P2P Trade With Prosumers' Actual Approximate Utility Functions Within Near-Potential Games Framework

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***Abstract*—There is a potential conflict between P2P trade and distribution network operator (DSO) decisions, which slows down the expansion of P2P trade. Expanding the P2P trade requires reducing this conflict and increasing DSO awareness of the actual behavior of prosumers. In other words, mitigating this conflict requires that DSO approximates prosumers' utility functions (PUFs) based on their actual behavior. On the other hand, PUFs approximated based on the actual behavior of prosumers have various parameters such as freedom in decision-making, collective influence, privacy, and marginal cost/utility. This is a mathematical challenge for DSO because this class of PUFs may not be convex or continuously differentiable. Hence, in this paper, a near-potential game (NPG) framework is proposed to support the design of P2P trade with PUFs belonging to this class. Also, to develop a realistic model of P2P trade, we classify prosumers into residential and non-residential classes and assume that prosumers have limited information about each other's decisions. Then within an NPG framework, we introduce a learning model, whereby each prosumer obtains an estimate of the prosumers' decisions in P2P trade.**

***Index Terms*—Peer-to-peer trade, prosumers' utility functions, near-potential game.**

NOMENCLATURE

Indices and Sets

$\mathbb{R}, \mathbb{N}, \mathbb{C}$	Sets of real, natural, and complex numbers.
$i, j, z, \mathcal{N}$	Indices and set of prosumers at P2P trade.
$-i$	All prosumers from set $\mathcal{N}$ except prosumer i.
r, ic	Indices of residential and non-residential prosumers.
$\mathcal{N}_r, \mathcal{N}_{ic}$	Sets of r and ic prosumers, respectively.
n_r, n_{ic}	Total number of r and ic prosumers, respectively.
$\mathcal{N}_S, \mathcal{N}_B$	Sets of seller and buyer prosumers, respectively.
S, B	Total number of seller and buyer prosumers.
fl, dg, sl	Indices of FLs, DGs, and static loads, respectively.
n_{fl}, n_{dg}, n_{sl}	Total number of FLs, DGs, and static loads, respectively.

Variables and Parameters

p_{ij}	Energy exchange tendency between prosumer i with prosumer j from point of view of prosumer i.
$\boldsymbol{p}_i$	Decision vector of prosumer i.
$\mathbf{p_i}$	Strategy space of prosumers i.
$\boldsymbol{p}_{-i}$	Decision vector of all prosumers except prosumers i for energy exchange with prosumers i.
$\mathbf{p_{-i}}$	Strategy space of all prosumers except prosumer i.
$\boldsymbol{p}_{ic}$	Maximum tradable energy in P2P trade by non-residential prosumers.
$\boldsymbol{p}_r$	Maximum tradable energy in P2P trade by residential prosumers.
$\hat{p}^z_{ji}$	Prosumer i's estimate of prosumer j's strategy for tendency to exchange energy with prosumer z.
$\hat{\pi}^j_i$	Prosumer i's estimate of the prosumer j's price for participating in P2P trade.
$\boldsymbol{p}_{fl}, \boldsymbol{p}_{dg}, \boldsymbol{p}_{sl}$	Active power vector of FLs, DGs, and static loads, respectively.
$\boldsymbol{\pi}_i$	Expected price of prosumer i from other prosumers.
$\boldsymbol{\pi}^d$	Energy price vector from DLMP.
c^1_i, c^2_i	cost of production (or consumer utility functions) of i in the utility function.
$c^{fix}_i, \boldsymbol{c}^{tr}_i$	Fixed and variable cost of i due to network utilization.
α	A number between zero and one to determine the impact of $\mathcal{N}_r$ or $\mathcal{N}_{ic}$ on P2P trade.
π^{DSO}_i	DSO's pricing strategy for prosumer i.
π^{Pro}_i	Motivation parameter i to participate in P2P trade.

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$\mathfrak{E}$	Discretized strategy space.
W_j	Weight matrix that makes $\hat{p}^z_{ji}(t) = p_{jz}(t)$ as $t \rightarrow \infty$.
$\boldsymbol{Y}, \boldsymbol{Z}$	Admittance and impedance matrix of the $\mathcal{G}$, respectively.
$\boldsymbol{R}, \boldsymbol{X}$	Resistance and reactance matrix of the $\mathcal{G}$, respectively.
$\underline{\boldsymbol{u}}, \underline{\boldsymbol{i}}, \underline{\boldsymbol{s}}$	Complex voltages, branch current and power injections, respectively.
p_{loss}, q_{loss}	Active and reactive losses in the $\mathcal{G}$, respectively.
λ, μ	Dual variables.
$\boldsymbol{c}_{fl}, \boldsymbol{c}_{dg}$	Positive price vectors for *fl* and *dg*, respectively.
$\boldsymbol{C}_{fl}, \boldsymbol{C}_{dg}$	Price sensitivity coefficients matrix for *fl* and *dg*, respectively.
fm	membership function for technical parameters of the network.

Operators and Symbols

$x, \boldsymbol{x}, \boldsymbol{X}, \underline{x}$	Number, vector, matrix and complex numbers.
$\boldsymbol{x}^\intercal, \lvert x \rvert$	Transpose and absolute of $\boldsymbol{x}$ and x respectively.
$\boldsymbol{X}^{-1}$	Inverse of the matrix $\boldsymbol{X}$
$\lVert \boldsymbol{x} \rVert$	Euclidean norm of $\boldsymbol{x}$, that is $\lVert x \rVert = \sqrt{x^\intercal x}$.
$\Pi_{\boldsymbol{x}}(\cdot)$	Euclidean projection operator on a set $\boldsymbol{x}$, that is $\Pi_{\boldsymbol{x}}(u) \triangleq \arg\min_{r \in \boldsymbol{x}} \lVert u - r \rVert$.
$\hat{x}$	Approximation of the actual value of x.
$\underline{x}^*, diag(\boldsymbol{x})$	Complex conjugate and diagonal matrix.
$real(\underline{x})$	Real value of $\underline{x}$.
$imag(\underline{x})$	Imaginary value of $\underline{x}$.
$\jmath x$	Imaginary part of x.

Abbreviations

NPG	Near potential game.
PUF	Prosumers' utility function.
PAAUFs	Prosumers' actual approximate utility functions.
DLMP	Distributed locational marginal pricing.
ADMM	Alternating direction method of multipliers.
FLs	Flexible loads.
DGs	Distributed generators.
NE	Nash equilibrium.

I. Introduction

A. Motivation and Goal

IN RECENT years, due to population growth and changing lifestyles, power systems have undergone significant rapid changes to coordinate, install, and operate renewable resources such as photovoltaics and wind turbines [1]. The expansion of renewable resources in power systems will bring many societal benefits, including environmental, and economic. An important non-technical force driving the use of renewable sources is the motivation of the end users [2]. Hence one of the important platforms for the expansion of renewable resources in distribution networks (DNs) that has received much attention from researchers is the peer-to-peer (P2P) trade platform, as it meaningfully accommodates the motivations of end users and their profits [3].

P2P trade is an energy exchange platform between end users, herein referred to as *prosumers*, which enables them to independently, and strategically, reduce their energy consumption costs by buying or selling energy from other prosumers [4]. The independence of the prosumers refers to the fact that they freely decide on the details of energy trade, such as the amount and price of energy, and as a result, this will increase their motivation to participate in the installation and operation of renewable resources.

One of the important factors preventing the development of P2P trade is the conflict between the goals, concerns, and priorities of DSOs and prosumers. In other words, from the point of view of DSO as the responsible entity of DNs, if the prosumers' behavior leads to an increase in technical parameters such as active losses and voltage deviation, DSO will prevent the development of P2P trade [5]. Consequently, DSOs' awareness of prosumers' behavior in P2P trade has a great impact on the development of P2P trade [6]. Hence, a critical research question arises in this context: what does DSO need to increase its understanding of prosumers' behavior?

In the literature, the interaction between DSO and P2P trade is based on well-known mathematical methods such as distributed locational marginal pricing (DLMP), alternating direction method of multipliers (ADMM), and leader-follower method (Stackelberg game). For example, in [7], a market plan based on DLMP and P2P trade has been proposed that leads to an increase in the DSO's revenue from P2P trade and maintains the technical parameters. In [8], a DLMP-based ancillary services market has been proposed that uses the ADMM method to regulate the behavior of prosumers in a friendly manner, in such a way that the network constraints are not violated. In [9], [10], [11], a bi-level model of the interaction between DSO and P2P trade is proposed, in which DSO pricing is based on DLMP on one level, and prosumers' behavior is modeled based on the ADMM approach on the other level. In this regard, recently in [12], based on a single-level mixed-integer quadratic programming, a pricing method for interaction between DSO and P2P trade has been proposed, which is useful for DSO and prosumers at the same time.

Despite extensive studies of the interaction between DSO and P2P trade, to the best of our knowledge, there is a significant gap between research and practice in this context. In fact, the prosumers' behavior in P2P trade, whether based on conventional models such as DLMP, ADMM, Stackelberg game, or any other model, is modeled based on their utility functions, and evaluating the degree of interaction between DSO and P2P trade is measured based on these utility functions. As a result, if prosumers' utility functions (PUFs) are inconsistent with their actual behavior in P2P trade, maximum interaction between DSO and P2P trade will not occur in reality. On the other hand, although the accurate determination of actual PUFs in different societies depends on a plethora of socio-techno-economic factors [13], numerous studies based on practical data allow the approximation of actual PUFs, herein referred to as *prosumers' actual approximate utility functions (PAAUFs)*, which constitutes an unusual mathematical form (a class of utility functions that may not be convex and continuously differentiable) [14], [15]. Consequently, from the theoretical point of view, increasing the interaction between DSO and

P2P trade in reality requires the modeling of PAAUFs, and on the other hand, these PAAUFs create important challenges to achieving convergence in optimization methods in P2P trade contexts.

Hence, this work aims to bridge this research gap: how can DSO calculate the asymptotic behavior of P2P trade if PAAUFs are modeled? As a result, a near-potential game framework for P2P trade design is proposed, which can calculate the asymptotic behavior of P2P trade by considering PAAUFs. The proposed framework of this paper leads to improved interaction between DSO and P2P trade in reality because DSO decisions are based on the calculated asymptotic behavior of P2P trade taking into account the PAAUFs.

B. Literature Review

As stated in [16], [17], PAAUFs have different dimensions. Firstly, the PAAUF of each prosumer is greatly affected by the collective behavior of all prosumers in P2P trade. This issue stems from the fact that from the point of view of microeconomics, the behavior of small players in markets is influenced by the collective behavior of other players [18]. This makes the issue of P2P trade optimization to be designed in the framework of a multiagent system with prosumers' strategic decisions that depend on each other's decisions. Secondly, PAAUF should be determined based on the price elasticity of each prosumer. In P2P trade, there is a wide variety of prosumers with different price elasticities. To model the actual behavior of prosumers, PAAUF should be proportional to the price elasticity of each prosumer. In this regard, in some articles such as [19], PAAUFs based on residential, industrial, and commercial classifications have been proposed, but other parameters, the collective influence of prosumers on each other, have been neglected. On the other hand, as stated in [20], PAAUF in the real world follows various functions such as linear, nonlinear, exponential, and logarithmic, which causes a lot of computational complexity in a multiagent system. Thirdly, privacy and freedom in decision-making should be considered in PAAUF modeling. Privacy refers to the fact that the information of each prosumer is not available to all other prosumers. This makes the calculation of the asymptotic behavior of P2P trade in a model based on a multiagent system with incomplete information [21]. On the other hand, freedom in decision-making refers to the fact that prosumers should have the possibility to change their decisions according to the decisions of other players, especially DSO, as a powerful institution of DNs [22].

Despite the fact that modeling PAAUF considering all these dimensions is very important in increasing DSO's understanding of prosumers' behavior in P2P trade, little attention has been paid to their mathematical modeling in the literature due to the computational complexity of PAAUF. For example, in [23], the PUFs have been classified into two parts, the first part of which has been related to the utility of energy usage and the second part has been related to the revenue and cost of P2P trade. Then, based on the Stackelberg game, they modeled the influence of the DSO on prosumers' decisions in the PUFs, but other parameters affecting the PAAUF were ignored. In [24], the quadratic function has been used to model PUFs in P2P

TABLE I
COMPARATIVE EVALUATION BETWEEN THIS STUDY AND PREVIOUS WORKS

Ref	**Classification of prosumers**	**Freedom in decision-making**	**privacy**	**Not be convex and continuously differentiable**	**Mathematical approach**
[23]	×	✓	×	×	Stackelberg game
[24]	×	×	×	×	Generalized aggregative game
[25]	×	×	×	×	ADMM
[26]	×	×	✓	×	Distributed generalized game
[27]	×	×	×	✓	Auction theory
[10]	×	×	✓	×	ADMM
[28]	×	×	×	×	ADMM
[8]	×	✓	×	×	DLPM and ADMM
[12]	×	✓	×	×	SLMIQP
This work	✓	✓	✓	✓	Near-potential game

trade, and a market-clearing mechanism has been proposed based on the generalized aggregative game. In this regard, in [25] ADMM-based P2P trade with the quadratic function of PUFs has been proposed. Their main focus has been on adding network usage charges to PUFs. The most common PUFs in the P2P trade are quadratic functions because these functions are strongly convex and well reflect the marginal cost (utility) of prosumers. However, it is important to note that prosumers' behavior is not only affected by the marginal cost (utility) and as a result, quadratic functions alone are not suitable for modeling PAAUF. In [26], the main focus is on the unification between prosumers, and as a result, the modeling of PUFs with a strongly monotone function has been proposed. In [27], it has been discussed that PUFs are non-convex in real conditions, therefore, an optimization model based on auction theory and game theory has been proposed that guarantees convergence in non-convex PUFs. The model proposed in [27] neglects issues such as privacy, price elasticity, and freedom in decision-making.

