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The Predictive Power of Shadow Rate Estimates

Forecasting Performance and Relevance for Discretionary Policymaking

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TIIVISTELMÄ:

Ohjauskorkojen keskeinen rooli rahapolitiikan työkaluna on heikentynyt viimeisten kolmen vuosikymmenen aikana. Useat keskuspankit ovat joutuneet laskemaan ohjauskorkonsa alarajalleen vuoden 2008 finanssikriisin myötä ja esimerkiksi Japanissa on turvauduttu nollakorkoympäristöön sekä muuhun epätavanomaiseen rahapolitiikkaan jo 1990-luvulta alkaen. Ohjauskorko on ollut perinteisesti myös keskuspankin rahapoliittisen asennoitumisen mittari, mutta vallinneissa nollakorkoympäristöissä sen käytettävyys on rajoittunut. Olemassa oleva kirjallisuus on ehdottanut kyseisen ongelman ratkaisuksi käsitettä varjokorko. Alun perin jo 1990-luvulla esitelty, mutta vasta 2010-luvulla analyyttisiin teoriakehyksiin puettu varjokorko on approksimaatio keskuspankin ohjauskorosta ilman alarajaa – tarkoituksenaan kuvastaa nollakorkoympäristön sekä muun epätavanomaisen rahapolitiikan yhteisvaikutusta ja täten keskuspankin rahapoliittista asennoitumista.

Tämä tutkielma sisältää kirjallisuuskatsauksen rahapolitiikan yleispiirteistä ja varjokoron teoreettisesta taustasta sekä empiirisen tutkimuksen varjokoron ennustekyvystä reaalisen bkt:n muutosta kohtaan kahdessa valtiossa, USA:ssa sekä Japanissa. Tutkimuksen tarkoituksena on selvittää voiko varjokorkoa hyödyntää nollakorkoympäristössä ennustamisessa ja täten rahapoliittisen päätöksenteon työkaluna. Tutkimus vertaa varjokoron ennustekykyä lyhyisiin pankkienvälisiin markkinakorkoihin yhdessä kontrollimuuttujien kanssa, luoden aitoja otoksen ulkopuolisia ennusteita reaalisen bkt:n muutoksesta sekä vektoriautoregressiivisiä että suoria ennusteita hyödyntäen. Tutkimus vertailee ennusteiden suorituskykyä niin virhetermien kuin visuaalisen analyysin avulla. Lisäksi kaikkia malleja verrataan satunnaiskulkuun sekä yksinkertaisen autoregressiivisen ennustemallin suorituskykyyn.

Teoreettisesta potentiaalistaan huolimatta, suoritetun tutkimuksen perusteella varjokorko ei tarjoa merkittävää parannusta reaalisen bkt:n ennustamisessa yli lyhyiden markkinakorkojen. Luodut ennusteet tuottavat autoregressiivistä vertailukohtaa sekä satunnaiskulkua huomattavasti parempia tuloksia, osoittaen kontrollimuuttujien onnistuneen valinnan ja tuotetuista ennusteista VAR-mallien dynaamiset ennusteet osoittautuvat kauttaaltaan parhaiksi. Vaikka osa malleista hyötyy varjokorosta, on parannus parhaillaan marginaalinen. Huolimatta heikosta ennustekyvystä reaalista toimintaa kohtaan, tarkasteltu kirjallisuus osoittaa varjokoron olevan erinomainen työkalu historialliseen tarkasteluun ja olemassa olevat tutkimukset ovat löytäneet varjokoron ennustavan erinomaisesti niin muita korkoja, kuin inflaatiotakin. Varjokoron voidaan siis katsoa olevan keskeinen työkalu niin historialliseen tarkasteluun kuin myös päätöksenteon työkaluna nollakorkoympäristön vallitessa.

AVAINSANAT: Central banks, Economic forecasts, Financial crises, Key interest rate, Monetary policy

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Abbreviations

AIC	Akaike information criterion
BIC	Schwarz Bayesian information criterion
CMP	Conventional monetary policy
CPI	Consumer price index
DFR	Direct multistep forecast with a recursive data pool
ECB	European Central Bank
Fed	The Federal Reserve System
FFR	Federal Funds Rate
HQIC	Hannan-Quinn information criterion
OLS	Ordinary least squares
OMO	Open market operations
PIR	Policy interest rate
UMP	Unconventional monetary policy
VAR	Vector autoregression
ZLB	The zero lower bound

1 Introduction

One of the core missions of a central bank is to maintain macroeconomic stability through the implementation of monetary policy. A central bank may apply expansionary or contractionary measures to influence the aggregate output of an economy, aiming to minimise fluctuations of business cycles, also referred to as the output gap. Conventionally, amongst the most prominent tools of transmitting monetary policy has been a central bank's policy interest rate, which affects other interest rates across the economy, thus, influencing borrowing, investment and consumption. Through raising its policy rate, a central bank aims to prevent an upwards trending economy from overheating and, vice versa, lowering the rate has the intent of providing monetary easing, effectively stimulating the economy to limit the length and depth of a downturn. As a key tool of monetary policy, the policy interest rate also serves as an indicator of a central bank's monetary policy stance.

The centric role of the policy rate for transmitting monetary policy has wavered, however, as the past three decades have borne witness to needs for unprecedented scales of monetary easing with many central banks having to lower their policy rates to their lower limits, the zero lower bound. In the turn of the 1990s, Japan faced the bursting of a vast asset price bubble and in the dire need of large-scale monetary easing, the Bank of Japan lowered their policy rate to the zero lower bound by the midway of the decade. Due to the severity of the crisis, the zero-interest-rate-policy was not enough on its own to aid the economy to rebound and additional monetary easing was applied. Other economies had to resort to similar means soon after, as the U.S. Subprime crisis quickly developed and spilled over into a global financial crisis, known as the Great Recession of 2008. The Great Recession saw the U.S. central bank, among many others, adopt zero-interest-rate-policies and additional monetary easing operations, commonly referred to as unconventional monetary policy.

With the policy rate, a key tool of monetary policy implementation, tied to its lower limit, a notable issue emerges. As an indicator, a policy rate limited to the zero lower bound no longer carries meaningful information of the monetary policy stance of a central bank. Large-scale operations of unconventional monetary policy are left unmeasured as the policy rate is bound by zero. Existing studies do propose a solution, however, the shadow rate. First conceptualised by Fisher Black in 1995 with extensive theoretical frameworks introduced later during the 2010s, the shadow rate is an approximation of a central bank's policy interest rate without a binding lower limit. During periods of positive policy rates, the shadow rate proxies the policy rate but extends to carry additional information when policy rates are bound by zero. Essentially, the shadow rate aims to capture the combined effects of zero interest rates and additional unconventional monetary policy, effectively measuring the monetary policy stance of a central bank when the policy rate is stuck at the zero lower bound.

The theoretical frameworks for the shadow rate primarily focus on the shadow rate estimations' ability to convey additional information on monetary policy within retrospective studies, examining historical data and the shadow rate's behaviour during periods of zero interest rates. Building the conceptual foundation of the study, this paper conducts a literature review on the theoretical background of the shadow rate and existing shadow rate frameworks – revised from the author's bachelor's thesis (Hakulinen, 2023). The review focuses not only on the properties of shadow rate estimates as representations of monetary policy stances using historical data but additionally raises a forward-looking proposition. As the policy rate of a central bank is utilised not only for the transmission of monetary policy but also within forecasting to support discretionary policy-making, the review examines the shadow rate as a complementary tool for monetary decision-making when the policy rate is at the zero lower bound.

It is the purpose of this paper to extend upon the author's bachelor's thesis by conducting an empirical study on the predictive power of the shadow rate over the real output of an economy. Said study is performed by creating ex-post out-of-sample forecasts using vector autoregressive and direct h-step forecasts to predict the growth rate of real GDP within two economies, the USA and Japan. The forecasts produced by shadow short rate estimates are compared to forecasts given by models substituting the shadow rate with short-term interbank rates closely following policy rates, in conjunction with a set of control variables utilised across the forecasting models. The intent of the study is to examine whether shadow short rate estimates could be utilised as a reliable tool for discretionary policymaking during periods of zero interest rates. The two economies selected were due to, first, Japan's extensive history with the zero lower bound, providing an excellent environment for the shadow rate to capture additional information over the policy rate, and second, the centric role of the U.S. for the widespread use of unconventional monetary policy as the epicentre of the Great Recession of 2008.

The study begins with a literature review, introducing the reader to the fundamentals of monetary policy while creating a distinction between conventional and unconventional monetary policy and by briefly exploring macroeconomic conditions leading to the zero lower bound for the two economies. The review then proceeds to show an analytical representation of one of the most prominent shadow rate frameworks, the shadow rate term structure model, and results of existing shadow rate approximations. Finally, the paper presents the conducted empirical research and a discussion on the implications of the results regarding the shadow rate as a forward-looking tool of monetary policy.

2 Monetary policy and the zero lower bound

Within this section of the paper lies a brief review of the fundamentals of monetary policies. For the sake of simplicity, the theories, and tools will be mainly explained from the perspective of the Federal Reserve System, the central bank of the United States, referred to as the Fed from this point onwards. Even though some tools share very similar characteristics, only parted by the scale of operations, this paper makes a distinction between conventional and unconventional monetary policy, CMP and UMP. Since the focus is on the perspective of the Fed, the Great Recession of 2008 will be considered as the starting point for UMP, holding true for many other central banks as well. The Great Recession as a start to UMP will be more closely discussed, as well as the definition of UMP during this section of the paper. First, some elementary theory behind CMP is discussed, followed by a brief introduction of CMP tools. Subsequently, UMP is presented in a similar fashion.

Even though each central bank has their own official mandates, one of the most universal objectives of any central bank is to maintain a stable economy. Central banks oversee that the banking system it upholds remains robust against any cyclical changes or shocks and sees to it by implementing monetary policy. One may also turn to Mankiw's collection (1994) where it is stated that price stability and small and consistent output gaps have been universal rules within central banking for quite some time. Regarding the official mandates, to take two examples, let us examine the Fed and the European Central Bank, the ECB. The primary objectives of the Fed are maximum employment and price stability, also known as the dual mandate (Federal Reserve Board, 2024). Quite similarly, the ECB aims to maintain price stability by targeting inflation close to 2 %, while also targeting secondary objectives set by the EU, such as balanced economic growth and an aim at full employment (Hartmann & Smets, 2018, pp. 3–5). These mandates exist to both communicate some reasoning behind monetary policy stances, as well as to enable other entities of the economy to follow the competence of their central bank.

2.1 Conventional monetary policy

Conventionally, before the Great Recession of 2008, the predominant tool for implementing monetary policy has been a central bank's policy interest rate (Krippner, 2020b, p. 3). This paper defines the PIR of a central bank to be the rate, which affects all other interest rates within the economy. This way, it is intuitive to consider that by lowering their PIR, a central bank is able to raise the aggregate demand of the economy, therefore incentivizing economic activity and minimizing the negative output gap of a downturn. Vice versa, by raising their PIR, a central bank may dilute the effects of a sharp economic boom by lowering aggregate demand, and once again keeping the positive output gap to a minimum. Conventionally, the PIR has also been a straightforward measurement of the monetary policy stance of a central bank. Looking at historical data, one is able to discern when a central bank has changed stances or kept taking a stronger foothold. The present day is no different with the stance of a central bank becoming apparent from frequent communication regarding their PIR. As an example, the Fed's decision-making body, the FOMC, decides on a new PIR target at least eight times a year (Federal Reserve Board, 2025a).

One of the most prevalent methods for determining a suitable level for the PIR is the Taylor rule (Taylor, 1993). Addressed in more detail within section 3.5, one may look to equation 13 and note that the Taylor rule is condensed into a simple form, intuitively consisting of existing inflation and the GDP, while giving a target PIR as its output. As Koenig , Leeson, & Kahn (2012, pp. 68–71) write, the rule is simple, intuitive, and most of all, a realistic depiction of how modern-day monetary policy has been implemented. The writers continue, that it took only two years for the FOMC to first quote the rule and that the creator of the rule had earlier found the actions of the Fed to already follow the rule. The rule quickly changed monetary policymaking, previously considered a topic of discretion, into an easy-to-comprehend mechanical form. Since its introduction, the Taylor rule has become a staple of monetary policy rules as even the Federal Reserve Board

(2018) mentions the Taylor rule as the best-known policy rule and lists it as the first equation on a list of monetary policy principles. Koenig, Leeson, & Kahn (2012, p. 70) also mention, that in addition to the Federal Reserve System, the Taylor rule quickly spread to become the corner stone of central banking around the world.

Let us now examine the primary tools of CMP by looking at the Fed and their PIR, the federal funds rate – an interbank lend rate for U.S. banks. Under conventional conditions, CMP tools primarily consist of open market operations, deposit rates and reserve requirements. These tools are used by central banks to affect both the demand as well as the supply of reserves available to private banks. At least in the Fed's case, an especially prominent series of tools are the open market operations, OMO, as stated by Ennis & Keister (2008, p. 236). OMO consist of a central bank, in this case, the Fed, buying and selling assets, such as government bonds, to influence the supply of private bank reserves. By selling bonds to private banks the Fed decreases the number of reserves that private banks hold, and by buying bonds, reserve supplies of private banks are increased. Then, to affect the demand of reserves, central banks typically pay interest on reserves, IOR. Ennis & Keister (2008, pp. 256–258) present that IOR enables a central bank to set a floor to its PIR and lower its volatility. In the Fed's case, IOR is named as interest on reserve balances, IORB, and has only been utilized since 2008 (Federal Reserve Board, 2025b). Intuitively, a rise in IOR shifts the demand curve to the right and a fall in IOR to the left. These shifts in supply and demand are used by the Fed to target the federal funds rate, their PIR. Reserve requirements are used in addition to these two to maintain a safe buffer of reserves within all private banks in addition to targeting the federal funds rate equilibrium. The base model of PIR targeting is illustrated in the figure (1) below.

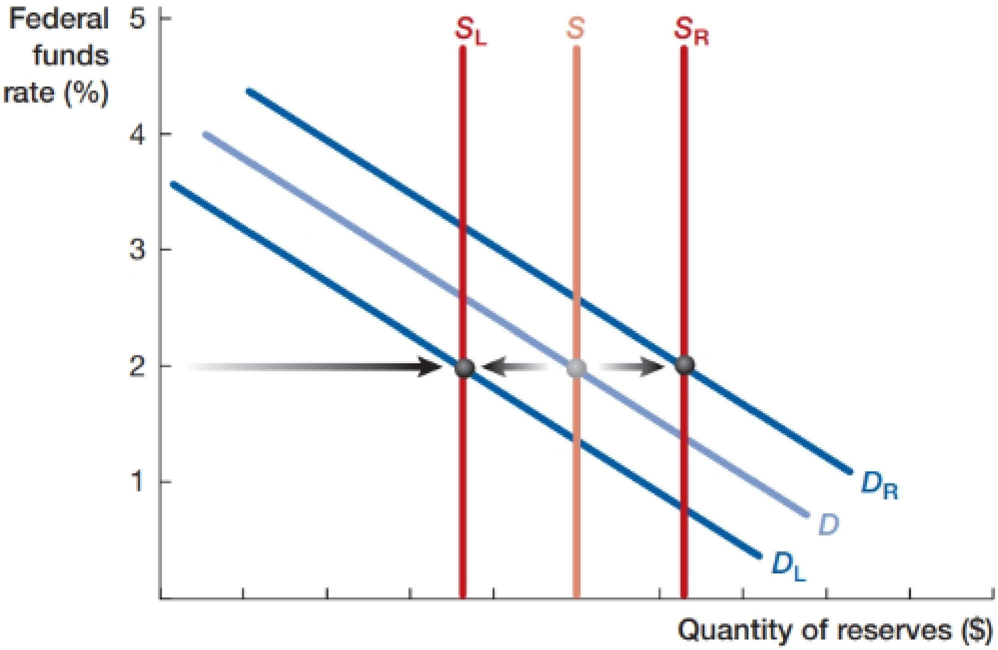


Figure 1. The federal funds market equilibrium (Acemoglu et al., 2022, p. 679).

Since central banks cannot access the domestic money supply belonging to companies and individuals within an economy, or adjust domestic inflation directly, they use CMP tools to control said variables indirectly. Even though controlling the money supply over the short term is not a part of the primary objectives of the Fed, per se, Acemoglu et. al. (2022, pp. 666–667) clarify that the quantity theory of money assumes that the GDP and the money supply grow in tandem over the long term. This implies that it is indeed in a central bank's best interests to attempt to control the money supply through monetary policy over the long term. The writers further imply that the inflation rate is the difference between the growth rate of money supply and the growth rate of real GDP in the long run. This indicates that if a central bank does not attempt to control the growth rate of money supply over the long term, it may not have adequate control of inflation over the long term either.

2.2 The need for unconventional monetary policy emerges

Before, and during, the early 2000's, the Fed was able to primarily control the economy through targeting a PIR via CMP tools, as briefly portrayed above. During this time, the Fed consistently sustained a healthy buffer between their interest rates and the zero lower bound, ZLB. To clarify, the ZLB is a term describing the lowest nominal value a central bank may set its PIR to. As discussed later during this paper, the lowest value changes by research, but as a good rule-of-thumb, it is always very close to, or zero. One may examine the Fed's interest rate history as far back as the 1910s and notice no mention of the ZLB. A St. Louis Fed publication by Bowsher (1965) describes the interest rates between 1914–1965 and, even though the economy faced the Great Depression in the 1930s, the Fed did not have to resort to unorthodox monetary policy in order for the economy to recuperate. The figure (2) below describes the federal funds rate between 1960–2021 and one may note that before the year 2008, the PIR of the Fed did not reach the ZLB.

