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Hybrid Stochastic/Robust Optimization Model for Resilient Architecture of Distribution Networks against Extreme Weather Conditions

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Abstract- This paper expresses the planning model of the backup distributed generation (DG) and lines hardening and tie lines in distribution networks according to resilient architecture (RA) strategy under natural disaster conditions such as earthquakes and floods. Indeed, the proposed deterministic problem of resilient distribution system planning considers the minimization of the daily investment, operation and resiliency (repair and load shedding) costs as objective functions subject to constraints of AC power flow equations, system operation limits, planning and operation model of backup DG and hardening and tie lines, as well as network reconfiguration formulation. The problem formulation is based on a mixed integer non-linear programming (MINLP) model, which is converted to a mixed integer linear programming (MILP) model on the basis of Benders decomposition (BD) approach using linearization approaches to achieve the optimal solution with the lower computational efforts and error. Besides, a hybrid stochastic/robust optimization (HSRO) based on the bounded uncertainty-based robust optimization (BURO) and a scenario-based stochastic optimization is used to model the uncertainties of load, energy price and availability of the network equipment under the extreme weather conditions. Finally, the proposed RA strategy is applied on 33-bus and 119-bus distribution test systems to investigate its capabilities in different case studies.

Keywords: Backup distributed generation; Hardening and tie lines; Resilient architecture; Hybrid stochastic/robust optimization.

Nomenclature

1) *Indices and Sets*

m, n_f	Index and total number of the iterations of the primal sub-problem to be feasible, respectively
(n,j), t, w, l, k	Indices of bus, time, scenario, linearization segments of voltage magnitude term and circular constraint, respectively
N, ST, S, L, K	Sets of bus, time, scenario, linearization segments of voltage magnitude term and circular constraint, respectively
n_l, n_k	Total number of linearization segments for voltage magnitude term and circular constraint, respectively
r, n_i	Index and total number of the iteration of the primal sub-problem to be infeasible, respectively

2) *Parameters*

A	Bus incidence matrix (if a line exists between buses b and j, $A_{b,j}$ is equal to 1, and 0 otherwise)
c^{dg}, c^{hl}, c^{tl}	Investment cost (in \$) for backup DG, hardening and tie lines, respectively
c^{rg}, c^{rl}	Repair cost (in \$) for backup DG and distribution line, respectively
du, Y	Duration time (in day) of extreme weather events, planning year, respectively
G, B	Line conductance and susceptance in per unit (pu), respectively
M	Large constant
N_{bus}	Total number of network buses
P^D, Q^D	Active and reactive loads in pu, respectively
S^{DGmax}	Maximum loading of backup DG in pu
$slop$	Line slope in the linearized segments for voltage magnitude function
S^{Smax}, S^{Lmax}	Maximum loading of distribution station and line in pu, respectively
$VOLL$	Value of lost load in \$/MWh
$\underline{V}, \overline{V}$	Minimum and maximum voltage magnitude in pu, respectively
$\Delta\alpha$	Angle deviation
π	Occurrence probability
κ, ρ^{dg}	Energy price and operation price of DG in \$/MWh

σ	Uncertainty level

3) *Variables*: All variables are in per unit (pu)

P^{DG}, Q^{DG}	Active and reactive power of backup DG, respectively
P^{L}, Q^{L}	Active and reactive power of distribution line, respectively
P^{NS}, Q^{NS}	Active and reactive power not supplied, respectively
P^{S}, Q^{S}	Active and reactive power of distribution station, respectively
V, ΔV, δ	Magnitude, deviation and angle of voltage (in rad), respectively
x^{dg}, x^{hl}, x^{tl}, x^{0}	Binary variables related to investment state of backup DG, hardening line, tie line and existing lines, respectively
y, y^{hl}, y^{tl}, y^{0}	Binary variables related to switch state of line, hardening line, tie line and existing lines, respectively
λ_{sub}, μ_{sub}	Dual variables of equality and inequality constraints in the primal sub-problem, respectively

1. Introduction

1. 1. Motivation

The distribution systems are developed generally according to normal weather conditions [1], but, this case is not suitable for these systems under natural disaster condition such as earthquake, flood, and storm [2-3]. Hence, the resilient architecture (RA) approach is necessary in this case to protect the distribution grids under extreme weather condition [4-5]. To improve the network resiliency, the RA strategy can employ the backup distributed generation (DG), hardening and tie lines, and other power devices in the network, where all of the equipment are resilient against the extreme weather events [6]. Therefore, to implement the proposed strategy, it is needed to develop optimal planning model for all of the equipment to place the optimal location of these devices in the distribution network while minimizing the planning cost and maximizing the network resiliency.

1.2. Literature review

In the area of power system resilience, there are different researches such as [6] to improve the resiliency in the distribution network using back-up DG, hardening and tie lines, wherein the linearized distribution flow (LinearDistFlow) method is used in the stochastic resilience-oriented design model. In [7], time-to-event models are combined to estimate the distribution system resilience as a probabilistic model according to various natural disaster conditions. In [8], a risk assessment method is expressed to investigate the probability of potential disturbances of the distribution networks and obtain an accurate model for trading renewable energy customers according to the resilience network capabilities. Also, the effects of the critical loads and variability and scarcity of DGs to improve the distribution system resiliency is investigated in [9] and [10], respectively. Moreover, the network reconfiguration method under the extreme weather conditions is modeled in [11] to improve the distribution system resiliency.

In [12], it models the resilience enhancement strategy in the coupled distribution network and urban transportation system to determine the optimal placement of lines hardening and DGs when outages occur in distribution lines and traffic lights. Moreover, the Great Britain distribution system operator has proposed various RA methods under flood conditions in [13]. In [14], a tri-level resilience enhancement strategy is modeled to minimize the grid hardening investment and load shedding costs under different natural disasters. In [15], a novel distribution system operation approach is proposed by forming multiple micro-grids energized by DGs. Also, the effects of different power devices such as power electronics, energy storage and power distribution architecture on the power network resiliency during extreme events are investigated in [16] and [17].

Proposing an effective objective function by utilizing optimal weighting factors plays an important role in the electrical networks to optimally enhance the quality and efficiency of evaluating the position and capacity of DGs. This case is investigated in [18] as a comprehensive study of different effective objective functions. Also, two-stage planning of DGs in active distribution networks including energy storages is presented in [19], wherein it determines the DG location and capacity in the first stage and it obtains DGs' impacts on the distribution network in the second stage. In [20], the planning of lines' hardening and renewable DG is presented to improve the network resiliency. In [21], the robust planning of DGs is modeled in this network to obtain the higher resiliency level. The authors of [22] have expressed a decision support method for power system operators to restore electricity to the critical loads in a distribution system after an earthquake. In [23], a multi-period traffic assignment model with time-shiftable traffic is used in the distribution network including electric vehicles' parking lot.

1.3. Contributions

According to the available literature, the research gaps of the current research in the area of the distribution network resiliency are as follows:

- There are different methods to improve the distribution system resiliency against natural disaster conditions such as network expansion planning by lines hardening, planning of backup DGs in the distribution grids, and reconfiguration method. Also, in [6], the combination of all methods have been considered to increase the resiliency.
- Besides, different research works have modeled the stochastic RA strategy as the mixed integer non-linear programming (MINLP). However, the stochastic modeling is based on the scenarios and it requires knowledge to specify probability distribution function (PDF) of the uncertain parameters to attain a guaranteed solution with the cost of the high calculation efforts. In addition, the drawback of the MINLP formulation lies in the fact that solving that by available MINLP algorithms is difficult and it usually takes a large computation time. Also, because of the existence of discrete variables and non-convex equations in the real world problems, the solution of a large-scale MINLP suffers from a lack of global optimality, robustness, reliability and efficiency. In this regard, the mixed integer linear programming (MILP) based on the LinearDistFlow method has been proposed by [6], but this method does not consider the power loss of active and reactive power in distribution lines as well as imaginary part of the voltage in the model. Therefore, the stochastic MINLP model of RA cannot be implemented on a large-scale distribution networks.