C. Our Contributions

According to the literature review, a gap in the literature has been identified, which was that most articles focused on the interaction between DSO and P2P trade and ignored the challenges of approximating the actual behavior of prosumers in P2P trade. Addressing these challenges and developing reliable mathematical models is crucial for real-world P2P trade development. In other words, if the PUFs are not consistent with their actual behavior, it may cause deviations in the results in theoretical modeling and the real world, and thus lead to preventing the development of P2P trade in the real world. Therefore, as summarized in Table I, this paper bridges the research gap by designing a P2P trade considering PAAUF based on near-potential games, where a potential function can model the actual behavior of prosumers.

The main contributions of this article are listed below.

1) This paper proposes a framework of P2P trade based on near-potential games (NPG) and best-response dynamics that deals with PAAUF, taking into account freedom in decision-making, collective influence, privacy, and marginal cost/utility. In this framework, prosumers are classified into residential (with a linear PAAUF) and non-residential (with a nonlinear PAAUF) categories, and the collective influence of prosumers on each other is modeled based on their expected price in P2P trade. Also, it is assumed that prosumers have limited information about each other's decisions. Thus, the NPG

developed also incorporates a learning model, whereby each prosumer obtains an estimate of the prosumers' decisions in P2P trade.

2) A leader-follower framework of the interaction between the DSO and P2P trade is discussed, in which the DSO adopts a price strategy based on the DLMP approach with the aim of increasing the amount of P2P trade and reducing the technical parameters of the DN. The proposed framework of this paper leads DSOs to be aware of the consequences of their decisions in the presence of P2P trade with strategic prosumers. This issue is important as the more the DSOs are aware of prosumers' decisions in P2P trade, the less conflict occurs between P2P trade and the DSO. Therefore, in the simulation section, it is shown that in the proposed framework, the DSO's understanding of P2P trade increases in the presence of PAAUFs.

As a sub-contribution of this paper, without imposing additional constraints, it is proven that the balance of sold and bought energy is guaranteed in P2P trade. In fact, from a theoretical point of view, guaranteeing the balance of the energy is considered a computational challenge because it requires a hard constraint in the optimization models. In some papers such as [25] (see [25, eq. (5b)]), relaxation approaches have been used to deal with this hard constraint, which increases the computational load. In this paper, it is proven that the proposed framework guarantees the balance of sold and bought energy in P2P trade without the need for hard constraints.

The remainder of the paper is organized as follows. In Section II, the structure of the proposed model is described. In Section III, designing P2P trade with PAAUFs based on near-potential games and best-response dynamics is discussed. In this section, we prove that near-potential games are well-equipped to model realistic P2P trade. Also in this section, the prosumers' near-realistic utility functions in P2P trade and the mathematical model of the estimation of prosumers from each other in P2P trade is discussed. In Section IV, the pricing strategy of the DSO for interacting with P2P trade is examined in detail. Also, the convergence of the leader-follower model is discussed. In Section V, the application of the proposed framework in the development of aware DNs is implemented in a standard network, and the obtained results are interpreted. Finally, Section VI concludes the paper with a summary and future directions of this research.

II. Model Description

As discussed in detail in Section I, approximating prosumers' utility functions based on their actual behavior is first of all a mathematical challenge. Because these functions may is not convex or continually differentiable, in which case it is difficult to calculate the asymptotic behavior of P2P trade. Second, it increases the efficiency of interaction between DSO and P2P trade. This is because if PAAUFs are used in conventional models of interaction between DSO and P2P trade such as DLMP, ADMM, and leader-follower (Stackelberg game), the interaction between DSO decisions and the actual behavior of prosumers will naturally increase. Therefore, in this paper, as shown in Fig. 1, a leader-follower model of interaction between DSO and P2P trade is considered to evaluate the design of P2P trade based on the framework proposed in this paper.

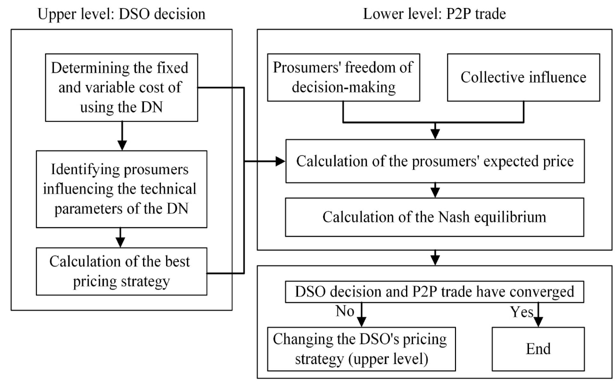


Fig. 1. Block diagram of the proposed work.

Modeling is done from the perspective of DSO. As a result, at the upper level, such as [8], [10], [12], [23], [25], DSO as the leader tends to converge the equilibrium of P2P trade to a point where trade between prosumers is maximized and active losses and voltage deviation are minimized.

Assumption 1: We assume that in the DN, as stated in [8], [10], in addition to prosumers, there are distributed generators (DGs), flexible loads (FLs), and static loads whose capacity DSO can use to increase social welfare.

According to Assumption 1, in the leader-follower model, the DSO should calculate the fixed and variable costs (its energy pricing strategy) that it imposes on the PAAUFs based on the impact of each prosumer on the technical parameters and the available capacities of DGs and FLs. For this purpose, we use the DLMP method for DSO's energy pricing strategy because it is proved in [29] that DLMP extracted from duality analysis provides a clear intuitive understanding of distribution network conditions.

At the lower level, P2P trade based on NPG is designed so that prosumers trade energy with each other considering various parameters such as freedom in decision-making, collective influence, privacy, and marginal cost/utility. In fact, prosumers calculate their expected price based on the collective influence and estimates they have obtained from the behavior of other prosumers. Then, according to the expected price, the utility function of each prosumer is calculated. Note that the parameter of prosumers' freedom in decision-making is a parameter influencing their expected price. In other words, any prosumer who has a higher freedom in decision-making can reduce the influence of the DSO on utility function.

In the following, we discuss the three main challenges of the leader-follower model, which are: 1) calculating the asymptotic behavior of P2P trade at the lower level, 2) determining the DSO energy pricing strategy at the upper level, and 3) the convergence of the leader-follower model.

III. Designing P2P Trade With PAAUFs Based on Near-Potential Games

In this paper, an agent with a flexible resource and/or load is identified as a prosumer, and the set of prosumers is $\mathcal{N} := \{1, 2, \ldots, N\}$. Prosumer i makes decisions based on the costs and benefits of participating in P2P trade and exchanging energy with other prosumers, which is denoted by $-i := \{1, 2, \ldots, i-1, i+1, \ldots, N\}$. The strategy space of prosumer i is denoted by $\mathbf{p_i}$, and strategy vector of prosumer i is denoted by $\boldsymbol{p_i} = (p_{i1}, p_{i2}, \ldots, p_{iN})^\intercal \in \mathbf{p_i}$. Also, the strategy space of all prosumers except prosumer i is denoted by $\mathbf{p_{-i}} := \prod_{j\in -i} \mathbf{p_j}$ and strategy vector of all prosumers for energy exchange with prosumer i is denoted by $\boldsymbol{p_{-i}} = (p_{1i}, p_{2i}, \ldots p_{ji} \ldots, p_{Ni})^\intercal \in \mathbf{p_{-i}}$. Furthermore, define $p_i(t)$ as the energy exchange tendency of prosumer i at time t. Hence, due to the competitive nature of the interactions between these prosumers the game $\Gamma = (\mathcal{N}, \{\mathbf{p_i}, u_i\}_{i\in\mathcal{N}})$ is played. In this game, the actual amount of energy trade between two arbitrary prosumers i and j is equal to $\min\{|p_{ij}|, |p_{ji}|\}$, since p_{ij} and p_{ji} defined as the tendencies to trade between these two prosumers from the viewpoints of i and j, respectively.

The utility function of prosumer i can be obtained as:

$$\arg\min_{\boldsymbol{p_i}\in\mathbf{p_i}} \ u_i(\boldsymbol{p_i}, \boldsymbol{p_{-i}}) \tag{1}$$

$$s.t.$$

$$p_i^{min} \le \mathbf{1}_{[1,N]}\boldsymbol{p_i} \le p_i^{max}, \tag{2}$$

$$\mathbf{1}_{[N,1]}0 \le |\boldsymbol{p_i}|, \tag{3}$$

where p_i^{max} and p_i^{min} represent the maximum and minimum energy for the prosumer i, respectively.

A. Learning Model for Estimating Strategy

Each prosumer may be aware of some other prosumers' strategies due to communication with relatives or neighbors, as discussed in [30]. In this paper, a learning model is proposed that leads each prosumer to obtain an estimate of the strategy of other prosumers.

Let us assume that the discretized strategy space is $\mathfrak{E}_{ij}$ which is a discrete version of the domain on which p_{ij} is defined, for all $i, j \in \mathcal{N}$. Suppose that $\mathfrak{E}_{ij} \subset \mathfrak{E}$ for all $i, j \in \mathcal{N}$, where $\mathfrak{E} = [0 : \epsilon : \mathfrak{m}]$ is a discrete estimate of $\mathfrak{M} = [0, \mathfrak{m}]$, such that $\boldsymbol{p}_i \subset \mathfrak{M}^N$ for all $i \in \mathcal{N}$ and ϵ is the step size for discrete strategy space $\mathfrak{E}$. In the following, we introduce a learning process for each prosumer to estimate the strategies of other prosumers. First, let us define $\hat{p}^z_{ji}(t)$ as the estimation by prosumer i of prosumer j's strategy for the tendency to exchange energy with prosumer z at time t. Additionally, the actual strategy variables of prosumer j is kept in the vector $\boldsymbol{p_j}(t) := (p_{j1}(t), \ldots, p_{jz}(t), \ldots, p_{jN}(t))$. We define for any $i, j, z \in \mathcal{N}$:

$$\begin{cases} \hat{p}^z_{ji}(t+1) = \Pi_{\mathfrak{E}}\Big[\sum_{l\in\mathcal{N}} w^i_{jl}\hat{p}^z_{jl}(t)\Big] = \\ \Pi_{\mathfrak{M}}\Big[\sum_{l\in\mathcal{N}} w^i_{jl}\hat{p}^z_{jl}(t)\Big] + \zeta_{ij} & \text{if } i \ne j \\ \hat{p}^z_{jj}(t+1) = \Pi_{\mathfrak{E}}\big[p_{jz}(t)\big] = p_{jz}(t) & \text{otherwise,} \end{cases} \tag{4}$$

where $\zeta_{ij} \le \epsilon$, $\Pi_{\mathfrak{E}}(\cdot)$ and $\Pi_{\mathfrak{M}}(\cdot)$ are Euclidean projection operator on $\mathfrak{E}$ and $\mathfrak{M}$ respectively. We denote the weight matrix elements by w^i_{jl} which is the weight that prosumer i puts on its neighboring prosumer l's estimate of prosumer j's strategy. The following assumption is made:

Assumption 2 [31, Assumption 3]: Assuming there is a scalar $0 < \eta < 1$ such that for all $i, j \in \mathcal{N}$,

(i) $w^i_{jl} \ge \eta$ only if $k \in \mathcal{N}(i, t) \bigcup \{i\}$, otherwise $w^i_{jl} = 0$.

(ii) $w^i_{ii} = 1$ for all t.

(iii) $\sum_{l\in\mathcal{N}} w^i_{jl} = 1$ for all t.

Given that we are looking for the dynamic properties of P2P trade, we define: $h(u_1, u_2, \ldots, u_n) = (\Pi_{\mathfrak{E}}(u_1), \Pi_{\mathfrak{E}}(u_2), \ldots, \Pi_{\mathfrak{E}}(u_n))$.

Thus we can write the following packed form for any $j, z \in \mathcal{N}$:

$$\hat{p}^z_{j:}(t+1) = h\left(\underbrace{W_j\Big(\hat{p}^z_{j:}(t) + \Big(\hat{p}^z_{jj}(t+1) - \hat{p}^z_{jj}(t)\Big)e_j\Big)}_{pack}\right), \tag{5}$$

where $W_{j_{[i,l]}} = w^i_{jl}$ collects weights of the population putting on prosumer j's estimate of its strategy. Note that Assumption 2 *(i)* guarantees that each prosumer can only assign positive weights to the estimates of other prosumers with which it is related. Assumption 2 *(ii)* guarantees that each prosumer's estimate of itself is equal to its own definite decision and tendency. Finally, Assumption 2 *(iii)* means that W_j is a row-stochastic at all times.