The time of UMP quickly surfaced, as the Great Recession of 2008, brought with it a global financial crisis. In retrospect, a massive U.S. crisis spreading globally is not surprising considering the size of the U.S. economy; as one may read from MacDonald & Popiel (2020, p. 1062), there exists evidence of asymmetric monetary policy spillovers, often one-directional from the U.S. to abroad. Even though the paper mostly discusses small open economies, the magnitude of the crisis stemming from the U.S. was so large, that it influenced even the world's largest economies. Mishkin (2010, pp. 4–9) agrees, writing that the collapse of the Lehman Brothers investment bank during September 2008 led to a series of events quickly spreading the crisis into a global phenomenon, with the ECB and the Bank of England having to also take instant action. The immediate effects of the crisis can be easily observed from the figures (2, 3) below which show both the Fed and the ECB dropping their PIRs very near the ZLB immediately at the end of 2008.

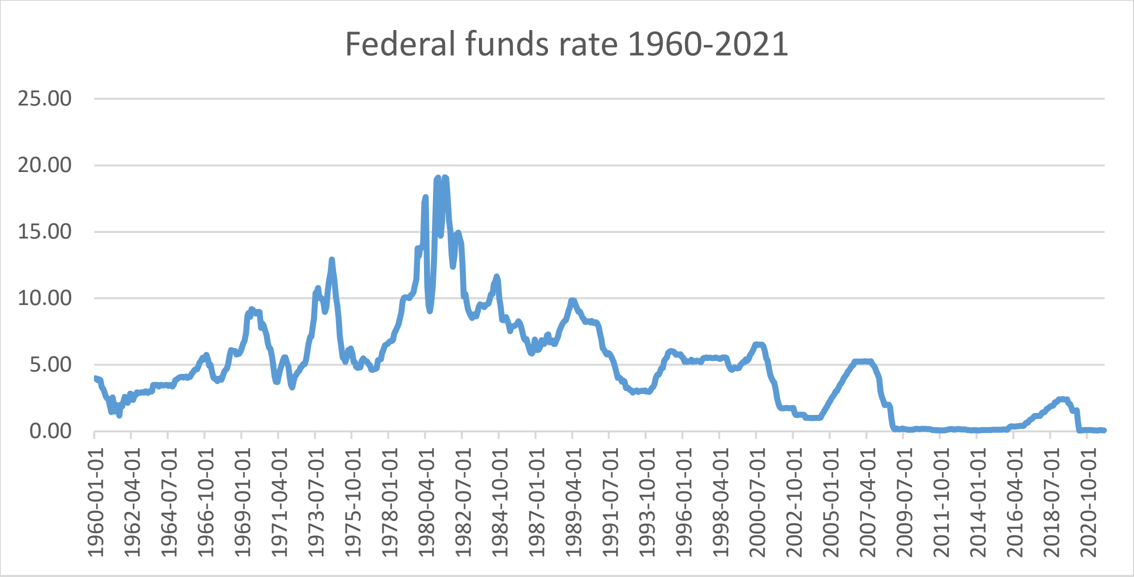
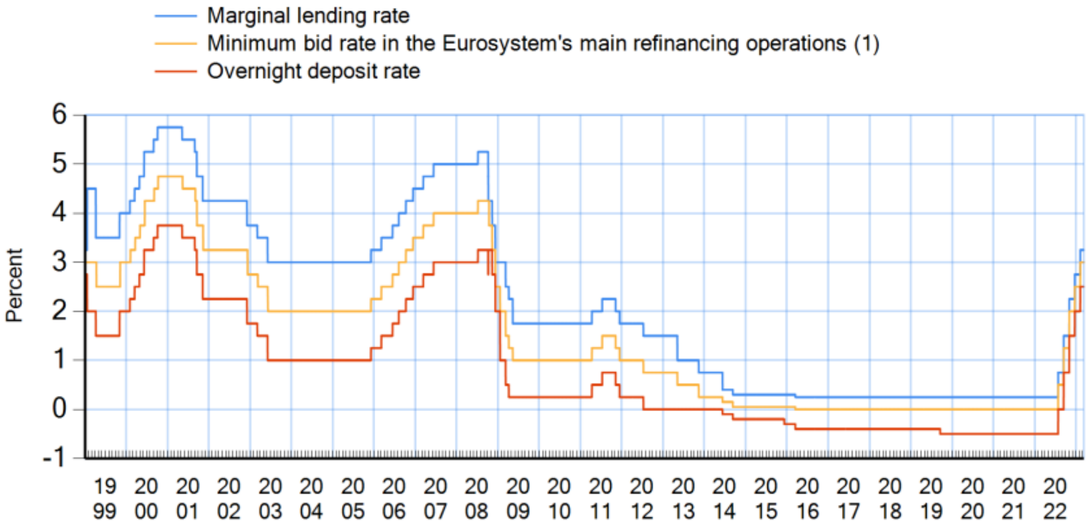


Figure 2. The Federal funds rate 2000-2014 (The Federal Reserve Bank of St. Louis, 2025c).



Source: European Central Bank
(1) ECB policy rate
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Figure 3. ECB Key interest rates 1999-2023 (Bank of Finland, 2023).

It is clear, that the effects of the Subprime crisis quickly spilled from the U.S. to the Euro area: the FFR dropped to 0,16 % in December 2008 and sticked to the ZLB for the remainder of the examination period with the ECB following soon by dropping the MROR to 1,00 % in May 2009 and the DFR to 0,25 in tandem, a new historical low according to Hartmann and Smets (2018, p. 26).

2.3 Unconventional monetary policy during and after the Great Recession

To begin with UMP, it is intuitive to consider that a central bank cannot set its PIR much lower than zero. Fischer Black (1995, pp. 1371–1372) illustrates this well by describing currency as an option to interest rates. The writer elaborates that if negative rates occurred, it would be more beneficial for consumers – or in this case, private banks – to hold money in cash rather than in instruments accruing a negative rate. This makes zero, the ZLB, a binding lower limit on interest rates. As stated by Bauer and Rudebusch (2016, p. 1440) the ZLB is not exactly zero due to factors, such as storing costs of currency, but, due to simplicity, it is referred to as the ZLB, signifying a lower limit to interest rates. Kortela (2016, pp. 2–5) states in his article that the numerical definition of the ZLB varies from study to study. He sums that typically the ZLB is set as a constant between the values -0,25–0,25 or that it can even be modelled as a time-varying limit, as the writer does in their own article.

In most instances, having originally been implemented as countermeasures to the severity of the Great Recession, UMP tools mostly consist of similar operations as CMP tools, but either at a larger expansionary scale, lower interest rate levels, or the operations span to address longer-term liquidity than CMP operations. This paper will divide UMP tools into three different categories: interbank liquidity tools, asset purchases, and forward guidance, which is a form of future-signalling of a central bank. One may even consider PIRs at the ZLB to be a form of UMP, sometimes referred to as a zero-interest-rate-policy, ZIRP, instead of simply being a threshold for unconventional policies. Some operations considered to be a part of UMP, do exists, such as bank bailouts, though they have been intentionally left out of discussion, since they do not directly relate to the topic of this paper.

To make yet another distinction, UMP has been used to serve two main purposes. Barnett & Jawadi (2015, p. 445) describe these purposes as firstly, to restore interbank money markets, and secondly, to implement further easing on the economy in a ZLB-environment. This means that UMP consist of tools, which further boost the liquidity of private banks, incentivize loan-taking among the banking system and affect the supply of money within the interbank system. It is also noteworthy to consider that a partial purpose of UMP is to not only have a real easing effect within the interbank market and the economy, but to also affect the sentiment of the public in order to raise the levels of cash flows within the economy.

Beginning with interbank liquidity, these tools consist of easing policies regarding interbank lending and central bank reserves. To promote interbank loans during a credit crunch, a central bank may conduct unprecedented cuts to its interbank lending rates. In addition, the central bank may also loosen its requirements on assets approved as collateral to said loans. Accepting a wider range of assets as collateral and lowering the rates at which banks can lend from the central bank, and each other, ought to cause a positive shift in the demand for money. This might not always be the case however, as presented by Mishkin (2010, pp. 11–13) who states, that abnormal amounts of discount lending from a central bank may be seen as a “bad omen” in terms of the state of a bank. As becomes apparent from the article, this was the case in the U.S., where the Fed had to create a temporary lending facility, the Term Auction Facility, in order for the banks to take out loans anonymously. Additionally, banks may strengthen the base for liquidity by offering higher-than-usual interest rates for reserves held within the central bank. Barnett & Jawadi (2015, p. 439) point out that the Fed has used such a tool, by paying an additional interest on excess reserves, IOER.

Continuing to asset purchases, these tools include notably large-scale asset purchases conducted by a central bank, often focusing on longer-term securities. As a central bank purchases large amounts of securities, such as treasury bonds, it creates two distinct effects. First, the purchases increase the money supply within the interbank system and secure long-term liquidity to banks. Second, since most purchases concern bonds, it evens out the yield spread of the assets purchased by raising the prices of long-term bonds in the market. The idea is, that these long-term bonds serve as better debt collateral, since they do not seem as volatile as before the asset purchase programs. Mishkin (2010, pp. 14–15) points out, however, that the real effect of purely increasing money supply and a central bank's balance sheet has been questioned as sole stimulators of the economy. Mishkin continues, that in the Fed's case, at least, the greatest influence was not derived from a $300 billion purchase of treasury bonds, but indeed from the Fed's $1,25 trillion purchases of mortgage-backed securities, MBSs. These MBS purchases aided the slowdown of the U.S. economy more drastically by impacting residential mortgage rates through lowering interest rates on the MBSs – in other words, by having a more direct effect on the real economy.

The third tool of UMP is forward guidance. Forward guidance refers to the information that a central bank releases to the public relating future monetary policy plans. Forward guidance may include plans of any length. The aim of forward guidance is to manage market expectations and further strengthen the transmission of other monetary policy. A central bank using forward guidance may, for example, state that its key rates will remain at the ZLB for as long as it is necessary, or at the very least until a said point in time. As an example, on February 7th, 2013, Mario Draghi implied that there will be no visible future reason for the ECB to raise its key interest rates (European Central Bank, 2013). Since a properly operated central bank has the trust of its markets, the statements alone are meant to influence the economy. For example, a central bank solidifying their PIRs to the ZLB for the next year through forward guidance will increase the demand for money, as low interest rates have been established through forward guidance.

2.4 Unconventional monetary policy in Japan

Although most instances of ZLB environments and practices of unconventional monetary policy have taken place post-2008, a notable exception lies within Japan. Making a comparison of the previous four decades between the U.S., Euro Area, and Japan, for example reveals differences in both, the macroeconomic performance, but also fundamental variation in monetary policymaking as, for example, before the current millennium the Bank of Japan did not pursue price stability, but rather aimed to maximise economic performance (Takatoshi & Mishkin, 2006, p. 141). The exceptional UMP history of the Japanese economy was set off by the bursting of a large asset price bubble by the end of the 1980s, brought on by the liberalisation of the Japanese financial markets (Tomfort, 2017, p. 133) as the much-hailed Japanese economy, with a long rising trend after WWII, experienced a steep turn for the worse. Through a combination of monetary policies, lending practices, external shocks, and international spillovers, the Bank of Japan, BoJ, was forced to experiment with large-scale easing monetary policies over a decade before the events of the Great Recession took place. Inspecting figures 4 and 5 below, one may note that not only has the Japanese economy seen UMP take place much sooner than most, but an extensive battle with deflation and a nearly uninterrupted period of low or zero interest rates spanning close to three decades.

Following WWII, the Japanese economy experienced an extreme upward trend, with sustained economic growth from the end of the war in 1945 all the way until the global oil crisis in 1973. Accredited to a combination of intensive investments into various industrial technologies through imports, the accumulation of household capital, increased quantity and quality of labour, and international trade (Shiohara, 2023) the period has been aptly named the Japanese Economic Miracle as the economic growth was not only sustained for a long period of time, but hovered near 10 % for much of the two-decade-period. Furthermore, as one inspects Japanese GDP growth figures (World Bank Data, 2025), it is to note that growth turned negative only once between 1945–1993, in 1974.

This period of extensive growth rapidly halted in the early 1990s, however, and was followed by a collapse of the Japanese economy, with a scale large enough to carry evidence of it all the way until the present day. As explained by Tomfort (2017, pp. 133–135) the period of growth began to evolve into an asset price bubble during the 1980s, as the Yen-Dollar agreement of 1984 increased foreign access to the Japanese financial markets. Within a span of less than five years, housing prices doubled and share prices tripled. Household debt also grew triple in size from 1979 until the early 1990s and the bubble was ready to burst. BoJ began to implement restrictive monetary policy with gradual interest rate increases from May 1989 (Tomfort, 2017, p. 138) which proved too much for the unstable economy to handle.

The beginning of the 1990s saw the Japanese economy experience a severe crash as the Nikkei 225 stock index lost more than 60 percent of its highest value within two and a half years, land value decline by 80 percent and inflation first touch zero by 1995 (Takatoshi & Mishkin, 2006, pp. 137–138; Tomfort, 2017, p. 138). Attempts were made to stabilise the economy and real estate lending from banks slowed considerably due to regulation, but a loophole in legislation allowed non-bank financial firms to continue with loose lending policies and nonperforming loans began to register as an issue by the mid-1990s. The combination of high debt-levels, non-performing loans, and a threatening deflationary environment led to the BoJ decision to lower their PIR near the ZLB and as the economy was aiming to recuperate, the late-adapted easing monetary policy along with vast amounts of non-performing loans led the country to not only fail at turning the economy around, but towards even more instability, made especially prominent by two banks failing in November 1997, as mentioned by Takatoshi & Mishkin (2006, p. 142). Poor monetary policymaking continued in the aftermath of the burst ICT bubble, as some minor improvements in the outlook of the Japanese economy were discernible and the ZIRP was lifted soon, only for the minor improvements to turn out as the peak of the carefully rising Japanese business cycle. The decision to raise interest rates was made against the advice of many economists and the demands of the Japanese government, as clarified by Takatoshi & Mishkin (2006, p. 132), acting as a prime example of the faulty discretion used within the Japanese economy during the turn of the millennium.

Upon lifting the ZIRP in August 2000, the Japanese economy turned back towards a recession by the end of the year and the Bank of Japan reverted to easing policies and ZIRP during 2001 for as long as necessary (Takatoshi & Mishkin, 2006, pp. 145–146). The strict clause regarding the ZIRP signified the BoJ finally yielding to accepting a monetary policy target related to price stability as the BoJ also announced a target of reaching a positive inflation rate in March 2001 (Takatoshi & Mishkin, 2006, p. 150). The ZIRP was implemented along with additional quantitative easing through excess reserves and large-scale bonds purchases during the early 2000s. Turning over to figure 4 and Japanese GDP growth data (World Bank Data, 2025), it becomes evident that the easing on the economy proved too little, too late, as interest rates had to be kept at the ZLB until 2006 and as the economy remained quite essentially stagnant with growth hovering mostly over only 1 percent during the years 1993–2006.

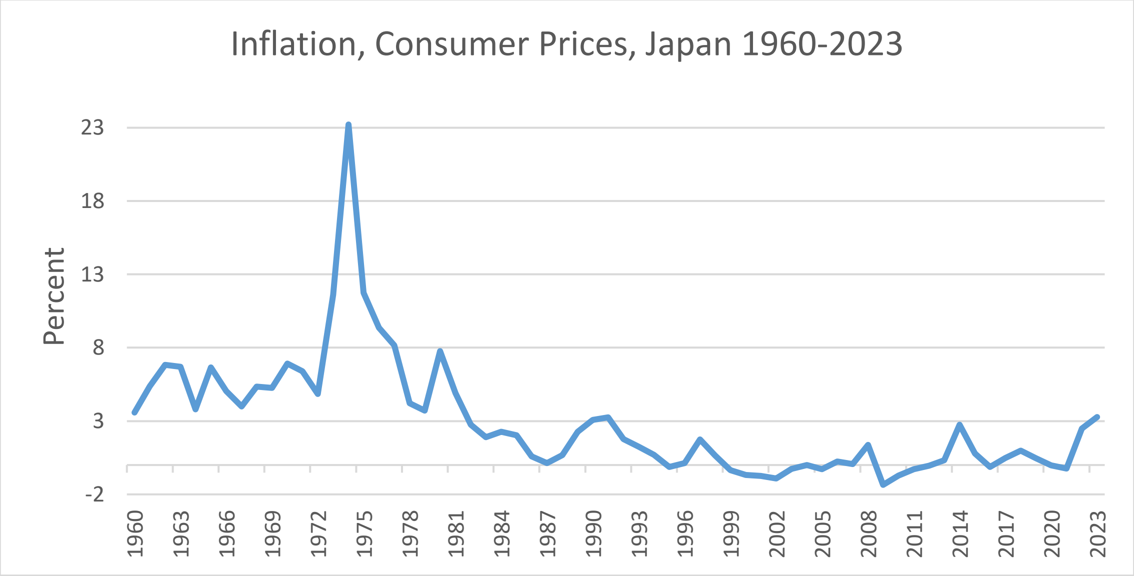

Figure 4. Inflation, consumer prices, Japan 1960-2023 (The Federal Reserve Bank of St. Louis, 2024a).

