Oliver Heaviside's Operational Calculus: The Foundations of Electrical Engineering [History]

Author(s): Godoy Simões, Marcelo

Title: Oliver Heaviside's Operational Calculus: The Foundations of Electrical Engineering [History]

Year: 2024

Version: Accepted manuscript

Please cite the original version:

Godoy Simoes, M. (2024). Oliver Heaviside's Operational Calculus: The Foundations of Electrical Engineering [History]. *IEEE Electrification Magazine* 12(1), 77-84. https://doi.org/10.1109/MELE.2023.3348335

Oliver Heaviside's Operational Calculus - Foundations of Electrical Engineering

By Marcelo Godoy Simões

WHAT happened to you in order to make you decide to become an electrical engineer ? I could fill a whole hundred pages (what would be the threshold of a diary to an essay) out of my memory lane I was still a child and observed the fiancée of my cousin changing the fuse of my house, which suddenly did not have lights, neither TV nor radio working, and suddenly, after he changed the fuse, started working again. My mother was just taking her shower, my cousin and her fiancée showed up for an impromptu visit to our home. In Brazil the showerheads are real-time water-heaters connected to the wall in the bathroom, with an internal 2 kW resistance immersed in the water flowing through the device, which heats as the person uses the shower, the cold water coming from the pipe. Therefore, at the same time the water heats, it accumulates inside the shower in a small reservoir, then flows down through gravity, for the individual who is showering. Those electric showers are connected to 220 volts, while the rest of the house is connected to 110 volts (at least in São Paulo, some other cities are only 220 volts in the local distribution). In all my travels around the world I never saw anywhere electric showers as they are in Brazil, people are afraid of electricity and water in the bath, I myself had several shocks of wires that were touching the metallic pipes, now everything is safer than when I was a child. When I got older, I started to blow transistors and diodes, even capacitors, but that is another story. I observed since very young how people would cook, and paid attention to the fact that beans were very hard seeds but somehow, they became soft and edible in my plate. Have you ever wondered about the word recipe ? It usually follows a format, with ingredients, their measurements, sometimes a prep on initial steps, then a method to transform all the ingredients into maybe casseroles, top stove or oven food, other ways of cooking, corn-breads, cakes, chocolate-chip cookies, all of them may be described how to make through a recipe. A receipt is something different, but long ago both words recipe and receipt were derived from *recipere,* or the Latin verb meaning "to receive or take" - receipt is the older word, dating at least back to the 14^{th} century, when it appeared in reference to a medicinal preparation, and now such a receipt that was a recipe appears in place of what we now call a prescription, commonly abbreviated to the present-day as "Rx". Recipes and methods are related, receipts might be good as well (purchasing things and having a receipt for payment is always a blessing), but prescriptions, we try to be far away from them. The general idea where a person willing to learn is called a student: she or he will be initially on rote learning and memorization of basic facts and rules -- the old paradigm that a student would be a vessel is no longer a good one, but it was very common in my teenage years when I was doing my middle and high school years. Then, through a teacher, a student would get to some critical thinking and logical reasoning, and probably after a lot of refinements, time, work, dedication, one would get to rhetorical stage, capable of persuasive communication. Then I found later in my life that there was a final degree conferred for that, and a student would eventually be certified with a diploma, capable of philosophizing in a certain domain, that is a Ph.D. or an equivalent doctoral degree for in-depth expertise. All of this is possible if we keep working hard, preparing elements for our transformation, keep things in our memory, using imagination to desire and create new forms, the recipe is applied to us, we become something else, or we think we do. In reality we get older and wiser. – In this TEW column, I thought it would be important to tell that Electromagnetism, Laplace Transforms, even Maxwell Equations, as well as much of terminology that we learn in the EE bachelor's degree would not be possible, without one more than a hero, a giant to us, Oliver Heaviside.

