Vaasan yliopisto
UNIVERSITY OF VAASA

Neema Lama

Volatility Forecasting Using Range-Based Estimators: Evidence from Five Major Cryptocurrencies

School of Accounting and Finance
Master’s thesis in Finance
Master's Degree programme in Finance

Vaasa 2026

UNIVERSITY OF VAASA
School of Accounting and Finance
Author: Neema Lama
Title of the Thesis: Volatility Forecasting Using Range-Based Estimators: Evidence from Five Major Cryptocurrencies
Degree: Master of Finance
Programme: Masters degree programme
Supervisor: Dr. Niranjan Sapkota
Year: 2026 **Sivumäärä:** 58

ABSTRACT:

This study evaluates the predictive efficacy of three range-based volatility estimators, Parkinson (1980), Garman and Klass (1980), and Rogers and Satchell (1991), within the Heterogeneous Autoregressive Realized Volatility (HAR-RV) framework, applied to five principal cryptocurrency markets: Bitcoin (BTC), Ethereum (ETH), Binance Coin (BNB), Ripple (XRP), and Tether USD (USDT). This study utilizes daily Open-High-Low-Close (OHLC) price data from January 2020 to December 2025, comprising approximately 2,162 observations per asset, to construct daily, weekly, and monthly realized variance components for estimating the HAR-RV model for each estimator across all five cryptocurrencies. Empirical findings indicate that lagged volatility components are statistically significant across all markets, with notable variations in volatility persistence patterns by asset: the daily component is predominant for BTC and ETH, while the monthly component is most significant for XRP. Of the three estimators, Parkinson (1980) regularly attains the greatest in-sample R^2 values, varying from 11.3% for Ripple to 26.8% for Binance Coin, surpassing the theoretically superior Garman-Klass estimator. This result is ascribed to microstructure noise resulting from the lack of centralized opening auctions in continuously traded digital asset markets, which distorts open price data and diminishes the trustworthiness of estimators that utilize opening prices. Rogers-Satchell exhibits the worst in-sample fit across all five assets. This research presents the inaugural systematic comparison of range-based volatility estimators inside a cohesive HAR-RV framework across many prominent cryptocurrencies, offering novel data for the most appropriate OHLC-based volatility measures for predicting cryptocurrency market volatility.

KEYWORDS: volatility forecasting, HAR-RV model, range-based estimators, cryptocurrency markets, OHLC data

Contents

Tables

Abbreviations

Autoregressive Conditional Heteroskedasticity	**ARCH**
Autoregressive Fractionally Integrated Moving Average	**ARFIMA**
Binance Coin	**BNB**
Bitcoin	**BTC**
Capital Asset Pricing Model	**CAPM**
Expected Shortfall	**ES**
Ethereum	**ETH**
Garman–Klass	**GK**
Generalized Autoregressive Conditional Heteroskedasticity	**GARCH**
Heterogeneous Autoregressive	**HAR**
Heterogeneous Autoregressive Realized Volatility	**HAR-RV**
Intertemporal Capital Asset Pricing Model	**ICAPM**
Open-High-Low-Close	**OHLC**
Ordinary Least Squares	**OLS**
Parkinson	**PK**
Realized Volatility	**RV**
Ripple	**XRP**
Rogers–Satchell	**RS**
Tether USD	**USDT**

1 Introduction

1.1 Background of the Study

Volatility forms one of the fundamental concepts in finance, which refers to the level of uncertainty in the returns of an asset over a period. Volatility is one of the basic inputs used in determining the price of assets, forming portfolios, and managing risks. There exists a strong link between risk and returns in the study of finance. The CAPM model was developed by Sharpe (1964) and Lintner (1975), highlighting the direct link between risk and returns. The work of Merton (1973) furthered this approach to volatility in the form of ICAPM, whereby he showed that risk premium should be attached to volatility as a proxy for opportunity set risk within equilibrium. However, despite the theoretical underpinning, research has shown that the risk-reward relationship is not always consistent throughout various financial securities markets. Higher levels of volatility have been associated with low levels of stock prices, which indicates that there is an inconsistent relationship between risk and return. Researchers such as Glosten, Jagannathan, and Runkle (1993) concluded that there was a complicated and inconsistent relationship between risk and returns. From their study, researchers observed that high levels of volatility were related to declining stock prices. Instability within markets defined by frequent trading, speculations, and changes in prices is a common characteristic in today's cryptocurrency market. It calls for accurate model for analyzing return dynamics and the risk-return relationship (Andersen, Bollerslev, Diebold, & Labys, 2003).

Development of risk measurement methodologies is based on an increasing understanding of the drawbacks inherent to traditional risk measurements. Value at Risk (VaR) became one of the popular methods of risk assessment because of its simplicity and clarity. However, VaR does not meet the subadditivity requirement, which is necessary for risk measurement techniques, and completely ignores losses above the quantile level (Artzner et al., 1999). Expected Shortfall (ES) has been developed as an improvement over these risk measures due to its coherence, which means that ES represents the expectation of losses, assuming that the VaR threshold has been breached (Acerbi & Tasche, 2002). It follows that the reliability of any risk measure depends greatly on the accuracy of the volatility model used to generate it (Engle, 2004). Cryptocurrency returns have heavier tails compared to those found in financial markets, and thus modeling them requires higher accuracy of the used volatility model (Gkillas & Katsiampa, 2018). In recent years, there have been quite a few scientific publications on the topic of cryptocurrency markets due to huge economic values involved (Fry & Cheah, 2016). Cryptocurrencies are an asset class with a completely different structure, which is based on decentralized p2p networks using blockchain technology as a decentralized register based on the application of cryptographical algorithms for transaction verification and prevention of duplicate payments (Nakamoto, 2008; Böhme et al., 2015). Cryptocurrencies, unlike regular financial assets, are not regulated by any governmental authority and have no relation to any national economy. The emergence of Bitcoin as a decentralized online payment system created by Nakamoto (2008) marked the beginning of blockchain-based financial systems.

The classical parametric models of volatility, especially the Autoregressive Conditional Heteroskedasticity (ARCH) modeling technique proposed by Engle (1982), and its variant Generalized ARCH (GARCH) proposed by Bollerslev (1986), are still used frequently to model volatility clustering in financial time series. Such methods are appreciated for their analytical tractability and ability to mimic stylized facts like volatility clustering and leptokurtosis (Bollerslev, Chou, & Kroner, 1992). Nevertheless, one major drawback of GARCH-type models is their distributional assumptions, which can misrepresent the structural changes and tails observed in the financial market data (Poon & Granger, 2003; Hansen & Lunde, 2005). Moreover, the GARCH models are usually estimated using daily close-to-close returns, which does not account for the price action taking place intra-day in the financial markets (Andersen & Bollerslev, 1998). In cryptocurrency markets that operate 24/7 and experience high volatility throughout the day (Makarov & Schoar, 2020), using only daily returns as the input for measuring volatility leads to a great deal of information loss, which may affect the precision of volatility forecasting and the reliability of estimator comparisons drawn from such measures (Andersen et al., 2003).