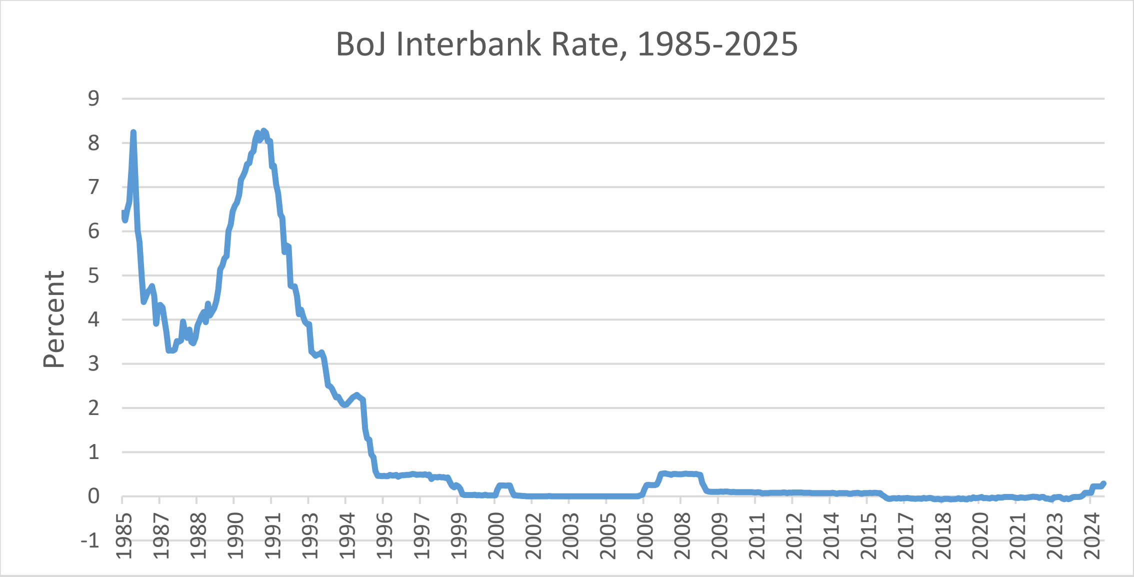

Figure 5. Bank of Japan's immediate interbank rate (The Federal Reserve Bank of St. Louis, 2025b).

By the year 2007, Japan had gone through over a decade of nearly non-existent growth, financial instability, and struggles with deflation, brought on by a combination of a combination of national and international macroeconomic shocks. Conditions did not improve going further and the Japanese economy was soon hit with more spillover shocks as the subprime crisis forced BoJ to re-enter a recently exited state of quantitative easing as noted by Ueda (2012, p. 6). The ZIRP was shortly abandoned in 2006 and quickly readapted before the beginning of 2009, coupled with the return of deflation. The UMPs implemented in Japan, post-2008 (Dell’Ariccia et al., 2018, pp. 163–164; Kawamoto et al., 2023, p. 210), became especially prominent with large-scale easing programs in conjunction with a new and straightforward inflation target of 2 % after Prime Minister Shinzo Abe took office in 2012. Two large easing programs consecutively in 2013 and 2014 showed a minor improvement within the economy, despite poor GDP growth, as stock prices grew, and a state of positive inflation was finally reached. The authors continue by stating that the success was short-lived, however, as a decline in emerging economies, notably China, saw the Yen appreciate and BoJ revert to not just ZIRP, but easing with a negative policy rate, visible in figure 5. The BoJ PIR (figure 5) along with the economy’s inflation (figure 4) and GDP growth data (World Bank Data, 2025) show that the

Japanese economy continued flatlining until the 2020s, with near-zero interest rates and a constant struggle with deflation.

From a national economic crisis to shocks caused by the ICT bubble, followed by the Great Recession, and struggles with international trade due to poor performance of surrounding emerging economies, Japan has recently faced nearly three decades of zero interest rates and other unconventional monetary policies, a notable exception from the U.S. and Euro economies. Especially atypical is the constant struggle with deflation, dragging the country's GDP growth down greatly from the previous decades of the Japanese Economic Miracle. Not only due to poor luck, the past three decades have shown more than one instance of poor monetary policy by the BoJ with decisions made against the advisement of economists and even the demands of the government. In hindsight, it is especially debatable that many instances of monetary policy were implemented much too late. As an example, Takatoshi & Mishkin (2006, pp. 139–140) note that some models have shown evidence that the BoJ would have indeed had the opportunity to noticeably shorten the duration of the initial asset bubble by implementing constricting policies sooner. While the past decades of monetary history within the Japanese economy are unfortunate, the exceptionally long period of unconventional monetary policy, especially the zero-interest-rate-environment make Japan an intriguing target for examining monetary policymaking and the effects of unconventional policies at the ZLB.

3 The shadow rate

This paper has now established that conventionally, without exceptionally poor economic conditions, monetary policy has been most prominently transmitted via a policy interest rate. In theory, and intuitively, a central bank raises its PIR when its purpose is to tighten monetary policy, and vice versa, as a response to the outlook of the economy. This section introduces some of the fundamental theoretical frameworks for the focal point of the study, the shadow rate. Following said theoretical presentation, the shadow rate is then utilised in an empirical study, presented within the following sections. The introduction of the shadow rate begins, by first, further illustrating the use of PIRs and yield curves as measurements of monetary policy stances and market expectations. Following the illustration, an analytical representation of one of the most prominent shadow rate models, the SRTSM is given.

3.1 Policy interest rate and yield curve as measurements of monetary policy stances

Examining the PIR as a measurement of monetary policy stances, one should turn their attention to the U.S. Federal funds rate illustrated in figure 2, as well as to the figures below (figure 4, figure 5), which demonstrate both, the U.S. inflation rate and output gap between 1960–2021. From figure 2, one may clearly observe times of contractionary and expansionary monetary policy; the figures below showcase, that the PIR seems to respond moderately well to at least one of the macroeconomic indicators below, at a time. Judd & Rudebusch (1998) try to model the Fed’s reaction function between the years 1970–1997 and find, that between those years, the Fed’s PIR changes seem to, indeed, reflect either of the two.

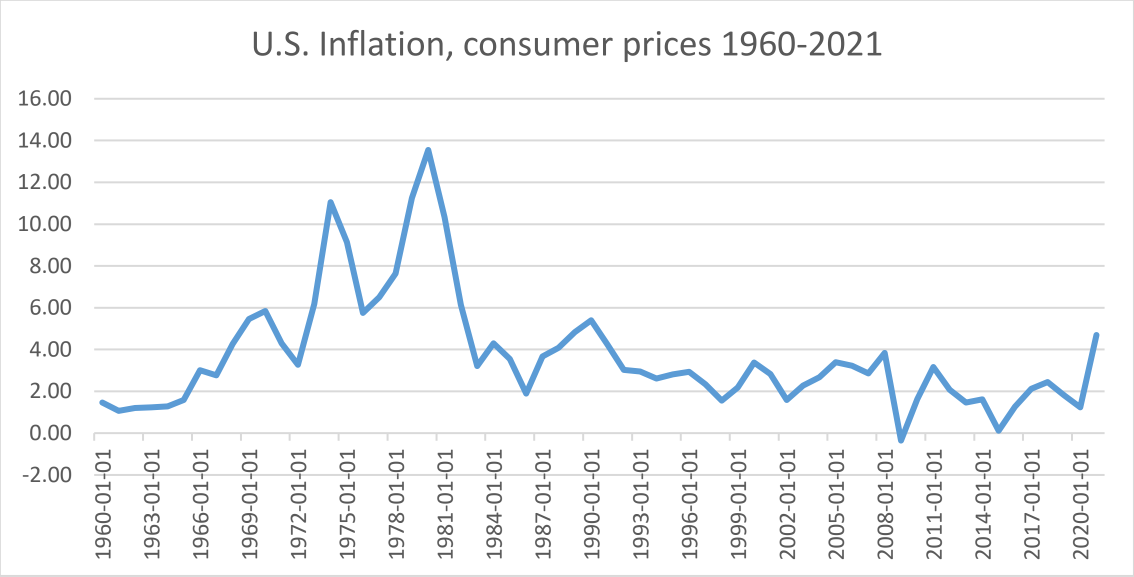

Figure 6. U.S. inflation, consumer prices 1960–2021 (The Federal Reserve Bank of St. Louis, 2024b).

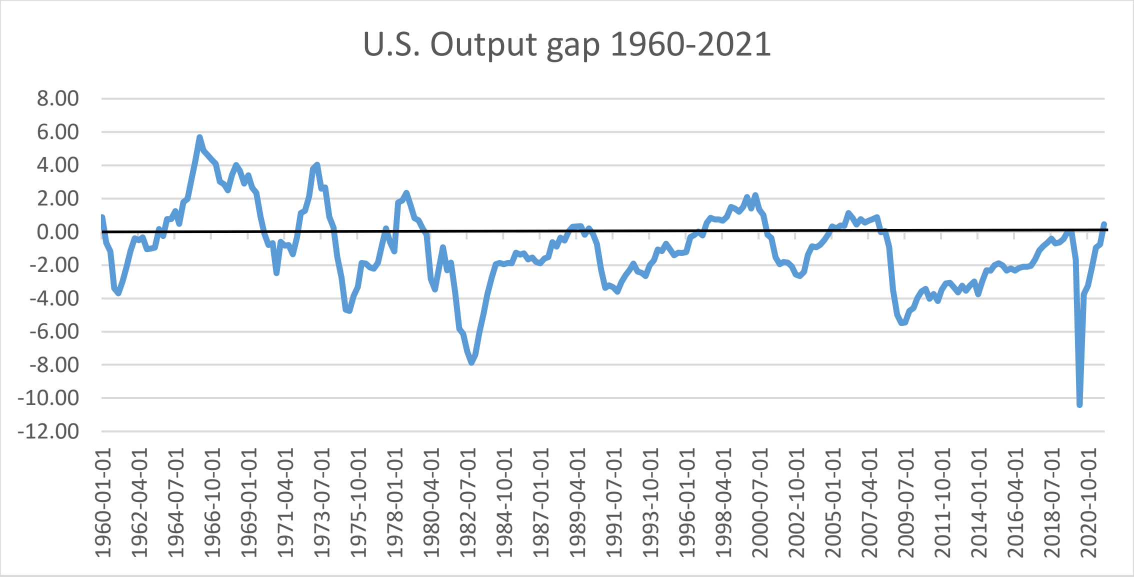

Figure 7. U.S. Output gap 1960–2021 (The Federal Reserve Bank of St. Louis, 2025a).

Another commonly referenced indicator of monetary policy is the yield curve. The yield curve refers to a graph showing the yield of bonds with varying maturities. Most bonds offer a fixed coupon rate, and that rate can be used to calculate the yield of a bond. The yield and market price of a bond have an inverse relationship, meaning that a price increase lowers the yield of a bond. Choudhry (2019, p. xiii) explains, that the main yield curve of any economy is the yield curve of government bonds, for example, in the U.S., it is the U.S. Treasury yield curve. Corporate bonds are often issued with higher yields than that of government bonds, and therefore they do not represent the behaviour of current, and future, interest rates to the same extent. Choudhry (2019, pp. 18; 172) continues, that a zero-coupon bond yield is regarded as the true indicator for market expectations. The writer further clarifies, that since most bonds have coupons, the zero-coupon yields need to be manually constructed. While most prominently associated with market expectations, the yield curve – especially the short end – reflects the transmission of PIRs into the financial system.

The theoretical basis for this interpretation can be drawn from Fisher (2004, p. 41), who defines asset values with the following equation (1).

$$asset\ value = \sum_{s \in S} (payout_s \times state\ price\text{–}\ deflator_s) \times probability_s \tag{1}$$

$$S = set\ states\ of\ the\ world\ at\ all\ future\ times$$
$$State\ price\text{–}\ deflator =\ the\ state$$
$$price\ divided\ by\ the\ probability\ of\ the\ state$$

Since the formula sets bond prices to be dependent on the expected future states of the economy, the market sentiment regarding the likelihoods of either a downturn or an upturn in the economy primarily affect the shape of the yield curve. If a rising trend within the economy is seen as likely, according to the formula, the yield curve of the government's bonds should be upwards sloping. Vice versa, a downwards sloping yield curve indicates negative market expectations. From this definition one can also deduce that a flat yield curve is often a signal of a yield curve transitioning from one slope to another. However, Choudhry (2019, p. 23) clarifies that a flat yield curve is a rare occurrence. In stable conditions, this structure allows the yield curve to serve as a complementary proxy for monetary policy stances.

However, after the Great Recession of 2008, and for Japan, during the 1990s, a key limitation emerged as both PIRs and yield curves became constrained by the ZLB. Since UMP has been applied as a complement to the PIRs already at the ZLB, it is intuitive to consider that the PIRs do not capture the full effect of a central bank's monetary policies at the ZLB. This view is also brought up by Wu & Xia (2014, p. 12), who state in their paper that, indeed, the FFR does not convey information about the easing policies of the FED after January 2009. Furthermore, since it has been established that short-term zero-coupon bond yields are derived from a central bank's PIR, when the PIR does not include all the easing policies of a central bank, neither does the yield curve. As the two prominent indicators and tools of transmitting monetary policy are thus hindered, the need for an alternative measurement becomes apparent. This is where the shadow rate is introduced.

3.2 The fundamentals of shadow rate models

Even though the need for unconventional monetary policies mostly arose only following the Great Recession of 2008, research papers about shadow rates and the ZLB have been published even before the current millennium. Fischer Black (1995) introduced the idea of currency as an option to negative rates and the concept of a shadow rate as early as 1995. The writer proposed that in addition to the nominal short rates, there exists a short rate which demonstrates what the rates would be, if the ZLB was not a binding lower limit. Black plotted two exemplary patterns, the first of which is a downwards concaving shadow yield moving to negative yields. The second included graphs illustrating the volatilities of nominal yields, and the shadow yield; the shadow yield having a higher and flatter line of volatility due to the fact, that its value may decrease further than that of the nominal rate. Black's paper does not, however, suggest any mathematical model to estimate shadow rates by calculation.

Krippner (2013) later followed up on Black's proposal by creating a shadow bond forward rate framework deriving a curve displaying negative yields on bonds. While the paper above also illustrated a shadow yield curve, Krippner (2013, p. 135) emphasizes, that the model introduced in his paper is much easier to implement into practical studies. The model returns tractable and seemingly valid results, when applied to Japan (Krippner, 2013, p. 136), but it is noteworthy, that the model in question does not include multiple macroeconomic factors at once. The next section of this paper introduces an analytical multi-factor approximation of the shadow rate term structure model (Wu & Xia, 2014). The reason behind choosing said model for presenting an approximation of shadow rates, is due to the fact that the model is quite comprehensible, and since it has functioned as a basis for later shadow rate studies, one of which will also be introduced in later sections of this paper.

3.3 The discrete shadow rate term structure model

Wu & Xia (2014) take the propositions above further by creating an Affine Term Structure model which allows for negative rates, called the Shadow Rate Term Structure Model. Please note, that the following equations (2–10) have been borrowed from Wu & Xia's (2014, pp. 5–9) paper, to properly construct the SRTSM for inspection of the model itself and its implemented results. The SRTSM begins with the assumption that the shadow rate s_t follows the conventional PIR r_t when positive but does not have an effective lower bound for the PIR. This is defined in the equation (2) below.

$$r_t = \max(r, s_t). \tag{2}$$

This simplistic definition is the foundation for the SRTSM. Equation 2 sees the shadow rate create a simple, yet informative PIR without a nominal lower limit. This definition alone suggests that the shadow rate contains more information about the level of monetary easing than the nominal interest rate when the PIR has reached its binding lower limit. The shadow rate $s_t$is then defined as an affine function to state variables X_t and the variables to follow a $VAR(1)$ process under a physical measure (P) in the equations (3–4) below.

$$s_t = \delta_0 + \delta_1' X_t. \tag{3}$$

$$X_{t+1} = \mu + \rho X_t + \Sigma \varepsilon_{t+1}, \qquad \varepsilon_{t+1} \sim N(0, I). \tag{4}$$

The SRTSM uses an affine log stochastic discount factor, as shown below (equation 5).

$$M_{t+1} = \exp\left(-r_t - \frac{1}{2}\lambda_t' \lambda_t - \lambda_t' \varepsilon_{t+1}\right). \tag{5}$$

Risk's price factors are defined as linear in the equation (6) below.

$$\lambda_t = \lambda_0 + \lambda_1 X_t \tag{6}$$

According to Wu & Xia (2014, p. 7) this implies that the risk neutral measure dynamics (Q) are also $VAR(1)$ as shown below (equation 7).

$$X_{t+1} = \mu^Q + \rho^Q X_t + \Sigma \varepsilon_{t+1}^{Q}, \qquad \varepsilon_{t+1}^Q \sim^Q N(0, I). \tag{7}$$

Finally, the parameters above are defined to have the following relations under P and Q (equations 8–9).

$$\mu - \mu^Q = \Sigma \lambda_0, \tag{8}$$
$$\rho - \rho^Q = \Sigma \lambda_1. \tag{9}$$

Having defined the SRTSM to follow the nominal PIR above the ZLB and to model what the PIR would be when the nominal rate is bound by the ZLB, and given the stochastic factors for the model, Wu & Xia (2014, pp. 6–7) construct an approximation for forward rates within the SRTSM described above. First, they define a linear yield function of risk-free n and $n + 1$ period zero-coupon bonds in the equation (10) below.

$$f_{n,n+1,t} = (n+1)y_{n+1,t} - ny_{nt}. \tag{10}$$

Where $f_{n,n+1,t}$ is the forward rate at time t for a loan starting at $t+n$ and the maturity is $t+n+1$. The approximation for the SRTSM is as follows (equation 11).

$$f_{n,n+1,t}^{SRTSM} = r + \sigma_n^Q g\left(\frac{a_n + b_n' X_t - r}{\sigma_n^Q}\right). \tag{11}$$

Where $(\sigma_n^Q)^2 \equiv Var_t^Q(s_{t+n})$. The state space SRTSM also receives an i.i.d. normal error term $\eta_{nt} \sim N(0, \omega)$. The final form of the SRTSM approximation defined above is in the equation (12) below.

$$f_{n,n+1,t}^{SRTSM} = r + \sigma_n^Q g\left(\frac{a_n + b_n' X_t - r}{\sigma_n^Q}\right) + \eta_{nt}. \tag{12}$$

This shadow forward rate approximation along with the preceding definitions includes the concept of a shadow rate following the nominal PIR above the ZLB, the shadow rate consisting of random state variables along a normal distribution, price factors for risk and of course, an error term. A large development from the previous propositions presented above, allowing for multiple factors.