Origin of the Operational Calculus

Towards the end of the 19th century Oliver Heaviside developed a formal calculus of differential operators in order to solve various physical problems. Oliver Heaviside was the man who wrote, in the first time in the history of science, the now-so-called "Maxwell’s four laws of Electricity and Magnetism". Maxwell had published his two-volume work A treatise on electricity and magnetism in 1873. Heaviside first came across this seminal work as part of his ‘self-study’ work while he was living in Newcastle upon Tyne in late 1873, he was working for the Great Northern Telegraph Company, which was his only paid work, he left in 1873 at the age of 23, to concentrate on his studies. He rearranged James Clerk Maxwell's 22 quaternionic equations into the four coupled partial differential equations as we know today. From those four Maxwell’s equations –written by Heaviside– it is easy to derive a simple wave equation that works for both the electric and the magnetic field, the basis for the modern wireless communications. Such work made by Heaviside has electro-magnetic wave equations that allow the description of both light rays as well as the radio waves. This work made Guglielmo Marconi to work in the radio, but the reality is that another genius and giant, Nikola Tesla, was the inventor of the radio, the US patent office did not want to recognize Tesla’s invention there was a convoluted layer of influences to have US keep in good terms with the Italians, but this is another record of our history.

Operational Calculus converts derivatives and integrals to operators that act on functions, and by doing so ordinary and partial linear differential equations can be reduced to purely algebraic equations that are much easier to solve. There have been a number of operator methods created as far back as Leibniz, and some operators such as the Dirac delta function created controversy at the time among mathematicians, but no one wielded operators with a sense of practical use, and relaxing the constraints of flair and abandon over the objections of mathematicians as Oliver Heaviside did. He was an applied mathematician, physicist and pioneer of electromagnetic theory, laying out the foundations to establish the early existence of electrical engineering as an independent field.

During the first decades of the 20th century several attempts were made to rigorize Heavside’s operational calculus, such a the time nascent Electrical Engineering (EE) was having some divergence in approaches. Some executives of that time wanted EE to become technician-oriented, giving skills to people in practicing technology, as it was necessary enough to have work-force individuals as resources of the major electricity and radio-oriented companies that were developing in the USA and UK. Therefore, many electrical engineering courses and studies were established initially during the transition of the 19th to the 20th century constrained as a track, or as a path, inside of the Physics of most notable universities. Heaviside made the operational calculus so simple and easy that the only way to have the birth of Electrical Engineering as a scientific and independent endeavor was to make it more complicated and more mathematically tedious.

There were two paths, one going along the support and explanation of the operational calculus in terms of integral transformations, and some mathematicians were more interested to have a two-way solution, from time to the operator domain (p or s), and the inverse, so a lot of efforts happened to formalize using the Bromwich contour and Cauchy theorem for the inverse domain, based on complex analysis. Some more electrical engineering-oriented theoreticians wanted to have a system domain analysis, the relationship with convolution was considered to be important; there was also the algebraic formulation developed by Mikusinski's "Operational Calculus", and also Schwartz's Theory of Distributions. The mathematicians Carson and Bromwich demonstrated that his operators were analogous to well-developed integral equations and contour integrals in the complex plane. Integral based transformations have been mostly adopted by professors and educators of the new 20th century, and there are at least four generations of electrical engineering departments that used the rebranded the Heaviside's operational calculus as a Laplace Transform, with there is all theory supporting it, incorporation of convolution in the whole theoretical framework, and further mathematical depths in complex variable analysis with contour and line integration.

During the Victorian age, there was a resistance in society about suppressing technological advancements. UK had their Luddites active and doing harsh actions on machinery, breaking industrial activities. It was in such industry, academia, erudite, and real life social fabric, when in 1893 Heaviside published the first of a three-part series describing his operator calculus in the Proceedings of the Royal Society. Later in the year the second part appeared, but this "was the last straw for mathematicians" his third part was rejected. What killed the third part was Heaviside's unconcerned use of divergent series, dismissing their tendencies to infinity while producing accurate results by manipulating them. Some historians say that Heaviside also produced a paper on similar ideas of convolution to complete his initial series of papers, and one of the most important "contributions" of the early scientists in the 20th century in addition of connecting his procedures to Laplace was to develop what they believe was "missing", i.e., the convolution approach, where the time domain response for an impulse would be the transfer function, in the s-domain, of that system. Heaviside was more concerned about the time domain solutions, instead of theorizing about a frequency domain, which became a whole paradigm of design in control systems, and analysis of linear systems throughout the 20th century. At the middle of the 20th century the Polish mathematician Jan Mikusinski (1913–1987) developed a direct algebraic approach to the Heaviside operational calculus and changed the viewpoint of many mathematicians to it. His calculus is known as Mikusinski's operational calculus, people who know both ways state that in practice there is absolutely no difference in Mikusinkig's approach when adopting the original Heaviside's operational calculus for solving electrical circuits.

