International Review of Financial Analysis 95 (2024) 103422

Contents lists available at ScienceDirect

International Review of Financial Analysis

journal homepage: www.elsevier.com/locate/irfa

A universal exponent governing foreign exchange rate risks

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A R T I C L E I N F O

JEL classification:
F31
C22
G11
G12
G13
G15

Keywords:
Foreign exchange rates
Pareto distributions
Power laws
Second moment
Variance

A B S T R A C T

Departing from previous studies, this paper uses power laws to model foreign exchange rate risks in terms of realized foreign exchange rate (FX) variances for daily and weekly data. Empirical tests based on daily data provide strong evidence for emergent market risk behavior manifested in a common power-law exponent governing the cross section of realized FX variances. We show that this emergent market risk behavior is invariant across various time frequencies. Based on modern bootstrapping techniques, we derive a novel joint test for investigating the presence of total invariance—that is, invariance of emergent market risk behavior across time frequencies and over time. Our novel test provides strong evidence for total invariance of realized FX variances. We argue that the results are in line with the theory of complex systems—that is, even though FX risk exhibits idiosyncratic features that may originate from market-distinct factors (inflation, interest rates, public debts, etc.), emergent market behavior manifests itself in a universal power-law exponent governing the cross section of FX risks.

1. Introduction

Foreign exchange (FX) rates are risky−perhaps riskier than earlier believed. To manage FX risk exposure, Opie and Riddiough (2020) argue that the relevant literature typically employs mean-variance optimization as the main tool. The authors highlight that despite its theoretically appealing features, when applied out of sample, this approach often suffers from severe estimation error in currency returns due to the well-known difficulty of predicting exchange rates, which results in poor overall currency hedging performance (Gardner & Stone, 1995; Larsen Jr. & Resnick, 2000; Meese & Rogoff, 1983). Taking the perspective of a US investor who invests in a portfolio of G10 developed economies, Opie and Riddiough (2020) propose a correlation-based method to dynamically hedge FX exposure in international equity and bond portfolios, which provides an improvement in estimation errors when compared to other leading alternative approaches to currency hedging.

In evaluating their dynamic method, the authors rely on commonly-used correlation-based performance measurements, such as Sharpe ratios. Opie and Riddiough (2020) emphasize that most sophisticated approaches studying global currency hedging employ the joint distribution of FX rates and underlying asset returns within a mean-variance framework. For instance, the findings of the study of Campbell, Serfaty-De Medeiros, and Viceira (2010) suggest that global equity investors minimize portfolio variance by leaving unhedged positions in currencies, such as the US dollar and euro, which are negatively correlated with global stock market returns. Therefore, investors are recommended to leave sizable exposure in "safe haven" currencies that seemingly provide a natural source of insurance. However, the question arises: how safe as these "safe havens"? Evidence shows that daily price variations in FX can be dreadful. On June 24, 2016, for example, the daily price range observed in the GBP/USD exchange rate corresponded to a 77-sigma event.[2] In fact, recent research provided evidence that these "outliers" are rather reoccurring events that are observable in a similar magnitude across all FX rates (Grobys, 2023a).

This study takes a novel perspective by exploring the risk of FX risk as measured in terms of FX variances and examines whether there exists a

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[1] This study was presented at the 2023 Finance Research Seminar at Aalto University. Valuable comments were received from Peter Nyberg and Matthijs Lof. Moreover, helpful comments were also received from the participants of the 2023 Mathematics & Statistics Research Seminar at the University of Vaasa. Special thanks are owed to Seppo Pynnönen, Tommi Sottinen, Hamidreza Almani, and Anupam Dutta. Furthermore, useful help was also received from John Kihn. Finally, constructive and helpful comments were received from an anonymous reviewer.

[2] On June 24, 2016, the high- and low-prices for the GBP/USD were according to investing.com quoted at 1.5020 and 1.3226, respectively.

https://doi.org/10.1016/j.irfa.2024.103422
Received 2 November 2023; Received in revised form 18 January 2024; Accepted 19 February 2024
Available online 29 June 2024

commonality among FX variances. Because intraday price ranges provide a higher level of information than closing prices (Chou, Chou, & Liu, 2010), this study follows earlier research (Grobys, 2021; Grobys, 2023a) in modeling FX variances using the intraday range-based variance estimator proposed by Parkinson (1980). Following Mandelbrot (1963a), G10 FX variances are modeled via power laws for daily and weekly data frequencies covering the period May 16, 2006, to May 31, 2023, for which data are publicly available. Power-law exponents are first estimated using Clauset et al.'s (2009) often-used approach based on maximum likelihood estimation (MLE). A novel feature of this study is that it employs Grobys and Junttila's (2021) proposed blocks bootstrap approach to estimate the covariance matrix of power-law exponents for G10 FX variances. In further analysis, the estimated covariance matrix of power-law exponents was employed to implement a joint test to explore whether a common source of risk across daily FX variances does exist. In this study, a manifestation of such a common factor, would be, a common power-law exponent governing seemingly unrelated FX variances. To test this hypothesis, the economically important power-law exponents ranging between $\alpha = 2.1$ and $\alpha = 3.1$ are iteratively tested.[3]

Since Mandelbrot (2008) points out that fractal-behavior is manifested in power-law exponents that are invariant across various time frequencies, a proposed extension of this test jointly tests whether a common source of risk across FX variances exists across both FX variances and time frequencies using the estimated covariance matrices for daily and weekly data derived from the blocks bootstraps. Finally, this study derives a further extension of this test which it terms the *test for total invariance*. Its purpose is to explore whether a common risk factor of FX variances is not only invariant across time frequencies but also invariant over subsamples for a given time frequency. To implement the test for total invariance, covariance matrices are estimated for both various time frequencies and various subsamples via blocks bootstraps.

The present study contributes to the literature in several important ways. Lustig, Roussanov, and Verdelhan (2011) employ correlation-based methodologies to derive common risk factors across currency returns. Using principal component analysis, the authors show that the first principal component is essentially the average excess return on all foreign currency portfolios (e.g., the dollar risk factor), whereas the second principal component is like the excess return on the carry trade risk factor. The two identified risk factors explain about 82% of the variation in returns on six currency portfolios sorted on average forward discounts. Other relevant risk factors are perhaps the currency momentum factor (Burnside, Eichenbaum, Kleshchelski, & Rebelo, 2011; Menkhoff, Sarno, Schmeling, & Schrimpf, 2012a; Okunev & White, 2003), and FX volatility (Menkhoff, Sarno, Schmeling, & Schrimpf, 2012b). Whereas these studies focus on identifying commonalities in the first moment of currencies, this study explores whether commonalities in the second moments do exist. A novel aspect in the present study is that it does not rely on correlation-based models as previous studies but makes use of power laws, which is in line with Mandelbrot (1963a), who advocates to model financial data using power laws. Some readers might wonder why we do not follow most studies and use GARCH-type models. Mandelbrot (2008) argues that GARCH-type models typically deliver sample-specific results and therefore recommends that scientists should search for invariances:

…many recent models of price variation try to explain the obviously shifting pattern of volatility by inserting parameters that change by the day, hour, and second; such are in the GARCH family … I would rather not dismiss the existence of invariances but continually look for them hiding in non-obvious places. Invariances make life easier. If you can find some market properties that remain constant over time or place, you can build better, more useful models and make sounder financial decisions. (Mandelbrot, 2008, p. 242).

Thus, contrary to previous studies, this is the first study that follows Mandelbrot's (2008) advice and explores commonalities in the second moment of FX rates using power laws. In doing so, we address the following main questions. First, are FX variances scale invariant? Second, is potential scale invariance manifested in a common behavior across FX variances? Third, is potential common scale invariance subject to sample specificity? These are important issues to clarify because invariance has serious implications, which will be elaborated on more in the following sections.

Next, this study contributes to the existing literature on global currency hedging. Managing FX exposure may offer potential benefits for investment performance. However, the question arises about which hedging strategy is optimal. Relying on the concept of correlation, Campbell et al. (2010) document that global equity investors minimize portfolio variance by leaving unhedged positions in currencies that are negatively correlated with international equity returns. Other literature is devoted to investigating optimal currency portfolios (e.g., Ackermann, Pohl, & Schmedders, 2018; Asness, Moskowitz, & Pedersen, 2013; Barroso & Santa-Clara, 2015; Della Corte, Sarno, & Tsiakas, 2009; Opie & Riddiough, 2020). However, as mentioned earlier, Opie and Riddiough (2020) document that these models are derived from mean-variance frameworks. Problems arise if mean-variance frameworks deliver sample-specific results. The poor out-of-sample performance of models based on mean-variance optimization (Opie & Riddiough, 2020) can be considered a manifestation of sample specificity. The "well-known difficulty in predicting exchange rates," as highlighted in Opie and Riddiough (2020, p. 781), is perhaps an issue arising from the absence of higher moments in foreign exchanges. In this regard, Mandelbrot (1963b) was the first to show that cotton price changes exhibit an infinite variance. Interestingly, Fama (1963) comments on Mandelbrot's (1963b) results in the following way:

… the infinite variance assumption of the stable Paretian model has extreme implications. From a purely statistical standpoint, if the population variance of the distribution of first differences is infinite, the sample variance is probably a meaningless measure of dispersion. Moreover, if the variance is infinite, other statistical tools (e.g., least-squares regression) which are based on the assumption of finite variance will, at best, be considerably weakened and may in fact give very misleading answers. (Fama, 1963, p. 421).

Potentially undefined variances imply that inferences from correlation-based models used to manage FX exposure may not only be biased but may indeed give misleading answers. Given the considerable amount of research in this area of finance, as well as the economic importance of the FX rate market, further investigation of this issue is warranted. Since the variance data exhibit fat tails, an important question arises: Are the tails too heavy to manage with tools derived from mean-variance frameworks?

A minor contribution of this study is that it explicitly tests whether power laws functions are reasonable to model FX variances. For instance, recent studies have found that FX variances exhibit relatively low tail indices (Grobys, 2021; Grobys, 2023a). The discovery of power laws with relatively low exponents for some measure of realized asset variance is in stark contrast to previous studies showing that the realized asset volatility is typically close to a log-normal distribution (e.g., Andersen, Bollerslev, Diebold, & Ebens, 2001; Andersen, Bollerslev, Diebold, & Labys, 2001a & Andersen, Bollerslev, Diebold, & Labys, 2001b). Therefore, this study employs various goodness-of-fit tests to explore the plausibility of power law models.

Finally, this study adds to the literature on providing practical approaches to estimate power-law exponents. For instance, White et al. (2008) discuss various approaches that have been introduced in the literature and argue that traditional approaches mainly rely on linear binning and log-log regressions. A study of Clauset et al. (2009) provides a novel approach in which the optimal power-law exponent is obtained by the optimized Kolmogorov-Smirnov distance. The authors show that

[3] Whereas $\alpha < 2.0$ in the research context of the present study would imply that the variance distribution would not have a theoretically defined mean, $2.0 < \alpha < 3.0$ would imply that the variance distribution would exhibit a defined mean but would not exhibit a defined second (or higher) moment.

their approach is superior to traditional ones. This study extends Clauset et al.'s (2009) approach in some important ways. First, using blocks bootstraps, the uncertainty of the power-law exponents can be estimated in a more robust manner.[4] Second, the blocks bootstrap procedure is implemented jointly for the data matrix of FX variances, and then individual FX variance data vectors are extracted from it, enabling us to estimate the blocks-bootstrap covariance matrix of power-law exponents. This is a novel aspect because previously proposed approaches to retrieve power-law exponents focused exclusively on single data series and, thus, did not account for potential correlations between power-law exponents estimated from seemingly unrelated data. However, joint tests for power-law exponents require the establishment of a covariance matrix, and therefore, this study remedies this important gap in the literature.

Using Clauset et al.'s (2009) approach to estimate the power-law exponents for the daily FX variances derived from the Parkinson (1980) estimator, the results indicate that the estimated power-law exponents vary between $\widehat{\alpha} = 2.2784$ for the AUD/USD variance and $\widehat{\alpha} = 2.9081$ for the SEK/USD variance. The fraction of the FX variance distribution governed by a power-law process varies between 4.60% for the SEK/USD variance and 33.06% for the AUD/USD variance. Whereas the economic magnitude for the estimated power-law exponents for weekly FX variance data is close to the figure for the daily data, the fraction of the distributions governed by a power-law process is often considerably larger, which is a counterintuitive finding because the effect of time aggregation would predict the opposite effect. Estimated covariance matrices derived from blocks bootstraps show that covariances between power-law exponents are statistically significantly positive. This result holds regardless of the time frequency, suggesting that covariances need to be accounted for when joint power-law behavior is subject of investigation.

Furthermore, testing the hypothesis about whether the daily FX risk shares a commonality manifested in a common power-law exponent governing the whole cross section of FX variances shows that this hypothesis cannot be rejected for exponents between $\alpha = 2.3$ and $\alpha = 2.9$. Further results, derived from an analysis of p-values, indicate that the most likely power-law exponent is $\alpha \approx 2.6$. Testing this issue jointly for daily and weekly FX variances strongly supports this finding. Specifically, the p-value for $\alpha \approx 2.6$ is 0.9813, indicating a type-2 error of $<5\%$, which is in exceptional outcome in applications of this type of hypothesis tests. Finally, our proposed test for total invariance shows that this emergent market risk behavior manifested in $\alpha \approx 2.6$ is present across time frequencies and over nonoverlapping subsamples, indicating a high level of stability.

Robustness checks provide evidence that the power-law model cannot be rejected for most FX variances. Using a one-sigma test provides further evidence that neither the lognormal nor the $\chi^2(1)$-distribution can accurately describe the underlying data-generating processes for FX variances, which is contrary to previous literature supporting lognormal distribution (e.g., Andersen, Bollerslev, Diebold, & Ebens, 2001; Andersen, Bollerslev, Diebold, & Labys, 2001a & Andersen, Bollerslev, Diebold, & Labys, 2001b). Furthermore, the documented research results are robust for (i) using alternative range-based estimators, (ii) imposing further restricting on the random block length used for the blocks bootstrap procedure or (iii) changing the database from which data are obtained. We argue that these results are in line with the theory of complex systems—that is, even though FX risk exhibits idiosyncratic features that may originate from market-distinct factors (inflation, interest rates, public debts, etc.), emergent market behavior manifests itself in a common component governing the risk of the overall FX market.

This study is organized as follows. The next section provides the background. The third section presents the data, and the fourth section describes the methodologies. The fifth section reports the results, and the sixth section discusses the results. The last section concludes the study.

[4] Note that the standard deviation proposed in Clauset et al. (2009) is derived under the assumption of independently distributed observations.

2. Background

Given that the worth of the entire global FX trading market is approximately 30 times larger than the global equity market capitalization, it is not surprising that the uncertainty in the FX market has been subject to intense study in the academic literature. For example, in an early study, Jorion (1995) analyzed the informational content and the predictive power of volatility implied in FX option prices. In doing so, he derived implied standard deviations (ISDs) from Chicago Mercantile Exchange options on foreign currency futures and compared them to time-series models, such as moving average and GARCH-type models. Moreover, in another early study on FX risk, Baillie and Bollerslev (1991) employed seasonal GARCH models to describe the time-dependent volatility apparent in the returns of four FX rate spot series recorded on an hourly basis for a six-month period in 1986. In line with this stream of research, Bollerslev and Melvin (1994) used GARCH-type models and found that the size of the bid-ask spread in the FX rate market was positively related to underlying exchange rate uncertainty.[5] Despite the widespread usage of GARCH-type models to investigate the uncertainty in the FX market, studies by Andersen, Bollerslev, Diebold, and Labys (2003) and Andersen, Bollerslev, and Meddahi (2004) provided evidence that simple reduced-form time series models for realized volatility (RV) outperform the often-used GARCH-type models for forecasting future volatility.

Further, Wang and Yang (2009) investigated the relationship between current daily RV and lagged daily returns in the FX market by implementing a variant of the heterogeneous autoregressive realized volatility (HAR-RV) model of Corsi (2004) and Andersen, Bollerslev, and Diebold (2005). In a similar manner, Bubák, Kočenda, Žikeš, and F. (2011) analyzed the dynamics of volatility transmission between Central European (CE) currencies and EUR/USD FX rate using model-free estimates of daily FX volatility derived from intraday data. Their proposed approach is, in essence, a multivariate generalization of the HAR-GARCH model by Corsi, Mittnik, Pigorsch, and Pigorsch (2008). Bubák et al. (2011) maintained that their analysis broke new ground because it did not rely on commonly-used GARCH-type models to investigate FX volatility. Interestingly, Wang and Yang (2009) have observed that models based on RV are better able to (i) capture underlying volatility and to (ii) test relevant hypotheses, as opposed to GARCH models, which are not able to efficiently capture the dynamics of underlying volatility due to their dependence on daily return series. An alternative to realized variance or volatility derived from either daily closing prices or intraday quotes is range-based estimators. In this regard, Chou et al. (2010) argued that range-based volatility estimators are more efficient and comprise more information than changes in closing prices. Therefore, range-based estimators provide a reasonable way to model the uncertainty in the FX market and have been used in recent studies on FX risk (e.g., Grobys, 2021; Grobys, 2023a).

However, there is a problem with the naïve usage of models derived from the empirical distribution of realized variances or volatilities. Taleb (2020) argued that "… the 'empirical distribution' is not empirical due to misrepresenting the expected realizations of the distribution in the tails. Future maximums are poorly tracked by past data without some intelligent extrapolation" (p. 33). Consequently, Taleb (2020) advocated the usage of power laws to address this issue because the power-law exponent captures via extrapolation low-probability deviations not seen in the data. Furthermore, in advocating the application of power laws in finance research, Taleb (2020) emphasized the following: "There are a lot of theories on why things should be power laws, as sort of exceptions to the way things work probabilistically. But it seems that the opposite idea is

[5] Alexander (1995) and Bauwens, Omrane, and Giot (2005) are other relevant studies. They used (G)ARCH-type models to investigate common volatility in the FX rate market as well as the impact of scheduled and unscheduled news announcements on foreign exchange rate return volatility.

never presented: power laws should be the norm, and the Gaussian a special case …" (p. 91). However, it is noteworthy that Taleb (2020) was not the first to advocate power laws for modeling financial data. In an early study, Mandelbrot (1963a, p. 438) argued that "…there is strong pragmatic reason to begin the study of economic distributions and time series by those that satisfy the law of Pareto." Whereas Mandelbrot (2008) criticized GARCH-type models due to yielding sample-specific model estimates, Calvet and Fisher (2004) and Lux, Morales-Arias, and Sattarhoff (2014) supported Mandelbrot (2008) proposition in the sense that they found that power-law models usually outperform GARCH-type models. Following Mandelbrot, 1963a, Mandelbrot, 2008) and Taleb (2020) recommendations, recent studies by Grobys (2021 & 2023a) used power laws to model the realized variances for the GBP/USD exchange rate and realized variances of the G10 currencies derived from Parkinson's (1980) range-based estimator. A commonality of these studies is that their findings indicate that the power-law null hypothesis cannot be rejected for most range-based FX variances.

Motivated by this literature, the present study employs Parkinson (1980) range-based estimator to obtain data on FX variances and then power-law models, which enable us to assess via extrapolation low-probability deviations not seen in the data. The present study also identifies commonalities among FX variances using various tests.

3. Data

Publicly available intraday prices for the AUD/USD, CAD/USD, CHF/USD, EUR/USD, GBP/USD, JPY/USD, NOK/USD, NZD/USD, and SEK/USD exchange rates were downloaded from finance.yahoo.com. Because the AUD/USD exchange rate data are only publicly available from May 16, 2006, onward, this study uses data from May 16, 2006, to May 31, 2023. Moreover, only the intersection was employed—that is, only daily data are accounted for where all FX rates were quoted on the same day, leaving us with 4410 daily observations.[6]

To estimate annualized daily variances, we employ the range-based variance estimator, proposed by Parkinson (1980), which is given by:

$$\sigma_{i,t}^2 = T\frac{1}{4ln(2)}\left(ln\left(H_{i,t}\right) - ln\left(L_{i,t}\right)\right)^2, \tag{1}$$

where $H_{i,t}$ and $L_{i,t}$ denote the highest and lowest price for the FX rate i on trading day t, and $\sigma_{i,t}^2$ denotes the FX rate i's corresponding annualized daily variance, where $T = 250$, as 250 trading days per annum are assumed.[7] Further, we commute the annualized weekly variances as:

$$\sigma_{i,w}^2 = W\frac{1}{4ln(2)}\sum_{t\in w}\left(ln\left(H_{i,t}\right) - ln\left(L_{i,t}\right)\right)^2, \tag{2}$$

where $H_{i,t}$ and $L_{i,t}$ denote the highest and lowest prices for the FX rate i on corresponding trading days $t \in w$, where w denotes the time index for weekly data, and $\sigma_{i,w}^2$ denotes the FX rate i's annualized weekly variance, where $W = 52$, as 52 trading weeks per annum, are assumed. Tables 1 and 2 report the descriptive statistics for the daily and weekly data. As shown in Table 1, the kurtosis values for the daily variance data vary between 153.19 for the CAD/USD exchange rate and 4408.00 for the EUR/USD exchange rate, strongly suggesting fat tails. From the data in Table 2, it becomes evident that the weekly data confirm this issue. The presence of fat tails in variance data is, of course, not surprising per se; the important question that arises is, though, how fat are the tails?

4. Methodology

4.1. Main analysis

4.1.1. Power laws

To investigate the fatness of the FX variance tails, FX variances are modeled using the following power-law function:

$$p(x) = Cx^{-\alpha}, \tag{3}$$

where $C = (\alpha - 1)x_{MIN}^{\alpha-1}$ with $\alpha \in \{\mathbb{R}_+ | \alpha > 1\}$, x denotes the respective annualized daily or weekly FX variance, provided that $x \in \{\mathbb{R}_+ | x_{MIN} \le x < \infty\}$, x_{MIN} is the minimum value governed by the power-law process, and α is the magnitude of the corresponding tail exponent.[8] Taleb (2020) argues that the tail exponent α of a power-law function captures via extrapolation the low-probability deviation not seen in the data; it plays a disproportionately large role in determining the mean of the process. Next, it can be shown that the conditional expectation of the FX variance, defined in this context as $E[X|x > x_{MIN}]$, is given by:

$$E[X|x > x_{MIN}] = \int_{x_{MIN}}^{\infty} xp(x)dx = \frac{(\alpha-1)}{(\alpha-2)}x_{MIN} \tag{4}$$

The second moment, $E[X^2]$, or the variance of the FX variance, is defined as:

$$E\left[X^2|x > x_{MIN}\right] = \int_{x_{MIN}}^{\infty} x^2\, p(x)dx = \frac{(\alpha-1)}{(\alpha-3)}x_{MIN}^2. \tag{5}$$

Higher moments of order k are analogously defined as:

$$E\left[X^k|x > x_{MIN}\right] = \frac{(\alpha-1)}{(\alpha-1-k)}x_{MIN}^k. \tag{6}$$

From Eqs. (4) and (5), we see that the theoretical mean only exists for $\alpha > 2$, whereas the variance only exists for $\alpha > 3$.

4.1.2. Maximum likelihood estimation

Following White et al. (2008) and Clauset et al. (2009), who conclude that MLE is the most accurate approach for estimating power-law exponents, the tail exponents are estimated as:

$$\widehat{\alpha} = 1 + N\left(\sum_{i=1}^{N} ln\left(\frac{x_i}{x_{MIN}}\right)\right)^{-1}, \tag{7}$$

where $\widehat{\alpha}$ denotes the MLE estimator, N is the number of observations exceeding x_{MIN}, and other notations are as previously defined. Clauset et al. (2009) note that determining the corresponding values for α and x_{MIN} is essential for estimating the most appropriate power-law model.

[6] As pointed out in footnote 3 in Grobys' (2023a) study, the data matching procedure, due to data availability, results in a reduced sample size by 39 lost observations. To give an illustrative example, the data base provided by yahoo.com does not offer FX quotations for some exchange rates for two periods during the global financial crisis. For example, between August 1, 2008, and August 7, 2008, as well as August 11, 2008 and August 25, 2008, the data base provided by yahoo.com does not provide quotations for the EUR/USD exchange rates, whereas the AUD/USD exchange rates were quoted in that period. Hence, a total of 16 observations for the two periods were deleted for all other foreign exchange rates.

[7] For instance, recent studies from Grobys (2021 & 2023a) employ this variance estimator. As mentioned earlier, Chou et al. (2010) support the view that range-based variance estimators comprise more information than changes in closing prices. Moreover, Wang and Yang (2009) have observed that unlike GARCH models that cannot efficiently capture the dynamics of underlying variance due to their dependence on daily return series, models based on realized variances are better able to capture underlying variance and test relevant hypotheses. Since range-based variance estimators are according to Chou et al. (2010) superior to realized variances based on closing prices, these estimators are consequently superior to GARCH-type models, too.

[8] Following Clauset et al. (2009), to simplify notation, index i, which denotes the respective individual FX variance, is dropped.

Table 1
Descriptive statistics of daily range-based variances using the Parkinson-estimator

	AUD/USD	CAD/USD	CHF/USD	EUR/USD	GBP/USD	JPY/USD	NOK/USD	NZD/USD	SEK/USD
Mean	0.1416	0.0067	0.0100	0.2005	0.0076	0.0078	0.0163	0.0164	0.0137
Median	0.0062	0.0035	0.0040	0.0034	0.0040	0.0036	0.0081	0.0067	0.0069
Maximum	282.2602	0.3105	9.9818	856.7995	0.8985	0.4658	2.4978	14.3093	1.2027
Minimum	2.43E-06	1.26E-05	1.45E-07	9.30E-08	1.44E-05	1.14E-06	8.67E-05	3.35E-05	1.23E-04
Std. Dev.	6.0006	0.0122	0.1518	12.9020	0.0188	0.0178	0.0580	0.2168	0.0338
Skewness	46.9208	9.1306	64.4175	66.3852	26.9962	12.7607	27.9110	65.0185	19.4378
Kurtosis	2202.7180	153.1909	4228.8620	4407.9950	1170.3830	257.2392	1000.5560	4286.1210	567.6574
Jarque-Bera (JB)	8.91E+08	4.21E+06	3.28E+09	3.57E+09	2.51E+08	1.20E+07	1.83E+08	3.37E+09	5.89E+07
(*p*-value JB)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
Observations	4410	4410	4410	4410	4410	4410	4410	4410	4410

Publicly available intraday prices for the AUD/USD, CAD/USD, CHF/USD, EUR/USD, GBP/USD, JPY/USD, NOK/USD, NZD/USD, and SEK/USD exchange rates were downloaded from finance.yahoo.com. The sample period is from May, 16, 2006 to May, 31, 2023. Only the intersection of the data is employed; that is, only daily data are accounted for where all FX rates were quoted on the same day, leaving us with 4410 daily observations. To estimate annualized daily variances, the range-based variance estimator proposed by Parkinson (1980) is employed which is given by:

$$\sigma_{i,t}^2 = T\frac{1}{4ln(2)}\left(ln(H_{i,t}) - ln(L_{i,t})\right)^2,$$

where $H_{i,t}$ and $L_{i,t}$ denote the highest and lowest price for foreign exchange rate market i on trading day t, and $\sigma_{i,t}^2$ denotes foreign exchange rate market i's corresponding annualized realized variance where $T = 250$, as 250 trading days per annum are assumed. This table reports the descriptive statistics.