To cope with the above issues, in this paper, as seen in Fig. 1, the backup DG and line hardening and tie lines planning in the distribution system has been developed based on a hybrid stochastic/robust resilient architecture (HSR-RA) method against earthquake and flood. Noted that the back-up DGs are able to provide on-site power for critical facilities and load centers, and contribute to energizing networks to restore load after an extreme weather events. Also, installing tie switches can improve network reconfiguration that can re-route power to on-outage portions of distribution networks, shorten the restoration time and enhance the restoration capability. Therefore, the proposed deterministic problem includes an objective function of minimizing the summation of investment, operation, repair and load shedding costs, and its constraints are AC optimal power flow equations, backup and network equipment's planning model, and reconfiguration formulation. Also, this paper converts the original MINLP model of RA strategy to MILP based on Benders decomposition (BD) method using conventional linearization approaches to achieve the optimal solution with the lower computational burden.

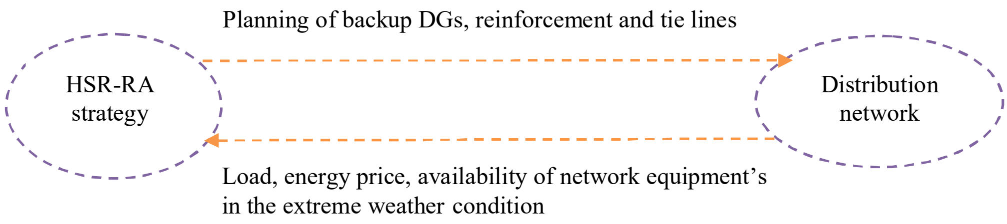


Fig. 1. The proposed RA framework in the distribution network

Moreover, in this problem, it is required to cope with the uncertainty of load, energy price and availability of network equipment under the extreme weather condition. Accordingly, the hybrid stochastic/robust optimization (HSRO) is implemented to model these uncertain parameters such that the first and second uncertain parameters are modeled based on the bounded uncertainty-based robust optimization (BURO) and the uncertainty of the availability of network equipment is modeled using the scenario-based stochastic optimization. Briefly, to the best of authors' knowledge, the main contributions of this paper can be summarized as follows:

- Presenting a RA strategy for the distribution system to obtain the optimal location of backup DG and line hardening lines and tie lines in the network to improve the system resiliency under extreme weather conditions.
- Modeling the RA strategy using MILP formulations based on the BD approach to achieve the optimal solution with the lower computational efforts in the large-scale distribution networks.
- Developing the hybrid stochastic/robust RA (HSR-RA) strategy to model the uncertainty of load, energy price and availability of network equipment.

1.4. Paper organization

The rest of the paper is organized as follows: Section 2 describes the deterministic RA model as MINLP and MILP models, and Section 3 presents the HSRO model based on the BD approach. Sections 4 and 5 address the numerical simulations and the main conclusions of the paper, respectively.

2. Proposed Problem Formulation

2.1. Original MINLP Model

The proposed RA strategy model in the distribution network is developed in this section. In the proposed optimization model, the summation of daily costs of planning, operation and resilience, i.e., repair and load shedding cost due to earthquake and flood, is considered as an objective function. The optimization model subjects to AC power flow constraints, system operation limits, sources and network equipment's planning model, and reconfiguration formulation. The proposed problem is based on the hybrid model of planning and operation studies while considering distribution network operation and backup DG, hardening, tie and existing lines planning. Also, this paper investigates the daily operation of the distribution network to assess the earthquake or flood. Hence, the proposed problem models the daily operation. However, this model can be developed for the higher time horizons by increasing the hours and reducing the value of *du*. Therefore, the proposed approach can be formulated as follows:

1) *Objective function*: The objective function of the RA strategy is expressed in equation (1), wherein the first and second parts of this equation refer to daily investment and repair costs of backup DG, hardening and tie lines, respectively [6]. Also, the operation cost of the network and DG is formulated in the third part [24], and load shedding against earthquake and flood is presented in the fourth part of (1) [25]. In this equation of the proposed problem, the repair and load shedding cost is considered as resiliency indices, so that the highest resiliency can be obtained in the distribution network in the case of zero costs for these terms. Noted that the proposed model is based on a day that the earthquake or flood is happening, thus, *du* is equal to the total days containing these extreme weather events. It should be noted that in the first row of (1), the investment cost of DGs and lines is over whole days of the planning years. But, their daily repair cost against extreme events, and daily operation and load shedding costs are modeled under extreme events conditions based on the second and third rows of this equation. Therefore, the total cost of the equipment is considered in this equation. Moreover, in (1), variables of x^{dg}, x^{hl} and x^{tl} are the states of the investments related to back-up DG, hardening and tie lines, respectively. For example, if $x^{dg} = 1$, thus, new back-up DG should be installed in the distribution network.

$$\min \quad \overbrace{\frac{1}{365\times Y}\left\{\sum_{n\in N} c_n^{dg} x_n^{dg} + \sum_{(n,j)\in N} c_{n,j}^{hl} x_{n,j}^{hl} + \sum_{(n,j)\in N} c_{n,j}^{tl} x_{n,j}^{tl}\right\}}^{\text{Daily investment cost}} + \overbrace{\frac{1}{du\times Y}.\left\{\sum_{n\in N} c_n^{rg} x_n^{dg} + \sum_{(n,j)\in N} c_{n,j}^{rl}\left(x_{n,j}^{0} + x_{n,j}^{hl} + x_{n,j}^{tl}\right)\right\}}^{\text{Daily repair cost}} + \overbrace{\sum_{t\in ST}\sum_{n\in N} \kappa_t P_{n,t}^{S} + \rho_n^{dg} P_{n,t}^{DG}}^{\text{Operational cost}} + \overbrace{\sum_{t\in ST}\sum_{n\in N} VOLL.P_{n,t}^{NS}}^{\text{Load sheding cost}} \tag{1}$$

2) *AC power flow constraints*: These constraints are expressed in (2)-(6) that are referred respectively to nodal active and reactive power balance, active and reactive power flow in distribution lines, voltage angle value in the slack bus [26-28].

$$P_{n,t}^{S} + P_{n,t}^{DG} - \sum_{j\in N} A_{n,j} P_{n,j,t}^{L} = P_{n,t}^{D} - P_{n,t}^{NS} \quad \forall\, n, t \tag{2}$$

$$Q_{n,t}^{S} + Q_{n,t}^{DG} - \sum_{j\in N} A_{n,j} Q_{n,j,t}^{L} = Q_{n,t}^{D} - Q_{n,t}^{NS} \quad \forall\, n, t \tag{3}$$

$$P_{n,j,t}^{L} = \left\{G_{n,j}\left(V_{n,t}\right)^2 - V_{n,t}V_{j,t}\left(G_{n,j}\cos\left(\delta_{n,t}-\delta_{j,t}\right) + B_{n,j}\sin\left(\delta_{n,t}-\delta_{j,t}\right)\right)\right\} y_{n,j,t} \quad \forall n, j, t \tag{4}$$

$$Q_{n,j,t}^{L} = \left\{-B_{n,j}(V_{n,t})^2 + V_{n,t}V_{j,t}\left(B_{n,j}\cos\left(\delta_{n,t}-\delta_{j,t}\right) - G_{n,j}\sin\left(\delta_{n,t}-\delta_{j,t}\right)\right)\right\} y_{n,j,t} \quad \forall n, j, t \tag{5}$$

$$\delta_{n,t} = 0 \quad \forall\, n = \text{Slack bus}, t \tag{6}$$

3) *System operation limits*: In this paper, the system operation limits include distribution line and station capacity limitations and voltage limit of buses, where these terms are modeled as constraints (7)-(9), respectively [29-31].

$$\left(P_{n,j,t}^{L}\right)^2 + \left(Q_{n,j,t}^{L}\right)^2 \le \left(S_{n,j}^{L\max}\right)^2 \quad \forall\, n, j, t \tag{7}$$

$$\left(P_{n,t}^{S}\right)^2 + \left(Q_{n,t}^{S}\right)^2 \le \left(S_{n}^{S\max}\right)^2 \quad \forall\, n = \text{Slack bus}, t \tag{8}$$

$$\underline{V} \le V_{n,t} \le \overline{V} \quad \forall\, n, t \tag{9}$$

4) *Planning and reconfiguration constraints*: The hybrid model of DGs and network equipment's planning and system reconfiguration is formulated in constraints (10) to (16). As shown in Fig. 2, the distribution line switch state, open or close, is modeled by (10), where this state depends on the distribution line construction state based on constraints (11) to (13). Moreover, the logical limit of (14) shows if there is an existing, hardening or tie line between buses *n* and *j*.