The advent of high-frequency intraday data has facilitated a much more accurate method of measuring volatility in terms of realized volatility, which is estimated by taking the sum of squared intraday returns observed over time intervals on a particular trading day for a certain financial asset, thereby allowing us to obtain an intrinsic integrated variance measure without making any distributional assumptions about the price process. Andersen, Bollerslev, Diebold, and Labys (2003) developed the theory of realized volatility in terms of continuous time series models, showing that it is an unbiased and efficient estimator of integrated variance, far superior to squared daily returns in estimating volatility. Barndorff-Nielsen and Shephard (2002) have further developed the asymptotic distribution theory that lies behind realized variance estimation, offering the necessary econometric framework for statistical inference. An important practical issue in realizing volatility estimates is the selection of intraday sampling periods. The problem with high-frequency sampling is the contamination by microstructure effects that emerge due to bid-ask bounce, asynchronous trading, and discontinuous price changes, thus causing upward bias in the realized variance estimate (Bandi & Russell, 2008). Consistent with Hansen and Lunde's (2006) finding that five-minute sampling periods provide the optimal trade-off between accuracy and microstructure contamination, this paper uses five-minute intraday returns to compute realized volatilities. The selection of realized volatility as the primary volatility measurement in the research can be explained by the statistical advantage of the former over daily return-based indicators, since the latter is highly biased and less informative than intra-day based realized volatility (Corsi, Pirino, & Reno, 2010).

In order to provide an alternative benchmark for realized volatility obtained through high frequency data returns, a series of range estimators has been developed based on the information present in open, high, low, and close prices available on a daily basis. The range estimators have shown themselves to be more efficient in measuring volatility compared with estimates based on closing price returns (Alizadeh, Brandt, & Diebold, 2002). Parkinson (1980) was the first researcher to introduce such an estimator, showing that the price range offers much more information on price fluctuations compared to the square of the close-to-close return and introducing a five times more statistically efficient estimator assuming a Brownian walk without drift. The approach was further enhanced by Garman and Klass (1980), who included the opening and closing prices into the range, thus generating a composite estimator of even higher statistical efficiency. Rogers and Satchell (1991) have rectified one important drawback of previous approaches by introducing a range-based estimator that is independent of the drift and does not require its knowledge for unbiased estimation. The estimation methods considered are Parkinson (1980), Garman and Klass (1980), and Rogers and Satchell (1991). The evaluation of their predictive ability under one and the same framework provides an opportunity to examine the potential of various types of intra-day price data in describing volatility behavior on cryptocurrency markets.

The HAR-RV model is a realization of the realized volatility approach to forecasting volatility proposed by Corsi (2009). It is one of the most extensively used models in financial research for forecasting realized volatility. The HAR-RV model represents the long memory effect found in financial volatility dynamics as the additive decomposition of realized volatility into three lags with different lengths of daily, weekly, and monthly realizations of volatility. The theory underlying the model is the Heterogeneous Market Hypothesis put forward by Muller et al. (1997), according to which heterogenous market participants, acting on various time horizons such as high frequency traders, day traders, and long term institutions, have their own unique expectations about volatility. The combination of the different volatilities produces the long persistence in the autocorrelation function seen in the empirical data, which can be captured in a natural way by the HAR-RV model. The HAR-RV model, despite being very simple, is estimated by ordinary least squares. Nonetheless, the model has shown better forecasting ability than other volatility models such as the GARCH family or even long memory specifications like the ARFIMA models (Andersen et al., 2007; Corsi, 2009). This mixture of theory-driven reasons, parsimony, and empirical evidence makes HAR-RV model the logical choice as a leading framework to analyze the intraday volatility forecasting in this paper.

In addition, the flexible nature of the HAR-RV regression model enables us to easily include more regressors other than the past realizations of realized volatility into the regression equation. Busch et al. (2011) showed the flexibility of the approach by including implied volatility as an extra regressor, resulting in improved forecasting accuracy relative to the basic equation. Building on the same principle, the current research seeks to extend the HAR-RV model by including Parkinson's (1980), Garman-Klass' (1980), and Rogers-Satchell's (1991) range-based volatility measures as extra variables for investigating the appropriate volatility model for capturing the volatility forecasting performance of major cryptocurrency markets.

1.2 Problem Statement

Although the large amount of research into range-based volatility estimates has been conducted in the classical stock market (Molnár, 2012; Alizadeh et al., 2002), the investigation of these estimates in relation to major cryptocurrency markets remains a relatively untrodden area of research where the efforts have been focused on Bitcoin only, but not other types of cryptocurrencies (Chaim & Laurini, 2018; Fakhfekh & Jeribi, 2020). Since these estimates have been initially proposed in the context of stock market analysis where there are certain sessions when markets open and close, they cannot fully be used in the cryptocurrency market with its round-the-clock trading. This means that the Parkinson volatility estimate may underestimate volatility, whereas the Garman and Klass estimate will be subject to opening prices (Sapkota, 2022). Moreover, return series in cryptocurrency markets exhibit strong skewness, high kurtosis, and frequent regime changes (Katsiampa, 2017; Baur et al., 2018; Makarov & Schoar, 2020). Such characteristics raise significant doubts about the validity of these estimators when applied to data on digital assets. Although Molnár (2012) provides a comparison between range-based estimators in stock markets and Catania & Grassi (2022) assesses volatility forecasting in cryptocurrencies using state-of-the-art econometric techniques, there is no empirical analysis of the use of these estimators as alternative inputs in a single framework like HAR for the forecasting of volatility in major cryptocurrency markets. The present study fills this void by empirically assessing the Parkinson (1980) and Garman-Klass (1980) estimators as alternative inputs in a HAR forecasting model.

HAR-RV models by Corsi (2009) have shown impressive out-of-sample predictive power for stocks and currency pairs, due to their theoretical foundation in the aggregation of investors' heterogenous horizons that corresponds well with institutional market structures. However, in cryptocurrency markets, the participants' composition is entirely different from traditional financial market segments, since retail trading accounts for the lion's share of transactions, while institutional involvement is minimal, and herding behavior has a significantly higher weight compared to regular asset markets (Dyhrberg et al., 2018 and Bouri et al., 2019). Nevertheless, whether the predictive abilities of the daily, weekly, and monthly volatility components in the HAR-RV model still apply in this new economic environment is not entirely clear (Ftiti et al., 2023). The evidence that suggests that the predictive power of the HAR-RV model over the GARCH models in predicting volatility is better for Bitcoin has some support empirically (Bergsli, Lind, Molnár, & Polasik, 2022); however, this study used only Bitcoin data without including the other top cryptocurrencies. Moreover, there was no investigation into the robustness of the predictive accuracy of the HAR-RV model depending on the type of range-based volatility estimator used.

While the correlation between risk and return on cryptocurrency investment has been explored to some extent (Koutmos, 2020; Liu & Tsyvinski, 2021), the main drawback that underpins the rationale behind this research is not the issue of risk and return but a lack of reliable instruments to measure volatility. According to Liu et al. (2022), cryptocurrency returns mostly depend on crypto-specific variables, thus rendering the task of estimating volatility more relevant than studying returns.

The use of volatility measurements that depend on the GARCH model or daily returns' variance rather than intraday volatility measures, which have been found to be statistically better according to Andersen et al. (2003), poses restrictions on the accuracy of the volatility forecast process. The question now is whether range-based measurements under the HAR-RV approach would offer better volatility predictions.

In addition to that, the existing studies tend to rely heavily on Bitcoin to assess overall cryptocurrency volatility, thus creating the risk of overlooking differences between different cryptocurrencies when it comes to volatility behavior, structure, and trading patterns (Ciaian, Rajcaniova, & Kancs, 2018). For instance, Bitcoin, Ethereum, Ripple, and Binance Coin represent different assets whose functions differ from one another and that have their own distinct levels of volatility that cannot be assessed solely by focusing on Bitcoin's performance. Whether the comparative forecasting performance of the Parkinson, Garman-Klass, and Rogers-Satchell estimators within the HAR framework carries over to crypto-assets beyond Bitcoin, each with different structural properties and microstructure characteristics, are issues not yet resolved by the existing literature.

