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Bitcoin growth amid climate change: Policy and investment implications

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ABSTRACT

Bitcoin contributes to global carbon emissions on a scale comparable to entire countries in order to secure its decentralized network. This exposes Bitcoin to climate policies aimed at reducing emissions. This paper develops a general equilibrium framework to examine how the stringency of climate policy affects Bitcoin's valuation and its relationship with the equity market. Our theoretical analysis delivers a key insight: a transition from a lenient to a stringent climate policy increases the conditional correlation between Bitcoin and equity returns, thereby compromising Bitcoin's appeal as a hedge or diversifier against equity market volatility. Empirical evidence supports this theoretical prediction.

1. Introduction

Do climate policies influence Bitcoin's value and its relationship with other assets? If so, how and through what mechanisms? These questions are timely and particularly important for several reasons. First, Bitcoin operates on a decentralized network with no central regulatory oversight. To secure this network, Bitcoin relies on an energy-intensive protocol known as Proof-of-Work (PoW), which consumes energy on a scale comparable to that of mid-sized industrialized countries such as Poland and New Zealand (Ren and Lucey, 2022).[1] A significant share of this energy is sourced from fossil fuels.[2] As a result, Bitcoin has emerged as a substantial contributor to global carbon emissions. According to Digiconomist, a single Bitcoin transaction generates approximately 764.67 kg of CO_2, resulting in an annualized carbon footprint rivaling that of entire nations like Qatar, a leading fossil fuel producer and exporter.[3]

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[1] The PoW protocol involves a lottery-like process among participants called miners, who use specialized hardware consuming vast amounts of electricity to generate hashes—fixed-length outputs produced by the SHA-256 algorithm. The process continues until one miner finds a hash that meets the target hash for a block. The winning miner receives newly minted Bitcoin (block rewards) and transaction fees. Because of the significant energy required and the probabilistic nature of mining, a malicious actor would need to control more than half of the network's hash rate—a so-called 51% attack—to compromise the blockchain. As the network's energy consumption and hash rate grow, the cost and difficulty of such an attack rise accordingly, enhancing Bitcoin's security (De Vries, 2018; Antonopoulos, 2014).

[2] According to the Cambridge Centre for Alternative Finance (CCAF), as of January 2022, fossil fuels such as coal and natural gas accounted for nearly 62% of Bitcoin's total electricity usage. (https://ccaf.io/cbnsi/cbeci/ghg).

[3] https://digiconomist.net/bitcoin-energy-consumption; Access date: 2025-07-08.

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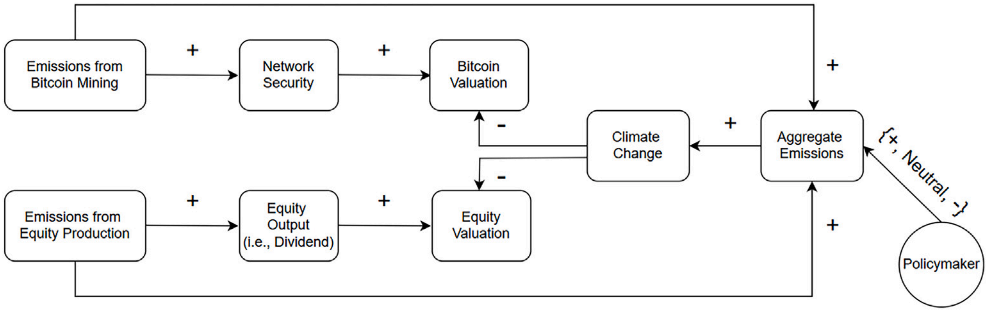

Fig. 1. This figure illustrates the key elements of our theoretical model. A "+" symbol indicates an increase or a positive relationship (e.g., enhanced network security increases Bitcoin's valuation), while a "−" symbol denotes a decrease or a negative relationship (e.g., climate change negatively affects the valuation of both Bitcoin and equities). Policymakers may decrease aggregate emissions through climate policies, maintain a neutral impact by abstaining from policy intervention, or increase aggregate emissions by lifting existing policies.

Second, in the absence of central regulatory oversight, Bitcoin must continually signal robust network security to encourage user adoption and preserve its status as both a mainstream cryptocurrency and a transactional medium. Because security in Bitcoin's PoW protocol is directly proportional to energy intensity, the prosperity of Bitcoin and its mining industry depends on sustained, large-scale energy consumption. Third, there is strong market interest and a solid demand base for Bitcoin, driven in part by popular narratives. These narratives portray Bitcoin as "digital gold" and a reliable store of value or hedge against financial instability. Coupled with speculative investment, such views have helped Bitcoin's market capitalization surpass $2 trillion as of May 2025. If this trajectory continues, it could lead to a significantly larger carbon footprint.

In this paper, we develop a stylized general equilibrium framework to address these questions. Specifically, we model an overlapping-generations economy in which investors in each generation can allocate their wealth among three assets: Bitcoin, equity, and a risk-free asset. Bitcoin provides both financial returns as an investment asset and transactional benefits as a medium of exchange. The process of Bitcoin mining requires energy consumption to maintain network security, and as security strengthens, Bitcoin's value increases. However, this enhanced security comes at an environmental cost: since a substantial share of energy used in Bitcoin mining comes from fossil fuels, higher energy consumption leads to greater emissions, thereby exacerbating climate change.

Equity refers to the shares of companies that produce goods and services. Equity production also consumes energy and generates emissions. The aggregate emissions from Bitcoin mining and equity production contribute to the rising concentration of greenhouse gases, particularly carbon dioxide, in the atmosphere—thereby accelerating global warming. Rising global temperatures trigger climate-related events, such as natural disasters, that damage or impair physical assets and destroy wealth. Following Barnett et al. (2020), among others, we incorporate a climate damage function to capture the economic impact of these physical climate shocks. A key property in our framework is that, all else being equal, the same level of investment in Bitcoin would result in higher emissions compared to equity. This is because energy is the sole input to Bitcoin mining and absorbs the entire investment, whereas equity production allocates investment across multiple forms of inputs, including energy, physical capital, intangible capital, and labor.

We then solve for equilibrium under various scenarios. Specifically, we compare a baseline scenario without climate policy (*no-policy*) to a scenario in which climate policy is enforced (*policy-enforced*). Within the policy-enforced scenario, we analyze two counterfactual cases: one with a lenient climate policy and another with a stringent one. Fig. 1 illustrates the key components and mechanisms of our model.

Our findings are as follows: In the baseline no-policy scenario, investors internalize the trade-off between the financial benefits of higher emissions and the associated welfare losses from climate change. A defining feature of the model is that emissions from one asset class generate a negative externality, which adversely affects the valuation of other assets. For instance, Bitcoin's emissions intensify climate damages, which adversely affect equity valuations. In the no-policy scenario, investors rationally account for these cross-asset externalities, reallocating capital to achieve the optimal financial outcomes. However, the resulting allocation may be socially suboptimal, as each generation lives for only a limited period and fails to fully internalize the long-term, cumulative climate damages their emission-intensive investments impose on future generations.

A noteworthy pricing effect in the no-policy scenario is a U-shaped relationship between Bitcoin's expected return and its carbon emissions. This U-shaped relationship emerges because Bitcoin valuation is subject to two opposing forces: on the one hand, higher emissions elevate exposure to climate risk, which lowers Bitcoin's value (climate risk effect); on the other hand, under the PoW protocol, higher energy use—and thus a greater emissions footprint—is associated with increased network security, which elevates Bitcoin's value (security effect). The interaction of these opposing forces (security effect vs. climate risk effect) gives rise to this U-shaped relationship: up to a certain emissions threshold, Bitcoin's value increases in response to higher emissions as the security effect dominates, leading to a lower expected return. Beyond this threshold, however, the climate risk effect overtakes the security effect, driving down Bitcoin's value (and increasing its expected return).

In the policy-enforced scenario, we incorporate climate policy into our model in the form of a cap (or restriction) on emissions. This policy can be viewed as a central planner's strategy to mitigate climate change by imposing a limit on permissible temperature increases.[4] The introduction of climate policy gives rise to a new market-clearing condition, which captures the trade-off involved in allocating the limited emissions budget between equity and Bitcoin. We then explore the model's implications under various scenarios.

First, we analyze a scenario in which miners are heavily reliant on fossil fuels (i.e., emissions-intensive) and climate policy is lenient (characterized by a high emissions cap). This scenario is reminiscent of Bitcoin's early days. In this setting, an incremental unit of emissions from mining raises Bitcoin's value, as the gain from enhanced network security outweighs the discount associated with Bitcoin's exposure to climate risk. In contrast, equity production, which competes with Bitcoin mining for a limited emissions budget, does not experience comparable output gains from an incremental unit of emissions, but still suffers a valuation discount due to Bitcoin's negative externality and its exposure to climate risk. Because Bitcoin and equity respond in opposite directions to an incremental increase in their respective emissions under a lenient climate policy, higher emissions lower the conditional correlation between Bitcoin and equity returns, suggesting that Bitcoin may serve as a hedge against equity.

Second, we examine a scenario in which miners remain emissions-intensive, but a stringent climate policy is in place. This setting resembles more recent years in Bitcoin's evolution. Under this scenario, the conditional correlation between Bitcoin and equity returns becomes positive. This shift occurs because under a stringent climate policy with a tight emissions cap: (1) miners' capacity to bolster network security through higher use of fossil fuels is reduced, thereby negatively affecting Bitcoin's value, and (2) equity valuation is also negatively impacted as stricter regulations constrain production. Consequently, both assets experience a discount in value and exhibit closer co-movement. Hence, Bitcoin's effectiveness as a hedge against equity erodes. This theoretical finding aligns with prior empirical evidence showing that, while Bitcoin used to be a hedge or a safe haven asset, this property has diminished in recent years (Bouri et al., 2017). It should be noted that our findings do not rule out other potential mechanisms that could explain the observed trend in Bitcoin's hedging and safe haven properties.

Third, we analyze a scenario in which Bitcoin miners transition to using renewable energy sources. This scenario reflects the contemporary trend in the Bitcoin ecosystem that is expected to continue. Under this scenario, given that miners shift their resources from fossil fuels to renewables, the emissions footprint of Bitcoin mining declines. Our analysis indicates that using renewable energy not only mitigates the externality imposed on equity but also increases the conditional correlation between Bitcoin and equity returns. As a result, with the widespread adoption of green energy in Bitcoin mining, Bitcoin's potential to serve as a hedge for equity diminishes.

Lastly, we examine the impact of a sudden and significant shift in climate policy. We theoretically show that an abrupt transition from a lenient to a stringent policy regime increases the conditional correlation between Bitcoin and equity returns. This suggests that Bitcoin's potential to serve as a hedge for equity diminishes in the face of such a sudden policy shift.

We provide empirical evidence to support our theoretical predictions. Our central hypothesis, based on the scenarios outlined above, is that a transition toward more stringent climate policy increases the conditional correlation between Bitcoin and equity returns, thereby weakening Bitcoin's role as a hedge against equity market fluctuations. To test this, we empirically analyze three notable events that reflect a transition to more stringent climate policies. The first event is the European Commission's issuance of the European Green Deal in December 2019. The second event targets Bitcoin emissions directly: in May 2021, China's State Council banned Bitcoin mining due to its significant energy consumption and climate impact, among other reasons. The third event is the U.S. Inflation Reduction Act (IRA), enacted in August 2022, where one of its primary goals was to address the climate crisis and reduce emissions levels by 50% to 52% below the 2005 level by 2030. For our empirical analysis, we compute the time-varying conditional correlation between Bitcoin and equity returns using the DCC-GARCH model (Engle, 2002). Our empirical analysis robustly shows that, consistent with theory predictions, the conditional correlation between Bitcoin and equity returns increased following these events. We conduct several robustness tests to (i) address endogeneity concerns and (ii) demonstrate that Proof-of-Stake (PoS) cryptocurrencies—which, unlike Bitcoin's PoW protocol, rely on an energy-efficient consensus mechanism—do not exhibit the same trend in their conditional correlations with equity returns.

2. Related literature

This paper bridges two strands of literature: (1) the economics of cryptocurrencies—particularly Bitcoin—and their interactions with financial markets; and (2) the economic and financial implications of climate change.

Within the first strand, several studies have explored Bitcoin's potential as a safe haven and its hedging properties for equity and other asset classes. Some studies find that Bitcoin can serve as a safe haven or hedging instrument for equities (Baur et al., 2018; Guesmi et al., 2019; Dyhrberg, 2016; Bouri et al., 2020), while others highlight its hedging capabilities for commodities (Bouri et al., 2017) and certain currencies (Urquhart and Zhang, 2019). These studies employ various methodologies, including GARCH-based models, quantile connectedness, and ordinary least squares (OLS) regressions, to assess Bitcoin's effectiveness as a hedging instrument. At the same time, other research suggests that Bitcoin is at best a weak hedge (Shahzad et al., 2019), whereas some studies conclude that it does not function as a safe haven at all (Conlon and McGee, 2020; Conlon et al., 2020). Our paper contributes to this literature in two ways. First, it develops a microfounded theoretical framework that explains Bitcoin's hedging role. Second, it helps reconcile the mixed empirical evidence by showing that Bitcoin's hedging effectiveness depends on the stringency of climate policy and therefore varies across time and policy regimes.

[4] Climate science shows a near-linear relationship between cumulative carbon emissions and the Earth's surface temperature (Fernández-Villaverde et al., 2024). Thus, a cap on emissions effectively implies a cap on temperature increases.

The second strand centers on the economic and financial implications of climate change, particularly those arising from negative externalities. Foundational contributions by Nordhaus (1992, 2008, 2017) quantify the welfare losses associated with global warming. More recent work has examined the social cost of carbon (SCC) and emphasized the need for policies such as emissions caps and carbon taxes to mitigate environmental damages (Hambel et al., 2021; Cai and Lontzek, 2019; Weitzman, 2012; Barnett et al., 2020). Further studies, including Edenhofer et al. (2024); Dietz et al. (2018), explore how transition risks and climate mitigation policies influence asset valuations. Our paper extends this literature by examining the environmental externalities generated by a novel technological innovation based on blockchain technology: cryptocurrencies. Specifically, we demonstrate that the energy-intensive nature of PoW cryptocurrencies can create negative spillovers that affect the valuation of other financial assets. We further show that such externalities can be mitigated through appropriately stringent climate policies.

In addition to these two strands, our study engages with a growing body of empirical research documenting the significant environmental harms (De Vries, 2018; De Vries and Stoll, 2021; de Vries, 2024) and adverse economic effects (Benetton et al., 2023) associated with Bitcoin's energy-intensive operations. For instance, Papp et al. (2023) estimate that a \$1 increase in the price of Bitcoin results in \$3.11–\$6.79 in external damages from carbon emissions. Building on these findings, our model makes two key contributions: (1) it provides a theoretical foundation that supports and explains these empirical observations, and (2) it offers policymakers a conceptual framework to understand and address Bitcoin's environmental challenges while minimizing disruptions to broader financial markets and economic activity.

3. Model

In this section, we develop a stylized general equilibrium model to investigate the effects of climate policy—particularly its varying levels of stringency—on Bitcoin and equity valuations, as well as their conditional correlation. We begin by developing a baseline model with no climate policy in place (No-policy scenario). Next, we incorporate climate policy into the model (Policy-enforced scenario) and assess how different levels of policy stringency affect asset valuations and their conditional correlation.

3.1. The economy

We model an economy populated by overlapping generations of investors.[5] Each generation lives for two dates, with generation t born at time t (young) and dies at time $t+1$ (old). Within generation t, there is a representative agent endowed with initial wealth W_t. The agent seeks to maximize the utility of terminal wealth while accounting for potential wealth destruction from climate change events such as natural disasters. The utility function and climate damage model will be specified later.

There are three assets available for investment: an equity (or stock), Bitcoin (or PoW cryptocurrency), and a risk-free asset which is in zero net supply. Let R_t^k denote the return on asset type k, where $k = b,e$, with e denoting equity and b denoting Bitcoin. Additionally, let R_f represent the return on the risk-free asset. The equity return reflects changes in the market value of a company producing goods and services. The Bitcoin return encompasses capital gains along with convenience yields derived from the transactional benefits offered by the Bitcoin platform.