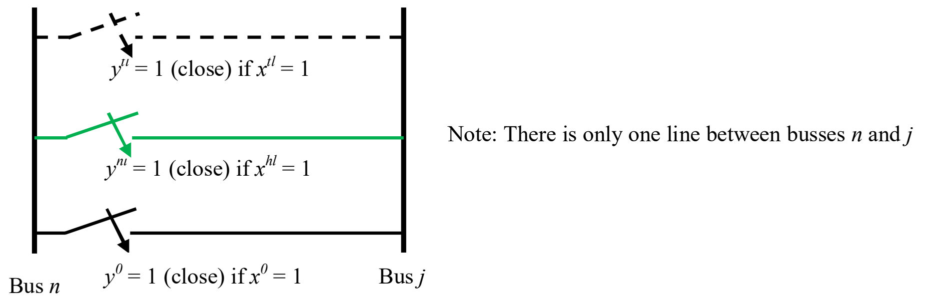


Fig. 2. Distribution line planning and network reconfiguration scheme

In (15), the constraint of the redial structure of the distribution network is modeled. Noted that in this paper as done in [32], it is considered that the distribution network includes one slack bus and several PQ buses. The slack bus is the distribution's substation bus, and loads and DGs are located in PQ buses. Hence, the radiality constraint implies that the total number of distribution lines is equal to the total number of PQ buses (total number of network busses – 1) [32]. Moreover, the backup DG planning with considering its capacity limit is formulated in (16). Finally, the constraints of this section should be merged by the network model, (2)-(9), using equations of (4) and (5).

$$y_{n,j,t} = y^{0}_{n,j,t} + y^{hl}_{n,j,t} + y^{tl}_{n,j,t} \quad \forall n, j, t \tag{10}$$

$$y^{0}_{n,j,t} \le x^{0}_{n,j} \quad \forall n, j, t \tag{11}$$

$$y^{hl}_{n,j,t} \le x^{hl}_{n,j} \quad \forall n, j, t \tag{12}$$

$$y^{tl}_{n,j,t} \le x^{tl}_{n,j} \quad \forall n, j, t \tag{13}$$

$$x^{0}_{n,j} + x^{hl}_{n,j} + x^{tl}_{n,j} \le 1 \quad \forall n, j \tag{14}$$

$$\sum_{(n,j)\in N} y_{n,j,t} = N_{bus} - 1 \quad \forall t \tag{15}$$

$$\left(P^{DG}_{n,t}\right)^2 + \left(Q^{DG}_{n,t}\right)^2 \le x^{dg}_{n} \left(S^{DG\max}_{n}\right)^2 \quad \forall\, n, t \tag{16}$$

2.2. Equivalent MILP model

The original RA formulation, (1)-(16), is as a non-convex MINLP, where the non-convexity refers to equations (4) and (5) [31, 33].

It should be mentioned that the linear formulation of this optimization problem generally includes following merits:

- Computational time is low; hence, the proposed optimization problem can be applied on large-scale problems [24].
- The optimal solution obtained by different solvers in the linear problem is unique for all the solvers. But, the occurrence probability of this statement is low in the non-convex NLP problems [28].
- All the solvers for the linear problem are able to certainly achieve the global optimal solution. However, some of them can obtain the global optimal solution for non-convex NLP problems, while their adjusting parameters will be changed if the model or data of the non-convex NLP problem is changed [30, 31].

Also, to apply Benders Decomposition model for the proposed RA strategy it is needed to have a linear formulation [33].

Accordingly, it is most probable that the MINLP finds the locally optimal solution in the best condition due to non-convexity and nonlinearity of some equations [26-31]. Therefore, this paper converts the proposed MINLP model to an equivalent MILP model with considering following linearization approaches to achieve the globally optimal solution at the lower calculation time:

1) *Linearized AC power flow equations*: In the AC power flow constraints, equations (4) and (5) are as mixed integer nonlinear formulations. However, the voltage angle difference across a line is less than 6 degrees [33], and the voltage magnitude can be approximated by $\underline{V} + \sum_{l \in L} \Delta V_l$ according to the piecewise linearization method [20]. Therefore, terms of $\cos\left(\delta_{n,t} - \delta_{j,t}\right)$, $\sin\left(\delta_{n,t} - \delta_{j,t}\right)$, V^2 and $V_n V_j$ can be approximated by 1, $\left(\delta_{n,t} - \delta_{j,t}\right)$, $\left(\underline{V}\right)^2 + \sum_{l \in L} slop_l \Delta V_l$ and $\left(\underline{V}\right)^2 + \underline{V}.\sum_{l \in L}\left(\Delta V_{n,l} + \Delta V_{j,l}\right)$, respectively, where $\Delta V \ll 1$ and the slope is the voltage deviation as the line slope. Moreover, equations (4) and (5) are as $a = b \times z$, while b and z are continuous and binary variables. According to Big M approach [24], this term can be linearized as $-M \times (1 - z) \leq a - b \leq M \times (1 - z)$ and $b_{min} \times z \leq a \leq b_{max} \times z$, where b_{min}, b_{max} and M refer to minimum/maximum value of b, and a large constant, respectively.

Noted that the linearized distribution flow (LinearDistFlow) can be used to obtain the linear format of the proposed problem, but it does not consider the power loss (active and reactive) of the distribution lines, and it is suitable for unidirectional radial power distribution network [6, 34]. But, the proposed linearization method here can be used for the distribution networks with different structures.

2) *Circular plane constraints*: The capacity limits of distribution line and station and backup DG follow the circular plane constraint as a non-linear equation $P^2+Q^2 \le S^2$. Noted that this plane can be approximated by a polygon plane [24, 33], where each edge of a polygon is a straight line and their equations are obtained from tangents of the circle at different points as $P\times\cos(k\times\Delta\alpha)+Q\times\sin(k\times\Delta\alpha)=S$ in the PQ plane and radius of S [33]. Therefore, the linear format of non-linear equation of $P^2+Q^2 \le S^2$ is expressed as $P\times\cos(k\times\Delta\alpha)+Q\times\sin(k\times\Delta\alpha)\le S$, where $\Delta\alpha = 2\pi/n_k$ is the angle deviation and n_k is the total number of the straight lines in a polygon. Finally, the index of k should be defined for the set of $K = \{1, 2, \ldots, n_k\}$.

As a result, The MILP method of the RA can be written as follows:

$$\min \quad \overbrace{\frac{1}{365\times Y}\left\{\sum_{n\in N} c_n^{dg} x_n^{dg} + \sum_{(n,j)\in N} c_{n,j}^{hl} x_{n,j}^{hl} + \sum_{(n,j)\in N} c_{n,j}^{tl} x_{n,j}^{tl}\right\}}^{\text{Daily investment cost}} + \overbrace{\frac{1}{du\times Y}.\left\{\sum_{n\in N} c_n^{rg} x_n^{dg} + \sum_{(n,j)\in N} c_{n,j}^{rl}\left(x_{n,j}^{0}+x_{n,j}^{hl}+x_{n,j}^{tl}\right)\right\}}^{\text{Daily repair cost}} + \overbrace{\sum_{t\in ST}\sum_{n\in N}\kappa_t P_{n,t}^{S} + \rho_n^{dg} P_{n,t}^{DG}}^{\text{Operational cost}} + \overbrace{\sum_{t\in ST}\sum_{n\in N} VOLL.P_{n,t}^{NS}}^{\text{Load sheding cost}} \tag{17}$$

