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STRATEGY MODELLING FOR AN ISSUER OF STRUCTURED BONDS

Fair Value, Marketing and Hedging

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ABSTRACT:
This study aims to contribute to the existing literature on structured products by exploring the engineering and issue process, providing a comprehensive literature review and a new classification approach, investigating various pricing methodologies, and providing new pricing methods. This study provides a new classification approach to structured products incorporating the existing approaches and contributing by adding new aspects such as the features or special conditions within the structured products. The research incorporates various pricing approaches that were either implemented alone or in a combination with other methods, including Black-Scholes model, Lattice models, Monte Carlo simulation, finite differences methods, bootstrapping and time-series models. Empirical results reveal that there is a significant difference between the resulting estimated intrinsic value of a structured product, moreover some of the methods could not be implemented in any way. Additionally, the study shows that certain types of publicly available structured products listed are overpriced by the market. Finally, the study provides an experiment carried out by implementing one of the incorporated approaches and models the issue of a Derivative Warrant.

KEYWORDS: Structured Product, Bonds, Modelling, Bond Issue, Bond's Fair Value, Hedging, Bond's Hedging, Issuer's Strategy

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ABBREVIATIONS

ECB	European Central Bank
EGARCH	Exponential Generalized AutoRegressive Conditional Heteroskedasticity
ESG	Environmental Social and Governance
ETF	Exchange-Traded Fund
EU	European Union
EUSIPA	European Structured Investment Products Association
FD	Finite Difference
MC	Monte Carlo
GARCH	Generalized AutoRegressive Conditional Heteroskedasticity
HKEX	Hong Kong Exchange
PDE	Partial Differential Equation
PV	Present Value
PIV	Pooled Investment Vehicle
SSPA	Swiss Structured Products Association
US	United States

1 INTRODUCTION

Over the past few decades, the financial landscape has undergone a significant transformation, particularly in the development and proliferation of financial products. Among these innovative instruments, structured securities have garnered substantial interest from both retail and institutional investors (Han & Liu, 2022; Hanaki, 2022). These customizable financial solutions enable investors to adapt their risk-return profiles, allowing for the optimization of investment strategies and the possible achievement of their desired financial objectives.

Structured products, as noted by Hanaki (2022), first appeared in 1990's. Since their inception, the market for structured securities has expanded considerably (Kallio et al. (2022), Baule & Muenchhalfen (2021));. European issuers alone have facilitated the sale of structured products valued at €3 trillion since the beginning of the 21st century (Célérier & Vallée, 2017). The growing demand for these securities can be attributed to several factors, such as the ability to access complex derivatives, simplified product creation for issuers, ease of investment for investors, versatility in hedging specific risks, and the potential to realize superior returns in comparison to traditional bonds while maintaining a lower risk profile than equities (Abdessamad Khaled, 2019).

While they may be indeed attractive, structured products are often perceived as complex, even for experienced financial professionals (Baker & Puttonen, 2017). This complexity arises from the numerous product types, underlying assets, and distinct features that characterize these financial instruments. Additionally, such intricacy may afford issuers the opportunity to capitalize on the cognitive biases of retail investors (Baule & Muenchhalfen, 2021). The pricing and subsequent hedging of structured products also pose challenges for issuers, necessitating the development of robust and reliable methodologies (Ascheberg et al. 2013).

Various approaches for pricing structured products have been explored previously in author's personal research. For example, EGARCH, GARCH, Cornish Fisher models were

used for evaluation of the fair value of those types of securities. EGARCH model proved to be the best at pricing of the predefined subset of structured securities resulting in a lower cost for an issuer. This study aims to provide a concise review of the relevant literature on pricing methodologies, offering valuable insights into the most effective techniques for determining the fair value of these instruments, as well as take a deep dive into the assessing of intrinsic value of more than 10000 products.

The primary motivation for this paper is not only to apply existing pricing models to a specific structured product but also to provide a design of structured product that incorporates diverse conditions. By addressing these objectives, this research will contribute to the ongoing discourse on structured products and offer valuable insights for both issuers and investors navigating in this dynamic market.

1.1 Purpose of the study

The primary purpose of this study is to contribute to the existing literature on structured products by addressing the following objectives.

Firstly, this study provides a comprehensive view and develops understanding of the factors influencing the demand for structured products among retail and institutional investors. This includes examining the role of risk management, return potential, access to complex derivatives, and simplicity of investment in driving the popularity of structured securities.

Secondly, this paper investigates the process of creating a structured product, including the selection of underlying assets, specific features, and custom conditions. This will enable a deeper understanding of the challenges and opportunities faced by issuers in designing tailored financial solutions for investors.

Thirdly, it critically analyses various pricing methodologies for structured products as discussed in the existing literature. This analysis will provide a foundation for exploring

the most suitable approach for pricing the various structured products that may exist in both developing and developed markets.

Fourthly, to price the specific structured product developed in this research using the chosen pricing methodology, and to assess the accuracy and reliability of the chosen method in determining the fair value of the product.

Next goal of this study is to explore the motivations and decision-making processes of investors when considering the purchase of structured products. As for the lack of access and the huge complexity of conducting interviews with the actual retail and institutional investors, this study will carry out a deep look into the existing literature of investor preferences, risk tolerance, and potential behavioral biases that may impact their investment decisions.

By addressing these objectives, this study seeks to provide valuable insights for both issuers and investors in the structured product market. Additionally, the findings of this research will contribute to the growing body of knowledge on the development, pricing, and demand for structured securities, paving the way for further research and innovation in this rapidly evolving field.

1.2 Research question and hypothesis

It is vital to highlight that this paper is novel, since the structured products are becoming more attractive among other financial instruments (H. Lee, Lee, et al. 2022), this field of studies is still developing and it can be said that the number studies on structured products is increasing year by year according to *Scopus* search results (see Figure 1) below.

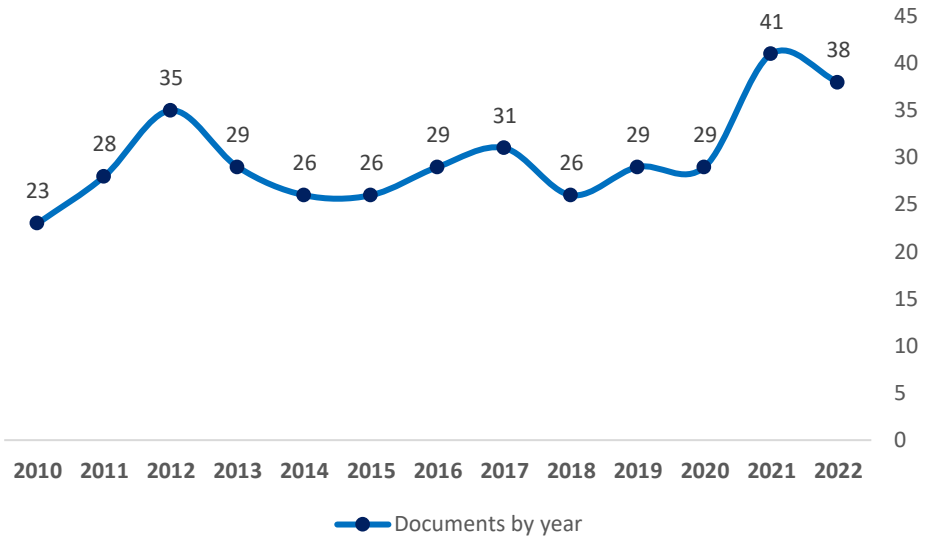

Figure 1 Number of documents published that mentions words: structured financial security, product, or bond together by year. (Scopus, 2023).

Investors technically can create and structured product for themselves, but when buying it either custom-made or publicly available product, an investor may spend much less time engineering a product, moreover a rational and highly experienced investor would need to do a lot of computations, when evaluating whether a product is fairly priced (Breuer & Perst, 2007).

That is one of the reasons why the first part of this study is devoted to different modelling approaches to evaluation of the fair value of structured products. On the other hand, in the present paper, the key stress is on the actual topic of issuers' engineering of the product, and how the product can be, from the author's perspective, relatively simply created. Thus, one of the key parts of this paper is devoted to the experiment of creating such structured product. And another goal of this paper is to implement already developed and tested previously approaches to modelling of fair-value of a structured securities to find out which approaches are the most suitable and whether some of existing structured products are fairly priced.

The first research question – what factors may drive the demand of structured products among retail and institutional investors are, it arises from studies by X. Chen et al. (2017), Kunz et al. (2017) where the retail investors' risk perceptions are observed, and a study by Baule & Muenchhalfen (2021) where product structure (cap) and costs are mentioned as the main drivers.

Second research question is also derived from the above-mentioned studies, and it expands the first question – what decision-making processes of investors are when considering the purchase of structured products.

While the first two questions will be discovered by a thorough literature review, the third research question is of personal interest of the author of this paper – to define how issuers can effectively design and create structured products with custom conditions that cater to investor preferences and needs is. Fourth research question coincides with the third one – what most suitable pricing methodologies for structured products are, and how these methodologies impact the pricing of a specific structured product.

Final research question and purpose of this paper – to provide a clear and up-to-the-minute classification of structured securities, the question arises from the Author's observations of the various classifications of structured products that can be found in e.g. Blümke (2009)*, EUSIPA – European Structured Investment Products Association*, T. Renz et al. (2014), since they may not be clear for a retail investor

The two sets of hypotheses were subsequently shaped as follows:

H_0 = The suitability of a pricing methodology for structured products depends on the specific product characteristics, underlying assets, and market conditions. The chosen pricing methodology will have a significant impact on determining the fair value of a structured product.

H_1 = The alternative to H_0, the suitability of a pricing methodology for structured products does not depend on the specific product characteristics, underlying assets, and market conditions. There is no significant difference between the pricing approaches

H_2 = HKEX's publicly available structured products consisting of over 10 000 warrants, callable contracts and inline warrants are on average overpriced.

H_3 = The alternative to H_2, HKEX's publicly available structured products consisting of over 10 000 warrants, callable contracts and inline warrants are on average priced correctly or underpriced.

These research questions and hypotheses will guide the study in exploring the development, pricing, and demand for structured products, ultimately contributing to the understanding of the structured product market.

1.3 Structure of the study

This chapter outlines the structure of this paper. The study starts with Chapter 1, where the topic, research questions, and hypotheses are introduced. In Chapter 2, the paper delves into the concept of structured products, providing an overview of their types, underlying assets, and features, as well as discussing the key motivations behind their increasing popularity among investors. This chapter serves as a conceptual framework for the study, presenting essential information regarding structured products and their intricacies.

Chapter 3 presents a literature review, summarizing relevant previous studies that provide a solid foundation for the conceptual and theoretical framework of the subsequent empirical analysis. In Chapter 4, the study explores the theoretical framework, examining the financial theories and methodologies used to price structured products, as well as the challenges associated with their valuation and hedging.

Chapter 5 details the data and methodology employed in this study, focusing on the design of the structured products, their payoffs structures, additional specifics on the pricing approaches, and specifics of an experimental structured product.

Following Chapter 5, the study continues to its empirical section in Chapter 6, where the results for various pricing models and the engineering of a structured product are presented. Chapter 7 discusses the empirical findings presented in the previous chapter, evaluating their implications for the understanding of different pricing methods' implemented in the study. Finally, the paper concludes with final words regarding this study and the conducted work.

2 STRUCTURED PRODUCTS

In this chapter, there will be an introduction to structured products, the description of these financial instruments, what are their key characteristics and what are they made of, providing a fundamental understanding of structured products. The subsequent subchapters will dive deep into the existing classifications of these securities, illustrating how they are usually perceived by the different financial and governmental institutions. For the purposes of this study, the author also provides a new classification approach to structured products.

2.1 Introduction to structured products

Structured products are innovative and sophisticated financial instruments designed to meet concrete investor needs that cannot be satisfied using traditional financial instruments such as stocks, bonds, or mutual funds (Fusai et al. 2022). These products are created by combining various financial assets or derivatives and customizing them to suit the preferences, goals, and risk profiles of investors.

In essence, structured products provide a customized investment tool that allows investors to gain exposure to a wide range of asset classes and market conditions. This flexibility makes them an attractive investment option for both retail and institutional investors, enabling them to achieve targeted returns, hedge against market risks, and enhance portfolio diversification (Yuan & Rieger, 2021).

The author of this study argues that there are several key components of structured products that contribute to their complexity and customization. Firstly, the underlying assets of these products can consist of almost any type of financial instrument or a combination of them. Common underlying assets include equities, bonds, commodities, indices, interest rates, currencies, and even other derivatives such as options or swaps.

Secondly, the payoffs of structured products are typically linked to the performance of these underlying assets, allowing investors to participate in the returns generated by various market scenarios. Depending on the product's design, payoffs can be linear, nonlinear, or even contingent upon specific events or conditions, such as the asset price reaching a certain threshold.

The third component is the product's structure, which can take various forms such as notes, certificates, or deposits. This structure determines the product's liquidity, marketability, and regulatory treatment, affecting the ease of trading and the level of investor protection.

Structured products can be further categorized based on their capital protection features. Capital protection refers to a certain protection level of the investor's initial capital, and structured products can offer varying levels of protection, such as full, partial, conditional, or no protection (Albeverio et al. 2018). Products with higher capital protection generally offer lower potential returns, while those with lower protection levels carry higher risks and potential returns (Climent Hernández & Cruz Matú, 2017).

The author of this study states that an essential aspect of structured products is their ability to incorporate specific features and conditions that cater to the unique requirements of individual investors. Based on the author’s observations, some of the features include leverage, which amplifies potential returns and risks; "memory effect," allowing for deferred payments if certain conditions are not met; and the use of exotic options, such as barrier options, cliquet options, or rainbow options, which introduce additional complexity and flexibility to the product's payoff structure (Neftci, 2008). These features and conditions are discussed in this paper further in this chapter.

Despite their many advantages, structured products also come with certain drawbacks. The complexity and customization of these products can result in limited transparency and increased difficulty in understanding and valuing them (Zongrun et al. 2020).

Additionally, they often have higher fees and commissions compared to traditional financial instruments, which may completely erase potential returns (R. Chen et al. 2020).

2.2 Existing classifications

Structured products can be not only difficult to evaluate and understand (Basu & Dulleck, 2020), but also to classify, which was one of the reasons to provide a thorough depiction of the various classifications and categorizations that govern the industry. In this sub-chapter, the author aims to provide a comprehensive overview of the major classification systems currently in use, thereby enabling readers to navigate easier within the landscape of structured products. By examining the existing classifications, this section serves as a crucial foundation for the subsequent novel classification schema that can be theoretically easier to understand for a retail investor.

The reason for the existence of so many different structured products is, from the author's perspective, simple – customization and an investor's interest, their goals, knowledge level, and biases (Arnold et al. 2021), all of the mentioned is connect with the fact that some institutional or retailer investor may desire to use a product for a specific strategy, in certain market, industry, or country (Baule & Muenchhalfen, 2021).

In the search of a rather complete classification, one may find a *EUSIPA – European Structured Investment Products Association* classification of structured product. Within this classification that can be used both by the issues and potential investors, the structured products are fragmented into 4 sections: capital protection, yield enhancement, participation, and leverage. Figure 1 demonstrates a part of the classification that can be found on the EUSIPA's website.

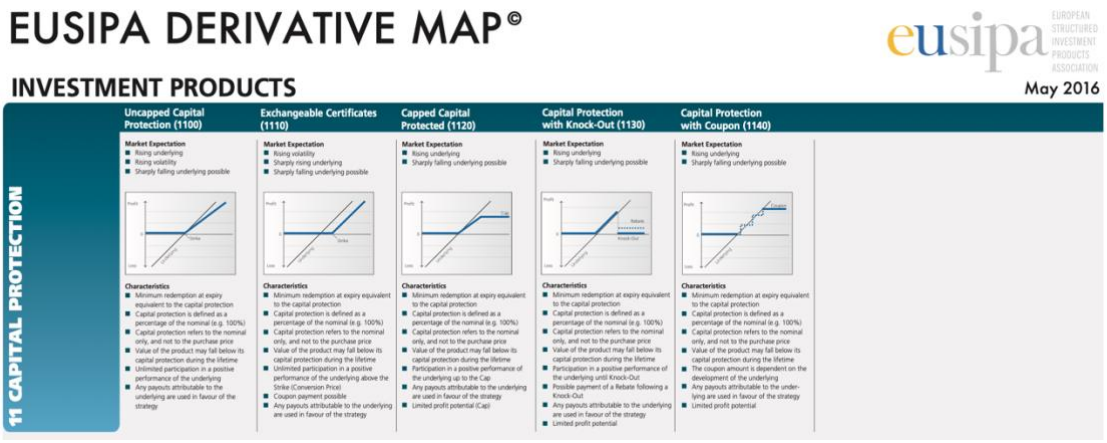

Figure 2 Part of USIPA DERIVATIVE MAP of structured investments products.

From the author's perspective, the classification provided by EUSIPA is fairly straightforward to use by both investors and issuers, as the classification gives an arguably clear description of structured product types as well as it provides the depiction of payoffs.

Another classification that is also reasonably comprehensive is by the *Swiss Structured Products Association*. As it can be seen in Figure 2 which is the top part of classification appears relatively the same.