Table 2
Descriptive statistics of weekly range-based variances using the Parkinson-estimator

	AUD/USD	CAD/USD	CHF/USD	EUR/USD	GBP/USD	JPY/USD	NOK/USD	NZD/USD	SEK/USD
Mean	0.1472	0.0070	0.0104	0.2085	0.0079	0.0081	0.0169	0.0171	0.0143
Median	0.0076	0.0042	0.0050	0.0042	0.0051	0.0048	0.0101	0.0085	0.0084
Maximum	117.2350	0.1388	2.1138	178.2245	0.2088	0.2333	1.3372	2.9817	0.4681
Minimum	3.45E-04	6.08E-05	4.70E-05	2.74E-05	5.06E-05	1.83E-04	4.26E-04	1.25E-04	4.92E-04
Std. Dev.	3.9472	0.0100	0.0721	6.0009	0.0126	0.0121	0.0489	0.1020	0.0238
Skewness	29.6448	6.4587	28.2390	29.6479	8.1498	8.9752	22.8001	27.9412	10.1194
Kurtosis	879.8778	64.0030	822.8317	879.9983	101.7616	144.9404	607.3170	811.3517	164.2234
Jarque-Bera (JB)	2.84E+07	1.43E+05	2.48E+07	2.84E+07	3.68E+05	7.52E+05	1.35E+07	2.41E+07	9.70E+05
(*p*-value JB)	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Observations	882	882	882	882	882	882	882	882	882

Publicly available intraday prices for the AUD/USD, CAD/USD, CHF/USD, EUR/USD, GBP/USD, JPY/USD, NOK/USD, NZD/USD, and SEK/USD exchange rates were downloaded from finance.yahoo.com. The sample period is from May, 16, 2006 to May, 31, 2023. Only the intersection of the data is employed; that is, only daily data are accounted for where all FX rates were quoted on the same day, leaving us with 4410 daily observations. To estimate annualized weekly variances, the range-based variance estimator proposed by Parkinson (1980) is employed which is given by:

$$\sigma_{i,w}^2 = W\frac{1}{4ln(2)}\sum_{t\in w}\left(ln(H_{i,t}) - ln(L_{i,t})\right)^2,$$

where $H_{i,t}$ and $L_{i,t}$ denote the highest and lowest prices for foreign exchange rate market i on corresponding trading days $t \in w$ where w denotes the time index for weekly data, and $\sigma_{i,w}^2$ denotes, accordingly, foreign exchange rate market i's annualized weekly variance where $W = 52$, as 52 trading weeks per annum are assumed. This table reports the descriptive statistics.

Clauset et al. (2009) show that the corresponding standard deviation of the estimated power-law exponent is derived as:

$$\widehat{\sigma} = \frac{\widehat{\alpha} - 1}{\sqrt{N}} + O\left(\frac{1}{N}\right). \quad (8)$$

From Eq. (7), it becomes evident that the MLE estimator depends on the chosen x_{MIN}, and consequently, there are different possible MLE estimators from which to select one. The question that arises is the following: Which is the most appropriate candidate for x_{MIN}?

Clauset et al. (2009) argue that it is a common practice to employ the $\widehat{\alpha}/x_{MIN}$ plot and select the value for x_{MIN} beyond which $\widehat{\alpha}$ is stable. Because this procedure is somewhat subjective and can be sensitive to noise or fluctuation in the tail of the distribution, the authors propose to select $\widehat{\alpha}$ based on the optimal Kolmogorov-Smirnov (KS) distance D, which measures the maximum distance between the cumulative density functions (CDFs) of the data and the fitted power-law model as defined by:

$$D = MAX_{x \geq x_{MIN}}|S(x) - P(x)|, \quad (9)$$

where $S(x)$ is the CDF of the data for the observation with a value of at least x_{MIN}, and $P(x)$ is the CDF for the power-law model that best fits the data in the region $x \geq x_{MIN}$. Estimate $\widehat{x}_{MIN}$ is then the value of x_{MIN} that minimizes D. While Clauset et al. (2009) provide strong evidence that their proposed technique to estimate α outperforms traditional log-log regression approaches by a substantial margin, their approach is only applicable to a single data series.

4.1.3. Blocks bootstraps for estimating the covariance matrix for power-law exponents

So far, the methodologies proposed in the literature to estimate power-law exponents were designed for analyzing a single data series or a seemingly unrelated data series. The statistical technique proposed by

Clauset et al. (2009), however, does not address the following issues. First, while the point estimates are unbiased in large samples, the estimated uncertainties are biased (e.g., underestimated) due to the unknown dependency structures in the data-generating processes. Second, if joint power-law behavior is subject of investigation—which is the main objective of this study—power-law exponents are presumably correlated. Hence, a new approach is needed to address these issues.

To address this issue, a blocks bootstrap procedure, as proposed in Grobys and Junttila (2021), is implemented. Denoting the selected block length as m, a blocks bootstrap procedure is chosen such that $E[m] = \sqrt{T}$. Then the Tx1 data vectors of FX variances $i = 1,\dots,N$, denoted as $\boldsymbol{x}_i$, are stacked into matrix $\boldsymbol{Y}$:

$$\boldsymbol{Y} = [\boldsymbol{x}_1, \boldsymbol{x}_2, \dots, \boldsymbol{x}_N].$$

The blocks of the dimension mxK are randomly drawn from matrix $\boldsymbol{Y}$ with respect to the time dimension $t = 1, \dots, T$. These blocks are governed by a geometric distribution—that is, $m \sim GEO(p)$ with $E[m] = \frac{(1-p)}{p}$. It is noteworthy that this procedure ensures stationarity, as detailed in Godfrey (2009). In the present research, we use $E[m] = 66$, $p = 0.0149$ for the daily data as $\sqrt{T} \approx 66$. Using this procedure, the blocks drawn from $\boldsymbol{Y}$ vary in lengths. The randomly drawn blocks, m, which have dimensions mxK from data matrix $\boldsymbol{Y}$, are stacked in matrix $\boldsymbol{Y}_b$ as:

$$\boldsymbol{Y}_b = \begin{bmatrix} m_1 \\ m_2 \\ m_3 \\ \vdots \end{bmatrix}.$$

The procedure is stopped when the length of the artificial matrix $\boldsymbol{Y}_b$ exhibits a length exceeding T. Observations exceeding T are cut off—that is, every artificial data matrix $\boldsymbol{Y}_b$ has the same length as the original data matrix $\boldsymbol{Y}$. This process corresponds to one iteration b of the blocks bootstrap procedure. Using this blocks bootstrap procedure, for each iteration b, the Tx1 vectors, $\boldsymbol{x}_{b,1}$, $\boldsymbol{x}_{b,2}$, …, $\boldsymbol{x}_{b,N}$ are extracted from matrix $\boldsymbol{Y}_b$, and the MLE estimators are estimated using the procedure described in section 4.1.2., giving us:

$$\begin{bmatrix} \widehat{\alpha}_{b,1} & \widehat{\alpha}_{b,2} & \cdots & \widehat{\alpha}_{b,N} \end{bmatrix}.$$

This blocks bootstrap procedure is performed for $b = 1, \dots, 1000$ iterations and point estimates for α.

are stacked in BxN matrix $\widehat{\boldsymbol{\alpha}}_{BOOT}$:

$$\widehat{\boldsymbol{\alpha}}_{BOOT} = \begin{pmatrix} \widehat{\alpha}_{1,1} & \widehat{\alpha}_{1,2} & \dots & \dots & \widehat{\alpha}_{1,N} \\ \widehat{\alpha}_{2,1} & \widehat{\alpha}_{1,2} & \dots & \dots & \widehat{\alpha}_{2,N} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \widehat{\alpha}_{B,1} & \widehat{\alpha}_{B,2} & \dots & \dots & \widehat{\alpha}_{B,N} \end{pmatrix}.$$

The corresponding bootstrapped standard errors $\widehat{\sigma}_{BOOT,i}$ are then given by:

$$\widehat{\sigma}_{BOOT,i} = \sqrt{\frac{1}{B}\sum_{b=1}^{B}\left(\widehat{\alpha}_{b,i} - \overline{\widehat{\alpha}}_{b,i}\right)^2}.$$

According to Grobys and Junttila (2021), this approach is robust to unknown dependency structures in the data, which are commonly observed for financial assets. Since this blocks bootstrap approach retains co-dependencies across the data, it enables us to compute the covariances between $\widehat{\alpha}_i$ and $\widehat{\alpha}_j$ for $i,j = 1, \dots, N$ using the matrix $\widehat{\boldsymbol{\alpha}}_{BOOT}$:

$\widehat{\sigma}_{BOOT,i,j} = \frac{1}{B}\sum_{b=1}^{B}\left(\widehat{\alpha}_{b,i} - \overline{\widehat{\alpha}}_{b,i}\right)\left(\widehat{\alpha}_{b,j} - \overline{\widehat{\alpha}}_{b,j}\right)$ with $i \neq j$.

If covariances between power-law exponents are significant, the covariance matrix needs to be accounted for when implementing statistical tests for detecting potential commonalties across the FX variances.

4.1.4. Is there a common component governing power-law behavior of FX variances?

To explore whether a common component governing power-law behavior of FX variances exists, the following test statistic is proposed:

$$\widehat{\lambda} = (\widehat{\boldsymbol{\alpha}} - q\mathbf{1})'\widehat{\boldsymbol{\Sigma}}^{-1}(\widehat{\boldsymbol{\alpha}} - q\mathbf{1}), \tag{10}$$

where the covariance matrix $\widehat{\boldsymbol{\Sigma}} = COV(\widehat{\boldsymbol{\alpha}}_{BOOT})$ has the dimension NxN, $\widehat{\boldsymbol{\alpha}}$ is a Nx1 vector of the estimated power-law exponents, $\mathbf{1}$ is a Nx1 vector consisting of ones, and q is the hypothesized common power-law exponent. The estimated test statistic denoted as $\widehat{\lambda}$ is under the null hypothesis distributed as $\chi^2(N)$. The test statistic is iteratively estimated covering the interval $q = (2.1, 2.2, \dots, 3.1)$. This is an economically important interval of power-law exponents. As an example, acceptance of the null hypothesis $\boldsymbol{\alpha}' = (3.1, 3.1, \dots, 3.1)'$ would indicate that FX variances are governed by a common power-law exponent. Moreover, we know from Eq. (5) that the variance of FX variances would exist if the null hypothesis was accepted for $\alpha = 3.1 \; \forall \; i = 1, \dots, N$, allowing us to employ standard statistical techniques for modeling the FX data. Yet rejecting the null hypothesis $\boldsymbol{\alpha}' = q\mathbf{1} \;\; \forall \;\; q = (2.1, 2.2, \dots, 3.1)$ would indicate that the FX variances exhibit heterogenous sources of risk manifested in FX-specific power-law behavior as opposed to common power-law behavior.

4.1.5. Is the power-law behavior of FX variances invariant across time?

The fractal behavior of financial assets is manifested in invariant power-law behavior across time frequencies. As pointed out from Mandelbrot (2008), "Statistically speaking, the risks of a day are much like those of a week, a month, or a year. But the price variations scale with time" (p. 239). Hence, the fractal behavior of FX variance is tested using the following approach. Let us define K as the dimension of time frequencies and N denote the number of FX variance vectors, each having the dimension of Tx1. Let the vector $\widehat{\boldsymbol{\alpha}}$ be defined as $\widehat{\boldsymbol{\alpha}} = (\widehat{\boldsymbol{\alpha}}_1, \widehat{\boldsymbol{\alpha}}_2, \dots, \widehat{\boldsymbol{\alpha}}_K)'$ where $\widehat{\boldsymbol{\alpha}}_j$ are Nx1 vectors of power-law exponents for the given frequency j and $j = 1, \dots, K$. Moreover, $\boldsymbol{q}$ is a NKx1 vector consisting of the corresponding common exponent, which is the subject to be tested, and $\widehat{\boldsymbol{\Sigma}}_{\widehat{\alpha}}$ is the estimated NKxNK covariance matrix of $\widehat{\boldsymbol{\alpha}} = (\widehat{\boldsymbol{\alpha}}_1, \widehat{\boldsymbol{\alpha}}_2, \dots, \widehat{\boldsymbol{\alpha}}_K)'$ obtained via blocks bootstrap and defined as:

$$\widehat{\boldsymbol{\Sigma}}_{\widehat{\alpha}} = \begin{pmatrix} \widehat{\boldsymbol{\Sigma}}_{\widehat{\alpha}_1} & \mathbf{0} & \dots & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \widehat{\boldsymbol{\Sigma}}_{\widehat{\alpha}_2} & \dots & \mathbf{0} & \mathbf{0} \\ \vdots & \vdots & & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \dots & \vdots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \dots & \mathbf{0} & \widehat{\boldsymbol{\Sigma}}_{\widehat{\alpha}_K} \end{pmatrix},$$

where $\widehat{\boldsymbol{\Sigma}}_{\widehat{\alpha}_j}$ are NxN covariance matrices for time frequency j, and $\mathbf{0}$ defines NxN matrices consisting of zeros. Thus, the following test statistic is proposed:

$$\widehat{\lambda} = (\widehat{\boldsymbol{\alpha}} - \boldsymbol{q})'\widehat{\boldsymbol{\Sigma}}_{\widehat{\alpha}}^{-1}(\widehat{\boldsymbol{\alpha}} - \boldsymbol{q}), \tag{11}$$

with $\boldsymbol{q} = q\mathbf{1}$, where $\mathbf{1}$ is the NKx1 vector of ones and q is the hypothesized common and time-frequency invariant power-law exponent. The estimated test statistic denoted as $\widehat{\lambda}$ is under the null hypothesis distributed as $\chi^2(NK)$. Again, the test statistic is iteratively estimated and covers the interval $q = (2.1, 2.2, \dots, 3.1)$. Accepting some of the null hypothesis would indicate that FX variances are governed by a common power-law exponent, which is, moreover, time-frequency invariant.

4.1.6. A test for total invariance

Mandelbrot (2008) criticizes the problem of sample-specific parameter estimates obtained from GARCH-type models and argues that "… many recent models of price variation try to explain the obviously shifting pattern of volatility by inserting parameters that change by the day, hour, and second; such are the GARCH family mentioned earlier" (p. 242). This begs the question of whether power-law behavior changes across time. Invariance should be manifested not only in the same power-law behavior across various time frequencies but also in the same power-law behavior over time, given each frequency. Hence, to test this issue, this study proposes a test that it terms the *test for total invariance*, which can be derived from the test described in section 4.1.5 as follows. Let us for simplicity only consider $K = 2$ time frequencies and $S = 2$ subsamples. We can then use the test described in section 4.1.5 to test jointly whether the power-law exponents are invariant across time and across time frequencies using:

$$\widehat{\mathbf{\Sigma}}_{\widehat{\alpha}^*} = \begin{pmatrix} \widehat{\mathbf{\Sigma}}_{\widehat{\alpha}_{1,1}} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \widehat{\mathbf{\Sigma}}_{\widehat{\alpha}_{1,2}} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \widehat{\mathbf{\Sigma}}_{\widehat{\alpha}_{2,1}} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \widehat{\mathbf{\Sigma}}_{\widehat{\alpha}_{2,2}} \end{pmatrix},$$

where $\widehat{\mathbf{\Sigma}}_{\widehat{\alpha}_{1,1}}$ and $\widehat{\mathbf{\Sigma}}_{\widehat{\alpha}_{1,2}}$ are the NxN covariance matrices for first and second non-overlapping subsample for daily data, whereas $\widehat{\mathbf{\Sigma}}_{\widehat{\alpha}_{2,1}}$ and $\widehat{\mathbf{\Sigma}}_{\widehat{\alpha}_{2,2}}$ are the NxN covariance matrices for the first and second non-overlapping subsample for weekly data. Again, $\mathbf{0}$ defines NxN matrices consisting of zeros. Thus, the following test statistic is proposed:

$$\widehat{\lambda} = (\widehat{\boldsymbol{\alpha}} - \boldsymbol{q})' \widehat{\mathbf{\Sigma}}_{\widehat{\alpha}^*}^{-1} (\widehat{\boldsymbol{\alpha}} - \boldsymbol{q}), \tag{12}$$

where $\widehat{\boldsymbol{\alpha}} = (\widehat{\boldsymbol{\alpha}}_{1,1}, \widehat{\boldsymbol{\alpha}}_{1,2}, \widehat{\boldsymbol{\alpha}}_{2,1}, \widehat{\boldsymbol{\alpha}}_{2,2})'$ and $\widehat{\boldsymbol{\alpha}}_{j,s}$ are Nx1 vectors for frequency j and subsample s, where $j = 1, \ldots, K$, and $s = 1, \ldots, S$. Note that here we consider only $K = S = 2$. Moreover, $\boldsymbol{q} = \mathbf{1}q$, where $\mathbf{1}$ is here a NSKx1 vector of ones, whereas q is again the hypothesized common power-law exponent, which is under the null hypothesis invariant across both markets, time, and time frequency. The estimated test statistic denoted as $\widehat{\lambda}$ is under the null hypothesis distributed as $\chi^2(NSK)$. Again, the test statistic is iteratively estimated and covers the interval $q = (2.1, 2.2, \ldots, 3.1)$. Accepting some of the null hypothesis would indicate that FX variances are governed by a common power-law exponent, which is invariant across both subsamples and time frequencies.

4.2. Robustness checks

4.2.1. Testing the informativeness of the Parkinson-estimator

Some readers might believe that the Parkinson-estimator could produce inflated variances. However, it is noteworthy that different views have been discussed in the relevant literature. For instance, Shu and Zhang (2006) investigate the relative performance of four range-based volatility estimators, including Parkinson, Garman-Klass, Rogers-Satchell, and Yang-Zhang estimators for the S&P 500 index data. The authors document that all those price range estimators perform very well. However, Molnár (2012) concludes that when the noise of range-based volatility estimators is accounted for, the best estimator is the Garman and Klass (1980) estimator. Hence, the first question that needs to be clarified is as follows: Does the Parkinson-estimator produce statistically the same information as other range-based estimators? To investigate whether the potential power-law behavior of FX variances is an artefact manifested in the possibly larger variance produced from the Parkinson estimator, the daily FX variances based on the Parkinson-estimator are compared with those obtained from using the Garman-Klass-estimator. Specifically, in line with Garman and Klass (1980), daily FX variances are computed as:

$$\sigma_{i,t}^2 = T\left(0.5\left[ln\left(\frac{H_{i,t}}{L_{i,t}}\right)\right]^2 - [2ln(2) - 1]\left[ln\left(\frac{C_{i,t}}{O_{i,t}}\right)\right]^2\right), \tag{13}$$

where $H_{i,t}$, $L_{i,t}$, $C_{i,t}$ and $O_{i,t}$ denote the highest, lowest, closing, and opening price for the FX market i on day t, and $\sigma_{i,t}^2$ denotes FX i's annualized daily variance, and $T = 250$. The power-law exponents and blocks bootstrapped covariance matrix are computed as detailed in sections 4.1.2 and 4.1.3. Next, we examine whether the point estimates for the power-law exponents obtained from FX variances based on the Parkinson-estimator are statistically the same as those obtained from employing the Garman-Klass-estimator. To test this issue, the following test statistic is proposed:

$$\widehat{\lambda} = (\widehat{\boldsymbol{\alpha}}_P - \widehat{\boldsymbol{\alpha}}_{GK})' \left(0.5\widehat{\mathbf{\Sigma}}_{\widehat{\alpha}_P} + 0.5\widehat{\mathbf{\Sigma}}_{\widehat{\alpha}_{GK}}\right)^{-1} (\widehat{\boldsymbol{\alpha}}_P - \widehat{\boldsymbol{\alpha}}_{GK}) \tag{14}$$

where $\widehat{\boldsymbol{\alpha}}_P$ is the Nx1 vector of point estimates for power-law exponents estimated via Clauset et al.'s (2009) MLE approach with FX variance data computed via the Parkinson-estimator, $\widehat{\boldsymbol{\alpha}}_{GK}$ is the Nx1 vector of point estimates for the power-law exponents estimated via Clauset et al.'s (2009) MLE approach with FX variance data computed via the Garman-Klass-estimator, $\widehat{\mathbf{\Sigma}}_{\widehat{\alpha}_P}$ and $\widehat{\mathbf{\Sigma}}_{\widehat{\alpha}_{GK}}$ are the corresponding covariance matrices obtained via blocks bootstrap.[9] The estimated test statistic is under the null hypothesis distributed as $\chi^2(N)$. Rejecting the null hypothesis would imply that the Parkinson-estimator and Garman-Klass-estimator produce estimator-specific power-law behavior.

Finally, the estimated standard deviations produced by employing the Parkinson-estimator, denoted as $\widehat{\sigma}_{i,P}$, are retrieved from the main diagonal of the estimated covariance matrix and regressed on the corresponding estimates produced by using the Garman-Klass-estimator, denoted as $\widehat{\sigma}_{i,GK}$. In doing so, the following regression equation is run:

$$\widehat{\sigma}_{i,P} = c + b\widehat{\sigma}_{i,GK} + \epsilon_i, \tag{15}$$

where ϵ_i is assumed to be an i.i.d. error term, and c and b are parameters to be estimated. Possible manifestations of inflated standard deviations produced by using the Parkinson-estimator are either $c > 0$ or $b > 0$.

4.2.2. Is the power-law hypothesis reasonable?

Despite the strong evidence of extremely fat tails, some readers could still wonder whether the power-law hypothesis is plausible. In this regard, it is important to note that earlier literature documented that realized asset volatility is close to a log-normal distribution (e.g., Andersen, Bollerslev, Diebold, & Ebens, 2001; Andersen, Bollerslev, Diebold, & Labys, 2001a & Andersen, Bollerslev, Diebold, & Labys, 2001b). Obviously, there are different ways to address this issue. Taleb (2020) argues that a typical manifestation of financial market data is that the probability of an event staying within one standard deviation of the mean is between 75 and 95%, whereas the corresponding probability derived from the normal distribution is 68% only. Because one-sigma events play an important role for financial market data, the following one-sigma test was employed to test whether the distribution governing FX variances is either log-normal or $\chi^2(1)$.[10] Hence, the following test statistic is proposed:

[9] The idea for the test-statistic rests on the two-sample-test statistic defined as $\widehat{z} = \frac{(\bar{y}_1 - \bar{y}_2)}{\sqrt{0.5\widehat{\sigma}_{y_1}^2 + 0.5\widehat{\sigma}_{y_2}^2)}}$, where y_1 and y_2 are random variables from the two samples, $\widehat{\sigma}_{y_1}^2$ and $\widehat{\sigma}_{y_2}^2$ are the estimated sample variances, and $\bar{y}_1$ and $\bar{y}_2$ are the corresponding sample averages.

[10] Note that if the traditional assumption often applied to finance theory holds—that is, financial returns are normally distributed—a standardized financial return would be, consequently, distributed as $N(0,1)$, whereas the corresponding variance process of such a standardized financial return should be distributed as $\chi^2(1)$ (see Grobys, 2021).

$$\lambda = \frac{\left(x_{\leq \pm 1\sigma} - m_{\leq \pm 1\sigma}\right)^2}{m_{\leq \pm 1\sigma}} + \frac{\left(x_{> \pm 1\sigma} - m_{> \pm 1\sigma}\right)^2}{m_{> \pm 1\sigma}}, \tag{16}$$

where $m_{\leq \pm 1\sigma}$ denotes the expected number of observations occurring within one standard deviation from the mean of some specified distribution, which can be either log-normal or $\chi^2(1)$, $m_{> \pm 1\sigma}$ denotes the expected number of observations exceeding one standard deviation from the mean, whereas $x_{\leq \pm 1\sigma}$ and $x_{> \pm 1\sigma}$ are the corresponding values for the observed distribution of some FX variance. In an early study, Pearson (1900) shows that this type of test statistic is distributed as $\chi^2(1)$ as $T \rightarrow \infty$. The one-sigma test is implemented using both distributions successively under the null hypothesis.

Since the one-sigma test requires that the distributions are standardized, which in turn requires that the theoretical mean and variance of the underlying distributions are defined, power-law models are tested using the goodness-of-fit (GoF) test proposed by Clauset et al. (2009) because $\alpha < 3$ suggests that the variance is undefined. This test can be summarized as follows. First, recall that the Kolmogorov-Smirnov (KS) distance is the maximum distance between the cumulative density functions (CDFs) of the data and the fitted power-law model, as defined by Eq. (9),

$$D = MAX_{x \geq x_{MIN}} |S(x) - P(x)|,$$

where $S(x)$ is the CDF of the data for the observation with a value of at least x_{MIN}, and $P(x)$ is the CDF for the power-law model that best fits the data in the region $x \geq x_{MIN}$. The estimate $\hat{x}_{MIN}$ is then the value of x_{MIN} that minimizes D. Using the parameter vector $(\hat{\alpha}, \hat{x}_{MIN})$ that optimizes D, Clauset et al.'s (2009) GoF test generates a p-value that quantifies the plausibility of the power-law null hypothesis. Specifically, this test compares D with the distance measurements for comparable synthetic data sets drawn from the hypothesized power-law model. The p-value is then defined as the fraction of synthetic distances that is larger than the empirical distance. Employing a standard significance level of 5%, the power-law null hypothesis is not rejected for p-values exceeding 5% because the difference between the empirical data and the model can be attributed to statistical fluctuations alone. The implementation of this test is detailed by Clauset et al. (2009, pp., 675–678). In the present study, power-law model parametrizations based on the original data sets are used as hypothesized power-law models when implementing the GoF tests.