S.to:

$$-M.\left(1-y_{n,j,t}\right) \le P_{n,j,t}^{L} - \left\{G_{n,j}\left(\sum_{l\in\varphi_l}\left(m_l-\underline{V}\right)\Delta V_{n,t,l} - \underline{V}.\Delta V_{j,t,l}\right) - \left(\underline{V}\right)^2 B_{n,j}\left(\delta_{n,t}-\delta_{j,t}\right)\right\} \le M.\left(1-y_{n,j,t}\right) \quad \forall n,j,t \tag{18}$$

$$-M.\left(1-y_{n,j,t}\right) \le Q_{n,j,t}^{L} - \left\{-B_{n,j}\left(\sum_{l\in\varphi_l}\left(m_l-\underline{V}\right)\Delta V_{n,t,l} - \underline{V}.\Delta V_{j,t,l}\right) - \left(\underline{V}\right)^2 G_{n,j}\left(\delta_{n,t}-\delta_{j,t}\right)\right\} \le M.\left(1-y_{n,j,t}\right) \quad \forall n,j,t \tag{19}$$

$$P_{n,j,t}^{L}\cos\left(k.\Delta\alpha\right) + Q_{n,j,t}^{L}\sin\left(k.\Delta\alpha\right) \le S_{n,j}^{L\max}.y_{n,j,t} \quad \forall\, n,j,t,k\in K \triangleq \{1,2,\ldots,n_k\}, \Delta\alpha = \frac{2\pi}{n_k} \tag{20}$$

$$P_{n,t}^{S}\cos\left(k.\Delta\alpha\right) + Q_{n,t}^{S}\sin\left(k.\Delta\alpha\right) \le S_n^{S\max} \quad \forall\, n = \text{Slack bus}, t, k \tag{21}$$

$$P_{n,t}^{DG}\cos\left(k.\Delta\alpha\right) + Q_{n,t}^{DG}\sin\left(k.\Delta\alpha\right) \le x_n^{dg} S_n^{DG\max} \quad \forall\, n, t, k \tag{22}$$

$$0 \le \Delta V_{n,t,l} \le \left(\overline{V}-\underline{V}\right)/n_l \quad \forall\, n,t,l\in L \triangleq \{1,2,\ldots,n_l\} \tag{23}$$

Constraints (2), (3), (6), (10) to (15) (24)

The objective function of the proposed MILP method, (17), is the same with (1), and constraints (18) and (19) refer to the linear model of (4) and (5) based on the first linearization method. Moreover, constraints (7), (8) and (16) are linearized respectively by (20)-(22) according to the second linearization approach. Noted that, the variable y appeared in the line capacity limit, (20), due to Big M approach in the proposed MILP model. Limitation (23) presents the voltage deviation limit in the new RA model as the voltage limit, because, the variable of voltage deviation is used in the proposed method. Finally, the constraint (24) considers all the linear equations of the original RA model, (1)-(16).

3. HSR-RA Model Based on BD Approach

A. Original HSRO model to RA: The parameters of active and reactive load, P^D and Q^D, energy price, κ, and repair cost, c^{rg} and c^{rl}, are the main sources of the uncertainty in the proposed RA strategy. Moreover, the repair cost of different sources and network equipment is dependent to availability of these devices in the extreme weather events such as earthquake or flood. Therefore, this cost will be imposed for an equipment if it is located in a zone including earthquake or flood. In this paper the HSRO model is adopted, because if the stochastic programming is deployed for all uncertain parameters, consequently, the calculation time will be increased for large scale networks and in some cases it results in infeasibilities. Also, to obtain accurate analysis for the load shedding cost and EENS, it is better to use the stochastic programming to model availability of the network equipment in the extreme weather events, but to achieve low calculation time, the robust model can be used for other uncertain parameters.

Noted that, in this paper, the HSRO method, as shown in Fig. 3, is used to model the uncertain parameters. In this regard, the BURO models the uncertainty of load and energy price, and the stochastic programming based on roulette wheel mechanism (RWM) generates different scenarios to model the availability of the different devices in the extreme weather events according to normal probability distribution function (PDF). In BURO, the true value of each uncertain parameter is between $(1-\sigma)\overline{P}$ and $(1+\sigma)\overline{P}$, where $\sigma \geq 0$ refers to the uncertainty level or the forecasting error and $\overline{P}$ is the normal or forecasted value of the uncertain parameter. Next, the uncertain parameter is equal to its upper or lower value depending on the optimization problem type, i.e., it is equal to $(1-\sigma)\overline{P}$ for *max* or *min* function and $(1+\sigma)\overline{P}$ for *min* function [35]. Finally, the proposed model to RA will be written as follows:

$$\min \quad \overbrace{\frac{1}{365\times Y}\left\{\sum_{n\in N} c_n^{dg} x_n^{dg} + \sum_{(n,j)\in N} c_{n,j}^{hl} x_{n,j}^{hl} + \sum_{(n,j)\in N} c_{n,j}^{tl} x_{n,j}^{tl}\right\}}^{\text{Daily investment cost}} + \tag{25}$$

$$\overbrace{\frac{1}{du\times Y}\cdot\sum_{w\in S}\pi_w\left\{\sum_{n\in N} c_{n,w}^{rg} x_n^{dg} + \sum_{(n,j)\in N} c_{n,j,w}^{rl}\left(x_{n,j}^{0} + x_{n,j}^{hl} + x_{n,j}^{tl}\right)\right\}}^{\text{Daily repair cost}} +$$

$$\overbrace{\sum_{w\in S}\pi_w\sum_{t\in ST}\sum_{n\in N}(1+\sigma)\kappa_t P_{n,t,w}^{S} + \rho_n^{dg} P_{n,t,w}^{DG}}^{\text{Operational cost}} + \overbrace{\sum_{w\in S}\pi_w\sum_{t\in ST}\sum_{n\in N} VOLL.P_{n,t,w}^{NS}}^{\text{Load sheding cost}}$$

S.to:

$$P_{n,t,w}^{S} + P_{n,t,w}^{DG} - \sum_{j\in N} A_{n,j} P_{n,j,t,w}^{L} = (1+\sigma)P_{n,t,w}^{D} - P_{n,t,w}^{NS} \quad \forall\, n,t,w \tag{26}$$

$$Q_{n,t,w}^{S} + Q_{n,t,w}^{DG} - \sum_{j\in N} A_{n,j} Q_{n,j,t,w}^{L} = (1+\sigma)Q_{n,t,w}^{D} - Q_{n,t,w}^{NS} \quad \forall\, n,t,w \tag{27}$$

Constraints (6), (10)-(15), (18)-(23) considering the index of w for continuous variables; (28)

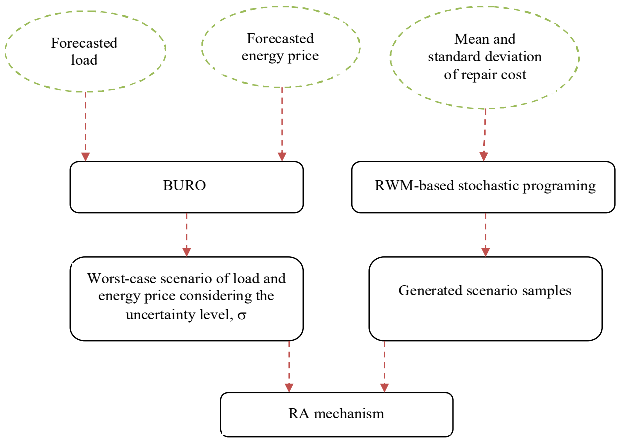


Fig. 3. The proposed HSRO scheme

In the above problem, the term of $(1 + \sigma)$ is used for energy price and load according to the BURO technique, also, these parameters and all proposed binary variables are not dependent of index w, because, they do not depend to uncertainty of availability of the different devices in the extreme weather events. In addition, in the proposed HSRO, the BURO is implemented in problem (25)-(28) to model the uncertainty of load and energy price and it is solved over different generated scenarios related to uncertainty of the availability of the different devices in the extreme weather events. Hence, in the proposed method both the stochastic and robust programming are implemented, simultaneously. The proposed HSRO based on BURO as a simple method benefits from the lower calculation time and suitable accuracy. However, in other robust models based on the adaptive robust programming, the method is complex and it includes more calculation time with respect to BURO [36].

It is noted that the proposed HSR-RA can be converted to the deterministic RA model by choosing zero values for the σ and the standard deviation.