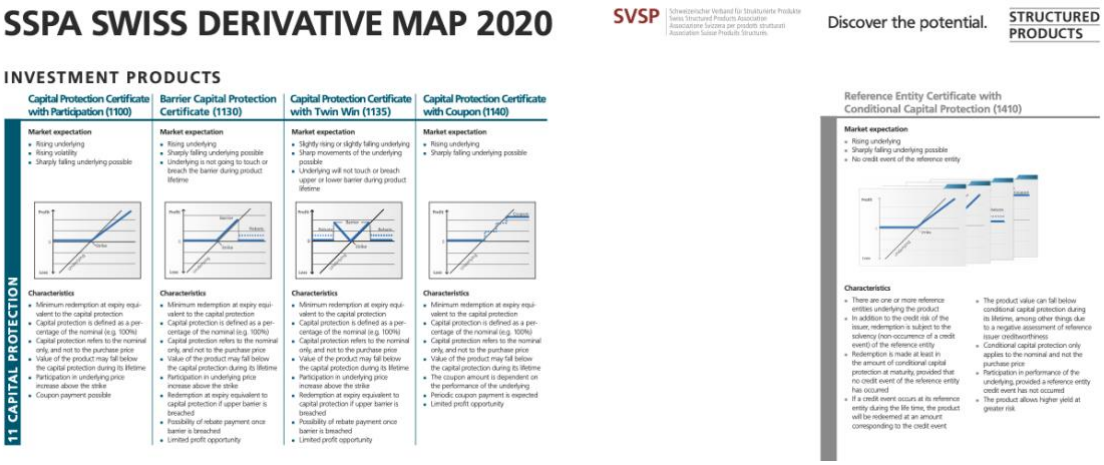

Figure 3 Part of SSPA DERIVATIVE MAP of structured investments products.

The author of this study assumes that the classification provided by SSPA has roughly the same level of detail provided regarding the structured products. The difference is that it provides two more categories: structured products with barrier conditions that will be covered later in this study and other products that practically includes everything else. SSPA is used specifically in Switzerland, while EUSIPA classification is used widely in Europe.

Another example of an implemented and widely used classification system is Hong Kong Structured Products Classification system provided by Hong Kong Exchange – *HKEX*. This classification system has no graphical representations of the possible payoffs, while two previous classifications provide such graphical representation, which can be beneficial for an investor (Anic & Wallmeier, 2020). Moreover, the classification has 9 major categories that are distinguishable by the underlying asset, and payoff structure.

The HKEX classification system is adequate as it serves the purpose of a standardized way of classification of structured products. However, lack of graphical representation of can be a significant drawback (Sheehan, 2017) in terms of attractiveness for potential investors as it may not be clear for them.

All three classifications have the following common characteristics:

- They are standardized and industry-accepted systems.
- They provide an arguably comprehensive outlook on a number of structured products.
- They may give a basic understanding of how structured products work.

However, apart from the mentioned drawback of a lacking graphical representation of the payoff structure as in the case of HKEX classification, the author of this study assumes that all three have one common pitfall – they lack flexibility in terms of accommodating new possible types of structured products that can emerge in the

market. With this limitation, the existing classification may not be as relevant for an investor or an issuer.

Another observed common disadvantage is that they display only a fraction of the unique features that can be within a structured product. The author of this study states that the classification system should be not only updated with new possible emerging structured products, but most certainly include the description of features and other configurations of structured products.

With the recommendation above, the classification can provide much more complete picture to both parties: investors and issuers, as, for example, investors can understand the mechanics and risks of purchasing structured products clearer, which is not always the case (Kunz et al. 2017); issuers, on the other hand, with an improved classification may or may not benefit from it when pricing or constructing the structured product.

2.3 New classification

One of the goals of this study is to provide a new classification approach that could be useful for both issuers and investors, a modern classification that will include not only the fundamentals of the existing ones, but also a revision of the beforementioned drawbacks.

The following author's own constructed approach for classification is rather different, the key components for the classification are:

Table 1 New Structured Products classification's components

Classification components and their definition	
Component	Definition
Type of product	The layout or the structure of a structured product, for example, a deposit, bond, etc.

The underlying asset	The underlying asset that will be used to assess the performance of the structured product taking into consideration other various additional agreed terms.
Capital protection	The level of protection of an initial investment in a structured product.
Investors' expectations	The investors' assessment of the future underlying asset's value movement.
Investors' goals	The investors' goals of investing in a structured product.
Features	The specific mechanics, features or options that will be embedded in a structured product.

The reasoning for such division is that it may give more clarity to the configuration of a structured product to both issuers and investors, as all of the mentioned components will define risk profile, its ease of sale, the legal obligations between interested parties, and groups of customers that a structured product will target, tax treatment, and ultimately nature of a structured product.

First crucial component of structured products is their layout i.e., in which form they are represented. For example, in case of a note the liquidity of this type is much higher, as the owner can buy more or sell directly on the exchange. Key differences of all 5 below mentioned forms are various liquidity and risk levels, return structures, etc. The major observed configurations are presented in Table 2.

Table 2 Structured Products' classification based on type of product

Observed structured products' types	
Type	Definition
Bonds or notes	Both terms can be used interchangeably. In structured securities, these refer to debt securities which payoff will depend on the performance of underlying assets.
Funds	In structured securities context, these refer to PIVs that have a portfolio of structured securities.
Deposits	In structured securities framework, these are technically a combination of a traditional bank deposit with an uplift option component that will depend on the performance of option's underlying asset performance.

Mutual Funds	In structured products context, these are similar to Funds, however, mutual funds are more regulated.
Fiduciary Management	In terms of structured products, this category implies that a structured product will be separately created and customized by an investment manager to suit an investor's needs.
Exchange-Traded Funds (ETFs)	In structured products perspective, these refer to PIVs that invest in a portfolio of structured products. These may provide not only an investment in structured products, but also more liquidity and transparency.

Next key difference between the structured products is the underlying asset or a group of them. The structured product can be created including a wide variety of asset types, moreover, it can have a combination of different assets. The major groups of assets that can be used in a structured product are represented in the Table 3.

Table 3 Structured Products' classification based on an underlying asset

Observed structured products' possible underlying assets	
Type of an asset	Example
Equities	Stocks, such as Apple Inc.
Fixed income securities	Debt securities, such as bonds, notes, etc.
Commodities	Commodities, such as gold, crude oil, etc.
Currencies	Currency exchange rates, such as USD against HKD.
Indices	Indices, such as STI Index, FTSE 100, etc.
Interest rates	Interest rates, such as ECB's interest rate.
Derivatives	Derivative instruments, such as futures.
Real estate	Various real estate assets performance.
Credit	Exposure to credit risks of default or bankruptcy of some entity.
Inflation	Exposure to certain inflation benchmarks.
ESG	Exposure to ESG standards benchmarks.
Basket	A combination of several assets.

Next essential component among the structured products is security of an initial investment, whether there are additional risks, whether the leverage is used. In the

industry it is called capital protection, and logic is that less capital protection means more risk and more hypothetical return on the initial investment. The examples of structured products within this component will be depicted below, it is important to notice that the Figures 4-6 were partially modelled from the ones that EUSIPA mapping has. Mainly there are 3 types that are represented in table 4.

Table 4 Structured Products' classification based on level of investment protection

Observed levels of protection	
Level of protection	Definition
Full protection	In the worst-case scenario, an investor may get back their initial investment.
Conditional or Partial	In the worst-case scenario, an investor may get back a portion of their initial investment.
No protection	In the worst-case scenario, an investor may lose all their initial investment.

Figure 4 depicts 4 examples of products' payoffs that have a full capital protection within them, which implies that in the worst-case scenario an investor will have its initial investment back with no additional return. As it can also be seen from the figure 4, products may have a variety of payoff structures which depends on the specific features that will be covered in this subchapter as well.

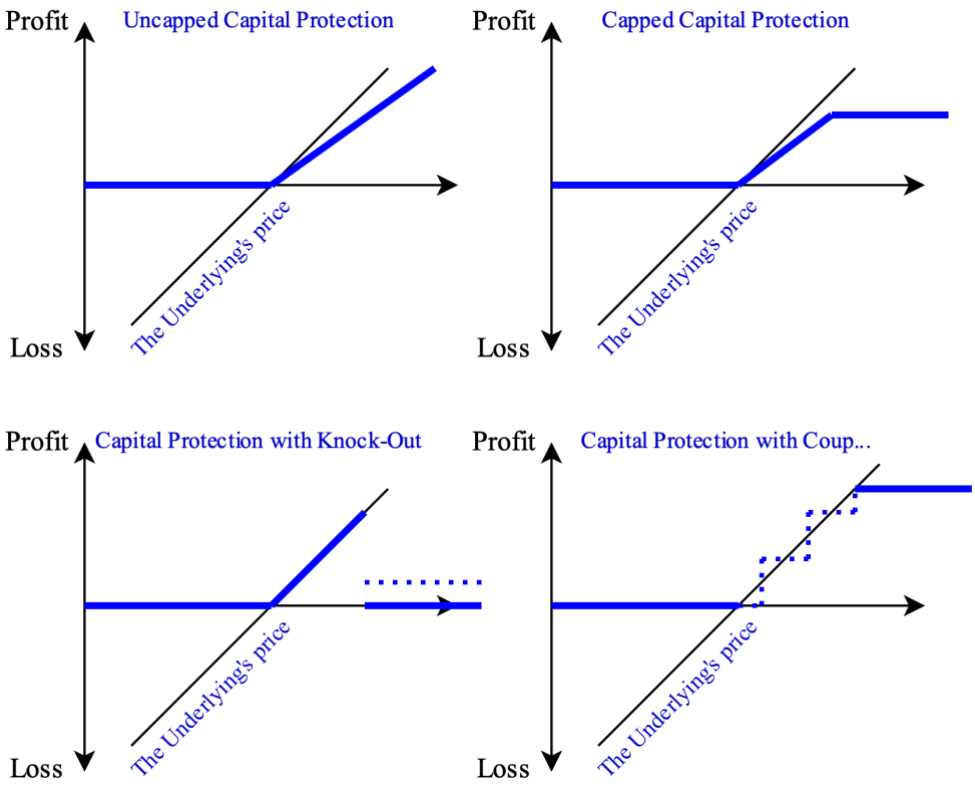

Figure 4 Full capital protection structured products.

Subsequently, Figure 5 displays the examples of structured products' payoffs that have a conditional protection or not full capital protection, which implies for an investor that when the underlying asset or a group of assets do not reach their target values, an investor may lose a part of their investment or in, some cases, get the worst performing of the assets from the structured product. These structured products are usually called as yield enhancement and more types of those are depicted in the figure below.

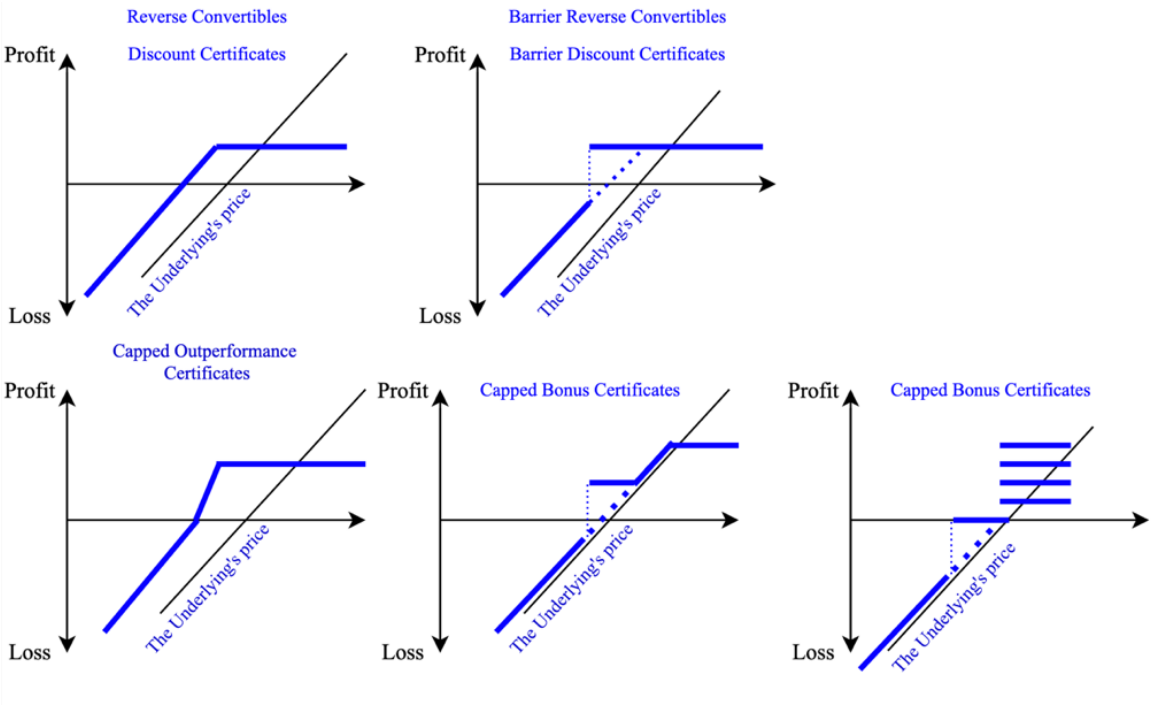

Figure 5 Yield enhancement structured products.

Another class are the riskiest of all structured products that have no capital protection or have "leverage". For example, in case of a product called warrant, an investor can have not only a massive loss, but lose all the initial investment when things go completely wrong. Figure 6 shows examples of the payoff structures for this class of products:

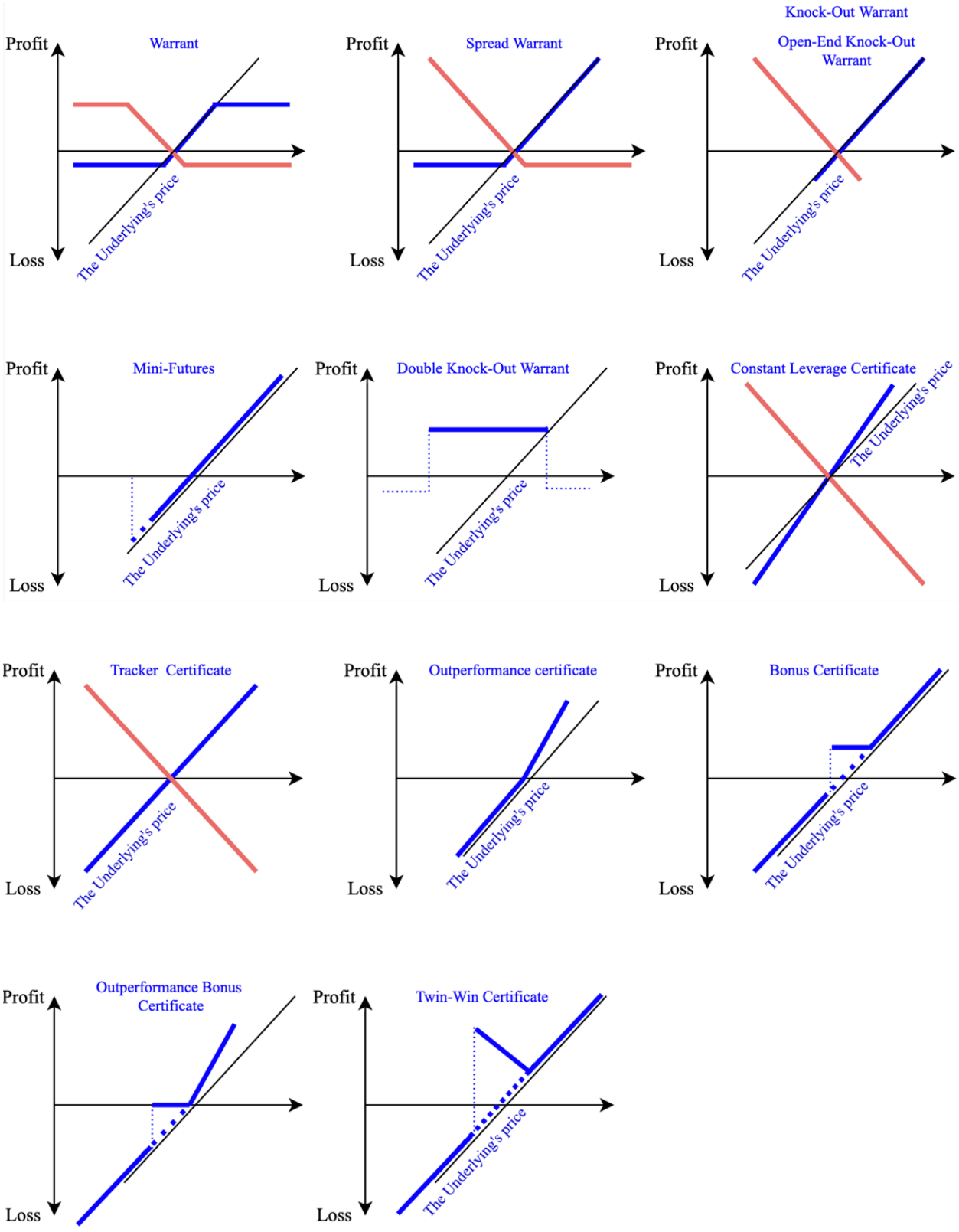

Figure 6 Leverage structured products.

Next component of a proposed classification is the investors' expectations. The reason behind this component is that investor's may expect various outcomes of their investment, and based on those expectations a structured product may be customized in a way to possibly meet those expectations, thus include some additional features and conditions to a product. The following are possible investors' expectations classification component represented in Table 5.

Table 5 Structured Products' classification based on investors' expectations.

Observed investor's expectations	
Expectation	Definition
Bull or Bear market	An investor may expect an increase or a decrease in value of an asset
Range of values	An investor may expect a certain range within which an underlying asset's value will be.
Occurrence or non-occurrence of certain events	An investor may expect an occurrence of specific event, such as default or any other.
Amount of volatility	An investor may expect a certain volatility of value of an underlying asset's value.

Moreover, the investors' goals also play a pivotal role when an issuer creates a structured product. This component influences heavily the product's form, features, underlying assets, and overall nature of the product. The possible investors goals' classification component is depicted in Table 6.

Table 6 Structured Products' classification based on investor's goals of investing in structured securities

Observed possible investor's goals of investing in structured products	
Goal	Definition
Hedging	An investor's goal is to hedge against specific risks.