4.2.3. Is the chosen block length for the implemented blocks bootstrap plausible?

Using a random block length governed by a geometric distribution, this study employs an expected block length of $\sqrt{T}$. Godfrey (2009) notes that the optimal block length depends upon the context in which the blocks bootstrap is being used and argues that block lengths of $T^{1/3}$ are often used for variance or bias estimation. However, the author documents that if the chosen block length is long enough, the dependency structure of the true data-generating process will be accurately reflected by the bootstrap samples. Since power-law behavior is manifested in a non-linear data-generation, the current research uses a longer block length of $T^{1/2}$ as opposed to $T^{1/3}$. However, to explore whether block lengths $T^{1/2}$ and $T^{1/3}$ produce estimates for power-law exponents that are statistically the same, the following test statistic is proposed:

$$\hat{\lambda} = \left(\hat{\boldsymbol{\alpha}}_{T^{1/2}} - \hat{\boldsymbol{\alpha}}_{T^{1/3}}\right)' \left(0.5\hat{\boldsymbol{\Sigma}}_{\hat{\alpha}_{T^{1/2}}} + 0.5\hat{\boldsymbol{\Sigma}}_{\hat{\alpha}_{T^{1/3}}}\right)^{-1} \left(\hat{\boldsymbol{\alpha}}_{T^{1/2}} - \hat{\boldsymbol{\alpha}}_{T^{1/3}}\right), \tag{17}$$

where $\hat{\boldsymbol{\alpha}}_{T^{1/2}} = \left(\bar{\hat{\alpha}}_{1,T^{1/2}}, \bar{\hat{\alpha}}_{2,T^{1/2}}, \ldots, \bar{\hat{\alpha}}_{N,T^{1/2}}\right)'$ is a Nx1 vector of point estimates of power-law exponents using daily FX variance data and an expected block length of $T^{1/2}$, whereas $\hat{\boldsymbol{\alpha}}_{T^{1/3}} = \left(\bar{\hat{\alpha}}_{1,T^{1/3}}, \bar{\hat{\alpha}}_{2,T^{1/3}}, \ldots, \bar{\hat{\alpha}}_{N,T^{1/3}}\right)'$ is a Nx1 vector of point estimates of power-law exponents using daily FX variance data and an expected block length of $T^{1/3}$.[11] For both blocks bootstrap procedures, the Parkinson-estimator is used for computing the FX variances. Moreover, $\hat{\boldsymbol{\Sigma}}_{\hat{\alpha}_{T^{1/2}}}$ and $\hat{\boldsymbol{\Sigma}}_{\hat{\alpha}_{T^{1/3}}}$ are the covariance matrices obtained via blocks bootstrap. The estimated test statistic is under the null hypothesis distributed as $\chi^2(N)$. Rejecting the null hypothesis would imply that ascertained power-law behavior depends on the chosen block length.

4.2.4. Testing the reliability of data provided from yahoo.com

Despite the fact that yahoo.com has been used in numerous studies as a reliable source for obtaining data, some readers might wonder if the main results are an artefact of measurement errors in the data. For instance, one could believe that, at times, daily low prices could be mistakenly quoted as the inverse of daily high prices which, in turn, might result in "extreme events" that never occurred. Therefore, to explore the reliability of the data provided from yahoo.com, we obtain data on daily FX rate quotations from another popular data provider, that is, investing.com. To investigate whether the potential power-law behavior of FX variances is an artefact manifested in possibly inflated variances produced from potential measurement errors, the daily FX variances based on the Parkinson-estimator derived from the database provided from yahoo.com are compared with those obtained from using the Parkinson-estimator derived from the database provided from investing.com. First, power-law exponents are estimated using the estimation approach as detailed in section 4.1.2. Next, the covariance matrix of power-law exponents is computed using blocks bootstraps as detailed in sections 4.1.2 and 4.1.3. To examine whether the point estimates for the power-law exponent are subject to database specificity, the following test statistic is proposed:

$$\hat{\lambda} = \left(\hat{\boldsymbol{\alpha}}_{yahoo} - \hat{\boldsymbol{\alpha}}_{investing}\right)' \left(0.5\hat{\boldsymbol{\Sigma}}_{\hat{\alpha}_{yahoo}} + 0.5\hat{\boldsymbol{\Sigma}}_{\hat{\alpha}_{investing}}\right)^{-1} \left(\hat{\boldsymbol{\alpha}}_{yahoo} - \hat{\boldsymbol{\alpha}}_{investing}\right) \tag{18}$$

where $\hat{\boldsymbol{\alpha}}_{yahoo}$ is the Nx1 vector of point estimates for power-law exponents estimated via Clauset et al.'s (2009) MLE approach with FX variance data computed via the Parkinson-estimator derived from the database provided from yahoo.com, $\hat{\boldsymbol{\alpha}}_{investing}$ is the Nx1 vector of point estimates for power-law exponents estimated via Clauset et al.'s (2009) MLE approach with FX variance data computed via the Parkinson-estimator derived from the database provided from investing.com, and $\hat{\boldsymbol{\Sigma}}_{\hat{\alpha}_{yahoo}}$ and $\hat{\boldsymbol{\Sigma}}_{\hat{\alpha}_{investing}}$ are the corresponding covariance matrices obtained via blocks bootstrap, as outlined in section 4.1.3. The estimated test statistic is under the null hypothesis distributed as $\chi^2(N)$. Rejecting the null hypothesis would imply that ascertained power-law behavior of FX variances is an artefact originating from measurement errors in the database provided from yahoo.com.

4.2.5. Evidence from principal component analysis

An often-used methodology to explore commonalities among financial data sets is principal component analysis (PCA). Common power-law behavior should be manifested in one dominant eigenvalue in the covariance matrix of $\hat{\boldsymbol{\alpha}}_{BOOT}$. Following common procedure, we standardize the Bx1 data vectors, $\hat{\boldsymbol{\alpha}}_{1,BOOT}$, $\hat{\boldsymbol{\alpha}}_{2,BOOT}$, …, $\hat{\boldsymbol{\alpha}}_{9,BOOT}$, such that $E\left(\hat{\boldsymbol{\alpha}}^S_{1,BOOT}\right) = E\left(\hat{\boldsymbol{\alpha}}^S_{2,BOOT}\right) = \ldots = E\left(\hat{\boldsymbol{\alpha}}^S_{9,BOOT}\right) = 0$ and $VAR\left(\hat{\boldsymbol{\alpha}}^S_{1,BOOT}\right) = VAR\left(\hat{\boldsymbol{\alpha}}^S_{2,BOOT}\right) = \ldots = VAR\left(\hat{\boldsymbol{\alpha}}^S_{9,BOOT}\right) = 1$. Using the eigenvectors, PCA transforms $\hat{\boldsymbol{\alpha}}^S_{1,BOOT}$, $\hat{\boldsymbol{\alpha}}^S_{2,BOOT}$, …, $\hat{\boldsymbol{\alpha}}^S_{9,BOOT}$ into nine orthogonal Bx1 vectors $s_1, s_2, \ldots, s_9$ as follows:

[11] For each FX rate variance i, $\bar{\hat{\alpha}}_i$ is the sample average of blocks bootstrapped power-law exponents for a given block length. For instance, from Table A.9 we observe that $\bar{\hat{\alpha}}_{1,T^{1/3}} = \bar{\hat{\alpha}}_{AUD/USD,T^{1/3}} = 2.3244$.

$$\delta_{1,1}\boldsymbol{s}_1 + \delta_{1,2}\boldsymbol{s}_2 + \ldots\delta_{1,9}\boldsymbol{s}_9 = \widehat{\boldsymbol{\alpha}}^S_{1,BOOT}$$
$$\delta_{2,1}\boldsymbol{s}_1 + \delta_{2,2}\boldsymbol{s}_2 + \ldots\delta_{2,9}\boldsymbol{s}_9 = \widehat{\boldsymbol{\alpha}}^S_{2,BOOT}$$
$$\cdots$$
$$\delta_{9,1}\boldsymbol{s}_1 + \delta_{9,2}\boldsymbol{s}_2 + \ldots\delta_{9,9}\boldsymbol{s}_9 = \widehat{\boldsymbol{\alpha}}^S_{9,BOOT}.$$

In our research context, we would expect that there is one dominant eigenvalue associated with $\boldsymbol{s}_1$ that explains the majority of variations in $\widehat{\boldsymbol{\alpha}}^S_{1,BOOT}$, $\widehat{\boldsymbol{\alpha}}^S_{2,BOOT}$, …, $\widehat{\boldsymbol{\alpha}}^S_{9,BOOT}$. We implement PCA for the matrix $\widehat{\boldsymbol{\alpha}}_{BOOT}$ based on (i) daily data, (ii) weekly data, and both (iii) daily and weekly data.

4.2.6. *Model validation: A scientific replication*

Finally, one could wonder whether our results still hold over expanded samples or model modifications. In this regard, Hou, Xue, and Zhang (2020) call for scientific replications of reported research. In their study, the authors refer to Hamermesh (2007), who distinguishes three categories of replication:

Pure replication (reproduction) is redoing a prior study in exactly the same way. Statistical replication is the same empirical model but different sample from the same underlying population. Scientific replication is different sample, different population, and similar, but not identical, statistical model. Hamermesh (2007, p. 716) argues that scientific replication "appears much more suited in type to our methods of research and, indeed, comprises most of what economists view as replication." The crux is that unlike natural sciences, economics, finance, and accounting are mostly observational in nature. As such, it is critical to evaluate the reliability of published results against "similar, but not identical," specifications. (Hou et al., 2020, p. 2021f)

Hence, to perform a scientific replication of our main results, we download daily price data on foreign exchange rates from investing.com covering an expanded sample from January, 3, 2001 to May, 31, 2023. Again, we employ only the intersection of the data, that is, only daily data are accounted for where all FX rates were quoted on the same day, leaving us with 5845 daily price observations. We then estimate weekly realized FX variances, using the following (modified) realized variance estimator ($\sigma^{*2}_{i,j}$)

$$\sigma^{*2}_{i,j} = \sum_{t\in j}\left(\frac{100\left(P_{i,t} - P_{i,t-1}\right)}{P_{i,t-1}}\right)^2,$$

where $P_{i,t}$ denote the price for foreign exchange rate market i on corresponding trading day t. It is assumed that every week exhibits five trading days; hence, the weekly realized FX variance corresponds to the sum of five consecutive and non-overlapping squared daily returns where $t \in j$ indicates the relevant trading days of respective week j. Further, $\sigma^{*2}_{i,j}$ denotes, accordingly, foreign exchange rate market i's weekly realized variance. Using the modified estimator for weekly realized FX variances, we perform the analyses as detailed in sections 4.1.2–4.1.4. If our results are robust we will expect that the outcome of our main analysis remains unchanged.

5. Results

5.1. *Main results*

Table 3 reports the results from estimating the power-law exponents for daily variances based on the Parkinson-estimator using the MLE approach proposed by Clauset et al. (2009). The sample period is from May 16, 2006, to May 31, 2023, comprising 4410 daily or 882 weekly observations. We observe from Table 3 that the estimated power-law exponents vary between $\widehat{\alpha} = 2.28$ for the AUD/USD variance and $\widehat{\alpha} = 2.91$ for the SEK/USD variance. The fraction of observations governed by power-law processes varies between 4.60% for the SEK/USD variance and 33.06% for the AUD/USD variance. Table 4 reports the corresponding estimates for the weekly FX variance data. The estimated power-law exponents were again estimated using the MLE approach proposed by Clauset et al. (2009). Based on Table 4, it becomes evident that even for the weekly data, the estimated power-law exponents vary between $\widehat{\alpha} = 2.35$ for the AUD/USD variance and $\widehat{\alpha} = 2.82$ for the CHF/USD variance. An interesting issue is that the fraction of observations governed by a power-law process is considerably larger for the weekly data as it is for the daily data, except for the NOK/USD variance. As an example, based on the daily data, 4.60% of the observations of SEK/USD variance data is governed by a power law, whereas this figure increases to 44.56% for the weekly data. Moreover, the corresponding estimated exponent is in its economic magnitude smaller for the weekly data as opposed to the daily data, suggesting that extreme events expected for weekly data are more severe than for daily data, which is a counterintuitive finding because it is contrary to the effect of time aggregation.[12] Overall, based on the single equation methodology, as proposed by Clauset et al. (2009), an important commonality between the results presented in Tables 3 and 4 is that it holds that $2 < \widehat{\alpha}_i < 3$ for all $i = 1, \ldots, 9$. From Eqs. (4) and (5), we know that $2 < \alpha < 3$ implies prima facie that the first moment (e.g., the theoretical mean of FX variances) is defined, whereas the second moment (e.g., the variance of FX variances) is not defined.

Since the estimation procedure proposed by Clauset et al. (2009) does first not account for dependency structures in the data and is not designed for joint hypothesis tests incorporating data of multiple time series, the covariance matrix of the estimated power-law exponents was estimated via blocks bootstrap, which was implemented using randomly selected block lengths following geometric distributions. The expected block length used in the main analysis is $E[m] = \sqrt{T}$, implying $E[m] = 66$ for the daily data and $E[m] = 30$ for the weekly data. For both frequencies (i.e., daily and weekly), $B = 1000$ bootstrap samples were generated, and the estimated power-law exponents were stored in BxN matrix $\widehat{\boldsymbol{\alpha}}_{BOOT}$. Tables 5 and 6 report the estimated covariances matrices $COV(\widehat{\boldsymbol{\alpha}}_{BOOT})$ for daily and weekly data using the FX variances derived from the Parkinson-estimator. Note that the square root of the elements on the main diagonal is the robust standard deviations. It becomes evident that the robust standard deviations are considerably larger than the ones derived from Clauset et al.'s (2009) i.i.d. assumption. For example, Table 3 shows that the estimated standard deviation for $\widehat{\alpha}_{AUD/USD}$ derived under the i.i.d. assumption is $\widehat{\sigma} = 0.0342$, whereas Table 4 gives us a robust estimate of $\widehat{\sigma}_{BOOT} = \sqrt{0.0475} = 0.2179$.

Moreover, at least two important results emerge from Tables 5 and 6. First, all covariances between power-law exponents are statistically significant on at least a 5% level. This result indicates that if joint power-law behavior was the subject of investigation, the hypothesis test would require us to account for the covariances too; otherwise, misleading conclusion could arise. Second, all covariances are positive implying that in the FX market, the level of uncertainty—as measured in terms of the economic magnitudes of power-law exponents—co-moves in a positive manner.

Furthermore, Tables 7 and 8 report the descriptive statistics of the distribution of power-law exponents for the individual FX variance samples derived from the bootstrap data. The average power-law exponents from the bootstrap samples are close to the estimated optimal power-law exponent based on Clauset et al.'s (2009) estimation approach, which is not surprising, as the point estimates derived from Clauset et al.'s (2009) methodology are unbiased. Again, regardless of which time frequency is considered, for all estimated power-law exponents, it holds that $2 < \overline{\widehat{\alpha}}_{BOOT,i} < 3$ for all $i = 1, \ldots, 9$, which confirms earlier results. Thus, the following question arises: Is there a common component across the FX variances manifested in joint power-law behavior?

[12] This issue is discussed in more detail later.

Table 3
Estimated power-law exponents for daily variances based on the Parkinson-estimator

	$\hat{\sigma}^2_{AUD/USD}$	$\hat{\sigma}^2_{CAD/USD}$	$\hat{\sigma}^2_{CHF/USD}$	$\hat{\sigma}^2_{EUR/USD}$	$\hat{\sigma}^2_{GBP/USD}$	$\hat{\sigma}^2_{JPY/USD}$	$\hat{\sigma}^2_{NOK/USD}$	$\hat{\sigma}^2_{NZD/USD}$	$\hat{\sigma}^2_{SEK/USD}$
$\hat{\alpha}$	2.2784	2.4220	2.7997	2.7937	2.5681	2.5702	2.4943	2.6328	2.9081
$\hat{x}_{min}$	0.0096	0.0067	0.0159	0.0144	0.0106	0.0133	0.0154	0.0298	0.0477
$\hat{\sigma}$	0.0342	0.0420	0.0865	0.0896	0.0589	0.0654	0.0442	0.0850	0.1388
N	1458 (33.06%)	1192	456	423	742	596	1190	392	203
(%)		(27.03%)	(10.34%)	(9.59%)	(16.83%)	(13.52%)	(26.98%)	(8.89%)	(4.60%)

FX variances are modeled using the following power-law function:

$$p(x) = Cx^{-\alpha},$$

where $C = (\alpha - 1)x_{MIN}^{\alpha-1}$ with $\alpha \in \{\mathbb{R}_+ | \alpha > 1\}$, x denotes the respective annualized daily or weekly FX variance provided $x \in \{\mathbb{R}_+ | x_{MIN} \leq x < \infty\}$, x_{MIN} is the minimum value governed by the power-law process, and α is the magnitude of the corresponding tail exponent. Following White et al. (2008) and Clauset et al. (2009), the tail exponents are estimated as:

$$\hat{\alpha} = 1 + N\left(\sum_{i=1}^{N} ln\left(\frac{x_i}{x_{MIN}}\right)\right)^{-1},$$

where $\hat{\alpha}$ denotes the MLE estimator, N is the number of observations exceeding x_{MIN}, and other notations are as previously defined. The estimate $\hat{\alpha}$ is selected based on the optimal Kolmogorov–Smirnov (KS) distance D measuring the maximum distance between the cumulative density functions (CDFs) of the data and the fitted power-law model as defined by:

$$D = MAX_{x \geq x_{MIN}} |S(x) - P(x)|,$$

where $S(x)$ is the CDF of the data for the observation with a value of at least x_{MIN}, and $P(x)$ is the CDF for the power-law model that best fits the data in the region $x \geq x_{MIN}$. Estimate $\hat{x}_{MIN}$ is then the value of x_{MIN} that minimizes D. This table reports the estimates $\hat{\alpha}$, $\hat{x}_{min}$, $\hat{\sigma}$, and N for daily FX variance data. The sample period is from May, 16, 2006 to May, 31, 2023 comprising 4410 daily observations.

Table 4
Estimated power-law exponents for weekly variances based on the Parkinson-estimator

	$\hat{\sigma}^2_{AUD/USD}$	$\hat{\sigma}^2_{CAD/USD}$	$\hat{\sigma}^2_{CHF/USD}$	$\hat{\sigma}^2_{EUR/USD}$	$\hat{\sigma}^2_{GBP/USD}$	$\hat{\sigma}^2_{JPY/USD}$	$\hat{\sigma}^2_{NOK/USD}$	$\hat{\sigma}^2_{NZD/USD}$	$\hat{\sigma}^2_{SEK/USD}$
$\hat{\alpha}$	2.3527	2.3867	2.8207	2.5242	2.6089	2.4515	2.7853	2.5330	2.4493
$\hat{x}_{min}$	0.0078	0.0043	0.0101	0.0054	0.0050	0.0063	0.0223	0.0102	0.0094
$\hat{\sigma}$	0.0670	0.0686	0.1366	0.0860	0.0768	0.0807	0.1545	0.1359	0.0757
N	438	437	192	337	465	348	146	362	393
(%)	(49.66%)	(49.55%)	(21.77%)	(38.21%)	(52.72%)	(39.46%)	(16.56%)	(41.04%)	(44.56%)

FX variances are modeled using the following power-law function:

$$p(x) = Cx^{-\alpha},$$

where $C = (\alpha - 1)x_{MIN}^{\alpha-1}$ with $\alpha \in \{\mathbb{R}_+ | \alpha > 1\}$, x denotes the respective annualized daily or weekly FX variance provided $x \in \{\mathbb{R}_+ | x_{MIN} \leq x < \infty\}$, x_{MIN} is the minimum value governed by the power-law process, and α is the magnitude of the corresponding tail exponent. Following White et al. (2008) and Clauset et al. (2009), the tail exponents are estimated as:

$$\hat{\alpha} = 1 + N\left(\sum_{i=1}^{N} ln\left(\frac{x_i}{x_{MIN}}\right)\right)^{-1},$$

where $\hat{\alpha}$ denotes the MLE estimator, N is the number of observations exceeding x_{MIN}, and other notations are as previously defined. The estimate $\hat{\alpha}$ is selected based on the optimal Kolmogorov–Smirnov (KS) distance D measuring the maximum distance between the cumulative density functions (CDFs) of the data and the fitted power-law model as defined by:

$$D = MAX_{x \geq x_{MIN}} |S(x) - P(x)|,$$

where $S(x)$ is the CDF of the data for the observation with a value of at least x_{MIN}, and $P(x)$ is the CDF for the power-law model that best fits the data in the region $x \geq x_{MIN}$. Estimate $\hat{x}_{MIN}$ is then the value of x_{MIN} that minimizes D. This table reports the estimates $\hat{\alpha}$, $\hat{x}_{min}$, $\hat{\sigma}$, and N for weekly FX variance data. The sample period is from May, 16, 2006 to May 31, 2023, comprising 882 weekly observations.

To explore this issue, we employed daily FX variances and estimated the test statistic given in Eq. (10). Thereby, the estimated covariance matrix from Table 5 was used for $\hat{\boldsymbol{\Sigma}} = COV(\hat{\boldsymbol{\alpha}}_{BOOT})$. The test statistic was implemented for the economically important power-law exponents $q = (2.1, 2.2, \ldots, 3.1)$ and was under the null hypothesis distributed as $\chi^2(9)$. The results are reported in Table 9. Recall that rejecting the null hypothesis $\forall q$ would indicate that the FX variances exhibit heterogenous sources of risk manifested in joint power-law behavior. Using a significance level of 5%, it becomes evident from Table 9 that the FX variances are indeed governed by the same underlying power-law behavior

Table 5
Covariance matrix for power-law exponents based on daily data

Covariance									
(t-Statistic)	$\widehat{\alpha}_{AUD/USD}$	$\widehat{\alpha}_{CAD/USD}$	$\widehat{\alpha}_{CHF/USD}$	$\widehat{\alpha}_{EUR/USD}$	$\widehat{\alpha}_{GBP/USD}$	$\widehat{\alpha}_{JPY/USD}$	$\widehat{\alpha}_{NOK/USD}$	$\widehat{\alpha}_{NZD/USD}$	$\widehat{\alpha}_{SEK/USD}$
$\widehat{\alpha}_{AUD/USD}$	0.0475								
	(——)								
$\widehat{\alpha}_{CAD/USD}$	**0.0395**	0.1568							
	(16.2461)	(——)							
$\widehat{\alpha}_{CHF/USD}$	**0.0190**	**0.0223**	0.0623						
	(11.7446)	(7.3274)	(——)						
$\widehat{\alpha}_{EUR/USD}$	**0.0311**	**0.0456**	**0.0196**	0.0879					
	(17.3233)	(13.3230)	(8.6663)	(——)					
$\widehat{\alpha}_{GBP/USD}$	**0.0492**	**0.0538**	**0.0385**	**0.0514**	0.1011				
	(31.8632)	(14.9132)	(17.5304)	(20.5425)	(——)				
$\widehat{\alpha}_{JPY/USD}$	**0.0190**	**0.0228**	**0.0091**	**0.0187**	**0.0282**	0.0201			
	(24.6108)	(14.0548)	(8.4459)	(15.6417)	(25.2766)	(——)			
$\widehat{\alpha}_{NOK/USD}$	**0.0197**	**0.0306**	**0.0047**	**0.0241**	**0.0348**	**0.0125**	0.0747		
	(11.0743)	(9.3064)	(2.1825)	(9.8276)	(13.8060)	(10.7692)	(——)		
$\widehat{\alpha}_{NZD/USD}$	**0.0433**	**0.0509**	**0.0266**	**0.0434**	**0.0635**	**0.0238**	**0.0342**	0.0786	
	(31.6805)	(16.2843)	(12.9525)	(19.3251)	(32.0988)	(23.5639)	(15.7803)	(——)	
$\widehat{\alpha}_{SEK/USD}$	**0.0390**	**0.0456**	**0.0292**	**0.0433**	**0.0616**	**0.0149**	**0.0276**	**0.0518**	0.1170
	(19.3827)	(11.2800)	(11.4951)	(14.9173)	(21.6810)	(10.2099)	(9.7420)	(20.2714)	(——)

The covariance matrix for power-law exponents are obtained via blocks bootstrap. Denoting the selected block length as m, a blocks bootstrap procedure is chosen such that $E[m] = \sqrt{T}$. Then Tx1 data vectors of FX variances $i = 1, \dots, N$, denoted as $\boldsymbol{x}_i$, are stacked into matrix $\boldsymbol{Y}$:

$$\boldsymbol{Y} = [\boldsymbol{x}_1, \boldsymbol{x}_2, \dots, \boldsymbol{x}_N].$$

Blocks m are randomly drawn from matrix $\boldsymbol{Y}$ with respect to the time dimension $t = 1, \dots, T$. These blocks are governed by a geometric distribution, that is, $m \sim GEO(p)$ with $E[m] = \frac{(1-p)}{p}$. Using this procedure, the blocks drawn from $\boldsymbol{Y}$ vary in lengths. Randomly drawn blocks m that have dimensions mxK from data matrix $\boldsymbol{Y}$ are stacked in matrix $\boldsymbol{Y}_b$ as:

$$\boldsymbol{Y}_b = \begin{bmatrix} m_1 \\ m_2 \\ m_3 \\ \vdots \end{bmatrix}.$$

The procedure is stopped when the length of the artificial matrix $\boldsymbol{Y}_b$ exhibits a length exceeding T. Observations exceeding T are cut off; that is, every artificial data matrix $\boldsymbol{Y}_b$ has the same length as the original data matrix $\boldsymbol{Y}$. This process corresponds to one iteration b of the blocks bootstrap procedure. Using this blocks bootstrap procedure, for each iteration b, Tx1 vectors, $\boldsymbol{x}_{b,1}$, $\boldsymbol{x}_{b,2}$, …, $\boldsymbol{x}_{b,N}$ are extracted from matrix $\boldsymbol{Y}_b$ and the MLE estimators are estimated using the procedure described in section 4.1.2, giving us:

$$[\, \widehat{\alpha}_{b,1} \quad \widehat{\alpha}_{b,2} \quad \cdots \quad \widehat{\alpha}_{b,N} \,].$$

This blocks bootstrap procedure is performed for $b = 1, \dots, 1000$ iterations and point estimates for α are stacked in BxN matrix $\widehat{\boldsymbol{\alpha}}_{BOOT}$:

$$\widehat{\boldsymbol{\alpha}}_{BOOT} = \begin{pmatrix} \widehat{\alpha}_{1,1} & \widehat{\alpha}_{1,2} & \dots & \dots & \widehat{\alpha}_{1,N} \\ \widehat{\alpha}_{2,1} & \widehat{\alpha}_{1,2} & \dots & \dots & \widehat{\alpha}_{2,N} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \widehat{\alpha}_{B,1} & \widehat{\alpha}_{B,2} & \dots & \dots & \widehat{\alpha}_{B,N} \end{pmatrix}.$$

This table reports the covariance matrix for power-law exponents based on daily data using the variance-estimator proposed by Parkinson (1980) and $E[m] = 66$. Data for the overall sample is from May, 16, 2006 to May, 31, 2023 corresponding to 4410 observations. Bold figures indicate statistical significance on at least a 5% level.

because the null hypothesis cannot be rejected for $2.2 < \alpha < 3.0$ as indicated by p-values >5%. The optimal power-law exponent governing the joint risk is estimated at $\widehat{\alpha}^* \approx 2.6$ as indicated by the highest p-value corresponding to 0.7861. In sum, the results from Table 9 indicate that the FX variances are governed by a common source of risk as measured in terms of a common power-law exponent; the infinite-theoretical-mean-hypothesis is rejected for the cross section, as the null hypothesis is rejected for exponents $\alpha < 2.3$; and the finite-theoretical-variance-hypothesis is also rejected on a cross-sectional level, as the null hypothesis is rejected for exponents $\alpha > 2.9$. The next question that arises is whether the power-law behavior of FX variances is invariant across both markets and time frequencies.

