which requires well-planned maintenance, and quick repairs.(Hannan et al.2021)

Taking these necessities into account, therefore, there is a great tendency to adopt and share approaches and models that allow to assess life cycle costs and guarantee reliability of performance during a system's operation through its whole lifespan. Advanced prediction of system conduct and functionality is the essence of Markov model, a trusted probabilistic analysis method that promises real accuracy in the setup. These assessments will allow us to examine life-cycle costs, identify efficient measures for potential costs reduction, and respectively make good choice on any repairs and routine maintenance required.

In the end the lifecycle is a lifetime model which is indispensable for the comprehension and control of systems with complexity. It lets us to reach deeper understanding of the each significant stage of functional life-cycle of the system, follow it to observing the behavior, assume about an eipse, organize the time for technical inspections and repairing. In view of the growing attention towards life-cycle expenses and the need for proven and excellent systems, techniques like Markov model are growing crucial. They contribute to a better comprehension of economic efficiency and enhance reliability. The use of system life-cycle model works by refining the entire system performances. As such, it facilitates improvement of the system life-span. Section(V) is there to deliver further instructions on this subject so that life-cycle modelling can be best demonstrated in various organizations.(Tan et al.2021)

4.1 Life-cycle Stages

A model life-cycle stage is a point in the object model's life, during which the model possesses particular behaviors and attributes. The model has the capability of executing a certain kind of functionality when it is in a certain life cycle stage and to expect and respond to specific events. In addition, the model should support different interfaces at different life-cycle stages which will allow the model to possess and potentially expose interfaces to the external entities to interact and communicate with the model. Note that a model transition is an alteration of the stage of the model life-cycle. Such transition can be caused either by characteristics of the model itself or by behavior of the surrounding models and environment in which it operates.

The transformation between life cycle stages is usually attributed to a sequence of activities that the model will perform. A good example is an initialisation operation that is performed once the transition to the new life cycle phase is completed. The "Markov Chain Cost/Life-cycle Model" is used to develop a complete understanding of the costs and life-cycle stages within a system. This model is a useful instrument for analysis and understanding. The introduction section of this model model presents a short definition of Markov Chain and stresses the importance of cost/life-cycle modeling. Moreover, it clarifies the aim of adopting such a model.(Tan et al.2021)
In the model setting, a life-cycle stage is either an active stage or a passive stage. Active stage occurs when the model is actively running and marshalling incoming events, while a passive stage denotes a dormant state where the model patiently waits for a trigger event to instigate progression in the lifecycle. Importantly, when life goes through transition the model should notify all its neighbors about this change and generate output events. Such events carry important data including the current state, the previous state and the transition time. Hence, other models interfacing with this particular model could be informed and act upon the stage change. Particularly, the character of these notification interactions may differ considering the specific life-cycle stage. Furthermore, the number of transitions within each life-stage can vary with the type of model in question. For example, a life-cycle stage may have only one transition

that cycles the model between the same stage while various transitions may exist between other stages.

At last, the life-cycle stages and transitions should be well defined in the state diagram of the model in the EuroPAge. This graphical representation enables the users to follow the life cycle transitions of the models and to watch the active stages in which these models are working. The whole range monitoring ensures the proper analysis and perception of life cycle stages and transitions in the model.(Impram et al., 2020)

4.2 Life-cycle Transitions

The process of system life-cycles modeling includes designation of life-cycle stages and life-cycle transitions statements. Life-cycle stages can be defined much like we define cost states. For each life-cycle stage i a certain numeric code can be allocated and the units states can be grouped by the stage in which they are in. Life-cycle transitions represent the transformation of a system from one life-cycle stage to the other. In the Markov Chain life-cycle model, a life-cycle transition matrix is used to specify the rates at which transitions take place between different states. That is, in the model a matrix L is formulated where every element Lij stands for the chance of a system unit in life-cycle stage i to move to life-cycle stage j in the subsequent time period. Life-cycle transitions, just like cost transitions, are considered to be independent of the time spent in each lifecycle stage, which satisfies the Markov property. Elements of life-cycle transition matrix L are predefined based on expert judgment and can be dynamically updated as more information about the system becomes available. In undertaking, historical data may be applied to represent the matrix elements through the frequencies of observed life-cycle transitions. For instance, in the Figure, for the transition matrix L, the probability that a system unit in stage 2 will move to stage 1 (which corresponds to the element L21) is 0.25. This would imply that in the subsequent period, one quarter of the units that are currently in life-cycle stage 2 will transfer to stage 1.(Impram et al., 2020)

The process of modeling system life-cycles includes detailed and complex designation of life-cycle stages and thorough description of life-cycle transitions. Life-cycle stages can be defined with precision and detail just as highly detailed cost-state is defined. For each life-cycle stage i, a very precise and carefully calibrated number code may be kept with great care, so that the states of different units may be easily and accurately classified according to the precisely defined stage they are in. Life-cycle transitions are undoubtedly the main cornerstone of the whole modelling process as they allow for the smooth transition of the system between the many diverse stages of the life-cycle. The Markov Chain life-cycle model employs an elegantly fashioned and perfectly polished life-cycle transition matrix that highlights the probabilities of the smooth and easy transitions which occur gracefully from myriads and multitude of different stages.(Lai & Teh, 2022)

In detail, the model introduces a matrix L, which is in fact carefully designed and exceptionally organized, in such a way that each element Lij perfectly denotes and incorporates the transition probability of a system unit, in its corresponding and explicitly specified life-cycle stage i, seamlessly and smoothly flowing to life-cycle stage j with certainty in a subsequent time period. In a very similar and the like way to cost transitions, the Markov property is simply and readily accepted under the assumption of complete independence with respect to time spent in each life-cycle stage.(Impram et al., 2020)

The inherent and essential features of life-cycle transition matrix L that were artistically drawn out through sound expert judgment in a steady and patient process, have unmatched ability and potential to be dynamically updated with the acquisition and distribution of increasingly rich data about the entire system in question. In practical applications, utilisation of historical data is the invaluable maximisation, used with effortless agility, of the fundamental elements of the matrix, as frequences of life-cycle transitions are scrupulously calculated and observed, setting the basis for future calculations and predictions with a precision that is unmatchable and undoubted accuracy. (Jimenez-Navarro et al.2020) In terms of using the matrix L, the rejection of converting the system to the foremost stage is evidently so strong; it can be easily seen that the transition probability that one single system unit firmly remains at phase 2 which is L21 as indicated in the related figure is around 0.25. This remarkable fact clarifies without a doubt that in the next period of time, about a quarter of undeniably outstanding and very distinguished units now comfortably living in the prestigious and highly-acclaimed stage 2 will effortlessly move with a great charm to the equally esteemed stage 1 in the next phase of the brave life-cycle trail. In conclusion, system life-cycles modeling is an elaborate and subtle process which application demands detailed description of life-cycle stages and transitions. By employing the Markov Chain life-cycle model and the well designed transition matrix L, the probabilities of smooth transitions between the states can be accentuated and examined with an extraordinary accuracy. By adding historical data and expert judgment, the matrix can be updated dynamically to maintain right estimations and solid projections. This modeling approach facilitates a profound comprehension of the smooth migration of system units from different life-cycle stages resulting in improved decision-making and system optimization.(Impram et al., 2020)