We assume that, within generation t, economic activities—such as company production and Bitcoin mining—occur at time t, generating a liquidating dividend in the form of terminal wealth, which is paid out to investors at time $t+1$. The representative investor optimally chooses a portfolio of these three assets to maximize the utility of terminal wealth. The representative investor's budget constraint is:

$$W_{t+1} = W_t \left(R_f + X_t^b (R_{t+1}^b - R_f) + X_t^e (R_{t+1}^e - R_f) \right) \tag{1}$$

where X_t^b and X_t^e are the portfolio weights of Bitcoin and equity in generation t.

Moving forward, we first characterize and microfound the economic forces underlying Bitcoin. We do the same for the equity. Subsequently, we derive the equilibrium returns for each asset by developing a CAPM-like pricing relation.[6] Building on this equilibrium framework, we develop propositions that elucidate how climate policy, and its degree of stringency, influence the expected returns and the conditional correlation between Bitcoin and equity returns.

Bitcoin. Bitcoin is a decentralized digital asset built on blockchain technology. Owing to its distinctive design, the Bitcoin network facilitates unique transactional activities, including rapid cross-border money transfers with minimal regulatory friction, user anonymity, and potential tax advantages. In Bitcoin, no central authority enforces honest behavior or validates transactions. Instead, decentralized trust is maintained through a cryptographic consensus mechanism known as Proof-of-Work (PoW). Under PoW, miners compete in a lottery-like process by expending computational resources to solve cryptographic puzzles, validate transactions, and append new blocks of transactions to the blockchain. This process requires substantial energy input to generate cryptographic hashes,

[5] We use the overlapping generations framework for two reasons. First, it facilitates the analysis of intergenerational externalities. Second, it is particularly suitable for our analysis because Bitcoin is an intrinsically worthless asset (Schilling and Uhlig, 2019), whose capital gains are driven by speculation. This implies that demand for Bitcoin as an investment asset—and the pursuit of capital gains—relies on the emergence of new generations willing to buy Bitcoin in anticipation of future financial gains. Consistent with this reasoning, prior studies have also employed the overlapping generations setting to analyze Bitcoin (Biais et al., 2023; Sockin and Xiong, 2023). It is worth noting that the claim that Bitcoin is intrinsically worthless does not preclude it from having a fundamental transactional value. For further discussion, see Biais et al. (2023) and Pagnotta (2022).

[6] The Capital Asset Pricing Model (CAPM) is a seminal framework for asset pricing originally developed by Sharpe (1964). Our setup (an OLG model with utility defined over terminal wealth) is particularly suited for deriving a CAPM-like relation, as demonstrated in prior work such as Acharya and Pedersen (2005).

and the competition continues until a miner finds a hash that matches a block's target. The rate of energy expenditure is captured by a metric called network hash rate, with a higher hash rate indicating greater energy use per unit of time.

Because success in the PoW mining competition depends on costly energy investment, attempts to launch an attack to manipulate the transaction ledger require substantial energy costs. As aggregate energy consumption rises, so does the energy cost of a successful attack, thereby enhancing network security. Network security thus evolves endogenously with aggregate mining energy consumption.[7]

This energy-intensive process is crucial for Bitcoin's transactional functionality and user adoption. Without adequate security, users and investors would be reluctant to hold Bitcoin or transact on its network, as their wealth could be compromised. Conversely, strong network security attracts more participants and enhances Bitcoin's efficiency as a transactional medium. In this sense, network security can be viewed as a public good supplied by miners: it is non-rival and non-excludable, benefiting all network users, while no single miner captures the full value it generates.

To formulate network security and its associated energy consumption, we follow Antonopoulos (2014) and define Bitcoin network security as the cost required to acquire a majority of the network's hash rate. This definition reflects the core mechanics of the PoW protocol, in which consensus power is proportional to each participant's contribution to total hash rate. If an attacker controls a majority of the hash rate, they can manipulate the blockchain's main chain in a so-called 51% attack.

The network hash rate is generated by the collective computing power of Bitcoin miners. Mining devices consume energy (electricity) as an input to produce hashes as an output. The efficiency of a mining device is determined by the number of hashes generated per unit of energy consumed. Defining g as the mining device efficiency,[8] the relationship between Bitcoin's energy consumption (E_t^b) and the network hash rate (H_t) can be expressed as:

$$H_t = gE_t^b \tag{2}$$

Given the above-mentioned definition of security and Eq. (2), the network security can be expressed as:

$$S_t = P_t^{eng} \cdot \frac{H_t}{g} \tag{3}$$

where P_t^{eng} represents the price of energy. In the main text, we assume that energy is supplied perfectly elastically, so that $P_t^{eng} \equiv \bar{P}^{eng}$. In the Online Appendix, we relax this assumption and endogenize the energy price by linking it to aggregate energy demand arising from both Bitcoin mining and equity production. Our baseline scenario assumes that miners rely exclusively on fossil fuels for hash generation. In Section 4.4, we extend the model to allow miners to use a mix of green and fossil fuel energy sources.

Emissions from Bitcoin mining are derived from the following relation:

$$Emission_t^b = \psi^b E_t^b \tag{4}$$

where $E_t^b = \frac{H_t}{g}$. Also, ψ^b represents the emission intensity of the energy sources used in Bitcoin mining.

As discussed above, network security can be interpreted as a form of public good essential for sustaining Bitcoin's role as a platform that delivers distinct transactional benefits. These benefits constitute a form of convenience yield associated with holding Bitcoin and reflect the interaction between the platform's intrinsic productivity[9] and its network security. Accordingly, we capture this relationship using a Cobb–Douglas aggregation of network security (S_t) and platform productivity (A_t):

$$\lambda_t = S_t^{\alpha} A_t^{1-\alpha} \tag{5}$$

where $0 < \alpha < 1$. The Cobb-Douglas framework is particularly well-suited for modeling transactional benefits, as it encapsulates the concept that when network security approaches zero within Bitcoin's decentralized structure, the transactional benefits also effectively reduce to zero. Formally speaking, network security and platform productivity are complementary to a certain degree: without sufficient network security, users cannot fully exploit Bitcoin's platform productivity. The Cobb-Douglas aggregator captures this relationship effectively.

Besides transactional benefits, Bitcoin also offers capital gain, which is speculative.[10] We assume that a representative investor benefits from the convenience yield of holding Bitcoin when young, then liquidates their holdings to consume the capital gain in old age. The aggregate gain in generation t, representing Bitcoin's overall return, is characterized as follows:

$$R_{t+1}^b = \lambda_t^{\theta} \left(R_{t+1}^{fb} \right)^{1-\theta} \tag{6}$$

where $0 < \theta < 1$. R_{t+1}^{fb} denotes the rate of capital gain for Bitcoin from t to $t+1$. We employ the Cobb-Douglas aggregator here for two reasons: first, it facilitates the derivation of a closed-form solution; second, it reflects the reality of Bitcoin as a digital asset. Intuitively, if Bitcoin offered no capital gain, investors might turn to alternative mechanisms for their transactional needs, such as traditional banks or payment systems like Visa and Mastercard. Such a shift would collapse demand for Bitcoin, negatively impacting its return. Conversely, if Bitcoin provided no unique transactional benefits tied to its blockchain technology, it would function purely

[7] For further discussion of PoW mining and its associated energy consumption in securing the Bitcoin network, see Pagnotta (2022) and Chapter 8 of Antonopoulos (2014).

[8] We assume a constant efficiency for simplicity. Our core findings are not affected by time-varying device efficiency.

[9] Productivity in this context refers to how Bitcoin's technological design and architecture enable certain transactional activities.

[10] In this paper, we do not focus on speculative dynamics within or across generations, as is evident from our choice of a representative agent framework. Instead, we simply treat the speculative nature of Bitcoin as a given fact.

as a speculative asset without fundamental support, eventually causing its value to converge to zero. Thus, Bitcoin's financial gains and transactional benefits are complementary to some degree—a relationship that the Cobb-Douglas functional form effectively captures.

Equity. Equity generates output using a combination of capital and energy inputs, which is conceptualized as dividends and characterized as follows:

$$D^e_{t+1} = (E^e_t)^\delta K_t^{1-\delta} \tag{7}$$

where $0 < \delta < 1$, and K_t denotes the capital stock,[11] and E^e_t denotes the energy consumption in equity production.

We also assume that investors do not speculate on equity, meaning the return on equity comes exclusively from fundamental sources. This assumption implies a constant price–dividend ratio.[12]

The emissions associated with equity have the following structure:

$$Emission^e_t = \psi^e E^e_t \tag{8}$$

where ψ^e represents the emission intensity of the energy sources used to produce equity output.

We now present a key property that underpins our analysis. Although we treat production quantities as exogenous for tractability, this property is formally derived in Appendix C by characterizing the optimal resource allocation for Bitcoin mining and equity production.

Property 1. *A given level of investment in Bitcoin mining, ceteris paribus, generates more emissions than the same level of investment in equity production. This implies that when $\psi^e = \psi^b$, we have:*

$$Emission^b_t > Emission^e_t \tag{9}$$

Moving forward, we assume that $\psi^e = \psi^b$ for tractability, unless stated otherwise.

3.2. Climate change

The accumulation of greenhouse gases, primarily carbon dioxide (CO_2) and methane (CH_4), in the atmosphere leads to an increase in Earth's temperature. This temperature rise, commonly referred to as climate change, can have substantial economic consequences.[13] Accordingly, the first step in modeling climate change is to quantify the aggregate emissions produced within the economy:

$$Emission^{agg}_t = X^b_t Emission^b_t + X^e_t Emission^e_t \tag{10}$$

Then, drawing on climate science studies such as Matthews et al. (2009), we posit an approximate linear relationship between aggregate emissions and Earth's temperature:

$$T_t \approx \kappa \left(Emission^{agg}_t + Emission^{stock}_{t-1} \right) \tag{11}$$

$$where \quad Emission^{stock}_{t-1} = \sum_{s=0}^{t-1} Emission^{agg}_s$$

where κ is the Transient Climate Response to cumulative Emissions (TCRE), and $Emission^{stock}_{t-1}$ denotes the stock of emissions inherited from the previous generations.

Climate Damages. An increase in Earth's temperature would trigger climate change events that result in asset damage or impairment. Following Golosov et al. (2014); Barnett (2024), we define the climate damage function as follows:

$$D(T_t) = \exp(-\eta T_t) \tag{12}$$

Eq. (12) demonstrates that the climate damage function serves as a discount factor, effectively capturing economic losses induced by climate change. The specification in (12) is deliberately stylized to facilitate the derivation of closed-form solutions. In Section 5, we generalize the formulation to allow for greater damage convexity and the possibility of climate tipping points.

3.3. The representative investor

The representative investor maximizes her utility, accounting for the fact that climate change events may cause financial losses. Accordingly, she incorporates the adverse impact of climate damages on her terminal wealth into her optimization framework and

[11] The capital stock here provides a broad measure that aggregates physical capital, human capital, and intangible assets such as patents.

[12] A constant price–dividend ratio implies that the equity return, $R^e_{t+1} = \frac{P^e_{t+1}+D^e_{t+1}}{P^e_t}$, is lognormally distributed. As discussed later, this facilitates the derivation of a closed-form solution for equilibrium expected returns.

[13] Predictions suggest that, under certain scenarios, climate change-induced events could even reduce global GDP by up to 25% by 2100 (IMF, 2019).

selects an optimal portfolio by solving the following problem:

$$\underset{X_t^b, X_t^e}{Max}\, E_t \left[U \left(D(T_t) W_{t+1} \right) \right] \tag{13}$$

subject to the budget constraint (1). The representative investor is assumed to have Constant Relative Risk Aversion (CRRA) preference over her terminal wealth:

$$U \left(D(T_t) W_{t+1} \right) = \frac{(D(T_t) W_{t+1})^{1-\gamma}}{1-\gamma} \tag{14}$$

where γ denotes the coefficient of relative risk aversion. The CRRA utility function is chosen for analytical convenience, as it allows for the derivation of a closed-form solution. In the Online Appendix, we also provide solutions using alternative preferences, such as Constant Absolute Risk Aversion (CARA).

It is important to note that in the representative investor's optimization problem, the damage function is indexed by time t (i.e., $D(T_t)$) while wealth is entered with time $t+1$ (i.e., W_{t+1}). To explain the rationale, recall that W_{t+1} represents a lump-sum liquidating dividend paid out to the representative investor, who is born at time t and exits the market at $t+1$. This dividend can be adversely affected by physical climate events (e.g., storms, floods, heat anomalies) that occur during the interval $[t, t+1)$, just before the dividend is paid at $t+1$ and the investor exits the market. In this discrete-time setting, the interval $[t, t+1)$ is captured by the time-t observation. Consequently, the lump-sum dividend is indexed by $t+1$ while the damage function is indexed by t.

3.4. Climate policy

In our model, climate policy is conceptualized as the effort of a policymaker or social planner to limit emissions, thereby slowing their accumulation in the atmosphere and mitigating the rise in Earth's temperature. The primary objective of these policies is to place an upper bound on the potential economic damages arising from climate change. In the literature, such efforts by policymakers and social planners to combat climate change are commonly referred to as a *transition*, and the risk it poses to economic activities—especially carbon-intensive ones—is termed *transition risk*.[14]

We characterize climate policy as an emissions cap imposed by the policymaker. Since our focus is not on optimal policy design, we treat the cap as exogenously given. Introducing an emissions cap establishes a market-clearing condition with the following structure:

$$X_t^b Emission_t^b + X_t^e Emission_t^e = Cap_t \tag{15}$$

A higher Cap_t indicates a less stringent policy, while a lower Cap_t reflects a more stringent policy aimed at reducing emissions.

4. Equilibrium

4.1. No-policy scenario

In this section, we derive the structure of equilibrium returns and develop a CAPM-like pricing relation in the absence of climate policy. We then compare these results with those in the subsequent section, where climate policy is incorporated into the model. Finally, we present a proposition demonstrating how the stringency of climate policy impacts the conditional correlation between Bitcoin and equity returns.

To derive equilibrium returns, we assume that all variables follow a lognormal distribution. This assumption aligns with the empirical distribution of most financial variables and allows for the derivation of closed-form, log-linear relationships for asset prices under the CRRA–lognormal framework.

Based on the characteristics of the CRRA-lognormal model, the representative investor's optimal portfolio takes the following structure:

$$X_t = \frac{1}{\gamma} \Sigma^{-1} \left(E_t(\mathbf{r_{t+1}} - r_f 1) + \frac{1}{2} \sigma^2_{\mathbf{rt}} - \eta \mathbf{T_t} \right) \tag{16}$$

where $X_t = (X_t^b, X_t^e)$ represents the vector of portfolio weights, $r_{t+1}^k = \log R_{t+1}^k$ denotes the log-return of asset k, and $\mathbf{T_t} = \left(T_t^b, T_t^e \right)'$ where $T_t^e = \kappa(Emission_t^e + Emission_{t-1}^{Stock})$ and $T_t^b = \kappa(Emission_t^b + Emission_{t-1}^{Stock})$ denote the earth's temperature if the entire wealth were invested in equity and Bitcoin, respectively. The variance–covariance matrix is given by $\Sigma = \sigma_{\mathbf{rt}} \sigma'_{\mathbf{rt}}$. Note that for Bitcoin, from Eq. (6), we have $r_{t+1}^b = \theta \log \lambda_t + (1-\theta) r_{t+1}^{fb}$. The addition of the one-half variance term ($\frac{1}{2}\sigma^2_{\mathbf{rt}}$) is necessary to convert log-return to simple return, which is the ultimate concern of the representative investor (Campbell and Viceira, 2002).

The optimal portfolio consists of two main components. The first component, $\Sigma^{-1} \left(E_t(\mathbf{r_{t+1}} - r_f 1) + \frac{1}{2}\sigma^2_{\mathbf{rt}} \right)$, represents the tangency portfolio, which offers the highest Sharpe ratio. The second component, $\Sigma^{-1} \left(-\eta \mathbf{T_t} \right)$ reflects the investor's hedging demand against climate damages. In fact, the deviation from the tangency portfolio is a strategic adjustment to hedge against climate damages. Therefore, the second component presents an important insight: in the absence of policy measures to protect against climate damages, the representative investor internalizes these potential damages and adjusts her portfolio accordingly.