B. Applying BD on HSR-RA modelBD approach: The proposed HSR-RA model is based on MILP modeling in (29), where the proposed binary/continuous variables are *x*/ *z*, and *a* to *g* refers to constants in the model of (25)-(28).

$$\min_{\Omega(x,z)} \quad a^T.x + b^T.z \quad \forall \Omega(x,z) \triangleq \left\{ x \in \{0,1\}, z \in \mathbb{R} \mid c.x \le d, e.x + f.z \le g \right\} \tag{29}$$

To accelerate the optimization solution procedure, the proposed MILP problem of (29) is decomposed by BD approach [37]. This approach includes the master problem (MP) and sub-problem (SP) [37], where the MP model of the proposed RA strategy in (29) is as follows:

$$\min_{\Xi(x)} \quad z_{lower} \quad \forall \Xi(x) \triangleq \left\{ \begin{array}{l} x \in \{0,1\} \mid 1) z_{lower} \ge a^T.x, \\ \qquad 2) c.x \le d, \\ \qquad 3) z_{lower} \ge a^T.x + J_{sub}^{(m)}(\lambda_{sub}^{(m)}) \,|_{m=1,2,\dots,n_f}, \\ \qquad 4) J_{sub}^{(r)}(\lambda_{sub}^{(r)}) \le 0 \,|_{r=1,2,\dots,n_i} \end{array} \right\} \tag{30}$$

z_{lower} is the MP's objective function that is equal to the first part of equation (29) according to constraint (1) in set of $\Xi(x)$, where it refers to the summation of daily investment (first row of (25)) and repair (second row of (25)) costs. Constraints (2)-(4) of this set refer respectively to logical limits in the proposed RA method, feasibility and infeasibility cuts of BD approach. Noted that constraint (2) of $\Xi(x)$ is equal to the distribution line planning and network reconfiguration models in equations (10)-(15). Finally, the MP output variable, *x*, as constant is applied to SP [37], where SP model is as (31) for the problem (29).

$$\min_{\Psi(z)} \quad J_{sub} = b^T.z \quad \forall \Psi(z) \triangleq \left\{ z \in \mathbb{R} \mid f.z \le g - e.x \right\} \tag{31}$$

J_{sub} is the SP's objective function that is equal to the second part of the equation (29), where it is the same as the summation of operational and load shedding costs in the third row of the equation (25). Constraint $f.z \le g - e.x$ refers to the linear and HSRO model of the constraints (2)-(9) and (16). Noted that the feasibility region of SP, $\Psi(z)$, depends on the value of *x*, therefore, it is changed in different iterations of the BD method [37]. To cope with this issue and obtain independent feasibility region from *x*, $\Pi(\lambda)$, the dual format of SP with the name of Dual Sub-Problem (DSP) is used as (32):

$$\max_{\Pi(\lambda)} \quad J_{sub} = (g - e.x)^T.\lambda \quad \forall \Pi(\lambda) \triangleq \left\{ \lambda \in \mathbb{R}^+ \mid f.\lambda (\le,=,\ge) b \right\} \tag{32}$$

The dual formulation has been expressed in [37]. Also, in (32), λ is the dual variable of constraint $f.z \le g - e.x$ in the problem (31). Moreover, the operators of ≤ /=/ ≥ are selected in constraint of $f.\lambda(\le,=,\ge)b$ if z is positive/free/negative [37]. Finally, there are three states for DSP based on the dual approach theory [37]:

1. *DSP has bounded value for its objective function*: The feasibility cut as (33) is added to the MP, (30), where $\hat{\lambda}_{sub}$ refers to the optimal value of λ in the problem (32).

$$z_{lower} \ge J_{sub}^{(m)}(\hat{\lambda}_{sub}) \qquad \forall J_{sub}^{(m)}(\hat{\lambda}_{sub}) = \text{Objective function of (32)}\Big|_{\hat{\lambda}_{sub}} \tag{33}$$

2. *DSP has unbounded value for its objective function*: The infeasibility cut as (34) is added to MP, so that $\hat{\lambda}_{sub}$ is achieved from (35).

$$J_{sub}^{(r)}(\hat{\lambda}_{sub}) \le 0 \qquad \forall J_{sub}^{(r)}(\hat{\lambda}_{sub}) = \text{Objective function of (35)}\Big|_{\hat{\lambda}_{sub}} \tag{34}$$

$$\max_{\Lambda(\lambda)} \quad J_{sub} = (g - e.x)^T.\lambda \quad \forall \Lambda(\lambda) \triangleq \left\{ \lambda \in \mathbb{R}^+ \mid f.\lambda(\le,=,\ge)b, \lambda \le 1 \right\} \tag{35}$$

3. *DSP is infeasible*: The proposed HSR-RA, (25)-(28), has the infeasible solution.

Finally, the proposed HSR-RA will be converged if the term $|z_{upper} - z_{lower}| \le \varepsilon$ is satisfied, where ε is the BD's convergence tolerance, and z_{upper} is calculated as (36). It is noted that the second part of (36) is the objective function of DSP and z_{lower} is determined by (32). The flowchart of the proposed algorithm is presented in Fig. 4.

$$z_{upper} = a^T.x + J_{sub}(\hat{\lambda}_{sub}) \qquad \forall J_{sub}(\hat{\lambda}_{sub}) = \text{Objective function of (32)} \tag{36}$$

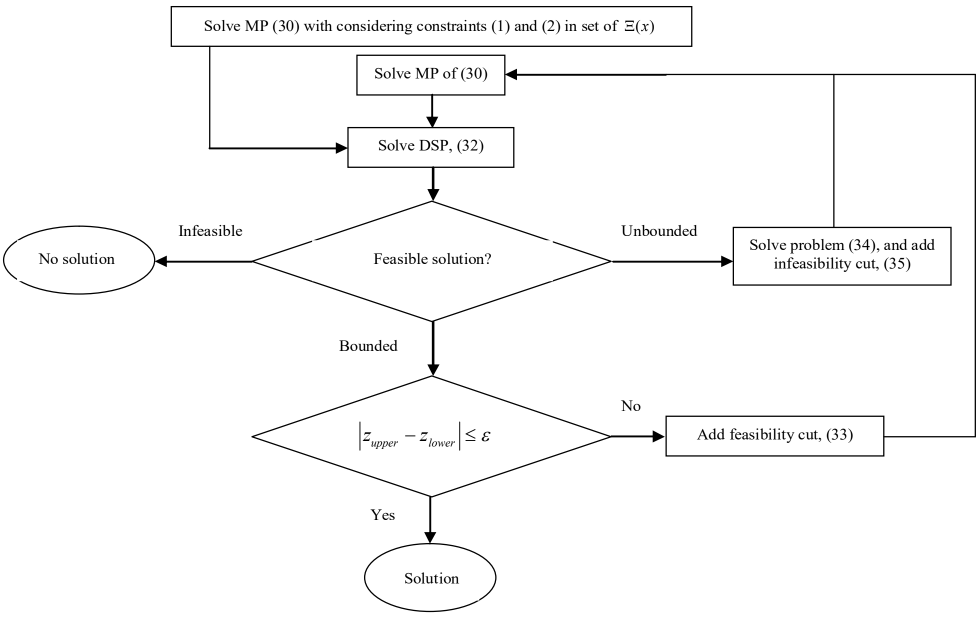
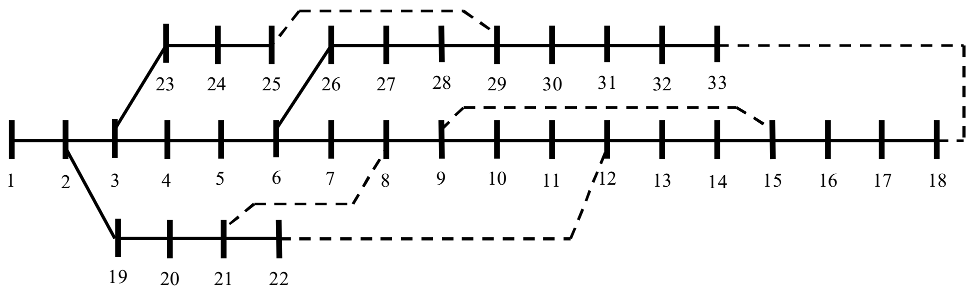