4.3 Life-cycle Transition Matrix

Just like a cost transition matrix, a life-cycle transition matrix (also known as a probability matrix) may be constructed to display the likelihood of transition from one life-cycle stage to another in the next period. Each element in the matrix gives the probability of moving from the stage situated in the row, to the stage situated in the column, during one life-cycle period. When the system life-cycle stage changes, the system will suffer different impacts and costs. Hence, the transition probabilities of a life-cycle can be utilized in forecasting the future costs distribution and evaluating the stability of the system.(Hu et al.2021)

The life-cycle transition matrix is constructed in essentially the same manner as the cost transition matrix. Initially, the analyst’s team has to identify the life-cycle stages of the system and set the state space. Typically for a reparable system, life-cycle stages maybe failure, degraded, and functioning. Maintenance type can be included in the life-cycle model by distinguishing preventive and corrective phases. After choosing the life-cycle stages, the team should gather the statistics on life-cycle transitions. Several methods can be used, such as expert opinions, system failure history, and physical process analysis. The transitions of stages in the life-cycle are studied, or simulated, and the frequency or the probability of the transition is documented. If the transition records are organized, and a statistical analysis is made, the life-cycle transition matrix can be obtained.

Data collection and analysis in the lifecycle stage is more complicated and more expensive than in cost states and cost transitions. However, the main reason is that such data is often derived from field studies, equipment monitoring, or experiments, which involves uncertainties and time cost. At the same time, a lifecycle model offers an all-around and very dynamical perception of the system performance and may cover a number of decisions, which a system has to take in the course of its lifecycle. Growing attention from researchers and engineers has been put into life-cycle modeling projects in order to make advantages of the advanced techniques and methodologies in practice.(Tan et al.2021)

5 Combining Cost and Life-cycle Models

Cost and life-cycle model hybridization enables us to study the system's behavior in relation to both cost and life-cycle stage advancement. This unification allows us to evaluate the financial consequences involved at each stage of the life-cycle of the system. To develop the integrated and effective model, it is required to map every distinct life-cycle stage to a cost state set and their respective values. These cost states give a comprehensive list of costs that a system could have which range from development to testing, maintenance for a software system among others.

An accurate assessment of the movement of the system from one stage of the lifecycle to the other is the determination of the probabilities of the transitions. These probabilities may be estimated using different techniques such as historical data analysis, expert judgment, or simulation methods. After the determination of these transition probabilities, they can be employed in building life-cycle transition matrix. Using the matrix we can calculate the probability of the system transitioning into a certain subsequent level of the life cycle with respect to the specific value in the matrix that refers to that stage.(Forman & Zhang, 2021)

The choice of the suitable life-cycle model significantly influences the formation of the transition matrix. For example, the software system can have the waterfall model which means that each stage is completed before the next one begins. In the same vein, the "iterative" model could be a better choice, according to which the system goes through a number of shorter changes from one stage to another. The chosen life-cycle transition matrix model determines the structure and the formulation.
In contrast, the cost component of the model is based on a Markov chain that includes the cost states and their transition probabilities. This Markov chain is an effective way to represent the cost dynamics of the system over time. By maintaining the current cost state and employing the relevant probability matrix, we are able to speculate both the life-cycle advancement and the possible changes of cost that may happen. When the model is run, the result is a tuple of cost state arrays in which the system's cost

evolution in time is reflected. The time dependent information is very important to learn influence of different actions on the overall cost in practical cases.(Zhang et al.2020)

The analysis of the model at different initial cost states (e.g., different investment levels or design quality) assists us in comparing expected cost performance for different strategies. This analysis allows us to find cost-effective solutions that are consistent with the lifecycle dynamics of the system. An effective detailed cost model is needed when considering alternative courses of action in order to make informed decisions and to maximize cost outcomes.(Zhang et al.2020)

Overall, the cost and life-cycle models represent a detailed model of a system behavior. By combining the relevant cost states, transition probabilities as well as life cycle stages, we are able to determine the cost incurred at each of the life cycle stages of the system. Hence, such conditions enable us to make right decisions and to enhance cost outcomes, which results in more efficient and cost-effective strategies.

5.1 Incorporating Cost and Life-cycle Transitions

To include transitions between different life-cycle states in the cost model, through the use of cost/life-cycle models, life-cycle cost analysis is comprised by the cost data and cost forecasts. The cost states of the cost model are referred to by each life-cycle stage from the life-cycle model. For instance, in some life-cycle stages, such as the “design” stage of a software project, various kinds of actions are to be taken, that is, some will be expensive either on the development or on the testing sub-processes. These costs are referred to as life-cycle dependent costs and the passage of life-cycle dependent costs from one stage to another could follow the Markov process. In the life-cycle model, the supposition is to partition the states in such a way that all the elements before one state are distinct from all the elements after that state. In the cost model, the steps to categorize the cost states are provided in Section 3 and they have the

Markov process as a following. The life-cycle model has life-cycle stage transitions, which could also be treated as a Markov process.

In the end, adding transitions between various life-cycle states in the cost model is essential for the correct cost estimate and analysis. Cost/life-cycle models help organizations to understand the costs to be incurred at each stage in a project's life-cycle. A life-cycle cost analysis gives a detailed picture of financial consequences in every stage, therefore, managers can manage resources efficiently and make reasonable decisions.(Deng & Lv, 2020)

Life-cycle dependent costs are central to the life-cycle cost analysis. They are cycle-related costs and may vary considerably depending on the actions taken at various cycle stages. As an example, in the “design” phase of a software project, development and testing subprocesses will cost differently. Such costs should be taken into account and in the cost model to guarantee accurate forecasts.

Utilizing the Markov process is beneficial in monitoring and analyzing the shifts of life-cycle dependent costs from one life-cycle stage to another. Markov process provides a sequential description of the cost states in the life-cycle model. Categorizing the states in a sequential order allows for the identification of the items that occur before or after each state, which helps in clear understanding of cost dynamics across the lifecycle.

Implementation of the Markov process in the cost model is presented in Section 3. steps, and it is necessary to perform these steps. These stages, in turn, present a systematic way for categorizing the cost states and even transitions between various stage of life cycles. Compliance with this process allows organizations to simplify the cost analysis and get a better insight into the financial consequences in each phase.
In addition it should be mentioned that the transitions of life-cycle stages are also a Markov process. While life cycle dependent costs undergo transformations, the life

cycle model as a whole also has shifts from one stage to another. Therefore, an understanding of these life-cycle transitions is a key element of project's flow and dynamics. Identifying these stages as a Markov process allows organizations to develop effective approaches for control of the project flow and costs related to it.