[14] For further discussion on transition risk, see Bolton and Kacperczyk (2023).

In the market equilibrium, the market portfolio weights represent the equilibrium portfolio weight of each risky asset. Let X_t^M denote the vector of market portfolio, which consists of the two risky assets: Bitcoin and equity. The equilibrium expected returns of Bitcoin and equity have the following structure:

$$\log E_t\left[\frac{R_{t+1}^{fb}}{R_f}\right] = \frac{\gamma}{1-\theta} e_1' \Sigma X_t^M + \frac{\eta}{1-\theta} T_t^b - \frac{\theta}{1-\theta} \log \lambda_t \tag{17}$$

$$\log E_t\left[\frac{R_{t+1}^{e}}{R_f}\right] = \gamma e'_2 \Sigma X_t^M + \eta T_t^e \tag{18}$$

where e_i represents the i^{th} basis vector. Eqs. (17) and (18) reveal how climate damages reshape the equilibrium expected returns. Specifically, the equilibrium expected return of equity has two components: The first component, $\gamma e'_2 \Sigma X_t^M$, represents the compensation for financial risk and is positive. The second component, ηT_t^e, captures the equity alpha,[15] which arises because the representative investor internalizes the equity's contribution to climate damages.

The equilibrium expected return of Bitcoin has three components: The first component, $\frac{\gamma}{1-\theta} e'_1 \Sigma X_t^M$, reflects the compensation for financial risk. Intuitively, the representative investor requires a risk premium for Bitcoin price volatility. The second component, $\frac{\eta}{1-\theta} T_t^b$, captures a shift in expected return to compensate for Bitcoin's contribution to climate damages. The third component, $-\frac{\theta}{1-\theta} \log \lambda_t$, reflects the gain from Bitcoin's transactional benefits, which are indirectly tied to emissions through network security, as discussed in Section 3.1. The second and third components together shape the Bitcoin alpha. Bitcoin alpha is subject to two opposing forces: higher emissions increase Bitcoin's contribution to climate damages (thus raising alpha), but simultaneously enhance network security (which lowers alpha). We analyze the implications of this interaction when studying the U-shaped relationship between Bitcoin alpha and Bitcoin emissions.

A key insight from the equilibrium expected return relations is that when internalizing climate damages in her portfolio decision, the representative investor only considers the impact of climate change on her own generation (as they are short-lived) and does not internalize its long-term consequences. This behavior gives rise to an intergenerational externality, highlighting the necessity of climate policy even when investors internalize the climate impact of their investment decisions. In the presence of such an externality, even low-emission assets may be excessively discounted due to inherited emissions.

U-Shaped Relationship. Considering Bitcoin alpha, denoted by $\alpha_b = \frac{\eta}{1-\theta} T_t^b - \frac{\theta}{1-\theta} \log \lambda_t$, an increase in Bitcoin emissions ($Emission_t^b$) generates two opposing forces: on one hand, higher emissions reduce Bitcoin's value by exposing investors to climate damages; on the other hand, they increase Bitcoin's value by enhancing network security. We intend to analyze the net effect of Bitcoin emissions ($Emission_t^b$) on Bitcoin alpha (α_b).

In the Online Appendix, we show that α_b and $Emission_t^b$ exhibit a U-shaped relationship, with α_b reaching a minimum at a specific level of emissions. This U-shape arises because higher emissions signal stronger network security for Bitcoin; however, investors simultaneously account for the climate impact of Bitcoin emissions. Initially, as the positive benefit of enhanced network security outweighs the negative impact of climate damages, α_b decreases until it reaches its minimum. Beyond this point, concerns over climate damages become more significant, causing α_b to rise as the adverse effects of emissions intensify.

Interestingly, we also show that when the stock of emissions inherited from previous generations is sufficiently low (i.e., $Emission_{t-1}^{Stock} < \overline{Emission}$),[16] α_b becomes negative, meaning that Bitcoin investors are rewarded for higher emissions as the benefits of stronger network security outweigh the adverse climate impacts of Bitcoin mining. This finding presents a significant concern, as it suggests that when a generation inherits a sufficiently low stock of emissions from previous generations, investors may be incentivized to allocate increasing amounts of capital to Bitcoin without adequately accounting for future intergenerational externalities.

We conduct a simulation, presented in Fig. 2, which demonstrates the U-shaped relationship between α_b and $Emission_t^b$ and illustrates the conditions under which α_b becomes negative when $Emission_{t-1}^{Stock} < \overline{Emission}$. Parameter values used for the simulation are provided in the Online Appendix. If Bitcoin network security were not associated with carbon emissions, the relationship between α_b and network security would have been linear, with α_b increasing linearly with $Emission_t^b$.

To establish a benchmark for comparison with conventional asset pricing models, we develop a CAPM-like pricing relation in the following proposition.

Proposition 1. *The expected excess return of assets in the absence of climate policy follows a CAPM-like pricing relation described below:*

$$\log E_t\left[\frac{R_{t+1}^{fb}}{R_f}\right] = \hat{\beta}_t^b \log E_t\left[\frac{R_{t+1}^{M}}{R_f}\right] + \hat{\eta}(T_t^b - \beta_t^b T_t) - \frac{\theta}{1-\theta} \log \lambda_t \tag{19}$$

$$\log E_t\left[\frac{R_{t+1}^{e}}{R_f}\right] = \beta_t^e \log E_t\left[\frac{R_{t+1}^{M}}{R_f}\right] + \eta(T_t^e - \beta_t^e T_t) \tag{20}$$

[15] In the CAPM literature, a shift in expected returns beyond the compensation for aggregate market volatility, which we refer to as financial risk, is known as "alpha."

[16] The threshold $\overline{Emission}$ is determined in the Online Appendix.

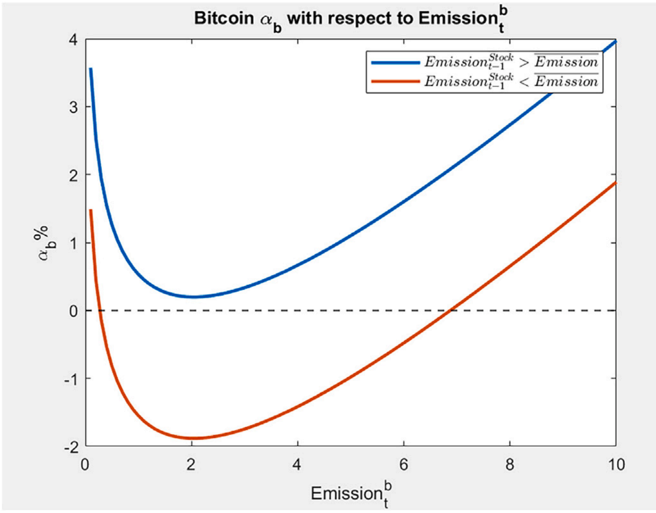

Fig. 2. This figure presents a simulation of the value of α_b for different levels of $Emission_t^b$. As is evident, when $Emission_{t-1}^{Stock} < \overline{Emission}$, the α_b becomes negative, meaning that investors will be rewarded from higher Bitcoin emissions.

where β_t^e and β_t^b *denote the (conditional) systematic risk exposure to the market portfolio for equity and Bitcoin, respectively. Additionally,* $T_t^e = \kappa(Emission_t^e + Emission_{t-1}^{Stock})$ and $T_t^b = \kappa(Emission_t^b + Emission_{t-1}^{Stock})$, *which represent the Earth's temperature if the entire wealth were invested in equity and Bitcoin, respectively.*[17]

Proof. *See Appendix C.*

A key takeaway from Proposition 1 is that climate damages impose a positive risk premium when an asset's contribution to climate change exceeds its financial contribution to the market. The financial contribution of an asset to the market is measured by the β_t^k coefficient, which reflects the asset's loading on the market portfolio. Specifically, if $T_t^k > \beta_t^k T_t$, then climate damages necessitate a positive risk premium on asset k.

Another key takeaway is that, for Bitcoin, higher emissions may result in a negative risk premium. This occurs because the component of Bitcoin's alpha associated with transactional benefits, $-\frac{\theta}{1-\theta}\log \lambda_t$, is negatively log-linearly related to Bitcoin's emissions. As emissions increase, this term becomes more negative, thereby increasing Bitcoin's valuation. The interaction between this negative force on Bitcoin's alpha and the positive force associated with climate damage—which gives rise to the U-shaped relationship discussed earlier—warrants further investigation to understand the impact of climate policy on Bitcoin's overall pricing and its relationship with equity. Specifically, incorporating climate policy could potentially alter this U-shaped relationship, with significant implications for asset prices. We explore this dynamic in the following section.

4.2. Policy-enforced scenario

We incorporate climate policy into the baseline model presented in the previous section and demonstrate that varying levels of policy stringency have notable implications for Bitcoin and equity valuations. Specifically, we show that the conditional correlation between Bitcoin and equity returns is influenced by the stringency of climate policy. This finding, as will be discussed further, has important implications for Bitcoin's hedging properties and its role as a safe haven asset.

To develop these findings, we first define the concept of a climate policy regime. Let the critical threshold $\overline{Cap}$ distinguish between stringent and lenient policy regimes. Specifically, the policy is classified as *stringent* when $Cap_t < \overline{Cap}$, and as *lenient* when $Cap_t > \overline{Cap}$, where $\overline{Cap}$ is derived in Appendix C. Also, let $e > 0$ denote a small, strictly positive value. Incorporating climate policy into the baseline model, we establish in Appendix C that the following proposition holds.

Proposition 2. *Variations in the stringency of climate policy influence the conditional correlation between Bitcoin and equity returns:*

- *When Bitcoin acts as a hedge for equity (i.e.,* $corr_t(r_t^b, r_t^e) < 0$ *), stricter climate regulations increase the conditional correlation between Bitcoin and equity, i.e.,* $\frac{\partial corr_t(r_t^b, r_t^e)}{\partial Cap_t} < 0$*;*
- *When Bitcoin acts as a diversifier for equity (i.e.,* $0 < corr_t(r_t^b, r_t^e) < e$*), we find that*[18]
  - *Under a stringent policy regime (i.e.,* $Cap_t < \overline{Cap}$ *), stricter climate regulations increase the conditional correlation between Bitcoin and equity, i.e.,* $\frac{\partial corr_t(r_t^b, r_t^e)}{\partial Cap_t} < 0$*;*

[17] Note that $\hat{\beta}_t^b = \frac{\beta_t^b}{1-\theta}$ and $\hat{\eta} = \frac{\eta}{1-\theta}$ where their derivations are explained in the Appendix.

[18] *The same results applies when Bitcoin and equity are strongly correlated meaning that* $e < corr_t(r_t^b, r_t^e) < 1$.:

– *Under a lenient policy regime (i.e,* $Cap_t > \overline{Cap}$ *), stricter climate regulations lower the conditional correlation between Bitcoin and equity, i.e.,* $\frac{\partial corr_t(r_t^b, r_t^e)}{\partial Cap_t} > 0$.

A shift from a lenient policy regime to a stringent policy regime, ceteris paribus, increases the conditional correlation between Bitcoin and equity returns:

$$corr_t^{Stringent}(r_t^b, r_t^e) - corr_t^{Lenient}(r_t^b, r_t^e) > 0 \tag{21}$$

Proof. *See Appendix C.*

Proposition 2 presents several important insights. First, when Bitcoin serves as a hedge for equity, a tighter emissions cap (i.e., stricter climate policy) increases the conditional correlation between the two assets, thereby weakening Bitcoin's effectiveness as a hedge. Second, when Bitcoin functions as a diversifier for equity—meaning it has a low correlation and may act as a safe haven during equity market fluctuations—a stringent policy regime also increases the conditional correlation, undermining Bitcoin's safe haven properties. However, a lenient policy regime helps Bitcoin retain its role as a safe haven asset. These findings offer a potential explanation for empirical evidence suggesting that Bitcoin initially exhibited hedging or safe haven characteristics (see, e.g., Dyhrberg (2016)), but that this role has gradually weakened over time (see, e.g., Shahzad et al. (2019)). Our analysis suggests that (at least part of) this trend may reflect the global transition toward stricter emissions regulations. Additionally, Proposition 2 shows that a sudden transition (i.e., a jump) from a lenient to a stringent policy regime increases the conditional correlation between Bitcoin and equity returns. We provide empirical evidence for these theoretical predictions in Section 6.

4.3. Externality with and without climate policy

Negative externality is a crucial aspect of climate change, referring to situations where certain activities contribute to climate change, but the resulting negative consequences affect the entire economy (Nordhaus, 1991, 2017). Our theoretical framework captures negative externality at the intergenerational level, as discussed in Section 4.1. In this section, we examine the asset pricing implications of this externality. In particular, we establish two main results: (1) Bitcoin emissions impose a negative externality on equity, and (2) implementing climate policies effectively mitigates this externality.

We aim to capture the following negative externality mechanism: As Bitcoin emissions increase relative to equity emissions, equity becomes more exposed to physical climate damages, since excessive Bitcoin emissions exacerbate climate change. This heightened exposure increases the perceived riskiness of equity, leading to a decline in its valuation and, consequently, a rise in the equity premium.

To formalize this mechanism, we define $\Delta Emission_t = Emission_t^b - Emission_t^e$ to represent the difference between Bitcoin and equity emissions. An increase $\Delta Emission_t$ indicates that Bitcoin is becoming more emission-intensive relative to equity. While higher $\Delta Emission_t$ directly affects Bitcoin, the presence of a negative externality implies an indirect effect on equity, meaning that the equity premium is also influenced by $\Delta Emission_t$.

Proposition 3. *When* $\Delta Emm_t > \Delta Emm^*$ *, then we have:*

$$\frac{\partial \log E_t\left[\frac{R_{t+1}^e}{R_f}\right]}{\partial \Delta Emission_t} > 0 \tag{22}$$

where $\Delta Emission^*$ *is defined in the Appendix.*

Proof. *See Appendix C.*

The intuition behind Proposition 3 is straightforward. When Bitcoin emissions substantially exceed those of equity, Bitcoin imposes a negative externality on equity, increasing the equity premium and thereby depressing equity valuation. The existence of the emission threshold ($\Delta Emission_t > \Delta Emission^*$) suggests that Bitcoin must be considerably more polluting than equity for the negative externality to take effect.

An important follow-up question is whether climate policy can mitigate this negative externality. We address this question in the following proposition. Before stating the proposition, let's define $\log E_t^{Policy}\left[\frac{R_{t+1}^e}{R_f}\right]$ as the equity premium when a binding climate policy is in place, and $\log E_t^{No-Policy}\left[\frac{R_{t+1}^e}{R_f}\right]$ as the equity premium in the absence of climate policy.

Proposition 4. *The equity premium is less sensitive to* $\Delta Emission_t$ *when a climate policy exists compared to a scenario where there is no climate policy:*

$$\frac{\partial \log E_t^{Policy}\left[\frac{R_{t+1}^e}{R_f}\right]}{\partial \Delta Emission_t} < \frac{\partial \log E_t^{No-Policy}\left[\frac{R_{t+1}^e}{R_f}\right]}{\partial \Delta Emission_t} \tag{23}$$

Proof. *See Appendix C.*

Proposition 4 demonstrates that implementing climate policy mitigates the negative externality Bitcoin imposes on the equity. This is evident from the reduced sensitivity of the equity premium to $\Delta Emission_t$ in the presence of climate policy compared to the scenario with no climate policy in place.

Limitation and Discussion. A key modeling assumption is the use of a representative agent in each generation. This setup is well suited to our primary objective of characterizing the implications of the intergenerational externality for Bitcoin and equity valuation, as well as for the relationship between the two assets. However, this approach does not capture within-generation externality, which arises when the portfolio choices of some agents generate disproportionately high emissions while the associated climate damages are borne collectively by all investors within the same generation. This channel is absent in our framework because the representative-agent structure effectively assumes a "universal owner" within each generation who holds both Bitcoin and equity. Extending the framework to incorporate heterogeneous agents with different portfolio allocations would provide a natural way to capture this dimension of the externality.