To investigate this issue, both daily and weekly FX variances were employed, and the test statistic given in Eq. (11) was estimated. Thereby, the estimated covariance matrices from Tables 5 and 6 were used for $\widehat{\boldsymbol{\Sigma}}_{\widehat{\alpha}_1} = COV(\widehat{\boldsymbol{\alpha}}_{BOOT}^{DAILY})$ and $\widehat{\boldsymbol{\Sigma}}_{\widehat{\alpha}_2} = COV(\widehat{\boldsymbol{\alpha}}_{BOOT}^{WEEKLY})$. Again, the test statistic was implemented for the economically important power-law exponents $q = (2.1, 2.2, \dots, 3.1)$ and was under the null hypothesis distributed as $\chi^2(18)$. The results are reported in Table 10. Recall that acceptance of some null hypothesis would indicate that FX variances are governed by a common power-law exponent, which governs the risk across time frequencies. Using a significance level of 5%, it becomes

Table 6
Covariance matrix for power-law exponents based on weekly data

Covariance									
(*t*-Statistic)	$\widehat{\alpha}_{AUD/USD}$	$\widehat{\alpha}_{CAD/USD}$	$\widehat{\alpha}_{CHF/USD}$	$\widehat{\alpha}_{EUR/USD}$	$\widehat{\alpha}_{GBP/USD}$	$\widehat{\alpha}_{JPY/USD}$	$\widehat{\alpha}_{NOK/USD}$	$\widehat{\alpha}_{NZD/USD}$	$\widehat{\alpha}_{SEK/USD}$
$\widehat{\alpha}_{AUD/USD}$	0.1294								
	(——)								
$\widehat{\alpha}_{CAD/USD}$	**0.1227**	0.3308							
	(23.2787)	(——)							
$\widehat{\alpha}_{CHF/USD}$	**0.0333**	**0.0528**	0.1283						
	(8.4397)	(8.3807)	(——)						
$\widehat{\alpha}_{EUR/USD}$	**0.0912**	**0.1341**	**0.0395**	0.1698					
	(24.6592)	(21.6757)	(8.7863)	(——)					
$\widehat{\alpha}_{GBP/USD}$	**0.1162**	**0.1275**	**0.0380**	**0.1030**	0.1739				
	(38.6938)	(19.8350)	(8.3194)	(23.6678)	(——)				
$\widehat{\alpha}_{JPY/USD}$	**0.0483**	**0.0579**	**0.0313**	**0.0338**	**0.0569**	0.0937			
	(15.4113)	(10.9982)	(9.4037)	(8.7945)	(15.7353)	(——)			
$\widehat{\alpha}_{NOK/USD}$	**0.0819**	**0.1166**	**0.0407**	**0.0859**	**0.0860**	**0.0402**	0.1489		
	(23.0967)	(19.4999)	(9.7410)	(20.2701)	(19.9662)	(11.4476)	(——)		
$\widehat{\alpha}_{NZD/USD}$	**0.0961**	**0.1084**	**0.0355**	**0.0801**	**0.1104**	**0.0465**	**0.0744**	0.1122	
	(41.7762)	(21.4952)	(9.7833)	(22.5089)	(40.7508)	(16.0954)	(22.2285)	(——)	
$\widehat{\alpha}_{SEK/USD}$	**0.0854**	**0.1074**	**0.0352**	**0.0908**	**0.1086**	**0.0434**	**0.0813**	**0.0815**	0.1481
	(24.7733)	(17.5450)	(8.3499)	(22.0599)	(29.0597)	(12.5221)	(20.6844)	(25.7929)	(——)

The covariance matrix for power-law exponents are obtained via blocks bootstrap. Denoting the selected block length as *m*, a blocks bootstrap procedure is chosen such that $E[m] = \sqrt{T}$. Then Tx1 data vectors of FX variances $i = 1, \ldots, N$, denoted as $\boldsymbol{x}_i$, are stacked into matrix $\boldsymbol{Y}$:

$$\boldsymbol{Y} = [\boldsymbol{x}_1, \boldsymbol{x}_2, \ldots, \boldsymbol{x}_N].$$

Blocks *m* are randomly drawn from matrix $\boldsymbol{Y}$ with respect to the time dimension $t = 1, \ldots, T$. These blocks are governed by a geometric distribution, that is, $m \sim GEO(p)$ with $E[m] = \frac{(1-p)}{p}$. Using this procedure, the blocks drawn from $\boldsymbol{Y}$ vary in lengths. Randomly drawn blocks *m* that have dimensions mxK from data matrix $\boldsymbol{Y}$ are stacked in matrix $\boldsymbol{Y}_b$ as:

$$\boldsymbol{Y}_b = \begin{bmatrix} m_1 \\ m_2 \\ m_3 \\ \vdots \end{bmatrix}.$$

The procedure is stopped when the length of the artificial matrix $\boldsymbol{Y}_b$ exhibits a length exceeding *T*. Observations exceeding *T* are cut off; that is, every artificial data matrix $\boldsymbol{Y}_b$ has the same length as the original data matrix $\boldsymbol{Y}$. This process corresponds to one iteration *b* of the blocks bootstrap procedure. Using this blocks bootstrap procedure, for each iteration *b*, Tx1 vectors, $\boldsymbol{x}_{b,1}$, $\boldsymbol{x}_{b,2}$, …, $\boldsymbol{x}_{b,N}$ are extracted from matrix $\boldsymbol{Y}_b$ and the MLE estimators are estimated using the procedure described in section 3.1.1, giving us:

$$[\,\widehat{\alpha}_{b,1} \quad \widehat{\alpha}_{b,2} \quad \cdots \quad \widehat{\alpha}_{b,N}\,].$$

This blocks bootstrap procedure is performed for $b = 1, \ldots, 1000$ iterations and point estimates for α are stacked in BxN matrix $\widehat{\boldsymbol{\alpha}}_{BOOT}$:

$$\widehat{\boldsymbol{\alpha}}_{BOOT} = \begin{pmatrix} \widehat{\alpha}_{1,1} & \widehat{\alpha}_{1,2} & \ldots & \ldots & \widehat{\alpha}_{1,N} \\ \widehat{\alpha}_{2,1} & \widehat{\alpha}_{1,2} & \ldots & \ldots & \widehat{\alpha}_{2,N} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \widehat{\alpha}_{B,1} & \widehat{\alpha}_{B,2} & \ldots & \ldots & \widehat{\alpha}_{B,N} \end{pmatrix}.$$

This table reports the covariance matrix for power-law exponents based on weekly data using the variance-estimator proposed by Parkinson (1980) and $E[m] = 30$. Data for the overall sample is from May, 16, 2006 to May, 31, 2023 corresponding to 882 observations. Bold figures indicate statistical significance on at least a 5% level.

evident from Table 10 that the FX variances are indeed governed by the same underlying power-law behavior, which is even present across time frequencies because the null hypothesis cannot be rejected for $2.1 < \alpha < 3.0$. Strikingly, the optimal power-law exponent governing the joint risk across time frequencies is, again, estimated at $\widehat{\alpha}^* \approx 2.6$ as indicated by the highest *p*-value corresponding to 0.9813, which is an intriguing finding. In sum, the results documented in Table 10 confirm the findings documented in Table 9 and, moreover, generalize them, as joint power-law behavior does not alter if the time frequency changes. Finally, the last question that arises is whether common power-law behavior is not only invariant across time frequencies but also invariant over time for a given time frequency.

To answer this question, this study proposes what it has termed the test for total invariance. Implementing this test, as given by Eq. (12), means that the whole sample was divided into two non-overlapping subsamples of equal length, leaving us with 2205 observations for daily data subsamples and 441 observations for weekly data subsamples. Data for the first subsample are from May 16, 2006, to December 10, 2014, whereas data for the second subsample are from December 11, 2014, to May 29, 2023. For each subsample and data frequency, point estimates for power-law exponents and corresponding covariance matrices are, again, estimated using the blocks bootstrap procedure as

Table 7
Descriptive statistics for bootstrapped power-law exponents using daily data

	$\hat{\alpha}_{AUD/USD}$	$\hat{\alpha}_{CAD/USD}$	$\hat{\alpha}_{CHF/USD}$	$\hat{\alpha}_{EUR/USD}$	$\hat{\alpha}_{GBP/USD}$	$\hat{\alpha}_{JPY/USD}$	$\hat{\alpha}_{NOK/USD}$	$\hat{\alpha}_{NZD/USD}$	$\hat{\alpha}_{SEK/USD}$
Mean	2.3378	2.6154	2.7961	2.7969	2.6435	2.5821	2.6940	2.5911	2.5924
Median	2.3063	2.5123	2.7550	2.7731	2.6110	2.5749	2.6567	2.5701	2.5764
Maximum	3.1879	4.7645	3.8552	4.4285	4.1516	3.1780	3.7614	3.7697	3.8879
Minimum	1.8382	1.9717	2.2784	2.0507	1.9136	2.1882	2.1357	1.9051	1.7931
Std. Dev.	0.2181	0.3962	0.2498	0.2967	0.3181	0.1419	0.2734	0.2805	0.3423
Skewness	0.7497	1.3566	0.9364	1.0908	0.6289	0.1525	0.7701	0.4413	0.4379
Kurtosis	3.7427	5.0519	4.2826	6.6151	3.4751	3.0222	3.6401	3.4393	3.1638
Jarque-Bera	116.6494	482.1653	214.6922	742.8210	75.3353	3.8966	115.9188	40.4922	33.0746
(*p*-value JB)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.1425)	(0.0000)	(0.0000)	(0.0000)
Observations	1000	1000	1000	1000	1000	1000	1000	1000	1000

This table reports the descriptive statistics for bootstrapped power-law exponents using variance-estimator proposed by Parkinson (1980) implemented for daily data. The results are based on $B = 1000$ blocks bootstrap replications and $E[m] = 66$. The blocks bootstrap procedure is detailed in section 4.1.2.

Table 8
Descriptive statistics for bootstrapped power-law exponents using weekly data

	$\hat{\alpha}_{AUD/USD}$	$\hat{\alpha}_{CAD/USD}$	$\hat{\alpha}_{CHF/USD}$	$\hat{\alpha}_{EUR/USD}$	$\hat{\alpha}_{GBP/USD}$	$\hat{\alpha}_{JPY/USD}$	$\hat{\alpha}_{NOK/USD}$	$\hat{\alpha}_{NZD/USD}$	$\hat{\alpha}_{SEK/USD}$
Mean	2.4741	2.8013	2.7931	2.7015	2.7584	2.6528	2.7871	2.5598	2.6212
Median	2.3995	2.6705	2.7494	2.6128	2.6633	2.6379	2.7325	2.5096	2.5717
Maximum	4.2188	5.5667	6.0934	4.7567	4.9611	4.3249	4.7447	4.1022	4.7506
Minimum	1.7535	1.8718	2.0186	2.0126	1.9352	1.8519	1.9994	1.9340	1.8665
Std. Dev.	0.3599	0.5754	0.3583	0.4122	0.4172	0.3062	0.3861	0.3352	0.3850
Skewness	1.1812	1.4538	2.2348	1.3961	0.9386	0.8208	0.7942	1.0109	1.0403
Kurtosis	4.8674	5.2984	15.9828	5.5020	4.1990	5.3754	4.0778	4.3679	5.2732
Jarque-Bera	377.8161	572.3850	7855.4000	585.6619	206.7434	347.3833	153.5181	248.2969	395.6696
(*p*-value JB)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
Observations	1000	1000	1000	1000	1000	1000	1000	1000	1000

This table reports the descriptive statistics for bootstrapped power-law exponents using variance-estimator proposed by Parkinson (1980) implemented for weekly data. The results are based on $B = 1000$ blocks bootstrap replications and $E[m] = 30$. The blocks bootstrap procedure is detailed in section 4.1.2.

described in section 3.1.3. Tables A.1–A.3 in the appendix report the estimated covariance matrices.[13] Tables A.1–A.3 show that virtually all covariances are positive and statistically significant on at least a 5% level, which support earlier results derived from estimations based on the whole sample. Using the covariance matrices from Tables A.1–A.3, the test statistic as given in Eq. 12 was employed. Again, the test statistic was implemented for the economically important power-law exponents $q = (2.1, 2.2, \ldots, 3.1)$. The results are reported in Table 11. Recall that accepting some of the null hypothesis would indicate that the FX variances are governed by a common power-law exponent, which is invariant across both subsamples: time and time frequencies. Using a significance level of 5%, it becomes evident from Table 11 that the FX variances are indeed governed by a common component that manifests those universal properties of invariance because the null hypothesis cannot be rejected for $2.3 < \alpha < 3.0$. Despite the evidence showing that the optimal power-law exponent is estimated at $\hat{\alpha}^* \approx 2.7$, as indicated by the highest *p*-value corresponding to 0.5737, the null hypothesis $\alpha^* \approx 2.6$ cannot be rejected either because the corresponding *p*-value exceeds the significance level of 5% by a substantial margin (*p*-value 0.5475). Furthermore, the results reported in Table 11 confirm earlier findings—that is, the infinite-theoretical-mean-hypothesis is rejected, and the finite-theoretical-variance-hypothesis is also rejected.

Table 9
Testing for a common power-law exponent

q	$\hat{\lambda}$	*p*-value
2.1	**24.5392**	**0.0035**
2.2	**17.7452**	**0.0383**
2.3	12.4471	0.1893
2.4	8.6448	0.4707
2.5	6.3385	0.7056
2.6	5.5281	0.7861
2.7	6.2135	0.7184
2.8	8.3949	0.4949
2.9	12.0721	0.2093
3.0	**17.2453**	**0.0450**
3.1	**23.9143**	**0.0044**

To explore whether exists a common component governing power-law behavior of FX variances, the following estimated test statistic is used:

$\hat{\lambda} = (\hat{\boldsymbol{\alpha}} - q\mathbf{1})'\hat{\boldsymbol{\Sigma}}^{-1}(\hat{\boldsymbol{\alpha}} - q\mathbf{1}),$

where the covariance matrix $\hat{\boldsymbol{\Sigma}} = COV(\hat{\boldsymbol{\alpha}}_{BOOT})$ has the dimension NxN, $\hat{\boldsymbol{\alpha}}$ is a Nx1 vector of estimated power-law exponents, $\mathbf{1}$ is a Nx1 vector of ones and q is the hypothesized common power-law exponent. The estimated test statistic denoted as $\hat{\lambda}$ is under the null hypothesis distributed as $\chi^2(N)$. The test statistic is iteratively estimated covering the interval $q = (2.1, 2.2, \ldots, 3.1)$. Since nine FX variances are tests, the corresponding test statistic is under the null hypothesis distributed as $\chi^2(9)$. Bold figures indicate statistical significance on a 5% level.

5.2. Results from robustness checks

To explore whether the Garman-Klass estimator would produce different results, the data covering the same sample as the data set used for the main analysis are used (e.g., May 16, 2006, to May 31, 2023). The

[13] To save space, the descriptive statistics for the point estimates are not reported. All data matrices are available upon request.

Table 10
Testing for invariance across time frequencies

q	$\widehat{\lambda}$	p-value
2.1	**31.8698**	**0.0228**
2.2	23.3535	0.1774
2.3	16.6898	0.5445
2.4	11.8785	0.8535
2.5	8.9198	0.9616
2.6	7.8135	0.9813
2.7	8.5597	0.9691
2.8	11.1584	0.8875
2.9	15.6096	0.6198
3.0	21.9132	0.2359
3.1	**30.0694**	**0.0368**

Fractal behavior of FX variances is tested using the following approach: K is defined as the dimension of time frequencies and N denotes the number of FX variance vectors, each having the dimension of Tx1. Further, $\boldsymbol{I}_{NK}$ is defined as NKxNK identity matrix, $\widehat{\boldsymbol{\alpha}} = (\widehat{\boldsymbol{\alpha}}_1, \widehat{\boldsymbol{\alpha}}_2, \ldots, \widehat{\boldsymbol{\alpha}}_K)'$ where $\widehat{\boldsymbol{\alpha}}_j$ are Nx1 vectors of power-law exponents for a given frequency j and $j = 1, \ldots, K$. Moreover, $\boldsymbol{q}$ is a NKx1 vector consisting of the corresponding common exponent that is subject to be tested, and $\widehat{\boldsymbol{\Sigma}}_{\widehat{\alpha}}$ is the estimated NKxNK covariance matrix of $\widehat{\boldsymbol{\alpha}} = (\widehat{\boldsymbol{\alpha}}_1, \widehat{\boldsymbol{\alpha}}_2, \ldots, \widehat{\boldsymbol{\alpha}}_K)'$ obtained via blocks bootstrap and defined as:

$$\widehat{\boldsymbol{\Sigma}}_{\widehat{\alpha}} = \begin{pmatrix} \widehat{\boldsymbol{\Sigma}}_{\widehat{\alpha}_1} & \mathbf{0} & \ldots & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \widehat{\boldsymbol{\Sigma}}_{\widehat{\alpha}_2} & \ldots & \mathbf{0} & \mathbf{0} \\ \vdots & \vdots & & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \ldots & \vdots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \ldots & \mathbf{0} & \widehat{\boldsymbol{\Sigma}}_{\widehat{\alpha}_K} \end{pmatrix},$$

where $\widehat{\boldsymbol{\Sigma}}_{\widehat{\alpha}_j}$ are NxN covariance matrices for time frequency j, and $\mathbf{0}$ defines NxN matrices consisting of zeros. Then, the following estimated test statistic is proposed:

$$\widehat{\lambda} = (\widehat{\boldsymbol{\alpha}} - \boldsymbol{q})'\widehat{\boldsymbol{\Sigma}}_{\widehat{\alpha}}^{-1}(\widehat{\boldsymbol{\alpha}} - \boldsymbol{q}),$$

with $\boldsymbol{q} = q\mathbf{1}$ where $\mathbf{1}$ is a NKx1 vector of ones and q is the hypothesized common and time-invariant power-law exponent. Denotes the interval $q = (2.1, 2.2, \ldots, 3.1)$. The estimated test statistic denoted as $\widehat{\lambda}$ is under the null hypothesis distributed as $\chi^2(NK)$. Again, the test statistic is iteratively estimated covering the interval $q = (2.1, 2.2, \ldots, 3.1)$. Since $N = 9$ FX variances are tested for $K = 2$ time frequencies (e.g., daily and weekly), the corresponding test statistic is under the null hypothesis distributed as $\chi^2(18)$. Bold figures indicate statistical significance on a 5% level.

Table 11
Testing for total invariance

q	$\widehat{\lambda}$	p-value
2.1	**94.7910**	**0.0000**
2.2	**75.0045**	**0.0002**
2.3	**59.0664**	**0.0090**
2.4	46.9769	0.1041
2.5	38.7358	0.3472
2.6	34.3431	0.5475
2.7	33.7990	0.5737
2.8	37.1033	0.4179
2.9	44.2561	0.1624
3.0	**55.2574**	**0.0210**
3.1	**70.1071**	**0.0001**

To test for total invariance of FX variances, this study proposes a test which can be derived from the test described in section 4.1.5. Considering $K = 2$ time frequencies, and $S = 2$ subsamples, we can use the test described in section 4.1.5 to test jointly whether the power-law exponents are invariant across time and across time frequencies using:

$$\widehat{\boldsymbol{\Sigma}}_{\widehat{\alpha}} = \begin{pmatrix} \widehat{\boldsymbol{\Sigma}}_{\widehat{\alpha}_{1,1}} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \widehat{\boldsymbol{\Sigma}}_{\widehat{\alpha}_{1,2}} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \widehat{\boldsymbol{\Sigma}}_{\widehat{\alpha}_{2,1}} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \widehat{\boldsymbol{\Sigma}}_{\widehat{\alpha}_{2,2}} \end{pmatrix},$$

where $\widehat{\boldsymbol{\Sigma}}_{\widehat{\alpha}_{1,2}}$ and $\widehat{\boldsymbol{\Sigma}}_{\widehat{\alpha}_{1,2}}$ are the NxN covariance matrices for first and second non-overlapping subsample for daily data, whereas $\widehat{\boldsymbol{\Sigma}}_{\widehat{\alpha}_{2,1}}$ and $\widehat{\boldsymbol{\Sigma}}_{\widehat{\alpha}_{2,2}}$ are the NxN covariance matrices for the first and second non-overlapping subsample for weekly data. Again, $\mathbf{0}$ defines NxN matrices consisting of zeros. Then, the following estimated test statistic is proposed:

$$\widehat{\lambda} = (\widehat{\boldsymbol{\alpha}} - \boldsymbol{q})'\widehat{\boldsymbol{\Sigma}}_{\widehat{\alpha}}^{-1}(\widehat{\boldsymbol{\alpha}} - \boldsymbol{q}),$$

where $\widehat{\boldsymbol{\alpha}} = (\widehat{\boldsymbol{\alpha}}_{1,1}, \widehat{\boldsymbol{\alpha}}_{1,2}, \widehat{\boldsymbol{\alpha}}_{2,1}, \widehat{\boldsymbol{\alpha}}_{2,2})'$ and $\widehat{\boldsymbol{\alpha}}_{j,s}$ are Nx1 vectors for frequency j, and subsample s, where $j = 1, \ldots, K$, and $s = 1, \ldots, S$. Here we consider only $K = S = 2$. Moreover, $\boldsymbol{q} = q\mathbf{1}$ where $\mathbf{1}$ is here a NSKx1 vector of ones, whereas q is again the hypothesized common power-law exponent which is under the null hypothesis invariant across markets, time, and time frequency. Again, the test statistic is iteratively estimated covering the interval $q = (2.1, 2.2, \ldots, 3.1)$. The estimated test statistic denoted as $\widehat{\lambda}$ is under the null hypothesis distributed as $\chi^2(NSK)$. Since $N = 9$, and $K = S = 2$, the test statistic is under the null hypothesis distributed as $\chi^2(36)$. Bold figures indicate statistical significance on a 5% level.

Garman-Klass-estimator for daily range-based FX variances was computed in line with Eq. (13). The descriptive statistics are reported in Table A.5 in the appendix.[14] Surprisingly, comparing the standard deviations produced from using the Parkinson-estimator (see Table 1) with the corresponding figures derived from using the Garman-Klass estimator (see Table A.5), it becomes evident that the Garman-Klass estimator produces six out of the nine FX variances with higher standard deviations.

Next, using the original data sets, the optimal power-law exponents are estimated using the approach proposed by Clauset et al. (2009) as outlined in section 4.1.2. The results are reported in Table A.6 in the appendix. From the data in Table A.6, it becomes evident that the optimal estimated power-law exponents vary between $\widehat{\alpha} = 2.44$ for the AUD/USD variance and $\widehat{\alpha} = 2.91$ for the SEK/USD variance. The fraction of observations governed by power-law process varies between 8.65% for the SEK/USD variance and 21.81% for the CAD/USD variance. After comparing the results reported in Table 3 with those from Table A.6, it becomes evident that the range of estimated power-law exponents is very close to one obtained from using the Parkinson-estimator. Overall, using the original data sets and Clauset et al.'s (2009) MLE approach to derive power-law exponents shows that a strong commonality exists—that is, both variance estimators suggest the power-law exponents $2 < \alpha_i < 3$ for all $i = 1, \ldots, 9$.

Next, the covariance matrix for power-law exponents is obtained from employing the blocks bootstrap procedure as detailed in section 4.1.3. The estimated covariance matrix for power-law exponents based on employing the Garman-Klass-estimator is reported in Table A.7 in the appendix. To explore whether the Parkinson-estimator would produce inflated estimates, the standard deviations of the power-law exponents are computed after collecting the elements on the covariances' main diagonals. The standard deviations produced from using the Parkinson-estimator are in line with Eq. (15) regressed on the corresponding estimates produced from using the Garman-Klass-estimator. Recall that manifestations of inflated standard deviations produced by using the Parkinson-estimator are either $c > 0$ or $b > 0$. Based on the regression

[14] Note that the sample includes five observations less than the sample for the Parkinson-estimator. One drawback of the Garman-Klass estimator may be that if $[2ln(2) - 1]\left[ln\left(\frac{C_{i,t}}{O_{i,t}}\right)\right]^2 > 0.5\left[ln\left(\frac{H_{i,t}}{L_{i,t}}\right)\right]^2$, the variance is undefined.

model (Eq. [15]), the point estimates of $\widehat{c} = 0.21$ and $\widehat{b} = 0.74$ are obtained with corresponding *t*-statistics of 0.69 and 15.40. The R-squared of the regression equation is 0.97, suggesting an excellent data fit. Overall, the evidence suggests that using the Parkinson-estimator does not result in inflated estimates for standard deviations of power-law exponents. If anything, the evidence suggests the opposite.

Finally, using point estimates and estimated covariance matrices for both estimators (e.g., Parkinson-estimator and the Garman-Klass-estimator), we tested whether the difference in point estimates is jointly statistically significant, which would indicate that using the Garman-Klass-estimator could potentially change derived inference based on using the Parkinson-estimator. To test this issue, we implemented the test proposed in Eq. (14). The test statistic is under the null hypothesis distributed as $\chi^2(9)$. Since $\widehat{\lambda} = 1.3669 < 16.9190 = \chi^2_{0.95}(9)$, the null hypothesis cannot be rejected (*p*-value is 0.9980). The evidence suggests that ascertained power-law behavior is robust to altering the range-based FX variance estimator.

Furthermore, the finding of power-law exponents with relatively low tail exponent for variances is contrary to the literature proposing that realized volatility is close to log-normally distributed. Is the power-law hypothesis reasonable? To answer this question, the one-sigma test in line with Eq. (16) was implemented. First, the lognormal distribution was used under the null hypothesis. For the standardized lognormal distribution, a fraction of 0.8189 of the observations is within one standard deviation from the mean.[15] Using the FX variances as reported in Table 1, the one-sigma test was run for all FX variances hypothesizing the lognormal as the null model. The results are reported in Table A.8 in the appendix. It becomes evident that the lognormal distribution is clearly rejected on any level for all FX variances.

Second, the chi-square distribution with one degree of freedom was used under the null hypothesis. For the standardized $\chi^2(1)$, a fraction of 0.8797 of the observations is within one standard deviation from the mean. Again, the one-sigma-test was run for all FX variances hypothesizing the $\chi^2(1)$ as the null model. The results reported in Table A.8 in the appendix show that the null model is rejected for all FX variances too. Because the one-sigma test requires that the theoretical mean and the theoretical variance of the distribution used as the null model be defined, this test cannot be implemented when using a power-law process with $\alpha \leq 3$ as the null model.

Hence, the GoF test proposed by Clauset et al. (2009) is used to explore the plausibility of the power-law null model. The GoF test was implemented first for the original FX variance data series based on the Parkinson-estimator in association with the parametrizations from Table 3. The results reported in Table A.8 suggests that the power-law null model cannot be rejected for at least six out of the nine FX variances. Second, the GoF test was implemented for the FX variance data series based on the Garman-Klass-estimator in association with the parametrizations from Table A.6. The results reported in Table A.8 indicate that the power-law null model cannot be rejected for eight out of the nine FX variances. Overall, the results from this robustness check strongly suggest that power-laws are indeed more reasonable models than the well-established lognormal distribution or the $\chi^2(1)$.