All together, these gaps in the unproven applicability of range-based volatility estimators to cryptocurrency markets, ambiguous HAR-RV model performance across different assets, unexplored cross-asset heterogeneity in volatility persistence, and Bitcoin-focused literature on cryptocurrency volatility suggest the main research problem of this paper: Which range-based volatility estimator better captures the volatility dynamics of major cryptocurrencies in the HAR framework? To investigate this problem, the present study examines the HAR-RV model, applying three different range-based estimators of volatility Parkinson (1980), Garman & Klass (1980), and Rogers & Satchell (1991) for BTC, ETH, USDT, BNB, and XRP, which are listed on CoinMarketCap (2024). Through rigorous comparative analysis of the estimators' performance under a single HAR framework, this research offers a broader and more comprehensive perspective on volatility dynamics than previous studies that focus solely on bitcoin volatility.

1.3 Research Objectives

This study focuses on understanding intraday volatility dynamics in major cryptocurrency markets through the application of the HAR-RV model with different range-based volatility estimators. Specifically, the objectives of the study include:

Objective 1: The estimation and comparison of the distributions of the realized variances of Parkinson (1980), Garman-Klass (1980), and Rogers-Satchell (1991) for the five cryptocurrencies: Bitcoin, Ethereum, Binance Coin, Ripple, and Tether USD between 2020 and 2025.
Objective 2: The assessment of which range-based measure yields the highest explanatory power in-sample within the context of the HAR-RV model among the five cryptocurrencies.
Objective 3: The investigation into the differences that exist, if any, between the daily, weekly, and monthly volatility components of the HAR model for the various types of markets among the five cryptocurrencies.

1.4 Rationale of the Study

This study is driven by four distinctly recognized deficiencies in the current literature regarding bitcoin volatility. Initially, notwithstanding the theoretical and empirical benefits of range-based volatility estimators, a comprehensive comparison of Parkinson (1980), Garman-Klass (1980), and Rogers-Satchell (1991) has not been performed across a wide array of prominent cryptocurrencies. Secondly, although the HAR-RV model has established itself as a standard framework for forecasting volatility, previous applications in cryptocurrencies have solely utilized high-frequency return-based realized volatility, so neglecting the possibility of range-based OHLC inputs within this framework. The predominant focus of cryptocurrency volatility studies is on Bitcoin, neglecting notable disparities in volatility structure, market microstructure, and trading behavior among assets like Ethereum, Binance Coin, Ripple, and Tether USD. Fourth, daily close-to-close return-based volatility measures continue to be the predominant method in bitcoin research, despite their proven inferiority to intraday-informed measures regarding statistical efficiency and informational quality. This study contributes methodologically to the volatility forecasting literature by offering the inaugural cross-asset assessment of range-based estimators within the HAR framework for prominent cryptocurrency marketplaces. From a pragmatic perspective, determining the most dependable OHLC-based volatility estimator has significant ramifications for financial professionals: enhanced volatility predictions support better-calibrated risk management instruments, such as Value at Risk (VaR) and Expected Shortfall (ES), which are becoming increasingly pertinent as institutional involvement in cryptocurrency markets expands. This study employs the HAR-RV model with three range-based volatility estimators across five cryptocurrency assets, including Tether USD as a near-zero volatility benchmark, thereby addressing the four identified gaps and enhancing the methodological framework for cryptocurrency volatility research. The results aim to benefit both academic researchers expanding the existing literature on cryptocurrency market dynamics and practitioners in investment management, portfolio construction, and financial risk assessment who need reliable and computationally feasible volatility models.

2 Literature Review

Cryptocurrencies, founded on cryptographic concepts, have garnered significant attention from both the media and economic stakeholders. They function as valid payment mechanisms and as alternatives to conventional government-issued currencies. monetary units. Among numerous cryptocurrencies, including Litecoin, Ethereum, Ripple, Peercoin, and Dogecoin, Bitcoin has emerged as the most dominant in the cryptocurrency market (Bouri, Gil-Alana, Gupta, & Roubaud, 2019).Notwithstanding these rapid reactions, the efficiency of markets continues to be contested, as numerous digital assets do not always adhere to the Efficient Market Hypothesis (Urquhart, 2016).

The development of cryptocurrencies has been significant since their invention, evolving from small online communities to being acknowledged as financial assets which can attract speculative investors along with researchers (Böhme et al., 2015). The functioning of cryptocurrencies has been analyzed by researchers to be that of currency commodities, much like precious metals (Baur et al., 2018 Due to the high levels of volatility and unique market characteristics of cryptocurrencies, accurate volatility measurement and forecasting has become a critical area of research; results demonstrate that cryptocurrency returns are mainly determined by market-specific factors such as time-series momentum and investor attention, while trading volumes could help in predicting extreme returns and volatility of the top cryptocurrencies (Bouri et al., 2019). New developments in the availability of high-frequency data have transformed the analysis of volatility by advancing from simple calculations using returns to complex range-based measures of volatility which generate valuable information about price movements (Andersen & Bollerslev, 1998).

The revolutionary work of Parkinson (1980) showed that by considering both the highest and lowest price levels on a day-to-day basis, the estimation of volatility is substantially more efficient than the traditionally employed close-to-close volatility estimators. In continuation with this approach, Garman and Klass (1980) developed a broader volatility estimator that includes the open, close, high, and low pricesRogers and Satchell (1991) then introduced the concept of drift-adjusted estimators specifically designed for trending markets, addressing an important drawback of earlier methods. The PK, GK, and RS estimators are theoretical and empirical models of great importance for measuring intraday volatility and remain relevant to this day in the field of finance research.

In parallel with developments in measuring volatility, the introduction of the Heterogeneous Autoregressive (HAR) model, developed by Corsi in 2009, has become an effective tool for modeling volatility persistence at different frequencies. Besides HAR-type models, models with jumps and semi-variances were identified to be more efficient to estimate the volatility of bitcoin than any other model specification (Ftiti, Louhichi, and Ben Ameur, 2023). However, even despite the abundant literature on volatility estimation in cryptocurrencies, there are many important research gaps that limit our knowledge of intraday volatility effects. Most of the current literature focuses on Bitcoin and, to a much smaller extent, Ethereum, while there is considerably lower focus on other cryptocurrencies such as Binance coin and Ripple. High-frequency intraday data studies are also rare, which poses a problem when trying to understand short-run volatility and persistence in different digital currencies (Ahmed, El Masry, Al Maghyereh, & Kumar, 2024).

Numerous empirical investigations depend on a singular volatility metric, typically realized volatility or close-to-close standard deviation, without systematically assessing how other volatility estimators may affect observed correlations. Data from range-based estimators, including Parkinson, Garman-Klass, Roger-Satchell, and Yang-Zhang, demonstrates that various measures yield differing forecasting efficacy, underscoring the necessity of evaluating multiple volatility proxies in risk modeling (Korkusuz, Kambouroudis, & McMillan, 2023). Although the use of high-frequency data in cryptocurrency research has increased, the intraday risk–return relationship remains insufficiently integrated into existing empirical frameworks, as much of the literature still relies on daily or lower-frequency data that may mask important high-frequency dynamics (Scaillet, Treccani, & Trevisan, 2020).

2.1 Theoretical Review

2.1.1 Volatility Concepts and Financial Risk

Volatility is the key statistic to measure uncertainties in financial markets. Volatility forms the bedrock on which efficient portfolios are optimized (Markowitz, 1952), derivatives are valued (Black & Scholes, 1973), and risk management systems are built (Christoffersen, 1998). Precise measurement of volatility is key to analyzing the changes in prices, thus facilitating better decision-making (Poon & Granger, 2003). Apart from its primary functions, volatility serves as an indicator of the amount of information flowing in, with higher volatility implying faster flow of information (Clark, 1973).