4.4. Green mining and its implications for externality

What are the implications of adopting green practices in Bitcoin mining? This question has become increasingly relevant as the mining industry shifts toward a stronger emphasis on sustainability.[19] In light of this, we investigate whether green practices can effectively mitigate Bitcoin's environmental impact and the resulting negative externality on equity.

Increasing the share of renewable energy in Bitcoin mining reduces its emission intensity, meaning the same level of energy consumption generates fewer emissions. To formalize this relationship, let μ represent the share of fossil fuels and $1-\mu$ the share of green resources used in Bitcoin mining where $0<\mu<1$. The aggregate Bitcoin energy consumption can then be expressed as:

$$E_t^b = \mu E_t^{b,fossil} + (1-\mu)E_t^{b,green} \tag{24}$$

Given that $\psi^{b,green}=0$ (the emission intensity of green resources is zero) and $\psi^{b,fossil}=\psi^b$, the aggregate emission intensity becomes $\hat{\psi}^b = \mu\psi^b < \psi^b$. We now introduce the concept of emission efficiency, defined as the inverse of emission intensity. Higher emission efficiency indicates reduced emissions for a given level of energy consumption. Let $f^b = \frac{1}{\psi^b}$ denote the emission efficiency of Bitcoin mining. As the share of green resources increases (i.e., μ decreases and $1-\mu$ increases), emission efficiency improves. As demonstrated in the following proposition, adopting green practices, which enhance emission efficiency, effectively mitigates the negative externality on equity.

Proposition 5. *There exists a negative association between the equity premium and the emission efficiency of Bitcoin mining, meaning that higher emission efficiency reduces the equity premium, thereby mitigating the negative externality on equity.*

$$\frac{\partial \log E_t\left[\frac{R^e_{t+1}}{R_f}\right]}{\partial f^b} < 0 \tag{25}$$

Proof. *See Appendix C.*

Proposition 5 emphasizes that green mining practices can mitigate the climate-related externality associated with Bitcoin mining. Consequently, it suggests that promoting green mining initiatives not only helps reduce Bitcoin's environmental footprint but also mitigates the negative externality generated by Bitcoin's emissions.

5. More reactive damage functions

We initially introduced the stylized exponential damage function (12) for analytical convenience. However, a growing body of literature emphasizes that the economic implications of climate change can be highly sensitive to the functional form of climate damages, particularly when damages are assumed to be moderate or weakly convex (e.g., Weitzman (2012)). In this section, we address this concern by considering a more general damage function, which allows us to analyze the implications of higher damage convexity and the presence of climate tipping points for our equilibrium results. Following Barnett et al. (2020) and Barnett (2023), we define the generalized climate damage function as:

$$D(T_t) = e^{-(\eta T_t + \frac{\delta}{2}T_t^2 + \frac{\delta^+}{2}(T_t-\tau)^2 \mathbf{1}_{\{T_t-\tau>0\}})} \tag{26}$$

Similar to Eqs. (12), (26) implies that the climate damage function acts as a discount factor. The parameters η, δ, and δ^+ characterize the sensitivity of damages to temperature changes. The third term in the exponent, $\frac{\delta^+}{2}(T_t-\tau)^2\mathbf{1}_{\{T_t-\tau>0\}}$, captures a tipping point or a more severe damage scenario where damages escalate significantly beyond a critical temperature threshold. This component divides the economy into two regimes: a pre-tipping regime ($T_t \le \tau$) and a post-tipping regime ($T_t > \tau$). Henceforth, we refer to (12) as the *exponential damage function* and to (26) as the *quadratic–exponential damage function*.

Considering the quadratic–exponential damage function (26), we re-derive the equilibrium results and present two remarks that highlight the key implications of introducing higher damage convexity.

[19] https://www.forbes.com/sites/digital-assets/2023/11/06/the-green-frontier-bitcoin-minings-path-to-sustainability-and-profitability/.

5.1. No-policy scenario under higher damage convexity

Under the no-policy scenario, the representative investor's optimal portfolio takes the following form:

$$X_t = \frac{1}{\gamma}\hat{\Sigma}^{-1}\left(E_t(\mathbf{r_{t+1}} - r_f 1) + \frac{1}{2}\sigma^2_{\mathbf{rt}} - \eta^*\mathbf{T_t}\right) \tag{27}$$

The perceived variance–covariance matrix is given by $\hat{\Sigma} = \Sigma + \frac{\delta^*}{\gamma}M$, where $\Sigma = \sigma_{\mathbf{rt}}\sigma'_{\mathbf{rt}}$ and $M = \mathbf{T_t}\mathbf{T_t}'$. Notably, each element of the perceived variance–covariance matrix under the quadratic-exponential damage function is larger than under the exponential damage specification, as the quadratic damage term adds a second-moment loading to the variance-covariance matrix. This implies that incorporating a quadratic term in the damage function amplifies baseline risk perceptions; under higher damage convexity, adverse market outcomes become more severe, leading investors to perceive greater aggregate risk.

The parameters η^* and δ^* capture the effective damage parameters and depend on whether the economy is in the pre- or post-tipping regime:

$$\eta^* = \begin{cases} \eta & if\ \ T_t \le \tau \\ \eta - \delta^+\tau & if\ \ T_t > \tau \end{cases} \qquad \delta^* = \begin{cases} \delta & if\ \ T_t \le \tau \\ \delta + \delta^+ & if\ \ T_t > \tau \end{cases}$$

As in the exponential damage case, the optimal portfolio under the quadratic-exponential damage function consists of two distinct components, though each component adjusts to reflect higher convexity. The first component, $\hat{\Sigma}^{-1}\left(E_t(\mathbf{r_{t+1}} - r_f 1) + \frac{1}{2}\sigma^2_{\mathbf{rt}}\right)$, represents a *modified* tangency portfolio, where the modification arises from the influence of climate damages on the variance–covariance structure. This modified tangency portfolio therefore maximizes the climate-adjusted Sharpe ratio. The second component, $\hat{\Sigma}^{-1}\left(-\eta^*\mathbf{T_t}\right)$, reflects the investor's hedging demand against climate damages. Notably, the scale of hedging demand depends on whether the economy is in the pre- or post-tipping regime.

The equilibrium excess returns of Bitcoin and equity have the following structure:

$$\log E_t\left[\frac{R^{fb}_{t+1}}{R_f}\right] = \frac{\gamma}{1-\theta}e'_1\hat{\Sigma}X^M_t + \frac{\eta^*}{1-\theta}T^b_t - \frac{\theta}{1-\theta}\log\lambda_t \tag{28}$$

$$\log E_t\left[\frac{R^{e}_{t+1}}{R_f}\right] = \gamma e'_2\hat{\Sigma}X^M_t + \eta^* T^e_t \tag{29}$$

where e_i represents the i^{th} basis vector. The overall structure of equilibrium expected returns remains similar to the exponential damage case, but several nuances emerge. In particular, in the first component of the equity expected return, $\gamma e'_2\hat{\Sigma}X^M_t$, the compensation for exposure to financial risk now depends on the curvature of the damage function. When the damage function is highly convex (i.e., when $\delta^* > 0$), adverse temperature realizations generate disproportionately large wealth destruction, thereby amplifying the financial risk. Accordingly, exposure to this amplified financial risk requires higher risk compensation. This risk amplification effect can be formally expressed as $\frac{\partial(\gamma e'_2\hat{\Sigma}X^M_t)}{\partial\delta^*} > 0$, for $\delta^* > 0$.

The second component, $\eta^* T^e_t$, which represents the equity alpha, is influenced by whether the economy is in the pre- or post-tipping regime.[20]

The equilibrium expected return of Bitcoin has three components: The first component, $\frac{\gamma}{1-\theta}e'_1\hat{\Sigma}X^M_t$, reflects the compensation for the exposure to the financial risk. As for equity, higher damage convexity increases this compensation because the financial risk is amplified. The second component, $\frac{\eta^*}{1-\theta}T^b_t$, captures a shift in expected return to compensate for Bitcoin's contribution to climate damages. The magnitude of this component depends on whether the economy is in the pre- or post-tipping regime. The third component, $-\frac{\theta}{1-\theta}\log\lambda_t$, reflects the gain from Bitcoin's transactional benefits.

A key intuition emerges when comparing the pre- and post-tipping regimes. In the post-tipping regime, the risk amplification mechanism becomes substantially more pronounced. Specifically, the effective linear loading η^* is larger in the pre-tipping regime, while the effective quadratic loading δ^* is larger in the post-tipping regime. This reflects the fact that, once the tipping point is crossed, climate damages transmit primarily through sharp increases in aggregate market risk rather than through smooth shifts in alphas.

We now re-derive Proposition 1, henceforth Proposition 1*, under higher damage convexity.

Proposition 1*. *Under damage function* (26)*, the expected excess return of assets in the absence of climate policy follows a CAPM-like pricing relation described below:*

$$\log E_t\left[\frac{R^{fb}_{t+1}}{R_f}\right] = \hat{\beta}^{b*}_t \log E_t\left[\frac{R^{M}_{t+1}}{R_f}\right] + \hat{\eta}^*(T^b_t - \beta^{b*}_t T_t) - \frac{\theta}{1-\theta}\log\lambda_t \tag{30}$$

[20] We note that the structure of expected returns derived in Eqs. (28) and (29), and the accompanying intuition, are based on our objective to present a CAPM-like framework. One could instead use the first-order conditions to rewrite the expected return relation in a different form—such as by separating M from Σ in the variance–covariance matrix. However, doing so would eliminate the ability to interpret the results within a CAPM framework.

$$\log E_t\left[\frac{R^e_{t+1}}{R_f}\right] = \beta^{e*}_t \log E_t\left[\frac{R^M_{t+1}}{R_f}\right] + \eta^*(T^e_t - \beta^{e*}_t T_t) \tag{31}$$

where β^{e*}_t and β^{b*}_t *denote the (conditional) systematic risk exposure to the market portfolio for equity and Bitcoin, respectively. Additionally,* $T^e_t = \kappa(Emission^e_t + Emission^{Stock}_{t-1})$ and $T^b_t = \kappa(Emission^b_t + Emission^{Stock}_{t-1})$*, which represent the Earth's temperature if the entire wealth were invested in equity and Bitcoin, respectively.*[21]

While the asset pricing structure in Proposition 1* retains the CAPM-like form presented in Proposition 1, two important distinctions arise. First, under the quadratic-exponential damage function, the economy is divided into pre-tipping and post-tipping regimes, implying that the magnitude of climate-related alpha depends on the prevailing regime. Second, and more importantly, higher damage convexity affects systematic risk exposure by altering the beta. This effect is summarized in the following remark.

Remark 1. *Under the quadratic–exponential damage specification, the systematic risk (beta) of asset k is endogenously reshaped by the extent to which asset k contributes to climate change. In particular, the effective beta is a weighted average of the asset's standard financial beta* (β^k_t) *and its relative contribution to climate change* ($\frac{T^k_t}{T_t}$)*. Specifically, in Appendix C.1.1, we show that*

$$\beta^{k*}_t = (1-\pi)\beta^k_t + \pi\frac{T^k_t}{T_t} \tag{32}$$

where $\pi \in (0,1) determined in the Appendix and increases with damage convexity, that is$

$$\frac{\partial \pi}{\partial \delta^*} > 0$$

implying that higher damage convexity increases the extent to which an asset's contribution to climate change magnifies its exposure to aggregate financial risk.

In summary, Remark 1 states that when climate damages are sufficiently convex, an asset's systematic risk (beta) is no longer determined solely by its financial covariance with the market. Instead, beta also reflects the asset's contribution to climate change, as assets that contribute more to climate change mechanically load more on the market downside risk.

5.2. Policy-enforced scenario under higher damage convexity

We re-derive the results associated with Proposition 2, henceforth Proposition 2*, under the quadratic-exponential damage function. An important distinction relative to the exponential damage case is that the endogenous critical threshold changes to reflect higher damage convexity. This adjustment arises because the representative investor views the policy regime in a different way when climate damages are highly destructive. Let the critical threshold $\overline{Cap^*}$ separate stringent from lenient climate policy regimes. Specifically, the policy is classified as *stringent* when $Cap_t < \overline{Cap^*}$, and as *lenient* when $Cap_t > \overline{Cap^*}$, where $\overline{Cap^*}$ is derived in Appendix C.[22] Also, as before, let $e > 0$ denote a small, strictly positive value. Incorporating climate policy into the baseline model, we establish in Appendix C that the following proposition holds.

Proposition 2*. *Under damage function* (26)*, variations in the stringency of climate policy influence the conditional correlation between Bitcoin and equity returns:*

- *When Bitcoin acts as a diversifier for equity (i.e.,* $0 < corr_t(r^b_t, r^e_t) < e$*), we find that*[23]
  - *Under a stringent policy regime (i.e.,* $Cap_t < \overline{Cap^*}$*), stricter climate regulations increase the conditional correlation between Bitcoin and equity, i.e.,* $\frac{\partial corr_t(r^b_t, r^e_t)}{\partial Cap_t} < 0$;
  - *Under a lenient policy regime (i.e,* $Cap_t > \overline{Cap^*}$*), stricter climate regulations lower the conditional correlation between Bitcoin and equity, i.e.,* $\frac{\partial corr_t(r^b_t, r^e_t)}{\partial Cap_t} > 0$.
- *When Bitcoin acts as a hedge for equity (i.e.,* $corr_t(r^b_t, r^e_t) < 0$*) and the damage function is mildly convex (i.e.,* $\delta^* < \bar{\delta}$*), stricter climate regulations increase the conditional correlation between Bitcoin and equity, i.e.,* $\frac{\partial corr_t(r^b_t, r^e_t)}{\partial Cap_t} < 0$;
- *When Bitcoin acts as a hedge for equity (i.e.,* $corr_t(r^b_t, r^e_t) < 0$*) and the damage function is highly convex (i.e.,* $\delta^* > \bar{\delta}$*), the conditional correlation responds to climate policy in the same way as if Bitcoin were a diversifier.*

A shift from a lenient policy regime to a stringent policy regime, ceteris paribus, increases the conditional correlation between Bitcoin and equity returns:

$$corr^{Stringent}_t(r^b_t, r^e_t) - corr^{Lenient}_t(r^b_t, r^e_t) > 0 \tag{33}$$

The core conclusions associated with the exponential damage (Proposition 2) continue to hold under higher damage convexity, with one important distinction highlighted in the following remark.

[21] Note that $\hat{\beta}^{b*}_t = \frac{\beta^{b*}_t}{1-\theta}$ and $\hat{\eta}^* = \frac{\eta^*}{1-\theta}$ where their derivations are explained in the Appendix.

[22] Note that the concept of climate policy regime differs from the climate change regime that separates the pre- and post-tipping regime.

[23] The same results applies when Bitcoin and equity are strongly correlated, meaning that $e < corr_t(r^b_t, r^e_t) < 1$.:

Remark 2. *When Bitcoin serves as a hedge for equity and the damage function is excessively convex, characterized by $\delta^* > \bar{\delta}$, the impact of climate policy on the conditional correlation between Bitcoin and equity returns mirrors that observed when Bitcoin acts as a diversifier for equity.*

The proof is provided in Appendix C.2.1. The intuition of Remark 2 is that when climate damages become sufficiently destructive, the representative investor prioritizes hedging against climate damages rather than smoothing financial returns by utilizing Bitcoin's hedging capacity. This mechanism becomes increasingly pronounced as the physical damages from climate change intensify, particularly once the economy enters the post-tipping regime.

The implications of higher damage convexity for Propositions 3 and 4—namely, Propositions 3* and 4*—are presented in Online Appendix.

6. Empirical evidence

The general equilibrium framework we developed predicts that a shift toward more stringent climate policies increases the conditional correlation between Bitcoin and equity returns (Proposition 2), thereby weakening Bitcoin's role as a hedging instrument for equity and as a potential safe haven. In this section, we provide empirical evidence in support of this theoretical prediction.

As a first step, we estimate the conditional correlation between Bitcoin and equity returns using the DCC-GARCH model. We then examine how the conditional correlation evolves around three major policy events, with the selected events representing critical instances in which policymakers introduced measures to restrict emissions and reduce reliance on fossil fuel-based energy as part of their climate change mitigation efforts.