It is also noteworthy that the finding of a common power-law exponent with $\alpha \approx 2.6$ is strikingly in line with recent research from Grobys (2023b), who extends Mandelbrot's (2008) multifractal model of asset returns to price realized variances. Specifically, Grobys' (2023b) multivariate model of asset invariances shows that a multifractal model of realized weekly asset variances based on binominal bending with $p = 0.6$ produces variance processes governed by power-law processes with $\alpha = 2.61$, on expectation. Overall, it seems to be self-evident that the weekly FX variances are in line with predictions derived from a multifractal model as opposed to any other distributional framework.

Next, it needs to be clarified whether the obtained results are sensitive to changing the block length for the blocks bootstrap procedure. Recall that this study employed a random block length with $E[m] = T^{1/2}$ as opposed to $E[m] = T^{1/3}$, which is often used for variance estimation as mentioned earlier. To explore this issue, the blocks bootstrap approach outlined in section 4.1.2 was implemented using a random block length selection following a geometric distribution with $E[m] = T^{1/3}$. Again, using the Parkinson-estimator for daily data, $B = 1000$ bootstrap samples were generated, and the estimated power-law exponents were stored in some BxN matrix $\widehat{\boldsymbol{\alpha}}_{BOOT}$. Whereas Table A.9 in the appendix reports the descriptive statistics for the bootstrapped power-law exponents, Table A.10 in the appendix reports the estimated covariance matrix. After comparing the average values for point estimates obtained from these two blocks bootstraps procedure (e.g., Table 7 and A.9), it becomes evident that the average point estimates for power-law exponents are close to one another, whereas the estimated standard deviations are larger for estimates derived from blocks bootstrap using $E[m] = T^{1/2}$. This is, of course, not surprising because using a longer block length allows for considering longer dependency structures in the data, as Godfrey (2009) points out.

As an example, considering the estimated standard deviation for $\widehat{\alpha}_{NOK/USD}$, from Tables 7 and A.9, it becomes evident that using $E[m] = T^{1/3}$ as opposed to $E[m] = T^{1/2}$ in the blocks bootstrap procedure shows that the estimated standard deviations are about the same.[16] This result indicates that the often-used (expected) block length of $T^{1/3}$ may sufficiently capture dependency structures. However, the impact becomes more pronounced when considering the estimated standard deviation for $\widehat{\alpha}_{GBP/USD}$. From the data in Tables 7 and A.9, it becomes evident that using $E[m] = T^{1/3}$ as opposed to $E[m] = T^{1/2}$ in the blocks bootstrap procedure shows that the estimated standard deviations decrease by approximately 30%, which could indicate that uncertainty in estimated power-law exponents may be underestimated when using $E[m] = T^{1/3}$.

Further, to test whether the point estimates for power-law exponents are sensitive to changing the (expected) block length in the blocks bootstrap procedure, the joint test proposed in Eq. (17) was implemented. Recall that rejecting the null hypothesis would imply that the assessed power-law behavior is not robust to changing the selected block length. Similar to a two-sample test, estimating the test statistic required to account for both covariance matrices in an equal-weighted manner. The test statistic is under the null hypothesis distributed as $\chi^2(9)$ and estimated at $\widehat{\lambda} = 0.0577 < 16.9190 = \chi^2_{0.95}(9)$. Thus, the null hypothesis cannot be rejected (*p*-value is 1.0000). Hence, the evidence suggests that ascertained power-law behavior is robust to altering the expected block length.

To explore whether the data provided from yahoo.com is reliable, we downloaded data from investing.com which provides another popular database. Unfortunately, investing.com did not provide data on NOK/USD FX rate quotations. Therefore, we only retrieved intraday prices for the AUD/USD, CAD/USD, CHF/USD, EUR/USD, GBP/USD, JPY/USD, NZD/USD, and SEK/USD FX rates. To ensure comparability of the data sets, data from May 16, 2006, to May 31, 2023 were downloaded. Again, only the intersection was employed—that is, only daily data are accounted for where all FX rates were quoted on the same day, leaving us with 4447 daily observations. The Parkinson-estimator for daily range-based FX variances was computed in line with Eq. (1). The descriptive statistics are reported in Table A.11 in the appendix. Comparing Tables 1 and A.11 it becomes evident that the FX rate variances based on data from yahoo.com produced considerably higher maxima for AUD/USD, EUR/USD, and NZD/USD, whereas data

[15] The corresponding figure for the standard normal distribution is 0.6827.

[16] The estimated standard deviations decrease from 0.2734 to 0.2599 when using $E[m] = T^{1/3}$ as opposed to $E[m] = T^{1/2}$ in the blocks bootstrap procedure, which is in relative terms about 5%.

retrieved from investing.com produced higher maximum for GBP/USD and SEK/USD. On the other hand, estimated values for all standard statistical metrics (e.g., mean, standard deviation, etc.) seem to be very close to each other for remaining FX variances (e.g., CAD/USD, CHF/USD, and JPY/USD). Overall, the data suggest that there is evidence for some inconsistencies. To examine whether our main results are an artefact of measurement errors in the data, we first used FX rate variances based on data obtained from investing.com and estimated the optimal power-law exponents using the approach proposed by Clauset et al. (2009) as outlined in section 4.1.2. The results are reported in Table A.12 in the appendix.

From the data in Table A.12, it becomes evident that the optimal estimated power-law exponents vary between $\widehat{\alpha} = 2.52$ for the AUD/USD variance and $\widehat{\alpha} = 3.34$ for the SEK/USD variance. Unlike the results documented Table 3, we observe from Table A.12 that the economic magnitude of at least three estimated power-law exponents exceeds three ($\widehat{\alpha} > 3$). Next, in Table A.13, 95% confidence intervals for the point estimates from Table 3 are reported. From Table A.13 it becomes evident that, using a 5% significance level, only the power-law exponent for SEK/USD FX rate variance is statistically different from the point estimate based on data retrieved from yahoo.com.

However, to draw cross-sectional conclusions, it is important to account for the covariance matrices $\widehat{\boldsymbol{\Sigma}}_{\widehat{\boldsymbol{\alpha}}_{yahoo}}$ and $\widehat{\boldsymbol{\Sigma}}_{\widehat{\boldsymbol{\alpha}}_{investing}}$. Using a block bootstrap procedure where the random block length is governed by a geometric distribution with $p = 0.0147$, gives us the covariance matrix $\widehat{\boldsymbol{\Sigma}}_{\widehat{\boldsymbol{\alpha}}_{investing}}$ reported in Table A.14.[17] Confirming earlier estimated covariance matrices for power-law exponents, at least eleven covariances are statistically significant on a 5% level. Next, we test whether point estimates for power-law exponents are statistically not different from each other, regardless the data provider (e.g., yahoo.com or investing.com). To do so, we implemented the test as shown in Eq. (18). The test statistic is under the null hypothesis distributed as $\chi^2(8)$. Since $\widehat{\lambda} = 5.2112 < 15.5073 = \chi^2_{0.95}(8)$, the null hypothesis cannot be rejected (p-value is 0.7348). Overall, the evidence suggests that ascertained power-law behavior of FX rate variances is not an artefact due to potential measurement errors in the database provided from yahoo.com.

Although this study lives true to the motto that scientific research should be replicable without imposing charges and therefore uses data that are available for free, a reader might think "you get what you pay for" and therefore, data obtained from data providers that impose charges are perhaps more reliable. To explore this issue, we obtained daily spot data on G10 currencies' high and low quotes from Bloomberg.[18] Again, to ensure comparability of the data sets, data from May 16, 2006, to May 31, 2023 were acquired. The Parkinson-estimator for daily range-based FX variances was computed in line with Eq. (1). The descriptive statistics are reported in Table A.15 in the appendix. Comparing Tables 1, A.11, and A.15 it becomes evident that the FX rate variances based on data from Bloomberg produced very similar FX rate variances as those based on data from investing. For this reason, there is evidence for some inconsistencies between data from yahoo.com and Bloomberg, but there are virtually no inconsistencies between data from investing.com and Bloomberg. Next, we used FX rate variances based on data obtained from Bloomberg and estimated the optimal power-law exponents using the approach proposed by Clauset et al. (2009) as outlined in section 3.1.2. The results are reported in Table A.16 in the appendix. Unsurprisingly, the point estimates are very close to the ones reported in Table A.12. To draw conclusions concerning the whole cross section, we needed to account for the covariance matrices $\widehat{\boldsymbol{\Sigma}}_{\widehat{\boldsymbol{\alpha}}_{yahoo}}$ and $\widehat{\boldsymbol{\Sigma}}_{\widehat{\boldsymbol{\alpha}}_{Bloomberg}}$. Using a block bootstrap procedure where the random block length is governed by a geometric distribution with $p = 0.0147$, gives us the covariance matrix $\widehat{\boldsymbol{\Sigma}}_{\widehat{\boldsymbol{\alpha}}_{Bloomberg}}$ reported in Table A.17. Strikingly, all estimated covariances between power-law exponents are statistically significant on a 1% level. Next, we test whether point estimates for power-law exponents are statistically not different from each other, regardless the data provider (e.g., yahoo.com or Bloomberg). To do so, we implemented the test as shown in Eq. (18) and replace data vectors and matrices estimated from data obtained from investing.com by data vectors and matrices estimated from data acquired from Bloomberg. The test statistic is under the null hypothesis distributed as $\chi^2(9)$. Since $\widehat{\lambda} = 6.5581 < 16.9190 = \chi^2_{0.95}(9)$, the null hypothesis cannot be rejected (p-value is 0.6830). Again, the overall, the evidence suggests that ascertained power-law behavior of FX rate variances is not an artefact due to potential measurement errors in the database provided from yahoo.com. The results of this robustness check also imply that despite of some data inconsistencies, results derived here from processing data that are available for free are statistically indistinguishable from the results derived from processing data that are subject to considerable charges.

Next, we make use of PCA to explore whether there exists a common factor across FX rate variances. Using the Bx1 data vectors, $\widehat{\boldsymbol{\alpha}}_{1,BOOT}$, $\widehat{\boldsymbol{\alpha}}_{2,BOOT}$, …, $\widehat{\boldsymbol{\alpha}}_{9,BOOT}$ derived from blocks bootstraps and daily data on FX rate variances (see Tables 1 and 5), the results from PCA reported in Table A.18 show that there is indeed one dominant eigenvalue explaining 51% of the variation in the common space spanned by the vectors $\widehat{\boldsymbol{\alpha}}_{1,BOOT}$, $\widehat{\boldsymbol{\alpha}}_{2,BOOT}$, …, $\widehat{\boldsymbol{\alpha}}_{9,BOOT}$. We consider this as strong evidence for the existence of a common exponent governing FX rate variances. Further, employing the Bx1 data vectors, $\widehat{\boldsymbol{\alpha}}_{1,BOOT}$, $\widehat{\boldsymbol{\alpha}}_{2,BOOT}$, …, $\widehat{\boldsymbol{\alpha}}_{9,BOOT}$ derived from blocks bootstraps and weekly data on FX rate variances (see Tables 2 and 6), the results from PCA reported in Table A.19 strongly confirm this finding because PCA shows that there is only one dominant eigenvalue. However, using both data sets jointly (e.g., estimated power-law exponents derived from daily and weekly data totaling 18 Bx1 data vectors), PCA shows that there are two dominant eigenvalues. The results reported in Table A.20 show that the dominant eigenvalues are in their economic magnitudes very close to the ones retrieved from estimations derived from single data sets (e.g., 5.21 versus 5.05 and 4.44 versus 4.59). The reason for why PCA cannot find one common factor here is perhaps that it is not designed to distinguish between various time frequencies. Using the blocks bootstraps procedure, as outlined in section 4.1.3, the data vectors of estimated power-law exponents are uncorrelated across time frequencies via construction.[19] However, PCA does not account for this issue, and hence, is not an appropriate methodology to test for invariance across time frequencies. The test statistics proposed in sections 4.1.4–4.1.6, however, remedy this issue.

Do our results hold up to scientific replication? To investigate this issue, we report in Table A.21 the descriptive statistics for weekly realized FX variance data over the expanded sample from January, 3, 2001 to May, 31, 2023 derived from a modified weekly realized variance estimator, incorporating squared daily returns, as detailed in section 4.2.6. In Table A.22 we report the corresponding estimated power-law exponents. We observe from Table A.22 that estimated power-law exponents vary between $\widehat{\alpha} = 2.49$ and $\widehat{\alpha} = 3.96$, whereas the estimates reported in Table 4 suggest that estimated exponents vary between $\widehat{\alpha} = 2.35$ and $\widehat{\alpha} = 2.82$. This implies that the estimated power-law exponents derived from our modified weekly realized FX variance estimator using an expanded sample are slightly larger with respect to their economic magnitudes. Nevertheless, for four-out-of-nine weekly realized FX variances, the estimated exponents are $\widehat{\alpha} < 3$, supporting our earlier argument that the variance of realized FX variance is often undefined. Next, in Table A.23 we report the estimated covariance matrix obtained via blocks bootstraps. Strikingly, all covariances are statistically significant,

[17] Note that because $E[m] = 67 = \sqrt{4447}$, it follows that $p = \frac{1}{1+E[m]} = 0.0147$.

[18] Note that Bloomberg provides data on all G10 currencies in the data base.

[19] Note that is accounted for in setting up the proposed test statistics in sections 4.1.3–4.1.6.

as indicated by *t*-statistics ranging between 4.39 and 31.93. This result is in line with Table 6 and supports our argument that the covariance matrix needs to be accounted for when implementing joint tests. Finally, in Table A.24 we report the results from joint tests which have the purpose to investigate whether there is a common exponent that governs the cross section of weekly realized FX variances. Strikingly, the results documented in Table A.24 provide strong evidence for the existence of such a universal exponent: Specifically, the null hypothesis cannot be rejected for $\hat{\alpha} > 2.3$ and $\hat{\alpha} < 3.3$. The optimum is reached for $\hat{\alpha} = 2.8$, validating our argument that the cross-sectional exponent governing realized FX variances is indeed $\alpha < 3$ implying that the second moment is undefined. Overall, we infer that our main results are robust.

6. Discussion

6.1. How do the results line up with the earlier literature?

Consistent with Grobys (2023a), this study examines the power-law behavior of FX variances. Furthermore, the power-law behavior does not alter as we move from daily to weekly frequency, suggesting the fractal-like behavior of FX risk (Mandelbrot, 2008). Whereas previous studies postulate that realized volatility is close to lognormally-distributed (e.g., Andersen et al., 2001; Andersen et al., 2001a & 2001b), the present study's findings do not support this claim. On the contrary, the lognormal model is clearly rejected for all FX variances, whereas the power-law null model holds for most of the FX variances. In line with earlier research (e.g., Grobys, 2021; Grobys, 2023a), the estimated power-law exponents for the FX variances are in the range of $2 < \hat{\alpha}_i < 3$ $\forall i = 1,\dots,9$, indicating that the second moment is not defined. Further, Segnon and Lux (2013) argue that despite the

…tail behavior would remain qualitatively the same under time aggregation, the asymptotic power law would apply in a more and more remote tail region only, and would, therefore, become less and less visible for finite data samples under aggregation. There is, thus, both convergence towards the Normal distribution and stability of power-law behavior in the tail under aggregation. While the former governs the complete shape of the distribution, the latter applies further and further out in the tail only and would only be observed with a sufficiently large number of observations. (Segnon and Lux, 2013, p. 5)

Contrary to the argument raised by Segnon and Lux (2013), the results of the present study indicate that as we move from higher to lower frequented data, the fraction of the sample observations governed by power-law processes increases.

6.2. Implications

The earlier literature explores how to eliminate the FX risk of international portfolios using various hedging strategies or how to minimize the risk in global bond or equity portfolios using hedging positions in foreign currencies (e.g., Glen and Jorion, 1993; Campbell et al., 2010). In contrast, relying on traditional correlation-based methodologies to evaluate FX risk, Kroencke et al. (2014) argue that using style-based management of the FX rates would generate economically large and significant diversification benefits. However, the finding of a common variance risk component, which exhibits the same properties regardless of the time frequency, implies that risk-diversification is perhaps more limited than earlier believed. In fact, the findings of the present study indicate that all FX variances are exposed to large standard deviations in the same manner, which manifests itself in $\alpha^* \approx 2.6$.

How can we use the estimated power-law models to simulate the behavior of FX variances? Given x_{MIN}, the empirical transition probability matrix $\boldsymbol{P}$ can be computed. To give an illustrative example, let us consider the AUD/USD variance and let us define s_1 as the state where observations are not governed by a power-law process and s_2 as the power-law state. Defining $s_{1,t}|s_{2,t-1}$ as the probability to switch from s_2 at time $t-1$ to s_1 in time t, and $s_{2,y}|s_{1,t-1}$ as the probability to switch from s_1 at time $t-1$ to s_2 in time t, the transition probabilities for the AUD/USD variance are then given by:

$$\hat{\boldsymbol{P}} = \begin{pmatrix} s_{1,t}|s_{1,t-1} & s_{1,t}|s_{2,t-1} \\ s_{2,y}|s_{1,t-1} & s_{2,t}|s_{2,t-1} \end{pmatrix} = \begin{pmatrix} 0.5223 & 0.1470 \\ (18.0355) & (-13.4947) \\ 0.1470 & 0.1837 \\ (1.3158) & (1.3517) \end{pmatrix}.$$ [20]

However, in case of independence, we would expect:

$$\boldsymbol{P} = \begin{pmatrix} 0.3347 & 0.3347 \\ 0.1653 & 0.1653 \end{pmatrix},$$

because we know from Table 3 that a fraction of 0.3306 of observations is governed by a power-law process. Hence, the *t*-statistics, which are given in parenthesis in $\hat{\boldsymbol{P}}$, are based on the null hypothesis about whether the difference between the expected probability and observed probability is equal. It becomes evident that the two transition probabilities are statistically significant, suggesting the presence of regime dependence. Next, using the transition probability matrix, the distribution of the AUD/USD variance can be determined. Specifically, let us define N as the number of observations exceeding x_{MIN}—that is, observations governed by a power-law. Then, if the state is s_1, x_t is drawn with the probability $\frac{1}{(T-N)}$ from the vector $(x_1, x_2, \dots, x_{T-N})$, where x_1, x_2, …, x_{T-N} are the sorted observations from the AUD/USD variance vector, where x_{T-N} is the largest observation for which $x_t < x_{MIN}$ is satisfied. Moreover, if the state is s_2, then x_t is drawn from $Cx^{-\alpha}$, where $C = (\alpha-1)x_{MIN}^{\alpha-1}$. Knowing that the common power-law exponent is $\alpha^* = 2.6$, it follows that $C = 9.46E-04$. In the same manner, the distributions for the other FX variances can be simulated.

6.3. Limitations

Range-based variance estimators, such as the ones proposed by Parkinson (1980) and Garman and Klass (1980), have, of course, shortcomings, too. For instance, Wiggins (1991) argues that the Parkinson-estimator and Garman-Klass-estimator are biased downward because the observed highs and lows may be smaller than the actual highs and lows. However, simulation analysis performed by Garman and Klass (1980) and Grammatikos and Saunders (1986) provides strong evidence that the bias decreases with an increasing number of transactions. Since the market capitalization of the FX market is about 30 times higher than that of the equity market, potential biases are perhaps negligible.[21] However, future research is still needed to clarify this issue.

Furthermore, referring to the dollar-Deutschmark FX rate covering the period 1973 to 1996, Mandelbrot (2008) argues that scaling behavior is robust between time intervals of two hours to 180 trading days. The present study employs only daily and weekly variance data over the period May 16, 2006, to May 31, 2023 because data for the AUD/USD exchange rate were only publicly available from May 16, 2006, onward. As a result, the results of the present study are replicable using publicly available data. However, future research is encouraged to test for the total invariance covering a wider spectrum of time frequencies in association with a longer sample period.

[20] Note that the *t*-statistics are given in parentheses and test the difference between point estimates and expected point estimates in case of independency. For example, the *t*-statistic corresponding to −13.4947 tells us that the difference (0.1470–0.3347) is statistically significantly different from zero on a common 5% level.

[21] Note that Shu and Zhang (2006) have analyzed the relative performance of four range-based volatility estimators—including the Parkinson-estimator and Garman-Klass-estimator, among others—for S&P 500 index data. The evidence suggests that the price range estimators all perform very well.

Finally, the present study identifies a commonality in power-law behavior across foreign exchange risks measured in terms of realized FX variances. While the approach of the present study relies on statistical modeling, there is some literature on foreign exchange risk that identifies potential factors that may affect foreign exchange rate risk. For instance, Hasselgren, Peltomäki, and Graham (2020) test the gradual information diffusion hypothesis, which postulates that information spreads gradually across asset markets. The authors' findings indicate that speculators play a vital role in enhancing informational efficiency in the FX market because in the presence of high speculator activity, the equity market's ability to predict the FX market dissipates, whereas this effect happens to be less pronounced for the commodity market. Further, a recent study of Kesse and Blenman (2024) investigates the link between foreign exchange rates and sovereign risk. Their findings demonstrate that sovereign risk affects advanced economies and emerging markets differently. Interestingly, local determinants of sovereign risk (e.g., macro risk and political risk) are priced in the FX markets. In another study on foreign exchange rate risk, Liu, Han, and Yin (2019) employ a GARCH-MIDAS-X component framework to explore the effect of news uncertainty on long-term exchange rate volatility. Their results indicate that news regarding financial intermediation, stock markets and government have a more significantly positive, long-run spillover impact on the volatilities of currencies. Interestingly, even though natural disasters and wars are low probability events, the news regarding them appears to exert an influence on relevant currencies. Finally, Omrane and Savaşer (2017) study the volatility reaction to macroeconomic news in major currency markets during the recent global financial crisis. Their findings show that even though volatility response to most news indicators is larger in expansion, the reaction of the foreign exchange rate market to new home sales and Fed funds rate news is larger in the crisis period. Future research is still needed to explore how these documented factors affect the tail risk of foreign exchange rate variances. This is, however, outside the scope of this paper, and therefore left for future research.

7. Conclusion

Power-law behavior is a manifestation of complex systems (Sornette, 2017; West, 2018). The present study shows that the FX variances are governed by a common component manifested in a joint power-law exponent of $\alpha^* \approx 2.6$. In line with the theory of complex systems, despite this commonality, the FX variances are subject to distinct idiosyncrasies. For instance, a recent study by Grobys (2023a) proposes the concept of co-fractality, which measures the co-dependence of processes governed by power laws. Let us consider two processes, X and Y, and, moreover, let us assume that process X exhibits a larger fraction of the sample observations obeying a power law. The strong co-fractality would imply that >50% of X's observations obeying a power law coincide with Y's observations governed by a power law. Grobys' (2023a) findings indicate that only 3 out of 36 FX variance pairs exhibit strong co-fractality over the full sample period (e.g., May 16, 2006, to November 19, 2021). Thus, the time intervals in which FX variances are governed by power laws must be manifestations of idiosyncrasies. Another market specificity is the fraction of the sample observations governed by power laws: The present study shows that this range is from 4.60% to 33.06% for the daily data. A novel finding of the present study is that it shows that although the FX variances may exhibit power-law behavior at different times, and once they do, they exhibit a common behavior ruled by a common power-law exponent corresponding to $\alpha^* \approx 2.6$. A manifestation of this issue is that the extreme events that have been observed at the cross-section of FX variances are enormous when measured in terms of their standard deviation (Grobys, 2023a).

Because the common power-law exponent governing FX variances is $\alpha^* < 3$, the second moment is not defined. What does that mean? An undefined second moment implies that in a finite sample, estimated sample variances are uninformative. In this regard, Taleb (2020) notes that for power-law exponents corresponding to $\alpha \approx 3$, the central limit operates slowly, and thus, n of the order of at least 10^6 is required so that the inference derived from the central limit becomes acceptable. Taleb (2020) shows that as the economic magnitude of power-law exponents decreases, the required n increases non-linearly. Overall, the results from the present study cast doubts about the validity of the research results derived from GARCH-type models, mean-variance optimizations, or other correlation-based metrics, which require that estimated sample variances are informative. Future research is encouraged to critically examine the validity of the earlier literature. Further, future research is encouraged to identify which sources of factors are associated with the common tail risk component, as suggested in the present study.

Data availability

No

Appendix A. Appendix

Table A.1
Covariance matrix for power-law exponents based on daily data for the first subsample.