Saving's protection	An investor's goal is to preserve capital in, for example, volatile market conditions.
Diversification	An investor's goal is to diversify their portfolio.
Optimization of tax base	An investor's goal is minimize the impact of taxes on their investment returns.
Derivative investment	An investor's goal is the actual possibility to invest in more complex instruments, such as options or futures.
Access to specific markets	An investor's goal is to gain exposure to specific markets that may be difficult to access otherwise.
Capital appreciation	A general investor's goal to to increase the value of their investment.

Finally, as all the above components are defined, the products can also have a number of specific mechanics, features or conditions. A lot of kinds of special conditions can be taken into account when the structured products are made or custom-made for a specific investor. As an example, one can mention the cliquet options, these allow for so-called floating parameters, i.e., the exercise price can vary from time to time. Relatively similar to cliquet options are rainbow options, which also have a "floating" price because the option possesses multiple at-the-money spot puts or calls. The table 7 provides a feature component of this study's classification.

Table 7 Structured Products' additional common features and conditions

Observed common structured products additional common features and conditions	
Condition	Definition
Knock-in	Condition to have that if the underlying asset should reach certain value level for the option to be triggered.
Knock-out	Condition to have that the option will be deactivated if certain asset's value level is reached.
Autocallability	Condition to have that if concrete conditions within prospectus or contract are met, an investor will receive a predetermined payment allowing for early maturity of a product.
Range	Condition to have that if asset's price remains within a particular range of values, an investor will receive a predetermined payment.

Conversion	Condition to have that if an underlying asset value goes below certain level, an investor will receive only the underlying asset.
Cliquet	Condition to have a set of options that go after one another with a strike price based on underlying's price at the end of previous option period asset value level allowing for locking-in the profit results.
Tracking	Condition to have payment be made completely reflecting the movements of the value of underlying asset or assets.
Dual currency	Condition to have exposure to exchange rate moves, allowing to receive e.g., coupon payments in one currency, and the principal in another currency.
Target Redemption	Condition to have in the form of notes, a specific target level of return from an underlying asset, when reaching one, the note pays the redemption amount.
Leverage	Condition to use leverage within a structured product, imposing higher possible return and subsequently possibility to lose all initial investment.
Basket	Condition to assess the performance of structured product based on the performance of a predefined number of specific equities, or other securities.
Worst-of	Condition to have payoff be based on the performance of the worst-performing asset in the group of assets.
Best-of	Condition to have payoff be based on the performance of the best-performing asset in the group of assets.
Lookback	Condition to have payoff be based on the best performance of an underlying asset over the period with call option, or the worst performance of an underlying asset over the period with put option.
Ladder	Condition to have multiple strike prices over the lifecycle of an option, the payoff will then depend on the best of reached strike prices.
Twin-win	Condition to have a positive outcome of an underlying asset going both up and down, however it comes with certain bottom barrier, when triggered may turn the product's payoff type into some other payoff structure, such as tracking condition.
Spread	Condition to have payoff be based on the relative difference of performances of two underlying assets.
Capping	Condition to have product carrying a "capped" or maximum possible return level.
Inverse float	Condition to have coupons paid inversely of an underlying interest rate embedded.

Rainbow	Generally, a condition to have assessment of the underlying asset's performance based on the average, weighted, and already mentioned best-of and worst-of performance of basket of assets.
Asian	Condition to have an assessment of underlying asset's performance based on an average value over a certain period.
Quanto	Condition to adjust currency risk, when an asset's value is denominated in one currency and the payments are made in the other currency.
Callable	Condition for issuer to call back the structured security in case an asset's value reach some predefined value.
Switchable	Condition to be able to exchange one asset for another during the lifecycle of a structured product.
Memory effect	Condition to have all missed coupons paid at the end of maturity of a structured product when other conditions are met. "Missed" here implies that in the past coupons were not paid, as an underlying asset's performance was not meeting other conditions.
Step-up	Condition to have an increase in the coupon rate over time based on predefined schedule when other conditions are met.
Shark-fin	Condition to have a fixed return on structured product when other conditions are met, as well as to have a participation in the underlying's asset appreciation or depreciation.
Buffering	Condition to have a specific buffer against losses, this feature works much like capital protection.
Participation rate	More general condition to define the share of an investment that can be lost or can generate an increased return in successful scenario.
Barrier	More general condition to have certain benchmarks levels of the performance of an underlying asset or assets.
Periodicity	More general condition stating the periodicity of the payments made when other conditions are met.
Life Period	More general condition of time period when an option is being activated or what is the structured product's maturity.
Fixed payoffs	More general condition of whether a product can make fixed coupon payments to an investor.

The payoff's structure of a structured securities can differ a lot based on the provided classification and make overall structured securities complex. For illustrative purposes of a structured product having various features, one can mention a hypothetical simplified structured product carrying "memory" effect and a barrier. Thus, in this case it implies for the structured product that when the payment is not being made as the conditions

were not met, for example, the asset did not reach certain target values, the payment is held for hypothetical future; thus, when the first payment is made, all previous non-paid payments are also included in this one. The example of a payoff is illustrated in Figure 7, in this case the barrier is at 80% of the value of an asset, two times in a row the asset did not reach the target value, and at third evaluation time, the structured product pays all three payments, including two initial ones.

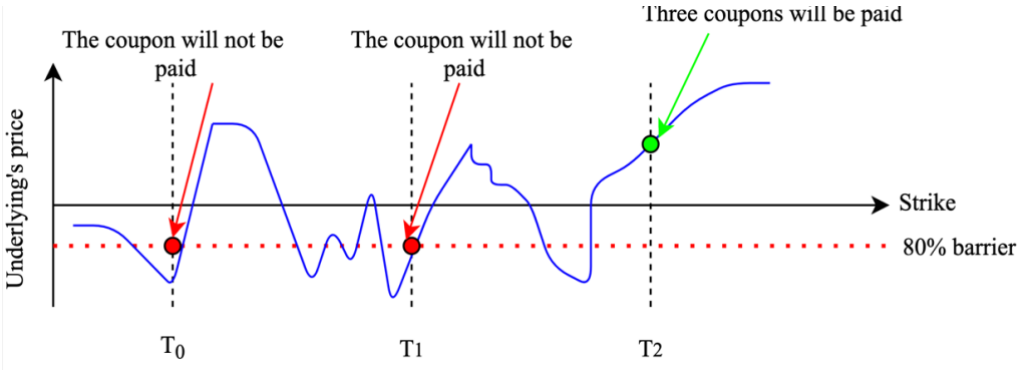

Figure 7 Example mechanism of memory feature within a structured product.

3 LITERATURE REVIEW

As it has been stated earlier in the first chapter of this study, the topic of structured products is gaining more attention among scholars in the recent years. Great number of articles have been written on the topic of structured securities and their specifics. This chapter presents four distinct major topics that are fairly related to this study, such as fair value estimation, engineering or creation of structured products, level of attractiveness of structured products to investors and finally the marketing-component of structured products' issue. Providing a literature review of these moderately covered topics by other researchers allows for the building up the theoretical framework of the current research as well as the deeper understanding of already explored ideas and hypotheses tested in the field.

The interest from academics to the topic of structured products is not coincidental, the author's assumption that the popularity of the topic among researchers is correlated with the overall increase in prevalence of these products on EU and US markets.

3.1 Fair value estimation

As the complexity of structured products grows (Célérier & Vallée, 2017), it is vital to understand how these securities can be priced for both issuers and potential investors. And as it was stated previously, the first structured products appeared already in 1990's (Hanaki, 2022), the first studies on them were done almost a decade after their introduction (Bekaert & Harvey, 2003; Fabozzi, 2005; Thonabauer & Nationalbank, 2004), however it is also possible that private or non-academic studies were conducted prior.

However, on the topic of fair value estimation, one may mention Stoimenov & Wilkens' (2005) study of plain-vanilla structured products, with equities as the underlying, in the German market. For valuation, the researchers used mainly classical Black & Scholes' (1973) approach as well as Reiner & Rubinstein's (1991) model. Their key finding was that depending on the life cycle of a structured product the pricing changed significantly

and most of the products were overvalued at the time. It is important to note that overvalued implies that the used model's estimation of a fair value for a structured product is less than the actual market price.

Fairly different from Stoimenov and Wilkens Eberlein's (2009) study, where it was decided to utilize Sato processes for valuation purposes (Carr et al. 2007, Sato, 1999). The authors stated using this approach the structured products are usually overvalued as well.

One may agree with their statement that it is better to have a distinct model according to a type of structured product. On the other hand, researchers were concerned that risk neutral pricing has arguably many downfalls, and thus should not be considered when evaluating the structured products. Apart from Sato, Eberlein and Madan considered using Lévy process, Merton jump model as well as Heston stochastic volatility model.

For this paper it was considered the best to use an exponential GARCH model to forecast volatility of the underlying asset, and there have been studies using the models from the same family. For example, a research done by T.-S. Lee & Lin (2009), where GJR-GARCH approach is used to give a fair value estimation of the structured products (Glosten et al. 1993).

In case of their paper, the authors defined a system for risk premium evaluation for bonds credit risks. Moreover, this paper as well as many more other papers implement Monte-Carlo approach, the list of previous research papers where it was used is not exhaustive (R. Chen et al. 2020; Deng et al. 2014; Kotadia, 2021; H. Lee, Choi, et al. 2022; H. Lee et al. 2019; H. Lee, Lee, et al. 2022; Song et al. 2023; Vokata, 2021; Yuan & Rieger, 2021, etc.)

As for the other approaches to model the fair values of structured products, one can mention more recent study conducted by (Climent Hernández & Cruz Matú, 2017), where the authors used log-stable and log-Gaussian methods in their paper as it is done in McCulloch & Lee's (2009) study. The key finding from this study was that log-Gaussian method overvalues the structured products in scope, whereas the log-stable does not do that. The researchers state that using the log-stable can be much more accurate, since the asymmetry and kurtosis are evaluated correctly in contrast to log-Gaussian.

Another evidence of using other methods, especially non-Gaussian methods, it is worth mentioning Zuev's (2017) study applying the Borland's (2002) approach as well as their own already mentioned Black & Scholes (1973) approach. The key take-away from this study is that the non-Gaussian methods such as univariate stochastic process can give much more accuracy when it comes to pricing of the structured products' options, while the industry-standard Black-Scholes is not that accurate.

Regarding structured products pricing, it is also worth mentioning an extensive and thorough research into using simulation, decomposition and numerical integration methods for pricing of the structured products (Deng et al. 2014). The authors used 20 000 distinct products to estimate their fair values, the result of the research includes a classification by the type of a structured product on which model was the appropriate one to use.

Another point to mention is that most of the products were either overvalued or undervalued, the researchers suggested that the reason for this was because of the discounting factor and transaction fees. The used approaches were a simulation (Glasserman, 2010), numerical integration (Ueberhuber, 1997), decomposition and partial differential equation approaches (Wilmott et al. 2000).

3.2 Engineering and issuance problems of structured products

The author of this study observed that one of the gaps in the existing literature is the managerial point of view on structured products, since the existing literature does not have much to offer, especially that considers the hedging and other costs that issuers may have, as this data is not publicly available. However, the financial engineering perspective and pricing is much more comprehensive. As the issuer's strategy is not considered anyhow in the existing literature, here will be presented the actual structuring of a product.

As a first example to mention is a thorough book by (Thonabauer & Nationalbank, 2004). In the book, one may find almost all kinds of structures existing at the time, the underlying assets include stocks, currencies, interest rates and other derivatives. From author's of this study standpoint, an issuer of structured securities can easily familiarize themselves with from the financial engineering aspect of these products. The terminology has changed drastically, but the basic mechanics remain the same till this day.

In a relatively similar manner, one may find a rather fundamental and comprehensive paper by (Neftci, 2008), where an issuer can indeed find the basic principles of structuring the securities. The study includes an overview of various derivative options that can be used to create a structured product. One may find an implementation of even exotic options, such as cliquet, rainbow, digital and touch derivatives. The paper also explores the theory about forward volatility.

The opposite example to the previous two is a study that takes into account an investor's perspective, as it is more aimed for professionals in the sphere of fiduciary management (Blümke, 2009). The book also considers all sorts of structured products, giving a comprehensive view on what are the structured products, and how they can be arranged in a portfolio. This study can be beneficial for both: investors and asset managers.

It is worth mentioning other not as holistic studies, where the hedging approach or only one particular type of product is considered. As an example, one can mention a paper by Abdelghafour (2009), where the hedging strategy is explored from a perspective of an insurance company that may use a structured product in their portfolio.

Two other studies worth mentioning include (Pelster & Schertler, 2019) and (Henderson et al. 2020). In the former one, the authors stated that issuers are exploiting the features of structured securities and various pricing approaches. The key finding revolves around the discount certificates which are usually bought in massive amounts so the prices of put warrants decrease, as issuer may try to hedge the risk exposure (Pelster & Schertler, 2019).

And of course, there are multiple costs involved when it comes to issuing structured products as it was outlined by Henderson et al. (2020). However, the exact values for management, marketing and hedging costs are not publicly available (Döbeli & Vanini, 2010).

Moreover, the margins of certain structured securities increase when an issuer has a lot of other structured securities in place, since it increases their risk exposure (Pelster & Schertler, 2019). It is also must be mentioned, that there are findings of a relatively identical to front-running techniques that issuers may use (Henderson et al. 2020). The authors debated in their study that purchasing large amounts of options in order to hedge themselves has a direct impact on pricing of retail structured securities. They are confident to state that the structures of these securities are exploited to gain benefit from altering of the stock prices.

3.3 Concerns and interests from investors

One of the major parts in this study is how the investors are affected by structured products, why they could be interested in these securities and what are the possible risks the investors may face. Custom-made nature of structured products may not only be

beneficial for an investor, but also can be exploited by issuers when creating a specific structured product using the general knowledge, risks' understanding and particular presentation formatting when presenting an opportunity for an investor (Baule et al. 2023).

The pioneers in this topic of attractiveness within structured products were Breuer & Perst (2007). The authors used hedonic framing provided by Thaler (1980) and cumulative prospect approach (TVERSKY & KAHNEMAN, 1992) in case of reverse convertible obligations. They stated that the demand for such products is dictated by investors who estimate the expected returns reasonably but underestimate the hypothetical volatility.

Regarding the presentation format, it was shown that structured securities are much more appealing to investors when a hypothetical payoff is shown using a probability distribution graphs (Anic & Wallmeier, 2020). In that paper, the experiment was set in a way that potential investors in structured securities could play with a number of different risk-return values and add specific conditions to a product. The researchers provided a proof that the structured products are more attractive when the presentation includes a variety different graphs explaining the mechanics of a structured product.

More recent studies explore how investors perceive risks regarding the structured products (Kunz et al. 2017). The authors took securities with baskets of assets as an example and debated that issuers have a lot of possibilities to exploit this type of structured products by creating a particular mix of assets within those products. They proved that a certain set of assets makes investors underestimate the volatility. The issuers can put together extremely risky assets with safer ones, and investors may tend to underestimate the risk that actually increases, even though a group of assets is thought to be diversified (Kunz et al. 2017). Finally, the authors showed that the knowledge level and expertise decrease the bias, but not entirely.

In another paper by X. Chen et al. (2017) it was shown that private information impacts the way the structured products are perceived by investors providing risk estimation bias. One of the results of the study was that investors tend to give less attention to public information, even though the private data has no substantial influence on the values of the assets within the structured security. Another key finding was that identification of risks is affected by investors' confidence levels since they tend to overvalue the probability of a beneficial outcome.

Finally, in the recent article by (Baule & Muenchhalfen, 2021) it was shown that issuer brand has no significance for an investor. It was stated in the study that the costs and structure of a security are both significant for investors with higher expertise and advanced knowledge, whereas investors with smaller expertise the costs are more crucial than the structure itself.

However, the lack of of publicly available costs makes it hard for both investors and researchers to make valuations of structured products (Döbeli & Vanini, 2010; Koh et al. 2015). Baule & Muenchhalfen (2021) used a risk evaluation numbers to the experiment, which played an important role for not so advanced investors, while highly qualified investors did not pay much attention to it. The conclusion was that the issuers can use these indicators to make the presentations of structured products more understandable and appealing to investors.

However, it is important to note that this study does not include research into non-structured product, as the primary focus is on the structured products only. From the author's perspective, there is a possibility that some of the investor's biases mentioned in the existing literature could be of the same nature and of the same or higher magnitude towards other financial instruments.

3.4 Marketing-component of structured products

When it comes to the actual marketing and selling of any kind of structured product, there is a number of questions that should be addressed, such as what kind of structured products drive the most attention, what key factors that drive the interest to structured products, how exactly the structured products are sold or marketed, etc. In the existing literature, there is only a small portion of researchers that have a take on the topic; thus, giving the possibility to explore more into this topic.

As it was mentioned before, there are dozens of types, subtypes, and conditions within the structured products. An investor's choice may indeed depend on several factors, such as:

- Investor's risk-aversity e.g., Basu & Dulleck (2020)
- Investor's knowledge of the market e.g., Kunz et al. (2017)
- Investor's knowledge of financial instruments e.g., Wang et al. (2011)
- Investor's desires regarding particular market and industry e.g., Raghunathan & Cecchini (2007)
- Investor's portfolio e.g., Basu & Dulleck (2020)

One of the key things to mention here is that structured products can be customized in a lot of ways according to an investor's desires, as it was mentioned by Chorafas (2006) . Since it is relatively easy for a bank or a broker comparing to a retail investor to create a structured product for a certain investor because an issuer may possess dozens of derivatives to create a structured product of. This way an issuer can create structured products, notes, obligations each week for both institutional and retail investors.

Thus, as it is relatively straightforward for an issuer to create a custom-made structured product, an investor can choose within a wide spectrum of features, for example, a currency in case of dual currency structured products. This appealing customization feature of a structured product is one of the key marketing instruments when an issuer

is marketing this idea to an investor, as it gives an investor an ability to work with complex financial products according to their interests (Raghunathan & Cecchini, 2007).