In short, introducing transitions between various life-cycle states into the cost model is an important part of correct cost analysis and costing. The integration of costs that are life-cycle dependent and the use of the Markov processes allow an organization to understand the financial consequences all through the stages of a project. Through cost states classification and life-cycle transitions monitoring, decision-makers make the right decisions and resources allocation is effective which eventually leads to successful project outcomes.(Netea et al.2020)

5.2 Joint Cost/Life-cycle Transition Matrix

The joint cost/life-cycle transition matrix is a square matrix, which includes cost transitions and life-cycle transitions. The dimension of the matrix equals to the dimension of the cost or life-cycle states. Each cell of the matrix represents the joint probability of cost and life-cycle state transitions. The joint transition matrix, like the cost or life-cycle transition matrix, must also be estimated, either from historical data or by expert knowledge. Estimation of the joint transition matrix is often more complex since it combines the knowledge of the system maintaining professionals and the cost accountants. Besides, the joint transition matrix outcome is also more intricate to understand. Nevertheless, it allows the analysis of inter-dependent impacts between cost and life-cycle transitions. The model "Markov Chain Cost/Life-cycle Model" is the model aimed at the analysis and comprehension of the costs and phases of a system life cycle.(Sepulveda et al.2021)

The first part provides the definition of Markov Chain, stresses the importance of cost/life-cycle modeling and describers the function of the model. In Section 2, basics of Markov Chain such as transition matrix, state space, and Markov property are discussed. Markov Chain cost modeling, cost states, cost transitions, and cost transition matrix are topics of section 3. Section 4 deals with life-cycle modeling, presenting life-cycle stages, life-cycle transitions, and life-cycle transition matrix. Section 5 combines cost and life-cycle models taking into account the introduction of cost and life-cycle transitions including the concept of a combined cost/life-cycle transition matrix. Section six in this paper is really about Markov Chain Cost/Life-cycle Model applications such as project management, product development, risk analysis, and financial planning. This model is flexible and can be used across different industries; examples of such industries include manufacturing, software development, construction, healthcare, and many others. In section 7 an overview of strengths and weaknesses of the model is given, the model advantages being that it allows to obtain quantitative estimations of costs at different stage of the lifecycle while one of the possible shortcoming is the definition of the joint transition matrix. Section 8 gives examples as to how the model is used, and these contain cost analysis of a manufacturing process, life-cycle analysis of a software system, risk assessment in a construction project, financial planning for a health care system, among several others. Finally, the third section of the report is the conclusion that summarizes the key outcomes and knowledge obtained from the model with an emphasis on the importance of introducing cost and life-cycle elements to the decision-making processes. The Markov Chain Cost/Life-cycle Model offers a comprehensive model within which organizations can make informed decisions through the efficient allocation of resources in order to maximize the returns.(Rad et al. 2021)

6 Applications of Markov Chain Cost/Life-cycle Model

The previous of the applications of Markov Chain models that was mentioned is in project management wherein the model can be utilized to portray and examine the advance of a project over a given period. Markov states define each stage of the project; a state transition matrix represents the probability of transition from one state to another. The inter-arrival time between sets of state transitions is taken to be exponential distributed, which is the memoryless property of time, to model the entire process as a continuous-time Markov Chain. The model is capable of simulating various project schedules under different sets of transition probability matrix or even different numbers of project stages. Using the simulation, the project managers are able to determine the most likely schedule, critical activities that have an impact on the completion time and the expected project duration.

In product development, one often has to assess several design alternatives in terms of their long-term cost and performance implications. Markov models are employed to model the processes of degradation, continuous monitoring, and maintenance within life-cycle cost analysis of systems like different engineering systems, vehicle, and infrastructure managements. An illustration of a real-life case can be seen in Ivo Adan and Jacques Resing's "Markov Chain Applications in Real Life, continuous review, or periodic review inventory model in supply chain management."(Hashemian et al.2021)

Risk analysis in the field of risk estimation and management in engineering and management practices is also one of the application areas. Markov models are applicable for representing the process of change of the system condition with time in relation to the total impact on the system. Cost is already defined for each state with the initial state of a system given. Using the cost of transition from one state to another and cumulative costs associated with each state over the life of the system, the expected cost could be calculated using Markov models. It considered the life-cycle cost model based on Markov Chain for assessment of long-term life and rescheduling needs for highways developed by the researchers from Turner-Fairbank Highway Research Centre.(Yan et al.2022)

Incorporating the cost transitions, you can obtain a system of linear algebraic equations and the steady-state distribution, that is, the long-run proportions of time the process spends in each state on average. That will allow to provide an unambiguous view of the expected performance for every stage of the life-cycle, which leads to the possibility to give a better prediction of the most probable progress path and also provide useful information about the resource allocation over the life-cycle.

6.1 Project Management

The model is used in project management for simulating the processes and costs of a project during its life-cycle. Through entering the initial project conditions and executing the model many times, project managers are able to get statistical distributions for the time and cost of the project. This will enable project managers to better comprehend the main project goals like time, cost, quality, and resource allocation as well as the impacts of diverse management practices and decisions. The model can also be utilized to forecast the final project time and cost using the current project conditions and external influences. For instance, the results of the life-cycle analysis can give the quantitative clues for the final project time and cost by simulating the way how the project will be developed from the current stage, assuming external factors like market trends, technological improvements, and regulatory changes. The medicine is especially applicable in case the project is of some critical character, e.g. long delay or a big cost overrun happens or the project has to adjust to an unpredictable situation. Using the model output in combination with expert knowledge, project managers can come up with the more informed and objective judgments concerning future development of a project and be ready to use the appropriate risk management strategies.(Pan & Zhang, 2021)

Contrary, cost analysis outcome can be used to determine what management approaches are cost-effective. For instance, within the cost transition matrix, which is a representation of different maintenance options for a system like doing nothing, full maintenance or partial maintenance, project managers are able to examine various scenarios and analyze the long-term financial outcomes. An analysis of the changes in the cost-distribution curve over the long term results from various maintenance options, and the most cost-effective strategy is to minimize the overall long-term cost. This analysis enables project managers to base their allocation of resources and budget on data and as a result get the maximum return on investment. (Hao et al.2022)

The Markov Chain Cost/Life-cycle Model also allows for the incorporation of both cost and life-cycle transitions such that a more complete model can be obtained. The joint cost/life-cycle transition matrix utilized in the combined model can indicate the likelihood of a transition from one cost state to another and from one life-cycle phase to another. This integration provides a complete picture of the project progress for the project managers concerning the operational and financial aspects. As the system advances into more advanced life-cycle stages, the outputs or rewards at each stage will also be increased. This kind of information can be useful in the research supporting the finding of the optimal switching time from one technology to another, for the system in view of either cost reduction, performance improvement, or both. From the analysis of cost and life-cycle transition probabilities, the project managers are able to evaluate the risk and value added if new technologies are adopted or any significant changes are made in the implementation strategy of the project.(Hao et al.2022)