To test the validity of the SRTSM in different PIR environments, Wu & Xia (2014, pp. 12–15) combine the two different SRTSM rates of r_t and s_t into s_t^o. This enables the application of a lot of historical data into the model, since the single approximation can be used for times of both, positive, and ZLB interest rates. The application begins with proposing a null hypothesis, that the SRTSM is constructed to react to the same macroeconomic variables as the FFR under conditions where the PIR is above the ZLB. This is done to see whether the SRTSM carries similar behaviour during the ZLB-period of UMPs in the U.S. as the FFR did before the Great Recession. Macroeconomic variables loaded include factors affecting both the real economy, as well as price levels. The test ends with a failure to reject the null hypothesis, which can be interpreted as a success in the sense, that the SRTSM does indeed seem to show similar relations to macroeconomic variables as the FFR before the period. Wu & Xia (2014, p. 16) conclude the test to indicate that the model will indeed show additional information about easing monetary actions when the PIR is bound by its effective lower limit, and that the approximation can also be applied to previous studies conducted by using historical FFR data.

3.4 Results of the SRTSM approximation

The earlier section went through the fundamentals of the SRTSM as well as briefly discussed the validity of the approximation proposed. Since the approximation seemed to, indeed, simulate the FFR when above the ZLB and contain more information about UMPs in a ZLB environment than the FFR, let us now observe the results and information unique to the s_t^o. First, a visual inspection of the approximation can be found below from figure 8. In the said figure, the blue line represents the shadow rate approximation s_t^o (equation 12) and the green dotted line is the actual FFR. From this figure, one may clearly see how well the approximation created follows the actual FFR before the ZLB-period, which is marked with the grey area. Also, the additional information carried during the ZLB-period is visualized as the two rates separate during the beginning of the Great Recession in late 2008.

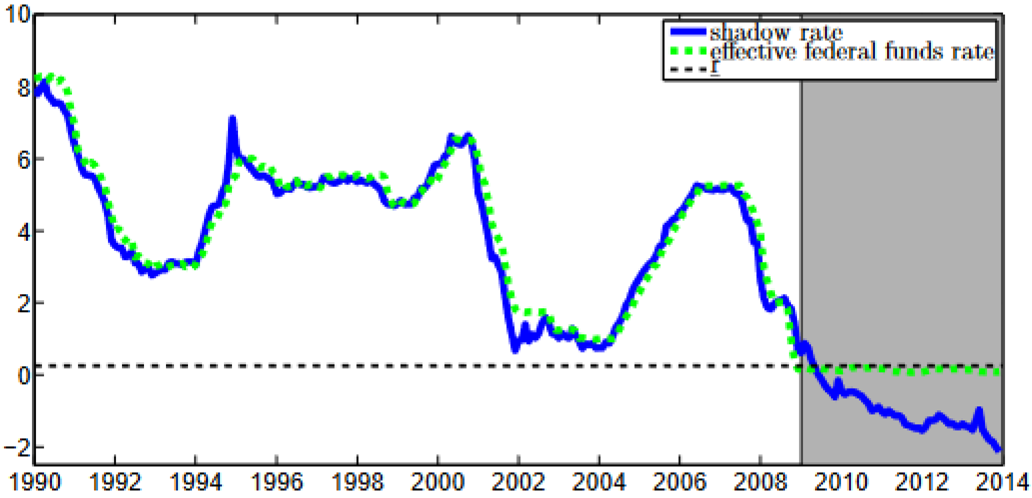

Figure 8. The shadow rate and the FFR (Wu & Xia, 2014, p. 41).

The macroeconomic effects of UMPs implied by the shadow rate during the ZLB period are also numerically estimated by Wu & Xia (2014, pp. 16–19). First, according to the results, UMPs conducted by the Fed have, in fact, actively lowered the PIR even though the nominal rate has been bound by zero since late 2008. Second, the results show that the UMPs conducted from late 2008 until December 2013 have in fact influenced the real economy. Compared to the estimates calculated with no expansionary monetary policies during the ZLB-period, unemployment was lowered by 0,13 %, capacity utilization increased by 0,3 % and housing starts increased by 11 000, or around 1,1 % compared. The writers also note an interesting phenomenon; the results would indicate that the stimulation transferred to the real economy did not result in higher inflation, but that the consumer price index instead dropped by 1.

This section of the paper introduced the reader to the idea of a shadow rate while also presenting a background as to why the concept is relevant in the context of modern central banking. Black (1995) proposed that a shadow rate exists alongside nominal PIRs, which is not bound by the ZLB and can show the real stance of monetary policies at the ZLB. Wu & Xia (2014) then built an analytical framework to model the shadow rate and applied it to historical data, both above and below the ZLB. The previous studies to showed, that the shadow rate can be a valid model for analysing historical data at the

ZLB. The next section of this paper examines whether the shadow rate could be applied as a tool of guidance for future monetary policies at the ZLB. This is done through the Taylor rule.

3.5 Using the Taylor rule for shadow rate modelling

The proposal of a shadow rate carrying information beyond the nominal PIR at the ZLB was first conceptualized by Black (1995) and has since been analytically described in multiple different publications, of which this paper introduced the SRTSM, modelled by Wu & Xia (2014), in the previous section. These publications suggest a valid argument stating, that the nominal PIRs used by central banks do seem to hide relevant information when rates are bound by the ZLB and that the shadow rate can be modelled to reveal this information while also working as a reliable substitute to the nominal PIRs by accurately tracing them above the ZLB. These properties raise the question whether the shadow rate could be used by central banks as a complementary tool when applying monetary policies at the ZLB in addition to the frameworks' potential for historical analyses. In this section, the paper will go through the widely used Taylor rule and compares it to a shadow Taylor rule proposed by Wu and Zhang (2016).

To begin the comparison, let us first examine the fundamentals of the original Taylor rule, as proposed by John Taylor in the Carnegie-Rochester Conference on Public Policy in 1993 (Taylor, 1993). Whereas the shadow rate frameworks previously reviewed were used for retrospective analyses of the effect of monetary policies, the Taylor rule is a policy tool proposed for determining the FFR in relation to set monetary objectives by the Fed. Taylor (1993, pp. 196–197) states, that at the time of the article, policymakers did not seem to follow any mechanical rules in setting monetary policies, despite recent and widespread discussion of such algebraic tools. While the writer admits that some aspects of the economy and its behaviour are difficult to dress up in a set of equations, the author insists that policy rules have great advantages when compared to discretionary policymaking. The rule Taylor (1993, p. 202) suggests is as follows (equation 13).

$$r = p + 0.5y + 0.5(p - 2) + 2. \tag{13}$$

$r = FFR,$
$p = rate\ of\ inflation\ over\ the\ previous\ four\ quarters,$
$y = \%\ of\ deviation\ of\ real\ GDP\ from\ a\ set\ target.$
$y = 100(Y - Y^*)/Y^*$
$Y = real\ GDP$
$Y^* = trend\ level\ of\ real\ GDP$

This means, that the Taylor rule is a simple tool for proposing a suitable target for the FFR, the Fed's PIR. The part $0.5.(p - 2)$ reflects the Fed's target inflation of 2 per cent, as defined by the Federal Reserve Board (2021). The rule is intuitive and follows the consensus among traditional monetary policies, since the target rate r rises, if inflation rises above 2 per cent or if the real GDP rises above trend level, meaning the economy is facing a positive output gap. Just as summarized in the beginning of section 3 of this paper, when the economy shows signs of overheating, monetary policy ought to be tightened by raising the FFR and vice versa; also presented by yield curves since an upward sloping yield curve shows expectations of a boom in the economy accompanied by a higher PIR. Koenig, Leeson, & Kahn (2012, pp. 68–69) discuss the appeal of the Taylor rule and praise its intuitiveness since the rule represents the common way policymakers view monetary policy and because it is easy to apply for mandates regarding both a steady price level and economic growth. The rule is intuitive, simple and manages to capture the logical approach monetary policymakers should take when issuing monetary policies.

According to the above, the Taylor rule is an excellent, simple monetary policy rule, which acts as a prominent tool amongst monetary policymakers around the world. The rule can be applied to mandates regarding both price levels and economic growth, and, as Koenig, Leeson, & Kahn (2012, p. 70) mention, it has managed to both describe monetary policies of the past as well as influence future policies. Of course, just as the PIRs of a central bank, the Taylor rule is bound by the ZLB. Since PIRs cannot be lowered below the ZLB, Taylor rule results have been limited to having the minimum value of the ZLB. Noting the wide use of the Taylor rule, another question to consider is whether the equation could provide valid results when the ZLB is removed and if the Taylor rule along with the shadow rate could be used as forward-looking tools of policymaking at the ZLB.

Wu & Zhang (2016) continue in the footsteps of Wu & Xia (2014) by proposing a New Keynesian Model without a binding ZLB called the Shadow Rate New Keynesian Model. The results of the SRNKM will be later examined as a whole, but this section focuses on the use of the Taylor rule. As previously suggested, Wu & Zhang (2016, p. 4) begin by stating, that they see the shadow rate, not as an actual borrowing or lending rate, but as a summary of combined easing monetary policies, which the central bank targets. Wu & Zhang (2016, p. 12) then take the s_t^0 data from Wu & Xia (2014) – which has its results introduced in this paper's section 3.6 – and implement the shadow Taylor rule, as shown in the figure (9) below.

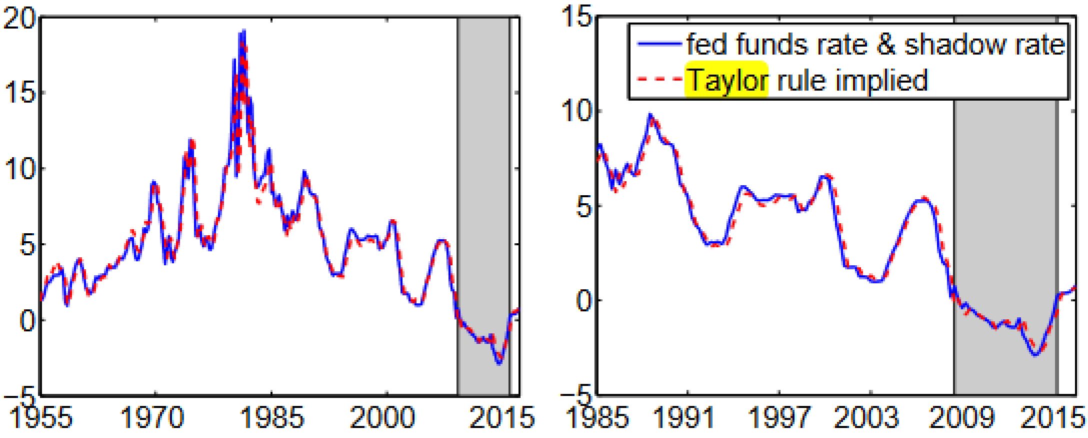

Figure 9. The FFR, shadow rate and the Taylor rule (Wu & Zhang, 2016, p. 13).

From the results, one may see that just as the SRTSM managed to both, follow the FFR above the ZLB and convey more information below the ZLB, the shadow Taylor rule seems to represent the entire history of the implemented s_t^o accurately. These combined results from the two articles indicate that the shadow rate can be modelled to accurately describe historical data and the effects of UMPs at the ZLB, and that the Taylor rule seems to remain as a valid policy rule without the ZLB. The shadow rate is then, an easily tractable, and a valid tool for modelling historical effects of monetary policies, as well as a potential tool for implementation of future monetary policies. In the next section, this paper examines empirical results and implications derived from different research papers conducted regarding the shadow rate.

3.6 Results of shadow rate models

So far, this paper has introduced the reader to the fundamentals of monetary policy, defined that the Great Recession of 2008 was the starting point for UMPs regarding the Federal Reserve – the 1990s for Japan – and showed that the shadow rate can be used to approximate the monetary policy stance of a central bank when interest rates are bound by zero. The research examined so far has shown, that PIRs do not convey all monetary information at the ZLB (Wu & Xia, 2014) and that shadow rate approximations can be constructed to follow the Taylor rule and PIRs quite accurately above zero. Furthermore, even a shadow Taylor rule without a binding lower limit can be proven to predict a shadow rate approximation accurately (Wu & Zhang, 2016). Having established a basis to see validity and relevance among shadow rate models, this paper will now examine results of some shadow rate models regarding the amount of monetary easing at the ZLB. Let us turn to the figure (10) below, which shows us Krippner's (2020b, p. 2) compilation of PIRs and two different shadow rate estimations of four great economies between the years 1995–2020. The estimations have been produced by using the Nelson-Siegel term structure model with an application of Krippner's own shadow rate framework, as mentioned by the writer (Krippner, 2020b, p. 5).

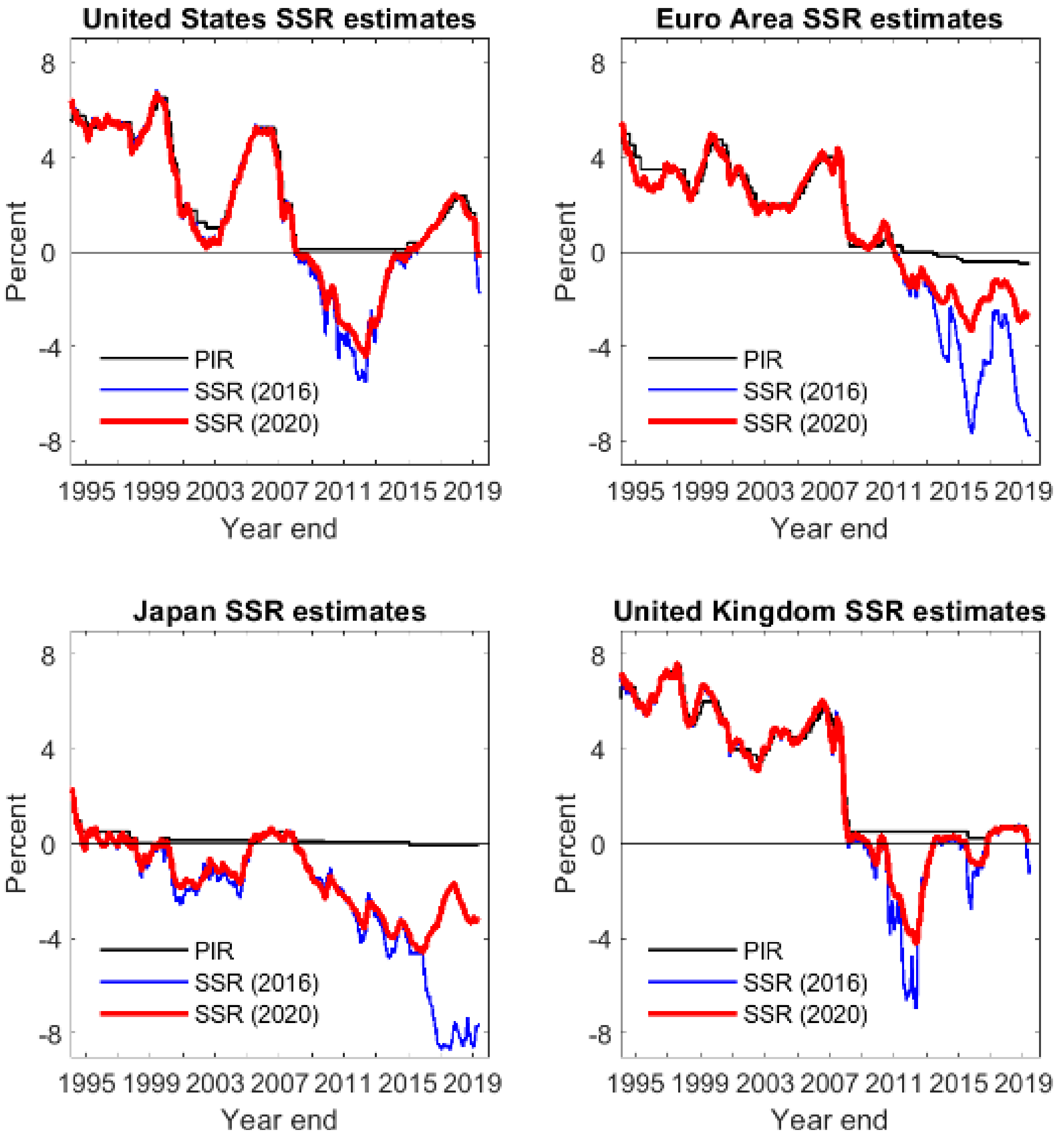

Figure 10. G4 SSR Estimates 1995–2020 (Krippner, 2020b, p. 2).