Traditional estimation of volatility relies on return variance of close to close prices, ignoring important intra-day prices and adding noise to the measurement process (Alizadeh, Brandt, & Diebold, 2002). Traditional estimation faces several constraints that include: not considering market structure and liquidity factors (Hasbrouck, 1991) and violating stationarity assumption during long periods of time. The weaknesses observed motivated the development of the Realized Volatility (RV), which employs intra-day high frequency prices to provide a consistent estimate of underlying integrated volatility (Andersen et al., 2001).

This approach is superior to the conventional approach because it has a reliable non-parametric estimate of integrated volatility, which will tend towards the quadratic variation of an asset as the sampling interval decreases, taking into consideration both the continuous price movements and discrete jumps of prices (Andersen, Bollerslev, Diebold, & Labys, 2003). Since cryptomarkets run round the clock without circuit breakers or trading suspensions, realized volatility measures at high frequency are important for separating real price appreciation from the volatility shocks that daily data will categorize wrongly (Scaillet, Treccani, & Trevisan, 2020). This accuracy is especially significant in cryptocurrency markets marked by severe volatility and frequent abrupt price fluctuations due to the lack of conventional market stabilizing mechanisms (Makarov & Schoar, 2020).

2.1.2 Range-Based Volatility Estimators: Theory and Properties

The use of range-based volatility estimators provides a theoretical advance and a practical improvement when calculating volatility using easily available data, such as daily opening, highest, lowest, and closing (OHLC) prices (Molnár, 2012). The basic idea behind using range-based estimations is that the high-low range observed during a day contains much more information regarding volatility than a simple calculation based on opening and closing prices (Parkinson, 1980). Under the assumption of continuous processes of pricing, the range will be directly proportional to the level of volatility.

The Parkinson (1980) Estimator

The Parkinson (1980) Estimator serves as the fundamental range-based volatility metric. Parkinson established that for a geometric Brownian motion exhibiting negligible drift, the volatility estimator derived from the scaled logarithmic high-low range is around five times more efficient than the traditional close-to-close return variance estimator. The Parkinson estimator is characterized as follows:

$$\sigma_{PK}^2 = \frac{1}{4\ln{(2)}n} \sum_{i=1}^{n} ln^2\left(\frac{H_i}{L_i}\right)$$

The reason for such great improvement in efficiency can be attributed to the ability of this range to identify intra-period movements that cannot be seen through the closing prices. Shu and Zhang (2006) show that the biases associated with Parkinson are great when there is deviation from the assumption of no drift in the movement of prices. Openings tend to underestimate volatility while strong directionality tends to overestimate it. These results have been obtained from S&P 500 returns, but they do highlight dangers associated with using Parkinson in trending assets like cryptocurrencies.

The Garman and Klass (1980) Estimator

Garman and Klass (1980) estimator improves upon the range methodology by combining open and close prices with high and low prices. The combination of all these elements is more efficient because it utilizes more of the available OHLC information. The formula for the estimator uses the variance of normalized log range along with some return components.

$$\sigma_{GK}^2 = \frac{1}{n}\left(\sum_{i=1}^{n} \frac{1}{2} ln^2\left(\frac{H_i}{L_i}\right) + (2\ln(2) - 1) ln^2\left(\frac{C_i}{O_i}\right)\right)$$

In fact, Garman and Klass (1980) proved that compared to the close-to-close variance estimator, this particular estimator has about eight times higher efficiency when prices follow a Brownian motion with no drift. According to Fuertes et al. (2009), this is supported by their study, which showed that intraday statistics like the Realized Range (RR) can be considerably more efficient than standard realized volatility. Garman-Klass makes use of daily extrema, while Fuertes et al. (2009) claim that a change to a range-based approach is the top performing one. The reason for this is that such estimators have greater ability to detect persistent volatility signals and are also "less noisy" compared to classical return-based calculations. However, similarly to Parkinson's estimator, the Garman-Klass method relies on the absence of any drift in prices.

The Rogers and Satchell (1991) Estimator

The Rogers and Satchell (1991) Estimator overcomes the zero-drift limitation by creating a volatility measure that stays unbiased even when prices drift away from zero. The RS estimator is defined as:

$$\sigma_{RS}^2 = \frac{1}{n}\sum_{i=1}^{n}\left(u_i(u_i - c_i) + d_i(d_i - c_i)\right)$$

This drift-stable feature is vital to the cryptomarket since it presents long memory and volatility persistence, implying that prices of cryptocurrencies are driven by memory effects and persistent trend movements.

On the other hand, Barndorff-Nielsen & Shephard (2002) provide a starting point for volatility measures from intraday data since they establish the asymptotic relationship between realized volatility measures derived from high-frequency data and integrated volatility with increasing sampling frequency. This feature holds even when there are jumps and stochastic volatility, thus presenting solid mathematical reasoning behind using range-based estimators with intraday data.

2.1.3 Volatility Persistence and Long-Memory Processes

The stylized fact that is most common in financial markets is volatility clustering, where high volatility tends to be succeeded by even higher volatility, while low volatility tends to persist for prolonged periods (Cont, 2001). Volatility clustering was made more explicit in the ARCH models by Engle (1982), illustrating the existence of high levels of temporal dependence in volatility. Short-memory models assume the presence of exponential decay in the shocks to volatility, which does not adequately account for the gradual and polynomial nature of the decay observed in empirical autocorrelation of absolute returns (Ding et al., 1993). This limitation necessitates the utilization of long-memory processes, which more accurately depict the enduring effects of volatility shocks on future market dynamics.

Models allowing fractional integration by using non-integer values of differencing are used to describe long-memory volatility properly. The FIGARCH model (Baillie et al., 1996) is based on fractional integration and adds an additional fractionally integrated parameter to standard GARCH models. This leads to a sequential persistence of conditional variance. In turn, models of stochastic volatility in continuous time that use fractional Brownian motions components (Comte & Renault, 1998) give a theoretical basis for describing assets which demonstrate long periods of higher or lower volatility. These frameworks are especially useful for capturing the slow mean reversion that is typical of financial market volatility.

The theory behind volatility persistence goes beyond mere statistics. As per the Heterogeneous Market Hypothesis (Müller et al., 1997), the reason behind such behavior lies within the behavior of market players, who operate in different periods of time, including high-frequency traders operating in a millisecond timeframe and investors with an outlook spanning several years ahead. The interaction between trading activity and information flow at different times naturally creates long-memory effects in volatility. It is due to such thinking that multivariate models of volatility are created in order to differentiate short-term and long-term variations in volatility.

This issue is not unique to the stock market, but it applies to cryptocurrency exchanges as well. Research proves that cryptocurrencies display long-term memory properties due to their highly persistent autocorrelations which set them apart from the traditional financial instruments (Katsiampa, 2017; Cheah et al., 2018). In light of such observations, it can be expected that the use of the FIGARCH model along with other long memory models will yield relevant results in modeling cryptocurrency volatilities. The chosen approach will adhere to the underlying theory of financial volatilities as well as practical properties of digital currencies.

2.1.4 The Heterogeneous Autoregressive (HAR) Model Framework

The Heterogeneous Autoregressive (HAR) model was first developed by Corsi (2009), who offered a very elegant approach to modeling the phenomenon of long-memory volatility, bypassing its computation burden related to the use of fractionally integrated specifications. In brief, the rationale of the methodology consists in the assumption that aggregation of volatility at different timescales, such as daily, weekly, and monthly, allows one to get close to the long-memory behavior while keeping the OLS estimation simple (Corsi, 2009). One of the most advantageous features of this approach is its outstanding computational efficiency. The HAR approach can be estimated via OLS and therefore, circumvent the numeric instability and burdensome optimization procedures required by most non-linear time series models (Buncic & Gisler, 2016). This makes the HAR model a strong and easy-to-use tool for looking at volatility on a large scale.