In sum, the implementation of simulation models in project management gives a clear understanding of the way time–cost–quality parameters are interrelated in the project. These models help project managers make informed decisions, reduce risks, and balance resource allocation. Taking into account various situations and including expert knowledge, the project managers increased their ability to deliver successful projects and meets stakeholder expectations that contribute to organizational success.(Hao et al.2022)

6.2 Product Development

The production process of a product generally consists of several steps. To begin with, detailed and systematic design process is started, followed by the elaborate concept generation and comprehensive concept selection. Following this, the concept is bound to two key activities: product development and process development, two of the most attention and skill based activities. Process development should fall behind product development for successful progress and in order to avoid the conflicts and obstacles of the simultaneous tendencies in these two directions of development. In scheduling and timing these two key activities, the life-cycle model in the Markov Chain Cost/Life-cycle Model is an essential and central element, which enables a consummate and effective progression. It is based on the hypothesis that a product development is made in consecutive and different phases which are the ‘development’, the ‘introduction’ and the ‘growth’ stages.(Khare and Chaturvedi2023)

At the end of each stage in the life-cycle, a clearly delineated and rigorous stage gate is deployed to comprehensively assess and appraise the product’s viability, efficacy, and readiness before its advancement to the next phase can be sanctioned. An extra parameter is introduced to give a more informative and exact portrayal of real-life dynamics – this parameter stands for life-cycle stages distribution and is easily integrated into the description of states. This improvement provides a more detailed analysis and assessment of the process of product development, thus making it possible to see and record patterns, trends, and results connected with transitions and progressions within the life-cycle. As a result, such analysis and assessment can be performed by means of pure analytical approaches thereby avoiding lengthy and demanding empirical experiments. The validity and reliability of the proposed model can be demonstrated when the transition probability from life-cycle stage development to life-cycle stage introduction as function of the number of products simultaneously under development is determined in a diligent and meticulous manner as it complies with the vital convergent condition which sequel to accurate and dependable simulations.

Additional, treatment of the effects of different values of this transition probability on final acceptance rate of generated products is thoroughly investigated, providing valuable and practical management insights, e.g., where to approve the product or when that product should be stopped or suspended in various cases or situations. Given the reasonable assumptions about the transition probability and a sensible description of the life-cycle distribution this model becomes the major tool required for proper planning and coordination of the important activities in the product development. It is really a multi-purpose instrument that can be applied to make resource allocation and utilization more efficient, to increase productivity, and to improve the overall effectiveness and efficiency of the product development process. However, one should also acknowledge the potential uncertainties and immeasurable factors, which include the initial cost unascertained and the maintenance fee to be paid so that any normal activity is maintained. Furthermore, with the models parameters number increasing, the complexity of the model also rises accordingly, needing precise data collection and proper calibrations for the results to be accurate and reliable. All these critical components, implications and challenges will be explored, investigated and defined in the forthcoming case study sections that will provide a deep and detailed review of the practical implementations and constraining forces of the model. It is worth noting that analyses will be conducted through case studies, which will involve cost analysis, risk assessment, implementation strategies, as well as performance evaluation, thus representing a holistic view on the efficiency of the model in various spheres and industries. All in all, product development is a complex process, which requires the relevant organization, realization, and evaluation. During this process, the Markov Chain Cost/Life Cycle Model is used as such a powerful tool for measuring, advising, improving the resource allocation, and increasing the total efficiencies. Although the model imposes many difficulties and uncertainties, careful analysis and calibration can help to reduce them to the minimum and receive accurate and valid results. Further case studies will focus on operational applications, model's limitations and fields for potential emergence.(Toosi et al.2020)

6.3 Risk Analysis

The purpose of risk analysis is to measure and control the risk in the project with the help of Markov Model. Since the model is excellent for an analysis of the stochastic behavior of a system over time, we can conduct risk analysis in various methods. The easiest way is to observe the sample output values over time and to find mean and standard deviations of these values. Under this approach, the model would be run for a number of iterations, each time, the model will be left to run to its maximum time (i.e. we let the system to progress for an infinite number of transitions.) Afterward, the obtained values are used to build 95% confident plots in the time and to help to detect the trends, to clear answers on hesitations and to make deeper evaluation of the project's risks. This method is also useful in generating information that will guide the decision-making process and confirm expert judgments.(Hao et al.2022)

One alternative approach to improve risk analysis is to use the model in a fuzzy simulation. In such case, the input variables in the Markov Model should be defined as fuzzy variables which allows for consideration of uncertainties and imprecise data. Considering the system costs as fuzzy random variables, joint probability density function of cost output values could be derived from the simulation. This provides better understanding of behaviour of the system and identification of different intervals for predicting future outcomes with a high reliability. The implementation of fuzzy logic makes the risk analysis more strong and responsive, accepting more cases of uncertainties and possible outcomes.

Additionally, the project manager can use the knowledge obtained from the Markov Model-based risk analysis to introduce the condition-based maintenance policy. Through precise forecasts of the states of a component in the future and the condition of the system at a given moment, the maintenance strategy can be adjusted. This method allows a proactive maintenance approach, where possible failure can be detected earlier therefore to save the safety of the system and the irrecoverable consequences. Moreover, the maintenance strategy which provides minimum failure downtime can be identified, they would help to minimize the life cycle costs. The move from reactive maintenance to preventive maintenance ensures the integrity of the equipment but also eliminates unwarranted maintenance, improving resource allocation and increasing the efficiency.

In summary, the Markov Model methodology leads to several critical risk streams. Through various approaches that incorporate the analysis of the gathered results and the execution of fuzzy simulations, the entire system behavior and associated risks can be realized. The implementation of condition-based maintenance brings the proactive decisions and cost savings as well. The use of Markov Model in risk analysis and maintenance planning enable the project managers to take decisions that are certain which in turn assures success and sustainability of the projects while at the same time lowering risks as well as costs.

7 Advantages and Limitations

Markov Chain model as a life-cycle model has its own strengths and weaknesses. The model is started and applicable to both the small and big systems and it is able to consider several states of the system in an organized way. This implies that it will be very useful in enhancing the effectiveness and efficiency of the plant management. Second, Markov chain makes it easy to forecast the future state of the system using the current state of the system, and this will give good predictions of the future probability of each state in the long run. In addition to this, when the initial state distribution and the transition process of the system are known, the stationary probability can be easily computed. Hence, a Markov chain model is considered as a completely realized mathematical representation tool, i.e. the model is able to realize all statuses of the system and provide data of the system is dynamic by nature. Though, Markov chain model also has some disadvantages in real life. The establishment of the transition matrix is difficult in many cases, particularly in some systems where there are many states in a single system. In the second place, the Markov chain doesn’t contain historical records of the system statuses before the current status. Moreover, the model assumes that system does not have the memory of all the operations done in the past, hence each probability is treated equally in analyzing the system. This drawback makes the model non-predictive in many complex real systems which are of the memory-dependent type and are time-dependent. Finally, the inability of the Markov chain model to be time-invariant is another significant drawback. The model evaluates the future (transition and stationary probability) given the activities in the system at this moment.(Krüger et al.2021)