How Heaviside Developed the Operational Calculus

Just after completely in rebranding Maxwell's equations, Heaviside developed multivariable calculus definitions to what today it is call as "Curl" and "Laplacian" operators, those definitions for the Maxwell's equation reformulation, became a motivational precursor for his operational calculus for ordinary differential equations. Heaviside was a self-taught mathematician and scientist; his main approach was to solve some practical problems (which today we identify as an engineer personality). At that time long-distance telegraphy with good transmission capability was a technical challenge. Companies involved in the commercialization of this technology were aiming for faster communication over longer distances, the investments were gigantic for wiring should reach the whole areas of services. It was very common to have squashed signaling at the receiving end, reducing the signal rate, and it was not only a matter of bigger copper cross section area for the wires. There were some unexpected situations, as when cabling for telegraphy submerged in the ocean, would have asymmetrical features, making the communication not completely half-duplex. Full-duplex communication was still unimaginable at that time.

Lord Kelvin, who was a famous scientist, made some strides in obtaining a theoretical analysis, in the form of a 'heat diffusion equation' where voltage V would be defined along x as a space coordinate, also involving and R and C for a distributed electrical resistance and capacitance. Heaviside revolutionized the theory of signaling along wires using his approach as assuming 'electromagnetic waves'. Those early days would be rather a breakthrough given the theoretical eruditeness of that period, to think on electromagnetic waves travelling in a cooper cable. Lord Kelvin missed a crucial component in the physical model, his 'telegraph equation' and that was — *electromagnetic inductance*. Limiting the model to electrostatic capacitance and electrical resistance missed such an element of Maxwell's theory, consequently fallen apart of a 'wave' description. Heaviside, in his article on 'The Extra Current' derived the now familiar differential equations, which formulate voltage V and current I on an electrical transmission line, as a function of distance x and time t — damped wave equation. In there a second order partial derivative in respect of the distance was equated to the components of a second order ordinary differential equation on the right-end-side. Heaviside understood the nature of this problem and defined that the speed and characteristics of the signaling would depend on the circuit parameters (currently described as the propagation/attenuation constants determined by line parameters R, L and C). In a second phase he applies the telegrapher's equations to derive closed-form solutions for numerous electric circuit configurations. Those problems and solutions were written in his volumes of *Electrical Papers,* including further studies with faults in cables, impact in signaling through telegraph circuits, all worked and done during his daily job as telegraph clerk at the Great Northern Telegraph Company in Denmark and Newcastle upon Tyne. He conceptualized the fundamentals of telegraphic propagation of signals.

When Heaviside's made a whole reinterpretation of Maxwell's theory, he started to have further deep insights for a comprehensive understanding of practical engineering problems of his time. Heaviside made several studies on Physics, found out that the ionosphere was a layer of charged ions in the upper atmosphere, and for many years in the 20th century the schoolbooks used to call that as Heaviside layer. His geophysics studies would then base surprisingly long distances for radio waves to travel through the air. He paved the way for the discovery and technological application of radio transmission, and the tale regarding Tesla versus Marconi. Heaviside did not socialize with others as a result also spending little time in his writing to justify his methods for a social elite. Absence of rigor is historically a very often peculiarity in many innovative ideas, on the other hand mathematical science progress mostly on abstractions and often proofs of those mathematical procedures lag many years in their application (for example Boolean algebra and the invention of the electronic computer one hundred years later).