The graph above is from Krippner's (2020b, p. 2) update on his shadow rate estimates of the U.S., Euro area, Japan and the U.K., the G4 economies. From both SSR estimations, one may clearly note a substantial spillover of financial shocks from the U.S. to the other great economies. All the central banks shown in the figure above, have clearly had to react to the shocks originating from the U.S. as all four PIRs hit either a zero or a near-zero lower bound from 2008 onwards. Comparing the central banks, the U.S. Fed was especially quick to bounce back from the ZLB with a single SSR trough and systematic return to a positive PIR, excluding 2020 which sees the FFR meet the ZLB due to the Covid-19 pandemic (Jane, E. Ihrig et al., 2020). These estimations are a good visual representation of the information hidden by the PIRs at the ZLB. One can interpret the shadow rates to show that, as an example, in the U.S., the combined easing effects of conventional and unconventional monetary policies amounted to the equivalent of a PIR of around -5 %. Without the shadow rate approximations, it would take unnecessary additional effort to discern the combined effects of all monetary policies at the ZLB to the economy.

Focusing again on the Fed, Krippner (2020b, pp. 11–15) states that the U.S. SSR consistently trending downwards after 2008 comes down to the QE1-program announced in late 2008 to combat the recently begun Great Recession with the trend primarily continuing until a sharp change as a response to Bernanke's statement about tapering QE in May 2013. This is a prime example of the shadow rate visualizing the effects of UMPs in yield curves otherwise invisible when bound by the ZLB. From the SSR estimates one can also see that, while the U.S. quickly rebounded from the shadow rate trough of early 2013, other central banks did not recover as easily. As an example, the ECB experienced two back-to-back crises as the Great Recession of 2008 spilling over from the U.S. was soon followed by the Euro crisis, which was then followed by a period of low inflation and deflation fears; ultimately forcing the ECB to continue expansive monetary policy for much longer than the Fed (Kok et al., 2022, pp. 12–22). From the four central banks compared, the Fed was the only one to raise their PIR back to levels of over 1 % between 2008–2020. This is an interesting phenomenon, since the PIR rebound of the Fed is somewhat contradicting: if one looks at figure 7, one may note that according to data by the Federal Reserve Bank of St. Louis (2025a), the output gap of the U.S. remained primarily negative throughout the 2010s.

Turning to alternative estimations, Alfaro & Piña (2023) have recently created new shadow rate estimations for the Fed based on multiple datasets and models previously created. While the previous results examined Krippner's (Krippner, 2020b) SSR estimations using the writer’s shadow rate framework applied to the Nelson-Siegel term structure model, Alfaro & Piña (2023, pp. 3–5) use a similar dynamic term structure model, but with a framework which has features from both, Krippner’s and Wu & Xia’s approximations. In addition, two different yield curve datasets are used for comparison. Alfaro & Piña (2023, pp. 8–11) then create eight different SSR comparisons between the two datasets, testing out the sensitivity of the shadow rate model by changing sample sizes, normalization methods, number of factors and by comparing yield curves from different maturity bonds. Sensitivity based on datasets seemed to be a noteworthy phenomenon, but its severity will be addressed within the next section of this paper. The main take regarding the results seems to once again be that the results draw very similar SSR estimations to Krippner’s (2020b) even within the different variations, once again meaning that the shadow rate does, indeed, seem to be easily tractable. Furthermore, Lombardi & Zhu (2014) have also built a robust and simple shadow rate model, indicating that the shadow rate can be modelled to reveal information beneath the ZLB and that it can be approximated quite well with the Taylor rule. Also, again consistent with Krippner’s (2020b) new and Alfaro & Piña’s (2023) SSR estimates, Lombardi & Zhu’s (2014, p. 20) results show similar entry and exit points for the ZLB and that the greatest monetary stimulus provided was on average, around -5 %, all before 2014. Even though the dataset only reaches 2014, it seems to react similarly to the previous graphs, with only the Wu & Xia (2014, p. 41) graph (figure 8) showing a more positive rate of around -2 %.

As a summary, especially the latest SSR estimations based on Krippner, Wu & Xia (2014) and Wu & Zhang (2016) seem to give quite consistent results indicating that the frameworks are easily tractable and that the estimations seem considerably similar between different papers with each paper claiming a robust result. The shadow rate is not without its issues, however, as there seems to be some sensitivity regarding the chosen datasets and sample sizes.

3.7 Critique and notes of caution

This paper has now examined some of the most well-known articles from many of the predominant researchers of the shadow rate. While others exist, most estimates seem to often lend datasets and frameworks from the studies previously presented. During the previous sections, this paper has found that the shadow rate is able to uncover monetary information no longer conveyed by the PIR at the ZLB, that it can be modelled to follow the Taylor rule very consistently even below the ZLB, and that many SSR estimations produce notably similar results, at the very least on the Fed's part. The field of shadow rate frameworks is quite new, however, which is most likely one of the reasons many of the estimations rely on the same frameworks and applications, mainly Krippner, Wu & Xia (2014) and Wu & Zhang (2016). Even though the models showed robustness on their own and though the shadow rate can be very tractable, the frameworks, and therefore also the estimations, are not without their problems. The main issue found has to do with the sensitivity of the shadow rate in relation to different datasets and factors.

Alfaro & Piña (2023, p. 2) begin their SSR estimation by stating that, already, multiple researchers have found shadow rate models to be sensitive to changes in the number of factors, the ZLB and even the estimation method regarding Kalman filtering. As stated before, the writers carry on conducting eight different SSR estimations. Even though the results show mainly similar graphs (Alfaro & Piña, 2023, pp. 8–11), with all shadow rates going below the ZLB in the end of 2008, exiting the ZLB near the end of 2015 and having overall the same shape throughout the years of data, the magnitude of negative rates seems to noticeably change depending on the characteristics of the SSR series and the dataset. The largest differences seem to be within the changes to bond maturities, sample size – in years – and from a 2-factor to a 3-factor model with all reaching the largest difference of around 2 percentage points between the years 2011–2015. This is the first signal, that the results of shadow rate models might need some scrutiny before direct interpretation.

A good place to continue examining the sensitivity of shadow rate models, is to review Krippner's (2020a) article written specifically to investigate said issue. During the article, the writer explains that for the paper written, a thorough testing of Wu & Xia's (2014) SRTSM has taken place with similar actions taken as in the previous article discussed. Krippner (2020a, pp. 952–953) begins by elaborating, that within Wu & Xia's (2014) SRTSM article, only a single SSR series along with a single dataset and a single ZLB were used to investigate the results of the SRTSM and that the article did not address the issue of sensitivity at all. A set of nine SSR series is then created (Krippner, 2020a, pp. 953–956) where all parameters see variation, from the ZLB, to the maturities within the yield curve. The results are consistent with Alfaro & Piña's (2023) article since, again, changing the parameters discussed seem to indeed cause even quite large variations to the results of the SRTSM. Krippner (2020a, p. 961) goes as far as to conclude that no single SSR series can be considered as a valid measure of the UMPs conducted at the ZLB since the sensitivity of the SSR leaves a lot of ambiguity regarding the real influence of the UMPs. However, this conclusion comes with an important note – that it is very much possible to find the most reliable results by creating multiple sets of SSR series and examining which of the series are the best at following the actual FFR.

Despite the presented issues, the reviewed literature has shown shadow rate frameworks to effectively produce estimates of the monetary policy stance of a central bank during zero-interest rate environments and existing potential for shadow rate estimates to be used within forward-looking analyses in addition to retrospective studies. Within the following sections, this paper is to empirically study the potential of shadow rate estimates as an additional tool for monetary policymaking within zero interest rate environments by examining the predictive power of shadow rate estimates in comparison to short interbank rates, proxying PIRs, within out-of-sample forecasts.

4 Method

Having introduced the theoretical concept of the shadow rate and its potential utility for monetary policymaking, this study employs a comparison of forecasting models predicting changes within the real GDP growth rates of two economies discussed previously, Japan and the USA. The comparison is between the predictive power of the economies' nominal interest rates and corresponding shadow rate estimates, using otherwise identical forecasting models. The purpose of the research is to examine whether the additional information carried by the shadow rate within a ZLB environment can be effectively utilised for out-of-sample forecasting and, thus, as an additional tool for monetary policymaking. The results are then compared to existing findings regarding shadow rate forecasting. The economies were chosen based on their history with zero interest rate policies. As already established, the U.S. Subprime crisis was the starting point for the Great Recession, and globally widespread ZLB environments, making the country a logical choice. Japan has an exceptional history of monetary policy, with a near–continuous ZLB environment covering almost three decades, making for an interesting focus of study.

This section of the paper consists of three parts, a description of the data used along with a brief examination of some relevant key characteristics, followed by a presentation of the forecasting models used, and, finally, an explanation of the performance metrics utilised for the comparison of said forecasts. The data section provides rationale for any data transformations and presents pre-estimation diagnostics common for all of the models used. The forecasting models section gives analytical presentations of the models used, along with any additional data diagnostics related to certain models used.

4.1 Data

The data is divided into national quarterly time series sets for USA and Japan, both including a list of either identical or comparable variables, consisting of macroeconomic indicators along with a national stock market index. The forecasting variables used is a combination of percentages and transformations of level information, described within table 1 below. The forecasted variable for the datasets is the growth rate of real GDP. For the comparison of forecasting performance, the datasets include 3-month interbank rates, 3MRs, substituting for national PIRs, and corresponding shadow rate estimates released by Leo Krippner (2024). The decision to replace national PIRs with 3-month interbank rates was made to improve continuity as the former are discretely set whereas the latter provide smoother, more continuous values. An additional shadow rate variable was created and included which is the spread between national 10-year government treasury bonds and shadow rate estimates. Said variable was built to be compared to the spread between the same bonds and national 3MRs.

Though most of the used variables have observations spanning over a longer horizon, the time series to be used for both economies is between 1995–2018. The chosen window is due to two factors: the quarterly shadow rate estimates used begin from Q1995 and data beyond 2019 is omitted due to abnormal levels of variance stemming from the global COVID-19 -pandemic – furthermore, the U.S. rates leave the ZLB after 2015. The chosen time window is most suitable for representing significant differences between the performance of the forecasting models as each model receives learning data from both outside of and during ZLB periods while primarily creating forecasts when the ZLB is met. The data used within this study is obtained from the database Federal Reserve Economic Data, FRED, maintained by the Federal Reserve Bank of St. Louis with the exception of the Japanese output gap, obtained from the Bank of Japan, and the stock indices S&P 500 (Investing.com, 2025b) and Nikkei 225 (Investing.com, 2025a). The full list of datasets used with sources can be found from appendix 3.

Table 1. Variables, descriptive information.

Variable name	USA	Japan	Transformation
3-month interbank rate**			$\Delta 3MR_t = 3MR_t - 3MR_{t-1}$
Shadow short rate **	SSR estimate (Krippner)	SSR estimate (Krippner)	$\Delta SSR_t = SSR_t - SSR_{t-1}$
Real GDP**	$Bn ch. se.adj.	¥Bn ch. se.adj.	$\Delta \ln(GDP_t) = \ln(GDP_t) - \ln(GDP_{t-1})$
CPI growth rate**	All items: total	All items: total	
Industrial production index**	Total industrial production	Total manufacturing	%-change
Unemployment rate**			$\Delta UR_t = UR_t - UR_{t-1}$
Output gap**			
Stock index**	S&P 500	Nikkei 225	%-change
T10Y3MR**	T10Y – 3MR	T10Y – 3MR	
T10YSSR*	T10Y – SSR	T10Y – SSR	

* = used in VAR
** = used in VAR and DFR

The chosen macroeconomic variables consist of some of the most prominent indicators of the current overall state of an economy with the intent of maximising the forecasting performance of each model, giving the varying interest rate variables the most suitable environment of showing significant differences between one another. Most variables are also naturally tied to the interest rate variables as they operate as the indicators which much of monetary policy aims to address – such as the output gap and CPI changes. The use of appropriate variables was also checked by referencing existing similar forecasting studies (Carriero et al., 2023, p. 28; Hännikäinen, 2017, p. 11) to ensure proper methodology. A complementary addition to the macroeconomic variables was made with the inclusion of a stock market index for both countries. Stock returns were first considered as a potential addition due to its theoretical potential for forecasting changes in output, as pointed out by Stock & Watson (2003, p. 12). Although the authors do state evidence of poor empirical performance shortly thereafter, other existing studies such as one by Kuosmanen & Vataja (2024, pp. 9–10) show contrary results with significant predictive power towards economic growth relative to other macroeconomic variables, such as interest rate variables.

All variables were tested to be suitable for the forecasting models and transformed accordingly. The Augmented Dickey-Fuller test was conducted to test for stationarity along with the Phillips-Perron test as a robustness check correcting for autocorrelation and heteroskedasticity. As one may examine from table 1 above, 3MR, SSR estimates, and the unemployment were transformed into their first differences to ensure stationarity. While the output gap and the two bond spreads, T10Y3MR, and T10YSSR, did not pass the mentioned unit root tests, it was considered preferrable to abstain from taking first differences from a conceptual standpoint. The index variables were transformed to present percentage changes from time $t-1$ to t and the dependant variable, real GDP, was transformed, first into its natural logarithm values and then its first difference. The choice of using natural logarithm was to ensure comparable results and to minimise unnecessary variance – with the final result representing an approximate growth rate for real GDP.

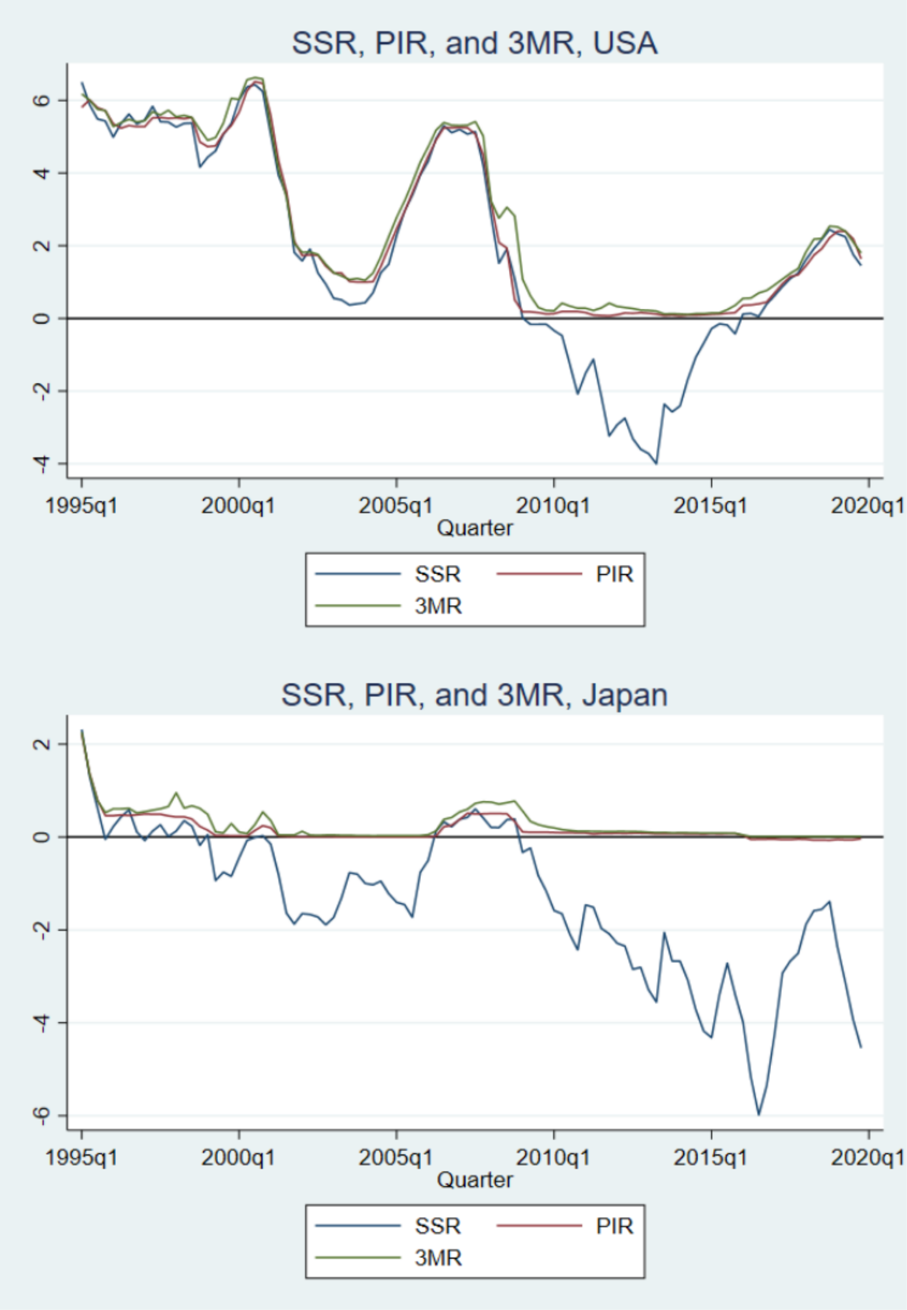

Figure 11. SSR, PIR, and 3MR.