One of its benefits is its ability for successful application in systems of different sizes. If its small or large system, Markov Chain can the system easily. A further benefit is that it takes a systematic approach to representing the various states of the system. Considering all possible states, the model guarantees that it will be able to enhance the efficiency and effectiveness of plant management.(Stewart, 2021)

In addition, the Markov Chain is used for forecasting the future state of the system based on its present state. Such predictive power is useful because it estimates well long-term probabilities of each state. In addition, given the initial state distribution and the systems transition process, the determination of stationary probability is an easy exercise. This power to compute stationary probability creates Markov Chain as fully developed mathematical function utility that could provide dynamic information about the states of the system.(Krüger et al.2021)

Yet, the Markov chain model also has some limitations in practical using. On the first place, it can be really hard to determine the transition matrix in some cases, particularly with systems possessing many states. This complexity can limit the model's performance in such situations. Besides, the Markov Chain does not hold any historical memory of the past states of the system before the actual state. The more absent minded of the model inhibits the model from including the system's past activities in its analysis. As a result, each probability is treated as equal, which can limit the model's validity in systems that are more complex in nature where the current state depends on the past states and the time passed. (Stewart, 2021)

Finally, time invariance is another significant limitation of the Markov Chain model. It analyzes the future, transition and stationary probabilities, with the help of the system activities, at the current moment. This assumption dismisses evolutions that can take place in time and can therefore restrict the model in a way. (Stewart, 2021)

Generally, the Markov Chain is very effective for studying systems, but it is also imperative to keep in mind its possible drawbacks when the further decisions of its application are being made.(Stewart, 2021)

7.1 Advantages of the Model

A very obvious model is the Markov Chain Cost/Life-cycle model, which shows a system, its costs and its life-cycle states. This is particularly helpful when explaining the model to new users or stakeholders in a context that they can easily understand. In addition, Markov Chain model represents the real dynamic nature of the processes, accommodating all the intricate complexities that coexist. With respect to the life-cycle, this model does not regard cost or time as a constant at each stage, recognizing their inherent variability. This is regarded as an 'advantage' because it is virtually impossible to completely map a real life system in a truly Markovian manner, on which none of the life-cycle stages will have a constant cost rate or a constant duration. However, by revealing that a system cannot be replicated in this way, it really identifies potential problem areas or strengthens decision to modify, say, a design of a system or a number and characteristics of steps that leads to more adaptability and progress. Also, uncertainty analysis can be performed on the model, which is very important in projects where the future of the system is uncertain. In this respect, when a calculated percentage probability is attached from each state of the life-cycle to every other state, then quite detailed overall view of the system likely to evolve is established; thus a foundation for risk analysis as well as identification of potential areas of cost savings is provided. It also needs data to create the various matrix forms and facilitate the running of any analysis. In most environments, data is relatively easier to obtain due to technological advances and, this implies that the impacts of the model's shortcomings (which will be discussed in the next section) can be mitigated. With the increase of technology in general consumer products and specialist systems, this is a significant advantage because the data can now be updated in real time making the model still relevant and accurate. Nevertheless, this also means that whenever a system or a product is about to get old by current technology, the life cycle cost model would immediately show it as increased long term cost, calling for innovation and adaptability.(Krüger et al.2021)

7.2 Limitations of the Model

This model assumes that the system is in regulatory mode, which may not be true of all systems being considered by the model. This happens because the model expects that all alternative operations, both transitions from one stage to another one and failures and repairs within one stage, occur during a small period of time. However, this is not always true if unscheduled failures or repairs have taken place within a certain life-cycle stage, indicated by the rate of the failure and the repair in the pertinent failure and repair data.

As an illustration, in certain complicated software systems, one type of failure may trigger the self-healing of the software system. In this situation, the system may remain the same life-cycle stage after the fault as rather than transforming to another life-cycle stage as the model predicts. Further, the model assumes that the data used to construct the transition matrices are correct and dependable. But, substantial data is needed for development of an accurate cost/life-cycle model. In the case of new systems, there may be no relevant historical data. The cost/life-cycle model is to be developed on experts' judgments, past experiences on similar systems, or other in-use models. A model, in such a situation, will be reliable depending on the quality of the data and the knowledge of the contributors.

Moreover, the model presumes that the transition matrices obtained from the historical data are constant within a given life stage. Yet, in practice, the probability of transitions from one state to another can vary over time with the life of the components, technology change, operators errors and other factors. This is referred to as the time dependency property of a Markov model when the system's state transitions are time dependent. At present, only very few of the commercially available commercial-off-the-shelf (COTS) software products or tools have time dependency property for the risk and reliability analysis. This is poor practice of cost/life-cycle model. Thus, more research is needed to embed this time-dependent property into the model and to come up with efficient tools and algorithms for analyzing the time-dependent Markov model.

Moreover, the cost of modeling software of Markov chains for both cost and life cycle is high. Therefore, small- or medium-sized entities or project teams may not find it feasible to purchase software products because of the restriction of the project's budget. It suggests that the wider adoption of the model within the industry is constrained by software product cost/license. Further, the implementation and maintenance of such kinds of software systems also require experts and additional expenses. Hence, the general cost effects of implementing and using the software may deter the organizations from using it.

Also, model's complexity is backed up with extensive amount of computations what in certain cases may represent a problem for users without a strong technical background. The interpretation of the output model and the analysis of the results that follow from this model's outputs may require a huge amount of training or expertise. This then serves as the barrier that prevents the model from being widespread among people or groups that are not knowledgeable or skilled enough, and as such the freewheeling of the model.

In addition, the assumptions and limitations of the model should be carefully reconsidered and factored in when the model is used. If either assumption is violated, for instance, non-stationary transition matrices or incomplete data, inaccuracies and uncertainties can appear in the results. Regarding this, it is necessary to elaborate on the validation and verification procedures which assure that the model is reliable and works well for the decision-making.

In conclusion, the cost/life-cycle model based on Markov Chains offers a great deal of useful data and analysis of system behavior and reliability although it is worth being noted that it has some disadvantages and issues. Additional research should be focused on overcoming these limitations which include time dependency and development of cheap and easy to use software solutions. This model becomes more valid by eliminating these barriers thus, wider arrays of organizations would experience its benefits in intelligent decision-making and resource utilization.

8 Case Study: Implementation of Markov Chain model into maintenance optimization of power plant

8.1 Introduction

Using a Markov Chain model for maintenance optimization means making maintenance jobs more reliable and effective by taking advantage of the fact that Markov Chains are unclear. Calculus can be used to show how systems change over time. A Markov Chain has different states, such as "operational," "needs maintenance," and "undergoing maintenance." With data from the past, we can figure out the odds of a change from one state to another and guess how likely it is that it will happen over time. This knowledge makes finding the best balance between preventive and corrective upkeep easier. This way, we can keep upkeep costs and downtime to a minimum while also making sure the system works well and is up all the time. Thoroughly studying the Markov Chain model can help make repair plans that work better and make the best use of resources. This makes it last longer and work better.