It is fairly important to mention another study by Döbeli & Vanini (2010) regarding marketing and selling of structured products, where the researchers dived deep into the analysis of behavior of an investor considering retail structured products. They made an experiment providing a group an example of a real structured product to invest in. They have found that when the description of the product was made in simple terms, hypothetical investors were more willing to invest in it, while the real product description was attracting much less people.

It is crucial to note here that when an issuer is selling or marketing a structured product using simple and easy communication style, explaining the payoffs, risks, structure, it may not only be beneficial for an issuer in terms of possible sell of a structured product (Sheehan, 2017), but it also decreases the risk that an investor buys a financial instrument that they do not understand.

Fairly recent study conducted by (Kallio et al. 2022) exhibits how different structured products are perceived by investors as well as it demonstrates portfolio performance with the structured products. In their paper, they observe so-called transparent structured products, which meet requirement of a transparent contract and “purity”. While structured products are said to be sophisticated and costly while being appealing for retail investors (Vokata, 2021), one of their findings was that structured products without full capital protection are indeed more appealing to individual retail investors, which can be possibly explained by an increased return perspective.

One may debate on the researchers’ proposition regarding this idea and scope, as all publicly available contracts are transparent in a way that all conditions and formulas are in prospectuses of those instruments. On the other hand, “purity” of a contract is fairly questionable for a reason that anything can be misleading in a contract creating a

situation, where an investor buys a structured product without understanding (Sheehan, 2017).

Another key point in marketing and selling of structured products is how they are priced. No doubt that it should be profitable for an issuer to sell a structured product. This study's perspective is the following: firstly, an issuer receives commission for the sell, secondly, they receive profit when the option is in-the-money.

The benefit of an issuer from selling a structured product would be then all profit from exercising of the option, apart from what was stated in the prospectuses. Thus, the question for an investor is whether he or she agrees with the stated price of the structured product, whether it is overpriced or is sold with a discount (Basu & Dulleck, 2020).

4 THEORETICAL FRAMEWORK

This chapter presents the models and approaches that could have been previously used in the existing literature as well as novel methods or combinations that were not found in academic literature – all the presented approaches were implemented in this study. Thus, in this study the classical approaches are tested along with new or experimental methodologies. However, it is important to note that the following models will consider fundamental valuation of the observed products' prices that will be useful to test both sets of the presented hypotheses in the Introduction chapter.

4.1 Ubiquitous and classic Black-Scholes approach

Black & Scholes' (1973) is indeed one of the fundamental and classic approaches used when approaching the valuation of structured products as it was pointed out in the literature chapter of this study. The list of literature where this approach was used is not exhaustive, but, for example in Deng et al. (2014); Entrop & Fischer, (2020); H. Lee et al. (2019); T.-S. Lee & Lin, (2009); Stoimenov & Wilkens (2005); Yuan & Rieger (2021); Zuev (2017), etc., studies. It is important to notice the important cornerstone assumptions for this approach, which are as follows:

- An underlying asset value should follow a log-normal distribution.
- Option only be exercised at expiration and there are no dividends paid.
- There is a constant volatility and continuous trading.
- There are no transaction costs, and interest rates are thought to be unchanging.
- Arbitrage is not considered.

The formula for call option as well as $d_{1,2}$ formulas are written below:

$$C(S_t, t) = N(d_1)S_t - N(d_2)Ke^{-r(T-t)}, \qquad (1)$$

Where:

$S_t =$ *price of an underlying asset at time t,*

$K =$ *strike price of an option,*

$r =$ *risk-free interest rate,*

$N(x) =$ *normal distribution function.*

$$d_1 = \frac{1}{\sigma\sqrt{T-t}}\left[\ln\left(\frac{S_t}{K}\right) + \left(r + \frac{\sigma}{2}\right)(T-t)\right], \quad (2)$$

$$d_2 = d_1 - \sigma\sqrt{T-t}, \quad (3)$$

Where:

$\sigma =$ *volatility of an underlying asset.*

Accordingly, the price for a put option is:

$$P(S_t, t) = N(-d_2)e^{-r(T-t)} - N(-d_{1)}S_t \quad (4)$$

The corresponding conditions are also applied:

$$C(0,t) = 0 \; for \; all \; t,$$
$$C(S,t) \to S - K \; as \; S \to \infty, \quad (5)$$
$$C(S,t) = \max\{S - K, 0\}$$

4.2 Lattice models

Lattice model is more generalized approach to pricing options, while binomial tree models are practically a subset, they are rarely used when it comes to valuation of structured products. From the observed literature, there were studies conducted by Abdelghafour (2009, Entrop & Fischer (2020), T.-S. Lee & Lin (2009). First formulated by Sharpe (1978) and subsequently generalized by Cox et al. (1979), to make this study more comprehensive taking into consideration a number of models and approaches, it was decided to as well take this method. The idea is fairly simple, one start with an initial stock price and then create a tree of possible stock price at future points in time; thus,

the tree is then used to value the options. In Figure 8, one can see an example of such tree structure.

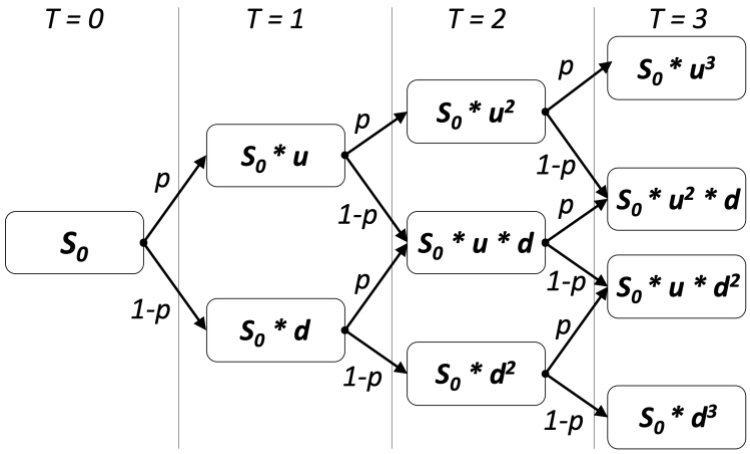

Figure 8 Binomial tree model example

Where:

$S_0 =$ *price of an underlying asset at start,*

$u =$ *upward movement factor of the value of an underlying asset,*

$d =$ *downward movement factor of the value of an underlying asset,*

$p =$ *probability of a movement.*

Subsequently, the upward, downward movement and probability will be defined as follows:

$$u = e^{\sigma\sqrt{\Delta t}},$$
$$d = e^{-\sigma\sqrt{\Delta t}}, \qquad (6)$$
$$p = \frac{e^{\frac{rt}{n}} - d}{u - d},$$

Where:

$\sigma =$ *volatility of an underlying asset.*

It should be pointed out that the presented example can be used for one single underlying asset, however the lattice approach can be as well used for the price evaluation of a basket of asssets.

4.3 Monte Carlo approach

Monte-Carlo simulation is widely used as well as Black-Scholes approach. One can mention, for example R. Chen et al. (2020), Dong et al. (2023), H. Lee et al. (2019), T.-S. Lee & Lin (2009), Song et al. (2023), Vokata's (2021) etc., studies.

Monte-Carlo simulation is a group of techniques that apply random sampling. There are a variations to the general approach, such as Quasi-Monte-Carlo that is used in T.-S. Lee & Lin's (2009) study, and Least Squares Monte-Carlo. The Quasi and general methods are conducted in similar manner, the difference would be the computation performance (Wei & Zhu, 2022), the latter is faster and less costly. The Least Squares framework uses the regression to fit and it is as well more efficient in terms of computational capacities than the general approach (Palupi et al. 2015).

There is no need for additional testing of the variations, nor there is a need for optimization of computational costs. Thus, for this study it was decided to use a general approach. And as it was pointed out, it is heavily used for pricing of structured products. An example of the Monte-Carlo algorithm for a European call option that will be used in this study is as follows:

1. Generating a certain number of sets with random values using normal distribution.
2. Using the randomly generated values to simulate a value path of an underlying asset through time with geometric Brownian motion, such as in the formula below:

$$S_t = S * e^{(r-0.5*\sigma^2)*T+\sqrt{T}*Z}, \quad (7)$$

Where:

$e^{(r-0.5*\sigma^2)*T+\sqrt{T}*Z} =$ *exponential function,*

$Z =$ *random variable following normal distribution,*

$\sigma =$ *volatility of an underlying asset's returns,*

$r =$ *risk-free interest rate.*

3. Computing the payoff of an option at maturity.
4. Averaging the payoff results and discounting them back to the PV with a predefined risk-free interest rate.

4.4 Finite difference methods

Moreover, it was decided to include at least one finite difference method as one of the approaches to evaluation of structured products' fair prices. For that it is needed a PDE to be solved. Practically, the general idea is to discretize some of the variables such as time and value and solve a PDE as it is described in Zvan et al. (2000) study. Regarding more modern literature and structured products, this method was used in studies by Deng et al. (2011), H. Lee, Lee, et al. (2022), T.-S. Lee & Lin (2009). The algorithm will be then demonstrated as follows:

1. Discretizing the time and asset price:

$$\Delta t = \frac{T}{L}\ ; \Delta P = \frac{2P}{N}, \quad (8)$$

Where:

$\Delta t =$ *time increment,*

$\Delta P =$ *price increment,*

$L =$ *number of price steps,*

$N =$ *number of time steps.*

2. Identifying the constants:

$$\lambda = \frac{r\Delta t}{2} \; ; \; \gamma = \frac{\sigma^2 \Delta t}{2}, \qquad (9)$$

Where:

$r =$ *risk-free interest rate,*

$\sigma =$ *an underlying asset's volatility,*

$\lambda, \gamma =$ *constants depending on r and σ respectively, and time increment.*

3. Creating the implicit matrix as in the formula below:

$$F = \begin{bmatrix} -(1+2\gamma+r\Delta t) & \gamma-\lambda & 0 & \dots & 0 \\ \gamma-\lambda & -(1+2\gamma+r\Delta t) & \gamma-\lambda & \dots & 0 \\ 0 & \vdots & -(1+2\gamma+r\Delta t) & \cdots & 0 \\ \vdots & 0 & \vdots & \ddots & \vdots \\ 0 & & 0 & \dots & -(1+2\gamma+r\Delta t) \end{bmatrix} \qquad (10)$$

4. Receiving the terminal condition:

$$P_{c,i} = \max(i\Delta P - K, 0), \qquad (11)$$
$$P_{p,i} = \max(K - i\Delta P, 0),$$

Where:

$P_{c,i} =$ condition for call option,

$P_{p,i} =$ condition for put option.

5. Iterating backwards through each step and getting the option price:

$$P_i^j = F^{-1} * P_i^{j+1}, \qquad (12)$$
$$P_N = P(S, 0)$$

4.5 GARCH, EGARCH approaches

Another approach to fair value estimation that will be used is GARCH approach (Bollerslev, 1986) – one of many stochastic volatility models such as ARCH. It is not as popular in scientific literature, among the observed papers, the application of this method was found only in studies performed by Dong et al. (2023), Entrop & Fischer (2020), T.-S. Lee & Lin, (2009), Song et al. (2023). For this study, this approach was mainly taken for experiment purposes. The volatility of an underlying asset, can be then extracted using the conditional variance of an underlying asset as in the following generalized equation:

$$\sigma_t^2 = \omega + \sum_{k=1}^{p} \beta_k \sigma_{t-k}^2 + \sum_{k=1}^{q} \alpha_k \varepsilon_{t-k}^2, \quad (13)$$

Where:

$\omega =$ *constant term,*

$\beta_k =$ *GARCH parameter with a number of lags p in a conditional variance equation,*

$\alpha_k =$ *GARCH parameter with a number of lags q in the conditional variance equation,*

$\varepsilon_{t-k}^2 =$ *squared errors.*

GARCH model (Bollerslev, 1986) is effective, however the shocks' direction is not considered in the model. Subsequently, EGARCH model (Nelson, 1991) takes into account the issue of shocks' sign and thus conditional variance can be defined as follows in the equation:

$$y_t = \mu_t + \sigma_t Z_t,$$
$$\hbar\sigma_t^2 = \omega + \sum_{k=1}^{p} \beta_i \hbar\sigma_{t-i}^2 + \sum_{k=1}^{q} \alpha_i (|Z_{t-i}| - E|Z_{t-i}|), \quad (14)$$

Where:

y_t *= Log return,*

μ_t *= Mean return,*

σ_t = *Volatility,*

Z_t = *White noise,*

$\hbar\sigma_t^2$ = *log of conditional variance,*

E = *weight given to lagged return.*

4.6 Cornish Fisher expansion

Finally, another approach – Cornish Fisher expansion (Cornish & Fisher, 1938) is proposed by the author of this study. There was no application of such method found in scientific literature in regard to structured products' pricing. This approach generates non-gaussian or non-normally distributed values, which can be especially valuable for exotic options (Zuev, 2017). It is important to notice, that it is still an approximation, however the advantage of this approach is that one can simply consider skewness and kurtosis in the return distribution. The general approach for the method is in below formula:

$$z \sim N(0,1) \quad E(z) = 0 \quad E(z^2) = 1 \quad E(z^3) = 0 \quad E(z^4) = 3,$$
$$Z = z + (z^2 - 1)\frac{S}{6} + (z^3 - 3z)\frac{K}{24} - (2z^3 - 5z)\frac{S}{36}\ ), \tag{15}$$

Where:

$z =$ *Standard normally distributed variable,*

$Z =$ *Non-Gaussian distributed variable,*

$K =$ *Kurtosis,*

$S =$ *Skewness.*

The reasoning for using this method for the study is ultimately for non-normality and the ability to control the "fat-tails" and "thin-heads" as it is could be a problem not only for structured products' underlying assets, but any other financial instruments. Additionally, this approach is added of pure academic interest, as it was not found to be used in the research at all; thus it brings additional novelty to this study.

5 DATA AND METHODOLOGY

This chapter represents the data as well as the methodology that were used to perform this study. The structured products' data was taken from HKEX's official website whereas share price data, indices' levels and exchange rates were provided by Refinitiv. The specifications and the usage of data are described below.

5.1 Structured products and underlying assets' data

In this study it was decided to take HKEX's publicly available structured products. These structured products include three major types: derivative warrants, inline warrants, callable bull, or bear contracts. This data was chosen for the study mainly because of its availability – anyone can access and download the data on those structured products if they have Internet connection and a computer, data was downloaded in April 2023. For the ease of conducting the study the prices of the underlying assets' were taken from Refinitiv, however anyone can as well get the pricing data from other publicly available resources. To ensure the proper testing of pricing methods and alleviate the possibility of not representative sample size, all 10367 available structured products were taken to test out the pricing approaches.

First, start on derivative warrants data, these are all European warrants, i.e., they are exercised at time of maturity, there are two main types: calls and puts. Their payoffs and other mechanics are more described in the methodology section. Table 8, however, presents some of descriptive statistics on these instruments. Within these structured products one may find indices, commodities, traditional stocks and exchange rates as underlying assets.

Table 8 Derivative Warrants Data – Descriptive Statistics

Derivative warrants – Descriptive Statistics of all publicly available structured products in Spring 2023

Statistic	Value
No. of Observations	6212
No. of Puts	1321
No. of Calls	4891
No. of unique underlying assets	258
Average time to maturity	0,6 years
Maximum time to maturity	4,06 years
Minimum time to maturity	0,02 years

Secondly, continue with callable bear and bull contracts' data, these are leverage instruments that have mandatory call feature that requires issuer to terminate the contract immediately if the underlying asset's price reaches the specified in the contract call price. Subsequently, in case when issuer "calls" the contract, an investor will either lose all of the initial investment or will receive a residual value that is also specified within a contract. In case when the call price is not reached within the life-period of a product, the product is exercised at maturity. There are as well two similar to derivative warrant's types: bull and bear. Their payoffs and other mechanics are more described in the methodology section. Table 9, however, presents some of descriptive statistics on these instruments.

Table 9 Callable bear and bull Data – Descriptive Statistics

Callable bear and bull contracts – Descriptive Statistics of all publicly available structured products in Spring 2023	
Statistic	Value
No. of Observations	4149
No. of Bears	1470
No. of Calls	2679

No. of unique underlying assets	47
Average time to maturity	1,26 years
Maximum time to maturity	4,3 years
Minimum time to maturity	0,01 years

Finally, the smallest portion of structured products represented by inline warrants. These have European style options. They pay X fixed amount of money at maturity if the underlying asset's value is within two barriers, however if the value of the underlying assets is out of a predefined range an investor will receive only X/4 fixed amount of money at maturity. Their payoffs and other mechanics are more described in the methodology section. There were only 6 such structured products found on HKEX's official website. Table 10, however, presents some of descriptive statistics on these instruments. They were only having Hang Seng index as the underlying.

Table 10 Inline Warrants Data – Descriptive Statistics

Inline Warrants – Descriptive Statistics of all publicly available structured products in Spring 2023	
Statistic	Value
No. of Observations	6
Underlying	Hang Seng Index
Average time to maturity	0,58 years
Maximum time to maturity	0,92
Minimum time to maturity	0,19

Regarding the underlying assets data, in total there were 259 unique underlying assets linked to the 10367 structured products, all structured products had only one asset assigned to them. The most frequent assets are presented in table 11. In order to get the payoffs for all structured products and subsequently their estimated fair values, it was decided to have, if existed, 5 years (2018-2023) of historical prices' data for each of the

underlying asset, as from author's of this study perspective this is a reasonable amount of historical data to evaluate, for example, volatility. Historical prices are then used to estimate volatility and simulate their price paths. Moreover, figure 9 shows the most frequent underlying type among the observed structured products – are stocks, while commodities and exchange rates are the least popular underlying types.