Baseline Specification

The standard HAR-RV model specifies future realized volatility as a linear function of past averages:

$$RV_{t+1} = \beta_0 + \beta_d RV_t^{(d)} + \beta_w RV_t^{(w)} + \beta_m RV_t^{(m)} + \varepsilon_{t+1}$$

This type of model follows the heterogeneity market hypothesis. The first component captures the high frequency actions, the second the medium frequency actions, and the third the low frequency actions. The combination of all the different behaviors creates the memory effect on the volatility which is directly related to the market structure (Müller et al., 1997).

2.1.5 Volatility Forecasting Models and Evaluation Frameworks

Prediction of volatility plays a key role in empirical finance, as it bears important applications in risk management, pricing of derivatives, and optimal asset allocation (Poon & Granger, 2003). Evaluation of volatility predictions necessitates both good proxy variables and loss functions that help differentiate among alternative models. As volatility is inherently hidden and unobservable, forecast evaluation cannot do without proxies, whereby realized volatility computed using intraday high-frequency returns becomes the standard proxy (Andersen & Bollerslev, 1998). The choice of proxy is consequential because a noisy volatility proxy can distort model rankings, leading to incorrect conclusions about relative forecasting performance (Hansen & Lunde, 2006).

Some of the common statistical loss functions utilized in volatility forecasts are Mean Squared Error (MSE), Mean Absolute Error (MAE), and the QLIKE loss function. Patton (2011) found out that only those loss functions that are resilient to substitution of the volatile variable with noisy proxies like MSE and QLIKE would consistently rank models irrespective of the proxy chosen. This observation is critical in the cryptocurrency market due to the fact that volatility levels are high, hence leading to increased probability of noise in the proxy variables. The Mincer and Zarnowitz (1969) regression approach, on the other hand, serves as a direct measure of forecast rationality and bias such that a perfectly rational forecast would have zero intercept and unit slope relative to the realized volatility.

Model comparison among competing specifications is performed formally by the Diebold and Mariano (2002) test, which determines whether the discrepancy in accuracy between two competing models is statistically significant. In the case of comparing several models at once, Hansen et al. (2011) proposed the Model Confidence Set (MCS) approach, which detects a group of models that are statistically indistinguishable from the best one at a particular confidence level. Such approaches are crucial for this paper since the main hypothesis to be tested concerns whether the Parkinson, Garman-Klass, or Rogers-Satchell range-based estimator yields better forecasting results when implemented into the HAR model among five cryptocurrencies.

2.2 Empirical Review

2.2.1 Empirical Evidence on Range-Based Volatility Estimators

Andersen and Bollerslev (1998) established that employing high-frequency data for volatility estimation significantly outperformed the conventional daily returns method. This facilitated the establishment of intraday volatility estimators. Shu and Zhang (2006) analyzed the S&P 500 index from 1995 to 1999 and found that the Parkinson estimator underestimates volatility when prices experience jumps at the start of trading, while it significantly overestimates volatility in instances of market drift. The Parkinson estimator has suboptimal performance for conditions with nonzero drifts, as it presupposes the existence of zero drifts.

Alizadeh, Brandt, and Diebold (2002) evaluated the impact of range-based estimation on five primary U.S. dollar exchange rates from 1978 to 1998 and found that range-based estimation significantly diminished latent volatility extraction errors compared to return-based proxies such as logarithmic absolute and logarithmic squared returns for all currencies examined. This presents empirical evidence about the efficacy of price range measurements in extracting volatility, initially identified by Garman and Klass (1980). Kumar and Maheswaran (2014) empirically demonstrated that among all range-based measures, the Rogers-Satchell measure is distinguished by its unbiased nature, independent of the drift parameter. In contrast, the Parkinson and Garman-Klass measures yield biased outcomes in the presence of a non-zero mean return, while their Conditional Autoregressive Rogers-Satchell measure exhibits superior forecasting performance when drift is present. Molnár (2012) indicated that an investigation of the 30 component stocks of the DJIA from 1992 to 2008 revealed that the Garman-Klass measure was the most effective range-based volatility metric for normalizing daily return rates, achieving normalcy akin to high-frequency transaction data. The Rogers-Satchell measure displayed pronounced heavy tails without drift, whereas the Parkinson measure produced a bimodal distribution with lower kurtosis relative to that of a normally distributed dataset. The results confirmed that the GK estimator most efficiently reproduces high-frequency accuracy using only daily OHLC data.

Furthermore, Andersen et al. (2003) demonstrated that volatility derived from high-frequency data exceeded that obtained from the GARCH model across many asset classes, hence highlighting the advantages of using intraday metrics for volatility assessment. The study conducted by Korkusuz, Kambouroudis, and McMillan (2023) utilized Parkinson, Garman-Klass, and Rogers-Satchell estimators, revealing that distinct estimators produce divergent forecasting capabilities, hence necessitating the selection of a suitable estimator.

2.2.2 Volatility Measurement in Cryptocurrency Markets

According to Baur, Hong, and Lee (2018), Bitcoin is much more volatile than traditional financial assets like stocks, currencies and commodities. It is highly volatile, and tends to fluctuate throughout its history, in times of stability and crisis. Liu and Tsyvinski (2021) state that the cryptocurrency markets are highly volatile compared to ordinary equities markets, with extreme changes between stable and extreme market conditions over time.

Barivieri et al. (2017) analyzed Bitcoin data during 2011-2017 using the Hurst exponent and found persistent long memory characteristics to Bitcoin volatility, with price volatility showing fractional integration and autocorrelations surviving over the long lags throughout the sample period. The analysis of their estimators showed that the range-based measures provided more efficient estimates, compared to return-based ones. In a study of the daily returns and volatility of Bitcoin, Chaim and Laurini (2018) found that the price discontinuities and volatility were much higher than traditional financial markets. Their results indicate that the major price changes are mostly associated with the formative market events, such as exchange hacks, unsuccessful fork attempts, and periods of increased public attention.

Makarov and Schoar (2020) studied 34 exchanges between 2016 and 2018 and found that spreads between exchanges vary significantly with mostly major exchanges showing small spreads, and with minor exchanges showing wide spreads. This heterogeneity of liquidity directly affects the best sample frequencies to evaluate volatility. In their study of 1,707 cryptocurrencies (Bitcoin, Ethereum, Ripple, Binance Coin, and Tether) Liu and Tsyvinski (2021) found that cryptocurrency markets have unique volatility characteristics that significantly outperform the volatility of traditional financial assets, and that volatility is significantly higher than in equity markets of all examined assets. Despite the growing evidence, studies based on high-frequency intraday data remain conspicuously limited to major cryptocurrencies beyond Bitcoin and Ethereum, which leads to significant gaps in understanding the dynamic of volatility in the short-term, asymmetric effects, and persistence dynamics of Binance Coin, Ripple, and Tether in particular (Ahmed et al., 2024).