Heaviside was ready (after all above) to make his contribution in developing a simpler way to solve electrical circuits, using his proposal of Operational Calculus. Nowadays people have no idea how difficult and cumbersome it was to solve transients of electrical circuits; many different voltage and current sources, sometimes switching or assuming a sinusoidal nature. Today we can realize that all of this can be considered with Algebraic Differential Equations and Multivariable Calculus. We need expanded infinite time-series, with homogeneous solutions, plus forcing functions, assuming linearity to aggregate the homogeneous plus input excitation conditions and need to take of initial conditions. However, such a thematic understanding was not clear. I learned a methodology of time-series in my courses of Physics III then Electromagnetism and Electrical Circuits in 1982 and 1983. Probably I was among one of the last generations to learn based in Heaviside's approach, in order to be prepared for another course where parallel courses in Circuits as well as in Linear Systems taught us (in 1983) the Laplace Transform methodology. It was a very heavy scientific preparation at *Escola Politécnica da Universidade de São Paulo* (in Brazil) my professors used to comment that "you will learn in Linear Systems as well as in the 3rd course of Electrical Circuits that you can adopt Laplace Transforms, and everything in time-domain, complex domain and s-domain will make sense". In fact, it still took me many years to figure out the relationships of Laplace Transforms, intertwined with Fourier Series, as well as their relationships to Fourier Transforms and the Z-Transform. I only comprehended after my course in Advanced Mathematics for Electrical Engineering in my Ph.D. program at the University of Tennessee in Knoxville (UTK) in 1991 and 1992, taught by Professor M.O. Pace in his mandatory course for the qualifying exam, based on the book by Oppenheim and Schafer on Discrete-Time Signal Processing, with extensive sets of handouts and notes, also Applied Math taught on the basis of integral equations. This happened a long time ago and students currently have a more simulation-based approach which facilitates their understanding. Notwithstanding, I was educated under the approach established by Oliver Heaviside, and to me there are many other champions for our modern 20^{th} century electrical engineering approach.

Step-by-Step Procedural Conversion

Heaviside proposed taking the basic differential equations for voltage $v(t)$ and current $i(t)$ for a resistance R, capacitance C and an inductance L, rewriting them using an operator $\boldsymbol{p}$, which performed the derivative with respect to time on the function to the right of it. Effectively he made the derivative $\frac{d}{dt}$ of a function such as $\frac{df(t)}{dt}$ to be $\boldsymbol{p}f(t)$ then he also assumed that the inverse of $\boldsymbol{p}$ i.e. $\frac{1}{\boldsymbol{p}}$ as the operator performing the integral of a function, so that . $\boldsymbol{p}\frac{1}{\boldsymbol{p}} = \frac{1}{\boldsymbol{p}}\boldsymbol{p} = 1$ because in practice the integral should be the inverse operation of a derivative. There are cases where this is not completely true, for example for non-linear cases, or with parameters dependent on time. A simple case and common case is when there is a constant term that might be considered, and derivative of any constant would be zero. This is regarded as initial conditions in differential equations. The proposed methodology are for problems where $f(0) = 0$. In practical electrical circuits, if the function $f(t)$ represents a variable that is associated to energy in a particular form, $f(0) = 0$ means the energy stored at the chosen instant $t = 0$ is zero. For example for the current in an inductor, we have this voltage and current relationship: $v = L\frac{di}{dt}$ with energy $E = \frac{1}{2}Li^2$, as well as the relationships for the current and voltage of a capacitor, where $i = C\frac{dv}{dt}$ with energy $E = \frac{1}{2}Cv^2$. It is elementary to validate, by just assuming an adequate time-shift, $t = t_0$ in order to comply with $f(t_0) = 0$, particularly for periodic systems. For non-periodic systems, an inspection must be made in the evolution of the response to define a new starting point with such a time shift. Then, the energy stored in that element would be zero, the input forcing function (or circuit excitation, such as a voltage source or a current source) would be shifted to that initial chosen time. Therefore, with such assumptions we have for resistors, capacitors, and inductors that:

$$v = iR$$
$$i = C\boldsymbol{p}v \text{ from } v = \frac{1}{c}\int idt$$
$$v = L\boldsymbol{p}i \text{ from } v = L\frac{di}{dt}$$