Covariance									
(t-Statistic)	$\widehat{\alpha}_{AUD/USD}$	$\widehat{\alpha}_{CAD/USD}$	$\widehat{\alpha}_{CHF/USD}$	$\widehat{\alpha}_{EUR/USD}$	$\widehat{\alpha}_{GBP/USD}$	$\widehat{\alpha}_{JPY/USD}$	$\widehat{\alpha}_{NOK/USD}$	$\widehat{\alpha}_{NZD/USD}$	$\widehat{\alpha}_{SEK/USD}$
$\widehat{\alpha}_{AUD/USD}$	0.0740 (−——)								
$\widehat{\alpha}_{CAD/USD}$	**0.0589** (14.8709)	0.2582 (−——)							
$\widehat{\alpha}_{CHF/USD}$	**0.0176** (7.1042)	0.0091 (1.9118)	0.0874 (−——)						
$\widehat{\alpha}_{EUR/USD}$	**0.0390** (15.2125)	**0.0487** (9.5750)	**0.0278** (9.3671)	0.1091 (−——)					
$\widehat{\alpha}_{GBP/USD}$	**0.0682** (22.3733)	**0.0828** (12.8129)	**0.0688** (20.0792)	**0.0743** (19.1685)	0.1880 (−——)				
$\widehat{\alpha}_{JPY/USD}$	**0.0266** (21.8370)	**0.0327** (12.7309)	**0.0221** (15.2161)	**0.0282** (18.0874)	**0.0493** (27.8640)	0.0296 (−——)			
$\widehat{\alpha}_{NOK/USD}$	**0.0446** (12.0741)	**0.0800** (11.5356)	−0.0048 (−1.1152)	**0.0508** (11.2417)	**0.0587** (9.7568)	**0.0235** (9.8664)	0.2106 (−——)		

(continued on next page)

Table A.1 (*continued*)

Covariance									
$\widehat{\alpha}_{NZD/USD}$	**0.0461**	**0.0638**	**0.0313**	**0.0490**	**0.0881**	**0.0349**	**0.0526**	0.0805	
	(23.5211)	(15.5986)	(12.6985)	(19.3746)	(32.4230)	(32.3958)	(13.9516)	(——)	
$\widehat{\alpha}_{SEK/USD}$	**0.0744**	**0.0903**	**0.0166**	**0.0877**	**0.1265**	**0.0257**	**0.0966**	**0.0727**	0.4572
	(13.9811)	(8.6077)	(2.6360)	(13.4885)	(15.1057)	(7.1641)	(10.3446)	(12.9282)	(——)

The covariance matrix for power-law exponents are obtained via blocks bootstrap. Denoting the selected block length as m, a blocks bootstrap procedure is chosen such that $E[m] = \sqrt{T}$. Then Tx1 data vectors of FX variances $i = 1, \ldots, N$, denoted as $\boldsymbol{x}_i$, are stacked into matrix $\boldsymbol{Y}$:

$$\boldsymbol{Y} = [\boldsymbol{x}_1, \boldsymbol{x}_2, \ldots, \boldsymbol{x}_N].$$

Blocks m are randomly drawn from matrix $\boldsymbol{Y}$ with respect to the time dimension $t = 1, \ldots, T$. These blocks are governed by a geometric distribution, that is, $m \sim GEO(p)$ with $E[m] = \frac{(1-p)}{p}$. Using this procedure, the blocks drawn from $\boldsymbol{Y}$ vary in lengths. Randomly drawn blocks m that have dimensions mxK from data matrix $\boldsymbol{Y}$ are stacked in matrix $\boldsymbol{Y}_b$ as:

$$\boldsymbol{Y}_b = \begin{bmatrix} m_1 \\ m_2 \\ m_3 \\ \vdots \end{bmatrix}.$$

The procedure is stopped when the length of the artificial matrix $\boldsymbol{Y}_b$ exhibits a length exceeding T. Observations exceeding T are cut off; that is, every artificial data matrix $\boldsymbol{Y}_b$ has the same length as the original data matrix $\boldsymbol{Y}$. This process corresponds to one iteration b of the blocks bootstrap procedure. Using this blocks bootstrap procedure, for each iteration b, Tx1 vectors, $\boldsymbol{x}_{b,1}$, $\boldsymbol{x}_{b,2}$, …, $\boldsymbol{x}_{b,N}$ are extracted from matrix $\boldsymbol{Y}_b$ and the MLE estimators are estimated using the procedure described in section 4.1.2, giving us:

$$[\, \widehat{\alpha}_{b,1} \quad \widehat{\alpha}_{b,2} \quad \cdots \quad \widehat{\alpha}_{b,N} \,].$$

This blocks bootstrap procedure is performed for $b = 1, \ldots, 1000$ iterations and point estimates for α are stacked in BxN matrix $\widehat{\boldsymbol{\alpha}}_{BOOT}$:

$$\widehat{\boldsymbol{\alpha}}_{BOOT} = \begin{pmatrix} \widehat{\alpha}_{1,1} & \widehat{\alpha}_{1,2} & \cdots & \cdots & \widehat{\alpha}_{1,N} \\ \widehat{\alpha}_{2,1} & \widehat{\alpha}_{1,2} & \cdots & \cdots & \widehat{\alpha}_{2,N} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \widehat{\alpha}_{B,1} & \widehat{\alpha}_{B,2} & \cdots & \cdots & \widehat{\alpha}_{B,N} \end{pmatrix}.$$

This table reports the covariance matrix for power-law exponents based on daily data using the variance-estimator proposed by Parkinson (1980) and $E[m] = 47$. Data for the first subsample is from May, 16, 2006 to December, 10, 2014 corresponding to 2205 observations. Bold figures indicate statistical significance on at least a 5% level.

Table A.2
Covariance matrix for power-law exponents based on daily data for the second subsample.

Covariance (*t*-Statistic)	$\widehat{\alpha}_{AUD/USD}$	$\widehat{\alpha}_{CAD/USD}$	$\widehat{\alpha}_{CHF/USD}$	$\widehat{\alpha}_{EUR/USD}$	$\widehat{\alpha}_{GBP/USD}$	$\widehat{\alpha}_{JPY/USD}$	$\widehat{\alpha}_{NOK/USD}$	$\widehat{\alpha}_{NZD/USD}$	$\widehat{\alpha}_{SEK/USD}$
$\widehat{\alpha}_{AUD/USD}$	0.0657								
	(——)								
$\widehat{\alpha}_{CAD/USD}$	**0.0321**	0.0958							
	(13.9801)	(——)							
$\widehat{\alpha}_{CHF/USD}$	**0.0099**	**0.0043**	0.0273						
	(7.5588)	(2.6553)	(——)						
$\widehat{\alpha}_{EUR/USD}$	**0.0049**	−0.0010	**0.0137**	0.0510					
	(2.6602)	(−0.4631)	(12.4636)	(——)					
$\widehat{\alpha}_{GBP/USD}$	**0.0362**	**0.0296**	**0.0083**	0.0001	0.1031				
	(15.4553)	(9.8759)	(5.0001)	(0.0301)	(——)				
$\widehat{\alpha}_{JPY/USD}$	**0.0103**	**0.0116**	**0.0024**	0.0007	**0.0157**	0.0177			
	(9.9772)	(9.2502)	(3.4556)	(0.7767)	(12.4935)	(——)			
$\widehat{\alpha}_{NOK/USD}$	**0.0412**	**0.0333**	**0.0074**	**0.0072**	**0.0339**	**0.0100**	0.0544		
	(29.9764)	(16.4272)	(6.1933)	(4.3696)	(16.0591)	(10.7716)	(——)		
$\widehat{\alpha}_{NZD/USD}$	**0.0435**	**0.0354**	**0.0085**	**0.0050**	**0.0408**	**0.0130**	**0.0455**	0.0653	
	(28.0506)	(15.8204)	(6.5125)	(2.7637)	(18.1227)	(13.0806)	(37.2268)	(——)	
$\widehat{\alpha}_{SEK/USD}$	**0.0332**	**0.0340**	0.0014	0.0018	**0.0340**	**0.0104**	**0.0366**	**0.0378**	0.0500
	(22.4698)	(17.8457)	(1.2249)	(1.1397)	(16.9698)	(11.8024)	(31.0744)	(27.8939)	(——)

The covariance matrix for power-law exponents are obtained via blocks bootstrap. Denoting the selected block length as m, a blocks bootstrap procedure is chosen such that $E[m] = \sqrt{T}$. Then Tx1 data vectors of FX variances $i = 1, \ldots, N$, denoted as $\boldsymbol{x}_i$, are stacked into matrix $\boldsymbol{Y}$:

$$\boldsymbol{Y} = [\boldsymbol{x}_1, \boldsymbol{x}_2, \ldots, \boldsymbol{x}_N].$$

Blocks m are randomly drawn from matrix $\boldsymbol{Y}$ with respect to the time dimension $t = 1,\dots,T$. These blocks are governed by a geometric distribution, that is, $m \sim GEO(p)$ with $E[m] = \frac{(1-p)}{p}$. Using this procedure, the blocks drawn from $\boldsymbol{Y}$ vary in lengths. Randomly drawn blocks m that have dimensions mxK from data matrix $\boldsymbol{Y}$ are stacked in matrix $\boldsymbol{Y}_b$ as:

$$\boldsymbol{Y}_b = \begin{bmatrix} m_1 \\ m_2 \\ m_3 \\ \vdots \end{bmatrix}.$$

The procedure is stopped when the length of the artificial matrix $\boldsymbol{Y}_b$ exhibits a length exceeding T. Observations exceeding T are cut off; that is, every artificial data matrix $\boldsymbol{Y}_b$ has the same length as the original data matrix $\boldsymbol{Y}$. This process corresponds to one iteration b of the blocks bootstrap procedure. Using this blocks bootstrap procedure, for each iteration b, Tx1 vectors, $\boldsymbol{x}_{b,1}$, $\boldsymbol{x}_{b,2}$, …, $\boldsymbol{x}_{b,N}$ are extracted from matrix $\boldsymbol{Y}_b$ and the MLE estimators are estimated using the procedure described in section 4.1.2, giving us:

$$[\,\widehat{\alpha}_{b,1} \quad \widehat{\alpha}_{b,2} \quad \cdots \quad \widehat{\alpha}_{b,N}\,].$$

This blocks bootstrap procedure is performed for $b = 1, \dots, 1000$ iterations and point estimates for α are stacked in BxN matrix $\widehat{\boldsymbol{\alpha}}_{BOOT}$:

$$\widehat{\boldsymbol{\alpha}}_{BOOT} = \begin{pmatrix} \widehat{\alpha}_{1,1} & \widehat{\alpha}_{1,2} & \dots & \dots & \widehat{\alpha}_{1,N} \\ \widehat{\alpha}_{2,1} & \widehat{\alpha}_{1,2} & \dots & \dots & \widehat{\alpha}_{2,N} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \widehat{\alpha}_{B,1} & \widehat{\alpha}_{B,2} & \dots & \dots & \widehat{\alpha}_{B,N} \end{pmatrix}.$$

This table reports the covariance matrix for power-law exponents based on daily data using the variance-estimator proposed by Parkinson (1980) and $E[m] = 47$. Data for the second subsample is from December, 11, 2014 to May, 31, 2023 corresponding to 2205 observations. Bold figures indicate statistical significance on at least a 5% level.

Table A.3
Covariance matrix for power-law exponents based on weekly data for the first subsample.

Covariance									
(t-Statistic)	$\widehat{\alpha}_{AUD/USD}$	$\widehat{\alpha}_{CAD/USD}$	$\widehat{\alpha}_{CHF/USD}$	$\widehat{\alpha}_{EUR/USD}$	$\widehat{\alpha}_{GBP/USD}$	$\widehat{\alpha}_{JPY/USD}$	$\widehat{\alpha}_{NOK/USD}$	$\widehat{\alpha}_{NZD/USD}$	$\widehat{\alpha}_{SEK/USD}$
$\widehat{\alpha}_{AUD/USD}$	0.1517 (——)								
$\widehat{\alpha}_{CAD/USD}$	**0.1198** (17.6996)	0.3956 (——)							
$\widehat{\alpha}_{CHF/USD}$	**0.0581** (8.5991)	**0.1364** (13.0485)	0.3226 (——)						
$\widehat{\alpha}_{EUR/USD}$	**0.0815** (13.1823)	**0.1707** (18.2361)	**0.1035** (11.2542)	0.2948 (——)					
$\widehat{\alpha}_{GBP/USD}$	**0.1002** (14.8276)	**0.1509** (13.6390)	**0.1288** (12.7662)	**0.1389** (14.7401)	0.3662 (——)				
$\widehat{\alpha}_{JPY/USD}$	**0.0506** (10.6174)	**0.0916** (12.0876)	**0.0714** (10.2449)	**0.0469** (6.8534)	**0.0760** (10.2333)	0.1660 (——)			
$\widehat{\alpha}_{NOK/USD}$	**0.1360** (18.7137)	**0.2558** (23.3262)	**0.1478** (12.9799)	**0.1635** (15.4707)	**0.2005** (17.4692)	**0.1041** (12.6981)	0.4689 (——)		
$\widehat{\alpha}_{NZD/USD}$	**0.0499** (13.2097)	**0.0724** (11.6743)	**0.0521** (9.0737)	**0.0467** (8.4733)	**0.0957** (17.1252)	**0.0408** (9.9953)	**0.0842** (12.6047)	0.1102 (——)	
$\widehat{\alpha}_{SEK/USD}$	**0.1167** (13.7889)	**0.1579** (11.2443)	**0.0524** (3.9257)	**0.1460** (12.1507)	**0.1418** (10.4120)	**0.0760** (8.1275)	**0.2646** (19.0349)	**0.0607** (7.9491)	0.5607 (——)

The covariance matrix for power-law exponents are obtained via blocks bootstrap. Denoting the selected block length as m, a blocks bootstrap procedure is chosen such that $E[m] = \sqrt{T}$. Then Tx1 data vectors of FX variances $i = 1,\dots,N$, denoted as $\boldsymbol{x}_i$, are stacked into matrix $\boldsymbol{Y}$:

$$\boldsymbol{Y} = [\boldsymbol{x}_1, \boldsymbol{x}_2, \dots, \boldsymbol{x}_N].$$

Blocks m are randomly drawn from matrix $\boldsymbol{Y}$ with respect to the time dimension $t = 1,\dots,T$. These blocks are governed by a geometric distribution, that is, $m \sim GEO(p)$ with $E[m] = \frac{(1-p)}{p}$. Using this procedure, the blocks drawn from $\boldsymbol{Y}$ vary in lengths. Randomly drawn blocks m that have dimensions mxK from data matrix $\boldsymbol{Y}$ are stacked in matrix $\boldsymbol{Y}_b$ as:

$$\boldsymbol{Y}_b = \begin{bmatrix} m_1 \\ m_2 \\ m_3 \\ \vdots \end{bmatrix}.$$

The procedure is stopped when the length of the artificial matrix $\boldsymbol{Y}_b$ exhibits a length exceeding T. Observations exceeding T are cut off; that is, every artificial data matrix $\boldsymbol{Y}_b$ has the same length as the original data matrix $\boldsymbol{Y}$. This process corresponds to one iteration b of the blocks bootstrap procedure. Using this blocks bootstrap procedure, for each iteration b, Tx1 vectors, $\boldsymbol{x}_{b,1}$, $\boldsymbol{x}_{b,2}$, …, $\boldsymbol{x}_{b,N}$ are extracted from matrix $\boldsymbol{Y}_b$ and the MLE estimators are estimated using the procedure described in section 4.1.2, giving us:

$$[\,\widehat{\alpha}_{b,1} \quad \widehat{\alpha}_{b,2} \quad \cdots \quad \widehat{\alpha}_{b,N}\,].$$

This blocks bootstrap procedure is performed for $b = 1, \ldots, 1000$ iterations and point estimates for α are stacked in BxN matrix $\widehat{\boldsymbol{\alpha}}_{BOOT}$:

$$\widehat{\boldsymbol{\alpha}}_{BOOT} = \begin{pmatrix} \widehat{\alpha}_{1,1} & \widehat{\alpha}_{1,2} & \ldots & \ldots & \widehat{\alpha}_{1,N} \\ \widehat{\alpha}_{2,1} & \widehat{\alpha}_{1,2} & \ldots & \ldots & \widehat{\alpha}_{2,N} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \widehat{\alpha}_{B,1} & \widehat{\alpha}_{B,2} & \ldots & \ldots & \widehat{\alpha}_{B,N} \end{pmatrix}.$$

This table reports the covariance matrix for power-law exponents based on daily data using the variance-estimator proposed by Parkinson (1980) and $E[m] = 21$. Data for the first subsample is from May, 16, 2006 to December, 10, 2014 corresponding to 441 observations. Bold figures indicate statistical significance on at least a 5% level.

Table A.4
Covariance matrix for power-law exponents based on weekly data for the second subsample

Covariance (*t*-Statistic)	$\widehat{\alpha}_{AUD/USD}$	$\widehat{\alpha}_{CAD/USD}$	$\widehat{\alpha}_{CHF/USD}$	$\widehat{\alpha}_{EUR/USD}$	$\widehat{\alpha}_{GBP/USD}$	$\widehat{\alpha}_{JPY/USD}$	$\widehat{\alpha}_{NOK/USD}$	$\widehat{\alpha}_{NZD/USD}$	$\widehat{\alpha}_{SEK/USD}$
$\widehat{\alpha}_{AUD/USD}$	0.3417 (−——)								
$\widehat{\alpha}_{CAD/USD}$	**0.1583** (21.5310)	0.2313 (−——)							
$\widehat{\alpha}_{CHF/USD}$	**−0.0128** (−2.0147)	**−0.0196** (−3.7754)	0.1183 (−——)						
$\widehat{\alpha}_{EUR/USD}$	**−0.0356** (−5.3357)	**−0.0498** (−9.3319)	**0.0770** (24.4838)	0.1334 (−——)					
$\widehat{\alpha}_{GBP/USD}$	**0.1082** (16.9323)	**0.0438** (7.5450)	0.0042 (0.9859)	−0.0025 (−0.5590)	0.1534 (−——)				
$\widehat{\alpha}_{JPY/USD}$	**0.0344** (5.7548)	**0.0303** (6.1655)	0.0056 (1.5681)	**−0.0093** (−2.4441)	**0.0219** (5.4611)	0.1080 (−——)			
$\widehat{\alpha}_{NOK/USD}$	**0.1047** (18.0218)	**0.0702** (13.9517)	**0.0109** (2.7671)	−0.0077 (−1.8526)	**0.0379** (8.7811)	**0.0166** (4.4501)	0.1307 (−——)		
$\widehat{\alpha}_{NZD/USD}$	**0.2975** (38.3441)	**0.1654** (19.3129)	0.0067 (0.9317)	**−0.0225** (−2.9597)	**0.0940** (12.3457)	**0.0438** (6.5181)	**0.1231** (19.0395)	0.4348 (−——)	
$\widehat{\alpha}_{SEK/USD}$	**0.0828** (20.9341)	**0.0477** (13.2684)	0.0050 (1.7978)	−0.0043 (−1.4398)	**0.0534** (19.8707)	**0.0161** (6.1329)	**0.0447** (17.4157)	**0.0877** (19.1667)	0.0657 (−——)

The covariance matrix for power-law exponents are obtained via blocks bootstrap. Denoting the selected block length as m, a blocks bootstrap procedure is chosen such that $E[m] = \sqrt{T}$. Then Tx1 data vectors of FX variances $i = 1, \ldots, N$, denoted as $\boldsymbol{x}_i$, are stacked into matrix $\boldsymbol{Y}$:

$$\boldsymbol{Y} = [\boldsymbol{x}_1, \boldsymbol{x}_2, \ldots, \boldsymbol{x}_N].$$

Blocks m are randomly drawn from matrix $\boldsymbol{Y}$ with respect to the time dimension $t = 1, \ldots, T$. These blocks are governed by a geometric distribution, that is, $m \sim GEO(p)$ with $E[m] = \frac{(1-p)}{p}$. Using this procedure, the blocks drawn from $\boldsymbol{Y}$ vary in lengths. Randomly drawn blocks m that have dimensions mxK from data matrix $\boldsymbol{Y}$ are stacked in matrix $\boldsymbol{Y}_b$ as:

$$\boldsymbol{Y}_b = \begin{bmatrix} m_1 \\ m_2 \\ m_3 \\ \vdots \end{bmatrix}.$$

The procedure is stopped when the length of the artificial matrix $\boldsymbol{Y}_b$ exhibits a length exceeding T. Observations exceeding T are cut off; that is, every artificial data matrix $\boldsymbol{Y}_b$ has the same length as the original data matrix $\boldsymbol{Y}$. This process corresponds to one iteration b of the blocks bootstrap procedure. Using this blocks bootstrap procedure, for each iteration b, Tx1 vectors, $\boldsymbol{x}_{b,1}$, $\boldsymbol{x}_{b,2}$, …, $\boldsymbol{x}_{b,N}$ are extracted from matrix $\boldsymbol{Y}_b$ and the MLE estimators are estimated using the procedure described in section 4.1.2, giving us:

$$[\,\widehat{\alpha}_{b,1} \quad \widehat{\alpha}_{b,2} \quad \cdots \quad \widehat{\alpha}_{b,N}\,].$$

This blocks bootstrap procedure is performed for $b = 1, \ldots, 1000$ iterations and point estimates for α are stacked in BxN matrix $\widehat{\boldsymbol{\alpha}}_{BOOT}$:

$$\widehat{\boldsymbol{\alpha}}_{BOOT} = \begin{pmatrix} \widehat{\alpha}_{1,1} & \widehat{\alpha}_{1,2} & \ldots & \ldots & \widehat{\alpha}_{1,N} \\ \widehat{\alpha}_{2,1} & \widehat{\alpha}_{1,2} & \ldots & \ldots & \widehat{\alpha}_{2,N} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \widehat{\alpha}_{B,1} & \widehat{\alpha}_{B,2} & \ldots & \ldots & \widehat{\alpha}_{B,N} \end{pmatrix}.$$

This table reports the covariance matrix for power-law exponents based on daily data using the variance-estimator proposed by Parkinson (1980) and $E[m] = 21$. Data for the second subsample is from December, 11, 2014 to May, 31, 2023 corresponding to 441 observations. Bold figures indicate statistical significance on at least a 5% level.

Table A.5
Descriptive statistics of daily range-based variances using the Garman-Klass-estimator

	AUD/USD	CAD/USD	CHF/USD	EUR/USD	GBP/USD	JPY/USD	NOK/USD	NZD/USD	SEK/USD
Mean	0.1263	0.0086	0.0127	0.2777	0.0094	0.0095	0.0197	0.0203	0.0168
Median	0.0083	0.0047	0.0052	0.0047	0.0052	0.0048	0.0105	0.0088	0.0089
Maximum	241.2387	0.3642	13.8377	1187.7760	1.2456	0.5358	2.2629	19.8368	1.6670
Minimum	3.36E-06	1.75E-05	2.01E-07	1.34E-05	8.51E-06	3.61E-07	1.12E-04	2.14E-05	1.14E-04
Std. Dev.	5.1229	0.0146	0.2100	17.8961	0.0240	0.0206	0.0579	0.2999	0.0430
Skewness	46.8942	9.1387	64.8507	66.3475	33.5581	11.7178	24.6947	65.5281	24.2019
Kurtosis	2200.2540	156.6508	4266.3580	4402.9970	1638.210	208.5783	784.4196	4330.0280	799.6652
Jarque-Bera	8.88E+08	4.39E+06	3.34E+09	3.56E+09	4.92E+08	7.86E+06	1.13E+08	3.44E+09	1.17E+08
Probability	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Observations	4405	4405	4405	4405	4405	4405	4405	4405	4405

Publicly available intraday prices for the AUD/USD, CAD/USD, CHF/USD, EUR/USD, GBP/USD, JPY/USD, NOK/USD, NZD/USD, and SEK/USD exchange rates were downloaded from finance.yahoo.com. The sample period is from May, 16, 2006 to May, 31, 2023. Only the intersection of the data is employed; that is, only daily data are accounted for where all FX rates were quoted on the same day, leaving us with 4410 daily observations. To estimate annualized daily variances, the range-based variance estimator proposed by Garman and Klass (1980) is employed which is given by:

$$\sigma^2_{i,t} = T\left(0.5\left[ln\left(\frac{H_{i,t}}{L_{i,t}}\right)\right]^2 - [2ln(2)-1]\left[ln\left(\frac{C_{i,t}}{O_{i,t}}\right)\right]^2\right)$$

where $H_{i,t}$ and $L_{i,t}$ denote the highest and lowest prices for foreign exchange rate market i on trading day t, and $\sigma^2_{i,t}$ denotes foreign exchange rate market i's annualized daily variance where $T = 250$ trading days per annum are assumed. This table reports the descriptive statistics.

Table A.6
Estimated power-law exponents for daily variances based on the Garman-Klass-estimator

	$\hat{\sigma}^2_{AUD/USD}$	$\hat{\sigma}^2_{CAD/USD}$	$\hat{\sigma}^2_{CHF/USD}$	$\hat{\sigma}^2_{EUR/USD}$	$\hat{\sigma}^2_{GBP/USD}$	$\hat{\sigma}^2_{JPY/USD}$	$\hat{\sigma}^2_{NOK/USD}$	$\hat{\sigma}^2_{NZD/USD}$	$\hat{\sigma}^2_{SEK/USD}$
$\hat{\alpha}$	2.4356	2.5552	2.8470	2.9785	2.7790	2.6366	2.6919	2.8130	2.9113
$\hat{x}_{min}$	0.0254	0.0103	0.0214	0.0182	0.0174	0.0176	0.0287	0.0396	0.0383
$\hat{\sigma}$	0.0587	0.0501	0.0985	0.0975	0.0810	0.0744	0.0655	0.1036	0.1005
N	632	1005	372	432	511	510	697	325	381
(%)	(14.35%)	(22.81%)	(8.44%)	(9.81%)	(11.60%)	(11.58%)	(15.82%)	(7.38%)	(8.65%)

FX variances are modeled using the following power-law function:

$$p(x) = Cx^{-\alpha},$$

where $C = (\alpha-1)x_{MIN}^{\alpha-1}$ with $\alpha \in \{\mathbb{R}_+ | \alpha > 1\}$, x denotes the respective annualized daily or weekly FX variance provided $x \in \{\mathbb{R}_+ | x_{MIN} \le x < \infty\}$, x_{MIN} is the minimum value governed by the power-law process, and α is the magnitude of the corresponding tail exponent. Following White et al. (2008) and Clauset et al. (2009), the tail exponents are estimated as:

$$\hat{\alpha} = 1 + N\left(\sum_{i=1}^{N} ln\left(\frac{x_i}{x_{MIN}}\right)\right)^{-1},$$

where $\hat{\alpha}$ denotes the MLE estimator, N is the number of observations exceeding x_{MIN}, and other notations are as previously defined. The estimate $\hat{\alpha}$ is selected based on the optimal Kolmogorov–Smirnov (KS) distance D measuring the maximum distance between the cumulative density functions (CDFs) of the data and the fitted power-law model as defined by:

$$D = MAX_{x \ge x_{MIN}} |S(x) - P(x)|,$$

where $S(x)$ is the CDF of the data for the observation with a value of at least x_{MIN}, and $P(x)$ is the CDF for the power-law model that best fits the data in the region $x \ge x_{MIN}$. Estimate $\hat{x}_{MIN}$ is then the value of x_{MIN} that minimizes D. This table reports the estimates $\hat{\alpha}$, $\hat{x}_{min}$, $\hat{\sigma}$, and N for daily FX variance data. The sample period is from May, 16, 2006 to May, 31, 2023 comprising 4405 daily observations.