Figure 11 above shows overlapping time series graphs comparing the historical values of SSR estimates, actual policy rates within each economy, and their counterparts, the 3-month interbank rates. The time series is limited to begin from 1995 as per the initial year for SSR data. As one may note, the U.S. first entered the ZLB environment as a reaction to the Subprime Crisis, from late 2009 onwards with the policy rate remaining near-zero until early 2016. The SSR estimate primarily manages to follow the policy rate above the ZLB, though the visual suggests a bias to somewhat underestimate the official rate while, during the ZLB era, shows a notable spread between itself the PIR with a maximum of roughly four percentage points. The data for Japan gives implication to the exceptional monetary history previously covered as the PIR falls near zero immediately upon the beginning of the time series and remains nearly static throughout the two and a half decades of monetary policymaking. The Japanese rate graph gives an especially apt visual presentation of the additional information carried by the SSR estimates as the spread between the two rates suggests quite distinctive monetary policy changes during various notable economic events such as the dotcom bubble, the global financial crisis, and the struggles of surrounding emerging countries while the PIR indicates much of nothing. The graphs also show that the 3-month interbank rates used within the empirical study closely follow national PIRs, maintaining the theoretical juxtaposition previously proposed between policy rates and shadow rate estimates.

These rate figures give precedent to the forecasting performance study conducted. The reviewed literature shows shadow rate estimates to convey additional information within such interest rate environments, from which one may draw the implication of additional predictive power. The economies used in the study show a tendency for the ZLB to be sticky as all periods remain for multiple years at a time meaning extended periods of time where short term interbank rates provide limited information to be used in ex-ante out-of-sample forecasts, thus providing motivation for the study. The assumption of a sticky ZLB is reasonable for any such future conditions, as zero-interest-rate-policies are an extreme measure, only to be implemented while necessary and once the economy is in a state requiring extended periods for recuperating.

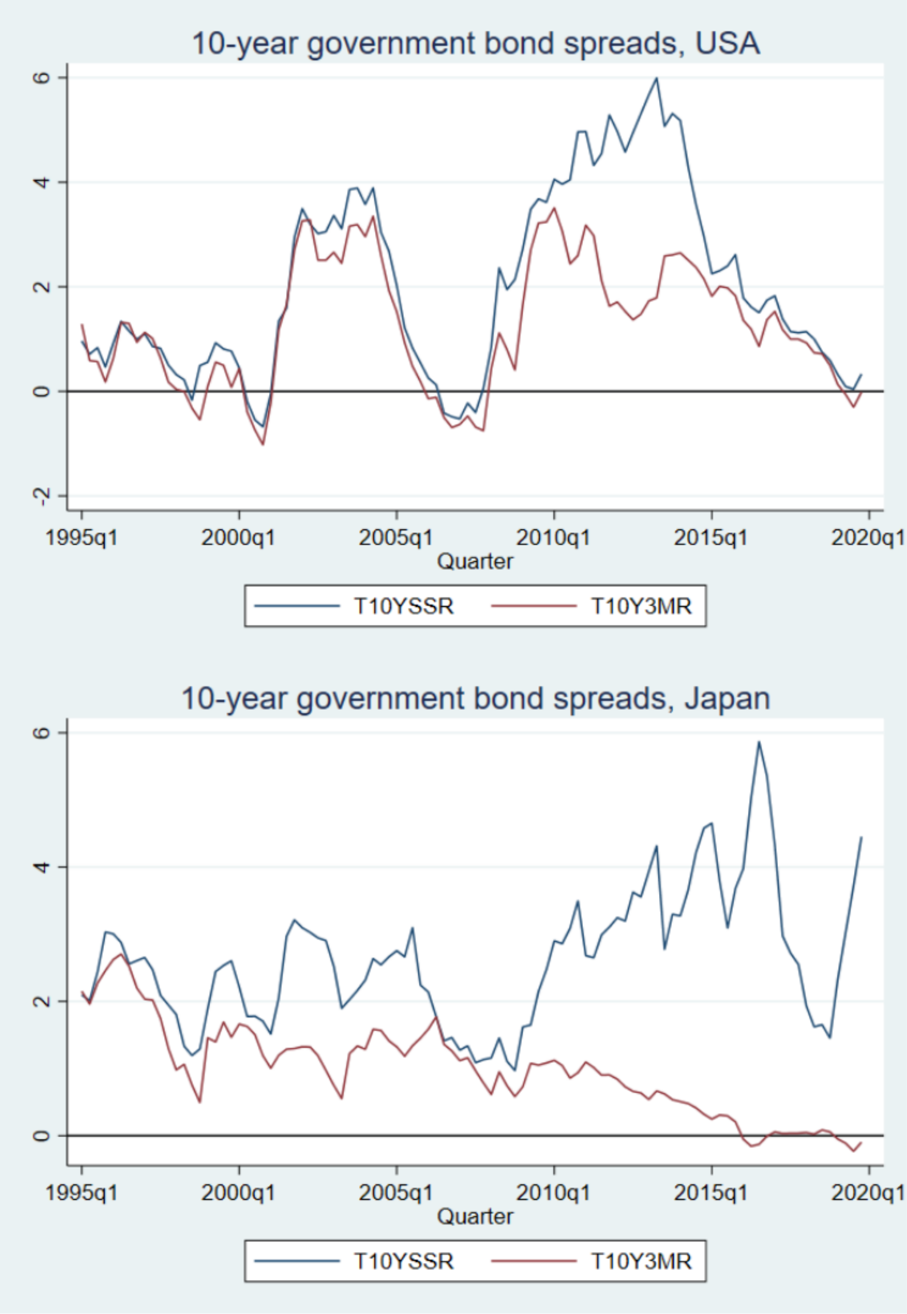

Figure 12. 10-year government bond spreads.

Above is a figure (12) showing graphs of the additional shadow rate variable and its counterpart, the spreads between 10-year government treasury bond yields, 3MRs, and SSR estimates. Although the spreads between interbank rates and long-term bond yields have the potential of reaching negative values just as the SSR counterparts, the additional variable was created to further highlight the difference between information carried by official rates and SSR estimates within a ZLB environment. As one may note from figure 12, the U.S. spreads created clearly exhibit a difference during the ZLB era of the Great Recession and the Japanese spreads present similar results over both the bursting of the dot-com bubble in the early 2000s and especially during the period following the Great Recession with the SSR spreads of both economies reaching around a six percentage point difference at their peaks.

4.2 Forecasting models

All forecasting models within this study were used to create ex-post out-of-sample forecasts to examine the predictive power of SSR estimates against official interbank rates over the quarterly growth rates of national real GDPs. The purpose of such forecasts was to study the potential benefits of utilising the shadow rate instead of short-term market rates during ZLB periods where future data was not available – in other words, to produce future forecasts of real GDP changes. The study used three different models with identical variables, a vector autoregressive model for dynamic one-step forecasts, a recursive model for direct h-step forecasts, and a univariate autoregressive model of the real GDP as a benchmark for the performance of the previous two. The first two models were further separated into comparisons between the performance of shadow rate estimates over policy rates and shadow rate spreads over policy rate spreads. All forecasting models were given the same learning window of 68 quarters from 1995Q1 to 2011Q4 after which all models were to produce forecasts until 2017Q4. Despite varying histories with ZLB environments, both datasets, and all models, were given the same learning and forecasting windows to ensure time for the models to observe periods of zero policy rates and to minimise the effects of any underlying historical differences affecting the forecast results. As previously mentioned, the choice to end the forecasting periods before 2018 was made as the U.S. rates leave the ZLB after 2015.

The study used four different vector autoregressive models with a lag of four quarters, $VAR(4)$, having only the difference of either using shadow rate values, interbank rate values, or a combination of the two with all the other variables previously listed (table 1) acting as control variables. The first model, named 3MR1, utilised the 3-month interbank rate and the spread between 10-year government treasury yields and the 3MR, T10Y3MR. The second model, SSR1, was given the substitute SSR estimates along with the same spread, T10Y3MR. The final two, named 3MR2 and SSR2, used the SSR spread counterpart, T10YSSR along with either the 3MR or SSR estimates, similar to the first two models. The appropriate number of lags was chosen using the combined suggestion of the Akaike Information Criterion, AIC, Schwarz Bayesian Criterion, BIC, and the Hannan-Quinn Information Criterion, HQIC. All the $VAR(4)$ models run were tested to satisfy the stability condition using the eigenvalue stability condition test and to be free of statistically significant autocorrelation using the Lagrange Multiplier Test. The detailed description of the VAR model used lies within equations 14 and 15 below.

$VAR(p)$:

$$\boldsymbol{Y_t} = \boldsymbol{C} + \sum_{\boldsymbol{i=1}}^{\boldsymbol{p}} \boldsymbol{A_i Y_{t-i}} + \boldsymbol{u_t} \tag{14}$$

$$\begin{bmatrix} \Delta SSR_t/\Delta PIR_t \\ Spread_t \\ Var3_t \\ \vdots \\ Var_{nt} \end{bmatrix} = \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ \vdots \\ c_n \end{bmatrix} + \sum_{i=1}^{t} \begin{bmatrix} a_{11,i} & a_{12,i} & \cdots & a_{1n,i} \\ a_{21,i} & a_{22,i} & \cdots & a_{2n,i} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1,i} & a_{n2,i} & \cdots & a_{nn,i} \end{bmatrix} \begin{bmatrix} \Delta SSR_{t-i}/\Delta PIR_{t-i} \\ Spread_{t-i} \\ Var3_{t-i} \\ \vdots \\ VAR_{n-(t-i)} \end{bmatrix} + u_t \ (15)$$

The first equation represents the generalised structure of the VAR model, where $\boldsymbol{C}$ is an $n \times 1$ vector of intercept terms, $\boldsymbol{A_i}$ are $n \times 1$ coefficient matrices corresponding to the lag $\boldsymbol{i}$**,** $\boldsymbol{Y_t}$ is an $n \times 1$ vector of the endogenous variables used, and $\boldsymbol{u_t}$ is a vector of error terms. The following equation further shows the structure of the model, where the first variable is either SSR estimates or 3-month interbank rates, depending on the model and the following variable, $Spread_t$, represents the two different 10-year spreads used between models. The rest of the matrix represents all the control variables used within all the models, listed within table 1. As presented above, the use of $VAR(p)$ models enable the use of a matrix of regressions, capturing the dynamic response of each variable to its own historic values and to those of all of the other variables at once. This study used the $VAR(4)$ to capture said relationships and to then create a dynamic forecast for the given forecasting window and then plotted graphs to see the forecasted values for the growth rate of real GDP.

In addition to the $VAR(4)$ model, separate linear regressions explaining the growth rate of real GDP were created using shadow short rate estimates, interbank rates, and the control variables specified in table 1. These linear regressions were then implemented into a direct h-step forecasting model to create direct forecasts, $DFR(h)$. The h-steps used were a 1-step forecast, 4-step forecast, and an 8-step forecast, effectively covering two years into the future, at most. The varying h-steps were chosen to represent typical short-term forecasting horizons that much of monetary policy aims to impact. The direct forecasting models were built as shown in equation 16 below.

$DFR(h)$:

$$\Delta \ln GDP_{t+h|t} = \beta_0 + \sum_{l=1}^{4} \beta_{1l} Z_{t-l} + \sum_{l=1}^{4} \boldsymbol{\beta_{2l} X_{t-l}} + \varepsilon_{t+h} \tag{16}$$

$$where,$$

$$\Delta \ln GDP_{t+h|t} = \ln GDP_{t+h} - \ln GDP_t \,, \mathrm{h} \in \{1, 4, 8\}$$

$$Z_{t-l} \in \{\Delta SSR_{t-l}, \Delta PIR_{t-l}\}, \boldsymbol{X}_{t-l} \in \{Var(1)_{t-l} \ldots Var(n)_{t-l}\}$$

The $DFR(h)$ models consisted of an ordinary least squares regression, OLS, where the $\Delta \ln GDP_{t+h|t}$ was the dependant variable for each model and the only variation within the independent variables was within Z_t which was either the SSR estimation or the 3-month interbank rate, 3MR. $\boldsymbol{X}_t$ represents the vector of control variables. Varying from the dynamic forecasts of the $VAR(4)$ where each step into the future was iterative, the $DFR(h)$ models were programmed to create direct forecasts after the given 68 quarters long training period and to continue making direct forecasts while including new periodic data gained after each h-step into the future. The possible benefit of direct forecasts, and the reason for the comparison, was that in case of large initial errors in forecasting values, the $DFR(h)$ models would not continue to build on said errors but instead had the potential of correcting them as new information was gained, and new direct forecasts were made.

To benchmark the forecasting ability of the previous models, a univariate autoregressive model, $AR(p)$, for $\Delta \ln GDP_t$ was also established with lag length of $AR(1)$, given by the Schwarz Bayesian Criterion, BIC. The benchmark was created to assess whether the models built, in total, provide additional forecasting power over the sole values of the real GDP itself. The $AR(1)$ was defined as shown within equation 17 below.

$AR(p)$:

$$\Delta \ln GDP_t = \alpha_0 + \sum_{i=1}^{p} \alpha_i \Delta \ln GDP_{t-1} + \varepsilon_t \tag{17}$$

The presented $AR(1)$ was then used to create dynamic one-step forecasts, mirroring the forecasting structure of the $VAR(4)$. This approach was chosen to enable comparisons to a minimalist baseline which each of the previous forecasting models ought to outperform within each scenario.

4.3 Performance metrics

A combination of a visual inspection of time series graphs and comparison of error terms was performed to examine the comparative performance of the forecasting models presented within the previous subsection of the paper. The following “Results” section includes a review of the visualised forecasts and tables created to present the forecasting errors. The forecasting errors measured are presented within the equations below and as follows, mean error, ME (equation 18), mean absolute error, MAE (equation 19), root mean squared error, RMSE (equation 20), and Theil's U, U (equation 21).

$$ME = \frac{1}{T}\sum_{t=1}^{T}(f_t - y_t) \tag{18}$$

$$MAE = \frac{1}{T}\sum_{t=1}^{T}|f_t - y_t| \tag{19}$$

$$RMSE = \sqrt{\frac{1}{T}\sum_{t=1}^{T}(f_t - y_t)^2} \tag{20}$$

$$U = \frac{RMSE_{model}}{RMSE_{naive}}, where\ RMSE_{naive} = \sqrt{\frac{1}{T}\sum_{t=1}^{T}(y_t - y_{t-1})^2} \tag{21}$$

$where,$

$f_t = forecasted\ value$

$y_t = actual\ value$

$T = number\ of\ periods\ in\ forecasting\ window$

The error metrics were implemented to aid in the comparison of forecasting performance between models as a visual inspection gives only approximate visual clues. ME, captures the average bias within forecasts, indicating whether the model systematically overpredicts, or vice versa. MAE reflects the average magnitude of deviation from actual values, whereas RMSE places greater weight on larger errors, penalising notable forecasting errors. Finally, Theil's U, implemented for the $VAR(4)$ and $DFR(h)$, is a metric used to determine whether the created forecasts provide superior results over random guessing. Theil's U has a threshold value of 1, with lower values indicating better performance and those higher suggesting guessing to be a better option.

5 Results

This section presents the reader with the empirical findings obtained using the methods detailed within the previous section. The results are structured into two parts of which the first contains an evaluation and a comparison between the forecasting accuracy of the different models and the two economies using the described error metrics. Following said comparison, a visual inspection of time series graphs between the different forecasts and actual values of real GDP is conducted. The comparisons are done separately for each country, followed by a summary of all notable findings within the results. Together, these results aim to assess whether shadow rate models offer meaningful improvements over conventional interest rates under zero lower bound conditions. To increase readability, the visual analysis includes the overall best-performing forecasts – the excluded $AR(1)$ benchmarks and the $DFR(4)$ forecasts can be found within appendices 1 and 2.

5.1 Evaluation of error metrics

Within table 2 below, one will find the previously presented performance metrics of all forecasting models for the U.S. dataset. Results for the $VAR(4)$ forecasts show only minor variation between the four different models used, although both shadow rate models do show the smallest forecasting error throughout with the only exception of mean error. The combination of shadow rate estimates and the spread between the 10-year government bond yield and the shadow rate, model SSR2, has a minor advantage over the other models with the metrics MAE, RMSE, and Theil's U. All models perform well relative to a "random walk" with values between 0.157 and 0.181 indicating a successful choice of control variables, also outperforming the $AR(1)$ benchmark in terms of MAE and RMSE. Between the $DFR(h)$ models, both the 3MR and SSR 1-step models manage to produce the smallest forecasting errors in terms of MAE, RMSE, and ME, which is an expected result, considering the shortest forecasting horizon. It is to be noted, however, that in terms of performance over a random walk, Theil's U, for both the 3MR and SSR models, the 8-step forecasts notably outperform the other models. All $DFR(h)$ models have higher MAE and RMSE values than the $AR(1)$ benchmark, though the ME of -0,0001 of the $AR(1)$ forecast suggests that the higher errors of the $DFR(h)$ forecasts might not signify a significant underperformance but instead misleading error figures of the $AR(1)$ benchmark due to highly stationary forecasts. If one turns their attention to appendix 1 showing the time series graph of the $AR(1)$ benchmark, it becomes indeed evident that the said benchmark forecast is unable to produce any meaningful information, as the forecasted growth of real GDP very quickly becomes stationary, only producing a stationary mean value.