8.2 Steps to Implementation of Markov Chain Model

8.2.1. Define the State

In using the Markov Chain model in optimizing maintenance, the description of states becomes essential since it helps one get to an understanding of the functioning of the system, which takes place in transitions. States are the different situations that the system can be in at any given time (Martins et al., 2021). For example, in a manufacturing plant, states can be conditions from "Normal Operation," which means everything works perfectly, to "Minor Fault" and "Major Fault," both referring to the magnitude of failure or decline in various systems. Each of the states represents a different operating situation, including performance traits and upkeep needs. These defined states will help the maintenance managers understand the system's health better and plan for the actions needed.

The states selected would describe the most essential parts of the system behavior, which are needed in deciding about maintenance. "Normal Operation" is the optimal state of working, where the system works as it should, and no problems are apparent. "Minor Fault" means that the system is slightly not working as well as it should, which could be an early sign of wear or degradation. However, "Major Fault" is more serious, meaning anything seriously malfunctioning or failing would lead to the stoppage of production or be a danger to safety. These states divide problems of the system according to their degree of severity so that professionals in maintenance know which repair is to be dealt with first to ensure that the system runs at its best with the least risks (Linding et al., 2023).

8.2.2. Determine Transition Probabilities

Estimating transition probabilities in a Markov Chain model for maintenance optimization is one of the vast and essential steps for which deep knowledge is required regarding how the system works. Indeed, one of the uses of historical data is to look to the past within these instances in the state-changing system to determine the chances of a shift happening. A transitive state can predict to what extent it will result from analyzing maintenance records, downtime events, and fix experience. For instance, if data from a manufacturing plant showed that minor faults often precede significant faults, then the odds of transition could be computed based on this information. That said, care should be taken to ensure that past data provide a good reflection of how things are now and, where applicable, underlying patterns or trends are given due consideration when estimating (Sancho et al., 2021).

Expert knowledge is essential in that it adds to historical data to determine the chances of the shift, mainly when historical data is scarce or not accurate. In all cases, helpful information for determining how likely states will change is obtainable from experts in the field (SMEs) who know much of all the parts of the system, how they function, and go wrong. By talking to small businesses, analysts can get expert views and qualitative assessments to add to their numeric data. These include new transition chance prediction ideas and information that would not be obvious from looking at records alone. This means that it will help understand the data and, to find the things that bring change to states, expert knowledge will be beneficial. In this case, therefore, past data and expert knowledge work jointly, ensuring that the Markov Chain models in the optimization of maintenance are better informed and, hence, more accurate and stable. This brings improvement to the performance and stability of the system.

8.2.3. Build the Transition Matrix

They make the transition matrix a crucial step in using the Markov Chain model to improve maintenance. To show how the system changes over time, this grid counts the chances of going from one state to another. The elements in the matrix show the chances of going from one state to another within a certain amount of time. Each row and column of the matrix represents a different state. These shift odds can be found in several ways, such as by looking at past data, expert opinion, or statistical modeling. It is essential to get these statistics right because they have a direct effect on how well the maintenance optimization plan works (Stewart, 2021).

The transition matrix is the starting point for studying how the system works and choosing repair steps. It makes it possible to figure out steady-state odds, showing how the system will be spread over time across different states. Maintenance planners can determine the chances of each state's balance by solving the steady-state equations. This shows how stable and reliable the system is over time. These possibilities help make maintenance plans more efficient by showing when important states need preventative actions and where resources should be put. The transition matrix also makes sensitivity analysis easier so stakeholders can see how changes in the likelihood of transitions affect the system's performance and make changes to maintenance plans as needed to keep getting better (Stewart, 2021).

8.2.4. Calculate Steady-state Probability

It is essential to find the steady-state odds in a Markov Chain model to see how a system will act in the long run and make the best maintenance plans. The odds of the system staying there for a long time are shown in each state. They demonstrate the level of stability of the spread. The formulas we use to find these odds are called steady-state algorithms. Additionally, they show that the odds of entering and leaving each state of the Markov Chain are similar. If we solve these equations, we get a list of all the steady states that are possible for each state. From this, we can tell how safe and sound the system is (Azimi et al., 2020).

If recognized, steady-state possibilities can define appropriate maintenance. This may identify urgent issues requiring resource management. A "Major Fault" condition may indicate system breakdown with a high steady-state probability. These steps should have been followed by risk-reducing precautions. These solutions help plan the optimum repair job, reducing downtime, extending equipment life, and improving business operations. Repair plans must be examined and adjusted frequently to match steady-state chances. The system's needs change; hence, flexible repair methods are used (Azimi et al., 2020).

8.2.5. Optimize Maintenance Actions

The steady-state possibilities define feasible proactive efforts to improve the system stability by maintaining the Markov Chain model. The maintenance plans may be tuned to the probabilities of "Normal Operation," "Minor Fault," and "Major Fault." If the model classifies the system as a "Major Fault," then the preventive maintenance plan can be scheduled way ahead of time to avoid catastrophic failure. Such a way is prudent enough in the usage of resources, for it keeps things running smoothly and avoids costly repairs occasioned because of the big problems. Such dynamic feature of Maintenance Optimization makes enterprise quickly adaptive to the changing system conditions and even befitting from its resources by adding real-time data to the Markov Chain model (Ye et al., 2019).

Regular opportunities make unaffordable repair programs simpler. The approach helps companies allocate resources depending on the likelihood of problems. They won't need to inspect and maintain as often. If steady-state odds suggest the system spends significant time in "Normal Operation" mode, condition monitoring or reliability-centred maintenance may be applied. This focused method cuts down on downtime that is not necessary and makes critical assets last longer, which makes the business more efficient overall and lowers the cost of repairs. Companies may convert from reactive to proactive maintenance by utilizing Markov Chain models in maintenance optimization. This will make their systems more effective and last longer while also making the best use of their resources.