The overall description of all circuit relationships will allow the separation (algebraically) of the operators from their functions, such as if they could have an independent existence, then Heaviside proposed an impedance defined as $\frac{v}{i}$ that he called Z (a terminology we still use today):

$$Z = R$$

$Z = \frac{1}{Cp}$

$Z = L\boldsymbol{p}$

In the Heaviside treatment it was assumed a constant voltage applied to a circuit at time that $t = 0$, because his initial interest was in the impulse (or step function) as the ones encountered as transient signals on telegraphic cables. In telegraphy these transient effects limit the signaling speed, while in telephony the transient effects limit the line length. In Electrical Engineering it is mostly assumed that for a particular initial time the differential equations for transient currents has an applied voltage 0 for $t < 0$, Heaviside wrote this step function as the bold symbol **1.** Such a ***Step Function*** has been defined in the areas of Controls, Signals, and Systems, as ***Heaviside Function***, usually denoted as **H(t)** or **U(t)** also has the property that $f(0) = 0$, a condition for the commutativity of the inverse operation. For example, in order to solve the differential equation $\frac{d^2y}{dt} + y = 1$ for t > 0, with zero initial conditions, that means $y(0) = 0$ and $\frac{dy(0)}{dt} = 0$, applying a ***Step Function***, Heaviside would rewrite this as :

$$\boldsymbol{p}^2 y + y = \mathbf{1}$$

the **"1"** at the right-hand side is the value of the Step Function for t > 0. The solution of y is 1-cos(t). Then, isolating *y* at the left side to solve for the solution, we have:

$$y = \frac{\mathbf{1}}{(\boldsymbol{p}^2 + 1)}$$

we kept so far ***p*** in ***bold italic*** and **1** in **bold** to emphasize that there is an operator and a forced function or an excitation from an input source, nonetheless from this point on, we can basically treat all of them algebraically, as simple numbers and variables.

$$y = \frac{p^{-2}}{(p^{-2} + 1)}\mathbf{1} = p^{-2}(1 - p^{-2} + p^{-4} - \cdots)\mathbf{1} = \frac{t^2}{2!} - \frac{t^4}{4!} + \frac{t^6}{6!} - \cdots = -\cos(t) + 1$$

The expression of y could expanded using the Binomial Series, then inverting to the time domain, as shown above.

Suppose a telegraph wiring section, made of a resistance R in series with an inductance L, which a step voltage would be applied, the current response would be:

$$i = \frac{v}{Z} = \frac{1}{(R + Lp)}\mathbf{1} = \frac{1}{R} \times \frac{R}{Lp}\frac{1}{\left(1 + \frac{R}{Lp}\right)}\mathbf{1} = \frac{1}{R} \times \frac{1}{\tau p}\frac{1}{\left(1 + \frac{1}{\tau p}\right)}\mathbf{1},$$

where $\tau = \frac{L}{R}$.

This term $(1+\frac{1}{\tau p})^{-1}$can be expanded:

$$\left(1+\frac{1}{\tau p}\right)^{-1} = 1-\frac{1}{\tau p}+\left(\frac{1}{\tau p}\right)^{2}-\left(\frac{1}{\tau p}\right)^{3}+\cdots$$

by the Binomial Series:

$$i=\frac{1}{R}\times\frac{1}{\tau p}\left[1-\frac{1}{\tau p}+\left(\frac{1}{\tau p}\right)^{2}-\left(\frac{1}{\tau p}\right)^{3}+\cdots\right]\mathbf{1}$$

distributing the outside term with the p into the fractions inside the brackets:

$$i=\frac{1}{R}\times\left[\frac{1}{\tau p}-\left(\frac{1}{\tau p}\right)^{2}+\left(\frac{1}{\tau p}\right)^{3}-\cdots\cdots\right]$$

but the Step Function, when integrated (since we are now doing the inverse) it means that the "integral."

$$\frac{\mathbf{1}}{\boldsymbol{p}}\mathbf{1}=t$$

and in general.