2.2.3 HAR Model Performance in Cryptocurrency

Empirical studies have indicated that the HAR-RV model works better than other specifications in predicting accuracy. As demonstrated by Corsi (2009), HAR-RV model achieves the same forecasting efficiency as complex ARFIMA models at a significantly lower estimation cost than traditional GARCH models, and captures long-memory properties that are not adequately represented by traditional GARCH models across S&P 500, USD/CHF, and Treasury bond futures. HAR-RV model provides a simple and elucidative approach to the volatility persistence, which is based on the Heterogeneous Market Hypothesis. This model and its extensions, including the HAR-RV-J, which also has jump terms (Andersen et al., 2007) and semivariance-based extensions that distinguish positive and negative volatility terms (Patton and Sheppard, 2015), have become widely used in the process of volatility prediction in a wide range of asset classes, such as exchange rates, equity indices, and bond fut Bollerslev, Patton, and Quaedvlieg (2016) extended this framework by proposing the HARQ model that showed significant improvements in out-of-sample forecast accuracy by performing time-varying measurement error. Buncic and Gisler (2017) evaluated the effect of jumps and leverage effect in 18 international equity markets and determined that leverage effects usually improved the accuracy of the forecast, but leap components did not add significant benefits to the S&P 500.

Within the context of cryptocurrency, Shen, Urquhart, and Wang (2020) have used 18 competing HAR specifications on Bitcoin using high-frequency data and have found that the addition of jump components and structural breaks to the HAR framework always improved the out of sample forecasting accuracy across all time horizons. Ftiti, Louhichi, and Ben Ameur (2023).This analysis was expanded to encompass four principal cryptocurrency markets: Bitcoin, Ethereum, Ethereum Classic, and Ripple, utilizing five-minute intraday data. It was proved that HAR extensions, having both positive and negative semi-variances, were better than standard specifications, particularly in the COVID-19 crisis. Bergsli et al. (2022) determined the superiority of HAR models over GARCH-family models, e.g. EGARCH and APARCH, in short-term volatility prediction of Bitcoin over one- and two-day horizons, attributing the superiority in HAR models to their ability to capture long-memory dynamics using high-frequency realized variance. Although HAR models have been widely implemented, almost all studies that applied such models used realized volatility based on high-frequency returns. No studies have examined the claim that the Parkinson, Garman-Klass or Rogers-Satchell estimators exhibit different persistence properties, or the magnitude of the HAR components, or accuracy in forecasting in cryptocurrency markets, and no studies have examined whether the choice of estimator affects the observed volatility persistence patterns and forecasting accuracy across different cryptocurrency markets.

2.2.4 Comparative Forecasting Performance Across Cryptocurrencies

Even with the swift development in the cryptocurrencies markets, studies concerning volatility prediction have been restricted to a limited number of assets, thus undermining the generalization of current results within the whole market. Generalization may be hindered by the restriction to a small range of assets, especially because of the significant differences between the markets regarding liquidity, structure, and volatility of the assets in the market (Makarov & Schoar, 2020). Therefore, comparison studies that go beyond Bitcoin will help to determine consistency across different assets.

Fakhfekh and Jeribi (2020) investigated the volatility characteristics for seven different cryptocurrencies like Bitcoin, Ethereum, and Ripple based on the use of GARCH-type models, which showed significant variations regarding the persistence of volatility and its asymmetry, which implies that no one model specification is likely to work optimally when applied to all cryptocurrency markets. Continuing this line of evidence, Ftiti et al. (2023) investigated Bitcoin, Ethereum, Ethereum Classic, and Ripple based on the five-minute frequency of intraday return data and showed that the HAR models with signed semivariance worked much better than the standard models especially during the period of the COVID-19 pandemic.

Despite this growing body of evidence, critical gaps remain. Ahmed et al. (2024) identified that high-frequency data research remains a critical gap in the cryptocurrency volatility literature, recommending broader multi-asset coverage as a priority for future research. Extending this work, a survey of the extant literature shows that there have been no studies that examine the relative forecasting power of various range-based estimators, including the Parkinson (1980), Garman-Klass (1980), and Rogers-Satchell (1991) estimators, under the HAR model. However, there are still important knowledge gaps. This is an important methodological gap since it is already well established in the case of traditional markets that range-based estimators provide greater efficiency than realized volatility based on returns (Alizadeh et al., 2002; Molnár, 2012). This study is able to contribute to bridging the above-mentioned gap as all three range-based estimators are considered using the HAR-RV model for the five leading cryptocurrencies namely Bitcoin, Ethereum, Ripple, Binance Coin, and Tether.

2.3 Research Gaps and Study Motivation

Gap 1: Range-Based Estimators PK, GK, and RS Absent from Cryptocurrency Volatility Research

As far as we know, there is no existing study on cryptocurrency volatility that provides a systematic comparison between range-based estimators (Parkinson, Garman-Klass, Rogers-Satchell) among different cryptocurrencies. In a systematic review of 164 papers on cryptocurrency volatility by Ahmed et al. (2024), it was confirmed that most of the research works focus on GARCH-type models and return-based realized volatility measures, but range-based estimators have been recognized as an unexplored field and have been recommended to be studied in future research. Examples of researches applying GARCH model to daily close-to-close return (Katsiampa, 2017; Dyhrberg, 2016); high-frequency return-based realized volatility in HAR model (Shen et al., 2020); and stochastic volatility models based on daily log-returns (Chaim & Laurini, 2018) do not include range-based estimators (Parkinson, Garman-Klass, or Rogers-Satchell). Conversely, any study that systematically contrasts the use of range-based estimators limits its scope solely to the conventional financial markets including stocks, currency, and futures, with no application of the range-based estimators to cryptocurrency markets (Molnár, 2012; Korkusuz et al., 2023; Alizadeh et al., 2002).

This limitation, however, creates a huge gap since cryptocurrency markets which are typified by constant trades, momentum, and price gaps are where range-based estimators need to be compared systematically.

Gap 2: Range-Based Measures Unexplored Within HAR Frameworks

In previous research regarding cryptocurrencies and their volatilities using the HAR approach, the input variables have been based mainly on high-frequency calculations of variables such as RV, Bi-power Variation, Realized Semivariance, and Jumps (Shen et al., 2020; Qiu et al., 2021). However, in the case of Sapkota (2022), where the estimators Parkinson (1980), Garman Klass (1980), and Rogers Satchell (1991) have been used as inputs to the HAR approach in order to study Bitcoin, comparing various models was only a means to establish a better benchmark in the presence of sentiment data.

Range-based estimator efficiency for use as key HAR inputs in a variety of cryptocurrencies, and if the benefits of efficiency as established in studies in the traditional financial markets (Alizadeh et al., 2002), where such estimators are efficient in improving forecasts by minimizing measurement error in inputs for realized volatilities (Bollerslev et al., 2016), have applicability within the context of HAR models, continues to remain empirically unexplored.

Gap 3: Cross-Asset Heterogeneity in Volatility Persistence Remains Underexplored

Empirical evidence increasingly indicates that the patterns of volatility persistence vary significantly among cryptocurrencies. Fakhfekh and Jeribi (2020) identify substantial disparities in volatility dynamics and persistence among various cryptocurrencies, including Ripple, Monero, and NEO, demonstrating that no singular model specification achieves optimal performance across all digital assets. Ftiti et al. (2023) bolster this assertion by demonstrating that the component weights of the HAR model and the relative contributions of daily, weekly, and monthly volatility exhibit significant variation among Bitcoin, Ethereum, Ethereum Classic, and Ripple under diverse market situations. No research has investigated whether these cross-asset disparities endure when range-based OHLC estimators are utilized as HAR inputs in place of high-frequency return-based metrics. The relative significance of short-term versus long-term volatility components within the HAR framework remains uncertain across different estimators and assets with fundamentally distinct microstructures, such as the near-zero variance stablecoin Tether USD compared to highly speculative assets like Ripple or Binance Coin.