The learning model presented in this paper in (4) results in all prosumers having an estimate of the strategy of other prosumers, hence we have:

Lemma 1: We have for all $z \in \mathcal{N}$ that $\|\hat{p}^z_{j:}(t) - p_{jz}(t).\mathbf{1}_{[N,1]}\| \le \frac{2\sqrt{N}\epsilon}{1-\rho}$ as $t \to \infty$, where $\rho = (1 - \eta^{(N-1)})^{\frac{1}{N-1}}$.

Proof: For proof, we follow a similar line from [31, Proposition 1]. Subtracting $\hat{p}^z_{jj}(t+1).\mathbf{1}_{[N,1]}$ form both sides of (5), and noting that:

$$\begin{aligned} &h\Big(\hat{p}^z_{jj}(t+1).\mathbf{1}_{[N,1]}\Big) = h\big(p_{jz}(t+1).\mathbf{1}_{[N,1]}\big) = \\ &\hat{p}^z_{jj}(t+1).\mathbf{1}_{[N,1]} = p_{jz}(t+1).\mathbf{1}_{[N,1]}, \end{aligned} \tag{6}$$

we can write:

$$\begin{aligned} \|y(t+1)\| &= \Big\|\hat{p}^z_{j:}(t+1) - \hat{p}^z_{jj}(t+1).\mathbf{1}_{[N,1]}\Big\| \\ &= \Big\|h(pack) - h\Big(\hat{p}^z_{jj}(t+1).\mathbf{1}_{[N,1]}\Big)\Big\| \\ &\le \Big\|pack - \hat{p}^z_{jj}(t+1).\mathbf{1}_{[N,1]}\Big\| + \sqrt{N}\epsilon \\ &= \|W(y(t) + \delta(t))\| + \sqrt{N}\epsilon, \end{aligned} \tag{7}$$

where $\delta(t) := (\hat{p}^z_{jj}(t+1) - \hat{p}^z_{jj}(t))(e_N - \mathbf{1}_{[N,1]})$. Note that we used the non-expansive property of projection $\Pi_{\mathfrak{m}}(\cdot)$. Hence, we can write from the above-mentioned recursive formula that

$$||y(t+1)|| \le \sum_{s=0}^{t-1} \rho^s ||\delta(t-s)|| + \sum_{s=0}^{t-1} \rho^s \sqrt{N}\epsilon. \tag{8}$$

Note that $\delta(t) \le N/t$. Defining $\delta_{avg}(t) := \frac{1}{t}\sum_{s=1}^{t} \frac{N+1}{s}$, we can conclude $||y(t+1)|| \le \frac{\delta_{avg}(t)\rho}{1-\rho} + \frac{2\sqrt{N}\epsilon}{1-\rho}$. Result follows by noting that $\delta_{avg}(t) = O(\frac{logt}{t})$. ∎

According to Lemma 1, the estimation vector of prosumer i from the strategies of prosumer j is equal to $\hat{\boldsymbol{p}}^j_i = (\hat{p}^1_{ji}, \ldots, \hat{p}^N_{ji})^\intercal$. We can extend the mathematical process described for calculating $\hat{\boldsymbol{p}}^j_i$ with an identical process to calculate $\hat{\pi}^j_i$, $\forall i, j \in \mathcal{N}$, which we define $\hat{\pi}^j_i$ to be the prosumer i's estimate of the prosumer j's price for participating in P2P trade. Since the process of $\hat{\boldsymbol{p}}_i^j$ and $\hat{\pi}_i^j$ are completely the same, we refrain from expressing it here for the sake of space.

B. Prosumers' Actual Approximate Utility Functions (PAAUFs)

As detailed in Section I, the purpose of this paper is to design P2P trade based on PAAUFs. On the other hand, as stated in [16], [17], PAAUFs are affected by several parameters, including freedom in decision-making, collective influence, privacy, and marginal cost/utility, which may cause PAAUFs to not be convex and continuously differentiable. Therefore, we consider the following assumption for PAAUFs based on the main purpose of the paper.

Assumption 3: (i) Based on empirical data, the price elasticity of non-residential prosumers follows a non-linear relationship due to reasons such as equipment shutdown costs and employees' work time, while the price elasticity of residential prosumers is usually linear [20].

(ii) In economic markets with strategic prosumers, small changes in the collective behavior of prosumers have a large influence on the behavior of individual prosumers [16], [17].

(iii) In economic models, the presence of a very powerful institution compared to other institutions can be challenging. In microeconomics, this will lead to a monopoly market in favor of the powerful entity, as a result of which all market operations will be performed under its supervision [18]. In P2P trade, DSO is a very powerful institution and its impact on prosumers behavior should be modeled.

(iv) The marginal cost/utility of prosumers should usually be modeled as a quadratic function.

Considering Assumption 3 *(i)*-*(iv)* leads to PAAUFs, which we will examine. Note that the main goal of this paper is to provide a robust and comprehensive framework based on which it is possible to calculate the asymptotic behavior of P2P trade for any form of PUFs. Therefore, while this paper approximates the general form of the actual PUFs (PAAUFs) from a mathematical point of view, the proposed model can be used for actual PUFs that are supported by practical data. The PAAUF of prosumer i is defined as

$$u_i = \mathbf{1}_{[1,-N]} c_i^{fix} |\boldsymbol{p}_{-\boldsymbol{i}}| + 0.5 \boldsymbol{c}_i^{tr} |\boldsymbol{p}_{-\boldsymbol{i}}| + \mathbf{1}_{[1,N]} c_i^1 |\boldsymbol{p}_{\boldsymbol{i}}| + 0.5 \boldsymbol{p}_{\boldsymbol{i}}^{\intercal} diag\Big(\mathbf{1}_{[1,N]} c_i^2\Big) |\boldsymbol{p}_{\boldsymbol{i}}| - \boldsymbol{\pi}_i |\boldsymbol{p}_{-\boldsymbol{i}}|, \tag{9}$$

where c_i^{fix} and $\boldsymbol{c}_i^{tr} = (c_1^{tr}, \ldots, c_{i-1}^{tr}, c_{i+1}^{tr}, \ldots, c_N^{tr})$ represent the fixed and variable costs of using the network, respectively. The inclusion of these parameters accounts for the fact that prosumers' use of the network is costly to the DSO, who is responsible for maintaining the network and fulfilling the transactions. We note that $\boldsymbol{c}_i^{tr}$ is determined by the extent to which each prosumer affects the technical parameters of the network, that is, as the technical parameters increase, $\boldsymbol{c}_i^{tr}$ also increases, discussed in detail in Section IV. Also, c_i^1 and c_i^2 represent the cost of production (or the consumer utility functions). Finally, $\boldsymbol{\pi}_i = (\pi_1, \ldots, \pi_{i-1}, \pi_{i+1}, \ldots, \pi_N)$ is the selling or buying vector price of energy in P2P trade. Note that $\boldsymbol{\pi}_i$ and c_i^1 are considered positive for seller's prosumers and negative for buyer's prosumers and $-N$ is equal to $N-1$. Also, c_i^1. In this paper, a pricing model for each prosumer is proposed, which reflects the effects of the collective behavior of prosumers, DSO pricing, and freedom in decision-making. Therefore, the steps of the proposed pricing model:

1) The First Step: Based on the learning model for strategy estimation discussed in detail in Section III-A, each prosumer estimates the price and energy of other prosumers in P2P trade.

2) The Second Step: Each prosumer calculates its expected price to participate in P2P trade. It is important to pay attention to the fact that the expected prices of the seller's prosumers and the buyer's prosumers in an economic market are different. Therefore, let $\mathcal{N_S} := \{1, 2, \ldots, S\}$ be the set of seller's prosumers and $\mathcal{N_B} := \{1, 2, \ldots, B\}$ be the set of buyer's prosumers. It should be clear that $\mathcal{N_S} \cap \mathcal{N_B} = \emptyset$ and $\mathcal{N_S} \cup \mathcal{N_B} = \mathcal{N}$. The prosumer's price i is defined as

$$\begin{cases} \begin{Bmatrix} \pi_i = \alpha\big(\pi_i^{DSO} + \pi_i^{Pro} \mathbf{1}_{[1,B]} \hat{\boldsymbol{p}}_i\big) + \\ (1-\alpha)\Big(\pi_i^{DSO} + \big(\mathbf{1}_{[1,B]} \hat{\boldsymbol{p}}_i\big)^{\pi_i^{Pro}}\Big) \end{Bmatrix}, \forall i \in \mathcal{N_S}, \\ \begin{Bmatrix} \pi_i = \alpha\big(\pi_i^{DSO} - \pi_i^{Pro} \mathbf{1}_{[1,S]} \hat{\boldsymbol{p}}_i\big) + \\ (1-\alpha)\Big(\pi_i^{DSO} - \big(\mathbf{1}_{[1,S]} \hat{\boldsymbol{p}}_i\big)^{\pi_i^{Pro}}\Big) \end{Bmatrix}, \forall i \in \mathcal{N_B}. \end{cases} \tag{10}$$

The equation (10) determines the expected price of prosumers based on three parameters, which we will explain below.

The first parameter (π_i^{DSO}) is derived from the price announced by the DSO for energy trade. In fact, as stated in [18], it is an accepted concept in microeconomics that the expected price of small players such as prosumers is highly influenced by powerful market players such as DSOs, which we discussed in detail in Section IV-D.

The second parameter represents the prosumer's freedom in decision-making to trade energy. In fact, this parameter, which consists of two parts $\hat{\boldsymbol{p}}_i$ (where if $i \in \mathcal{N_S}$, $\hat{\boldsymbol{p}}_i$ represents the vector of prosumer i's estimate of the energy strategy of the buyer's prosumers) and π_i^{Pro}, allows prosumers to adjust their expected prices based on their estimation of the behavior of others. If a prosumer, for example, a seller's prosumer, estimates that the participation of buyer's prosumers in P2P trade is low ($\hat{\boldsymbol{p}}_i$ is small), its expected price will be closer to the price announced by the DSO. Because there is a lower probability of success in trade due to the reduced participation of buyer's prosumers. On the other hand, if the participation of buyer's prosumers increases ($\hat{\boldsymbol{p}}_i$ is big), due to the increase in the probability of succeeding in the trade, the seller's prosumer can deviate its expected price from the price announced by DSO in order to earn more profit. Consequently, in this paper, we define: if $mean(\hat{\boldsymbol{p}}_i) \leq \frac{p_i^{min}+p_i^{max}}{2}$, we will have $\pi_i^{Pro} = 0$, otherwise, π_i^{Pro} in (10) should be enough to satisfy the condition $\pi_i = mean(\hat{\boldsymbol{\pi}}_i)$. Where $mean(.)$ represents the average of a vector and, similar to $\hat{\boldsymbol{p}}_i$, if $i \in \mathcal{N_S}$, $\hat{\boldsymbol{\pi}}_i := (\hat{\pi}_i^1, \ldots, \hat{\pi}_i^B)$ represents the vector of prosumer i's price estimates of the pricing strategy of buyer's prosumers. Note that in actuality, π_i^{Pro} is a parameter that is affected by several factors such as risk-taking, motivation, and competitiveness of prosumers, and is naturally bound to a minimum and maximum range. Therefore, in this article, if the condition $\pi_i = mean(\hat{\boldsymbol{\pi}}_i)$ is outside the range of bounds of π_i^{Pro}, we consider $\pi_i^{Pro} = 0$. As a result, the effect of π_i^{DSO} in (10) is significant. For example, consider a seller's prosumer who has $\pi_i^{DSO} > mean(\hat{\boldsymbol{\pi}}_i)$ and $\pi_i^{Pro} > 0$, therefore $\pi_i = \pi_i^{DSO}$ will always be.

The third parameter (α) is based on the collective influence of other prosumers, as stated in Assumption 3, in an economic market like P2P trade, two economic principles can be identified. Firstly, prosumers can be classified based on their different price elasticity, which helps differentiate them. Secondly, the overall market behavior at different times can be modeled based on the class that exerts the most influence. For instance, as stated in [20], residential prosumers have linear price elasticity and non-residential prosumers have exponential price elasticity. On the other hand, intuitively, due to the different energy levels, the influence of non-residential prosumers on the market price is greater from 10 to 14 am, while the effect of residential prosumers is greater from 6 to 10 pm. Consequently, according to which class has the greatest impact on the market price, each prosumer calculates the deviation from the price announced by DSO linearly (the influence of residential prosumers is greater) and exponentially (the influence of non-residential prosumers is greater). Finally, it is clear that α should be between zero and one, continuous and smooth so that if the collective influence of residential prosumers is greater than non-residential prosumers, then $\alpha \approx 1$ and otherwise $\alpha \approx 0$. Hence let vectors $\boldsymbol{p}_{ic} \in \mathbb{N}^{n_{ic}}$ and $\boldsymbol{p}_r \in \mathbb{N}^{n_r}$ denote the maximum tradable energy in P2P trade by non-residential and residential prosumers, respectively, which satisfy $\mathcal{N}_{ic} \cap \mathcal{N}_r = \emptyset$ and $\mathcal{N}_{ic} \cup \mathcal{N}_r = \mathcal{N}$. In this paper, α is defined as

$$\alpha = \log_2\left(2 - \frac{e^{\left(\mathbf{1}_{[1,n_{ic}]}|\boldsymbol{p}_{ic}| - \mathbf{1}_{[1,n_r]}|\boldsymbol{p}_r|\right)}}{1 + e^{\left(\mathbf{1}_{[1,n_{ic}]}|\boldsymbol{p}_{ic}| - \mathbf{1}_{[1,n_r]}|\boldsymbol{p}_r|\right)}}\right), \tag{11}$$

where in (11) we have: $\{\alpha \to 0|\ |\boldsymbol{p}_{ic}| > |\boldsymbol{p}_r|\}$ and $\{\alpha \to 1|\ |\boldsymbol{p}_r| > |\boldsymbol{p}_{ic}|\}$.