$$\frac{\mathbf{1}}{\boldsymbol{p}^{n}}\mathbf{1}=\frac{t^{n}}{n!}$$

therefore,

$$i=\frac{1}{R}\times\left[\frac{t}{\tau}-\frac{1}{2!}\left(\frac{t}{\tau}\right)^{2}+\frac{1}{3!}\left(\frac{t}{\tau}\right)^{3}-\cdots\right]$$

with ad-hoc knowledge, it is possible to compare with the power series expansion of e^{-t} :

$$e^{-t}=1+t+\frac{t^{2}}{2!}+\frac{t^{3}}{3!}+\cdots$$

finally, the current can be rewritten in the time domain as:

$$i=\frac{1}{R}\left[1-e^{\left(-\frac{t}{\tau}\right)}\right]$$

Then, by using the superposition property on the original problem for other waveforms, making the solution per linear combination of waveforms, then adding the solutions, allowed Heaviside to analyze distortion at all signal frequencies, plus making recommendations in how to match segments of telegraphic wiring with extra capacitances, he was solving the problem of long-distance telegraphic losses in the line communication through an ad-hoc solution of his just invented operational calculus.

Let us do in solving a second order differential equation, where it happens that to have a minus on the coefficient, opposed to the previously positive coefficient exemplified above:

$$\frac{d^2y}{dt} - y = 1$$

for t > 0, with zero initial conditions, that means $y(0) = 0$ and $\frac{dy(0)}{dt} = 0$, we would simply apply the operator p to have the algebraic solution in y, expanding through a Binomial Series as well, still assume that $\left(\frac{1}{pn} 1 = \frac{tn}{n!}\right)$ then as before, rearranging the in y terms in rising powers, to eventually find out the time-domain solution for such a second order differential equation with ad-hoc resemblance of exponential series expansion as:

$$y = \frac{t2}{2!} + \frac{t4}{4!} + \frac{t6}{6!} + \cdots = \frac{1}{2}(e^t - e^{-t}) - 1$$

It is easy to just get in the habit of dropping the 1 altogether working with p as a simple variable, once a routine is established. For circuits with continuously distributed impedance such as telegraph lines, and nowadays for distribution power lines, sometimes having an impulse, or a particular theoretical infinite current appears instantaneously at t=0, it is possible to expand with the Kron or Dirac delta functions, plus also having the possibility of fractional powers of p such as $p\,{}^1\!/_2$ cases that have been extensively studied in the past fifty years.

Heaviside proposed the operator function:

$$\left[\frac{p}{p+B}\right]\frac{1}{2}$$

acting on a ***Step Function***, he divided through by p to get

$$\left[\frac{1}{1+\frac{B}{p}}\right]\left(-\frac{1}{2}\right)$$

and expanded into a power series, arriving at a solution in terms of modified-Bessel functions, supporting such methodology provides viable and right solutions. He also found a technique in using Fourier series methods (for the diffusion equation) and applied it to a problem for which $p\,{}^{1}/{}_{2}$ showed in the formulation, then he made an operator formulation for current from a step voltage in an ***infinitely long cable***. Heaviside equated the form of his solution to the Fourier solution in order to deduce the fractional p-operator term, and he declared results to be true for all practical purposes. Later, Heaviside presented a direct derivation based on the gamma function, which can also be deducted using Carson integrals and other methods. Heaviside often produced two versions of his power series solution, a convergent one that was useful for small t but was too slow to converge for large t, and a divergent one that was useful for large t when it was taken to a small number of terms. His treatise works on all these and other possible cases, but the 20^{th} century newly organized cohort of school of electrical engineering people did not take his methodologies further.

Other Techniques Derived by Heaviside

Heaviside developed the Heaviside Expansion Theorem to convert Z into partial fractions to simplify his work. For i=1/Z and Z a polynomial in p, the roots of Z can be found and i expressed as a sum of terms consisting of constants divided by the simpler factors. A similar procedure is done when using Laplace Transforms. Heaviside developed this method of calculating those coefficients for the expansion in partial fractions, it is called Heaviside Cover-Up Method, there are some discussions in the literature for some classes of problems that this methodology does not work, particularly for non-linear and time varying coefficients, or systems that do not allow the approach of convolution, but those are not often found in regular electrical engineering problems, so they are not discussed here.