Table 11 Top-10 most frequent underlying among 10367 structured products.

Underlying assets – Top 10 most frequent underlying assets within publicly available HKEX's structured products in Spring 2023			
Asset ticker symbol	Name	Asset type	No. of Observations
HSI	Hang Seng Index	Index	1967
0700.HK	Tencent Holdings Limited	Equity	1032
9988.HK	Alibaba Group Holding Limited	Equity	617
3690.HK	Meituan	Equity	533
0388.HK	Hong Kong Exchanges and Clearing Limited	Equity	408
1211.HK	BYD Company Limited	Equity	343
0005.HK	HSBC Holdings plc	Equity	265
2318.HK	Ping An Insurance (Group) Company of China Ltd.	Equity	251
9618.HK	JD.com, Inc.	Equity	234
0941.HK	China Mobile Limited	Equity	209

Figure 9 Frequency of underlying assets by type within 10367 structured products.

5.2 Methodology

5.2.1 Structured product payoffs

First of all, it is suggested to present the exact payoff structures for each of three structured products, since the algorithms for calculation of structured products' fair values is relatively the same.

Derivative warrants' payoff is as follows:

$$Call\ type\ payoff\ at\ maturity = \frac{\max(S_t - K, 0)}{entitlement}, \qquad (16)$$
$$Put\ type\ payoff\ at\ maturity = \frac{\max(K - S_t, 0)}{entitlement},$$

Where:

$S_t =$ *price of the underlying asset at maturity,*

$K =$ *strike price,*

$Entitlement =$ *a value conversion factor.*

Consequently, the schematically payoff structure if plotted can be displayed in Figure 10. As it can be seen from both graphs, in the worst-case scenario may lose all initial investment, while in the best-case scenario the payoff will be unlimited.

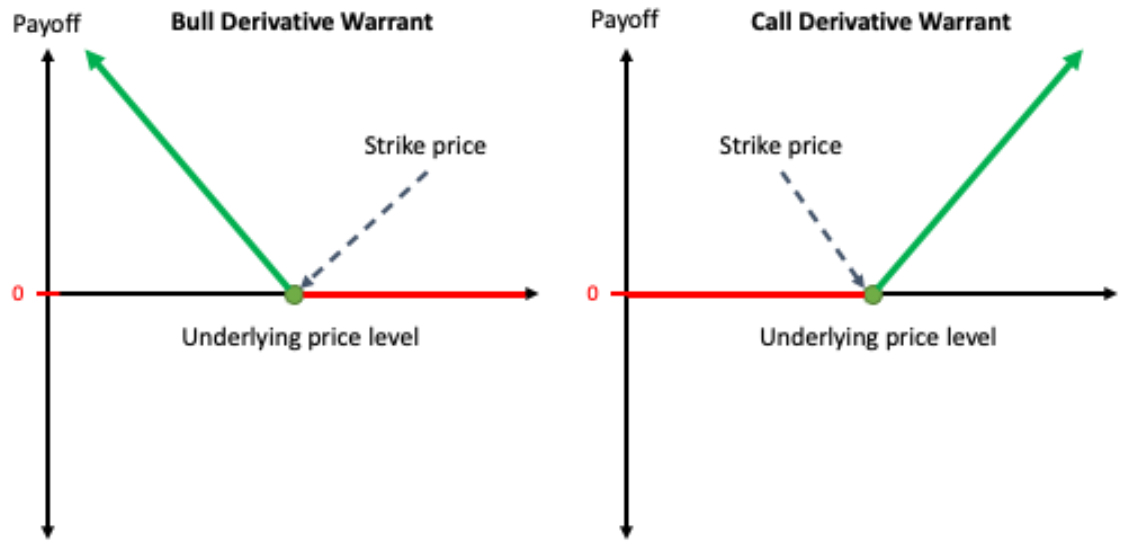

Figure 10 Derivative warrants payoff structures.

Callable bear or bull contracts' payoff is as follows:

$$\begin{matrix} Bull\ type\ payoff \\ at\ maturity \end{matrix} = \frac{(\mathrm{S_t} - K)}{entitlement}, if\ S_t \leq Call_p; \tag{17}$$

$$\begin{matrix} Bull\ type\ payoff \\ if\ call\ happened \end{matrix} = \begin{cases} \frac{(\mathrm{S_t} - K)}{entitlement} * 0, if\ S_t > Call_p\ and\ category = N; \\ \frac{(\mathrm{S_t} - K)}{entitlement}, if\ S_t > Call_p\ and\ category = R; \end{cases} \tag{18}$$

$$\begin{matrix} Bear\ type\ payoff \\ at\ maturity \end{matrix} = \frac{(\mathrm{S_t} - K)}{entitlement}, if\ S_t \geq Call_p; \tag{19}$$

$$\begin{matrix} Bear\ type\ payoff \\ if\ call\ happened \end{matrix} = \begin{cases} \frac{(\mathrm{S_t} - K)}{entitlement} * 0, if\ S_t < Call_p\ and\ category = N; \\ \frac{(\mathrm{S_t} - K)}{entitlement}, if\ S_t < Call_p\ and\ category = R; \end{cases} \tag{20}$$

Where:

$Call_p =$ *Call price level or barrier,*

$Category\ (N, R) =$ *category of a product that will indicate whether an investor will receive residual value in case of call happening,*

$R =$ *category of products with residual value,*

$N =$ *category of products without residual value,*

$S_t =$ *the same as in derivative warrants,*

$K =$ *the same as in derivative warrants.*

Consequently, the payoff structure of callables if plotted can be displayed in Figure 11. As it can be seen from the four graphs, the category “R” is less risky, as the “calling” event termination provides the residual value, whereas category “N” provides 0 return.

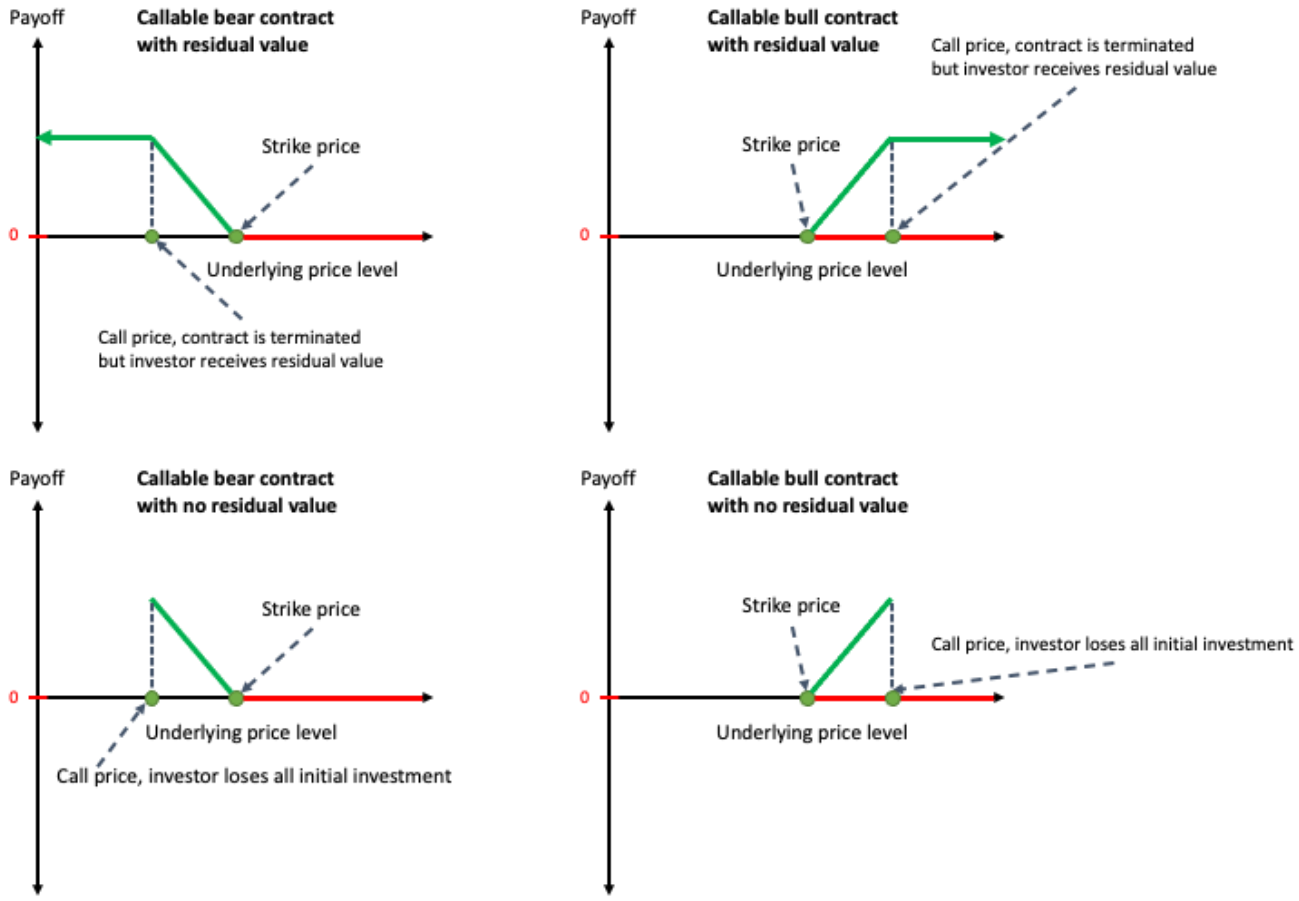

Figure 11 Callable bear and bull payoff structures.

Finally, inline warrants' payoff will be the following:

$$Inline\ payoff = \begin{cases} X, & if\ S_t \geq bottom\ barrier\ and\ S_t \leq top\ barrier \\ 0{,}25 * X, & if\ S_t > top\ barrier\ or\ S_t < bottom\ barrier \end{cases}, \quad (18)$$

Where:

$X =$ *natural money value such as €1,*

$bottom\ barrier =$ *bottom barrier price level of underlying below which structured products provide only 0.25 of the X value,*

$top\ barrier =$ top barrier *price level of underlying above which structured products provide only 0.25 of the X value,*

$S_t =$ *the same as in derivative warrants.*

Thus, plotting the payoff structure of an inline warrant will be as in Figure 12. In the worst-case scenario, an investor will receive a payment even though 4 times less than in the best-case scenario when the underlying price level is within the barriers.

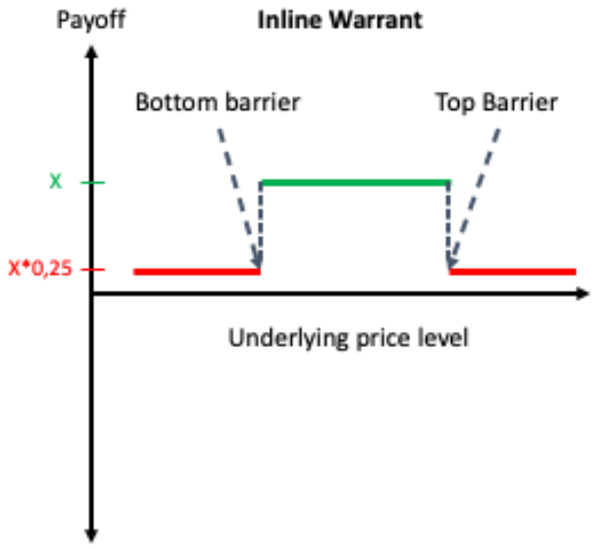

Figure 12 Inline warrants payoff structure.

5.2.2 Structured product's fair value estimation

Monte Carlo simulation was used the most, and it was used alone, as well as in combinations with Cornish Fisher Expansion, GARCH and EGARCH approaches. Each product underlying asset's price paths was simulated 10000 times to make sure that the estimated fair value is reasonable.

With Monte Carlo simulation alone the algorithm for structured product price would be as follows:

1. Set the number of simulations – 10000 times and set risk free interest rate – individual to assets' geographical regions.
2. Generate random numbers using normal distribution and subsequently price paths as shown in Figure 13.

3. Based on the simulated prices, calculate the payoffs accordingly to the type of product, taking into account entitlement, strike price, call price, barriers if included and underlying price.
4. Derive the structured product price by discount the average payoff to today's present value using risk free interest rate and the time to maturity such that:

$$SP_p = e^{-rT} * \frac{1}{number\ of\ simulations} \sum_{j=1}^{number\ of\ simulations} SP_i \ , \quad (18)$$

Where:

$T =$ *Time to maturity in years,*

$SP_{i,p} =$ *Structured product prices.*

It can be as well seen from Figure 14 that the resulted prices do indeed follow normal distribution, and in case of that product, which was a put derivative warrant with an underlying – Meituan stock, Monte Carlo simulation suggests that the product is overpriced.

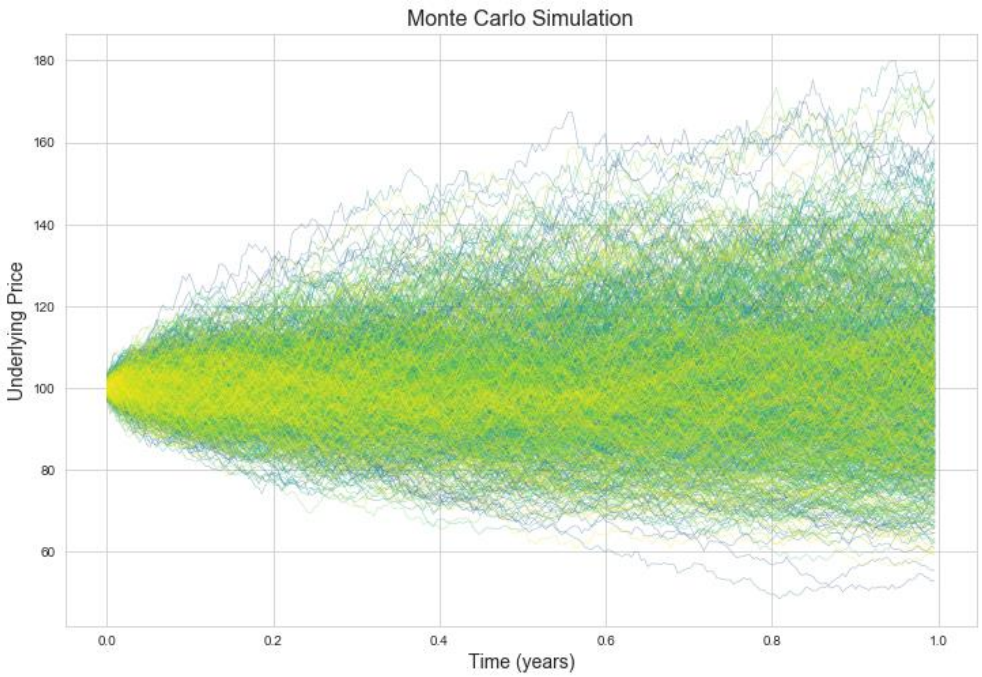

Figure 13 Monte Carlo example simulated price paths of Meituan's stock price – for some of the structured products observed within this study.

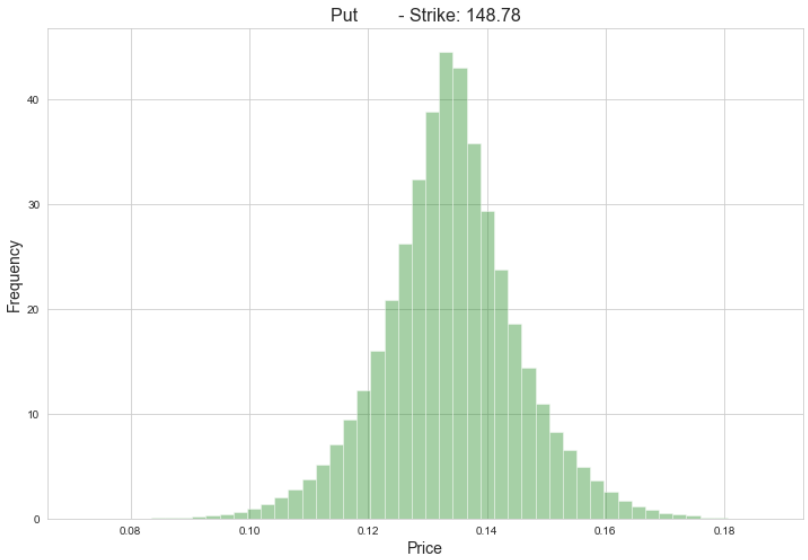

Figure 14 Monte Carlo price distribution based on the example of simulating Meituan's stock price for one of the structured products.

Subsequently, a combination of Monte Carlo with Cornish-Fisher Expansion was applied, the difference in the approach lies in the fact that the random variables are adjusted as per Cornish-Fisher Expansion, whereas in classical Monte Carlo approach a standard normal distribution is used.

Moreover, for the case of GARCH and Monte Carlo combination, the GARCH model is applied to evaluate the volatility of the underlying asset's returns to subsequently simulate the returns of the underlying asset and the paths for its prices. Thus, the same algorithm is applied in case of EGARCH model application.

Besides Monte Carlo and its combinations with other approaches, a relatively similar – bootstrap approach was used as well. The bootstrap method takes random samples from historical returns data to generate future price paths. The reasoning behind incorporating a bootstrap method is that this approach is computationally less heavy and does not require as much assumptions as other methods, however that is as well its downside, since it may not capture all underlying's price features and specifics.

To make this study more comprehensive, apart from mentioned implicit finite differences method, the explicit finite differences method was applied accordingly. With the former one, the algorithm incorporates backward time steps with discretizing the Black-Scholes on a grid, resulting in a grid that gives the intrinsic value at each step. When the values are calculated, the structured product price will be the middle point in the grid.

All steps with the intrinsic values and stock prices can be then as well plotted as in the Figure 15, in this case as well the method applied provides the suggestion that the price of this structured product is much higher than the actual, however implicit method provides a even higher price than Monte Carlo approach.