Fig. 4. Flowchart of the proposed BD solution method

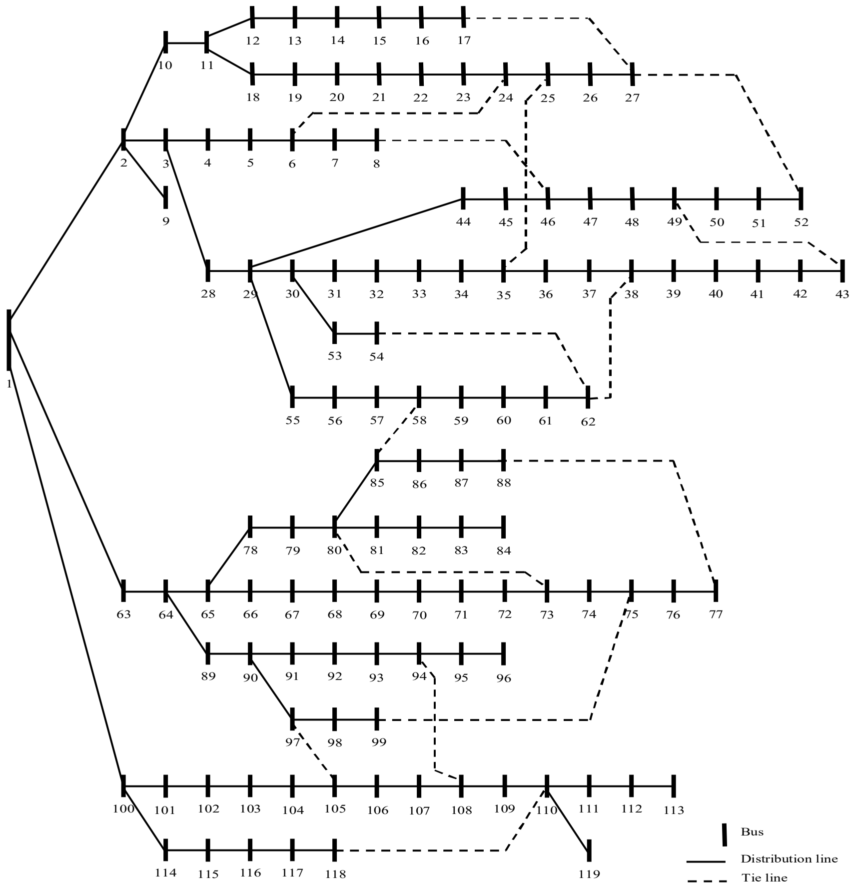
4. Numerical Results and Discussion

4.1. Case studies

The proposed HSR-RA strategy is studied on 33-bus and 119-bus radial distribution test networks depicted by Fig. 5 [38]. The line characteristics and peak load data are expressed in [38], but daily load data is considered as the multiplication of peak load value and daily load factor that is based on data shown in Fig. 3(a) [27]. Moreover, the characteristics of the backup DG, hardening and tie lines are presented in Table 1. It is assumed that the backup DG, hardening and tie lines are resistant against the natural disasters. Therefore, it is possible that their repair cost is very low, which is omitted in this study. Hence, this paper considers that the repair cost is zero for these devices and it is 3211$/pole for existing line. It is possible that buses (11-16), (20-22), (23-24), (29-31) in 33-bus test are exposed by the earthquake, flood, earthquake and flood, respectively. Also, it is anticipated that buses (21-25), (28-30, 53, 53), (39-43), (70-73), and (114-118) in 119-bus system are endangered respectively by earthquake, earthquake, flood, flood, and flood, respectively. In addition, daily curve of energy price is presented in Fig. 6(b) [39], but VOLL is considered to 100 $/MWh to achieve a network with high resilience. Finally, RWM generates 30 scenario samples to model the uncertainty of availability of the DGs and network equipment's under extreme weather events based on the normal PDF with standard deviation of 10%. Also, the uncertainty of energy price and load is based on BURO model in the worst-case scenario.

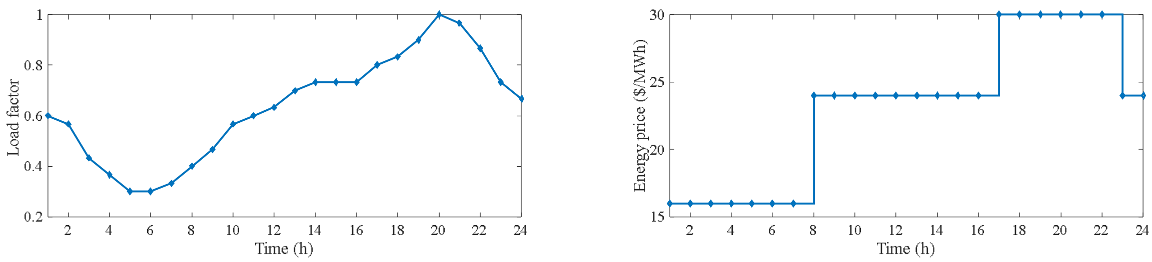


(a)



(b)

Fig. 5 Distribution test networks, (a) 33-bus, (b) 119-bus [38]



(a) (b)

Fig. 6. Daily curve of, (a) load factor [27], (b) energy price [39]

Table 1: Characterizes of the backup DG, hardening and tie line [6]

Device	Candidate location	Capacity	Investment cost	Operation cost
Backup DG	All buses	500 kVA	1500 $/kVA	20 $/MWh
Lines hardening	All line section	All data is same with existing line	5924 $/pole	-
Tie line	Dashed line in Fig. 3	-	15000 $	-

4.2. Results

The proposed problem of (25)-(28) based on the BD approach is programmed in GAMS, employing the CPLEX solver to investigate the capabilities of this method [40].

1) *Comparison of different model results*: Table 2 presents the results of the deterministic RA model by different approaches, i.e. MINLP, MILP and BD-based MILP, for 33-bus and 119-bus networks. According to Table 2, the calculation error of MILP method with respect to the original MINLP method ([variable value in MINLP – variable value in MILP]/variable value in the MINLP) for the power and voltage is about 2.3% and 0.45%, respectively, in different distribution networks. It is noted that the MILP method can obtain the optimal solution at 64 and 127 seconds for 33-bus and 119-bus networks, respectively, while MINLP solves the deterministic RA problem at 794 and 2106 seconds for these networks, respectively. Moreover, the MILP model based on BD approach is able to achieve the optimal solution at 17 and 31 seconds with 13 and 18 convergence iteration numbers for these distribution systems. Therefore, this method is a faster solver and approach for the proposed RA strategy with the low calculation error in comparison with the original RA model in the different distribution grids. Noted that the proposed BD approach can obtain the optimal solution at 31 seconds in the 119-bus distribution network while the original MINLP model is executed by 2106 seconds. It is possible that the original model cannot be able to achieve the optimal solution for a large-scale real distribution network due to complexities of the original model. Thus, to cope with this issue, it is necessary to use relaxation or decomposition methods. Also, the Benders decomposition method can be applied to linear formulations, accordingly, this paper has proposed the MILP format for the proposed problem. The proposed problem can obtain optimal solution in different sizes of the distribution network based on Table 2. Moreover, benefits of the linearized AC optimal power flow in this paper are:

- The total network variables such as active and reactive power, active and reactive power loss, magnitude and angle of voltage can be calculated in this method, while the linearized distribution flow method in [6] ignores the active and reactive power loss and imaginary part of voltage, or DC method does not consider reactive power, power loss and voltage drop.
- Also, the proposed method is so accurate where its results are close to the original AC optimal power flow results based on Table 2.

Consequently, these explanations demonstrate the benefits of the second contribution in section 1.3.

Table 2: Comparison between deterministic MINLP and MILP models at peak load hour (20:00)

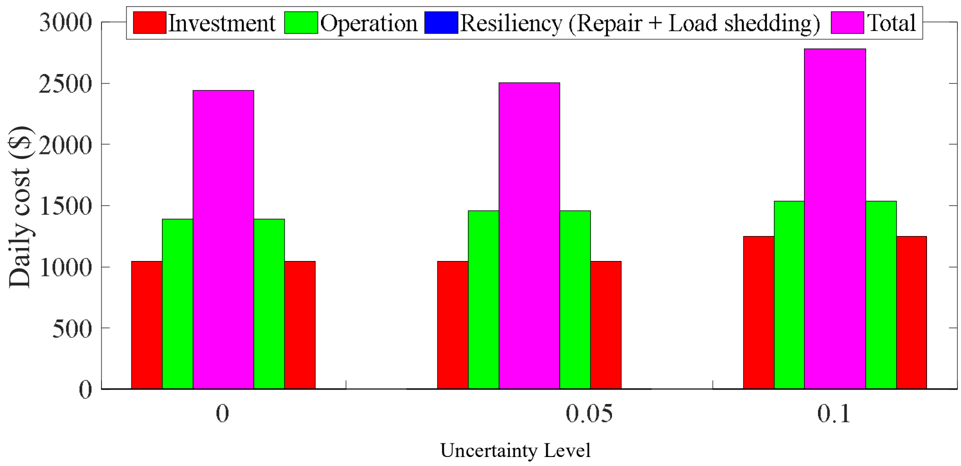
33-bus distribution network					
Parameter	MINLP	MILP		MILP based on BD method (ε = 5$)	
		Value	Calculation error (%)	Value	Calculation error (%)
Total generation active power (MW)	3.850	3.764145	2.23	3.75683	2.242
Total generation reactive power (MVAr)	2.393	2.335329	2.41	2.33496	2.425
Mean of voltage magnitude (p.u)	0.957	0.9615936	0.48	0.961622	0.483
Mean of voltage angle (rad)	-0.0005	-0.00049795	0.41	-0.00049794	0.411
Calculation time (seconds)	794	64	-	17	-
Convergence iteration number	-	-	-	13	-
119-bus distribution network					
Parameter	MINLP	MILP		MILP based on BD method (ε = 5$)	
		Value	Calculation error (%)	Value	Calculation error (%)
Total generation active power (MW)	23.277	22.7626	2.21	22.7579	2.23
Total generation reactive power (MVAr)	17.552	17.1325	2.39	17.1290	2.41
Mean of voltage magnitude (p.u)	0.946	0.950541	0.48	0.950978	0.484
Mean of voltage angle (rad)	-0.0009	-0.00089635	0.405	-0.00089622	0.42
Calculation time (seconds)	2106	127	-	31	-
Convergence iteration number	-	-		18	-