Heaviside's had an ingenious solutions using his operators for the time-varying current in circuits with additional continuously distributed parameters such as found in actual telegraph lines. He adds a section of cable to the beginning of an infinite line, finds the current as a function of time for that configuration, and then "removes" the initial section to end up with the solution for the original cable. In fact, Heaviside used his operator calculus to design a transmission line with zero distortion (but with exponential attenuation over distance), and this is a possible system decoupling that would require some theoretical explanations. When Z is a polynomial of degree greater than 4, its roots are difficult or impossible to find directly. Also, the Expansion Theorem does not work for a Z with a root of zero or with repeated roots, a situation not encountered in passive networks but one that can occur when for example an active amplifier (with transistors) will source energy from a different path of the regular input.

Heaviside treats equal roots as being unequal, and solve for the transient current, letting those roots approach equality as a limit, then apply Calculus based theorems of limit approaching zero or infinity. Heaviside removed these difficulties by expanding Z in their inverse powers of p and then replacing p by tn/n! exactly as discussed before. This approach is called *Heaviside's Extended Expansion Theorem*. Probably if such method had a fancier name it would have been preserved in the history of electrical engineering. However, we are forever fortunate that Heaviside coined many terms, those were disliked in his days, but now they are adopted and frequent, such as impedance, inductance, conductance, admittance and reluctance. He called as "algebraizing" a differential equation with his operators and "logarising" when taking a logarithm, and he called the e-ph the "Spotting function" because it isolates, or spots, a certain value of the function, plus also the "Cover-Up Method" taught in Laplace Transforms, all those new terms invented by him became memorable names. Heaviside used physical intuition to guide him in handling these series, and he was unparalleled in his electromagnetic intuition. Because Heaviside's work was results-oriented, he sometimes provided ad hoc arguments to support his derivations. He often suggested in rather abrupt prose that mathematicians should provide rigorous proofs for what he did. Electrical Engineering education as whole has been influenced by the operational calculus based on Laplace – the original ideas of Heaviside have been deemed as simply historical – but he is indeed the Father of Operational Calculus, and of many techniques used in Electrical Engineering until now.
The Laplace Transform has the advantage of incorporating initial conditions in the formulation. The Heaviside Operational Calculus and the Laplace Transform formulations are pretty much identical for vanishing initial and boundary value problems, the constructions for Heaviside in (i), and Laplace in (ii), are expressions in $\boldsymbol{p}$ and in s :

(i) $v(t) = G(\boldsymbol{p})u(t)$ for all t; (expanding in time series, then associating those time series with recognized exponential functions).

(ii) $V(s) = G(s)U(s)$ when $Re[s] > \sigma$; (theoretically it would be possible to perform the inverse-Laplace operation, in reality most engineers simply look to a Table of Laplace Inverses).

Conceptually there is a difference between G(p) as an operator and G(s) as a complex number, but we can mathematically formulate, on an easy assumption, that G(s) is the evaluation of G(p) at p = s.