The first event occurred in December 2019, when the European Commission announced the European Green Deal. This policy aims to reduce net greenhouse gas emissions by at least 55% by 2030 and pursues three primary goals: (1) net-zero greenhouse gas emissions by 2050, (2) economic growth decoupled from resource use, and (3) social and geographic inclusivity, ensuring no person or place is left behind.

The second event reflects China State Council's regulatory measure targeting Bitcoin mining due to its significant energy consumption and environmental impact, among other concerns. In May 2021, the State Council issued a statement announcing a crackdown on Bitcoin mining and highlighted the need to address the risks associated with cryptocurrency activities, especially those related to Bitcoin mining.[24]

The third event is the Inflation Reduction Act (IRA) by the U.S. government on August 16, 2022. One of the primary IRA goals was to address climate crises by cutting U.S. climate pollution by 50% to 52% below 2005 levels by 2030.[25]

Our analysis intentionally focuses on two types of policy events: one that specifically targets Bitcoin (the Mining Ban) and others that encompass broader climate policies affecting both Bitcoin and equity markets (the Green Deal and the IRA). This approach enhances the robustness of our empirical findings and addresses a potential concern: Bitcoin is fundamentally disconnected from the equity market and possesses unique characteristics—such as the pseudonymity of miners, which allows them to operate with minimal regulatory oversight and relocate easily without the constraints of corporate bureaucracy—that may insulate Bitcoin mining from the effects of broader climate policies. By examining both types of events, we effectively mitigate this concern.

We also address endogeneity concerns in Section 7.2. Specifically, policy decisions may be endogenous if they are influenced by conditions or characteristics of the Bitcoin market, the equity market, or both. To account for this possibility, we employ a Double-Selection LASSO estimator, as described in Section 7.2. This approach allows us to control for a high-dimensional set of confounding variables and helps mitigate biases arising from endogenous policy adoption.

6.1. Empirical strategy

Our empirical investigation consists of two building blocks. First, we apply the DCC-GARCH model developed by Engle (2002) to estimate the conditional correlation between Bitcoin and equity returns. Second, we assess how this conditional correlation changes around the three aforementioned policy events. As discussed earlier, we anticipate an increase in the conditional correlation between Bitcoin and equity returns following each policy event.

A key consideration in our empirical analysis, which will be discussed in detail, is the strong autoregressive pattern observed in the conditional correlation between Bitcoin and equity. Specifically, regressing the log of conditional correlation on its lagged value yields an R-squared of 77% (for the whole sample). This pattern guides our empirical analysis. Specifically, we examine whether there is an upward shift in the autoregressive trend, implying that the conditional correlation rises more sharply than its pre-existing trend following a policy event. In other words, we test whether the autoregressive coefficient increases in post-event periods relative to pre-event periods.

However, this approach raises a potential concern: the autoregressive coefficient may be time-varying, meaning any observed change could reflect natural dynamics rather than a policy effect. To address this issue, we employ a Logit regression framework, which helps mitigate this concern. Further details on this methodology are provided in subsequent sections.

[24] https://www.loc.gov/item/global-legal-monitor/2022-02-08/china-national-development-and-reform-commission-issues-notice-restricting-cryptocurrency-mining/.

[25] https://www.whitehouse.gov/cleanenergy/inflation-reduction-act-guidebook/.

In our analysis, Bitcoin return data are available on all weekdays, while equity indices are not traded on weekends, resulting in an unbalanced dataset when using daily time series. To resolve this, we construct a balanced dataset by calculating the average weekly returns for both Bitcoin and the equity indices.

6.1.1. DCC-GARCH model

We employ the Dynamic Conditional Correlation Generalized Autoregressive Conditional Heteroskedasticity (DCC-GARCH) model (Engle, 2002) to calculate the conditional correlation between Bitcoin and equity returns.

To apply DCC-GARCH, the first step is to compute the time series of Bitcoin and equity returns. While Bitcoin returns are straightforward to compute, the construction of equity returns requires the selection of an appropriate proxy for the equity market. Because our analysis examines policy interventions across three different regions, we construct a composite equity market index that captures equity market dynamics within these regions. Specifically, we select a well-established benchmark equity index from each region: the S&P 500 for the United States, the MSCI Europe Index for Europe, and the MSCI China Index for China.[26] We then extract the first principal component of these three return series to form a single composite equity index.

This composite index is particularly useful because (1) although each policy is implemented within a specific region, such interventions are likely to generate cross-border spillovers affecting global equity markets, and (2) many companies are either cross-listed in these indices or have international operations, making them exposed to policy shifts in other regions. For instance, approximately 13% of S&P 500 revenue originates from Europe, while about 19% comes from East Asia, underscoring how policies in Europe or China can influence U.S. equity markets. The principal-component-based index therefore provides a parsimonious measure that aggregates both common and orthogonal equity market movements across these regions, capturing spillover effects as well as firms' global operational exposure. We also conduct robustness tests by computing conditional correlations using alternative equity benchmarks, including the STOXX Europe 600, the Shanghai Composite Index, and the Russell 2000. Furthermore, we show that our main empirical results remain robust when region-specific equity indices, rather than the composite index, are used to compute the conditional correlations.

The DCC-GARCH model is implemented in two main steps:

Step 1: Modeling the Conditional Variance. Let $\sigma^2_{k,t}$ represent the conditional variance of asset k where $k = e,b$. The time series of returns for Bitcoin and equity follows the following structure:

$$r^k_t = \mu_k + \varepsilon^k_t \;\; where \;\; \varepsilon^k_t \sim N(0,\sigma^2_{k,t}) \tag{34}$$

$$\sigma^2_{k,t} = \alpha_k + \sum_{i=1}^{q} \beta_k (\varepsilon^k_{t-i})^2 + \sum_{l=1}^{p} \gamma_k \sigma^2_{k,t-l} \tag{35}$$

Step 2: Modeling the Conditional Correlation. Let C_t denote the conditional correlation matrix. C_t is modeled using the DCC model:

$$C_t = (Q^*_t)^{-1} Q_t (Q^*_t)^{-1} \tag{36}$$

$$Q_t = (1 - \sum_{m=1}^{M} \alpha_m - \sum_{n=1}^{N} \beta_n)\bar{Q} + \sum_{m=1}^{M} \alpha_m (\varepsilon_{t-m}\varepsilon'_{t-m}) + \sum_{n=1}^{N} \beta_n Q_{t-n}$$

$$Q^*_t = \begin{pmatrix} \sqrt{q_{11}} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \sqrt{q_{kk}} \end{pmatrix}$$

Q_t refers to the time-varying unconditional correlation matrix. $\bar{Q}$ represents the unconditional covariance of the standardized residuals. Q^*_t is a diagonal matrix that consists of the square roots of the diagonal elements from matrix Q_t.

A key practical consideration in implementing the DCC-GARCH model is the selection of appropriate GARCH orders. We do so by conducting ARCH tests and comparing information criteria (AIC and BIC) across alternative specifications; details are provided in the Online Appendix. Fig. 3 presents the conditional correlation between equity returns and Bitcoin returns as calculated from the DCC-GARCH model.

6.1.2. Impact of policy events on conditional correlation

We examine the impact of three major policy events on the conditional correlation between Bitcoin and equity returns: the European Commission's Green Deal (hereafter, Green Deal), China's ban on Bitcoin mining (hereafter, Mining Ban), and the Inflation Reduction Act (hereafter, IRA).

Based on our theoretical prediction, we hypothesize that the conditional correlation increases following these events, as each represents a shift toward more stringent climate policies. To test this hypothesis, we conduct an OLS analysis with the (log of) conditional correlation between Bitcoin return and equity return as the dependent variable. We consider the interaction of its lagged value with indicator variables representing the pre- and post-policy periods as explanatory variables. Additionally, we control for

[26] S&P stands for Standard & Poor's and MSCI stands for Morgan Stanley Capital International.

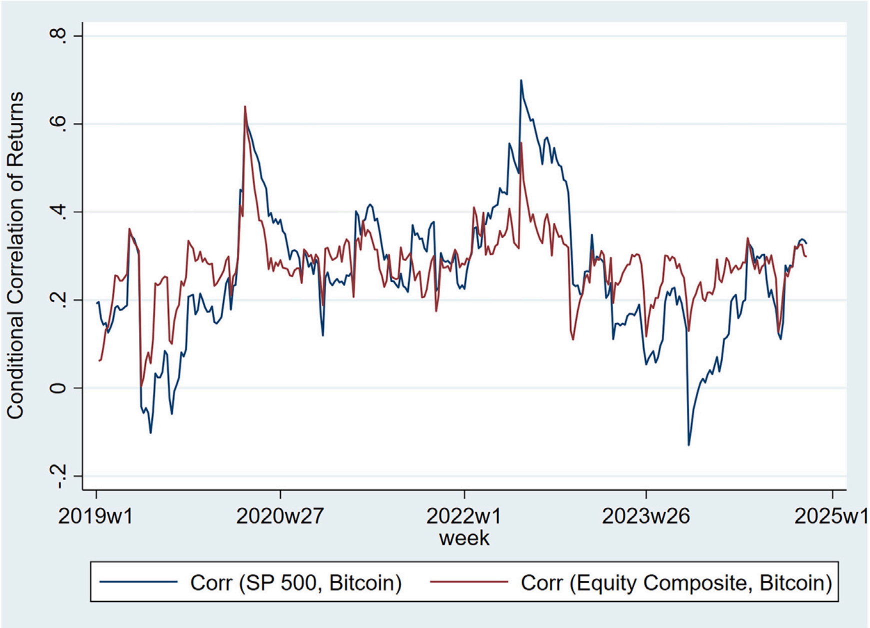

Fig. 3. Conditional correlation between Bitcoin returns and returns of the S&P 500 and a composite equity index constructed from the S&P 500, MSCI Europe, and MSCI China indices.

Table 1
OLS Regression Analysis.

VARIABLES	(1) c_t	(2) c_t	(3) c_t	(4) c_t	(5) c_t	(6) c_t
c_{t-1}^{Before} − *Green Deal*	0.433**	0.362**				
	(2.787)	(2.307)				
c_{t-1}^{After} − *Green Deal*	0.495***	0.419**				
	(3.334)	(2.770)				
c_{t-1}^{Before} − *Mining Ban*			0.590**	0.667***		
			(2.636)	(3.863)		
c_{t-1}^{After} − *Mining Ban*			0.628***	0.699***		
			(3.047)	(4.293)		
c_{t-1}^{Before} − *IRA*					0.596**	0.324
					(2.511)	(1.457)
c_{t-1}^{After} − *IRA*					0.613**	0.383*
					(2.816)	(1.869)
Constant	−0.658***	−0.788***	−0.506*	−0.400*	−0.395	−0.649***
	(−3.291)	(−3.897)	(−1.890)	(−1.941)	(−1.745)	(−3.051)
Controls	YES	NO	YES	NO	YES	NO
Observations	23	23	23	23	23	23
R-squared	0.571	0.364	0.570	0.511	0.501	0.213

Notes: This table reports the coefficients obtained from estimating Eq. (37) using ordinary least squares (OLS). t-statistics are reported in parentheses.*** p<0.01, ** p<0.05, * p<0.1.

the (log of) Bitcoin return and equity return. Logarithmic transformations are applied to ensure stationarity, making the variables suitable for OLS analysis. The regression specification is as follows:

$$c_t = \alpha_0 + \alpha_1 c_{t-1} \times \mathbf{1}_{Before} + \alpha_2 c_{t-1} \times \mathbf{1}_{After} + Controls_t + \varepsilon_t \tag{37}$$

where $c_t = \log C_t$. $\mathbf{1}_{Before}$ and $\mathbf{1}_{After}$ are indicator functions representing the periods before and after the policy event, respectively. Specifically, if the policy event occurs at time t^*, the indicator functions are defined as follows:

$$\mathbf{1}_{Before} = \begin{cases} 1 & if \;\; t < t^* \\ 0 & otherwise \end{cases} \qquad \mathbf{1}_{After} = \begin{cases} 1 & if \;\; t \geq t^* \\ 0 & otherwise \end{cases}$$

Our hypothesis posits that $\alpha_2 > \alpha_1$, indicating an upward shift in the autoregressive trend and implying that the conditional correlation increases more sharply than its pre-existing trend following a policy event. The OLS regression results in Table 1 confirm this theoretical prediction: $\alpha_2 > \alpha_1$ holds across policy events, both with and without control variables, supporting our argument that a shift to a more stringent policy would increase the conditional correlation between Bitcoin and equity returns and thereby weaken Bitcoin's hedging capacity.

To isolate the policy effects, we use a six-month window (12 weeks before and after each event). A wider window could contaminate the policy effect with unrelated incidents, while a narrower window would reduce observations, potentially compromising the statistical reliability of our results. For robustness, Appendix B presents additional tests using a longer, eight-month window, which yield consistent results. Finally, due to the relatively small sample size, we test the normality of c_t using the Shapiro–Wilk test. The resulting p-value is significantly greater than 0.05, leading us to reject the null hypothesis of non-normality. This confirms that c_t is normally distributed over the six-month interval, thereby validating the use of OLS in our setting.

We present a series of robustness tests and supplementary empirical evidence in the following section to support our main hypothesis. First, we construct an energy-efficient counterfactual for Bitcoin using Proof-of-Stake (PoS) cryptocurrencies and show that these assets do not exhibit patterns similar to those observed for Bitcoin in response to climate policy shifts. This result supports our argument that the observed patterns are driven by Bitcoin's energy- and emissions-intensive architecture (Section 6.2). Second, instead of estimating the impact of the policy shift on conditional correlations computed using a composite equity index, we analyze the impact of policy shifts in each region on conditional correlations computed using major equity indices specific to that region and show that our main findings remain robust (Section 7.1). Third, we address potential endogeneity concerns (Section 7.2). Fourth, we test whether the observed changes in conditional correlation stem from time trends in the AR coefficient (Section 7.3).

6.2. PoS cryptocurrencies vs. Bitcoin

Our theoretical findings and empirical results hinge on Bitcoin's reliance on the energy-intensive Proof-of-Work (PoW) consensus protocol. This implies that, in a counterfactual scenario where Bitcoin were energy-efficient, the relationships documented in the previous section would not persist. One way to approximate this counterfactual is by using cryptocurrencies that rely on Proof-of-Stake (PoS), an alternative consensus mechanism that is substantially more energy-efficient. PoS cryptocurrencies employ the same decentralized blockchain architecture, offering comparable transactional functionalities and exposure to similar economic shocks as Bitcoin. However, rather than relying on energy-intensive mining, PoS systems rely on validators who stake cryptocurrency to secure the network and validate transactions. This fundamental difference results in PoS networks consuming over 99% less energy than PoW networks such as Bitcoin (Gallersdörfer et al., 2022).

Accordingly, energy-efficient PoS cryptocurrencies should not exhibit the same pattern as the energy-intensive Bitcoin; specifically, their conditional correlation with equity returns should not increase following a shift toward more stringent climate policy. To test this, we employ two complementary approaches. First, we examine whether the conditional correlation between leading PoS cryptocurrencies and equity returns responds to climate policy shifts in the same way as Bitcoin. Second, we treat these PoS cryptocurrencies as a control group and implement a synthetic control analysis to construct a *synthetic Bitcoin* in the pre-policy period, and then assess whether this counterfactual asset replicates Bitcoin's post-policy behavior.

We focus on Cardano, Solana, and Algorand, which are among the largest PoS cryptocurrencies by market capitalization. Additionally, Cardano and Algorand were active during all three policy events considered, while Solana was present during two (the Mining Ban and the IRA). Other PoS cryptocurrencies were excluded either due to market capitalizations below one billion USD—which makes them more susceptible to speculative activity and excess volatility—or due to their absence during the policy events.

Under our first approach, we re-estimate Eq. (37), replacing the Dynamic Conditional Correlation (DCC) between Bitcoin and equity returns with the DCC between each PoS cryptocurrency (Solana, Algorand, and Cardano) and equity returns. Given the substantial spillover effects from Bitcoin to other cryptocurrencies—largely because Bitcoin consistently represents more than 50% of the total cryptocurrency market capitalization—we include the DCC between Bitcoin and equity returns as an additional control variable. We also control for equity returns and the returns of the respective PoS cryptocurrencies.