Figure 15 Implicit Finite Difference implementation using a put derivative warrant product with Meituan's stock as underlying.

In case of the latter – explicit approach, forward time steps are used to evaluate the fair value of the structured product. Applying the explicit finite differences approach the resulting price was considerably higher than the actual price of the structured product. The grid with the intrinsic values and the option, i.e., structured product price is displayed in Figure 16.

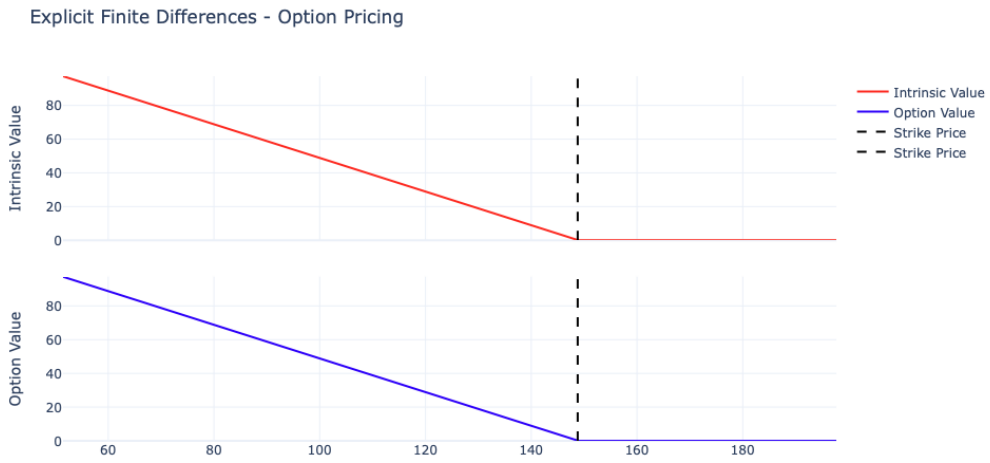

Figure 16 Explicit Finite Difference implementation using a put derivative warrant product with Meituan's stock as underlying.

Binomial trees implementation was done in the following way. All of the underlying assets prices are calculated in the tree using 100 steps with previously calculated probabilities of movements and their factors. The option tree is calculated at the same time for the structured product. When the option tree is constructed, the backward induction is started providing the structured product's a discounted intrinsic value at the earliest node which provides the present value of a structured product today. The figure 17 shows the example of implementation of such approach with the same product as in the previous examples. As it can be seen from the graphs, the binomial tree approach gives a fairly lower price in comparison to the actual one.

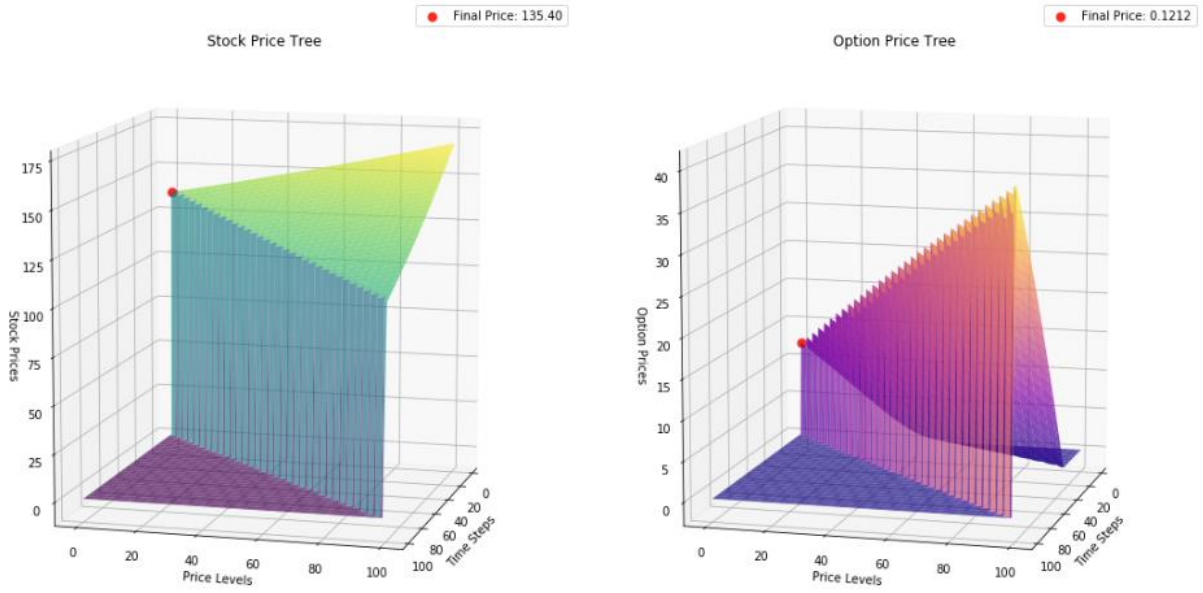

Figure 17 Binomial trees implementation using a put derivative warrant product with Meituan's stock as underlying.

5.2.3 Structured product issue modelling

Considering the actual experiment of creating a structured product, here will be presented the process of modelling a structured product issue using a combination of Monte Carlo simulation and EGARCH volatility model.

First of all, there should be the conditions and duration of the contract to be defined. In this case the structured product, it was decided to model an issue of a derivative warrant, as they comprised the majority of the actual structured products observed in this study.

As for the parameters of the structured product to be modelled, it was decided to take Hang Seng Index as the underlying, as it was the most frequently faced underlying asset in this study, other parameters were chosen more freely and serve as an experimental case study. Thus, the parameters are:

- Type of the contract – Derivative Warrant
- The underlying asset: HSI index.

- Duration of the contract – 1 years.
- Entitlement: 7000.
- Strike price: 21000.

The pricing process was done using Monte Carlo and EGARCH approaches' combination, that was as well used in this study and, as it will be discovered in the Empirical Results chapter proved to be the most accurate to the actual prices of this type of structured products. The EGARCH model serves a purpose of modelling of the volatility in the market, whereas Monte Carlo generates 10000 potential price paths for the underlying asset over the time to maturity. The figure 18 provides visually the relationship between the historical HSI's level with the estimated EGARCH conditional volatility.

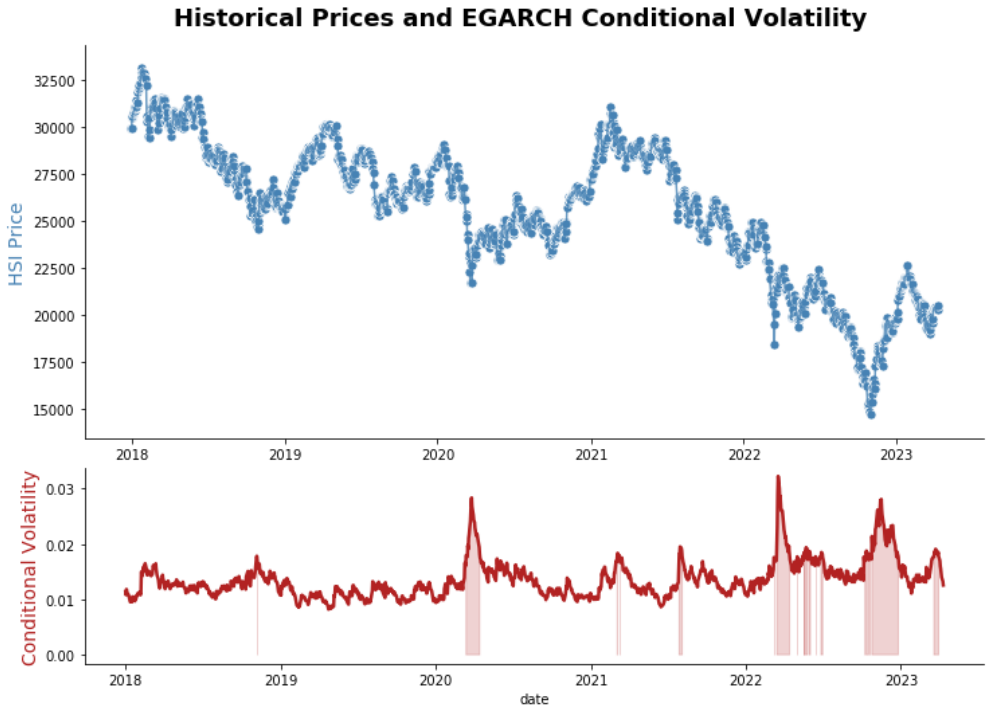

Figure 18 Historical prices of HSI Index in comparison with the estimated EGARCH conditional volatility.

From the author's of this study perspective, the key concern is the hedging of a structured product, apart from hypothetical marketing costs, transaction, maintenance and computational costs.

The hedging of the structured product can be achieved by firstly incorporating the Black-Scholes option pricing model by which the option price can be calculated. And it is important to notice that there can be more effective ways of hedging, however for the simplicity of the experiment, it was decided to use this approach. For Black-Scholes Formula 1 can be used, as for the delta or the sensitivity of the option price to a change in the underlying asset's price as well as the hedge ratio or the number of units of other derivatives an issuer should hold to alleviate potential changes in the payoff of the structured product, they can be defined as follows:

$$\Delta = N(d_1), \tag{18}$$

$$hedge = -\Delta, \tag{19}$$

Where:

$\Delta =$ *delta,*

$hedge =$ *hedge ratio.*

Thus, the expected payoff for the structured product and subsequently its price can be calculated as follows:

$$Expected\ payoff = \frac{1}{N}\sum_{i=0}^{N} \max\left(\frac{S_t - K}{Entitlement}, 0\right) \tag{20}$$

A hypothetical fee rate of 2.5% for purchasing the structured product can be set as well:

$$Fee = Rate * Expected\ payoff \tag{21}$$

The profit and loss for an issuer will then be then calculated as follows in the formula:

$$PL_{issuer} = (Hedge\ Payoff + Fee - Expected\ payoff - Call\ Option\ Price * hedge) \tag{22}$$

6 EMPIRICAL RESULTS

Taking into consideration everything mentioned in the theoretical framework and the methodology, in this chapter of the study the empirical results from assessing all of the 10367 structured products with all of the beforementioned methods and combinations of approaches as well as the comparison and all relevant statistics for the assessment will be provided.

Moreover, since one of the goals and research questions was to produce and engineer a structured product from the position of an issuer, the results of such experiment will also be discovered here.

6.1 Derivative Warrants estimation

The Table 12 shows the descriptive statistics of all estimated and retrieved data. It is important to notice that there are 6124 products mentioned instead of 6212 firstly mentioned, since there were products with no price data. It can be also seen that GARCH with MC (Monte Carlo) has fairly huge maximum estimated value for one of the products and average value, it may suggest that overall that the method is not as suitable to evaluate the price for these structured products.

Table 12 Descriptive statistics of estimated prices with actual prices of Derivative Warrants

Descriptive statistics observed from the resulting estimated prices for each method										
Statistic	Black-Scholes	Binomial Trees	Cornish-Fisher with MC	Implicit FD	Explicit FD	GARCH with MC	EGARCH with MC	Bootstrap	MC	Actual Price
count	6124	6124	6124	6124	6124	6124	6124	6124	6124	6124
mean	0,30	0,30	0,32	0,35	0,37	1,22	0,31	0,40	0,32	0,15
std	0,43	0,43	0,45	0,46	0,54	0,88	0,41	0,44	0,45	0,21
min	0,00	0,00	0,00	0,00	0,00	0,07	0,00	0,00	0,00	0,01
25 %	0,00	0,00	0,00	0,00	0,00	0,60	0,01	0,09	0,00	0,03
50 %	0,12	0,12	0,14	0,18	0,16	0,95	0,14	0,26	0,14	0,09
75 %	0,44	0,44	0,48	0,51	0,54	1,63	0,44	0,55	0,48	0,18
max	3,04	3,04	3,10	3,15	3,71	7,25	2,91	3,12	3,10	2,45

An example of an actual observed product could be the following:

- Underlying: JD Health International Inc.
- Type: Call Derivative Warrant
- Maturity date: 13-09-2024
- Actual (at time of valuation) price: 0.034
- Strike price: 105.89
- Conversion factor or entitlement: 100

To test the H_1 which states that structured products depending on a product and its features should have a distinct suitable approach for setting the issue price from an issuer perspective as well as the evaluation price from an investor perspective, there will be statistical tests implemented, namely – ANOVA, Kruskal-Wallis and Tukey's HSD (Honestly Significant Difference) tests.

The Table 13 shows three tests' results – ANOVA, Kruskal-Wallis and Tukey's HSD tests. ANOVA or Analysis of Variance test demonstrates an immensely significant F-Statistic of 1992,883 with 0 p-value combined with Kruskal-Wallis test that exhibited H-Statistic of 9886,131 and 0 p-value together they provide a clear proof that the there is a significant difference in evaluation among the selected methods. On the other hand, the Tukey's HSD test provides a more in-depth view on all provided pairs within the test, from which it can be stated that some of the methods can be used interchangeably in case the H_0 of Tukey's test is rejected.

Table 13 Difference significance among the estimation methods statistical tests implementation for Derivative Warrants

ANOVA, Tukey's HSD and Kruskal-Wallis tests identifying whether there is a significant difference between the estimated prices

ANOVA		**Tukey's HSD**						
F-statistic	**p-value**	group1	group2	meandiff	p-adj	lower	upper	reject
1992,883	0	Bin_tree	Boot	0,10	0,00	0,07	0,13	True
		Bin_tree	Bin_tree	BS	0,00	0,90	-0,03	0,03
		Bin_tree	CF_MC	0,02	0,37	-0,01	0,05	False

Kruskal-Wallis								
H-statistic	**p-value**	Bin_tree	EG_MC	0,01	0,90	-0,02	0,04	False
9886,131	0	Bin_tree	EFD	0,07	0,00	0,04	0,10	True
		Bin_tree	GA_MC	0,91	0,00	0,88	0,94	True
		Bin_tree	IFD	0,04	0,00	0,01	0,07	True
		Bin_tree	MC	0,02	0,36	-0,01	0,05	False
		Boot	BS	-0,10	0,00	-0,13	-0,07	True
		Boot	CF_MC	-0,08	0,00	-0,10	-0,05	True
		Boot	EG_MC	-0,09	0,00	-0,12	-0,06	True
		Boot	EFD	-0,03	0,09	-0,06	0,00	False
		Boot	GA_MC	0,82	0,00	0,79	0,85	True
		CF_MC	Boot	IFD	-0,05	0,00	-0,08	-0,03
		Boot	MC	-0,08	0,00	-0,10	-0,05	True
		BS	CF_MC	0,02	0,38	-0,01	0,05	False
		BS	EG_MC	0,01	0,90	-0,02	0,04	False
		BS	EFD	0,07	0,00	0,04	0,10	True
		BS	GA_MC	0,91	0,00	0,88	0,94	True
		BS	IFD	0,04	0,00	0,01	0,07	True
		BS	MC	0,02	0,36	-0,01	0,05	False
		CF_MC	EG_MC	-0,01	0,82	-0,04	0,01	False
		CF_MC	EFD	0,05	0,00	0,02	0,08	True
		CF_MC	GA_MC	0,89	0,00	0,86	0,92	True
		CF_MC	IFD	0,02	0,32	-0,01	0,05	False
		CF_MC	MC	0,00	0,90	-0,03	0,03	False
		EG_MC	EFD	0,06	0,00	0,03	0,09	True
		EG_MC	GA_MC	0,91	0,00	0,88	0,94	True
		EG_MC	IFD	0,04	0,00	0,01	0,07	True
		EG_MC	MC	0,01	0,80	-0,01	0,04	False
		EFD	GA_MC	0,84	0,00	0,81	0,87	True
		EFD	IFD	-0,03	0,10	-0,06	0,00	False
		EFD	MC	-0,05	0,00	-0,08	-0,02	True
		GA_MC	IFD	-0,87	0,00	-0,90	-0,84	True
		GA_MC	MC	-0,89	0,00	-0,92	-0,86	True
		IFD	MC	-0,02	0,34	-0,05	0,01	False

Where:

Bin_tree = Binomial Tree method,

BS = Black-Scholes method,

CF_MC = Cornish-Fisher combined with Monte Carlo method,

GA_MC = GARCH combined with Monte Carlo method,

EG_MC = EGARCH combined with Monte Carlo method,

EFD = explicit finite difference method,

IFD = implicit finite difference method,

Boot = Bootstrap method,

MC = Monte Carlo method.

It was also important to in the study to evaluate the overall accuracy of each of the methods, two key statistics were used for this matter – MAE (Mean Absolute Error) and MSE (Mean Squared Error). The Table 14 and provides all values for all 9 methods implemented in case of Derivative Warrants. It can be clearly seen that the closest to the actual prices methods are a combination of EGARCH with Monte Carlo, Black-Scholes and Binomial Trees, while the GARCH with Monte Carlo, finite difference methods provided the worst accurate results.

Table 14 MSE and MAE for all implemented methods for Derivative Warrants

Mean Squared Errors and Mean Absolute Errors for each method for Derivative Warrants		
Method	MAE	MSE
EGARCH with MC	0,31	0,26
Black-Scholes	0,36	0,30
Binomial Trees	0,36	0,30
Cornish-Fisher with MC	0,38	0,32
MC	0,38	0,32
Bootstrapping	0,38	0,34
Implicit FD	0,39	0,35
Explicit FD	0,42	0,43
GARCH with MC	1,08	1,93

However, the H_2 which states that the structured products are generally overpriced is rejected in this case, as it can be seen from the Table 15 that on average the Derivative Warrants are underpriced using all of the methods to evaluate their intrinsic value. The table also shows that GARCH with MC method mostly overvalues them.