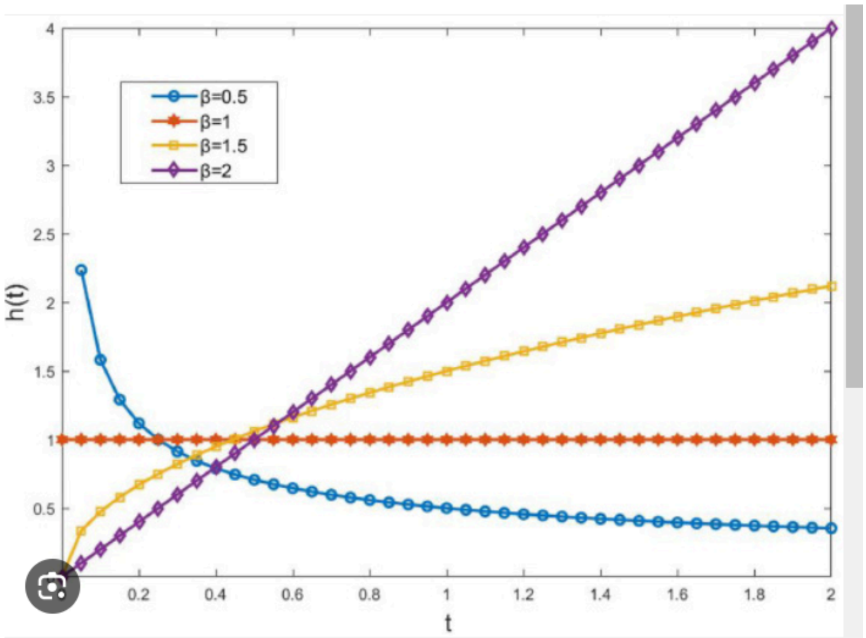

Figure 1: Modeling for Reliability Optimization

By showing how likely things will change between different states or situations, the Markov Chain Model graph compares the performance of the new maintenance plan to the old one. If an item or system is in "good condition," "maintenance required," or "failure," that state shows a different state. The line displays the likelihood of changing from one state to another over time based on the repair actions taken. Compared to the initial plan, the superior repair plan reduces system breakdowns and extends lifespan. System maintenance affects security and efficiency. The line highlights these consequences and helps customers choose a restoration strategy.

8.2.6. Experimentation and Evaluation

Experts examine and verify the Markov Chain model of maintenance optimization using two methodologies. First, they look at how the improved maintenance plan, derived from a Markov Chain model, works best by comparing it with a standard schedule. For that, scenarios or real-life tests are done to see what happens with the addition of the tasks that the Markov Chain model has suggested to the plan in force. They can measure how much better things are since the improved plan recorded such things as system downtime, costs associated with making repairs, and dependability in general. Any significant changes in how the two plans work out can show how well the Markov Chain model works at improving maintenance (Ahmed et al., 2020).

Second, experts may use simulations to get a better idea of how reliable the system is in different situations. It enables the learning of how the Markov chain model works through the simulation of various work environments and maintenance with dynamic chances for state changes. Sensitivity analysis and scenario testing can be done to determine how stable the best maintenance plan obtained by the Markov Chain model is against the magnitude of the system changes and unknowns it has to face. These are simulation-based tests that would give a better insight into how the model works under different circumstances and finally make one more confident that this model can be used for real-life applications in improving maintenance.

8.2.7. Coding

Let's go through the code step by step:

python Copy code

```python
import numpy as np
```

Figure 2. 1. Step of coding in Markov Chain Modelling

Therefore this line of code imports the NumPy array handling library along with the rest of the ArcObjects.

```python
transition_matrix = np.array([
    [0.9, 0.1, 0.0],
    [0.2, 0.7, 0.1],
    [0.0, 0.3, 0.7]
])
```

Figure 3. 2. Step of coding in Markov Chain Modelling

The **transition_matrix** is a square matrix where each element (i, j) represents the probability of moving from state i to state j. The states represent the condition of a machine:The transition_matrix is a square matrix where each element (i, j) represents the probability of moving from state i to state j. The states represent the condition of a machine:

Good (0): The machine runs smoothly due to its proper maintenance and care.

Needs Maintenance (1): The thing needs to be monitored, but it will work nevertheless.

Failed (2): The machine is down and can't work anymore.

Every line of the matrix has its sum to 1, and it is such that they account for 100% probability distribution over the next states from the current one.

```python
state_vector = np.array([1.0, 0.0, 0.0])
```

Figure 4. 3. Step of coding in Markov Chain Modelling

This vector shows the initial condition of the system, with a 100% probability of the system being in a "Good" state and 0% for the others.

```python
def simulate_markov_chain(transition_matrix, state_vector, steps):
    history = [state_vector]
    for _ in range(steps):
        state_vector = np.dot(state_vector, transition_matrix)
        history.append(state_vector)
    return np.array(history)
```

Figure 5. 4. Step of coding in Markov Chain Modelling

The function **simulates_markov_chain**, uses transition matrix, initial state vector, and the set number of steps to simulate.

A state vector, generated in a **history**, is stored and saved from the moment of the beginning till the end.

Inside the loop, the state vector is updated by uniting the transition matrix and the state vector via **np.dot()**, which perform a matrix multiplication. This multiplication replaces the olds state vector by calculating transition probabilities.

The above state vector in the updated form is added to the **history** list after each step.

The function will output **history**, which is a NumPy array containing all state vectors calculated during all steps of the simulation.

```python
simulation_result = simulate_markov_chain(transition_matrix, state_vector, 10)
```

Figure 6. 5. Step of coding in Markov Chain Modelling

This line runs the simulation for 10 steps and stores the resulting series of state vectors in **simulation_result**.

Modeling a Markov chain with 10 steps beginning at a point where the system is in a good state (100% probability), brings into play how the likelihood of being in each state (Good, Needs Maintenance, Failed) is followed over time. Here are the results:

Initial state:	100% Good		
1 step	90% Good	10% Needs Maintenance	
2 steps:	83% Good	16% Needs Maintenance	1% Failed
3 steps:	77.9% Good	19.8% Needs Maintenance	2.3% Failed
4 steps:	74.07% Good	22.34% Needs Maintenance	3.59% Failed
5 steps:	71.13% Good	24.12% Needs Maintenance	4.75% Failed
6 steps:	68.84% Good	25.42% Needs Maintenance	5.74% Failed
7 steps:	67.04% Good	26.40% Needs Maintenance	6.56% Failed
8 steps:	65.62% Good	27.15% Needs Maintenance	7.23% Failed
9 steps:	64.49% Good	27.74% Needs Maintenance	7.78% Failed
10 steps:	63.59% Good	28.20% Needs Maintenance	8.22% Failed

Table1: Simulation results with 10 steps

This simulation exhibits the shift from a good state into a state that may require maintenance or failure, therefore, emphasizing the importance that proactive maintenance is in monitoring the system's health.