Under our second approach, we re-estimate Eq. (37) using the conditional correlation between *synthetic Bitcoin* and equity returns. We adopt the synthetic control method rather than a conventional Difference-in-Differences framework because an equally weighted portfolio of the above-noted PoS cryptocurrencies does not exhibit a parallel trend with Bitcoin returns prior to each policy shift. To address this issue, following Abadie et al. (2010) and related studies, we employ the synthetic control approach to closely replicate Bitcoin's pre-policy behavior. We report the associated regression results both with and without control variables; when included, these controls consist of equity and Bitcoin returns.

We present the results for our first approach in Panels A–C of Table 2 and for our second approach in Panel D of Table 2. The results indicate that neither PoS cryptocurrencies nor the synthetic Bitcoin exhibit the same pattern as Bitcoin—namely, an increase in conditional correlation with equity markets in response to climate policy shifts. On the contrary, in some instances we observe a negative shift in the conditional correlation. These findings reinforce our theoretical argument that the energy-intensive nature of the PoW consensus mechanism plays a central role in shaping the relationship between Bitcoin and equity returns in the context of climate-related regulatory changes.

7. Robustness tests

7.1. Alternative proxies for equity

This robustness test aims to demonstrate that our findings are not driven by the construction of the proposed composite equity index. Specifically, rather than focusing on a composite equity index, we use the major equity indices in each region that experienced

Table 2
PoS cryptocurrencies.

VARIABLES	Panel A: Cardano			Panel B: Algorand			Panel C: Solana		Panel D: Synthetic Bitcoin					
	(1)	(2)	(3)	(1)	(2)	(3)	(1)	(2)	(1)	(2)	(3)	(4)	(5)	(6)
$c_{t-1}^{Before} - Green\ Deal$	−0.0650			0.111					0.216	0.147				
	(−0.281)			(0.637)					(0.856)	(0.626)				
$c_{t-1}^{After} - Green\ Deal$	−0.0334			0.0114					0.273	0.222				
	(−0.177)			(0.0609)					(1.209)	(1.043)				
$c_{t-1}^{Before} - Mining\ Ban$		0.456*			0.0432		−0.0764				0.0819	0.183		
		(2.073)			(0.202)		(−0.264)				(0.313)	(0.840)		
$c_{t-1}^{After} - Mining\ Ban$		0.376			0.0270		−0.00976				0.106	0.200		
		(1.667)			(0.128)		(−0.0383)				(0.423)	(0.938)		
$c_{t-1}^{Before} - IRA$			−0.324**			0.630***		−0.148					0.401*	0.299
			(−2.221)			(2.931)		(−0.746)					(2.011)	(1.413)
$c_{t-1}^{After} - IRA$			−0.394**			0.605***		−0.269					0.387*	0.330
			(−2.690)			(3.393)		(−1.199)					(2.007)	(1.605)
Constant	0.449	−0.379	−0.838***	0.152	−0.529*	−0.245	−0.444	0.650	−0.755***	−0.862***	−1.148***	−1.015***	−0.611***	−0.687***
	(0.719)	(−0.907)	(−4.492)	(0.614)	(−1.934)	(−0.681)	(−0.350)	(1.355)	(−2.990)	(−3.640)	(−3.587)	(−3.771)	(−3.086)	(−3.277)
Controls	YES	YES	YES	YES	YES	YES	YES	YES	YES	NO	YES	NO	YES	NO
Observations	23	23	23	23	23	23	23	23	23	23	23	23	23	23
R-squared	0.512	0.295	0.688	0.742	0.550	0.612	0.083	0.544	0.302	0.113	0.134	0.060	0.448	0.147

Notes: Panels A–C report OLS estimates of Eq. (37), where the dependent variable is the dynamic conditional correlation (DCC) between equity returns and the returns of a PoS cryptocurrency—Cardano (Panel A), Solana (Panel B), and Algorand (Panel C). The core explanatory variable is the interaction of its lagged value with indicator variables representing the pre- and post-policy periods. Panel D reports OLS estimates of Eq. (37) using the DCC between equity returns and *synthetic Bitcoin*, constructed from PoS cryptocurrencies using the synthetic control method prior to each policy event. t-statistics are reported in parentheses. *** p<0.01, ** p<0.05, * p<0.1

Table 3
Alternative Proxies for Equity Market.

VARIABLES	(1) c_t^{EU}	(2) c_t^{EU}	(3) c_t^{China}	(4) c_t^{China}	(5) c_t^{US}	(6) c_t^{US}
$c_{t-1}^{EU,\,Before} - Green\ Deal$	−0.339*	−0.352*				
	(−1.879)	(−1.776)				
$c_{t-1}^{EU,\,After} - Green\ Deal$	−0.257	−0.245				
	(−1.511)	(−1.315)				
$c_{t-1}^{China,\,Before} - Mining\ Ban$			0.434*	0.473**		
			(1.798)	(2.113)		
$c_{t-1}^{China,\,After} - Mining\ Ban$			0.459*	0.492**		
			(2.063)	(2.361)		
$c_{t-1}^{US,\,Before} - IRA$					0.458*	0.447**
					(1.884)	(2.107)
$c_{t-1}^{US,\,After} - IRA$					0.514**	0.502**
					(2.352)	(2.512)
$\Delta\left(\frac{EUR}{USD}\right)$	21.74**	21.66**				
	(2.463)	(2.387)				
$\Delta\left(\frac{CNY}{USD}\right)$			0.0892	0.0787		
			(0.785)	(0.747)		
Constant	−1.653***	−1.689***	−1.455**	−1.336**	−0.314**	−0.334**
	(−7.188)	(−6.669)	(−2.476)	(−2.425)	(−2.264)	(−2.589)
Controls	YES	NO	YES	NO	YES	NO
Observations	23	23	23	23	23	23
R-squared	0.542	0.332	0.389	0.269	0.515	0.276

Notes: This table reports coefficient estimates from Eq. (37) estimated by ordinary least squares (OLS), where region-specific equity indices are used to compute the conditional correlation between Bitcoin and equity returns. t-statistics are reported in parentheses.*** p<0.01, ** p<0.05, * p<0.1.

a policy shift and compute the conditional correlation accordingly. Then, we run a similar regression as (37). Below, we provide the details of these indices.

European Equity Indices: To capture the European equity market movements, we utilize two European equity indices: the MSCI Europe Index and the STOXX 600. We use them to test our hypothesis for the Green Deal, which denotes a policy shift in Europe. To aggregate these indices into a single representative variable for DCC-GARCH estimation and regression analysis, we utilize their first principal component.

Chinese Proxies: To capture the Chinese equity market movements, we utilize two European equity indices: the MSCI China Index and the Shanghai Composite. Accordingly, they are used to test our hypothesis for the Mining Ban which represents a policy shift in China. We also use the first principal component to create a representative proxy for the Chinese equity market.

U.S. Proxies: To capture the U.S. equity market movements, we utilize S&P 500 Index along with two other U.S. equity indices: the MSCI U.S. Index and the Russell 2000 Index. We use their first principal component as a representative proxy for the U.S. equity market. This combined proxy is then used to test our hypothesis for the IRA, which is a policy shift that occurred in the U.S.

In the previous section, the equity index was constructed using the S&P 500 and MSCI Europe and China indices, all of which are expressed in U.S. dollars. In this section, we expand the set of equity indices to include additional benchmarks that are reported in their local currencies (for example, the STOXX 600 is expressed in euros). Because Bitcoin prices are also denominated in U.S. dollars, and our analysis in this section includes regional equity indices expressed in local currencies, we control for exchange rate fluctuations of these local currencies against the U.S. dollar when examining the conditional correlations between Bitcoin and the European and Chinese equity markets.[27]

The results reported in Table 3 further support the claim that the conditional correlation between Bitcoin and equity returns increases after a policy shift, thereby further supporting the robustness of our findings.

7.2. Addressing endogeneity concerns

A critical consideration in empirical investigations is the potential endogeneity of shifts in climate policy. Stricter policies may emerge as endogenous responses by policymakers to evolving conditions that shape the relationship between Bitcoin and equity markets. For example, in periods of heightened economic uncertainty, a low conditional correlation between Bitcoin and equity returns may lead investors to increasingly view Bitcoin as a hedge or store of value. This increased demand can elevate Bitcoin's price, incentivizing more mining activity, which in turn raises energy consumption and carbon emissions. In response to these environmental consequences, policymakers may adopt stricter climate policies.

[27] Exchange rate data were missing from the Wall Street Journal (WSJ) database for certain dates; we impute the missing values using the average of the exchange rates from one week before and one week after the missing dates.

Table 4
Double-Selection LASSO Analysis.

VARIABLES	(1) c_t	(2) c_t	(3) c_t
$c_{t-1}^{Before} - Green\ Deal$	−0.488***		
	(−2.758)		
$c_{t-1}^{After} - Green\ Deal$	−0.296*		
	(−1.816)		
$c_{t-1}^{Before} - Mining\ Ban$		0.497**	
		(2.231)	
$c_{t-1}^{After} - Mining\ Ban$		0.631***	
		(2.775)	
$c_{t-1}^{Before} - IRA$			0.234*
			(1.836)
$c_{t-1}^{After} - IRA$			0.368***
			(3.361)
Wald chi2	13.35	9.44	18.36
Number of Controls	37	37	37
Number of Selected Controls	5	6	9
Observations	23	23	23

Notes: This table reports coefficient estimates obtained using the Double-Selection LASSO estimator. Robust z-statistics are reported in parentheses. *** p<0.01, ** p<0.05, * p<0.1

Considering the scenario described above, along with many other potential scenarios, addressing endogeneity is crucial to ensure the robustness of our findings. To this end, we employ the Double-Selection LASSO estimator, a recently developed methodology introduced by Belloni et al. (2014). This approach uses a two-stage process that systematically identifies relevant control variables within a high-dimensional state space and effectively mitigates biases caused by endogeneity, particularly omitted variable bias.[28] This method is well-suited to our analysis for two key reasons: (1) it mitigates endogeneity concerns, especially omitted variable bias, which is our primary source of endogeneity; and (2) unlike equities, which have well-established valuation frameworks, Bitcoin's valuation remains relatively understudied, making the identification of relevant control variables more uncertain. The Double-Selection LASSO provides a data-driven approach to identifying relevant controls in a complex, high-dimensional setting, reducing the subjectivity and bias inherent in manual variable selection.

In our analysis, we incorporate a comprehensive set of 37 control variables: 21 Bitcoin-specific trading and network characteristics, 4 variables related to energy prices and indices, and 3 macroeconomic and aggregate market indicators, along with several additional factors such as public attention to global warming and climate change and longer lags of the conditional correlation between Bitcoin and equity returns. A detailed explanation of these variables is provided in Appendix A. These controls are integrated into the Double-Selection LASSO framework, allowing us to robustly address omitted variable bias and effectively manage the challenges of a high-dimensional state space. The results of this analysis are presented in Table 4.

The findings from the Double-Selection LASSO analysis are consistent with our theoretical predictions, showing that all three policy shifts are associated with a statistically significant increase in the conditional correlation between Bitcoin and equity returns. Specifically: (1) After the Green Deal, the autoregressive coefficient increased from a pre-policy value of -0.488 to a post-policy value of -0.296, with the increase being statistically significant; (2) After the Mining Ban, the autoregressive coefficient rose from a pre-policy value of 0.497 to a post-policy value of 0.631, with the increase being statistically significant; and (3) After the IRA, the autoregressive coefficient increased from a pre-policy value of 0.234 to a post-policy value of 0.368, with the increase being statistically significant. The Wald Chi-square statistics for these policy events are 13.35, 9.44, and 18.36, respectively, confirming that the policy events are relevant explanatory variables for the conditional correlation.

7.3. Time-varying autoregressive coefficient

A potential concern with the OLS analysis presented in Eq. (37) is that the autoregressive (AR) coefficient may vary over time. If so, the documented effect could simply be capturing this time variation rather than a genuine shift in the trend induced by policy. To address this concern, rather than comparing the AR coefficients before and after each event, we adopt a different approach. Specifically, we ask what is the likelihood that this week's conditional correlation is higher than the previous week's value? Our expectation is that following the policy events, the likelihood of observing an increase in conditional correlation should significantly rise.

[28] The Double-Selection LASSO has gained traction in economics and finance literature, as demonstrated by studies such as Feng et al. (2020); Liu et al. (2020); Kluender (2024), which use this estimator to address endogeneity in high-dimensional empirical analyses.

Table 5
Logit Regression analysis.

VARIABLES	(1) logit($pr(D_t=1)$)	(2) logit($pr(D_t=1)$)	(3) logit($pr(D_t=1)$)	(4) logit($pr(D_t=1)$)	(5) logit($pr(D_t=1)$)	(6) logit($pr(D_t=1)$)
$C_t^{Before} - Green\ Deal$	9.168	11.28				
	(0.561)	(0.748)				
$C_t^{After} - Green\ Deal$	15.79	16.85				
	(0.829)	(0.980)				
$C_t^{Before} - Mining\ Ban$			43.74**	17.88		
			(2.019)	(1.390)		
$C_t^{After} - Mining\ Ban$			51.17**	21.15		
			(2.043)	(1.414)		
$C_t^{Before} - IRA$					37.12*	32.97**
					(1.706)	(1.994)
$C_t^{After} - IRA$					48.66*	42.94**
					(1.813)	(2.095)
Constant	−2.634	−3.297	−14.87**	−6.384	−17.43*	−15.70**
	(−0.551)	(−0.755)	(−2.081)	(−1.533)	(−1.853)	(−2.150)
Controls	YES	NO	YES	NO	YES	NO
Observations	23	23	23	23	23	23
$Pseudo - R^2$	0.1282	0.0736	0.3219	0.0787	0.4148	0.3498

Notes: This table reports Logit regression estimates of Eq. (39). z-statistics are reported in parentheses. *** p<0.01, ** p<0.05, * p<0.1

To test this, we employ a Logit regression with an interaction term. Our objective is to assess whether the policy events increase the likelihood of a rise in the conditional correlation. As a first step, we define a dummy variable, D_t, indicating whether the conditional correlation increased relative to the prior week:

$$D_t = \begin{cases} 1 & if\ C_t > C_{t-1} \\ 0 & otherwise \end{cases} \tag{38}$$

Similar to the OLS analysis, we use indicator functions that capture the pre- and post-event trends. We then estimate the following Logit regression:

$$\text{logit}(pr(D_t = 1)) = \beta_0 + \beta_1 C_t \times \mathbf{1}_{Before} + \beta_2 C_t \times \mathbf{1}_{After} + \varepsilon_t \tag{39}$$

If our theoretical prediction holds, we expect $\beta_2 > \beta_1$, indicating that the policy event increases the likelihood of a rise in conditional correlation. Table 5 presents the results, corroborating our theoretical prediction.

8. Conclusion

Amid global efforts to reduce carbon emissions and mitigate climate change, Bitcoin has emerged and expanded as a technology with a substantial carbon footprint. This paper examines the economic implications of Bitcoin's recent growth, particularly in the context of intensifying climate action. We develop a stylized general equilibrium framework in which overlapping generations of investors optimally allocate their wealth across Bitcoin, equity, and a risk-free asset. A distinguishing aspect of our model is that investment decisions are shaped by two key considerations: (1) emissions from Bitcoin mining and equity production contribute to climate change, which in turn leads to asset impairment and wealth destruction; and (2) higher emissions from Bitcoin mining may signal greater network security—due to its PoW consensus mechanism—which can increase Bitcoin's value. To mitigate the adverse effects of climate change, climate policies are introduced.

Theoretically, we find that a shift toward more stringent climate policies, while effective in reducing Bitcoin's environmental impact, diminishes its effectiveness as a hedge or safe haven in equity markets by increasing the conditional correlation between Bitcoin and equity returns. Our empirical analysis supports this prediction: major climate policy events—including the European Green Deal, China's mining ban, and the U.S. Inflation Reduction Act—were followed by a statistically significant increase in Bitcoin-equity correlation.