Growth of Electrical Engineering Based on the Operational Calculus

There was a tremendous growth of Electrical Engineering from 1930 to 1960, establishing the basis of the curriculum that eventually educated many people in the second half of the century. Under the leadership of Vannevar Bush a modernization of the EE curriculum at MIT was launched in the late 1930s and 1940s , more based on sciences like physics and mathematics than on craftmanship. Ernst Guillemin moved temporarily from MIT to the University of Munchen, Germany, for a doctorate in mathematical physics in 1926 with Arnold Sommerfeld. Ernst Guillemin is generally considered as the founding father of circuit theory education, upon returning to MIT he was invited to assist in the development of a communications option for undergraduate students. Then revised and expanded communication transmission lines, telephone repeaters, balancing networks, and filter theory. Guillemin's had a radical approach to the teaching of his approach, he published many important books, and in 1953 his Introductory Circuit Theory begins with the concepts of graphs, networks and trees, cut-sets, duality and so on before even mentioning Kirchhoff, loops and nodes, then he shows what became the most common way to teach electrical circuits throughout the whole 20th century with the impulse and step response, sinusoidal steady-state response, then he published The Synthesis of Passive Networks (1957) and The Theory of Linear Physical Systems (1963), in many ways the first coherent and cohesive presentation of the new discipline of network synthesis, with realization theory and methods, and a presentation of Butterworth, Chebyshev and elliptic approximation techniques for filter design. The book again offers us insights into Guillemin's teaching style, filling some of the gaps not covered by the earlier books, and partly offers alternative approaches and theoretical consolidation. The book "Operational Calculus Based on the Two-Sided Laplace Integral" by Balthasar van der Pol and H. Bremmer, was published by the University of Michigan in 1955, with an unique treatment with applications to mathematics, physics as well as circuit theory.

The development of inexpensive computers and computational facilities together with strong circuit simulation programs, like SPICE, and system analysis programs, like MATLAB, offered new opportunities for designers and the education of design. In the last 30 years several universities dropped or reduced their circuit analysis in favor of a SPICE simulations, but that made several professionals to go in their career thinking that design is merely a trial-and-error ad-hoc approach with SPICE simulations for some random values of components until they go through a more complex or real-life circuit. Nowadays with the advent of Matlab based books in electrical engineering, students learn directly using toolboxes and functions in Matlab how to expand partial fractions, it is not even necessary to memorize a long Table of Laplace Transforms, we can use computational based software such as Matlab, Mathematica, Maple and many others, because those tools became embedded and ready to use.

With the interests in digital signal processing there has been a move to reverse the traditional order in EE education of analog circuits and signal processing first and then digital signal processing DSP. Today, most of the engineering computational software will have some sort of blockset, or a connection to embed a microcontroller, a DSP, with promotional videos and toolboxes ready-made by the company that developed the simulation system. But there is still some educational paths that EE should deal with discrete time and continuous time in a interlaced way, at the same time with a push towards "coding" as necessary to prepare the workforce. Hardware-in-the-loop (HIL) and possibilities of Digital Twin became emphasized in the past few years. However, a good electrical engineering education should approach the building mind concepts of circuits and systems to integrate continuous time and discrete time models, based on the fact that we will have controls, communications, energy conversion, and recently the enhanced Artificial Intelligence based requirements for huge data systems. We have to define today what is the Electrical Engineering education for the next generation, and Energy Transformation and Sustainability should be taken into account.

Heaviside was a mathematical pioneer, but mostly an electrical engineer ahead of his time. He was an early adopter of vector analysis and calculus and developed much of his contributions to suit engineering and physics problems. In 1922, three years before his death, he was awarded the very first Faraday Medal by the IEE. He was an outstanding polymath, self-taught, with humble origins of lower middle-class, son of a wood engraver from Stockton-on-Tees. Heaviside was a remarkable man, an original thinker with brilliant mathematical powers and physical insights. We owe him the foundations of modern electrical engineering and the contemporary horizons that we have today. Thank you for reading "*The Elektron Whisperer*" (TEW) by Marcelo Godoy Simões.

Further Reading

- J. R. Carson, "Electric Circuit Theory and the Operational Calculus", New York, McGraw-Hill (1926)
- Balthasar van der Pol and H. Bremmer; "Operational Calculus Based on the Two-Sided Laplace Integral", University of Michigan (1955)
- E. A. Guillemin, "Teaching of Circuit Theory and its Impact on Other Disciplines", Proceedings of the IRE, May pp. 872-878 (1962)
- M.E. Van Valkenburg, "Teaching Circuit Theory: 1934-1984", IEEE Trans. Circuits and Systems, Vol. CAS-31, N0. 1, January, pp. 133-138 (1984)
- Paul J. Nahin; "Oliver Heaviside: The Life, Work, and Times of an Electrical Genius of the Victorian Age", Johns Hopkins University Press (1988)