3) The Third Step: In the third step, each prosumer calculates $\boldsymbol{\pi}_i$ according to its expected price π_i. It is clear that the calculation of $\boldsymbol{\pi}_i$ should be such that sellers trade with buyers who have a higher expected price and conversely buyers trade with sellers who have a lower expected price. Hence we have:

$$\begin{cases} \begin{Bmatrix} \boldsymbol{\pi}_{i_{[j]}} = \hat{\pi}_i^j & \text{if } \pi_i \le \hat{\pi}_i^j \\ \boldsymbol{\pi}_{i_{[j]}} = 0 & \text{otherwise,} \end{Bmatrix}, \forall i \in \mathcal{N}_{\mathcal{S}} \text{ and } j \in \mathcal{N}_{\mathcal{B}} \\ \begin{Bmatrix} \boldsymbol{\pi}_{i_{[j]}} = \hat{\pi}_i^j & \text{if } \pi_i \ge \hat{\pi}_i^j \\ \boldsymbol{\pi}_{i_{[j]}} = 0 & \text{otherwise,} \end{Bmatrix}, \forall j \in \mathcal{N}_{\mathcal{S}} \text{ and } i \in \mathcal{N}_{\mathcal{B}}, \end{cases} \tag{12}$$

where $\boldsymbol{\pi}_{i_{[j]}}$ is the j-th element of the vector $\boldsymbol{\pi}_i$.

C. P2P Trade Design Based on NPGs

The closer the P2P trade design is to reality (modeling PAAUFs), the more accurate the DSO's decisions will be to the optimal operation of network distribution. On the other hand, designing P2P trade considering PAAUFs is a mathematical challenge, because the game Γ, in general, may have an unknown asymptotic behavior due to its generic form. Therefore, in this section, a P2P trade design with PAAUFs based on near-potential games is proposed, which we discuss in detail below.

Definition 1 (Potential Game): If in a given non-cooperative game, there is a potential function (u) that corresponds to the prosumer unilateral deviation of prosumer i to increase the profit, it is called a potential game, i.e., as [32]: $u(\boldsymbol{p_i}', \boldsymbol{p_{-i}}) - u(\boldsymbol{p_i}, \boldsymbol{p_{-i}}) = u_i(\boldsymbol{p_i}', \boldsymbol{p_{-i}}) - u_i(\boldsymbol{p_i}, \boldsymbol{p_{-i}})$, for every $i \in \mathcal{N}$, $\boldsymbol{p_i}', \boldsymbol{p_i} \in \mathbf{p_i}$ and $\boldsymbol{p_{-i}} \in \mathbf{p_{-i}}$.

Theorem 1: If the variables in the utility function (9) are not considered constant, then the game Γ is not a potential game because one cannot find a potential function (u) in the game Γ that satisfies the following equation: $\frac{\partial u_i}{\partial \boldsymbol{p_i}'} = \frac{\partial u}{\partial \boldsymbol{p_i}'}$, $\forall i \in \mathcal{N}, \boldsymbol{p_i}' \in \mathbf{p_i}$, where $\frac{\partial u}{\partial \boldsymbol{p_i}'} := (\frac{\partial u}{\partial p_1'}, \ldots, \frac{\partial u}{\partial p_N'})^\intercal$.

Proof: For proof, we follow [32, Lemma 4.4 and Th. 4.5]. It is easy to conclude that according to the characteristics of (9), this equality is not established and the game Γ is not a potential game. ■

In this paper, a near-potential game framework is proposed that supports the design of P2P trade with PAAUFs. More precisely, if the game Γ is not a potential game, the proposed framework can ensure that we reach a point close to the Nash equilibrium of the potential game based on the framework of close-to-potential games. As a result, the proposed framework is an efficient method for designing P2P trade in the presence of greedy prosumers and communication networks, which we will examine its mathematical modeling in the rest of this section.

Assumption 4: We assume that the utility functions of (9) and potential games in this paper are all Lipschitz-continuous with some Lipschitz constant $\mathbf{L}_1$ and $\mathbf{L}_2$, respectively.

Definition 2 (Maximum Pairwise Difference [MPD]): For two games Γ and $\hat{\Gamma}$ with utility functions $\{u_i\}_{i\in\mathcal{N}}$ and $\{\hat{u}_i\}_{i\in\mathcal{N}}$, we can define the maximum pairwise difference (MPD) between these two games as

$$d\left(\Gamma, \hat{\Gamma}\right) \triangleq \max_{i\in\mathcal{N}, \boldsymbol{p_i}'\in\mathbf{p_i}, \boldsymbol{p_{-i}}\in\mathbf{p_{-i}}} \left|\left(u_i\left(\boldsymbol{p_i}', \boldsymbol{p_{-i}}\right) - u_i\left(\boldsymbol{p_i}, \boldsymbol{p_{-i}}\right)\right)\right.$$
$$\left. - \left(\hat{u}_i\left(\boldsymbol{p_i}', \boldsymbol{p_{-i}}\right) - \hat{u}_i\left(\boldsymbol{p_i}, \boldsymbol{p_{-i}}\right)\right)\right|. \tag{13}$$

We note that according to [33, Proposition 2.1] for any arbitrary dynamics in potential games along a closed path, which is a sequence of strategies in action space with the same beginning and end point, the improvement of the utility of a prosumer is 0.

Definition 3 (Best Response Dynamics [BRD]): The best response dynamics is an updating process in which prosumer i with strategy space $\mathbf{p_i}$ changes its strategy if its utility function improves, that is, we define for $i \in \mathcal{N}$ and $\boldsymbol{p_{-i}} \in \mathbf{p_{-i}}$

$$\begin{cases} \text{No change of strategy if } u_i = \max_{\boldsymbol{p_i}} u_i(\boldsymbol{p_i}, \boldsymbol{p_{-i}}) \\ \arg\max_{\boldsymbol{p_i}} u_i(\boldsymbol{p_i}, \boldsymbol{p_{-i}}) \quad \text{otherwise.} \end{cases} \tag{14}$$

Note that for the BRD process, a prosumer is randomly selected and we define the *trajectory of the dynamics* as a sequence of the strategy space $\{\mathbf{p_i}(t)\}_{t=0}^{\infty}$ [33].

As mentioned in Section I, we assume that all prosumers behave rationally and seek to maximize their utility function in P2P trade, on the other hand, BRD is a powerful tool for Nash equilibrium in such trade. Therefore, in the following, according to Definitions 1, 2, and 3, we utilize the above procedure to guarantee that for an NPG $\Gamma = (\mathcal{N}, \{\mathbf{p_i}, u_i\}_{i\in\mathcal{N}})$, which is close to the potential game $\hat{\Gamma} = (\mathcal{N}, \{\mathbf{p_i}, \hat{u}_i\}_{i\in\mathcal{N}})$ with MPD $(\Gamma, \hat{\Gamma}) \leq \delta_{mpd}$ (see [33]), the BRD converges to some $\epsilon - NE$. Let us consider the BRD introduced in Definition 3.

Theorem 2: This BRD converges to $(|\mathfrak{E}|^N\delta_{mpd} + \frac{4\mathbf{L}\sqrt{N}\epsilon}{1-\rho}) - NE$ of potential game $\hat{\Gamma}$. Where $\mathbf{L} = \max\{\mathbf{L}_1, \mathbf{L}_2\}$ is the greatest Lipschitz-continuity coefficient among $u_i, \hat{u}_i$ for all $i \in \mathcal{N}$.

Proof: We prove the claim by modeling the update process as a Markov chain and employing the concept of improvement path conditions for PGs [32]. Using [33, Definition 3.1] we can represent the strategy updates in the BRD as the transition in the following Markov chain: firstly, each state corresponds to a strategy profile and secondly, there is non-zero transition probability from a state s' to state s'', where $s' \neq s''$. if s' and s'' differ in a strategy of a single prosumer, say i, and $\boldsymbol{p}_i{}^*$ is a strict best response of prosumer i to s'_{-i}. the transition probability from s' to s'' is equal to the probability that in state s', we choose prosumer i for update and it choose s'_i as her new strategy. Since there is a finite number of states, one of the recurrent classes of the Markov chain is reached in a finite time with probability 1. Thus it is sufficient to prove any state belonging to a recurrent class of this Markov chain is an approximate Nash equilibrium.

The recurrent classes can be either a cycle or singleton. Let us assume that it is a cycle with a length of (L_c), while the singleton case will be addressed later on. Along this cycle, suppose that a prosumer i at step $k_\circ$ can improve its utility by an amount of β, which means that: $u_{i_{k_\circ}}(p_i^{k_\circ+1}, \hat{p}_{-i}^{k_\circ}) - u_{i_{k_\circ}}(p_i^{k_\circ}, \hat{p}_{-i}^{k_\circ}) = \beta$. Assuming that prosumer i_k is the prosumer which is selected at time k to best respond, we know that $u_{i_k}(p_i^{k+1}, \hat{p}_{-i}^k) - u_{i_k}(p_i^k, \hat{p}_{-i}^k) \geq 0$ for other elements of this cycle, which leads to $\sum_{k=1}^{L_c} u_{i_k}(p_i^{k+1}, \hat{p}_{-i}^k) - u_{i_k}(p_i^k, \hat{p}_{-i}^k) \geq \beta$. Additionally, along the same path, we conclude that: $\sum_{k=1}^{L_c} \hat{u}_{i_k}(p_i^{k+1}, \hat{p}_{-i}^k) - \hat{u}_{i_k}(p_i^k, \hat{p}_{-i}^k) = 0$. Considering the Lipschitz-continuity of u_i and $\hat{u}_i$, we can write for sufficiently large time t,

$$\underbrace{\sum_{k=1}^{L_c}\left(u_{i_k}\left(p_i^{k+1}, p_{-i}^k\right) - u_{i_k}\left(p_i^k, p_{-i}^k\right)\right)}_{TH1} \geq \beta - \frac{2N\mathbf{L}\sqrt{N}\epsilon}{1-\rho}, \tag{15}$$

and

$$\underbrace{\sum_{k=1}^{L_c}\left(\hat{u}_{i_k}\left(p_i^{k+1}, p_{-i}^k\right) - \hat{u}_{i_k}\left(p_i^k, p_{-i}^k\right)\right)}_{TH2} \leq \frac{2N\mathbf{L}\sqrt{N}\epsilon}{1-\rho}. \tag{16}$$

As a result, remembering that MPD $(\Gamma, \hat{\Gamma}) \leq \delta_{mpd}$, we have $|\mathfrak{E}|^N\delta_{mpd} \geq L_c\delta_{mpd} \geq TH1 - TH2 \geq \beta - \frac{4N\mathbf{L}\sqrt{N}\epsilon}{1-\rho}$. Since $(p_i^{k_\circ}, \hat{p}_{-i}^{k_\circ})$ can be any arbitrary component of the recurrent class, this strategy is $(|\mathfrak{E}|^N\delta_{mpd} + \frac{4N\mathbf{L}\sqrt{N}\epsilon}{1-\rho}) - NE$. For a singleton case similarly, we can prove it is $(\frac{4N\mathbf{L}\sqrt{N}\epsilon}{1-\rho}) - NE$. Therefore, the algorithm converges to an approximate Nash equilibrium of the PGs. ■

Algorithm 1 P2P Trade With PAAUFs Based on NPG

1: Initialize the weighted matrix $W_{j[i,l]} = w_{jl}^i$, $\forall j \in \mathcal{N}$ so that: $0 \leq w_{jl}^i \leq 1, \forall l \in \mathcal{N}(i)\bigcup\{i\}$, $w_{jl}^i = 0, \forall l \notin \mathcal{N}(i)\bigcup\{i\}$, $w_{ii}^i = 1$, and $\sum_{l\in\mathcal{N}} w_{jl}^i = 1$.