CRediT authorship contribution statement

Mohammadhossein Lashkaripour: Writing – review & editing, Writing – original draft, Visualization, Software, Resources, Methodology, Investigation, Funding acquisition, Formal analysis, Data curation, Conceptualization. **Seyedmehdi Hosseini:** Writing – review & editing, Validation, Supervision, Project administration, Conceptualization. **Rizwan Ahmed:** Writing – review & editing, Validation, Supervision.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Appendix A. Variable description

Table 6
Variables, Definitions, and Sources.

Variable	Description	Source
WEI	Weekly Economic Index (Lewis-Mertens-Stock)	FRED
Oil Price	price of crude oil (WTI) per barrel	FRED
lagged Oil Price	lagged value of Oil Price	FRED
Gas Price	Henry Hub Natural Gas Spot Price	FRED
lagged Gas Price	lagged value of Gas Price	FRED
BTC price	Average weekly price of Bitcoin	Coinmetrics
BTC Market Cap	Average weekly market capitalization of Bitcoin	Coinmetrics
lagged BTC Market Cap	lagged value of BTC Market Cap	Coinmetrics
BTC hash rate	Average weekly hash rate of Bitcoin	Coinmetrics
lagged BTC hash rate	lagged value of BTC hash rate	Coinmetrics
BTC 1-year ROI	The return on investment for Bitcoin assuming a purchase 1 year prior	Coinmetrics
UTXO	The sum count of unspent transaction outputs	Coinmetrics
Mean TX Size	The sum USD value of native units transferred divided by the count of transfers (i.e., the mean size in USD of a transfer) between distinct addresses	Coinmetrics
TX Count	The sum count of transactions	Coinmetrics
Miners' Revenue	The sum USD value of all miner revenue (fees plus newly issued native units)	Coinmetrics
Miners' Revenue per Hash	The USD value of the mean miner reward per estimated hash unit performed during the period, also known as hashprice	Coinmetrics
NVT	The ratio of the network value (or market capitalization, current supply) divided by the adjusted transfer value. Also referred to as NVT.	Coinmetrics
RVT	The ratio of the network's realized value to the adjusted transfer value. Also referred to as RVT.	Coinmetrics
MVRV	The ratio of the sum USD value of the current supply to the sum realized USD value of the current supply	Coinmetrics
Exchange Withdrawal	The sum USD value withdrawn from exchanges that interval, excluding exchange to exchange activity	Coinmetrics
Exchange Deposit	The sum USD value sent to exchanges that interval, excluding exchange to exchange activity	Coinmetrics
TX Fee	The USD value of the mean fee per transaction	Coinmetrics
Difficulty	The mean difficulty of finding a hash that meets the protocol-designated requirement (i.e., the difficulty of finding a new block) that interval. The requirement is unique to each applicable cryptocurrency protocol. Difficulty is adjusted periodically by the protocol as a function of how much hashing power is being deployed by miners	Coinmetrics
New Address Count	The sum count of unique addresses that were newly created that interval	Coinmetrics
Active Address Count	The sum count of unique addresses that were active in the network (either as a recipient or originator of a ledger change) that interval. All parties in a ledger change action (recipients and originators) are counted. Individual addresses are not double-counted if previously active.	Coinmetrics
30-Day Realized Volatility	The 30D volatility, measured as the standard deviation of the natural log of daily returns over the past 30 days	Coinmetrics
S&P 500 Index	The SPX index	WSJ Market Data
STOXX 600 Index	The SXXP index	WSJ Market Data
Shanghai Composite Index	The SHCOMP index	WSJ Market Data
Russell 2000 Index	The RUT index	WSJ Market Data
Euro/U.S. Dollar	The Euro to U.S. Dollar exchange rate	WSJ Market Data
Chinese Yuan/U.S. Dollar	The Chinese Yuan to U.S. Dollar exchange rate	WSJ Market Data
MSCI Europe	The MSCI Europe Index, representing large- and mid-cap equity performance across developed European markets	FactSet
MSCI China	The MSCI China Index, representing large- and mid-cap equity performance of Chinese companies listed domestically and internationally	FactSet
MSCI U.S.	The MSCI USA Index, representing large- and mid-cap equity performance in the United States equity market	FactSet
VIX	The CBOE Volatility Index, measuring the market's expectation of 30-day forward-looking volatility implied by S&P 500 index options	WRDS

Appendix B. Further empirical tests

B.1. Details of double-selection LASSO

The Least Absolute Shrinkage and Selection Operator (LASSO) regression is a type of regularized linear regression that incorporates a penalty for the magnitude of coefficients. This regularization process not only prevents overfitting but also allows certain coefficients to shrink to zero, effectively performing variable selection. The strength of this penalty is controlled by a parameter, typically denoted as λ. Applying LASSO enables us to account for a wide range of potential factors that may influence the dynamic conditional correlation (DCC) between Bitcoin and equities, beyond the effects of policy shifts.

Given that our focus extends beyond control variable selection to addressing endogeneity concerns, we employ a refined approach called Double-Selection LASSO. This method mitigates estimation bias in the presence of high-dimensional control variables and potential confounding factors (Belloni et al., 2014). Double-Selection LASSO extends the conventional LASSO framework through a two-step process. In the first step, LASSO identifies relevant control variables that influence the outcome variable, which in this context is the DCC between Bitcoin and equities. In the second step, LASSO is applied again to select variables correlated with the explanatory variable, specifically climate policy shifts. The union of covariates selected in both steps is then used in a final ordinary least squares (OLS) regression to estimate the relationship between the DCC of Bitcoin and equity returns and climate policy stringency. This dual selection process mitigates omitted variable bias (Belloni et al., 2014; Liu et al., 2020), resulting in more reliable estimates of the relationship.

Before applying LASSO, we logarithmically transform the non-stationary variables. Additionally, as part of the standard approach to ensure robust results, we perform z-score normalization. The set of control variables is detailed in Table 6. To select the optimal value of the LASSO penalty parameter (λ), we use an iterative process known as the Plugin method, with the number of iterations set to 100.[29]

B.2. Using eight-month window

In this section, we repeat regression (37) using an 8-month window. The regression results presented in Table 7 are consistent with the 6-month window analysis in the main text and provide further support for our theoretical arguments.

Table 7
Eight-Month Window Analysis.

VARIABLES	(1) c_t	(2) c_t	(3) c_t	(4) c_t	(5) c_t	(6) c_t
$c_{t-1}^{Before} - Green\ Deal$	0.645***	0.763***				
	(7.522)	(6.928)				
$c_{t-1}^{After} - Green\ Deal$	0.687***	0.765***				
	(7.185)	(6.076)				
$c_{t-1}^{Before} - Mining\ Ban$			0.490**	0.532***		
			(2.451)	(3.173)		
$c_{t-1}^{After} - Mining\ Ban$			0.561***	0.595***		
			(3.016)	(3.812)		
$c_{t-1}^{Before} - IRA$					0.650***	0.692***
					(3.606)	(3.883)
$c_{t-1}^{After} - IRA$					0.831***	0.811***
					(6.150)	(6.052)
Constant	-0.404***	−0.259	−0.602**	−0.557**	−0.273	−0.290*
	(−3.289)	(−1.626)	(−2.415)	(−2.743)	(−1.632)	(−1.741)
Controls	YES	NO	YES	NO	YES	NO
Observations	31	31	31	31	31	31
R-squared	0.837	0.638	0.427	0.414	0.715	0.638

t-statistics in parentheses *** p<0.01, ** p<0.05, * p<0.1

Appendix C. Proofs

C.1. Proof of Proposition 1

Restructuring Eq. (16) yields[30]:

$$\gamma\Sigma X = E_t(\mathbf{r_{t+1}} - \mathbf{r_f}\mathbf{1}) + \frac{1}{2}\sigma^2_{\mathbf{rt}} - \eta\mathbf{T_t} \tag{40}$$

Multiplying the transpose of the portfolio vector $X' = (X^b, X^e)$ on both sides of Eq. (16) gives:

$$\gamma X'\Sigma X = X'(E_t(\mathbf{r_{t+1}} - \mathbf{r_f}\mathbf{1}) + \frac{1}{2}\sigma^2_{\mathbf{rt}}) - \eta X'\,\mathbf{T_t} \tag{41}$$

From Eqs. (10) and (11), we have $T_t = X'\,\mathbf{T_t}$ where $\mathbf{T_t} = \left(T_t^b, T_t^e\right)$ and $T_t^e = \kappa(Emission_t^e + Emission_{t-1}^{Stock})$ and $T_t^b = \kappa(Emission_t^b + Emission_{t-1}^{Stock})$ denote the Earth's temperature if the entire wealth were invested in equity and Bitcoin, respectively. Note also that

[29] The Plugin method leverages the statistical characteristics of the data to determine a near-optimal value for λ, effectively suppressing noise in the estimating equations and reducing the likelihood of including variables that do not belong to the true model (see https://www.stata.com/manuals/lassolasso.pdf).

[30] We drop the time subscripts from X_t and Σ_t for notational simplicity.

$X'(E_t(\mathbf{r_{t+1}}-\mathbf{r_f 1})+\frac{1}{2}\sigma^2_{\mathbf{rt}})$ represents the market portfolio return. This implies:

$$\gamma X'\Sigma X = E_t(r^M_{t+1} - r_f 1) + \frac{1}{2}\sigma^2_{Mt} - \eta T_t \tag{42}$$

Applying a simple algebra, it's straightforward that:

$$\log E_t\left[\frac{\mathbf{R_{t+1}}}{R_f}\right] = \beta \log E_t\left[\frac{R^M_{t+1}}{R_f}\right] + \eta(\mathbf{T_t} - \beta T_t) \tag{43}$$

where $\beta = \frac{\Sigma X}{X'\Sigma X}$.

Considering that Bitcoin returns have both pecuniary and non-pecuniary components (Eq. (6)), we can further simplify the above pricing relation:

$$\log E_t\left[\frac{R^{fb}_{t+1}}{R_f}\right] = \hat{\beta}^b \log E_t\left[\frac{R^M_{t+1}}{R_f}\right] + \hat{\eta}(T^b_t - \beta^b T_t) - \frac{\theta}{1-\theta}\ln \lambda_t \tag{44}$$

$$\log E_t\left[\frac{R^e_{t+1}}{R_f}\right] = \beta^e \log E_t\left[\frac{R^M_{t+1}}{R_f}\right] + \eta(T^e_t - \beta^e T_t) \tag{45}$$

where $\hat{\beta}^b = \frac{\beta^b}{1-\theta}$ and $\hat{\eta} = \frac{\eta}{1-\theta}$. Next, we prove Remark 1.

C.1.1. Proof of Remark 1

From Sections 5.1 and 4.1, we have:

$$\beta^* = \frac{\hat{\Sigma}X}{X'\hat{\Sigma}X} = \frac{\Sigma X + \frac{\delta^*}{\gamma}MX}{X'\Sigma X + \frac{\delta^*}{\gamma}X'MX} \tag{46}$$

$$\beta = \frac{\Sigma X}{X'\Sigma X} \tag{47}$$

For better tractability, let $c = \frac{\delta^*}{\gamma} > 0$. We can then write:

$$\Sigma X = \left(X'\Sigma X\right)\beta \tag{48}$$

$$T_t = \mathbf{T'}_t X \tag{49}$$

$$T_t^2 = X'\,\mathbf{T}_t\mathbf{T'}_t X \tag{50}$$

Using (48), (49), and (50), we can rewrite β^* as follows:

$$\beta^* = \frac{\left(X'\Sigma X\right)\beta + cT_t\mathbf{T}_t}{X'\Sigma X + cT_t^2} \tag{51}$$

After straightforward algebraic manipulation, Eq. (51) can be expressed as

$$\beta^* = (1-\pi)\beta + \pi\frac{\mathbf{T}_t}{T_t} \tag{52}$$

$$where \quad \pi = \frac{cT_t^2}{X'\Sigma X + cT_t^2}$$

The parameter π determines the weight assigned to each component in the effective beta, with $\pi \in (0,1)$. Given that $c = \frac{\delta^*}{\gamma}$, it follows that

$$\frac{\partial \pi}{\partial \delta^*} > 0$$

which completes the proof.

C.2. Proof of Proposition 2

Consider the optimal portfolio structure in (16). The variance-covariance matrix (Σ) has the following structure:

$$\Sigma = \begin{pmatrix} \sigma_b^2 & \sigma_{eb} \\ \sigma_{eb} & \sigma_e^2 \end{pmatrix} \tag{53}$$

Substituting the inverse of Σ, we can rewrite the optimal portfolio structure as follows:

$$X = \frac{1}{\gamma(\sigma_b^2\sigma_e^2 - \sigma_{eb}^2)} \begin{pmatrix} \sigma_e^2 & -\sigma_{eb} \\ -\sigma_{eb} & \sigma_b^2 \end{pmatrix} \begin{pmatrix} ER^b - \eta k(Emission_t^b + Emission_{t-1}^{Stock}) \\ ER^e - \eta k(Emission_t^e + Emission_{t-1}^{Stock}) \end{pmatrix} \tag{54}$$

where $ER^k = \log E_t\left[\frac{R_{t+1}^k}{R_f}\right]$. Considering this, the weights of Bitcoin and equity in the optimal portfolio are given by the following structure:

$$X^b = \frac{1}{\gamma(\sigma_b^2\sigma_e^2 - \sigma_{eb}^2)}\left[\sigma_e^2(ER^b - \eta k(Emission_t^b + Emission_{t-1}^{Stock})) - \sigma_{eb}(ER^e - \eta k(Emission_t^e + Emission_{t-1}^{Stock}))\right] \tag{55}$$

$$X^e = \frac{1}{\gamma(\sigma_b^2\sigma_e^2 - \sigma_{eb}^2)}\left[-\sigma_{eb}(ER^b - \eta k(Emission_t^b + Emission_{t-1}^{Stock})) + \sigma_b^2(ER^e - \eta k(Emission_t^e + Emission_{t-1}^{Stock}))\right] \tag{56}$$

Additionally, considering the market-clearing condition (15) and the risky assets market-clearing condition ($X^e + X^b = 1$), it is straightforward to show that the weights of Bitcoin and equity in the portfolio have the following structure:

$$X^b = \frac{Cap_t - Emission_t^e}{Emission_t^b - Emission_t^e} \tag{57}$$

$$X^e = \frac{Emission_t^b - Cap_t}{Emission_t^b - Emission_t^e} \tag{58}$$

In equilibrium, these weights must be equivalent. This implies that, for instance, the weight of Bitcoin in Eq. (55) and (57) must represent the same portfolio weight, leading to the following relationship:

$$\frac{1}{\gamma(\sigma_b^2\sigma_e^2 - \sigma_{eb}^2)}\left[\sigma_e^2(ER^b - \eta k(Emission_t^b + Emission_{t-1}^{Stock})) - \sigma_{eb}(ER^e - \eta k(Emission_t^e + Emission_{t-1}^{Stock}))\right] = \frac{Cap_t - Emission_t^e}{Emission_t^b - Emission_t^e} \tag{59}$$

To simplify the notation, let's say $\sigma_{eb} \equiv Y$ and $Cap_t \equiv X$. Additionally, define A, B, and C as follows:

$$A = \gamma\sigma_b^2\sigma_e^2 > 0 \tag{60}$$

$$B = \sigma_e^2(ER^b - \eta k(Emission_t^b + Emission_{t-1}^{Stock})) > 0 \tag{61}$$

$$C = ER^e - \eta k(Emission_t^e + Emission_{t-1}^{Stock}) > 0 \tag{62}$$

Using these definitions, we can rewrite Eq. (59) as follows:

$$\frac{1}{A - \gamma Y^2}[B - CY] = \frac{X - Emission_t^e}{Emission_t^b - Emission_t^e} \tag{63}$$

which is equivalent to the following equation which is quadratic equation with respect to Y:

$$\gamma(X - Emission_t^e)Y^2 - C\Delta Emission_t Y + (B\Delta Emission_t - AX + AEmission_t^e) = 0 \tag{64}$$

Where $\Delta Emission_t = Emission_t^b - Emission_t^e$. It follows from Property 1 that $\Delta Emission_t > 0$. This is because Bitcoin mining relies exclusively on energy as an input, whereas equity production utilizes both energy and capital. Therefore, if all wealth were invested in Bitcoin, total emissions would exceed those in a scenario where all wealth is allocated to equity.