Table A.7
Covariance matrix for power-law exponents based on daily data and Garman-Klass- estimator

Covariance									
(t-Statistic)	$\hat{\alpha}_{AUD/USD}$	$\hat{\alpha}_{CAD/USD}$	$\hat{\alpha}_{CHF/USD}$	$\hat{\alpha}_{EUR/USD}$	$\hat{\alpha}_{GBP/USD}$	$\hat{\alpha}_{JPY/USD}$	$\hat{\alpha}_{NOK/USD}$	$\hat{\alpha}_{NZD/USD}$	$\hat{\alpha}_{SEK/USD}$
$\hat{\alpha}_{AUD/USD}$	0.0686								
	(——–)								

(continued on next page)

Table A.7 (*continued*)

Covariance									
$\widehat{\alpha}_{CAD/USD}$	**0.0748**	0.2261							
	(23.7167)	(–——)							
$\widehat{\alpha}_{CHF/USD}$	**0.0122**	**0.0136**	0.0483						
	(6.8622)	(4.1329)	(–——)						
$\widehat{\alpha}_{EUR/USD}$	**0.0189**	**0.0383**	**0.0041**	0.0440					
	(11.5828)	(13.1357)	(2.8542)	(–——)					
$\widehat{\alpha}_{GBP/USD}$	**0.0379**	**0.0577**	**0.0169**	**0.0230**	0.0609				
	(22.8244)	(17.8128)	(10.3575)	(15.6464)	(–——)				
$\widehat{\alpha}_{JPY/USD}$	**0.0137**	**0.0215**	**0.0052**	**0.0063**	**0.0133**	0.0125			
	(16.6205)	(13.9264)	(6.8408)	(8.8419)	(17.3374)	(–——)			
$\widehat{\alpha}_{NOK/USD}$	**0.0237**	**0.0338**	−0.0001	**0.0111**	**0.0258**	**0.0114**	0.0728		
	(11.2630)	(8.6375)	(−0.0356)	(6.3155)	(13.2510)	(12.8828)	(–——)		
$\widehat{\alpha}_{NZD/USD}$	**0.0393**	**0.0596**	**0.0138**	**0.0205**	**0.0378**	**0.0143**	**0.0243**	0.0658	
	(22.8173)	(17.7097)	(7.9629)	(13.0522)	(23.5165)	(18.0909)	(11.8264)	(–——)	
$\widehat{\alpha}_{SEK/USD}$	**0.0352**	**0.0637**	**0.0059**	**0.0269**	**0.0384**	**0.0143**	**0.0384**	**0.0370**	0.1035
	(14.5454)	(14.4677)	(2.6275)	(13.7095)	(17.4402)	(13.6159)	(15.5497)	(15.8575)	(–——)

The covariance matrix for power-law exponents are obtained via blocks bootstrap. Denoting the selected block length as m, a blocks bootstrap procedure is chosen such that $E[m] = \sqrt{T}$. Then Tx1 data vectors of FX variances $i = 1, \ldots, N$, denoted as $\boldsymbol{x}_i$, are stacked into matrix $\boldsymbol{Y}$:

$$\boldsymbol{Y} = [\boldsymbol{x}_1, \boldsymbol{x}_2, \ldots, \boldsymbol{x}_N].$$

Blocks m are randomly drawn from matrix $\boldsymbol{Y}$ with respect to the time dimension $t = 1,\ldots,T$. These blocks are governed by a geometric distribution, that is, $m \sim GEO(p)$ with $E[m] = \frac{(1-p)}{p}$. Using this procedure, the blocks drawn from $\boldsymbol{Y}$ vary in lengths. Randomly drawn blocks m that have dimensions mxK from data matrix $\boldsymbol{Y}$ are stacked in matrix $\boldsymbol{Y}_b$ as:

$$\boldsymbol{Y}_b = \begin{bmatrix} m_1 \\ m_2 \\ m_3 \\ \vdots \end{bmatrix}.$$

The procedure is stopped when the length of the artificial matrix $\boldsymbol{Y}_b$ exhibits a length exceeding T. Observations exceeding T are cut off; that is, every artificial data matrix $\boldsymbol{Y}_b$ has the same length as the original data matrix $\boldsymbol{Y}$. This process corresponds to one iteration b of the blocks bootstrap procedure. Using this blocks bootstrap procedure, for each iteration b, Tx1 vectors, $\boldsymbol{x}_{b,1}$, $\boldsymbol{x}_{b,2}$, …, $\boldsymbol{x}_{b,N}$ are extracted from matrix $\boldsymbol{Y}_b$ and the MLE estimators are estimated using the procedure described in section 4.1.2, giving us:

$$[\,\widehat{\alpha}_{b,1} \quad \widehat{\alpha}_{b,2} \quad \cdots \quad \widehat{\alpha}_{b,N}\,].$$

This blocks bootstrap procedure is performed for $b = 1, \ldots, 1000$ iterations and point estimates for α are stacked in BxN matrix $\widehat{\boldsymbol{\alpha}}_{BOOT}$:

$$\widehat{\boldsymbol{\alpha}}_{BOOT} = \begin{pmatrix} \widehat{\alpha}_{1,1} & \widehat{\alpha}_{1,2} & \ldots & \ldots & \widehat{\alpha}_{1,N} \\ \widehat{\alpha}_{2,1} & \widehat{\alpha}_{1,2} & \ldots & \ldots & \widehat{\alpha}_{2,N} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \widehat{\alpha}_{B,1} & \widehat{\alpha}_{B,2} & \ldots & \ldots & \widehat{\alpha}_{B,N} \end{pmatrix}.$$

This table reports the covariance matrix for power-law exponents based on daily data using the variance-estimator proposed by Garman and Klass (1980) and $E[m] = 66$. Data for the overall sample is from May, 16, 2006 to May, 31, 2023 corresponding to 4405 observations. Bold figures indicate statistical significance on at least a 5% level.

Table A.8
Testing standard distributions and hypothesized power-law models

	AUD/USD	CAD/USD	CHF/USD	EUR/USD	GBP/USD	JPY/USD	NOK/USD	NZD/USD	SEK/USD
$prob(x \leq \pm 1\sigma)$	0.9995	0.9317	0.9986	0.9998	0.9587	0.9506	0.9780	0.9980	0.9535
$H_0 : \sigma^2_{i,t} \sim LGN$	970.3944	378.6708	960.6742	972.8322	581.4213	515.5164	752.7563	953.4161	538.8582
(*p*-value)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
$H_0 : \sigma^2_{i,t} \sim \chi^2(1)$	598.5342	112.8791	589.5088	600.8013	260.2700	209.2788	402.7036	582.7847	227.0516
(*p*-value)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
p-value for GoF test[a] $\left(H_0 : \sigma^2_{i,t} \sim PL\left(\widehat{\alpha}, \widehat{\sigma}^2_{i,min}\right)\right)$	**0.2710**	0.0000	**0.4420**	**0.8470**	0.0170	**0.4500**	0.0180	**0.8060**	**0.4170**
p-value for GoF test[b] $\left(H_0 : \sigma^2_{i,t} \sim PL\left(\widehat{\alpha}, \widehat{\sigma}^2_{i,min}\right)\right)$	**0.3380**	0.0000	**0.7150**	**0.8670**	**0.3850**	**0.3990**	**0.0830**	**0.1420**	**0.1870**

[a] These *p*-values are based on Clauset et al.'s (2009) GoF test using the Parkinson estimator for $\sigma^2_{i,t}$.
[b] These *p*-values are based on Clauset et al.'s (2009) GoF test using the Garman-Klass estimator for $\sigma^2_{i,t}$.

The following one-sigma-test is employed to test whether the data-generating process governing FX variances is either log-normal, $\chi^2(1)$, or a power law:

$$\lambda = \frac{(x_{\le\pm1\sigma} - m_{\le\pm1\sigma})^2}{m_{\le\pm1\sigma}} + \frac{(x_{>\pm1\sigma} - m_{>\pm1\sigma})^2}{m_{>\pm1\sigma}},$$

where $m_{\le\pm1\sigma}$ denotes the expected number of observations occurring within one standard deviation from the mean of some specified distribution which can be either log-normal or $\chi^2(1)$, $m_{>\pm1\sigma}$ denotes the expected number of observations exceeding one standard deviation from the mean, whereas $x_{\le\pm1\sigma}$ and $x_{>\pm1\sigma}$ are the observed values for the distribution of some FX variance. Pearson (1900) showed that this type of test statistic is distributed as $\chi^2(1)$, as $T \longrightarrow \infty$. Note that it can be shown that a fraction of 0.8189 is within one standard deviation from the mean for a standardized lognormal distribution (LGN), whereas the corresponding figure for a standardized $\chi^2(1)$ distribution is 0.8797. For testing the power-law models, the goodness-of-fit (GoF) test proposed by Clauset et al. (2009) is employed which is summarized in section 4.2.2. The power-law model parametrizations are based on the original data sets are used as hypothesized power-law models for running the GoF tests. Bold figures indicate that the null hypothesis cannot be rejected on a common 5% significance level. The tests are implemented for daily data covering the whole sample from May, 16, 2006 to May, 31, 2023.

Table A.9
Descriptive statistics for bootstrapped power-law exponents using daily data and an expected block length of $T^{1/3}$

	$\hat{\alpha}_{AUD/USD}$	$\hat{\alpha}_{CAD/USD}$	$\hat{\alpha}_{CHF/USD}$	$\hat{\alpha}_{EUR/USD}$	$\hat{\alpha}_{GBP/USD}$	$\hat{\alpha}_{JPY/USD}$	$\hat{\alpha}_{NOK/USD}$	$\hat{\alpha}_{NZD/USD}$	$\hat{\alpha}_{SEK/USD}$
Mean	2.3244	2.5547	2.7917	2.7765	2.6122	2.5852	2.6579	2.5692	2.5673
Median	2.3043	2.4682	2.7735	2.7610	2.5969	2.5808	2.5893	2.5486	2.5201
Maximum	3.1035	4.2316	3.8544	4.7237	3.3297	3.1687	3.8397	3.4965	4.0708
Minimum	1.9308	1.9949	2.1541	2.0846	2.0211	2.1965	2.1665	2.0219	1.9706
Std. Dev.	0.1524	0.3067	0.2151	0.2631	0.2205	0.1256	0.2599	0.2191	0.2796
Skewness	0.9717	1.6698	0.3936	0.9409	0.3372	0.4099	1.2664	0.4395	0.9553
Kurtosis	5.3719	6.2128	3.9452	6.5492	2.9098	4.0342	4.8047	3.2779	4.4241
Jarque-Bera	391.7706	894.7718	63.0388	672.4225	19.2900	72.5588	402.9887	35.4092	236.5837
(*p*-value)	0.0000	0.0000	0.0000	0.0000	0.0001	0.0000	0.0000	0.0000	0.0000
Observations	1000	1000	1000	1000	1000	1000	1000	1000	1000

This table reports the descriptive statistics for bootstrapped power-law exponents using variance-estimator proposed by Parkinson (1980) implemented for daily data. The results are based on $B = 1000$ blocks bootstrap replications and $E[m] = 16$. The blocks bootstrap procedure is detailed in section 4.1.2.

Table A.10
Covariance matrix for power-law exponents based on daily data and an expected block length of $T^{1/3}$

Covariance									
(*t*-Statistic)	$\hat{\alpha}_{AUD/USD}$	$\hat{\alpha}_{CAD/USD}$	$\hat{\alpha}_{CHF/USD}$	$\hat{\alpha}_{EUR/USD}$	$\hat{\alpha}_{GBP/USD}$	$\hat{\alpha}_{JPY/USD}$	$\hat{\alpha}_{NOK/USD}$	$\hat{\alpha}_{NZD/USD}$	$\hat{\alpha}_{SEK/USD}$
$\hat{\alpha}_{AUD/USD}$	0.0232 (——)								
$\hat{\alpha}_{CAD/USD}$	**0.0185** (13.6627)	0.0939 (——)							
$\hat{\alpha}_{CHF/USD}$	**0.0076** (7.4956)	**0.0080** (3.8482)	0.0462 (——)						
$\hat{\alpha}_{EUR/USD}$	**0.0150** (12.7455)	**0.0233** (9.5211)	**0.0125** (7.1322)	0.0691 (——)					
$\hat{\alpha}_{GBP/USD}$	**0.0196** (22.6632)	**0.0283** (14.5637)	**0.0149** (10.4816)	**0.0249** (15.0677)	0.0486 (——)				
$\hat{\alpha}_{JPY/USD}$	**0.0086** (15.8444)	**0.0109** (9.2932)	**0.0027** (3.1940)	**0.0103** (10.4022)	**0.0118** (14.8755)	0.0158 (——)			
$\hat{\alpha}_{NOK/USD}$	**0.0119** (10.0017)	0.0153 (6.1933)	0.0003 (0.1637)	**0.0100** (4.6758)	**0.0189** (11.0510)	**0.0071** (7.0319)	0.0675 (——)		
$\hat{\alpha}_{NZD/USD}$	**0.0188** (21.5464)	**0.0268** (13.7325)	**0.0119** (8.2452)	**0.0232** (13.9324)	**0.0284** (23.0077)	**0.0114** (14.3370)	**0.0208** (12.4243)	0.0479 (——)	
$\hat{\alpha}_{SEK/USD}$	**0.0153** (12.1370)	**0.0231** (8.8523)	**0.0079** (4.1790)	**0.0169** (7.4780)	**0.0253** (14.2152)	**0.0076** (7.0233)	**0.0214** (9.7506)	**0.0255** (14.5101)	0.0781 (——)

The covariance matrix for power-law exponents are obtained via blocks bootstrap. Denoting the selected block length as m, a blocks bootstrap procedure is chosen such that $E[m] = T^{1/3}$. Then Tx1 data vectors of FX variances $i = 1, \ldots, N$, denoted as $\boldsymbol{x}_i$, are stacked into matrix $\boldsymbol{Y}$:

$$\boldsymbol{Y} = [\boldsymbol{x}_1, \boldsymbol{x}_2, \ldots, \boldsymbol{x}_N].$$

Blocks m are randomly drawn from matrix $\boldsymbol{Y}$ with respect to the time dimension $t = 1, \ldots, T$. These blocks are governed by a geometric distribution, that is, $m \sim GEO(p)$ with $E[m] = \frac{(1-p)}{p}$. Using this procedure, the blocks drawn from $\boldsymbol{Y}$ vary in lengths. Randomly drawn blocks m that have dimensions mxK from data matrix $\boldsymbol{Y}$ are stacked in matrix $\boldsymbol{Y}_b$ as:

$$\boldsymbol{Y}_b = \begin{bmatrix} m_1 \\ m_2 \\ m_3 \\ \vdots \end{bmatrix}.$$

The procedure is stopped when the length of the artificial matrix $\boldsymbol{Y}_b$ exhibits a length exceeding T. Observations exceeding T are cut off; that is, every artificial data matrix $\boldsymbol{Y}_b$ has the same length as the original data matrix $\boldsymbol{Y}$. This process corresponds to one iteration b of the blocks bootstrap procedure. Using this blocks bootstrap procedure, for each iteration b, Tx1 vectors, $\boldsymbol{x}_{b,1}$, $\boldsymbol{x}_{b,2}$, …, $\boldsymbol{x}_{b,N}$ are extracted from matrix $\boldsymbol{Y}_b$ and the MLE estimators are estimated using the procedure described in section 4.1.2, giving us:

$$[\,\widehat{\alpha}_{b,1} \quad \widehat{\alpha}_{b,2} \quad \cdots \quad \widehat{\alpha}_{b,N}\,].$$

This blocks bootstrap procedure is performed for $b = 1, \ldots, 1000$ iterations and point estimates for α are stacked in BxN matrix $\widehat{\boldsymbol{\alpha}}_{BOOT}$:

$$\widehat{\boldsymbol{\alpha}}_{BOOT} = \begin{pmatrix} \widehat{\alpha}_{1,1} & \widehat{\alpha}_{1,2} & \cdots & \cdots & \widehat{\alpha}_{1,N} \\ \widehat{\alpha}_{2,1} & \widehat{\alpha}_{1,2} & \cdots & \cdots & \widehat{\alpha}_{2,N} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \widehat{\alpha}_{B,1} & \widehat{\alpha}_{B,2} & \cdots & \cdots & \widehat{\alpha}_{B,N} \end{pmatrix}.$$

This table reports the covariance matrix for power-law exponents based on daily data using the variance-estimator proposed by Parkinson (1980) and $E[m] = 16$. Data for the overall sample is from May, 16, 2006 to May, 31, 2023 corresponding to 4410 observations. Bold figures indicate statistical significance on at least a 5% level.

Table A.11
Descriptive statistics of daily range-based variances using the Parkinson-estimator and data retrieved from investing.com

	AUD/USD	CAD/USD	CHF/USD	EUR/USD	GBP/USD	JPY/USD	NZD/USD	SEK/USD
Mean	0.0172	0.0086	0.0118	0.0086	0.0100	0.0101	0.0179	0.0157
Median	0.0090	0.0051	0.0056	0.0050	0.0054	0.0052	0.0104	0.0095
Maximum	1.0733	0.3215	9.7223	0.1977	1.4589	0.5085	0.6342	0.5342
Minimum	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Std. Dev.	0.0406	0.0130	0.1469	0.0120	0.0297	0.0212	0.0295	0.0219
Skewness	13.9958	8.7125	64.9786	5.6163	32.6829	12.1572	8.7995	7.0404
Kurtosis	282.5061	148.8101	4291.7250	53.5741	1436.7600	226.0462	127.6905	103.7815
Jarque-Bera	1.46 + E08	4.00 + E07	3.41 + E09	4.97 + E06	3.82 + E08	9.33 + E07	2.94 + E07	1.92 + E07
(*p*-value)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
Observations	4447	4447	4447	4447	4447	4447	4447	4447

Publicly available intraday prices for the AUD/USD, CAD/USD, CHF/USD, EUR/USD, GBP/USD, JPY/USD, NZD/USD, and SEK/USD exchange rates were downloaded from investing.com. The sample period is from May, 16, 2006 to May, 31, 2023. Only the intersection of the data is employed; that is, only daily data are accounted for where all FX rates were quoted on the same day, leaving us with 4447daily observations. To estimate annualized daily variances, the range-based variance estimator proposed by Parkinson (1980) is employed which is given by:

$$\sigma_{i,t}^2 = T\frac{1}{4ln(2)}\left(ln(H_{i,t}) - ln(L_{i,t})\right)^2,$$

where $H_{i,t}$ and $L_{i,t}$ denote the highest and lowest price for foreign exchange rate market i on trading day t, and $\sigma_{i,t}^2$ denotes foreign exchange rate market i's corresponding annualized realized variance where $T = 250$, as 250 trading days per annum are assumed. This table reports the descriptive statistics.

Table A.12
Estimated power-law exponents for daily variances based on the Parkinson-estimator using data retrieved from investing.com

	$\widehat{\sigma}^2_{AUD/USD}$	$\widehat{\sigma}^2_{CAD/USD}$	$\widehat{\sigma}^2_{CHF/USD}$	$\widehat{\sigma}^2_{EUR/USD}$	$\widehat{\sigma}^2_{GBP/USD}$	$\widehat{\sigma}^2_{JPY/USD}$	$\widehat{\sigma}^2_{NZD/USD}$	$\widehat{\sigma}^2_{SEK/USD}$
$\widehat{\alpha}$	2.5248	3.1045	2.9218	3.1560	2.6103	2.7190	2.8766	3.3382
$\widehat{x}_{min}$	0.0209	0.0213	0.0212	0.0228	0.0134	0.0191	0.0397	0.0519
$\widehat{\sigma}$	0.0530	0.1145	0.1001	0.1251	0.0560	0.0778	0.0978	0.1649
N	865 (19.45%)	355	388	313	765	514	389	213
(%)		(7.98%)	(8.72%)	(7.04%)	(17.20%)	(11.56%)	(8.75%)	(4.79%)

FX variances are modeled using the following power-law function:

$$p(x) = Cx^{-\alpha},$$

where $C = (\alpha - 1)x_{MIN}^{\alpha-1}$ with $\alpha \in \{\mathbb{R}_+ | \alpha > 1\}$, x denotes the respective annualized daily or weekly FX variance provided $x \in \{\mathbb{R}_+ | x_{MIN} \le x < \infty\}$, x_{MIN} is the minimum value governed by the power-law process, and α is the magnitude of the corresponding tail exponent. Following White et al. (2008) and Clauset et al. (2009), the tail exponents are estimated as:

$$\widehat{\alpha} = 1 + N\left(\sum_{i=1}^{N} ln\left(\frac{x_i}{x_{MIN}}\right)\right)^{-1},$$

where $\widehat{\alpha}$ denotes the MLE estimator, N is the number of observations exceeding x_{MIN}, and other notations are as previously defined. The estimate $\widehat{\alpha}$ is selected based on the optimal Kolmogorov–Smirnov (KS) distance D measuring the maximum distance between the cumulative density functions (CDFs) of the data and the fitted power-law model as defined by:

$$D = MAX_{x \geq x_{MIN}} |S(x) - P(x)|,$$

where $S(x)$ is the CDF of the data for the observation with a value of at least x_{MIN}, and $P(x)$ is the CDF for the power-law model that best fits the data in the region $x \geq x_{MIN}$. Estimate $\widehat{x}_{MIN}$ is then the value of x_{MIN} that minimizes D. This table reports the estimates $\widehat{\alpha}$, $\widehat{x}_{min}$, $\widehat{\sigma}$, and N for daily FX variance data. The sample period is from May 16, 2006 to May, 31, 2023 comprising 4447 daily observations.

Table A.13
Comparison of estimated power-law exponents for daily variances based on the Parkinson-estimator using different data obtained from yahoo.com and investing.com

Data source	Metric	$\widehat{\sigma}^2_{AUD/USD}$	$\widehat{\sigma}^2_{CAD/USD}$	$\widehat{\sigma}^2_{CHF/USD}$	$\widehat{\sigma}^2_{EUR/USD}$	$\widehat{\sigma}^2_{GBP/USD}$	$\widehat{\sigma}^2_{JPY/USD}$	$\widehat{\sigma}^2_{NZD/USD}$	$\widehat{\sigma}^2_{SEK/USD}$
yahoo	$\widehat{\alpha}$	2.3378	2.6154	2.7961	2.7969	2.6435	2.5821	2.5911	2.5924
	$\widehat{\sigma}_{BOOT}$	0.2181	0.3962	0.2498	0.2967	0.3181	0.1419	0.2805	0.3423
	$\widehat{\alpha} \pm 1.96\widehat{\sigma}_{BOOT}$	[1.9103; 2.7653]	[1.8388; 3.3920]	[2.3065; 3.2857]	[2.2154; 3.3784]	[2.0200; 3.2670]	[2.3040; 2.8602]	[2.0413; 3.1409]	[1.9215; 3.2633]
Investing	$\widehat{\alpha}$	2.5248	3.1045	2.9218	3.1560	2.6103	2.7190	2.8766	3.3382**

** Statistically significant on a 5% level.

This tables compares the estimated power-law exponents reported in Table 3 (e.g., derived from data provided from yahoo.com) with the ones reported in Table A.12 (e.g., derived from data provided from investing.com). Moreover, using the robust standard deviations, as reported in Table 7, this tables reports a 95% confidence interval for the point estimates from Table 3. Bold figures indicated statistical significance on a 5% level.

Table A.14
Covariance matrix for power-law exponents based on daily data using data retrieved from investing.com

Covariance (*t*-Statistic)	$\widehat{\alpha}_{AUD/USD}$	$\widehat{\alpha}_{CAD/USD}$	$\widehat{\alpha}_{CHF/USD}$	$\widehat{\alpha}_{EUR/USD}$	$\widehat{\alpha}_{GBP/USD}$	$\widehat{\alpha}_{JPY/USD}$	$\widehat{\alpha}_{NZD/USD}$	$\widehat{\alpha}_{SEK/USD}$
$\widehat{\alpha}_{AUD/USD}$	0.1200 (−——)							
$\widehat{\alpha}_{CAD/USD}$	**0.1110** (33.6641)	0.1929 (−——)						
$\widehat{\alpha}_{CHF/USD}$	−0.0011 (−0.0782)	−0.0156 (−0.8390)	1.7869 (−——)					
$\widehat{\alpha}_{EUR/USD}$	**0.0709** (28.6205)	**0.0818** (24.3614)	0.0009 (0.0663)	0.0930 (−——)				
$\widehat{\alpha}_{GBP/USD}$	**0.0636** (34.1987)	**0.0700** (26.1731)	−0.0034 −0.3247	**0.0453** (23.3791)	0.0624 (−——)			
$\widehat{\alpha}_{JPY/USD}$	0.0000 (−0.0077)	0.0001 (0.0583)	−0.0029 (−0.5084)	−0.0005 (−0.4173)	−0.0008 (−0.7282)	0.0183 (−——)		
$\widehat{\alpha}_{NZD/USD}$	**0.0779** (41.9725)	**0.0806** (27.1308)	0.0006 (0.0496)	**0.0532** (24.9182)	**0.0495** (31.3506)	0.0000 (0.0364)	0.0792 (−——)	
$\widehat{\alpha}_{SEK/USD}$	0.0297 (1.9257)	0.0228 (1.1673)	**1.0756** (22.0080)	0.0091 (0.6698)	0.0130 (1.1684)	−0.0082 (−1.3580)	0.0162 (1.2960)	1.9814 (−——)

The covariance matrix for power-law exponents are obtained via blocks bootstrap. Denoting the selected block length as m, a blocks bootstrap procedure is chosen such that $E[m] = \sqrt{T}$. Then Tx1 data vectors of FX variances $i = 1, \ldots, N$, denoted as $\boldsymbol{x}_i$, are stacked into matrix $\boldsymbol{Y}$:

$$\boldsymbol{Y} = [\boldsymbol{x}_1, \boldsymbol{x}_2, \ldots, \boldsymbol{x}_N].$$

Blocks m are randomly drawn from matrix $\boldsymbol{Y}$ with respect to the time dimension $t = 1, \ldots, T$. These blocks are governed by a geometric distribution, that is, $m \sim GEO(p)$ with $E[m] = \frac{(1-p)}{p}$. Using this procedure, the blocks drawn from $\boldsymbol{Y}$ vary in lengths. Randomly drawn blocks m that have dimensions mxK from data matrix $\boldsymbol{Y}$ are stacked in matrix $\boldsymbol{Y}_b$ as:

$$\mathbf{Y}_b = \begin{bmatrix} m_1 \\ m_2 \\ m_3 \\ \vdots \end{bmatrix}.$$

The procedure is stopped when the length of the artificial matrix $\boldsymbol{Y}_b$ exhibits a length exceeding T. Observations exceeding T are cut off; that is, every artificial data matrix $\boldsymbol{Y}_b$ has the same length as the original data matrix $\boldsymbol{Y}$. This process corresponds to one iteration b of the blocks bootstrap procedure. Using this blocks bootstrap procedure, for each iteration b, Tx1 vectors, $\boldsymbol{x}_{b,1}$, $\boldsymbol{x}_{b,2}$, …, $\boldsymbol{x}_{b,N}$ are extracted from matrix $\boldsymbol{Y}_b$ and the MLE estimators are estimated using the procedure described in section 4.1.2, giving us:

$$[\,\widehat{\alpha}_{b,1} \quad \widehat{\alpha}_{b,2} \quad \cdots \quad \widehat{\alpha}_{b,N}\,].$$

This blocks bootstrap procedure is performed for $b = 1, \ldots, 1000$ iterations and point estimates for α are stacked in BxN matrix $\widehat{\boldsymbol{\alpha}}_{BOOT}$:

$$\widehat{\boldsymbol{\alpha}}_{BOOT} = \begin{pmatrix} \widehat{\alpha}_{1,1} & \widehat{\alpha}_{1,2} & \dots & \dots & \widehat{\alpha}_{1,N} \\ \widehat{\alpha}_{2,1} & \widehat{\alpha}_{1,2} & \dots & \dots & \widehat{\alpha}_{2,N} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \widehat{\alpha}_{B,1} & \widehat{\alpha}_{B,2} & \dots & \dots & \widehat{\alpha}_{B,N} \end{pmatrix}.$$

This table reports the covariance matrix for power-law exponents based on daily data using the variance-estimator proposed by Parkinson (1980) and $E[m] = 67$. Data were downloaded from investing.com. The sample period is from May 16, 2006 to May, 31, 2023 comprising 4447 daily observations. Bold figures indicate statistical significance on at least a 5% level.