2) *Economic and expansion planning results of the proposed RA strategy*: The expansion planning results based on the proposed HSR-RA model, (25)-(28) are expressed in Table 3 for 33-bus and 119-bus distribution grids. In this Table, uncertain level of 5%, i.e. $\sigma = 0.05$, is considered to the load and energy price, and standard deviation for the availability of the network equipment in the extreme weather condition is 10%. According to the comparison between this table and Fig. 5, it is seen that in these networks, lines hardening are installed in zones that are included extreme weather events such as earthquake or flood to obtain high resiliency for these networks at the proposed natural disaster conditions. Because, these lines are strong and the probability of their outage are low under these conditions with respect to conventional existing lines. Moreover, 3 and 5 backup DGs are installed respectively to 33-bus and 119-bus distribution networks to improve the operation and resiliency indices such as voltage profile, load shedding condition, so that these systems are located in buses and zones that are placed farther from slack bus. Also, as shown in Table 3, the proposed RA strategy suggests 4 and 12 tie lines for 33-bus and 119-bus grids to improve the economic, operational and resiliency indices based on the minimization of the planning, operation and resiliency costs. The impacts of these planning results on the distribution network operation and resiliency conditions are expressed in sub-section 4.2.3.

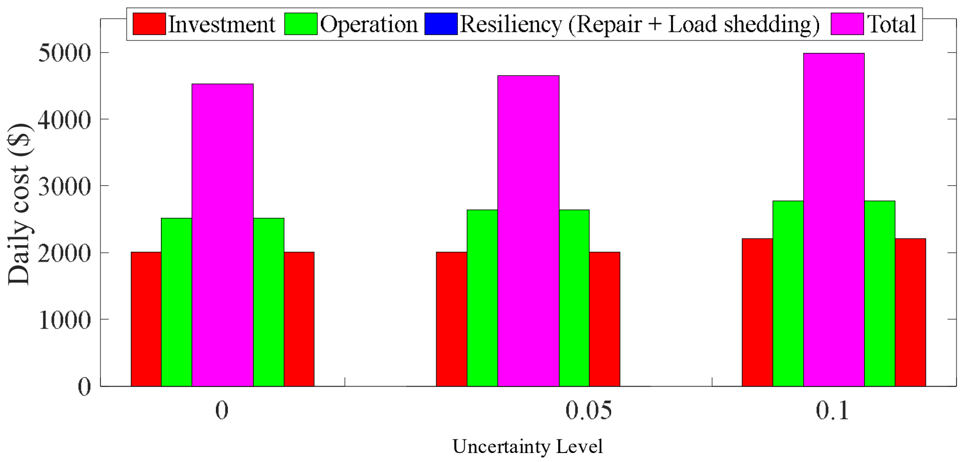
Table 3: Expansion planning results in different distribution networks in the HSR-RA model with considering σ = 0.05

Network	Optimal location of		
	Backup DG in buses	Lines hardening	Tie line between buses
33-bus	13, 17, 30	Between buses 1-3, 28-31, 11-16, feeder 3-25, 2-22	(9,15), (12,22), (18,33), (25,29)
119-bus	25, 29, 41, 73, 110	Between buses, 1-2, 1-63, 1-100, 21-26, 41-43, 70-74, feeders 3-30, 30-54, 100-118	(6,24), (8,46), (25,35), (54,62), (43,49), (38,62), (58,85), (73,80), (75,99), (94,108), (97,105), (110,118)

In addition, the economic results of the proposed HSR-RA approach according to different values of uncertainty level of load and energy price, and 10% standard deviation for the availability of the network equipment in the extreme weather condition for the different proposed networks are shown in Fig. 7. Based on this figure, this approach can obtain zero resiliency, i.e., repair and load shedding cost in these networks, therefore, the VOLL with considering 100 \$/MWh is suitable for these systems. Because, in this condition with the high VOLL, the system planner uses the high number of backup DGs, hardening and tie lines in the distribution network to minimize the load shedding cost according to (1). Also, the system repair cost will be minimized in this condition because these DGs and lines that are installed generally in the zones containing earthquake or flood include zero repair cost based on Section 4.1. Noted that in this case, based on the Roulette Wheel Mechanism (RWM) 30 scenario samples have been generated to model the uncertainty of availability of the network equipment in the extreme weather condition for each uncertainty level of load and energy price. In each scenario, several network equipments such as distribution line and back-up DG in zone including earthquake or flood should be disconnected from the network if their outage probability is high. But, since that strong equipments against natural disasters are installed in the network based on Table 3, where their outage probability is about zero in this paper, hence, the load shedding and repair cost is about zero. Therefore, these networks have high resiliency in this case and at different conditions of uncertainty levels. Moreover, the operation cost is increased with increasing σ, because, the load and energy price values will be increased if the uncertainty level is increased based on the model (25)-(28). But, the investment cost is the same for $\sigma = 0$ and $\sigma = 0.05$, and it is increased in $\sigma = 0.1$ with respect to other values of σ. This statement refers that upstream network and backup DGs in Table 3 are able to supply total network demand at cases with $\sigma = 0$ and $\sigma = 0.05$, but, the proposed networks needs more backup DGs at $\sigma = 0.1$ in comparison with other cases to supply the increased load in this condition. Accordingly, the total RA cost based on equation (1) will be increased with increasing the uncertainty level.



(a)



(b)

Fig. 7 Economic results of the proposed HSR-RA in distribution network of (a) 33-bus, (b) 119-bus test networks

3) *Investigating operation and resiliency capability*: Table 4 presents the operation indices results of 33-bus and 119-bus distribution networks in cases I and II that are referred respectively to network power flow analysis and the proposed HSR-RA problem. In the case of $\sigma = 0$, the proposed RA can reduce the maximum voltage deviation (MVD) and energy loss (EL) in the 33-bus and 119-bus networks, accordingly the MVD (EL) is reduced respectively about to 39% (31%) and 29% (33%) in these networks for Case II with respect to Case I. This statement is due to using local sources such as backup DGs and optimal operations of tie lines in these distribution systems. In addition, MVD and EL is increased by increasing the uncertainty level according to Table 4, because, the load value will be raised if the σ is increased based on the model (25)-(28). In addition, the curve of the resiliency indices, i.e., expected energy not supplied (EENS) as $\sum_{w \in S} \pi_w \sum_{t \in ST} \sum_{n \in N} P^{NS}_{n,t,w}$, repair and load shedding cost, and planning cost

(investment + operation costs) versus VOLL according to different values of σ in 33-bus and 119-bus distribution grids are plotted in Figs. 8 and 9, respectively. In the case of $\sigma = 0$, according to these figures, these systems contain low resiliency in the VOLL = 0 (no incentive) due to the high value for ENNS and repair cost in this condition. Noted that the EENS and repair cost will be reduced if the VOLL is increased, but, the load shedding cost is increased/reduced if the VOLL is increased between 0 to 20 $/MWh / 20 to 100 $/MWh. Therefore, the high resiliency condition, i.e. EENS, repair and load shedding cost are zero, is obtained for 33-bus and 119-bus networks at the VOLL of 70 and 80 $/MWh, respectively. But, it is seen that increasing the resiliency is based on the increasing of planning cost, where this statement implies that the system needs the high number of backup DGs and hardening and tie lines to obtain the higher resiliency. Finally, increasing the uncertainty level causes that the value of the resiliency indices and planning cost are increased due to increasing the load and energy price in this condition. Therefore, it is seen that based on these explanations, the proposed RA scheme can obtain high resiliency (zero load shedding and repair costs) in the distribution network considering suitable value for the VOLL, i.e., 60 / 80 $/MWh for 33-bus / 119-bus networks. Also, this approach is able to improve the network operation indices (EL and MVD) based on the management and coordination of the local DGs in distribution systems by optimal planning of back-up DGs, hardening and tie lines according to the proposed RA model in (1)-(16), where these objectives demonstrate the benefits of the first contribution in section 1.3. In addition, the HSRO model in the proposed approach is able to consider the variation of load, energy price, and availability of the network equipment in the extreme weather condition based on the results of sub-sections 4.2.2 and 4.2.3. Hence, the proposed HSR-RA can attain the robust and guaranteed planning for the back-up DGs, hardening and tie lines in the distribution network as the profits of the third contribution in section 1.3.