Gap 4: Limited Research Beyond Bitcoin

Studies related to volatility in cryptocurrency markets have tended to pay more attention to Bitcoin compared to other major cryptocurrencies. In this regard, Köse et al. (2024) use the framework of structural vector autoregression to investigate the impact of global macroeconomic factors on the price volatility of Bitcoin, whereas Sapkota (2022) uses range-based volatilities and news sentiment in HAR models only for Bitcoin. Moreover, another study conducted by Klein et al. (2018) focuses on conditional variance properties and portfolio return using BEKK-GARCH framework only for Bitcoin. It is important to note here that such a focus limits studies to a particular asset, while Bitcoin, Ethereum, Binance Coin, and Ripple have different microstructures and functionalities in cryptocurrency markets. Hence, no prior research exists which analyzes major cryptocurrencies using range-based HAR models.

Study Motivation and Research Question

These four gaps are centered on a common theme in that current studies of volatility and cryptocurrency risk-reward relationships lack sufficient methodology and scope to provide meaningful and theoretically sound risk-reward analysis. Range-based estimators have not been incorporated into the cryptocurrency HAR framework, risk-return results are not strong and are devoid of time horizons, and the literature is overwhelmingly biased towards Bitcoin. This paper attempts to address these gaps by applying range-based HAR models based on Parkinson (1980), Garman and Klass (1980), and Rogers and Satchell (1991) estimators for Bitcoin, Ethereum, Binance Coin, and Ripple to examine whether a theoretically sound risk-reward relationship emerges.

The main research question is:

Which range-based volatility estimator best captures volatility dynamics across major cryptocurrency markets within the HAR-RV framework?

3 Research Methodology

3.1 Introduction

Research methodology provides a systematic framework for addressing a research problem by outlining the plan and procedures adopted in a study. It explains the structured steps researchers follow to explain, clarify, and predict observed phenomena. In this sense, research methodology represents the scientific examination of how research is conducted, rather than the findings themselves.

A comprehensive methodology typically includes research strategy, research approach and methods, data collection techniques, sample selection, research process, data presentation procedures, analytical techniques, software tools used for analysis, ethical considerations, and study limitations. By clearly defining these elements, the methodology ensures transparency and rigor in the research process.

Importantly, the methodology section precedes data presentation and analysis, as it establishes the foundation upon which empirical findings are interpreted. Without a well-defined methodological framework, research conclusions may lack clarity, validity, and reliability.

3.2 Methodology of the Study

This paper examines volatility forecasting in prominent cryptocurrency markets using three range-based volatility estimators: Parkinson (1980), Garman-Klass (1980), and Rogers-Satchell (1991), within the Heterogeneous Autoregressive Realized Volatility (HAR-RV) framework. Specifically, it aims to determine the most suitable range-based volatility measure for analyzing intraday price dynamics and predicting volatility for five principal cryptocurrencies: Bitcoin, Ethereum, Binance Coin, Ripple, and Tether USD.

3.3 Range-Based Volatility Estimation

Daily realized volatility is estimated using three range-based estimators that exploit intraday price information more efficiently than close-to-close measures.

3.3.1 Parkinson (1980) Estimator

The paper by Parkinson (1980) provided an innovative method for calculating the measure of volatility by including both the maximum and minimum prices instead of only the closing price. High low prices would provide an improved model compared to close to close prices since the former takes into account variations in prices during trading hours, which are not included in the latter measure.

The Parkinson variance estimator is defined as:

$$\sigma_{PK}^2 = \frac{1}{4\ln(2)\,n}\sum_{i=1}^{n} ln^2\left(\frac{H_i}{L_i}\right)$$

3.3.2 Garman and Klass (1980) Estimator

Garman & Klass (1980) sought to overcome this drawback by coming up with an estimator of volatility that utilizes all four OHLC data. Their method takes into account price action at the opening and closing of markets, which makes their approach more precise in estimating volatility.

$$\sigma_{GK}^2 = \frac{1}{n}\left(\sum_{i=1}^{n}\frac{1}{2}ln^2\left(\frac{H_i}{L_i}\right) + (2\ln(2)-1)ln^2\left(\frac{C_i}{O_i}\right)\right)$$

where σ_{GK}^2 is the Garman and Klass (1980) variance estimator, H_i is the highest and L_i the lowest intraday price, O_i is the opening and C_i the closing price of asset *i.*

3.3.3 Rogers and Satchell (1991) Estimator

The Rogers and Satchell volatility estimator is a volatility estimator based on the opening, highest, lowest, and closing price of the day that includes a drift term to account for the trend in the prices. This estimator gives better results for volatility estimation when there is an underlying trend in the prices.

3.4 HAR Model for Volatility Forecasting

The study utilizes the Heterogeneous Autoregressive model of Realized Volatility (HAR-RV) to model and predict volatility. The HAR model captures volatility persistence across diverse time periods, mirroring the conduct of market participants with various trading timeframes.

Realized variance for each of daily, weekly, and monthly frequencies is expressed as:

• Daily volatility: $RV_t^{(d)} = RV_t$

• Weekly volatility (7 days):

$$RV_t^{(w)} = \frac{1}{7}\sum_{i=0}^{6} RV_{t-i}$$

• Monthly volatility (30 days):

$$RV_t^{(m)} = \frac{1}{30}\sum_{i=0}^{29} RV_{t-i}$$

The HAR-RV model is specified as:

$$RV_{t+1} = \beta_0 + \beta_d RV_t^{(d)} + \beta_w RV_t^{(w)} + \beta_m RV_t^{(m)} + \varepsilon_{t+1}$$

3.5 Data Source

This study makes use of daily Open-High-Low-Close (OHLC) prices and trading volume data of the following five popular cryptocurrencies: BTC, ETH, USDT, BNB, and XRP. The data has been collected via the investing.com website, which is a widely used platform for financial information that acquires prices of various cryptocurrencies from different exchanges. It must be emphasized that all the prices have been expressed in terms of United States Dollars (USD). Selection of the above-mentioned cryptocurrencies (BTC, ETH, USDT, BNB, and XRP) has been done based on market capitalization at CoinMarketCap (2024).

The sample period spans from January 1, 2020 to December 31, 2025, yielding 2,162 daily observations for Bitcoin, Ethereum, and Binance Coin, and 2,161 observations for Ripple and Tether USD, with the minor difference attributable to data availability across exchanges. The period under study was selected based on data availability, ensuring a sufficiently large sample for reliable estimation of the HAR-RV model and systematic comparison of range-based volatility estimator performance across all five cryptocurrency markets.

The Table 1 shows the descriptive statistics of the range-based volatility measures for five major cryptocurrencies Bitcoin (BTC), Ethereum (ETH), Binance Coin (BNB), Tether USD (USDT), and Ripple (XRP). Table 1 includes three range-based volatility measures that include Parkinson (PK), Garman Klass (GK), and Rogers Satchell (RS) over four horizons which include daily returns (RT), one-day realized variance (RV1), seven-day realized variance (RV7), and thirty-day realized variance (RV30), resulting in 2,162 daily observations for BTC, ETH, and BNB and 2,161 daily observations for XRP and USDT.