Table A.15
Descriptive statistics of daily range-based variances using the Parkinson-estimator and data retrieved from Bloomberg

	AUD/USD	CAD/USD	CHF/USD	EUR/USD	GBP/USD	JPY/USD	NOK/USD	NZD/USD	SEK/USD
Mean	0.0163	0.0083	0.0115	0.0085	0.0098	0.0101	0.0180	0.0175	0.0150
Median	0.0082	0.0048	0.0053	0.0049	0.0053	0.0051	0.0102	0.0098	0.0085
Maximum	1.0639	0.3197	9.3580	0.1917	1.4506	0.5210	1.6150	0.6938	0.5312
Minimum	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Std. Dev.	0.0395	0.0132	0.1415	0.0121	0.0274	0.0209	0.0405	0.0314	0.0219
Skewness	14.1143	8.8139	64.8255	5.6374	35.0017	11.7773	21.9569	10.3371	6.9170
Kurtosis	290.6720	147.8430	4277.7150	53.4786	1745.3820	221.6816	732.1324	172.3910	101.3036
Jarque-Bera	1.55E+07	3.94E+06	3.39E+09	4.96E+05	5.63E+08	8.96E+06	9.89E+07	5.40E+06	1.83E+06
(p-value JB)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
Observations	4447	4447	4447	4447	4447	4447	4447	4447	4447

Publicly available intraday prices for the AUD/USD, CAD/USD, CHF/USD, EUR/USD, GBP/USD, JPY/USD, NOK/USD, NZD/USD, and SEK/USD exchange rates were downloaded from Bloomber. The sample period is from May, 16, 2006 to May, 31, 2023. This data set comprises 4447 observations. To estimate annualized daily variances, the range-based variance estimator proposed by Parkinson (1980) is employed which is given by:

$$\sigma^2_{i,t} = T\frac{1}{4ln(2)}\left(ln(H_{i,t}) - ln(L_{i,t})\right)^2,$$

where $H_{i,t}$ and $L_{i,t}$ denote the highest and lowest price for foreign exchange rate market i on trading day t, and $\sigma^2_{i,t}$ denotes foreign exchange rate market i's corresponding annualized realized variance where $T = 250$, as 250 trading days per annum are assumed. This table reports the descriptive statistics.

Table A.16
Estimated power-law exponents for daily variances based on the Parkinson-estimator using data retrieved from Bloomberg

	$\widehat{\sigma}^2_{AUD/USD}$	$\widehat{\sigma}^2_{CAD/USD}$	$\widehat{\sigma}^2_{CHF/USD}$	$\widehat{\sigma}^2_{EUR/USD}$	$\widehat{\sigma}^2_{GBP/USD}$	$\widehat{\sigma}^2_{JPY/USD}$	$\widehat{\sigma}^2_{NOK/USD}$	$\widehat{\sigma}^2_{NZD/USD}$	$\widehat{\sigma}^2_{SEK/USD}$
$\widehat{\alpha}$	2.5221	3.0439	2.9015	3.0605	2.5981	2.7071	2.6997	2.8260	2.4664
$\widehat{x}_{min}$	0.0203	0.0209	0.0182	0.0205	0.0127	0.0194	0.0224	0.0373	0.0142
$\widehat{\sigma}$	0.0535	0.1111	0.0862	0.1089	0.0568	0.0778	0.0563	0.0911	0.0406
N	845	356	509	376	827	507	947	423	1353
(%)	(19.00%)	(8.01%)	(11.45%)	(8.46%)	(18.60%)	(11.40%)	(21.30%)	(9.51%)	(30.43%)

FX variances are modeled using the following power-law function:

$$p(x) = Cx^{-\alpha},$$

where $C = (\alpha - 1)x^{\alpha-1}_{MIN}$ with $\alpha \in \{\mathbb{R}_+ | \alpha > 1\}$, x denotes the respective annualized daily or weekly FX variance provided $x \in \{\mathbb{R}_+ | x_{MIN} \leq x < \infty\}$, x_{MIN} is the minimum value governed by the power-law process, and α is the magnitude of the corresponding tail exponent. Following White et al. (2008) and Clauset et al. (2009), the tail exponents are estimated as:

$$\widehat{\alpha} = 1 + N\left(\sum_{i=1}^{N} ln\left(\frac{x_i}{x_{MIN}}\right)\right)^{-1},$$

where $\widehat{\alpha}$ denotes the MLE estimator, N is the number of observations exceeding x_{MIN}, and other notations are as previously defined. The estimate $\widehat{\alpha}$ is selected based on the optimal Kolmogorov–Smirnov (KS) distance D measuring the maximum distance between the cumulative density functions (CDFs) of the data and the fitted power-law model as defined by:

$$D = MAX_{x \geq x_{MIN}} |S(x) - P(x)|,$$

where $S(x)$ is the CDF of the data for the observation with a value of at least x_{MIN}, and $P(x)$ is the CDF for the power-law model that best fits the data in the region $x \geq x_{MIN}$. Estimate $\widehat{x}_{MIN}$ is then the value of x_{MIN} that minimizes D. This table reports the estimates $\widehat{\alpha}$, $\widehat{x}_{min}$, $\widehat{\sigma}$, and N for daily FX variance data. The sample period is from May 16, 2006 to May, 31, 2023 comprising 4447 daily observations.

Table A.17
Covariance matrix for power-law exponents based on daily data using data retrieved from Bloomberg

Covariance									
(*t*-Statistic)	$\widehat{\alpha}_{AUD/USD}$	$\widehat{\alpha}_{CAD/USD}$	$\widehat{\alpha}_{CHF/USD}$	$\widehat{\alpha}_{EUR/USD}$	$\widehat{\alpha}_{GBP/USD}$	$\widehat{\alpha}_{JPY/USD}$	$\widehat{\alpha}_{NOK/USD}$	$\widehat{\alpha}_{NZD/USD}$	$\widehat{\alpha}_{SEK/USD}$
$\widehat{\alpha}_{AUD/USD}$	0.1012								
	(——–)								
$\widehat{\alpha}_{CAD/USD}$	**0.0948**	0.1710							
	(32.8064)	(——–)							
$\widehat{\alpha}_{CHF/USD}$	**0.0145**	**0.0284**	0.0539						
	(6.3093)	(9.7820)	(——–)						
$\widehat{\alpha}_{EUR/USD}$	**0.0677**	**0.0789**	**0.0139**	0.0843					
	(34.0378)	(27.5483)	(6.6783)	(——–)					
$\widehat{\alpha}_{GBP/USD}$	**0.0577**	**0.0651**	**0.0179**	**0.0469**	0.0668				
	(31.0826)	(24.2688)	(9.9031)	(25.3145)	(——–)				
$\widehat{\alpha}_{JPY/USD}$	**0.0228**	**0.0279**	**0.0028**	**0.0185**	**0.0143**	0.0183			
	(19.6564)	(18.1836)	(2.8481)	(16.8682)	(14.1700)	(——–)			
$\widehat{\alpha}_{NOK/USD}$	**0.0345**	**0.0377**	**0.0058**	**0.0174**	**0.0263**	**0.0095**	0.0859		
	(12.5699)	(10.3329)	(2.6896)	(6.5957)	(11.6846)	(7.7838)	(——–)		
$\widehat{\alpha}_{NZD/USD}$	**0.0747**	**0.0842**	**0.0177**	**0.0534**	**0.0496**	**0.0186**	**0.0329**	0.0870	
	(41.5563)	(30.1642)	(8.4578)	(25.1618)	(27.0262)	(16.6896)	(13.0239)	(——–)	
$\widehat{\alpha}_{SEK/USD}$	**0.0873**	**0.1023**	**0.0251**	**0.0783**	**0.0667**	**0.0130**	**0.0182**	**0.0734**	0.3319
	(17.1106)	(15.0174)	(6.0337)	(16.7325)	(15.8225)	(5.3404)	(3.4198)	(15.1326)	–

The covariance matrix for power-law exponents are obtained via blocks bootstrap. Denoting the selected block length as m, a blocks bootstrap procedure is chosen such that $E[m] = \sqrt{T}$. Then Tx1 data vectors of FX variances $i = 1,\dots,N$, denoted as $\boldsymbol{x}_i$, are stacked into matrix $\boldsymbol{Y}$:

$$\boldsymbol{Y} = [\boldsymbol{x}_1, \boldsymbol{x}_2, \dots, \boldsymbol{x}_N].$$

Blocks m are randomly drawn from matrix $\boldsymbol{Y}$ with respect to the time dimension $t = 1,\dots,T$. These blocks are governed by a geometric distribution, that is, $m \sim GEO(p)$ with $E[m] = \frac{(1-p)}{p}$. Using this procedure, the blocks drawn from $\boldsymbol{Y}$ vary in lengths. Randomly drawn blocks m that have dimensions mxK from data matrix $\boldsymbol{Y}$ are stacked in matrix $\boldsymbol{Y}_b$ as:

$$\boldsymbol{Y}_b = \begin{bmatrix} m_1 \\ m_2 \\ m_3 \\ \vdots \end{bmatrix}.$$

The procedure is stopped when the length of the artificial matrix $\boldsymbol{Y}_b$ exhibits a length exceeding T. Observations exceeding T are cut off; that is, every artificial data matrix $\boldsymbol{Y}_b$ has the same length as the original data matrix $\boldsymbol{Y}$. This process corresponds to one iteration b of the blocks bootstrap procedure. Using this blocks bootstrap procedure, for each iteration b, Tx1 vectors, $\boldsymbol{x}_{b,1}$, $\boldsymbol{x}_{b,2}$, …, $\boldsymbol{x}_{b,N}$ are extracted from matrix $\boldsymbol{Y}_b$ and the MLE estimators are estimated using the procedure described in section 4.1.2, giving us:

$$[\,\widehat{\alpha}_{b,1} \quad \widehat{\alpha}_{b,2} \quad \cdots \quad \widehat{\alpha}_{b,N}\,].$$

This blocks bootstrap procedure is performed for $b = 1,\dots,1000$ iterations and point estimates for α are stacked in BxN matrix $\widehat{\boldsymbol{\alpha}}_{BOOT}$:

$$\widehat{\boldsymbol{\alpha}}_{BOOT} = \begin{pmatrix} \widehat{\alpha}_{1,1} & \widehat{\alpha}_{1,2} & \dots & \dots & \widehat{\alpha}_{1,N} \\ \widehat{\alpha}_{2,1} & \widehat{\alpha}_{1,2} & \dots & \dots & \widehat{\alpha}_{2,N} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \widehat{\alpha}_{B,1} & \widehat{\alpha}_{B,2} & \dots & \dots & \widehat{\alpha}_{B,N} \end{pmatrix}.$$

This table reports the covariance matrix for power-law exponents based on daily data using the variance-estimator proposed by Parkinson (1980) and $E[m] = 67$. Data were downloaded from Bloomberg. The sample period is from May 16, 2006 to May, 31, 2023 comprising 4447 daily observations. Bold figures indicate statistical significance on at least a 5% level.

Table A.18
Principal component analysis for estimated power-law exponents derived from daily data

Eigenvalues:				Cumulative	Cumulative
Number	Value	Difference	Proportion	Value	Proportion
1	**4.5906**	**3.6284**	**0.5101**	**4.5906**	**0.5101**
2	0.9622	0.2291	0.1069	5.5528	0.6170
3	0.7331	0.0654	0.0815	6.2859	0.6984
4	0.6677	0.0455	0.0742	6.9536	0.7726
5	0.6222	0.0632	0.0691	7.5758	0.8418
6	0.5590	0.2160	0.0621	8.1347	0.9039
7	0.3430	0.0655	0.0381	8.4778	0.9420

(*continued on next page*)

Table A.18 (*continued*)

Eigenvalues: Number	Value	Difference	Proportion	Cumulative Value	Cumulative Proportion
8	0.2775	0.0327	0.0308	8.7552	0.9728
9	0.2448	–	0.0272	9.0000	1.0000

This table reports the results from principal component analysis as detailed in section 4.1.5. Dominant eigenvalues are defined as $eig(.) > 1$ and marked in bold.

Table A.19
Principal component analysis for estimated power-law exponents derived from weekly data

Eigenvalues: Number	Value	Difference	Proportion	Cumulative Value	Cumulative Proportion
1	**5.0528**	**4.1326**	**0.5614**	**5.0528**	**0.5614**
2	0.9202	0.1606	0.1022	5.9731	0.6637
3	0.7597	0.2017	0.0844	6.7327	0.7481
4	0.5579	0.0683	0.0620	7.2907	0.8101
5	0.4897	0.0559	0.0544	7.7803	0.8645
6	0.4338	0.0505	0.0482	8.2141	0.9127
7	0.3833	0.1755	0.0426	8.5975	0.9553
8	0.2078	0.0131	0.0231	8.8053	0.9784
9	0.1947	–	0.0216	9.0000	1.0000

This table reports the results from principal component analysis as detailed in section 4.1.5. Dominant eigenvalues are defined as $eig(.) > 1$ and marked in bold.

Table A.20
Principal component analysis for estimated power-law exponents derived from both daily and weekly data

Eigenvalues: Number	Value	Difference	Proportion	Cumulative Value	Cumulative Proportion
1	**5.2137**	**0.7763**	**0.2896**	**5.2137**	**0.2896**
2	**4.4374**	**3.4476**	**0.2465**	**9.6511**	**0.5362**
3	0.9898	0.0725	0.0550	10.6409	0.5912
4	0.9172	0.1040	0.0510	11.5581	0.6421
5	0.8132	0.1011	0.0452	12.3713	0.6873
6	0.7121	0.0787	0.0396	13.0834	0.7269
7	0.6333	0.0072	0.0352	13.7167	0.7620
8	0.6261	0.0405	0.0348	14.3428	0.7968
9	0.5856	0.0656	0.0325	14.9284	0.8294
10	0.5200	0.0253	0.0289	15.4483	0.8582
11	0.4946	0.0673	0.0275	15.9430	0.8857
12	0.4273	0.0475	0.0237	16.3703	0.9095
13	0.3798	0.0387	0.0211	16.7501	0.9306
14	0.3411	0.0696	0.0189	17.0912	0.9495
15	0.2715	0.0311	0.0151	17.3627	0.9646
16	0.2404	0.0365	0.0134	17.6031	0.9780
17	0.2039	0.0110	0.0113	17.8070	0.9893
18	0.1930	–	0.0107	18.0000	1.0000

This table reports the results from principal component analysis as detailed in section 4.1.5. Dominant eigenvalues are defined as $eig(.) > 1$ and marked in bold.

Table A.21
Descriptive statistics of weekly realized foreign exchange rate variances derived from squared daily returns

	AUD/USD	CAD/USD	CHF/USD	EUR/USD	GBP/USD	JPY/USD	NOK/USD	NZD/USD	SEK/USD
Mean	3.1248	1.4892	2.2282	1.7094	1.7103	1.9301	3.0323	3.1219	2.7544
Median	1.7208	0.9136	1.2239	1.1294	1.1118	1.1220	1.9722	1.9293	1.8244
Maximum	165.1185	28.3084	359.2435	25.9233	78.9068	40.0209	74.0531	74.0994	41.1842
Minimum	0.0899	0.0259	0.0174	0.0363	0.0379	0.0208	0.0363	0.0694	0.0650
Std. Dev.	7.8302	2.0772	10.9376	1.8958	3.1082	2.7585	4.3210	4.3990	3.4180
Skewness	13.7712	6.3136	30.1551	4.1613	14.8711	5.0132	7.8238	6.8347	4.6547
Kurtosis	240.4900	63.4720	975.6902	35.6315	336.9650	45.0758	99.0305	79.9453	36.6948
Jarque-Bera	2.78E+06	1.86E+05	4.62E+07	5.52E+04	5.47E+06	9.10E+04	4.61E+05	2.97E+05	5.95E+04
Probability	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Observations	1168	1168	1168	1168	1168	1168	1168	1168	1168

Publicly available prices for the AUD/USD, CAD/USD, CHF/USD, EUR/USD, GBP/USD, JPY/USD, NOK/USD, NZD/USD, and SEK/USD exchange rates were downloaded from investing.com. The sample period is from January, 3, 2001 to May, 31, 2023. Only the intersection of the data is employed; that is, only daily data are accounted for where all FX rates were quoted on the same day, leaving us with 5845 daily price observations. To estimate weekly realized variances, the following variance estimator is employed:

$$\sigma_{i,j}^{*2} = \sum_{t\in j}\left(\frac{100\left(P_{i,t}-P_{i,t-1}\right)}{P_{i,t-1}}\right)^2,$$

where $P_{i,t}$ denote the price for foreign exchange rate market i on corresponding trading day t. It is assumed that every week exhibits five trading days; hence, the weekly realized FX variance corresponds to the sum of five consecutive and non-overlapping squared daily returns where $t \in j$ indicates the relevant trading days of respective week j. Further, $\sigma_{i,j}^{*2}$ denotes, accordingly, foreign exchange rate market i's weekly realized variance. This table reports the descriptive statistics.

Table A.22
Estimated power-law exponents for weekly variances derived from squared daily returns

	$\widehat{\sigma}^2_{AUD/USD}$	$\widehat{\sigma}^2_{CAD/USD}$	$\widehat{\sigma}^2_{CHF/USD}$	$\widehat{\sigma}^2_{EUR/USD}$	$\widehat{\sigma}^2_{GBP/USD}$	$\widehat{\sigma}^2_{JPY/USD}$	$\widehat{\sigma}^2_{NOK/USD}$	$\widehat{\sigma}^2_{NZD/USD}$	$\widehat{\sigma}^2_{SEK/USD}$
$\widehat{\alpha}$	2.4902	3.0654	3.0129	3.9644	2.8598	2.4525	3.1531	3.0672	2.9636
$\widehat{x}_{min}$	2.5173	3.0421	2.6145	4.6978	2.0708	1.6773	6.4478	5.7339	4.4509
$\widehat{\sigma}$	0.0778	0.1952	0.1305	0.3532	0.1178	0.0747	0.2144	0.1773	0.1515
N	392	122	253	76	266	404	110	147	181
(%)	(33.56%)	(10.45%)	(21.66%)	(6.51%)	(22.77%)	(33.59%)	(9.42%)	(12.59%)	(15.50%)

FX variances are modeled using the following power-law function:

$$p(x) = Cx^{-\alpha},$$

where $C = (\alpha-1)x_{MIN}^{\alpha-1}$ with $\alpha \in \{\mathbb{R}_+|\alpha > 1\}$, x denotes the respective annualized daily or weekly FX variance provided $x \in \{\mathbb{R}_+|x_{MIN} \leq x < \infty\}$, x_{MIN} is the minimum value governed by the power-law process, and α is the magnitude of the corresponding tail exponent. Following White et al. (2008) and Clauset et al. (2009), the tail exponents are estimated as:

$$\widehat{\alpha} = 1 + N\left(\sum_{i=1}^{N} ln\left(\frac{x_i}{x_{MIN}}\right)\right)^{-1},$$

where $\widehat{\alpha}$ denotes the MLE estimator, N is the number of observations exceeding x_{MIN}, and other notations are as previously defined. The estimate $\widehat{\alpha}$ is selected based on the optimal Kolmogorov–Smirnov (KS) distance D measuring the maximum distance between the cumulative density functions (CDFs) of the data and the fitted power-law model as defined by:

$$D = MAX_{x\geq x_{MIN}}|S(x) - P(x)|,$$

where $S(x)$ is the CDF of the data for the observation with a value of at least x_{MIN}, and $P(x)$ is the CDF for the power-law model that best fits the data in the region $x \geq x_{MIN}$. Estimate $\widehat{x}_{MIN}$ is then the value of x_{MIN} that minimizes D. This table reports the estimates $\widehat{\alpha}$, $\widehat{x}_{min}$, $\widehat{\sigma}$, and N for weekly FX variance data, as detailed in section 4.2.6. The sample period is from January 2, 2001 to May, 31, 2023 comprising 1168 weekly observations.

Table A.23
Covariance matrix for power-law exponents based on weekly variances derived from squared daily returns

Covariance (*t*-Statistic)	$\widehat{\alpha}_{AUD/USD}$	$\widehat{\alpha}_{CAD/USD}$	$\widehat{\alpha}_{CHF/USD}$	$\widehat{\alpha}_{EUR/USD}$	$\widehat{\alpha}_{GBP/USD}$	$\widehat{\alpha}_{JPY/USD}$	$\widehat{\alpha}_{NOK/USD}$	$\widehat{\alpha}_{NZD/USD}$	$\widehat{\alpha}_{SEK/USD}$
$\widehat{\alpha}_{AUD/USD}$	0.1977								
	(——)								
$\widehat{\alpha}_{CAD/USD}$	**0.0964**	0.1593							
	(20.4508)	(——)							
$\widehat{\alpha}_{CHF/USD}$	**0.0257**	**0.0307**	0.0794						
	(6.6078)	(8.9561)	(——)						
$\widehat{\alpha}_{EUR/USD}$	**0.0412**	**0.0299**	**0.0232**	0.2962					
	(5.4560)	(4.3900)	(4.8350)	(——)					
$\widehat{\alpha}_{GBP/USD}$	**0.0848**	**0.0534**	**0.0147**	**0.0421**	0.1106				
	(22.1059)	(13.8763)	(5.0011)	(7.5597)	(——)				
$\widehat{\alpha}_{JPY/USD}$	**0.0620**	**0.0425**	**0.0198**	**0.0274**	**0.0472**	0.0760			
	(18.5169)	(13.2392)	(8.3465)	(5.8660)	(18.9904)	(——)			
$\widehat{\alpha}_{NOK/USD}$	**0.1144**	**0.0812**	**0.0116**	**0.0791**	**0.0928**	**0.0541**	0.2475		
	(19.0879)	(14.1545)	(2.6147)	(9.6454)	(21.4152)	(13.5582)	(——)		
$\widehat{\alpha}_{NZD/USD}$	**0.2008**	**0.1129**	**0.0506**	**0.0697**	**0.1281**	**0.0906**	**0.1802**	0.4697	
	(27.6819)	(14.3140)	(8.5814)	(6.0051)	(21.4605)	(17.2592)	(19.6691)	(——)	
$\widehat{\alpha}_{SEK/USD}$	**0.1646**	**0.1087**	**0.0464**	**0.0589**	**0.1221**	**0.0781**	**0.1495**	**0.2589**	0.2825
	(30.6570)	(18.8544)	(10.2854)	(6.5733)	(30.1686)	(19.8945)	(21.6429)	(31.9274)	(——)

The covariance matrix for power-law exponents are obtained via blocks bootstrap. Denoting the selected block length as m, a blocks bootstrap procedure is chosen such that $E[m] = \sqrt{T}$. Then Tx1 data vectors of FX variances $i = 1, \ldots, N$, denoted as $\boldsymbol{x}_i$, are stacked into matrix $\boldsymbol{Y}$:

$\boldsymbol{Y} = [\boldsymbol{x}_1, \boldsymbol{x}_2, \ldots, \boldsymbol{x}_N]$.

Blocks m are randomly drawn from matrix $\boldsymbol{Y}$ with respect to the time dimension $t = 1, \ldots, T$. These blocks are governed by a geometric distribution, that is, $m \sim GEO(p)$ with $E[m] = \frac{(1-p)}{p}$. Using this procedure, the blocks drawn from $\boldsymbol{Y}$ vary in lengths. Randomly drawn blocks m that have dimensions mxK from data matrix $\boldsymbol{Y}$ are stacked in matrix $\boldsymbol{Y}_b$ as:

$$\boldsymbol{Y}_b = \begin{bmatrix} m_1 \\ m_2 \\ m_3 \\ \vdots \end{bmatrix}.$$

The procedure is stopped when the length of the artificial matrix $\boldsymbol{Y}_b$ exhibits a length exceeding T. Observations exceeding T are cut off; that is, every artificial data matrix $\boldsymbol{Y}_b$ has the same length as the original data matrix $\boldsymbol{Y}$. This process corresponds to one iteration b of the blocks bootstrap procedure. Using this blocks bootstrap procedure, for each iteration b, Tx1 vectors, $\boldsymbol{x}_{b,1}$, $\boldsymbol{x}_{b,2}$, …, $\boldsymbol{x}_{b,N}$ are extracted from matrix $\boldsymbol{Y}_b$ and the MLE estimators are estimated using the procedure described in section 4.1.2, giving us:

$$[\, \widehat{\alpha}_{b,1} \quad \widehat{\alpha}_{b,2} \quad \cdots \quad \widehat{\alpha}_{b,N} \,].$$

This blocks bootstrap procedure is performed for $b = 1, \ldots, 1000$ iterations and point estimates for α are stacked in BxN matrix $\widehat{\boldsymbol{\alpha}}_{BOOT}$:

$$\widehat{\boldsymbol{\alpha}}_{BOOT} = \begin{pmatrix} \widehat{\alpha}_{1,1} & \widehat{\alpha}_{1,2} & \ldots & \ldots & \widehat{\alpha}_{1,N} \\ \widehat{\alpha}_{2,1} & \widehat{\alpha}_{1,2} & \ldots & \ldots & \widehat{\alpha}_{2,N} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \widehat{\alpha}_{B,1} & \widehat{\alpha}_{B,2} & \ldots & \ldots & \widehat{\alpha}_{B,N} \end{pmatrix}.$$

This table reports the covariance matrix for power-law exponents based on weekly data, as detailed in section 4.2.6, and $E[m] = 34$. Data were downloaded from investing.com. The sample period is from January, 2, 2001 to May, 31, 2023 comprising 1168 weekly observations. Bold figures indicate statistical significance on at least a 5% level.

Table A.24
Testing for a common power-law exponent using weekly variances derived from squared daily returns

q	$\widehat{\lambda}$	p-value
2.1	**25.0715**	**0.0029**
2.2	**21.4785**	**0.0107**
2.3	**18.4308**	**0.0305**
2.4	15.9284	0.0684
2.5	13.9715	0.1234
2.6	12.5599	0.1836
2.7	11.6936	0.2314
2.8	11.3727	0.2512
2.9	11.5972	0.2368
3	12.3671	0.1932
3.1	13.6823	0.1342
3.2	15.5429	0.0771
3.3	**17.9488**	**0.0358**
3.4	**20.9001**	**0.0131**
3.5	**24.3968**	**0.0037**

To explore whether exists a common component governing power-law behavior of FX variances, the following estimated test statistic is used:

$$\widehat{\lambda} = (\widehat{\boldsymbol{\alpha}} - q\mathbf{1})' \widehat{\boldsymbol{\Sigma}}^{-1} (\widehat{\boldsymbol{\alpha}} - q\mathbf{1}),$$

where the covariance matrix $\widehat{\boldsymbol{\Sigma}} = COV(\widehat{\boldsymbol{\alpha}}_{BOOT})$ has the dimension NxN, $\widehat{\boldsymbol{\alpha}}$ is a Nx1 vector of estimated power-law exponents, $\mathbf{1}$ is a Nx1 vector of ones and q is the hypothesized common power-law exponent. The estimated test statistic denoted as $\widehat{\lambda}$ is under the null hypothesis distributed as $\chi^2(N)$. The test statistic is iteratively estimated covering the interval $q = (2.1, 2.2, \ldots, 3.5)$. Since nine FX variances are tests, the corresponding test statistic is under the null hypothesis distributed as $\chi^2(9)$. Bold figures indicate statistical significance on a 5% level.

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