2: Each prosumer $(j \in \mathcal{N})$ estimates the strategy of all prosumers based on the communication with some prosumers. So that:

$$\begin{cases} \hat{p}_{ji}^z(t+1) = \Pi_{\mathfrak{E}}\left[\sum_{l\in\mathcal{N}} w_{jl}^i \hat{p}_{jl}^z(t)\right] = \\ \Pi_{\mathfrak{M}}\left[\sum_{l\in\mathcal{N}} w_{jl}^i \hat{p}_{jl}^z(t)\right] + \zeta_{ij} & \text{if } i \neq j \\ \hat{p}_{jj}^z(t+1) = \Pi_{\mathfrak{E}}\left[p_{jz}(t)\right] = p_{jz}(t) & \text{otherwise.} \end{cases}$$

3: Each prosumer $(j \in \mathcal{N})$ calculates the their expected price for participating in P2P trade. So that:

$$\begin{cases} \left\{\begin{array}{l} \pi_i = \alpha\left(\pi_i^{DSO} + \pi_i^{Pro}\mathbf{1}_{[1,B]}\hat{\boldsymbol{p}}_i\right) + \\ (1-\alpha)\left(\pi_i^{DSO} + \left(\mathbf{1}_{[1,B]}\hat{\boldsymbol{p}}_i\right)^{\pi_i^{Pro}}\right), \end{array}\right\}, \forall i \in \mathcal{N}_{\mathcal{S}}, \\ \\ \left\{\begin{array}{l} \pi_i = \alpha\left(\pi_i^{DSO} - \pi_i^{Pro}\mathbf{1}_{[1,S]}\hat{\boldsymbol{p}}_i\right) + \\ (1-\alpha)\left(\pi_i^{DSO} - \left(\mathbf{1}_{[1,S]}\hat{\boldsymbol{p}}_i\right)^{\pi_i^{Pro}}\right), \end{array}\right\}, \forall i \in \mathcal{N}_{\mathcal{B}}. \end{cases}$$

4: Each prosumer calculates $\boldsymbol{\pi}_i$ according to π_i. So that:

$$\begin{cases} \left\{\begin{array}{l} \boldsymbol{\pi}_{i[j]} = \hat{\pi}_i^j \text{ if } \pi_i \leq \hat{\pi}_i^j\} \\ \boldsymbol{\pi}_{i[j]} = 0 \quad \text{otherwise,} \end{array}\right\}, \forall i \in \mathcal{N}_{\mathcal{S}} \text{ and } j \in \mathcal{N}_{\mathcal{B}} \\ \left\{\begin{array}{l} \boldsymbol{\pi}_{i[j]} = \hat{\pi}_i^j \text{ if } \pi_i \geq \hat{\pi}_i^j\} \\ \boldsymbol{\pi}_{i[j]} = 0 \quad \text{otherwise,} \end{array}\right\}, \forall j \in \mathcal{N}_{\mathcal{S}} \text{ and } i \in \mathcal{N}_{\mathcal{B}}, \end{cases}$$

5: **for** $t = 1$ to t_{Max} **do**
6: Randomly select prosumer i and calculate its best response u_i^*.
7: Update steps 2 and 4.
8: t :=t+1
9: **end for**

The pseudo-code for the algorithm for designing P2P trade with PAAUFs based on NPG is given in Algorithm 1.

D. Discussion

1) Nash Equilibrium: Suppose game $\Gamma := (\mathcal{N}, \{\mathbf{p_i}, u_i\}_{i\in\mathcal{N}})$ is a PG, then it is clear that P2P trade will converge exactly to the NE. On the other hand, if game Γ is not a PG, we can calculate $\mathfrak{E}$, which is close to the NE, and we proved that it is equal to $(|\mathfrak{E}|^N\delta_{mpd} + \frac{4N\mathbf{L}\sqrt{N}\epsilon}{1-\rho}) - NE$. From a mathematical point of view, as discussed in Theorem 1, (1) is not intrinsically a PG, but based on Algorithm 1, it can be considered as a PG because at every step of Algorithm 1, where c_i^{fix}, $\boldsymbol{c}_i^{tr}$, c_i^1, c_i^2, and $\boldsymbol{\pi}_i$ given a fixed estimate of prosumer i on the decision variables of other prosumers, $\hat{\boldsymbol{p}}_i^j$ and $\hat{\pi}_i^j, \forall i, j \in \mathcal{N}$, are fixed. Therefore, by checking the condition proposed in [32, Th. 4.5], we have for all i, j in the network: $\frac{\partial^2 u_i}{\partial p_{ij}\partial p_{ji}} = \frac{\partial^2 u_j}{\partial p_{ji}\partial p_{ij}}$. This happens since, for the linear part of (9), the mixed partial derivative is zero, and for the nonlinear part, it only depends on the decision of prosumer i itself, which means the mixed partial derivative is zero for this term. As a result, we have developed the framework proposed in [33] so that in each step of Algorithm 1, game Γ is a potential game and based on the estimation of each prosumer from other prosumers, game Γ converges to NE.

2) Value of ϵ: As it is stated in Theorem 2, the algorithm 1 converges to a $(|\mathfrak{E}|^N \delta_{mpd} + \frac{4N\mathbf{L}\sqrt{N}\epsilon}{1-\rho}) - NE$ where the $\epsilon > 0$ is the discretization factor. In general $|\mathfrak{E}|^N$ get larger as ϵ get smaller. Therefore, there is a trade-off between these two terms. However, in the case of potential games where $\delta_{mpd} = 0$, there is no such trade-off and one can arbitrarily make ϵ small to get a better approximation of a NE.

3) Approximation of δ_{mpd}: In general, the δ_{mpd} (MPD) value of two games may be large. However, in the framework of the proposed model of this paper, since the PAAUFs in each iteration of Algorithm 2, a potential game is formed. Consequently, δ_{mpd} will be zero. In other words, for a class of utility functions that have potential games similar to those introduced in this paper, δ_{mpd} approximation is not identified as a challenge for the convergence of the proposed model.

4) Lipschitz-Continuity: As stated in Section III-D1, the estimates of $\hat{p}_{ij}^z$ make (1) a PG. As a result, since these utility functions are Lipschitz the potential game also has Lipschitz continuity property with similar Lipschitz constant. Therefore, Assumption 4 covers nontrivial cases where the game is potential.

5) Sensitivity of Asymptotic Behavior of P2P Trade to Approximation of PUFs (A Qualitative Perspective): In general, if the approximation of PUFs satisfies the condition $\frac{\partial^2 u_i}{\partial p_{ij}\partial p_{ji}} = \frac{\partial^2 u_j}{\partial p_{ji}\partial p_{ij}}$, the asymptotic behavior of P2P trade converges to a specific point (NE). Otherwise, if the approximation of PUFs does not satisfy the condition $\frac{\partial^2 u_i}{\partial p_{ij}\partial p_{ji}} = \frac{\partial^2 u_j}{\partial p_{ji}\partial p_{ij}}$, but the action space of all prosumers is bounded, the asymptotic behavior of P2P trade can be defined in a bounded set $((|\mathfrak{E}|^N \delta_{mpd} + \frac{4N\mathbf{L}\sqrt{N}\epsilon}{1-\rho}) - NE)$. The size of this bounded set depends on the size δ_{mpd} and the trade-off between $\epsilon > 0$ and $|\mathfrak{E}|^N$. Consequently, the more accurate our approximation of PUFs is based on the actual behavior of prosumers, the condition $\frac{\partial^2 u_i}{\partial p_{ij}\partial p_{ji}} = \frac{\partial^2 u_j}{\partial p_{ji}\partial p_{ij}}$ is not satisfied, and subsequently the size of the bounded set is also enlarged. Therefore, the sensitivity of the asymptotic behavior of P2P trade to the approximation of PUFs based on the actual behavior of prosumers is significant, and as a result, the difference between research and practical results also increases. Note that as stated in Sections III-D1 and III-D2, the asymptotic behavior of P2P trade converges to a NE in the framework of the model proposed in this paper.

6) Energy Balance: The existence of energy balance in P2P trade means that the decisions of the seller's prosumers are equal to the decisions of the buyer's prosumers. Therefore the energy exchanged tendency p_{ij} between prosumer i and prosumer j has the following property: $\{p_{ij} = -p_{ji}, \ \forall i, j \in \mathcal{N}\}$. Hence, generally, the energy trade between sets $\mathcal{N}_{\mathcal{S}}$ and $\mathcal{N}_{\mathcal{B}}$ can be defined as the $\boldsymbol{P} \in \mathbb{R}^{N\times N}$ matrix where to prevent energy imbalances, we define $\boldsymbol{P} = -\boldsymbol{P}^\mathsf{T}$. Energy balance $(\boldsymbol{P} = -\boldsymbol{P}^\mathsf{T})$ is known as a hard constraint in the optimization problem because it makes the decisions of each prosumer dependent on the decisions of the other prosumers. In the proposed framework, the asymptotic behavior of P2P trade is calculated based on the BRD, and thus energy imbalance is avoided without the need for an additional constraint in (1) because when each prosumer wants to calculate the best response, all other prosumers must agree.

IV. DSO'S ENERGY PRICING STRATEGY

A. Grid Model

We have a radial distribution network with graph $\mathcal{G}(\mathcal{B}, \mathcal{L})$ that $\mathcal{B} := \{n_1, n_2, \ldots, n_b\}$, where n_1 is the slack node. Let's FLs, DGs, and static loads are defined by $\boldsymbol{p}_{fl} \in \mathbb{N}^{n_{fl}}$ and $\boldsymbol{p}_{dg}/\boldsymbol{q}_{dg} \in \mathbb{N}^{n_{dg}}$, $\boldsymbol{p}_{sl} \in \mathbb{N}^{n_{sl}}$, respectively. Also, $\boldsymbol{p}_S/\boldsymbol{q}_S \in \mathcal{N}_{\mathcal{S}}$ and $\boldsymbol{p}_B/\boldsymbol{q}_B \in \mathcal{N}_{\mathcal{B}}$ also represent the active and reactive power of seller and buyer prosumers involved in the P2P trade, where $n_{fl} + n_{dg} + n_{sl} + S + B = b$. Let $\underline{u}_n = v_n e^{j\theta}$ and $\underline{s}_n = p_n + jq_n$ be considered as complex voltages and power injections for each node, respectively, and in this regard, $\underline{\boldsymbol{u}} := (\underline{u}_{n_1}, \ldots, \underline{u}_{n_b})^\mathsf{T}$ and $\underline{\boldsymbol{s}} := (\underline{s}_{n_1}, \ldots, \underline{s}_{n_b})^\mathsf{T}$ complex voltages and power injections for the whole network. Let $\mathcal{B}/n_1$ represent the total number of nodes PQ in $\mathcal{G}$ and is $-b = b - 1$.

B. Load Flow

In a PQ network, the relationship between $\underline{\boldsymbol{u}}$ and $\underline{\boldsymbol{s}}$ can be defined based on the network structure as $\underline{\boldsymbol{s}}_{[-b,1]} := diag(\underline{\boldsymbol{u}}_{[-b,1]})(\underline{\boldsymbol{Y}}^*_{[-b,1]}\underline{u}^*_{n_1} + \underline{\boldsymbol{Y}}^*_{[-b,-b]}\underline{\boldsymbol{u}}^*) \in \mathbb{C}^b$, where $\mathbf{Y} \in \mathbb{C}^{b\times b}$ is the network admittance matrix. We follow an iterative solution of the fixed-point from [34] for calculating the $\underline{\boldsymbol{u}}$ with respect to $\underline{u}_{n_1}$ constant, which is defined:

$$\hat{\underline{\boldsymbol{u}}}_{[-b,1]} = \underbrace{-\underline{\boldsymbol{Y}}^{-1}_{[-b,-b]}\underline{\boldsymbol{Y}}_{[-b,1]}\underline{u}_{n_1}}_{\boldsymbol{y}_1} + \underbrace{\underline{\boldsymbol{Y}}^{-1}_{[-b,-b]}diag\left(\underline{\boldsymbol{u}}^*_{[-b,1]}\right)^{-1}}_{\boldsymbol{y}_2} \left(\underline{\boldsymbol{s}}^*_{[-b,1]}\right), \tag{17}$$

where $\hat{\underline{\boldsymbol{u}}}$ is the next iteration of $\underline{\boldsymbol{u}}$. If $\hat{\underline{\boldsymbol{u}}}'$ is the converged voltage vector from (17), complex branch currents ($\underline{\boldsymbol{i}}$), active losses ($p_{loss}$), and reactive losses ($q_{loss}$) are calculated in the following form:

$$\underline{\boldsymbol{i}} = \underline{\boldsymbol{Y}}^f\left[\underline{u}_{n_1}; \hat{\underline{\boldsymbol{u}}}'_{[-b,1]}\right] \in \mathbb{C}^{-b\times 1}, \tag{18}$$

$$p_{loss} = \mathbf{1}_{[1,-b]} real\left(\underline{\boldsymbol{Z}}\right)\, diag\left(\underline{\boldsymbol{i}}\right)\underline{\boldsymbol{i}}^*, \tag{19}$$

$$q_{loss} = \mathbf{1}_{[1,-b]} imag\left(\underline{\boldsymbol{Z}}\right)\, diag\left(\underline{\boldsymbol{i}}\right)\underline{\boldsymbol{i}}^*, \tag{20}$$

where $\underline{\boldsymbol{Y}}^f := [\underline{\boldsymbol{y}}^f_{[-b,1]}, \underline{\boldsymbol{Y}}^f_{[-b,-b]}] \in \mathbb{C}^{-b\times b}$ creates a proper arrangement of π-model shunt and branch admittance and $\underline{\boldsymbol{Z}} := (\boldsymbol{R} + j\boldsymbol{X}) \in \mathbb{C}^{-b\times -b}$ is the impedance of the branches of the $\mathcal{G}$.