Table 15 Overpriced and underpriced Derivative Warrants

Mean relative, absolute differences and % of underpriced, overpriced products for each method				
Method	Mean relative difference	Mean difference, HK$	Underpriced products, %	Overpriced products, %
On average	8,5200	0,1700	74,23 %	25,77 %
Black-Scholes	10,1640	0,1541	50,62 %	49,38 %
Binomial Trees	10,1640	0,1541	50,62 %	49,38 %
Cornish-Fisher with MC	10,5144	0,1752	52,22 %	47,78 %
Implicit FD	10,8678	0,1970	54,11 %	45,89 %
Explicit FD	11,5597	0,2238	53,25 %	46,75 %
GARCH with MC	28,2994	1,0675	97,84 %	2,16 %
EGARCH with MC	8,4200	0,1608	46,34 %	53,66 %
Bootstrapping	11,1054	0,2510	66,59 %	33,41 %
MC	10,5184	0,1754	52,22 %	47,78 %

6.2 Callable Bear and Bull Contracts estimation

The Table 16 shows the descriptive statistics of all estimated and retrieved data for Callable contracts. It is important to notice that there are 4129 products mentioned instead of 4149 firstly mentioned, since there were products with no price data. It can be also seen that GARCH with MC (Monte Carlo) has fairly huge maximum estimated value for one of the products and average value, it may suggest that overall that the method is not as suitable.

Table 16 Descriptive statistics of estimated and actual prices of Callable contracts

Descriptive statistics observed from the resulting estimated prices for each method									
Statistic	Binomial Trees	Cornish-Fisher with MC	Implicit FD	Explicit FD	GARCH with MC	EGARCH with MC	Bootstrapping	MC	Actual Price
count	4129	4129	4129	4129	4129	4129	4129	4129	4129
mean	0,03	0,03	0,04	0,20	0,65	0,05	0,05	0,04	0,22
std	0,07	0,06	0,05	0,14	0,53	0,09	0,09	0,07	0,16
min	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,01
25 %	0,00	0,00	0,00	0,08	0,20	0,00	0,00	0,00	0,10
50 %	0,00	0,00	0,02	0,14	0,46	0,00	0,00	0,00	0,17
75 %	0,03	0,01	0,05	0,31	1,11	0,09	0,08	0,05	0,29
max	0,42	0,41	0,26	0,55	2,46	0,60	0,59	0,52	1,98

An example of an actual observed product could be the following:

- Underlying: Tencent Holdings Limited.
- Type: Callable bear contract.
- Maturity date: 22-03-2024.
- Actual (at time of valuation) price: 0.17.
- Strike price: 453.
- Conversion factor: 450.

Testing the H_1, the Table 17 shows three tests' results – ANOVA, Kruskal-Wallis and Tukey's HSD tests for Callable contracts. The test again demonstrates an immensely significant F-Statistic of 4620,212 with 0 p-value combined with Kruskal-Wallis test that exhibited H-Statistic of 15580,93 and 0 p-value together they provide a clear proof that the there is a significant difference in evaluation among the selected methods. On the other hand, the Tukey's HSD test provides a more in-depth view on all provided pairs within the test, from which it can be stated that some of the methods can be used interchangeably as well for Callable contracts.

Table 17 Difference significance among the estimation methods statistical tests implementation for Callable contracts

ANOVA, Tukey's HSD and Kruskal-Wallis tests identifying whether there is a significant difference between the estimated prices

ANOVA		**Tukey's HSD**						
F-statistic	**p-value**	group1	group2	meandiff	p-adj	lower	upper	reject
4620,212	0	Bin_tree	Boot	0,02	0,00	0,00	0,03	True
		Bin_tree	CF_MC	0,00	0,90	-0,02	0,01	False
		Bin_tree	EG_MC	0,02	0,00	0,01	0,03	True
		Bin_tree	EFD	0,17	0,00	0,15	0,18	True
		Bin_tree	GA_MC	0,62	0,00	0,60	0,63	True
		Bin_tree	IFD	0,01	0,90	-0,01	0,02	False
		Bin_tree	MC	0,01	0,71	-0,01	0,02	False
		Boot	CF_MC	-0,02	0,00	-0,03	-0,01	True
		Boot	EG_MC	0,00	0,90	-0,01	0,02	False
		Boot	EFD	0,15	0,00	0,14	0,16	True

Kruskal-Wallis								
H-statistic	**p-value**	Boot	GA_MC	0,60	0,00	0,58	0,61	True
		Boot	IFD	-0,01	0,20	-0,02	0,00	False
		Boot	MC	-0,01	0,37	-0,02	0,00	False
		CF_MC	EG_MC	0,02	0,00	0,01	0,04	True
		CF_MC	EFD	0,17	0,00	0,16	0,18	True
15580,93	0	CF_MC	GA_MC	0,62	0,00	0,60	0,63	True
		CF_MC	IFD	0,01	0,48	0,00	0,02	False
		CF_MC	MC	0,01	0,28	0,00	0,02	False
		EG_MC	EFD	0,14	0,00	0,13	0,16	True
		EG_MC	GA_MC	0,59	0,00	0,58	0,61	True
		EG_MC	IFD	-0,02	0,02	-0,03	0,00	True
		EG_MC	MC	-0,01	0,05	-0,03	0,00	True
		EFD	GA_MC	0,45	0,00	0,44	0,46	True
		EFD	IFD	-0,16	0,00	-0,17	-0,15	True
		EFD	MC	-0,16	0,00	-0,17	-0,14	True
		GA_MC	IFD	-0,61	0,00	-0,62	-0,60	True
		GA_MC	MC	-0,61	0,00	-0,62	-0,59	True
		IFD	MC	0,00	0,90	-0,01	0,01	False

The Table 18 demonstrates all values for all 8 methods implemented in case of Callable contracts, and it be immediately seen that the closest to the actual prices' methods are a combination of explicit finite differences method, a combination of EGARCH with Monte Carlo and Bootstrap, whereas Cornish-Fisher with Monte Carlo and GARCH with Monte Carlo scored the worst.

Table 18 MSE and MAE statistics for all implemented methods for Callable contracts

Mean Squared Errors and Mean Absolute Errors for each method for Derivative Warrants		
Method	MAE	MSE
Explicit FD	0,145	0,037
EGARCH with MC	0,167	0,062
Bootstrapping	0,170	0,063
MC	0,180	0,064
Binomial Trees	0,187	0,066
Implicit FD	0,187	0,062
Cornish-Fisher with MC	0,190	0,066
GARCH MC	0,475	0,447

Testing out H_2 in case of Callable contracts, Table 19 suggests that on average the Callable contracts are overpriced using all of the provided methods apart from Black-Scholes

which could not be implemented in case of this type of structured products. The table also shows that GARCH with MC method mostly overvalues these structured products.

Table 19 Overpriced and underpriced Callable contracts

Mean relative, absolute differences and % of underpriced, overpriced products for each method				
Method	Mean relative difference	Mean difference, HK$	Underpriced products, %	Overpriced products, %
On average	-0,0900	-0,0830	34,56 %	65,44 %
Binomial Trees	-0,8148	-0,1865	0,61 %	99,39 %
Cornish-Fisher with MC	-0,8353	-0,1896	0,56 %	99,44 %
Implicit FD	-0,7198	-0,1807	7,22 %	92,78 %
Explicit FD	0,3616	-0,0211	44,20 %	55,80 %
GARCH with MC	3,4360	0,4285	79,66 %	20,34 %
EGARCH with MC	-0,6615	-0,1657	3,63 %	96,37 %
Bootstrapping	-0,7053	-0,1696	1,48 %	98,52 %
MC	-0,7720	-0,1793	0,90 %	99,10 %

6.3 Inline Warrants estimation

Although the inline warrants were presented the least among all 10367 structured products – only 6 products, it was still important to try out all of the presented methods on this type of structured products the same way as for all other structured products.

The table 20 provides the descriptive statistics on the available data on Inline Warrants. In case of Inline Warrants, it can be seen that the finite differences methods as well as the Monte Carlo and Cornish-Fisher combination had the highest maximum value when estimating the intrinsic value of inline warrants, which may suggest that they very roughly overvalue these structured products when implemented.

Table 20 Descriptive statistics of estimated and actual prices of Inline Warrants

Descriptive statistics observed from the resulting estimated prices for each method									
Statistic	Binomial Trees	Cornish-Fisher with MC	Implicit FD	Explicit FD	GARCH with MC	EGARCH with MC	Bootstrapping	MC	Actual Price
count	6	6	6	6	6	6	6	6	6
mean	0,34	0,37	0,37	0,38	0,26	0,30	0,34	0,37	0,41
std	0,12	0,30	0,30	0,31	0,01	0,04	0,10	0,30	0,16
min	0,25	0,24	0,24	0,25	0,25	0,26	0,25	0,24	0,25
25 %	0,27	0,25	0,24	0,25	0,25	0,28	0,27	0,25	0,30
50 %	0,30	0,25	0,25	0,25	0,26	0,30	0,34	0,25	0,36
75 %	0,37	0,25	0,25	0,25	0,26	0,32	0,37	0,25	0,49
max	0,57	0,98	0,97	1,01	0,26	0,36	0,51	0,98	0,68

An example of an actual observed product could be the following:

- Underlying: Hang Seng Index.
- Type: Inline Warrant.
- Maturity date: 29-11-2023.
- Bottom strike price: 22000.
- Top strike price: 26000.
- Actual (at time of valuation) price: 0.53.
- Conversion factor or entitlement: 1.

The results for ANOVA, Kruskal-Wallis and Tukey's HSD tests cannot be representable for such a small sample of structured products to derive conclusions from. Thus, it was decided to not include those in this study and suggested for a future study to have a much more significant sample of such structured products.

Regarding other metrics, the table 21 shows MAE and MSE metrics for Inline Warrants. In case of this products, the most accurate to the actual prices were binomial trees method and bootstrapping, while GARCH and finite differences methods scored the worst.

Table 21 MSE and MAE statistics for all implemented methods for Callable contracts

Mean Squared Errors and Mean Absolute Errors for each method for Derivative Warrants		
Method	MAE	MSE
Binomial Trees	0,07	0,007058
Bootstrapping	0,07	0,009542
EGARCH with MC	0,111667	0,028342
Cornish-Fisher with MC	0,143333	0,033675
MC	0,143333	0,033675
Implicit FD	0,145	0,033192
Explicit FD	0,146667	0,035875
GARCH with MC	0,158333	0,047708

Testing out H_2 in case of Inline Warrants, Table 22 suggests that on average the Inline Warrants are overpriced using all of the provided methods apart from Black-Scholes which could not be implemented in case of this type of structured products. The table also shows that GARCH with MC method mostly undervalues these structured products.

Table 22 Overpriced and underpriced Inline Warrants

Mean relative, absolute differences and % of underpriced, overpriced products for each method				
Method	Mean relative difference	Mean difference, HK$	Underpriced products, %	Overpriced products, %
On average	-0,17	-0,0830	16,67 %	83,33 %
Binomial Trees	-0,1528	-0,0700	7,00 %	0,00 %
Cornish-Fisher with MC	-0,1465	-0,0433	4,33 %	16,67 %
Implicit FD	-0,1598	-0,0483	4,83 %	16,67 %
Explicit FD	-0,1360	-0,0367	3,67 %	16,67 %
GARCH with MC	-0,3073	-0,1583	15,83 %	0,00 %
EGARCH with MC	-0,1966	-0,1100	11,00 %	16,67 %
Bootstrapping	-0,1386	-0,0700	7,00 %	0,00 %
MC	-0,1465	-0,0433	4,33 %	16,67 %

6.4 Structured product issue modelled

As for the results of the experiment, one can mention that the experiment is fairly simplified and was done on a more theoretical level rather than taking into consideration the actual fees and costs from both issuer and investor's side.

Firstly, the Figure 19 simply shows another example of implementation of Monte Carlo simulation but this time with the HSI index and EGARCH model fitted for the product parameters:

- Current underlying index level (HSI): 20485,24
- Strike price: 21000
- Time to maturity: 1 year
- Risk-free rate 0,0279
- Entitlement: 7000

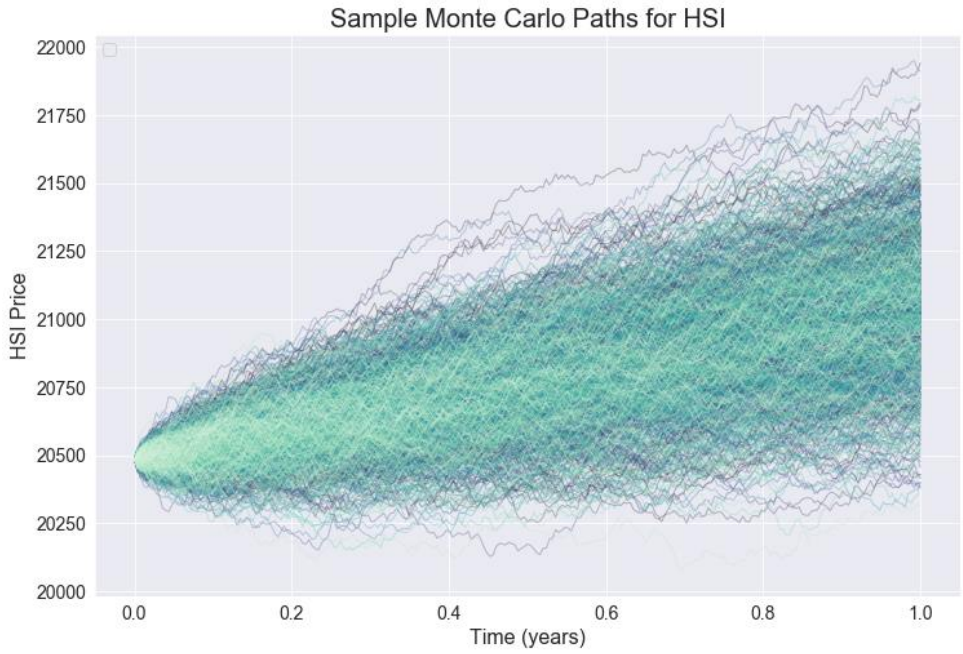

Figure 19 Future index levels of HSI Index simulated using Monte Carlo approach in combination with the estimated volatility from EGARCH model.

- Issuer's profit and loss for one product without hedge: 0,0004 HK$
- Issuer's profit and loss with hedge: 164,4207 HK$
- Investor's payment for the product: 0,0204 HK$
- Investor's return (%): -0.96

In this experiment, an investor will receive a loss if investing in this product, whereas issuer will receive a substantial profit when selling a structured product. However, there can be other fees and costs included.

7 DISCUSSION ON THE EMPIRICAL RESULTS

This chapter presents a discussion on the empirical results achieved from the analysis conducted to test the 4 hypotheses shaped in the introduction to this study. The H_0 and H_1 concerned the suitability of a pricing methodology for structured products, while H_2 and H_3 were aimed to identify whether HKEX's publicly available structured products are on average overpriced.

To test the first two, various pricing approaches including the classic Black-Scholes model, lattice framework, Monte Carlo simulation, finite difference methods, bootstrapping, time-series models approaches were implemented either distinctively or in combination with others.

Conducted ANOVA, Kruskal-Wallis and Tukey's HSD tests have revealed that there is a significant difference in the selected approach to pricing of a structured product. Although there still can be no significant difference between specific pairs such as in combinations of EGARCH with Monte Carlo and GARCH with Monte Carlo, there is a significant difference overall. Moreover, in case of Inline Warrants and Callable contracts, approaches such as Black-Scholes could not be implemented at all, as those are more exotic options rather than classic European style options, whereas Monte Carlo combinations with EGARCH or finite differences methods or bootstrap were relatively simply implemented, thus showing a better adaptability for estimating the intrinsic value of structured products.

To test the last two hypotheses, an extensive analysis was performed on the same sample of more than 10000 products listed on HKEX. Each product was analyzed using each suggested method, in total the evaluation part included creating over 100 billion price paths. The results of this analysis demonstrated that on average Inline Warrants and Callable contracts were overpriced by the market whereas the Derivative Warrants were underpriced. It is worth noting that the factors of overpricing were not discussed and were not part of the goals to achieve in this study.

In summary, the empirical results obtained provide significant support for H_0 and thus rejecting H_1, whereas for H_2 and H_3 the situation is dubious. It cannot be simply said that all products on average are overpriced, however it can be said that on average each type can be overpriced such as Callable contracts and Inline Warrants, while Derivative Warrants are underpriced. Future research could dive deep into investigation of the reasons behind the overpricing and underpricing of structured products.

As for the limitations and further research, one crucial limitation of the study is the assumption that the chosen pricing approaches are comprehensive and accurate enough to capture the intrinsic value of the structured products observed. Even though, the methods incorporated in this study are widely recognized and used by the market participants, there is still a possibility of an alternative or other far more advanced approaches that could give radically different results.

Additionally, the study's focus was on the HKEX's publicly available structured products, thus it may limit the generalization of the finding. Future research could increase the scope of the analysis and include more structured products types and different markets which may allow for a more holistic understanding of the most suitable pricing approach as well as yield different results on whether the structured products are overpriced.

Moreover, the study does not take into account the hypothetical behavioral aspects of investors even though mentioning the existing literature on that topic. Future research could as well investigate how other behavioral factors such as risk aversion or cognitive biases may affect the pricing and demand for structured products.

Finally, the present study assumes that the market conditions and the underlying assets' dynamics remain constant for the analysis period. However, it is widely known that the financial markets' volatility and other possible changes in market's conditions may play a crucial role on the pricing and valuation of structured products.

8 CONCLUSIONS

In conclusion, this study contributes to the existing literature on structured products by exploring the factors that influence the demand for them among retail and institutional investors, investigating the engineering and issuance process, providing a modern classification approach, and taking a deep dive into the various pricing approaches. The existing literature analysis shows that for the most part the role of risk management, return potential, access to complex derivatives and investor biases are driving the popularity of structured products. It is crucial to remind of the presented hypotheses in this study. The first set of hypotheses was designed to evaluate the suitability of pricing methodology for structured products, whereas the second set's goal was to evaluate whether the market overvalues or undervalues the observed structured products.