Table 2. Forecasting performance, USA.

Forecasting performance - USA

VAR(4)	3MR1	SSR1	3MR2	SSR2
MAE	0.132	0.125	0.143	0.119
RMSE	0.318	0.303	0.333	0.288
Mean error	0.039	0.032	0.026	0.031
Theil's U	0.173	0.165	0.181	0.157

DFR	3MR h1	3MR h4	3MR h8
MAE	0.553	1.239	1.179
RMSE	1.127	1.767	1.670
Mean Error	-0.041	-0.111	-0.436
Theil's U	0.631	0.532	0.313

DFR	SSR h1	SSR h4	SSR h8
MAE	0.523	1.227	1.245
RMSE	1.126	1.782	1.778
Mean Error	-0.035	-0.154	-0.657
Theil's U	0.631	0.536	0.334

AR	AR(1)
MAE	0.304
RMSE	0.376
Mean error	-0.0001

The following table (table 3) provides the same results for the Japanese dataset. The results are quite similar in terms of relative performance, though most models produce somewhat higher error terms than the ones produced for the U.S. data. Especially for the 3MR models this was to be expected as the time series used shows much longer periods of ZLB for the training data, however, against expectations, the SSR forecasts did not prove to be superior. As with the U.S. results, the $VAR(4)$ models show only marginal variation between the performance metrics used, however, as a contrast to the U.S. results, the 3MR1 model produces the smallest forecasting errors in terms of MAE and

RMSE along with the best Theil's U, with only the exception of SSR2 showing the smallest mean error. The results for the $DFR(h)$ models show an interesting contrast to the U.S. results as the Theil's U values show even smaller variation between the different forecasting horizons. As expected, MAE, RMSE, and ME show once again increasing errors as the forecasting horizon grows in length. Once again, one of the SSR estimate models, SSR h1, produces the best Theil's U, though marginally. Similar to the U.S. results, the $AR(1)$ benchmark outperforms the $DFR(h)$ models in terms of forecasting errors though, once again, turning over to the visual representation of the benchmark forecast within appendix 1 it is evident that the said benchmark is unable to produce nothing but a stationary mean value following the initial forecasting periods.

Table 3. Forecasting performance, Japan.

Forecasting performance - Japan

VAR(4)	3MR1	SSR1	3MR2	SSR2
MAE	0.145	0.154	0.176	0.164
RMSE	0.338	0.396	0.409	0.385
Mean error	0.066	0.075	0.024	0.009
Theil's U	0.171	0.201	0.207	0.195

DFR	3MR h1	3MR h4	3MR h8
MAE	0.657	1.143	1.630
RMSE	1.111	1.603	2.173
Mean Error	-0.194	-0.287	0.555
Theil's U	0.679	0.647	0.694

DFR	SSR h1	SSR h4	SSR h8
MAE	0.677	1.231	1.493
RMSE	0.999	1.591	1.988
Mean Error	-0.092	-0.187	0.462
Theil's U	0.611	0.642	0.634

	AR(1)
MAE	0.531
RMSE	0.722
Mean error	0.125

Overall, the dynamic one-step $VAR(4)$ models outperform the other forecasts for both economies in terms of forecasting errors, showing especially good performance in comparison to a “random walk” – however, with only marginal differences between the different SSR and 3MR variables. It is to be also noted that even though the $DFR(h)$ models produce more notable forecasting errors than the $AR(1)$ benchmark, said benchmarks only produce stationary mean values almost throughout the forecasting window. Despite a minor advantage in forecasting errors in some respects, especially within U.S. data, so far, the SSR estimates have not proven to provide significantly better forecasting performance during periods of zero interest rates. Said result is unexpected especially within the Japanese forecasts as the 3MR values hover close to the ZLB for most of the dataset. Within the next section, a visual analysis of the forecasts is conducted to examine whether there are notable differences in results outside of the performance metrics used. The forecasts to be visually analysed are all of the $VAR(4)$ results, along with $DFR(1)$ as the closest comparison to the dynamic $VAR(4)$ forecasts along with $DFR(8)$ due to the advantage amongst the Theil's U values within U.S. forecasts. One may find time series graphs of the remaining $DFR(4)$ forecasts within appendix 2 along with the already discussed visual representations of the $AR(1)$ benchmarks (appendix 1).

5.2 Visual analysis

Figure 13 below shows the time series graphs for all four $VAR(4)$ forecasts for the USA, graphs above the 3MR1 and SSR1 containing said rates and the spread between the 10-year government bond yield and the 3MR and the latter showing the 3MR2 and SSR2 also utilising said rates and the spread between the 10-year government bond yield and the SSR estimates. As an initial note, all of the $VAR(4)$ forecasts seem to perform relatively well, giving indication of a successful choice of control variables, with the most prominent exception being the failure of forecasting the notable dip in 2014Q1 amongst all forecasts. While all forecasts show only quite minor variations, models 3MR1 and SSR1 utilising the T10Y3MR seem to group tighter together throughout than their T10YSSR counterparts, 3MR2 and SSR2. Said result is an interesting one, considering the SSR2 marginally outperformed the other forecasts overall. Upon visual inspection, the SSR2 also seems to provide the best overall results, despite some overcorrections in relation to 3MR2. Despite marginally smaller forecasting errors, between the two graphs, both instances would suggest the SSR models to have slightly higher variation with both positive and negative over corrections in relation to the 3MR counterparts. Overall, the differences seem as negligible as the previous performance metrics suggested with all of the models performing well in comparison to a random walk and with only minor variations.

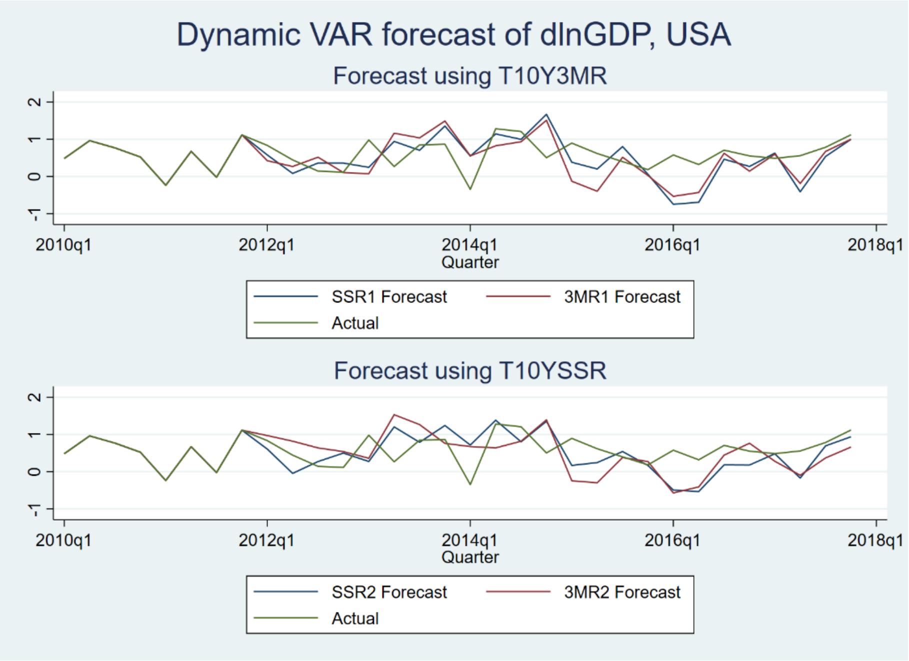

Figure 13. Dynamic VAR forecast of dlnGDP, USA.

Comparing the graphs of the $VAR(4)$ forecasts (figure 13) with the visuals of $DFR(1)$ and $DFR(8)$ below (figure 14), one may note some intriguing differences. Despite higher forecasting errors, the direct one-step forecasts seem to react better to larger variations within the actual growth rate of real GDP. Especially prominent are the changes between 2014Q1 and the two following quarters where the SSR forecast seems to be especially reactive to the changes. The smoother forecasts of the $VAR(4)$ models is somewhat to be excepted the forecasts in question are iterative and use their own previous forecasting values for the following quarters whereas the $DFR(h)$ forecasts utilise a recursive data pool, expanding after each forecasting step. Greater ability to react does seem to also bring greater variance as becomes evident both from the time series graph for $DFR(1)$ as well as the forecasting errors previously presented. Same does not seem to hold true for $DFR(8)$, however as the two-year long forecasting horizon seems to notably smoothen out the forecasts, though same holds true for the actual values. The far better Theil's U of the U.S.' eight-step forecast is also visible within the visual presentation as both the SSR and 3MR forecasts seem to follow the actual values quite well. So far, said instance is the first where the performance of the SSR model is distinctly better than its 3MR counterpart.

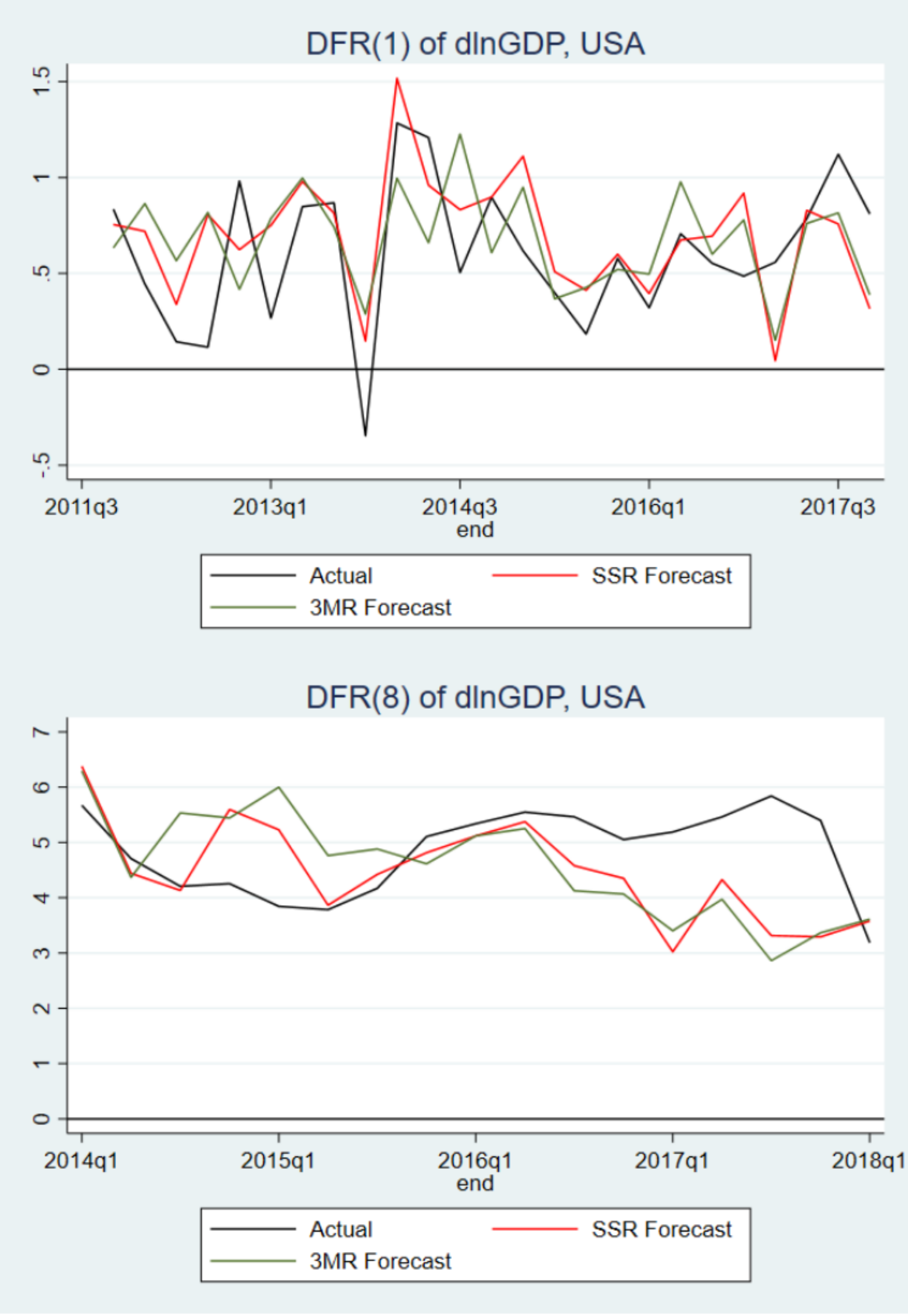

Figure 14. DFR(1) and DFR(8) of dlnGDP, USA.

Turning over to the Japanese $VAR(4)$ forecasts (figure 15) below, one may find results quite similar to those of the U.S. models. All of the $VAR(4)$ forecasts seem to perform well and, despite higher forecasting errors than their U.S. counterparts, the visual inspection would suggest somewhat better forecasting ability regarding larger changes within the actual growth rate of real GDP. Opposite to the U.S. results, models SSR2 and 3MR2, utilising the T10YSSR spread seem to be grouped closer together than the forecasts SSR1 and 3MR1 which utilise the spread between 10-year government bond yields and the 3-month interbank rate. Also, on the contrary, as noted within the forecasting errors comparison before, the 3MR1 forecast seems to have produced marginally better forecasts than the other $VAR(4)$ models and once turning to the graphic representation, the difference seems to be primarily due to a slightly smoother graph, indicating a smaller variance. The results are in line with the U.S. forecasts and the performance metrics as the different models seem to produce only slightly different results between each other. Said result is, once again, especially the results for Japan are against expectations considering the long periods policy and interbank rates have been stuck at the ZLB.

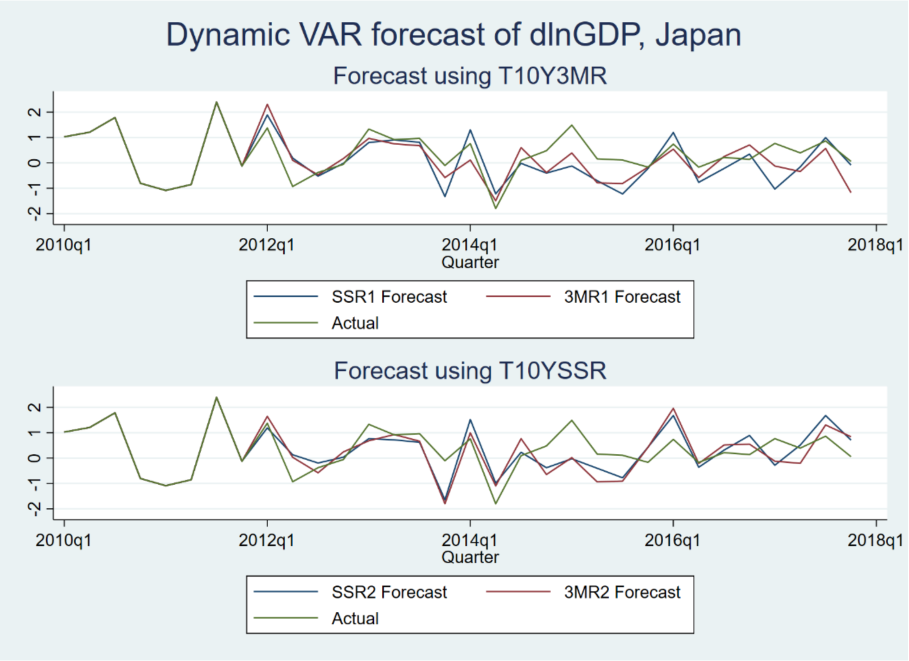

Figure 15. Dynamic VAR forecast of dlnGDP, Japan.

Moving to the $DFR(h)$ forecasts for Japan (figure 16), comparing the dynamic $VAR(4)$ forecasts to $DFR(1)$, the visual representation seems to complement the forecast errors as the dynamic forecasts seem to produce better forecasts than the direct one-step model. Regarding $DFR(1)$, a particularly prominent shortcoming is the inability to predict the periods of larger positive GDP growth between 2014Q3 and 2016Q1. Both the SSR and 3MR forecasts seem to have the ability to predict larger negative movements, however, with the values seemingly having a negative bias – also represented within the mean errors of the models. Performance between the SSR and 3MR forecasts seems also otherwise similar with only very slight changes, also shown within the only marginally different performance metrics. While the Japanese $DFR(8)$ forecasts do not share the U.S.' result of relatively better Theil's U values comparing to the other direct forecasts, the Japanese forecast does share a similar visual appearance with a notably smoother graph, although in Japan's case the SSR forecast does not have a clear advantage over the 3MR model.