C. Distribution Locational Marginal Price

The purpose of the DSO is to create the maximum compatibility between the network parameters and the prosumers with the lowest cost, in which case the social welfare will be maximized, thus we define:

$$DSO_{ps} = \max_{\boldsymbol{x}}\left(\boldsymbol{c}_{fl}^{\top}\boldsymbol{p}_{fl} + 0.5\boldsymbol{p}_{fl}^{\top}\boldsymbol{C}_{fl}\boldsymbol{p}_{fl}\right) - \left(\boldsymbol{c}_{dg}^{\top}[\boldsymbol{p}_{dg};\boldsymbol{q}_{dg}] + 0.5[\boldsymbol{p}_{dg};\boldsymbol{q}_{dg}]^{\top}\boldsymbol{C}_{dg}[\boldsymbol{p}_{dg};\boldsymbol{q}_{dg}]\right) \tag{21}$$

s.t.

$$\left(\lambda_p^{bal}\right) : \mathbf{1}_{[1,-b]}\overbrace{[\boldsymbol{p}_{dg};\boldsymbol{p}_S;\boldsymbol{p}_{fl};\boldsymbol{p}_B;-\boldsymbol{p}_{sl}]}^{\boldsymbol{z}_1} = \hat{p}_{loss}, \tag{22}$$

$$\left(\lambda_p^{bal}\right) : \mathbf{1}_{[1,-b]}\underbrace{[\boldsymbol{q}_{dg};\boldsymbol{q}_S;\boldsymbol{q}_{fl};\boldsymbol{q}_B;-\boldsymbol{q}_{sl}]}_{\boldsymbol{z}_2} = \hat{q}_{loss}, \tag{23}$$

$$(\mu_{br}): \left|\hat{\boldsymbol{i}}\right| \le \left|\boldsymbol{i}^{max}\right|, \tag{24}$$

$$\left(\mu_{\underline{u}}^{+}, \mu_{\underline{u}}^{-}\right): \boldsymbol{u}^{min} \le \hat{\boldsymbol{u}} \le \boldsymbol{u}^{max}, \tag{25}$$

$$\left(\mu_p^{dg+}, \mu_p^{dg-}\right): \boldsymbol{p}_{dg}^{min} \le \boldsymbol{p}_{dg} \le \boldsymbol{p}_{dg}^{max}, \tag{26}$$

$$\left(\mu_q^{dg+}, \mu_q^{dg-}\right): \boldsymbol{q}_{dg}^{min} \le \boldsymbol{q}_{dg} \le \boldsymbol{q}_{dg}^{max}, \tag{27}$$

$$\left(\mu_p^{fl+}, \mu_p^{fl-}\right): \boldsymbol{p}_{fl}^{min} \le \boldsymbol{p}_{fl} \le \boldsymbol{p}_{fl}^{max}, \tag{28}$$

where $\boldsymbol{c}_{fl} \in \mathbb{R}^{n_{fl}}$ and $\boldsymbol{c}_{dg} \in \mathbb{R}^{2n_{dg}}$ are positive price vectors and $\boldsymbol{C}_{fl} \in \mathbb{R}^{n_{fl}\times n_{fl}}$ and $\boldsymbol{C}_{dg} \in \mathbb{R}^{2n_{dg}\times 2n_{dg}}$ are diagonal, symmetric, and positive definite matrices. Equations (22) and (23) represent the balance of active and reactive power, respectively. Equations (24) and (25) limit the branches' current and nodes' voltage, and ultimately limit the trading energies of each of the FLs and DGs by (26)-(28). We note that in this work we consider the reactive power for all prosumers (except $\boldsymbol{q}_{dg}$) as a constant multiple of the active power. Also, the Greek letters to the left of equations represent dual variables. We use $\hat{p}_{loss}$, $\hat{q}_{loss}$, $|\hat{\boldsymbol{i}}|$, and $\hat{\boldsymbol{u}}$ to linearize non-convex constraints, given as:

$$\hat{\boldsymbol{u}} = \lfloor y_1 \rfloor + \boldsymbol{M}_{\boldsymbol{z}_1}^{\boldsymbol{u}_{[-b,1]}}\boldsymbol{z}_1 + \boldsymbol{M}_{\boldsymbol{z}_2}^{\boldsymbol{u}_{[-b,1]}}\boldsymbol{z}_2, \tag{29}$$

$$|\hat{\boldsymbol{i}}| = \underline{Y}_{[n_1]}^{f}\underline{u}_{n_1} + \underline{Y}_{[-b,-b]}^{f}y_1 + \boldsymbol{M}_{\boldsymbol{z}_1}^{|\underline{\boldsymbol{i}}|}\boldsymbol{z}_1 + \boldsymbol{M}_{\boldsymbol{z}_2}^{|\underline{\boldsymbol{i}}|}\boldsymbol{z}_2, \tag{30}$$

$$\hat{p}_{loss} = \boldsymbol{M}_{\boldsymbol{z}_1}^{p_{loss}}\boldsymbol{z}_1 + \boldsymbol{M}_{\boldsymbol{z}_2}^{p_{loss}}\boldsymbol{z}_2, \tag{31}$$

$$\hat{q}_{loss} = \boldsymbol{M}_{\boldsymbol{z}_1}^{q_{loss}}\boldsymbol{z}_1 + \boldsymbol{M}_{\boldsymbol{z}_2}^{q_{loss}}\boldsymbol{z}_2, \tag{32}$$

where

$$\boldsymbol{M}_{\boldsymbol{z}_1}^{\boldsymbol{u}_{[-b,1]}} := real\left(|diag(y_1)|\left(diag(y_1)^{-1}y_2\right)\right), \tag{33}$$

$$\boldsymbol{M}_{\boldsymbol{z}_2}^{\boldsymbol{u}_{[-b,1]}} := real\left(|diag(y_1)|\left(diag(y_1)^{-1} - \mathrm{j}y_2\right)\right), \tag{34}$$

$$\boldsymbol{M}_{\boldsymbol{z}_1}^{|\underline{\boldsymbol{i}}|} := diag(|\boldsymbol{i}|)real\left(diag(\underline{\boldsymbol{i}})\boldsymbol{M}_{\boldsymbol{z}_1}^{\boldsymbol{i}}\right), \tag{35}$$

$$\boldsymbol{M}_{\boldsymbol{z}_2}^{|\underline{\boldsymbol{i}}|} := diag(|\boldsymbol{i}|)real\left(diag(\underline{\boldsymbol{i}})\boldsymbol{M}_{\boldsymbol{z}_2}^{\boldsymbol{i}}\right), \tag{36}$$

$$\boldsymbol{M}_{\boldsymbol{z}_1}^{\boldsymbol{i}} := \underline{Y}_{[-b,-b]}^{f}y_2, \tag{37}$$

$$\boldsymbol{M}_{\boldsymbol{z}_2}^{\boldsymbol{i}} := -\mathrm{j}\underline{Y}_{[-b,-b]}^{f}y_2, \tag{38}$$

$$\boldsymbol{M}_{\boldsymbol{z}_1}^{p_{loss}} := \mathbf{1}_{[1,-b]}\boldsymbol{R}(2(\boldsymbol{M}_1 + \boldsymbol{M}_2)), \tag{39}$$

$$\boldsymbol{M}_{\boldsymbol{z}_1}^{q_{loss}} := \mathbf{1}_{[1,-b]}\boldsymbol{X}(2(\boldsymbol{M}_1 + \boldsymbol{M}_2)), \tag{40}$$

$$\boldsymbol{M}_1 := diag\left(real(\underline{\boldsymbol{i}})\right)real\left(\boldsymbol{M}_{\boldsymbol{z}_1}^{\boldsymbol{i}}\right), \tag{41}$$

$$\boldsymbol{M}_2 := diag\left(imag(\underline{\boldsymbol{i}})\right)imag\left(\boldsymbol{M}_{\boldsymbol{z}_1}^{\boldsymbol{i}}\right). \tag{42}$$

Finally, using the Karush-Kuhn-Tucker conditions, DLMP can be obtained after calculating the optimal points of the decision variables $\boldsymbol{x} \in \{\boldsymbol{p}_{fl}, \boldsymbol{p}_{dg}, \boldsymbol{q}_{dg}\}$. DLMPs are generally composed of four components, which are: 1) energy price per node ($\boldsymbol{\pi}_{ep} := up_c^p\mathbf{1}_{[b]}$), where up_c^p is the price of active power in the upstream market; 2) impact of each node on active and reactive losses in the network $\boldsymbol{\pi}_{loss} := -((\boldsymbol{M}_{\boldsymbol{z}_1}^{p_{loss}})^{\top}up_c^p + (\boldsymbol{M}_{\boldsymbol{z}_1}^{q_{loss}})^{\top}up_c^q)$, where up_c^q is the price of reactive power in the upstream market; 3) impact of each node on voltage $\boldsymbol{\pi}_{\underline{u}} := ((\boldsymbol{M}_{\boldsymbol{z}_1}^{\boldsymbol{u}_{[-b,1]}})^{\top}(\mu_{\underline{u}}^{-} - \mu_{\underline{u}}^{+}))$; and 4) branch congestion sensitivity to each node $\boldsymbol{\pi}_{con} := ((\boldsymbol{M}_{\boldsymbol{z}_1}^{|\underline{\boldsymbol{i}}|})^{\top}\mu_{br})$. Finally, the DLMP is defined for each $n \in \mathcal{B}$ as

$$\pi_i^d = \pi_{ep_i} + \overbrace{\pi_{loss_i} + \pi_{\underline{u}_i} + \pi_{con_i}}^{c_i^{tr}}, \quad \forall i \in \mathcal{B}, \tag{43}$$

where π_i^d is the pricing strategy for each node by DSO. We can also define $\boldsymbol{\pi}^{\boldsymbol{d}} := (\pi_1^d, \ldots, \pi_b^d)^{\top}$. Furthermore, we note that the value of $\boldsymbol{c}_i^{tr}$ must support the NE, thus we have:

Proposition 1: BRD algorithm converges to $p_{ij} = -p_{ji}$ for game Γ, if $\pi_i \gg \boldsymbol{c}_{i_{[:]}}^{tr}$, $\forall i, j \in \mathcal{N}$, where : is all elements of vector $\boldsymbol{c}_i^{tr}$.

Proof: To prove the proposition, we first consider two prosumers $i, j \in \mathcal{N}_B$, where $\boldsymbol{c}_i^{tr} = \mathbf{0}, \boldsymbol{c}_j^{tr} > \mathbf{0}$, and $\pi_i, \pi_j > \pi_s$, $\forall s \in \mathcal{N}_S$, and use induction. In the first step, prosumer i trades with $\{s_1 | \pi_{s_1} + \boldsymbol{c}_{i_{[s_1]}}^{tr} < \pi_{s_{-1}} + \boldsymbol{c}_{i_{[s_{-1}]}}^{tr}, s_{-1} = \mathcal{N}_S / s_1\}$, and then prosumer i can sell the purchased energy to $\{j | \pi_{s_1} + \boldsymbol{c}_{i_{[s_1]}}^{tr} + \boldsymbol{c}_{j_{[i]}}^{tr} + \gamma < \pi_{s_{-1}} + \boldsymbol{c}_{j_{[s_{-1}]}}^{tr}, 0 < \gamma \ll 1\}$. As a result, $p_{ij} = -p_{ji}$ is violated since prosumer i can participate in the P2P trade as a seller and buyer of energy. ■

D. DSO Pricing Strategy (π_i^{DSO})

As stated previously, the goal of the DSO is to increase the compatibility of P2P trade with the technical parameters of the network. Therefore, the DSO should take increasing the compatibility into account, along with Proposition 1 for pricing strategy. Firstly, the participation of those prosumers who have the largest effect in increasing the technical parameters of the network should be limited in P2P trade, and secondly, the amount of energy exchanged in P2P trade should be increased. Therefore, for the first concept, the DSO determines $\pi_i^{DSO}, \forall i \in \mathcal{N}$ according to which technical parameter (voltage deviation, and active losses) it wants to reduce. Based on (43), the DSO determines the π_i^{DSO} prosumers who have the largest effects in the technical parameters in such a way that they participate the least in P2P trade in Algorithm 1. As a result, we define:

$$\begin{cases} \pi_i^{DSO} > \frac{\mathbf{1}_{[1,b]}\boldsymbol{\pi}^{\boldsymbol{d}}}{b} & \forall i \in \mathcal{N}_S \\ \pi_i^{DSO} < \frac{\mathbf{1}_{[1,b]}\boldsymbol{\pi}^{\boldsymbol{d}}}{b} & \forall i \in \mathcal{N}_B. \end{cases} \tag{44}$$

Equation (44) is only calculated for the prosumers that the DSO wants to affect. For the rest of the prosumers, $\pi_i^{DSO} = \pi_i^d$. On the other hand, regarding the second concept, the DSO determines the pricing strategy based on the total transactions of the prosumers in the P2P trade. As a result, we define the DSO pricing strategy as

$$DSO_p^{st} := max\left\{DSO_p^{st}(it)\right\}_{it=0}^{it_{end}}, \tag{45}$$

Algorithm 2 DSO's Energy Pricing Strategy

1: Select *FLs* and *DGs* and initialize to $\boldsymbol{\pi_i} = 0, \forall i \in \mathcal{N}$.
2: Calculation of $\boldsymbol{p_i} \rightarrow u_i^* = \arg\min_{\boldsymbol{p_i}} u_i, \ \forall i \in \mathcal{N}$ based on Algorithm 1.
3: Solve based on the load flow
$p_{loss} = \mathbf{1}_{[-b,1]} real\left(\underline{\boldsymbol{Z}}\right) diag\left(\underline{\boldsymbol{i}}\right)\underline{\boldsymbol{i}}^*$,
$q_{loss} = \mathbf{1}_{[-b,1]} imag\left(\underline{\boldsymbol{Z}}\right) diag\left(\underline{\boldsymbol{i}}\right)\underline{\boldsymbol{i}}^*$.
4: DSO calculates $\pi_i^{DSO}, \forall i \in \mathcal{N}$ based on $\boldsymbol{\pi^d}$ obtained from DLMP.
5: **while** $iter = iter_{end}$ **do**
6: The second to fourth steps continue until $iter = iter_{end}$.
7: **end while**

where

$$DSO_p^{st}(it) := \frac{CO(it)}{mean(CO)_{it=0}^{it_{end}}} + \frac{\sum fm(it)}{mean(fm(it))_{it=0}^{it_{end}}}, \quad (46)$$

where it is equal to the iterations of upper-level model, and $CO := \sum_{i\in\mathcal{N}} \mathbf{1}_{[1,N]}[\boldsymbol{p}_S; \boldsymbol{p}_B]$. Also, fm is the fuzzy membership function of the technical parameters of the network, which are defined for each parameter as

$$fm = \begin{cases} 0 & fm \leq fm^{min} \\ 1 & fm \geq fm^{max} \\ \frac{fm^{max} - fm}{fm^{max} - fm^{min}} & fm^{min} \leq fm \leq fm^{max}, \end{cases} \quad (47)$$

where fm^{max} and fm^{min} are the upper and lower limits of each of the technical parameters of the network, respectively [35].