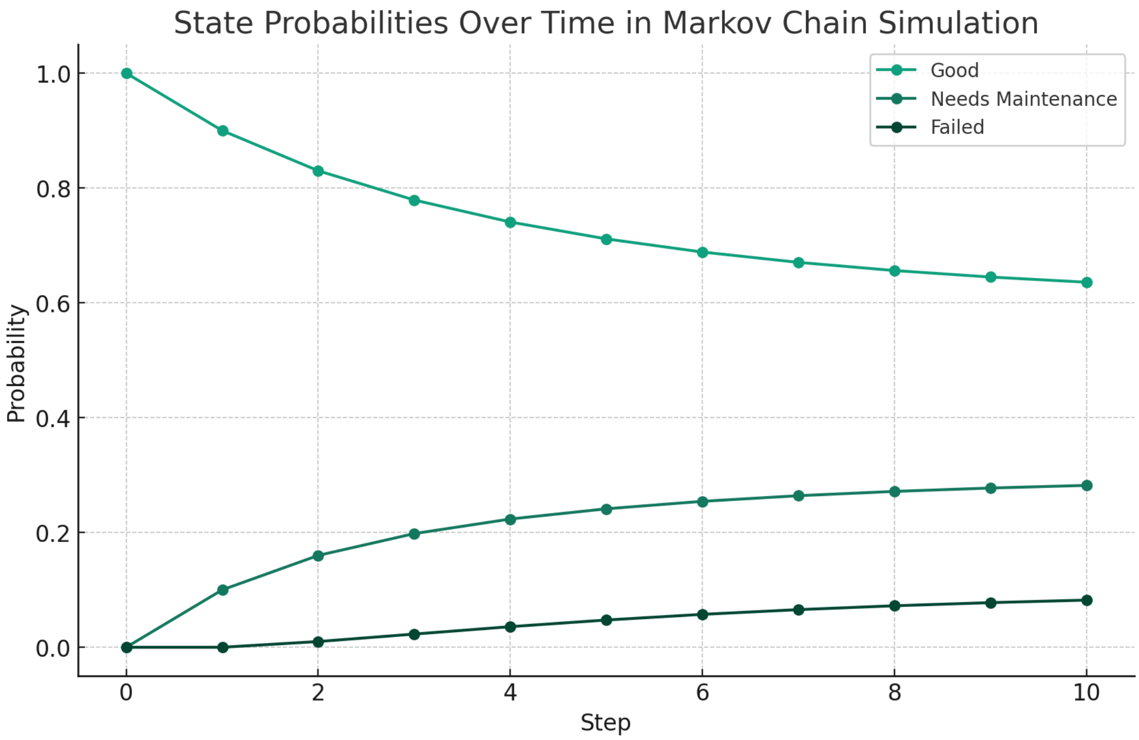

Figure 7: State Probabilities over time in Markov Simulation

Above is the chart that displays the evolution of the probabilities to be in each state (Good, Needs Maintenance, Failed) over 10 steps of the Markov chain simulation. This can be seen from a reduction of the probability that the system is in a “Good” state and a grow-up of probabilities that it is in a “Needs Maintenance” and “Failed” states. This chart improves understanding of the system's temporal dynamics.

8.2.8. Predictive Maintenance and Decision-Making

A Markov Chain model can be used in maintenance optimization to help with more than just making maintenance plans better. Guo and Liang (2022) say it can also help with maintaining things and making intelligent choices. The model looks at the chances of going from one state to another to guess how the system will act in the future. Also, it can help you figure out how something might go wrong or where you need to take action. So, preventative maintenance is possible. In this type of maintenance, jobs are done before a failure happens based on how likely it is to happen. This Markov Chain model gives people an opportunity to make informed decisions about when to organize the resources or plan for the maintenance exercises or operations funding. The model can also help to compare the various repair schemes through the modeling of the different situations and view their impact on the performance and dependability of the system. This lets people who have to make decisions use data to help them and make repair plans that are the best in terms of cost-effectiveness, system uptime, and lowering risk. Maintenance optimization with Markov Chain modeling makes management more reliable and preventative. This helps businesses run better and have less downtime.

To sum up, using a Markov Chain model in maintenance optimization is a solid way to make maintenance work more reliable and efficient. Because Markov Chains are not clear, maintenance managers can better understand how systems change over time and plan the best ways to cut down on downtime and maintenance costs while also increasing uptime and efficiency. In this process, you describe states, figure out the chances of changes, make transition models, figure out the chances of steady-states, and then use what you have learned to find the best upkeep steps. Experiments and tests, such as scenario testing and computer simulation, are some of the methods that can also fully assess the capability of the Markov Chain model in enhancing maintenance plans. This model helps in the optimization of forecasts, not just in maintenance for an already existing one, together with intelligent decision-making by guessing how the system will be able to behave and finding possible failure modes, hence easing the way to an economically valid plan of repair. Finally, Markov Chain model applications in optimization underline more proactive maintenance management that continues with the improvement of operating efficiency and downtime reduction, hence running the business more safely and efficiently with time.

9 Conclusion

This report is an in-depth analysis of the highly esteemed and influential Markov Chain Cost/Life-cycle Model and its application in various areas. Having an advanced model structure and a clear and systematic approach, this model is capable of giving a deep understanding of the complicated dynamics and various life stages of a system at all times. Next, it effectively captures all the intricate stages of changes occurring between them and the gradual costs of each stage. However, it is important to mention that cost performance evaluation of a certain project or software development process is really complicated and usually uncertain due to which the Markov model introduced below possesses the potential of giving much more accurate results and wider applications than the traditional model.

The main power of Markov model is its unique ability to introduce uncertainty which is peculiar to complex systems, allowing a wide range of possible outcomes and systemic likelihood, while accommodating expert judgment, as well as various stakeholders perspectives. This great flexibility is exploited as a mighty arm in accurate forecasting and evaluation of project costs due to the possibility to rapidly accommodate and consider some peculiarities that every separate project has.

Thorough knowledge and deep understanding of all stages of the life-cycle of a project are necessary to successfully apply a Markov model. Through suitable constriction of a very informative Markov model that incorporates all the knowledge and historical data that exists, invaluable information regarding the steps and transformations of a particular system can be got. It should be noted that Markov model has been used with great practical success in numerous areas of engineering which ensures remaining significance of Markov model in project management, product development, risk analysis and others, among others.

Although, it should be noted that the current application of the Markov model in project cost estimate and life cycle cost analysis is restricted only to homogeneous flow of life systems and those systems conforming to the basic assumptions of Markov processes. The overwhelming flexibility and the capability of the model to be modified suggest interesting prospects to apply the model in studying more advanced systems. Therefore, more research on the model's practical applications is necessary, particularly when considering new technologies like building information modeling (BIM) and geospatial information systems (GIS). The Markov model and these emerging technologies have a deep synergistic interrelation with them due to which they are capable of providing new strategies and precious insights for engineers.

Personally, a burning curiosity and deep aspiration remain to utilize the potential of the Markov model in advanced research and powerful development in the domain of smart cities. Along with the detailed simulation of different stages in the life-cycle of an infrastructure system and their respective costs, the Markov model will significantly change the decision-making process, providing the essential help and direction. Ultimately, thereby, allowing the intelligent development of modern, technologically advanced infrastructure systems. The implications of this research go way beyond any particular discipline and are reflected in a wide range of engineering areas, such as project management, product development, and risk analysis among others.

The next part of this deep research will compare enlightening literature by the classic statistics with the unique Markov model. In addition, the prevailing lacunae within the current literature will be thoroughly analyzed and critiqued, while offering a pragmatic potential solution that exploits the tremendous potential of the Markov model for the vital areas of project cost estimation and life-cycle cost analysis. The focal points of this artistically crafted research project reveal the unalloyed strength and broad utilizations of the Markov model.

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