Table 4: Network operation indices in the proposed HSR-RA on the different distribution test networks

Case	Network	Index	Uncertain level		
			0	0.05	0.1
I	33-bus	MVD (p.u)	0.087	0.092	0.098
		EL (MWh)	3.077	3.231	3.385
	119-bus	MVD (p.u)	0.092	0.098	0.105
		EL (MWh)	25.64	26.92	28.20
II	33-bus	MVD (p.u)	0.053	0.055	0.0575
		EL (MWh)	2.111	2.216	2.322
	119-bus	MVD (p.u)	0.065	0.0667	0.069
		EL (MWh)	17.12	17.97	18.83

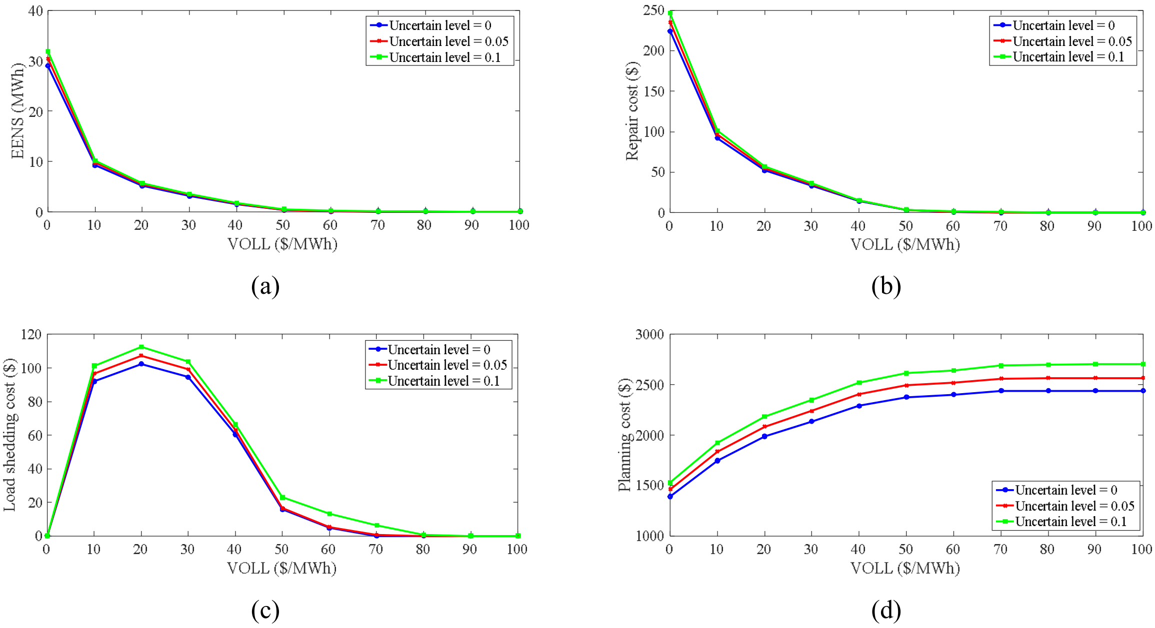


Fig. 8 Economic and resiliency indices curve versus VOLL in the 33-bus distribution test network

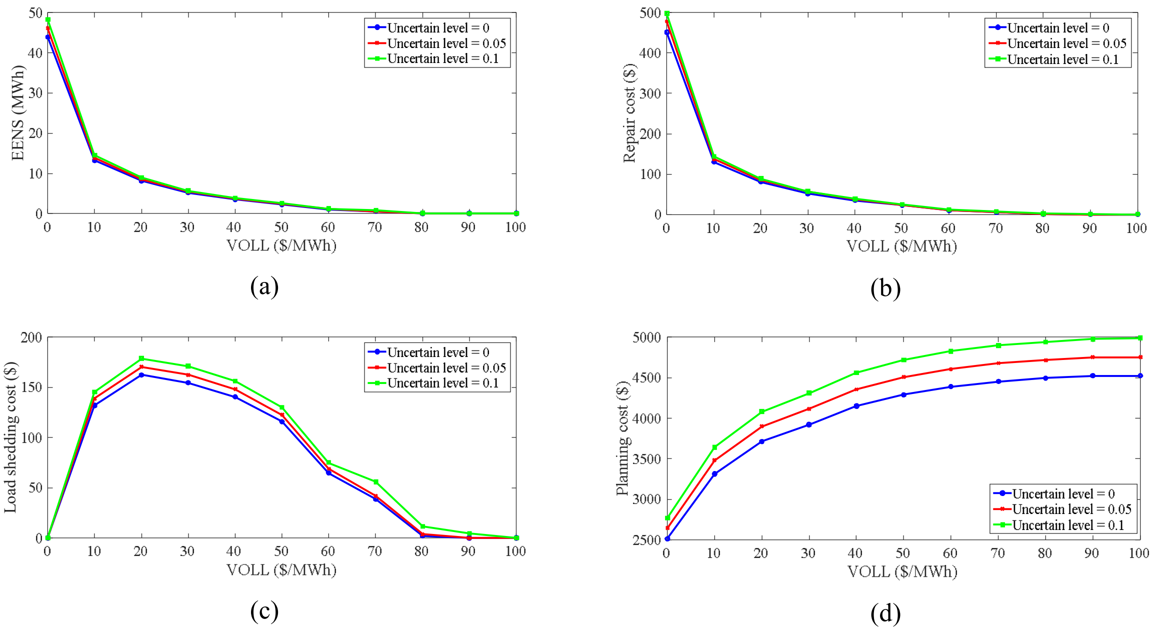


Fig. 9 Economic and resiliency indices curve versus VOLL in the 119-bus distribution test network

5. Conclusions

In this paper, backup DG and hardening and tie lines planning model has been presented in the distribution networks using the RA strategy against the extreme weather events, such as earthquakes and floods. In the proposed deterministic model, the objective function is to minimize the daily investment, operation, repair and load shedding costs subject to the constraints of the network operation model, DGs and line planning, as well as network reconfiguration formulation. The linear formulation has been developed based on the BD approach. To deal with the uncertainty sources of the problem, the HSRO based on BURO and scenario-based stochastic optimization have been developed to cope with the uncertainty of load, energy price and availability of network equipment under the earthquake and flood conditions. According to the simulation results, the proposed BD method-based MILP HSR-RA problem is able to achieve the optimal solution at the lowest calculation time and calculation error with respect to the original proposed problem. Accordingly, this approach obtains the feasible solution in the 119-bus test network at 31 seconds while the original MINLP model of the RA solves at 2106 seconds. In addition, the proposed strategy could obtain the higher resiliency level with zero value for EENS, repair and load shedding cost based on the optimal value of VOLL of 60 / 80 $/MWh in the 33-bus / 119-bus radial test distribution networks, while it achieved the optimal location of backup DGs and hardening and tie lines in the distribution network under the earthquake and flood conditions. Moreover, it was able to improve the operational and economic indices in the distribution system for different uncertainty levels of load and energy price, consequently it can reduce the network energy loss and maximum voltage deviation about 30% with respect to the case without the RA strategy.

In addition, this paper has not considered the impacts of the truck-mounted mobile emergency generator, power electronics devices, energy storage, power distribution architecture, and lifeline dependencies on the resilient response against natural disasters, hence, this case will be considered in the future works. Moreover, the HSR-RA model for the low voltage distribution network can be developed to the unbalanced distribution networks as a new research work.

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