1) DSO's Energy Pricing Strategy Algorithm: An iterative approach is proposed to determine the DSO's pricing strategy. In this algorithm, whose pseudo-code is given in Algorithm 2, the DSO calculates the utility function of each prosumer according to the past data of prosumers' behavior in P2P trade and without considering the pricing strategy. According to the calculated utility function, the DSO calculates the technical parameters of the network based on the load flow. Then, the DSO calculates the sensitivity of the technical parameters of the network to each prosumer based on DLPM and determines the pricing strategy. The pricing strategy leads to a change in the behavior of prosumers, which will change the utility function of each prosumer. Again, the DSO calculates the technical parameters of the network based on the prosumers' new utility functions, and this process continues until there is no change in DSO's pricing strategy. In other words, the termination condition of the algorithm is calculated based on the value of DSO_p^{st} in several iterations, which is

$$it_{end} := \left\{ it | \frac{1}{it} \sum_{it=1}^{it} DSO_p^{st}(it) \leq DSO_p^{st}(it+1) \& it \geq it_{min} \right\}. \quad (48)$$

E. Convergence of the Leader-Follower Model

In every step of Algorithm 2, where Algorithm 1 is running, Theorem 2 guarantees that this latest algorithm, independent of initial conditions coming from Algorithm 2, converges to some ϵ- Nash equilibrium set in general, and to a Nash equilibrium for the case of $\delta_{mpd} = 0$ (the class of the utility functions introduced in the current study). As a result, running the entire Algorithm 2 results in convergence to a subset of Nash equilibria of the game corresponding to the utility functions as (9).

V. Simulation and Results

In this section, we use the IEEE 141-bus radial distribution system to evaluate the proposed algorithm.

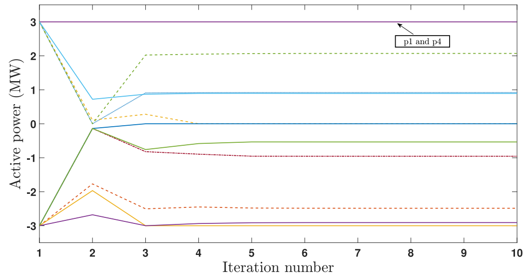
A. Setting Parameters

The total active and reactive power of the static loads of this network is equal to 11.4496 MW and 7.4026 MVAR, where the information about consumption and branches are set based on [8]. We consider 12 nodes as prosumers, sellers in nodes 32, 34, 52, 95, 130, and 138, and buyers in nodes 2, 6, 43, 50, 64, and 99, respectively. The trading range of each prosumer lies in [0, 3] MW while the cost of production (or consumer utility functions) is $c_i^1 = 5, c_i^2 = 0.1, \ \forall i \in \mathcal{N}_S$, and $c_i^1 = 15, c_i^2 = 1, \ \forall i \in \mathcal{N}_B$ (\$/MWh). Also, $c^{fix} = 0.5$ (\$/MWh) is considered, and at the beginning of the model, $\boldsymbol{c}_{[:]}^{tr} = 0$ and $\boldsymbol{\pi}_{[:]}^{DSO} = 10$ (\$/MWh) are considered for all prosumers. Nodes 1, 30, 42, and 80 are DGs, which are considered as $\boldsymbol{p}_{dg}^{min} := \mathbf{1}_{[1,4]}0$, $\boldsymbol{p}_{dg}^{max} := (20, 3.5, 3.5, 3.5)^\intercal$ MW and $\boldsymbol{q}_{dg}^{min} := \mathbf{1}_{[1,4]}0$, $\boldsymbol{q}_{dg}^{max} := (20, 2.5, 2.5, 2.5)^\intercal$ MVar. Nodes 74, and 128 are FLs, which are considered as $\boldsymbol{p}_{fl}^{min} := (-0.3, -0.6)^\intercal$, and $\boldsymbol{p}_{fl}^{max} := \mathbf{1}_{[1,2]}0$ MW. Finally, 10 (\$/MWh) and 3 (\$/MVarh) are considered as active and reactive marginal costs, respectively, and 1.10^{-4} \$/MWh^2(MVarh^2) is the price sensitivity coefficient.

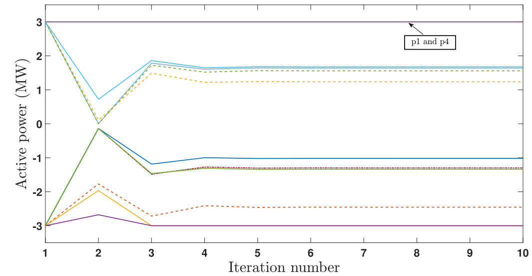
B. Evaluation of the Proposed Framework in Interaction Between DSO and P2P Trade

Increasing the DSO's understanding of prosumers' reactions to the pricing strategy can help the DSO select the best pricing strategy. In other words, if the DSO would like P2P trade to reach a specific NE, it must know how prosumers react to the pricing strategy. Prosumers' reaction to DSO's pricing strategy depends on various parameters of collective influence, freedom in decision-making, and marginal cost/utility, each of which is evaluated below. Before discussing the effect of different parameters on the behavior of DSO and P2P trade, it is important to mention that this paper proposed a framework based on NPGs to calculate the Nash equilibrium of P2P trade with PAAUFs. In Section III-D1, we proved in detail that our proposed model converges to Nash equilibrium, and in this section, we focus on the application of the proposed model in distribution networks. In other words, in studying the effect of each of the parameters, the convergence to the Nash equilibrium based on the proposed model will be shown.

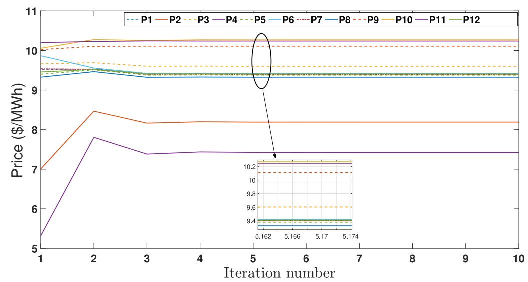
1) Collective Influence: To evaluate the collective influence of prosumers' behavior on the P2P trade equilibrium, we assume that the DSO's pricing strategy is determined according to DLMP and $-1 \leq \pi_i^{Pro} \leq 1, \ \forall i \in \mathcal{N}$, and we calculate the proposed framework for $\alpha = 1$, $\alpha = 0$, and $\alpha = 0.5$. Figure 2 shows the equilibrium of P2P trade, DSO's pricing strategy, and π_i^{Pro} in 10 iterations for different α. Figure 2(c) shows the DSO pricing strategy, while Figure 2(a) and 2(b)



(a) Equilibrium of P2P trade at $\alpha = 0$.



(b) Equilibrium of P2P trade at $\alpha = 1$ and $\alpha = 0.5$.



(c) DSO's pricing strategy.

Fig. 2. Equilibrium of P2P trade with constant pricing strategy, $-1 \leq \pi_i^{Pro} \leq 1, \ \forall i \in \mathcal{N}$, and $\alpha \in \{0, 0.5, 1\}$.

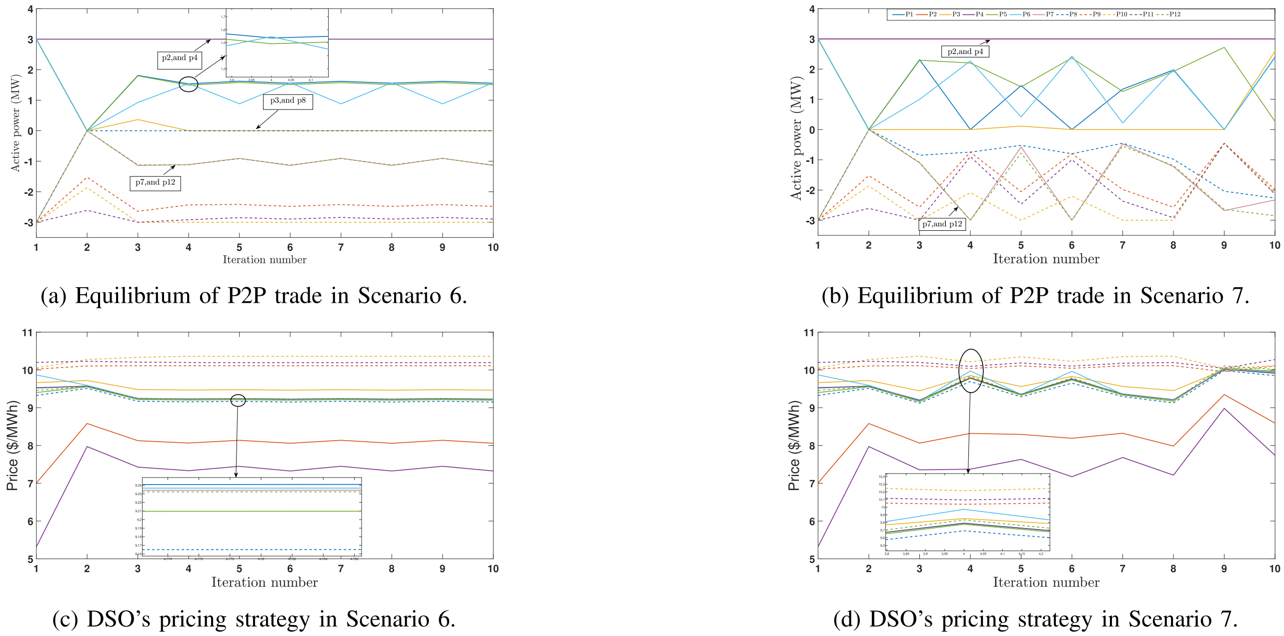
show the P2P trade equilibrium at $\alpha = 0$ and $\alpha = \{0.5, 1\}$, respectively. Interaction between DSO and P2P trade occurs in both situations after 5 iterations, but in each situation, the P2P trade converges to a different Nash equilibrium. This proves that the collective influence of prosumers affects the asymptotic behavior of P2P trade. In other words, before adopting a price strategy, DSO must first be aware of the existence of the majority of prosumers' classification and calculate the behavior of prosumers according to the collective influence of prosumers on their expected price. This is important because, as stated in Section III-B, the collective influence of prosumers varies over 24 hours. This means that from 10 to 14 pm, the expected price of prosumers tends to be calculated based on a non-linear function, and as a result, the DSO's pricing strategy during these hours should be adopted according to the non-linear behavior of prosumers.

2) Freedom in Decision-Making: Another important parameter in the approximation of PUFs based on the actual behavior of prosumers is related to the parameter of prosumers' freedom in decision-making. In fact, as detailed in Section III-B, in most papers the interaction between DSO and P2P trade has been modeled by adding a cost, which is inconsistent with the realistic behavior of prosumers. On the other hand, the DSO has to have a sufficient understanding of the prosumers' freedom in decision-making to adopt an optimal pricing strategy. Therefore to better understand the impact of the prosumers' freedom in decision-making on the DSO's pricing strategy, let us set $\alpha = 1$ and define π_i^{Pro} according to Table II. Figures 3 show the equilibrium of P2P trade with the DSO's pricing strategy in 10 iterations for Scenarios 1 and 2. It can be seen that the change in π^{Pro} has caused a change in the equilibrium of P2P trade and the DSO's pricing strategy. In other words, in Scenario 1, Figures 3(a) and 3(c), there is greater convergence in DSO pricing strategy

TABLE II
THE VALUE OF π^{Pro} IN TWO SCENARIOS 1 AND 2

Prosumers	32	34	52	95	130	138
Scenario 1	0.1	0.15	0	0.5	0.1	0.2
Scenario 2	0.5	0.5	0.5	0.05	0.1	0.5
Prosumers	2	6	43	50	64	99
Scenario 1	0.05	0	0.2	0.5	0.5	0.1
Scenario 2	0.5	0.5	0.5	0.01	0.01	0.5

and P2P trade equilibrium than in Scenario 2, Figures 3(b) and 3(d). This shows that the value of prosumers' freedom in decision-making in P2P trade is very important. For example, prosumer 32 participates in P2P trade in scenario 1 with $\pi^{Pro} = 0.1$ and in scenario 2 with $\pi^{Pro} = 0.5$. As can be seen in Figures 3(a) and 3(b), the standard deviation of this prosumer's energy value in each iteration from the average energy value of its behavior in 10 iterations in Scenario 1 is much less than the standard deviation of this prosumer's energy value in Scenario 2. In other words, by reducing the freedom in the decision-making parameter of prosumer 32 in scenario 1, this prosumer has a more coordinated behavior with the decisions of DSO compared to Scenario 2. Similar to the same analysis, we can see in the behavior of prosumer 138 that with the increase of freedom in decision-making, the standard deviation of this prosumer's energy value in each iteration from the average energy value of its behavior in 10 iterations in Scenario 1 is much less than the standard deviation of this prosumer's energy value in Scenario 2. Therefore, if the DSO does not have a sufficient understanding of prosumers' freedom in decision-making in P2P trade, it cannot adopt the optimum pricing strategy for maximum interaction between P2P trade and DNs technical parameters.