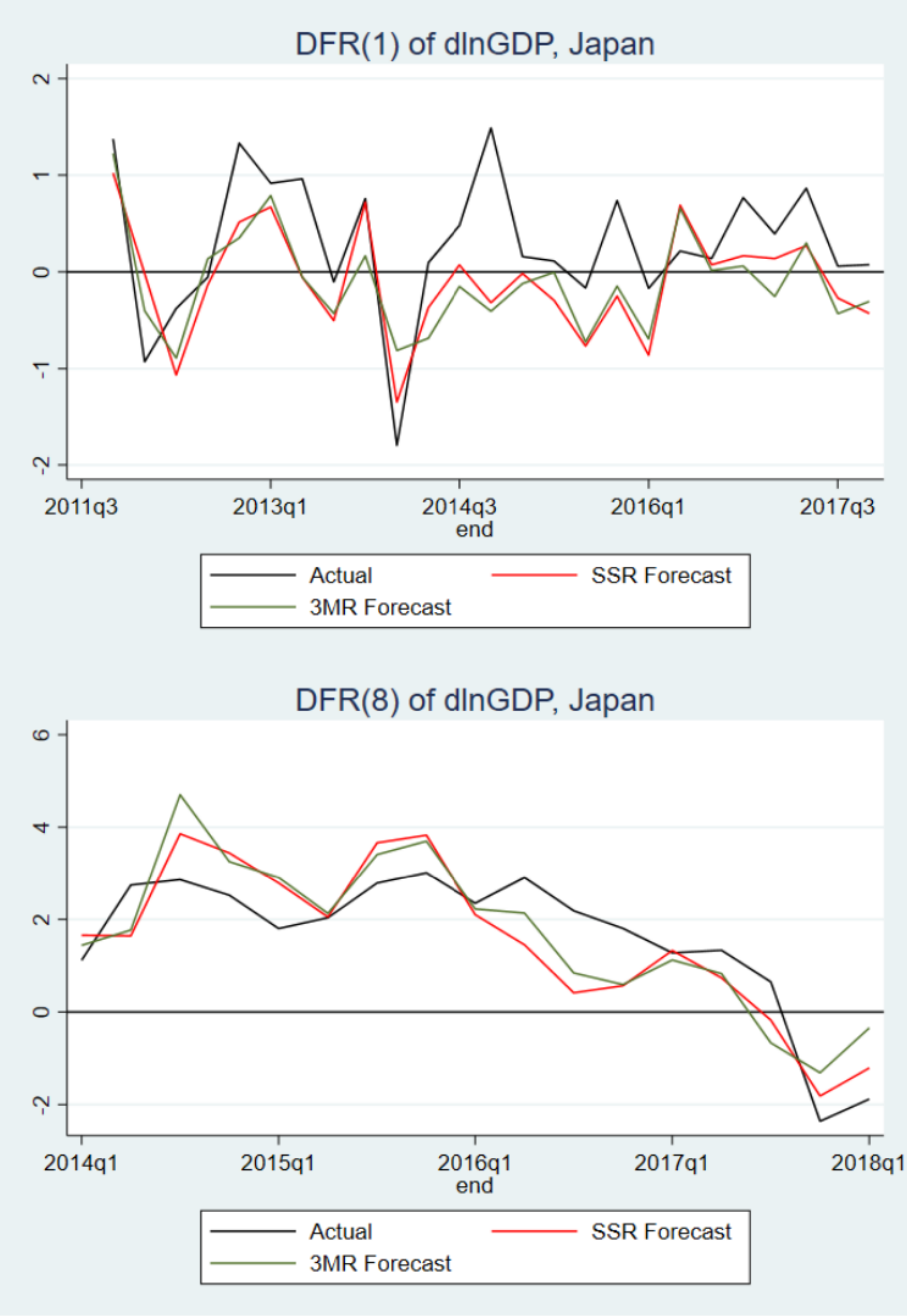

Figure 16. DFR(1) and DFR(8) of dlnGDP, Japan.

Overall, the $VAR(4)$ and $DFR(h)$ forecasting models performed well during the given forecasting period, especially against a random walk. In terms of forecasting errors, the best results were achieved via the $VAR(4)$ models for both economies, although the visual inspection proved the $DFR(h)$ models to also have some redeeming characteristics, such as better responses to larger changes in actual values for $DFR(1)$. Utilising SSR estimates instead of short interbank rates did not prove to give consistently superior or, in some instances, even marginally better results in forecasting real GDP growth rates despite placing the forecasting period in the middle of zero-interest rate environments. An especially surprising discovery is the lack of improvement in forecasting results for Japan, considering the country's extensive history with the zero lower bound. The only notable advantage in terms of forecasting with SSR estimates over the 3MR was with $DFR(8)$ in within the U.S. forecasts.

While said results do not show a significant benefit in utilising shadow rate estimates for real GDP growth forecasts in a zero-interest rate environment as per existing findings such as ones by Hännikäinen (2017) and Kuosmanen & Vataja (2024, p. 9), some existing studies do suggest potential benefits in using the shadow rate for monetary decision-making. Despite similar results regarding real activity for the USA, Hännikäinen (2017, pp. 5–9) did find SSR estimates to produce significant predictive power over inflation at the ZLB with out-of-sample forecasts. Furthermore, Carriero et al. (2021, pp. 26–27) found vector autoregressive models utilising the shadow rate to produce superior results for interest rate forecasts for the USA during the ZLB period post-2009. For real activity, the results of this study and are in line with other previous findings as well, as mentioned by Stock & Watson (2003, p. 9) who state that other previous studies have also found that interest rates have shown poor predictive content overall.

6 Conclusions

The Great Recession of 2008 shook the foundations of many economies, being called the once-in-a-century credit crisis. With banks collapsing in the U.S., the Federal Reserve System had to rapidly adjust and implement immediate expansionary monetary policies. By the end of 2008, the federal funds rate had been dropped to the zero lower bound and with the economy still spiralling the Fed was forced to implement additional monetary easing. Whereas the Great Recession was the start of unconventional monetary policy for many economies, Japan practiced similar procedures already during the previous decade. Even with the newly adopted unconventional monetary policies, economic recovery was slow, and policy rates remained stuck at the zero lower bound for several years. One of the most prominent tools of the central banks, and an indicator of monetary policy stances, the policy interest rate, was tied to zero and no longer carried accurate information on the amounts of monetary easing.

The conducted literature review presented a proposition to measuring the effects of monetary policy in a zero lower bound environment, the shadow rate. The review showed that the produced shadow rate estimations provided consistent and robust results conveying information about the easing effects of unconventional monetary policy hidden by policy rates at the zero lower bound, essentially indicating the shadow rate to be an effective tool for historical analyses. Furthermore, the application of the shadow rate to the Taylor rule indicated potential for the shadow rate to be used as a forward-looking tool within monetary decision-making.

Expanding on existing research, the empirical study performed within this paper examined the predictive power of shadow short rate estimates over the growth rate of real GDP during periods at the zero lower bound against short-term interbank rates. Both the vector autoregressive dynamic forecasts and direct forecasts performed well, all being superior to the autoregressive benchmark and against a random walk. Out of the models used, the dynamic forecasts created using the vector autoregressive model produced the best results within each economy, Japan and the USA. Despite the theoretical potential of the shadow rate, the use of shadow short rate estimates over the short-term interbank rates did not exhibit consistent or notable improvements within forecasting results. While the U.S. $VAR(4)$ and the Japanese $DFR(h)$ models did benefit from the shadow rate, the improvements were marginal and were not present across all forecasts. The results for the Japanese forecasts proved especially surprising, as the use of the shadow rate showed only minor improvement in the performance metrics despite the extensive periods of short interest rates hovering near or stuck at the zero lower bound.

The results are consistent with existing studies also showing the shadow rate to hold only minimal additional predictive power whilst forecasting real GDP or national output. This does not completely invalidate the shadow rate as a forecasting tool, however, as recent studies have shown the estimates to produce excellent results in forecasting interest rates and inflation. Overall, the shadow rate’s ability to visualise the effects of easing monetary policy in a zero-interest-rate-environment, combined with the predictive power outside of real activity prove the concept to be a valuable asset for historical retrospect and a notable tool for future forecasts.

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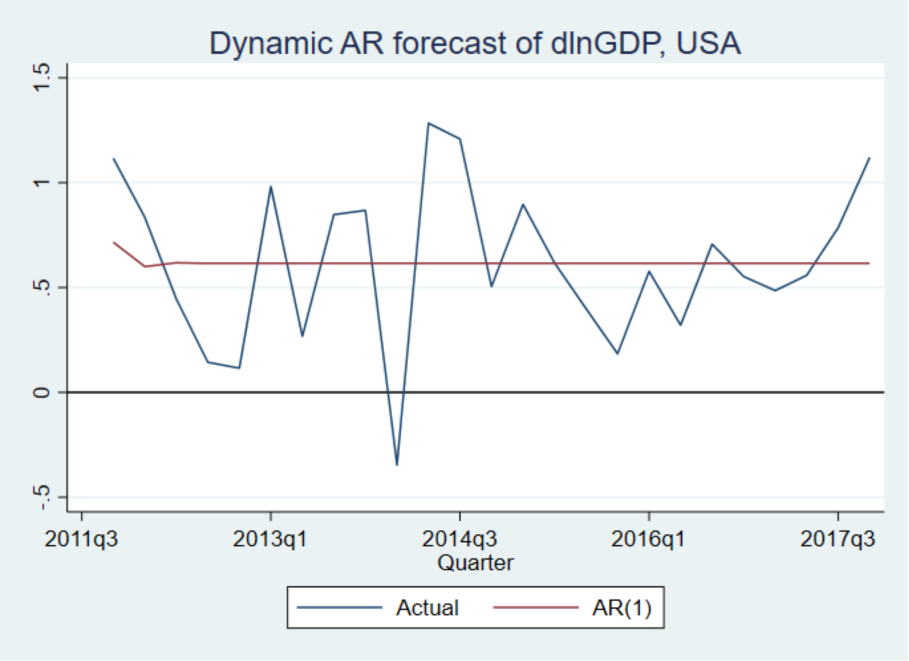
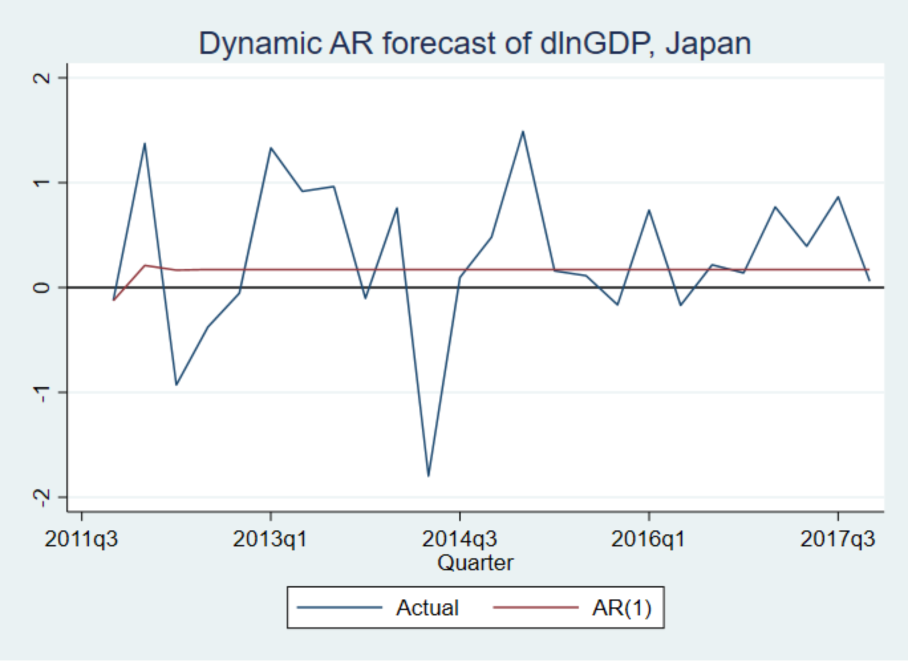
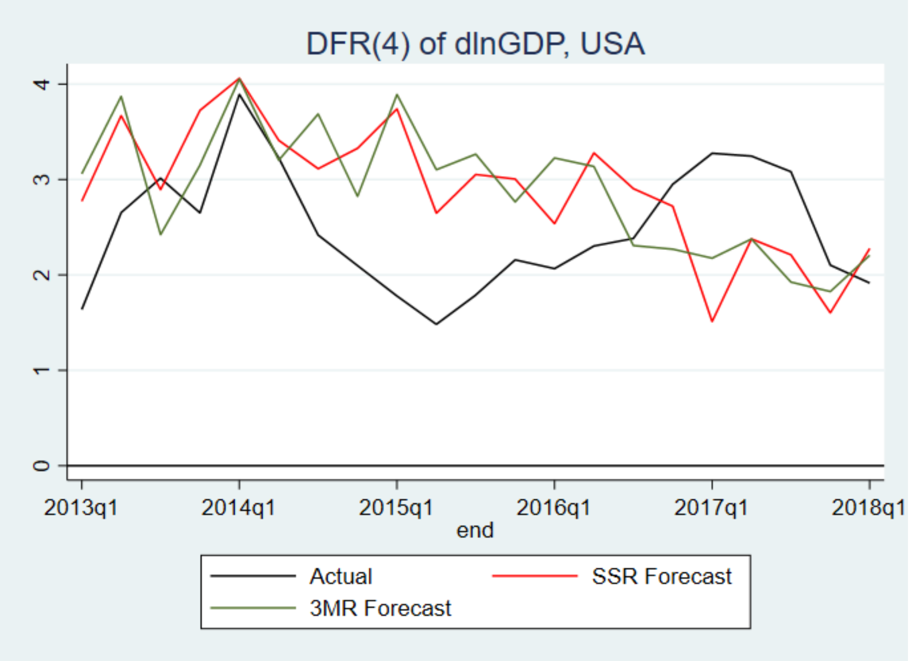
Appendices

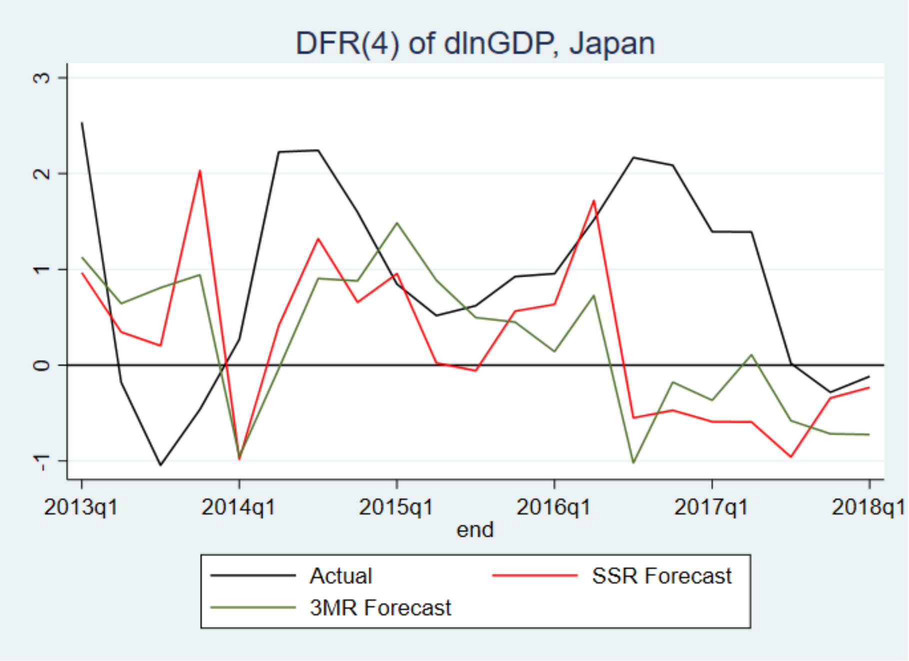
Appendix 1. Dynamic AR forecasts of dlnGDP, USA and Japan





Appendix 2. DFR(4) forecasts of dlnGDP, USA and Japan





Appendix 3. Complete reference list of variables used

Variable name	USA	Source
3-month interbank rate**		The Federal Reserve Bank of St. Louis (FRED)
Shadow short rate **	SSR estimate (Krippner)	ljkmfa.com
Real GDP**	$Bn ch. se.adj.	The Federal Reserve Bank of St. Louis (FRED)
CPI growth rate**	All items: total	The Federal Reserve Bank of St. Louis (FRED)
Industrial production index**	Total industrial production	The Federal Reserve Bank of St. Louis (FRED)
Unemployment rate**		The Federal Reserve Bank of St. Louis (FRED)
Output gap**		The Federal Reserve Bank of St. Louis (FRED)
Stock index**	S&P 500	Investing.com
T10Y3MR**	T10Y – 3MR	The Federal Reserve Bank of St. Louis (FRED)
T10YSSR*	T10Y – SSR	

Variable name	Japan	Source
3-month interbank rate**		The Federal Reserve Bank of St. Louis (FRED)
Shadow short rate **	SSR estimate (Krippner)	ljkmfa.com
Real GDP**	¥Bn ch. se.adj.	The Federal Reserve Bank of St. Louis (FRED)
CPI growth rate**	All items: total	The Federal Reserve Bank of St. Louis (FRED)
Industrial production index**	Total manufacturing	The Federal Reserve Bank of St. Louis (FRED)
Unemployment rate**		The Federal Reserve Bank of St. Louis (FRED)
Output gap**		Bank of Japan
Stock index**	Nikkei 225	Investing.com
T10Y3MR**	T10Y – 3MR	The Federal Reserve Bank of St. Louis (FRED)
T10YSSR*	T10Y – SSR	