What we aim to compute is $\frac{dY}{dX}$. Taking the first-order derivative of both sides of Eq. (64) with respect to X yields:

$$\gamma Y^2 + 2\gamma(X - Emission_t^e)Y\frac{dY}{dX} - C\Delta Emission_t\frac{dY}{dX} - A = 0 \tag{65}$$

Solving for $\frac{dY}{dX}$, we obtain:

$$\frac{dY}{dX} = \frac{A - \gamma Y^2}{2\gamma(X - Emission_t^e)Y - C\Delta Emission_t} \tag{66}$$

Now, our goal is to determine the sign of $\frac{dY}{dX}$. The numerator in Eq. (66) is always positive. Specifically:

$$A - \gamma Y^2 = \gamma\sigma_b^2\sigma_e^2 - \gamma\sigma_{eb}^2 > 0 \tag{67}$$

Inequality (67) holds because the correlation between the two assets is between -1 and 1 and is defined as $corr = \frac{\sigma_{eb}}{\sqrt{\sigma_b^2 \sigma_e^2}}$. Taking the square of both sides gives:

$$0 < corr^2 = \frac{\sigma_{eb}^2}{\sigma_b^2 \sigma_e^2} < 1 \tag{68}$$

Considering Eq. (68), and given the assumption that investors' relative risk aversion is positive (i.e., $\gamma > 0$), it follows that $A - \gamma Y^2 = \gamma \sigma_b^2 \sigma_e^2 - \gamma \sigma_{eb}^2 > 0$.

Therefore, the sign of $\frac{dY}{dX}$ is determined entirely by the denominator. To assess this, we need to identify the thresholds for X at which the denominator becomes positive or negative. Based on this, four potential cases emerge:

Case 1: If $Y > 0$ and the denominator is positive:

$$2\gamma(X - Emission_t^e)Y - C\Delta Emission_t > 0$$

In this case, it follows that for $X > \frac{C\Delta Emission_t}{2\gamma Y} + Emission_t^e$, we have $\frac{dY}{dX} > 0$.

Case 2: If $Y > 0$ and the denominator is negative:

$$2\gamma(X - Emission_t^e)Y - C\Delta Emission_t < 0$$

In this case, it follows that for $X < \frac{C\Delta Emission_t}{2\gamma Y} + Emission_t^e$, we have $\frac{dY}{dX} < 0$

Cases 1 and 2 correspond to situations in which Bitcoin acts as a diversifier ($0 < Y < e$) or becomes highly correlated with equity ($Y > e$).

Case 3: If $Y < 0$ and the denominator is positive:

$$2\gamma(X - Emission_t^e)Y - C\Delta Emission_t > 0$$

This case cannot occur. Recall that the emissions cap, X, is the weighted average of $Emission_t^b$ and $Emission_t^e$. Since we previously assumed that $\Delta Emm_t > 0$, then it follows that $X > Emission_t^e$. However, the inequality characterizing Case 3 implies: $X < \frac{C\Delta Emission_t}{2\gamma Y} + Emission_t^e$. Given that $Y < 0$ the term $\frac{C\Delta Emission_t}{2\gamma Y} < 0$, which means the right-hand side is less than $Emission_t^e$. This contradicts the condition $X > Emission_t^e$, and therefore, Case 3 is not feasible under our assumptions.

Case 4: If $Y < 0$ and the denominator is negative:

$$2\gamma(X - Emission_t^e)Y - C\Delta Emission_t < 0$$

It follows that for $X > Emission_t^e + \frac{C\Delta Emission_t}{2\gamma Y}$, we have $\frac{dY}{dX} < 0$.

Since $X > Emission_t^e + \frac{C\Delta Emission_t}{2\gamma Y}$ always holds in this case, we conclude that $\frac{dY}{dX} < 0$ always holds when $Y < 0$.

Case 4 corresponds to a situation in which Bitcoin acts as a hedge against equity. Thus, when Bitcoin is a hedge, more stringent climate policies (i.e., higher X) always increase the correlation between Bitcoin and equity returns.

Finally, by defining $\overline{Cap} = \frac{C\Delta Emission_t}{2\gamma Y} + Emission_t^e$, we complete the first part of the proof.

Next, we aim to prove that a shift from a lenient to a more stringent climate policy—ceteris paribus—increases the conditional correlation. Based on the above findings, when $Y < 0$, it is straightforward to see that:

$$corr^{Stringent} - corr^{Lenient} > 0$$

Now, consider the case where $Y > 0$. Suppose the emissions cap is tightened, which we denote as $\Delta Cap < 0$. When policy in effect is lenient ($X > \overline{Cap}$), the equilibrium covariance $Y^{Lenient}$ satisfies $F(Y^{Lenient}, X^{Lenient}) = 0$, where $F(Y,X)$ denotes the left-hand side of Eq. (64).

In a counterfactual world, holding everything else constant, evaluating the equilibrium condition at the lenient covariance $Y^{Lenient}$ under the stricter cap ($\Delta Cap < 0$) gives:

$$F(Y^{Lenient}, X^{Lenient} + \Delta Cap) - F(Y^{Lenient}, X^{Lenient}) = \Delta Cap \cdot (\gamma (Y^{Lenient})^2 - A) \tag{69}$$

Using $A - \gamma (Y^{Lenient})^2 > 0$ from inequality (67) yields:

$$F(Y^{Lenient}, X^{Lenient} + \Delta Cap) > 0 \tag{70}$$

Under the stringent regime, $\partial F / \partial Y < 0$, so to restore equilibrium the covariance must rise:

$$Y^{Stringent} > Y^{Lenient} \tag{71}$$

which simplifies to:

$$\frac{Y^{Stringent} - Y^{Lenient}}{\sigma_b \sigma_e} > 0 \tag{72}$$

Since $\sigma_b, \sigma_e > 0$, ceteris paribus, it follows that:

$$corr^{Stringent} - corr^{Lenient} > 0$$

This completes the proof.

Quadratic-Exponential Damage Function. Under the quadratic-exponential damage function, the endogenous emissions cap—which divides the climate policy regime into lenient and stringent categories—shifts to reflect higher convexity relative to the exponential damage case.

To prove this, note that under the quadratic–exponential damage function, the covariance between Bitcoin and equity shifts from σ_{eb} to $\sigma_{eb} + \frac{\delta^*}{\gamma} T_t^b T_t^e$. Let the perceived covariance under the quadratic–exponential specification be defined as $\hat{Y} = \sigma_{eb} + \frac{\delta^*}{\gamma} T_t^b T_t^e$. Also, recall that earlier we defined $\sigma_{eb} \equiv Y$. Replacing $\hat{Y}$ with Y, we have the term $2\gamma(X - Emission_t^e)\hat{Y} - C\Delta Emission_t$ which can be used to determine our $\overline{Cap^*}$. For example, $Cap_t < \overline{Cap^*}$ represents the stringent regime and $Cap_t > \overline{Cap^*}$ represents the lenient regime where

$$\overline{Cap^*} = \frac{C\Delta Emission_t}{2\gamma \hat{Y}} + Emission_t^e$$

Given that $\hat{Y} > Y$, then we have $\overline{Cap^*} < \overline{Cap}$.

C.2.1. Proof of Remark 2

Under the quadratic–exponential damage function, the covariance between Bitcoin and equity shifts from σ_{eb} to $\sigma_{eb} + \frac{\delta^*}{\gamma} T_t^b T_t^e$. Let the perceived covariance under the quadratic–exponential specification be defined as $\hat{Y} = \sigma_{eb} + \frac{\delta^*}{\gamma} T_t^b T_t^e$. Also, recall that earlier we defined $\sigma_{eb} \equiv Y$.

It is then straightforward to show that, in a scenario where Bitcoin serves as a financial hedge against equity (i.e., $Y < 0$), the perceived covariance ($\hat{Y}$)—which replaces Y in the numerator and denominator of Eq. (66)—will be positive if and only if $\delta^* > \bar{\delta}$ where

$$\bar{\delta} = \frac{-\gamma \sigma_{eb}}{T_t^b T_t^e} > 0$$

C.3. Proof of Proposition 3

Considering Eqs. (20), (10), and (11), we can write the CAPM-like pricing relation for equity as follows. Given that, in equilibrium, the aggregate weight of equity and Bitcoin must sum to one in each generation, we have:

$$\log E_t\left[\frac{R_{t+1}^e}{R_f}\right] = \beta^e \log E_t\left[\frac{R_{t+1}^M}{R_f}\right] + \eta\kappa(Emission_t^e - \beta^e(Emission_t^e X^e + Emission_t^b(1 - X^e)) + (1 - \beta^e)Emission_{t-1}^{Stock}) \tag{73}$$

Simple algebra yields:

$$\log E_t\left[\frac{R_{t+1}^e}{R_f}\right] = \beta^e \log E_t\left[\frac{R_{t+1}^M}{R_f}\right] + \eta\kappa((1 - \beta^e)Emission_t^e + \beta^e X^e \Delta Emission_t - \beta^e \Delta Emission_t + (1 - \beta^e)Emission_{t-1}^{Stock}) \tag{74}$$

Now, holding the risk structure of the market fixed, we take the first-order derivative of the above relation with respect to $\Delta Emission_t$ which results in the following structure:

$$\frac{\partial \log E_t\left[\frac{R_{t+1}^e}{R_f}\right]}{\partial \Delta Emission_t} = \eta\kappa\left((1 - \beta^e)\frac{\partial Emission_t^e}{\partial \Delta Emission_t} + (\beta^e X^e - \beta^e) + \beta^e \Delta Emission_t \frac{\partial X^e}{\partial \Delta Emission_t}\right) \tag{75}$$

It is straightforward that $\frac{\partial Emission_t^e}{\partial \Delta Emission_t} = -1$. Now, we want to determine the condition that gives us $\frac{\partial \log E_t\left[\frac{R_{t+1}^e}{R_f}\right]}{\partial \Delta Emission_t} > 0$ which is:

$$(-1 + \beta^e) + (\beta^e X^e - \beta^e) + \beta^e \Delta Emission_t \frac{\partial X^e}{\partial \Delta Emission_t} > 0 \tag{76}$$

Note that from Eq. (16) and the explanations in the main text, we know that $\frac{\partial X^e}{\partial \Delta Emission_t} > 0$. Intuitively, as the relative emissions of Bitcoin increase compared to equity, the equilibrium weight of equity rises due to climate hedging demand. In light of this, the inequality $\frac{\partial \log E_t\left[\frac{R_{t+1}^e}{R_f}\right]}{\partial \Delta Emission_t} > 0$ holds if the following condition is satisfied:

$$\Delta Emission_t > \frac{1 - \beta^e X^e}{\beta^e \frac{\partial X^e}{\partial \Delta Emission_t}} \tag{77}$$

Defining $\frac{1 - \beta^e X^e}{\beta^e \frac{\partial X^e}{\partial \Delta Emission_t}} = \Delta Emission^*$, the proof is complete.

C.4. Proof of Proposition 4

With policy in effect, the equilibrium CAPM-like pricing relation has the following structure:

$$\log E_t^{Policy}\left[\frac{R_{t+1}^e}{R_f}\right] = \beta^e \log E_t\left[\frac{R_{t+1}^M}{R_f}\right] + \eta(\kappa Emission_t^e - \beta^e \kappa Cap_t) \tag{78}$$

From Eq. (15) and the equilibrium condition $X^e + X^b = 1$, we can write:

$$Cap_t = -X^e \Delta Emission_t + Emission_t^b \tag{79}$$

We have the following relation for the first-order derivative:

$$\frac{\partial \log E_t^{Policy}\left[\frac{R_{t+1}^e}{R_f}\right]}{\partial \Delta Emission_t} = \eta\kappa\left(\frac{\partial Emission_t^e}{\partial \Delta Emission_t} - \beta^e \frac{\partial Cap_t}{\partial \Delta Emission_t}\right) \tag{80}$$

Which simplifies to:

$$\frac{\partial \log E_t^{Policy}\left[\frac{R_{t+1}^e}{R_f}\right]}{\partial \Delta Emission_t} = \eta\kappa(-1 + \beta^e X^e) \tag{81}$$

From the previous proposition, the first-order derivative with no policy in effect has the following structure:

$$\frac{\partial \log E_t^{No-Policy}\left[\frac{R_{t+1}^e}{R_f}\right]}{\partial \Delta Emission_t} = \eta\kappa\left(-1 + \beta^e X^e + \beta^e \Delta Emission_t \frac{\partial X^e}{\partial \Delta Emission_t}\right) \tag{82}$$

The additional term compared to the case with policy in effect is $\beta^e \Delta Emission_t \frac{\partial X^e}{\partial \Delta Emission_t}$ which is positive for the following reasons. First, the equilibrium β^e is positive because the equilibrium weight of equity in the market portfolio is positive. $\Delta Emission_t$ is also positive, as shown in the previous proposition, where it was established that $\Delta Emission_t > \Delta Emission^*$ holds and $\Delta Emission^*$ itself is positive since both its numerator and denominator are positive (see Proposition 3). Finally, as discussed in Proposition 3, $\frac{\partial X^e}{\partial \Delta Emission_t}$ is also positive. Therefore we have:

$$-1 + \beta^e X^e < -1 + \beta^e X^e + \beta^e \Delta Emission_t \frac{\partial X^e}{\partial \Delta Emission_t} \tag{83}$$

which implies that:

$$\frac{\partial \log E_t^{Policy}\left[\frac{R_{t+1}^e}{R_f}\right]}{\partial \Delta Emission_t} < \frac{\partial \log E_t^{No-Policy}\left[\frac{R_{t+1}^e}{R_f}\right]}{\partial \Delta Emission_t} \tag{84}$$

C.5. Proof of Proposition 5

To prove this proposition, we use the findings of Proposition 3 and apply the chain rule as follows:

$$\frac{\partial \log E_t\left[\frac{R_{t+1}^e}{R_f}\right]}{\partial f^b} = \underbrace{\frac{\partial \log E_t\left[\frac{R_{t+1}^e}{R_f}\right]}{\partial \Delta Emission_t}}_{>0} \times \underbrace{\frac{\partial \Delta Emission_t}{\partial Emission_t^b}}_{>0} \times \underbrace{\frac{\partial Emission_t^b}{\partial f^b}}_{<0} < 0 \tag{85}$$

C.6. Proof of Property 1

Consider an investment level denoted by I_t. It is straightforward that for a Bitcoin miner, given that the only input is energy, the following relation holds for the investment:

$$I_t = \bar{P}^{eng} E_t^b \tag{86}$$

In contrast, equity production involves two inputs: energy and capital. Optimal resource allocation requires that the marginal contribution of each input to output be equal. This condition implies:

$$\frac{\partial D_{t+1}^e}{\partial E_t^e} = \frac{\partial D_{t+1}^e}{\partial K_t} \tag{87}$$

Considering the above equation and Eq. (7), we derive the relationship between capital and energy inputs:

$$K_t = \frac{1-\delta}{\delta} E_t^e \tag{88}$$

The investment I_t must be allocated between the two inputs:

$$I_t = K_t + P^{eng} E_t^e \tag{89}$$

Substituting Eq. (88) into the expression above yields:

$$I_t = (\frac{1-\delta}{\delta} + P^{eng})E_t^e \tag{90}$$

From Eq. (86), we have:

$$E_t^b = \frac{I_t}{\bar{P}^{eng}}$$

From Eq. (90), we have:

$$E_t^e = \frac{I_t}{\frac{1-\delta}{\delta} + P^{eng}}$$

Since $\frac{1-\delta}{\delta} > 0$, it follows that:

$$E_t^b > E_t^e$$

If $\psi^b = \psi^e$, then we have:

$$Emission_t^b > Emission_t^e \tag{91}$$

This completes the proof.

Appendix D. Supplementary data

Supplementary data for this article can be found online at doi:10.1016/j.jeem.2026.103351.

Data availability

Data will be made available on request.

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