By investigating more than 10000 structured products with diverse conditions, by creating a new modern classification for structured products, this study has shed light on the complexity of structured products. This research critically analyzes the various pricing methodologies, such as EGARCH, GARCH, Monte Carlo, Black-Scholes and other models and their combinations, and, most importantly, their effectiveness in determining the intrinsic value of structured products on a sample of publicly available structured products. Providing the results of over 100 billion simulations and implementing widely recognized statistical tests, it can be stated that the empirical results reveal that there is a significant difference in the estimated fair value when implementing different approaches, thus it indeed depends on which model a potential investor or an issuer will use for either deciding on the purchase of a product or an issue price respectively.

Moreover, this research provides proof that certain types of publicly available structured products listed on HKEX, such as callable contracts, are on average, overpriced, whereas derivative warrants are underpriced when considering their intrinsic value. These findings can be important for both investors and issuers, which again emphasizes the

need for accurate and reliable pricing methodologies according to the type of a product and its specific features or conditions.

By analyzing the existing literature, classification approaches and creating a new classification approach, this study gives a modern outlook on the structured products market and their mechanisms.

Considering the experiment of creating a structured product that was provided as well, it was mainly of the author's interest, however it could be helpful as well for an investor to get a clear, simple, high level picture of how a structured product can be created from the issuer's perspective.

Finally, this research offers valuable insights for both retail and institutional investors as well as for investors in the structured product market, providing to the growing body of knowledge on the engineering of structured products, their pricing and demand for them.

REFERENCES

Abdelghafour, M. O. (2009). *Structured products: Pricing hedging*. 54.

Abdessamad Khaled, K. F. (2019, October 16). Sure Time to grasp the potential of structured products. *Bloomberg Professional Services*. https://www.bloomberg.com/professional/blog/sure-time-to-grasp-the-potential-of-structured-products/

Albeverio, S., Steblovskaya, V., & Wallbaum, K. (2018). The volatility target effect in structured investment products with capital protection. *Review of Derivatives Research*, *21*(2), 201–229. https://doi.org/10.1007/s11147-017-9138-2

Anic, V., & Wallmeier, M. (2020). Perceived Attractiveness of Structured Financial Products: The Role of Presentation Format and Reference Instruments. *Journal of Behavioral Finance*, *21*(1), 78–102. https://doi.org/10.1080/15427560.2019.1629441

Arnold, M., Schuette, D., & Wagner, A. (2021). Neglected Risk in Financial Innovation: Evidence from Structured Product Counterparty Exposure. *European Financial Management*, *27*(2), 287–325. https://doi.org/10.1111/eufm.12281

Ascheberg, M., Bick, B., & Kraft, H. (2013). Hedging structured credit products during the credit crisis: A horse race of 10 models. *Journal of Banking & Finance*, *37*(5), 1687–1705. https://doi.org/10.1016/j.jbankfin.2013.01.002

Basu, A. K., & Dulleck, U. (2020). Why do (some) consumers purchase complex financial products? An experimental study on investment in hybrid securities. *Economic Analysis and Policy*, *67*, 203–220. https://doi.org/10.1016/j.eap.2020.07.005

Baule, R., & Muenchhalfen, P. (2021). What is your desire? Retail investor preferences in structured products. *Review of Behavioral Finance*, *ahead-of-print*(ahead-of-print). https://doi.org/10.1108/RBF-10-2020-0254

Baule, R., Münchhalfen, P., Shkel, D., & Tallau, C. (2023). Fair-washing in the market for structured retail products? Voluntary self-regulation versus government regulation. *Journal of Banking & Finance*, *148*, 106749. https://doi.org/10.1016/j.jbankfin.2022.106749

Bekaert, G., & Harvey, C. R. (2003). Emerging markets finance$. *Journal of Empirical Finance*, 53.

Black, F., & Scholes, M. (1973). The Pricing of Options and Corporate Liabilities. *The Journal of Political Economy*, *81*(3), 637–654.

Blümke, A. (2009). *How to invest in structured products: A guide for investors and investment advisors*. Wiley.

Bollerslev, T. (1986). Generalized autoregressive conditional heteroskedasticity. *Journal of Econometrics*, *31*(3), 307–327. https://doi.org/10.1016/0304-4076(86)90063-1

Borland, L. (2002). Option Pricing Formulas Based on a Non-Gaussian Stock Price Model. *Physical Review Letters*, *89*(9), 098701. https://doi.org/10.1103/PhysRevLett.89.098701

Breuer, W., & Perst, A. (2007). Retail banking and behavioral financial engineering: The case of structured products. *Journal of Banking & Finance*, *31*(3), 827–844. https://doi.org/10.1016/j.jbankfin.2006.06.011

Carr, P., Geman, H., Madan, D. B., & Yor, M. (2007). SELF-DECOMPOSABILITY AND OPTION PRICING. *Mathematical Finance*, *17*(1), 31–57. https://doi.org/10.1111/j.1467-9965.2007.00293.x

Célérier, C., & Vallée, B. (2017). Catering to Investors Through Security Design: Headline Rate and Complexity*. *The Quarterly Journal of Economics*, *132*(3), 1469–1508. https://doi.org/10.1093/qje/qjx007

Chen, R., Zhou, H., Jin, C., & Liu, J. (2020). Discount or premium? Pricing of structured products: An analysis of Chinese market. *International Review of Financial Analysis*, *70*, 101493. https://doi.org/10.1016/j.irfa.2020.101493

Chen, X., Wang, Z., Deng, S., Fang, Y., ,School of Business, Central South University, Changsha, China, ,School of Mathematics and Statistics, Central South University, Changsha, China, & ,Academy of Mathematics and Systems Science, Chinese Academy of Sciences, Beijing, China. (2017). Risk measure optimization:

Perceived risk and overconfidence of structured product investors. *Journal of Industrial & Management Optimization*, *13*(5), 1–20. https://doi.org/10.3934/jimo.2018105

Chorafas, D. N. (2006). Currency exchange structured products. In *Wealth Management* (pp. 338–358). Elsevier. https://doi.org/10.1016/B978-075066855-2.50016-7

Climent Hernández, J. A., & Cruz Matú, C. (2017). Pricing of a structured product on the SX5E when the uncertainty of returns is modeled as a log-stable process. *Contaduría y Administración*, *62*(4), 1160–1182. https://doi.org/10.1016/j.cya.2017.06.012

Cornish, E. A., & Fisher, R. A. (1938). Moments and Cumulants in the Specification of Distributions. *Revue de l'Institut International de Statistique / Review of the International Statistical Institute*, *5*(4), 307. https://doi.org/10.2307/1400905

Cox, J. C., Ross, S. A., & Rubinstein, M. (1979). Option pricing: A simplified approach. *Journal of Financial Economics*, *7*(3), 229–263. https://doi.org/10.1016/0304-405X(79)90015-1

Deng, G., Husson, T., & McCann, C. (2014). Valuation of Structured Products. *The Journal of Alternative Investments*, *16*(4), 71–87. https://doi.org/10.3905/jai.2014.16.4.071

Deng, G., Mallett, J., & McCann, C. (2011). Modeling autocallable structured products. *Journal of Derivatives*, *17*.

Döbeli, B., & Vanini, P. (2010). Stated and revealed investment decisions concerning retail structured products. *Journal of Banking & Finance*, *34*(6), 1400–1411. https://doi.org/10.1016/j.jbankfin.2009.12.007

Dong, Z., Tang, D., & Wang, X. (2023). Pricing vulnerable basket spread options with liquidity risk. *Review of Derivatives Research*, *26*(1), 23–50. https://doi.org/10.1007/s11147-022-09192-0

Engle, R. F. (1982). Autoregressive Conditional Heteroscedasticity with Estimates of the Variance of United Kingdom Inflation. *Econometrica*, *50*(4), 987. https://doi.org/10.2307/1912773

Entrop, O., & Fischer, G. (2020). Hedging costs and joint determinants of premiums and spreads in structured financial products. *Journal of Futures Markets*, *40*(7), 1049–1071. https://doi.org/10.1002/fut.22109

EUSIPA – European Structured Investment Products Association. (n.d.). Retrieved March 19, 2023, from https://eusipa.org/

Fabozzi, F. J. (2005). The Structured Finance Market: An Investor's Perspective. *Financial Analysts Journal*, *61*(3), 27–40. https://doi.org/10.2469/faj.v61.n3.2725

Fusai, G., Longo, G., & Zanotti, G. (2022). Interest rate structured products: Can they improve the risk–return profile? *The European Journal of Finance*, *28*(13–15), 1481–1512. https://doi.org/10.1080/1351847X.2021.1967180

Glasserman, P. (2010). *Monte Carlo methods in financial engineering*. Springer.

Glosten, L. R., Jagannathan, R., & Runkle, D. E. (1993). On the Relation between the Expected Value and the Volatility of the Nominal Excess Return on Stocks. *The Journal of Finance*, *48*(5), 1779–1801. https://doi.org/10.1111/j.1540-6261.1993.tb05128.x

Han, X., & Liu, W. (2022). Investors' Heterogeneous Preferences for Structured Financial Products in China: The Impact of Demographic Characteristics. *Review of Business Management*, *24*(3), 458–471. https://doi.org/10.7819/rbgn.v24i3.4187

Hanaki, N. (2022). Risk misperceptions of structured financial products with worst-of payout characteristics revisited. *Journal of Behavioral and Experimental Finance*, *33*, 100604. https://doi.org/10.1016/j.jbef.2021.100604

Henderson, B. J., Pearson, N. D., & Wang, L. (2020). Pre-trade hedging: Evidence from the issuance of retail structured products. *Journal of Financial Economics*, *137*(1), 108–128. https://doi.org/10.1016/j.jfineco.2020.02.004

HKEX. (n.d.). Retrieved March 25, 2023, from https://www.hkex.com.hk/?sc_lang=en

Kallio, M., Halme, M., Dehghan Hardoroudi, N., & Aspara, J. (2022). Transparent structured products for retail investors. *European Journal of Operational Research*, *302*(2), 752–767. https://doi.org/10.1016/j.ejor.2022.01.014

Koh, B. S. K., Koh, F., Chuen, D. L. K., Guan, L. K., Ng, D., & Fai, P. K. (2015). A Risk- and Complexity-Rating Framework for Investment Products. *Financial Analysts Journal*, *71*(6), 10–28. https://doi.org/10.2469/faj.v71.n6.2

Kotadia, C. (2021). Evaluation of Equity-linked structured products and pricing. *International Journal for Innovation Education and Research*, *9*(1), 314–335. https://doi.org/10.31686/ijier.vol9.iss1.2902

Kunz, A. H., Messner, C., & Wallmeier, M. (2017). Investors' risk perceptions of structured financial products with worst-of payout characteristics. *Journal of Behavioral and Experimental Finance*, *15*, 66–73. https://doi.org/10.1016/j.jbef.2017.07.005

Lee, H., Ahn, S., & Ko, B. (2019). Generalizing the reflection principle of Brownian motion, and closed-form pricing of barrier options and autocallable investments.

The North American Journal of Economics and Finance, *50*, 101014. https://doi.org/10.1016/j.najef.2019.101014

Lee, H., Choi, Y. H., & Lee, G. (2022). Multi-step barrier products and static hedging. *The North American Journal of Economics and Finance*, *61*, 101676. https://doi.org/10.1016/j.najef.2022.101676

Lee, H., Lee, M., & Ko, B. (2022). A semi-analytic valuation of two-asset barrier options and autocallable products using Brownian bridge. *The North American Journal of Economics and Finance*, *61*, 101704. https://doi.org/10.1016/j.najef.2022.101704

Lee, T.-S., & Lin, H.-Y. (2009a). The pricing of structured notes with credit risk. *Investment Management and Financial Innovations*, *6*(4), 13.

Lee, T.-S., & Lin, H.-Y. (2009b). The pricing of structured notes with credit risk. *Investment Management and Financial Innovations*, *6*(4), 13.

McCulloch, J. H., & Lee, S. H. (2008). *Estimation of Risk Neutral Measures using the Generalized Two-Factor Log-Stable Option Pricing Model*.

Neftci, S. N. (2008). Essentials of Structured Product Engineering. In *Principles of Financial Engineering* (pp. 513–546). Elsevier. https://doi.org/10.1016/B978-0-12-373574-4.50020-2

Nelson, D. B. (1991). Conditional Heteroskedasticity in Asset Returns: A New Approach. *Econometrica*, *59*(2), 347. https://doi.org/10.2307/2938260

Palupi, I., Sitorus, I. U., & Umbara, R. F. (2015). Using least-square Monte Carlo simulation to price American multi underlying stock options. *2015 3rd International Conference on Information and Communication Technology (ICoICT)*, 504–509. https://doi.org/10.1109/ICoICT.2015.7231476

Pelster, M., & Schertler, A. (2019). Pricing and issuance dependencies in structured financial product portfolios. *Journal of Futures Markets*, *39*(3), 342–365. https://doi.org/10.1002/fut.21978

Raghunathan, C., & Cecchini, A. (2007). A Primer on Structured Products. *The RMA Journal*, *89*(6), 30–33. ABI/INFORM Collection; Accounting, Tax & Banking Collection.

Reiner, E. S. (1991). Breaking down the barriers. Risk Magazine, 4(8), 28–35.

Sato, K. (1999). *Lévy processes and infinitely divisible distributions*. Cambridge University Press.

Scopus—Analyze search results. (n.d.). Retrieved March 19, 2023, from https://www-scopus-com.proxy.uwasa.fi/term/analyzer.uri?sort=plf-f&src=s&sid=82f5eec1adff37ff07b09f22664b3635&sot=a&sdt=a&cluster=scosubjabbr%2c%22ECON%22%2ct&sl=113&s=TITLE-ABS-

KEY%28structured+AND+financial+AND+securities+OR+products+OR+Bonds%29+AND+PUBYEAR+%3e+2009+AND+PUBYEAR+%3c+2023&origin=resultslist&count=10&analyzeResults=Analyze+results

Sharpe, W. F. (1978). *Investments*. Prentice-Hall.

Sheehan, M. (2017). Selling structured products to retail investors in the US. *Journal of Investment Compliance*, *18*(3), 59–63. https://doi.org/10.1108/JOIC-06-2017-0041

Song, S., Tang, D., Xu, G., & Yin, X. (2023). An analytical GARCH valuation model for spread options with default risk. *International Review of Economics & Finance*, *83*, 1–20. https://doi.org/10.1016/j.iref.2022.08.013

SSPA | Swiss Structured Products Association. (n.d.). Retrieved March 25, 2023, from https://sspa.ch/en/

Stoimenov, P. A., & Wilkens, S. (2005). *Are structured products ÔfairlyÕ priced? An analysis of the German market for equity-linked instruments*. 23.

T. Renz, H., Kalisch, I., Pfister, S., Axford, S., & M. Williams, Jr., G. (2014). ESMA opinion on structured retail products – good practices for product governance arrangements. *Journal of Investment Compliance*, *15*(4), 19–24. https://doi.org/10.1108/JOIC-09-2014-0043

Thaler, R. (1980). Toward a positive theory of consumer choice. *Journal of Economic Behavior & Organization*, *1*(1), 39–60. https://doi.org/10.1016/0167-2681(80)90051-7

Thonabauer, G., & Nationalbank, Ö. (2004). *Financial Instruments: Structured products handbook*. OENB. https://books.google.ru/books?id=K_hYXwAACAAJ

Tversky, A., & Kahneman, D. (1992). Advances in Prospect Theory: Cumulative Representation of Uncertainty. *Journal of Risk and Uncertainty*, *5*(4), 297–323. JSTOR.

Ueberhuber, C. W. (1997). *Numerical computation: Methods, software, and analysis*. Springer.

Vokata, P. (2021). Engineering lemons. *Journal of Financial Economics*, *142*(2), 737–755. https://doi.org/10.1016/j.jfineco.2021.04.035

Wang, M., Keller, C., & Siegrist, M. (2011). The Less You Know, the More You Are Afraid of—A Survey on Risk Perceptions of Investment Products. *Journal of Behavioral Finance*, *12*(1), 9–19. https://doi.org/10.1080/15427560.2011.548760

Wei, W., & Zhu, D. (2022). Generic improvements to least squares monte carlo methods with applications to optimal stopping problems. *European Journal of Operational Research*, *298*(3), 1132–1144. https://doi.org/10.1016/j.ejor.2021.08.016

Wilmott, P., Dewynne, J., & Howison, S. (2000). *Option pricing: Mathematical models and computation* (Repr. with corrections). Oxford Financial Press.

Yuan, S., & Rieger, M. O. (2021). Diversification with options and structured products. *Review of Derivatives Research*, *24*(1), 55–77. https://doi.org/10.1007/s11147-020-09169-x

Zongrun, W., Tangtang, H., & Xuekai, N. (2020). A Portfolio Optimization Perspective Based on Tri-Reference Point Theory for Explaining the Attractiveness of Structured Products. *ECONOMIC COMPUTATION AND ECONOMIC CYBERNETICS STUDIES AND RESEARCH*, *54*(1/2020), 177–196. https://doi.org/10.24818/18423264/54.1.20.12

Zuev, D. (2017). A Non-Gaussian Pricing Model for Structured Products. *Journal of Corporate Finance Research / Корпоративные Финансы | ISSN: 2073-0438*, *11*(3), 45–58. https://doi.org/10.17323/j.jcfr.2073-0438.11.3.2017.45-58

Zvan, R., Vetzal, K. R., & Forsyth, P. A. (2000). PDE methods for pricing barrier options. *Journal of Economic Dynamics and Control*, *24*(11–12), 1563–1590. https://doi.org/10.1016/S0165-1889(00)00002-6