(a) Equilibrium of P2P trade in Scenario 6.

(b) Equilibrium of P2P trade in Scenario 7.

(c) DSO's pricing strategy in Scenario 6.

(d) DSO's pricing strategy in Scenario 7.

Fig. 3. Equilibrium P2P trade, and DSO's pricing strategy for Scenarios 6 and 7.

On the other hand, prosumers' freedom in decision-making it possible for prosumers to converge the equilibrium of P2P trade to another point, and as a result, DSO's power in facing prosumers in P2P trade is reduced. In fact, these results prove the fact that the closer we get to realistic modeling of prosumers' behavior in P2P trade, the more precisely we need to model their priorities and concerns.

3) DSO's Pricing Strategy: As stated previously, one of the challenges in the implementation of P2P trade is the potential conflict between the P2P trade and optimization criteria for the technical parameters of the network (such as voltage deviation and active losses). To evaluate of proposed framework in this dichotomy and determine the optimal DSO pricing strategy, we consider five scenarios and assume that each prosumer is aware of the deterministic decisions of other prosumers, while the expected price is considered equal to the DSO pricing strategy.

- *Scenario 3:* Without considering P2P trade.
- *Scenario 4:* The DSO pricing strategy is considered with the aim of reducing voltage deviation.
- *Scenario 5:* The DSO pricing strategy is considered with the aim of reducing active losses.
- *Scenario 6:* Without considering the technical parameters of the network.
- *Scenario 7:* Increasing the interaction between the technical parameters of the network and the participation of prosumers in P2P trade.

Table III lists the results of Scenarios 3 to 7. The lowest voltage deviation and active losses are in Scenario 3, and on the other hand, the highest voltage deviation and active losses are in Scenario 6. As a result, the expansion of P2P trade and the reduction of network technical parameters are in conflict with each other. On the other hand, the comparison of Scenarios 4 and 5 shows that the active losses reduction strategy has a great impact on the reduction of P2P trade in the DNs, which leads to a decrease in interaction between the DSO and P2P trade. In Scenario 7, DSO enhances P2P trade in addition to simultaneously reducing active losses and voltage deviation. This Scenario seems to be the best DSO pricing strategy compared to other Scenarios as it increases the interaction between technical parameters and P2P trade. Scenarios 3 to 7 prove that the conflict between the DSO and the P2P trade is great, and as a result, each of them must be fully aware of each other's priorities and concerns to achieve optimal results. In other words, it is inevitable to model PAAUFs to increase interaction between the DSO and the P2P trade.

TABLE III

COMPARISON OF INTERACTION BETWEEN TECHNICAL PARAMETERS AND P2P TRADE IN SCENARIOS 1 TO 5. WHERE P2P TRADE: $\sum_{i\in\mathcal{N}} \mathbf{1}_{[1,N]}\boldsymbol{p}_i^*$ (MW), DSO TRADE: $\mathbf{1}_{[1,n_{fl}]}\boldsymbol{p}_{fl} + \mathbf{1}_{[1,n_{dg}]}\boldsymbol{p}_{dg}$ (MW), ACTIVE LOSSES: p_{loss} (MW), AND VOLTAGE DEVIATION: MAX$|\mathbf{1}_{[b,1]} - \hat{\boldsymbol{u}}|$ (P.U.) ARE CONSIDERED

Scenario	P2P trade	DSO trade		Active losses	Voltage deviation
		p_{fl}	p_{dg}		
Scenario 1	0	-0.03	0.2373	0.026	0.0218
Scenario 2	23.2924	-0.06	0.4699	0.1443	0.0731
Scenario 3	12	-0.06	0.3507	0.0895	0.0745
Scenario 4	36	-0.06	0.5253	0.1874	0.0902
Scenario 5	24.8329	-0.06	0.4730	0.147	0.0752

C. Evaluation of Proposed Learning Model

Another challenge in modeling P2P trade is related to the communication between prosumers. In real DNs, prosumers are not made aware of decisions made by all other prosumers, and protecting their privacy is important. A learning model is proposed in which each prosumer only communicates

TABLE IV
THE PROSUMERS AND THE PROSUMERS' NEIGHBORS

Prosumers	Prosumers' neighbors	Prosumers	Prosumers' neighbors
32	34, 2, 6, 64	34	32, 130, 138, 99
52	43, 50, 99	95	138, 50, 64
130	52, 2, 6	138	6, 9, 50
2	32, 130, 6	6	130, 138, 64
43	52, 138, 99	50	52, 95, 138
64	32, 95, 6	99	34, 52, 43

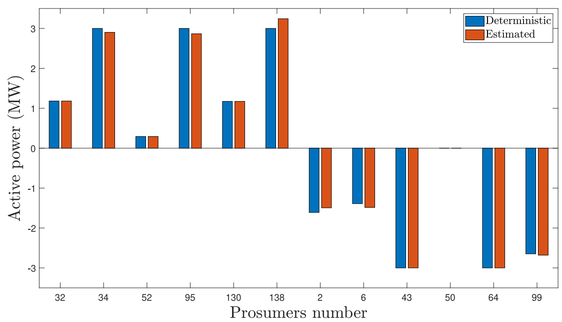


Fig. 4. Equilibrium of P2P trade in deterministic and estimated states with constant π^{DSO}.

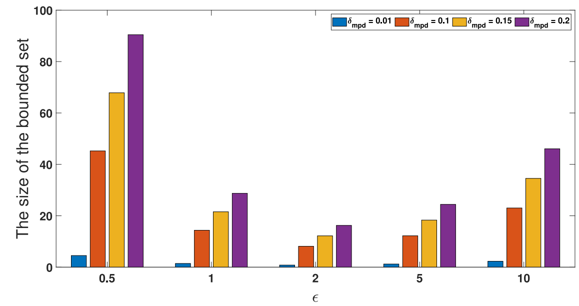


Fig. 5. The size of the bounded set in different ϵ and δ_{mpd}.

with the neighboring prosumers, and based on the exchanged information, estimates the decisions of other prosumers. The evaluation of the effectiveness of the proposed learning model has compared the asymptotic behavior of P2P trade in the deterministic and estimated state with fixed π^{DSO}, while in Table IV, prosumers and each one's neighbors are listed.

Figure 4 shows a comparison of the asymptotic behavior of P2P trade in deterministic and estimated states and the total difference between deterministic and estimated states is equal to 0.0261 MW. More precisely, prosumers 32, 52, 130, 43, 50, and 64 have equal deterministic and estimated states, while prosumers 34, 95, 138, 2, 6, and 99 have deterministic definite and estimated states. The ratio of the difference between the deterministic and estimated state with the total energy traded shows the effectiveness of the proposed learning model, whereby prosumers can get an acceptable estimate of the behavior of other prosumers without violating privacy.

D. Sensitivity of Asymptotic Behavior of P2P Trade to Approximation of PUFs (A Quantitative Perspective)

As stated in Section I, the approximation of PUFs plays an important role in reducing the gap between research results and practice. In Section III-D5, the sensitivity of the asymptotic behavior of P2P trade to the approximation of PUFs has been examined from a qualitative point of view, and it has been stated that the bounded set size depends on the $\epsilon > 0$, and δ_{mpd}. As a quantitative investigation of the sensitivity of the asymptotic behavior of P2P trade to the approximation of PUFs, let us assume a game where the condition $\frac{\partial^2 u_i}{\partial p_{ij}\partial p_{ji}} = \frac{\partial^2 u_j}{\partial p_{ji}\partial p_{ij}}$ holds and the action space of all prosumers is bounded, while we have $N = 2$, $\mathbf{L} = \delta_{mpd} = \{0.01, 0.1, 0.15, 0.2\}$, and $\mathfrak{E} = [0, 10]$. Figure 5 shows the size of the bounded set in different ϵ and δ_{mpd}. As can be seen, the smaller the size of δ_{mpd} is, the smaller the size of the bounded set is, and as a result, the solution result is closer to NE. In other words, as stated in Definition 2, the maximum pairwise difference (MPD) between two games Γ and $\hat{\Gamma}$ is represented by δ_{mpd}, while the condition $\frac{\partial^2 u_i}{\partial p_{ij}\partial p_{ji}} = \frac{\partial^2 u_j}{\partial p_{ji}\partial p_{ij}}$ is satisfied in Γ and not in $\hat{\Gamma}$. As a result, when δ_{mpd} is small, it means that the sensitivity of the asymptotic behavior of P2P trade to the approximation of PUFs is low, and the solution result is closer to NE. Furthermore, the effect of different ϵ on the size of the bounded set shows that when the discretization factor becomes very small and approaches the continuous action space, the asymptotic behavior of P2P trade is placed in a larger bounded set. In other words, the closer the prosumers' approximate utility functions are to their actual behavior, the bigger $|\mathfrak{E}|^N$ becomes, and assuming that the prosumers' action space is still limited, the asymptotic behavior of P2P trade is placed in a larger bounded set. This means that what was happening in practice could have a significant deviation from what was calculated in the research.

VI. CONCLUSION

In this paper, the conflict between distribution network operator (DSO) decisions and P2P trade was discussed. It was shown that the technical parameters of distribution networks (DNs) such as active losses and voltage deviations are at their lowest value without the presence of prosumers, while these technical parameters increase with the development of prosumers' participation in P2P trade. In other words, it was proven that to reduce the conflict between the DSO's decisions and P2P trade, their understanding of each other's concerns and priorities should be increased. On the other hand, it was stated that the interaction between DSO's decisions and P2P trade requires an approximation of the prosumers' actual utility function. Therefore, In this paper, prosumers' actual approximate utility functions (PAAUFs) were proposed, which include various parameters such as freedom in decision-making, collective influence, and marginal cost/utility. The main advantage of using such functions is to increase the interaction between the DSO and the P2P trade in reality, which, as a result, increases the speed of the development of P2P trade. In fact, if the modeling of the interaction between the DSO and the P2P trade is based on unrealistic utility functions of prosumers, the results obtained in the modeling will be inconsistent with the results obtained in reality. Therefore, the modeling of these functions brings the theoretical modeling results closer to reality and thus avoids unknown operational challenges. For example, two influential parameters were been analyzed in PAAUFs, i.e., freedom in decision-making and collective influence, and it was been shown that the asymptotic behavior of P2P trade is strongly dependent on these two parameters. Therefore, DSO should consider these two parameters to make optimal decisions in order to have the least conflict with the P2P trade.

On the other hand, the biggest disadvantage in PAAUF modeling is their mathematical complexity, because, in general, these functions have an unusual shape that may not be convex or continuously differentiable, resulting in unknown asymptotic behavior. To deal with this complexity, A framework based on game theory is proposed. Indeed, in this paper, it was proved that based on the near-potential game and best-response dynamics, the asymptotic behavior of P2P trade can be calculated for such actual utility functions.

Privacy is also a challenging issue in the P2P trade. In this paper, A learning model was proposed in which each prosumer estimates the decisions of other prosumers, which then influences their decision-making. It was shown that the asymptotic behavior of P2P trade in this learning model is very close to the deterministic situation of prosumers, where all decisions are assumed to be available to all prosumers, corresponding to a case where privacy is not protected.

As future directions for this research, we shall consider more elaborated prosumer behavior. In fact, the main goal of the proposed framework of this paper is to deal with the mathematical challenges caused by the unknown asymptotic behavior of P2P trade. Hence, the PAAUFs considered in this paper may still not match the actual behavior of prosumers. Note that, as discussed in detail in Section III, this mismatch does not harm the robustness and comprehensiveness of the proposed framework. Therefore, the proposed framework provides a suitable fundamental approach to calculating unknown asymptotic behavior based on prosumers' behavior in reality. We can also consider the retailers' profits as active players, and prosumers with energy storage systems as other next challenges.

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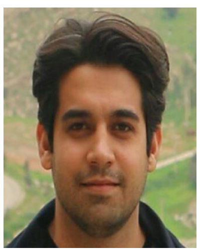

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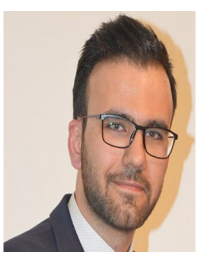

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