Table 1. Summary Statistics of Five Cryptocurrency Currencies and RV Estimates (Jan 2020-Dec 2025)

Variable	Nobs	Min	Max	Mean	Median	StDev	Skewness	Kurtosis
	Bitcoin							
PK_RT	2162	0.000005	0.113285	0.001105	0.00047	0.003433	21.051	603.096
PK_RV1	2162	0.000005	0.113285	0.001105	0.00047	0.003433	21.051	603.118
PK_RV7	2162	0.000055	0.02951	0.001105	0.000662	0.001951	9.174	112.36

PK_RV30	2162	0.000135	0.009155	0.001106	0.000759	0.001236	3.868	18.174
GK_RT	2162	0.000006	0.252576	0.001949	0.000771	0.006755	26.102	904.884
GK_RV1	2162	0.000006	0.252576	0.001949	0.000771	0.006755	26.103	904.912
GK_RV7	2162	0.000086	0.056512	0.001951	0.001177	0.00359	10.217	136.484
GK_RV30	2162	0.000217	0.01722	0.001953	0.001333	0.002219	4.201	21.985
RS_RT	2162	0.000004	0.092912	0.001136	0.000484	0.003361	15.848	355.37
RS_RV1	2162	0.000004	0.092912	0.001136	0.000485	0.003361	15.849	355.382
RS_RV7	2162	0.000056	0.024265	0.001136	0.000645	0.001964	6.882	61.179
RS_RV30	2162	0.000149	0.007828	0.001136	0.000765	0.001282	3.278	11.576
	Ethereum							
PK_RT	2162	0.000006	0.148644	0.001911	0.00084	0.005316	17.166	405.119
PK_RV1	2162	0.000006	0.148644	0.001912	0.00084	0.005316	17.167	405.137
PK_RV7	2162	0.00005	0.038485	0.001913	0.001169	0.003101	7.429	71.28
PK_RV30	2162	0.00012	0.013024	0.001915	0.001431	0.00199	3.491	14.076
GK_RT	2162	0.000008	0.340345	0.003397	0.001404	0.010397	20.733	590.557
GK_RV1	2162	0.000008	0.340345	0.003399	0.001405	0.010397	20.733	590.575
GK_RV7	2162	0.000076	0.072744	0.0034	0.002063	0.005712	7.984	81.794
GK_RV30	2162	0.000193	0.022933	0.003405	0.002567	0.00359	3.609	14.957
RS_RT	2162	0.000008	0.139173	0.001923	0.000833	0.005242	13.811	280.436
RS_RV1	2162	0.000008	0.139173	0.001924	0.000835	0.005242	13.811	280.453
RS_RV7	2162	0.000062	0.045082	0.001924	0.001133	0.003131	7.086	72.159
RS_RV30	2162	0.000151	0.014687	0.001925	0.001444	0.002022	3.474	15.273
	BNB							
PK_RT	2162	0.000002	0.137755	0.002034	0.000675	0.006872	12.891	211.37
PK_RV1	2162	0.000002	0.137755	0.002035	0.000675	0.006872	12.891	211.377
PK_RV7	2162	0.000062	0.050629	0.002035	0.000916	0.004448	6.773	54.583
PK_RV30	2162	0.00015	0.019462	0.002041	0.00111	0.003006	3.65	14.124
GK_RT	2162	0.000003	0.321438	0.003592	0.001105	0.013376	15.15	294.325
GK_RV1	2162	0.000003	0.321438	0.003593	0.001107	0.013376	15.15	294.335
GK_RV7	2162	0.000098	0.090344	0.003593	0.001576	0.008257	7.065	58.332
GK_RV30	2162	0.000257	0.037125	0.003603	0.001902	0.005528	3.781	15.178
RS_RT	2162	0	0.193664	0.002104	0.000685	0.007103	14.263	296.572
RS_RV1	2162	0	0.193664	0.002104	0.000685	0.007103	14.263	296.58
RS_RV7	2162	0.000068	0.051359	0.002105	0.000948	0.004271	6.249	48.96
RS_RV30	2162	0.000165	0.017183	0.002111	0.001173	0.002822	3.305	11.871
	Tether							
PK_RT	2161	0	0.001319	0.000002	2.31E-07	0.000032	33.59	1298.77
PK_RV1	2161	0	0.001319	0.000002	2.31E-07	0.000032	33.589	1298.76
PK_RV7	2161	3.09E-08	0.000281	0.000002	3.02E-07	0.000016	14.636	234.014
PK_RV30	2161	5.38E-08	0.000069	0.000002	3.66E-07	0.000008	7.116	53.313
GK_RT	2161	0	0.001838	0.000003	3.55E-07	0.000045	33.248	1275.78
GK_RV1	2161	0	0.001838	0.000003	3.55E-07	0.000045	33.247	1275.76
GK_RV7	2161	4.73E-08	0.0004	0.000003	4.74E-07	0.000022	14.657	234.747
GK_RV30	2161	8.06E-08	0.000099	0.000003	5.84E-07	0.000012	7.12	53.418
RS_RT	2161	0	0.002944	0.000004	3.20E-07	0.000072	33.818	1307.67

RS_RV1	2161	0	0.002944	0.000004	3.20E-07	0.000072	33.817	1307.66
RS_RV7	2161	3.71E-08	0.00059	0.000004	4.35E-07	0.000033	14.256	223.593
RS_RV30	2161	7.46E-08	0.000143	0.000004	5.42E-07	0.000017	6.976	51.483
	XRP							
PK_RT	2161	0.000017	0.194633	0.003223	0.000946	0.010649	11.198	161.517
PK_RV1	2161	0.000017	0.194633	0.003223	0.000947	0.010649	11.198	161.52
PK_RV7	2161	0.000139	0.057176	0.003224	0.001304	0.005964	4.85	30.064
PK_RV30	2161	0.00036	0.021527	0.003228	0.00187	0.003892	2.58	6.795
GK_RT	2161	0.000025	0.381245	0.005592	0.001525	0.019357	11.604	173.265
GK_RV1	2161	0.000025	0.381245	0.005592	0.001535	0.019357	11.604	173.268
GK_RV7	2161	0.000217	0.097692	0.005593	0.00223	0.010556	4.847	29.456
GK_RV30	2161	0.000594	0.037328	0.005601	0.003149	0.006778	2.507	6.298
RS_RT	2161	0	0.305971	0.003507	0.001055	0.012826	14.075	270.591
RS_RV1	2161	0	0.305971	0.003507	0.001055	0.012826	14.076	270.595
RS_RV7	2161	0.00018	0.070452	0.003507	0.001399	0.006727	5.123	33.601
RS_RV30	2161	0.000398	0.0258	0.003512	0.002066	0.00433	2.726	7.837

Note: PK (Parkinson), GK (Garman-Klass), RS (Rogers-Satchell), RT (daily return), RV1 (1-day realized variance), RV7 (7-day realized variance), RV30 (30-day realized variance).

3.6 Correlation

Table 2 shows the correlation matrix of the range-based volatility estimators for four levels of aggregation – daily return (RT), 1-day realized variance (RV1), 7-day realized variance (RV7), and 30-day realized variance (RV30) – corresponding to Bitcoin, Ethereum, Binance Coin, Tether USD, and Ripple, respectively. Correlation tests are performed to analyze the relationship between the Parkinson (PK), Garman-Klass (GK), and Rogers-Satchell (RS) estimators under different levels of aggregation and check the presence of multicollinearity between variables before performing regression estimation.

Table 2. Pairwise correlation matrix of range-based volatility estimators

Bitcoin												
Variable	**PK_RT**	**PK_RV1**	**PK_RV7**	**PK_RV30**	**GK_RT**	**GK_RV1**	**GK_RV7**	**GK_RV30**	**RS_RT**	**RS_RV1**	**RS_RV7**	**RS_RV30**
PK_RT	1											
PK_RV1	0.415	1										
PK_RV7	0.292	0.505	1									

PK_RV30	0.193	0.284	0.589	1								
GK_RT	0.984	0.327	0.238	0.162	1							
GK_RV1	0.434	0.984	0.453	0.252	0.339	1						
GK_RV7	0.298	0.523	0.995	0.567	0.242	0.477	1					
GK_RV30	0.196	0.294	0.595	0.997	0.164	0.264	0.577	1				
RS_RT	0.805	0.573	0.387	0.238	0.69	0.605	0.396	0.243	1			
RS_RV1	0.254	0.805	0.547	0.31	0.208	0.69	0.54	0.311	0.324	1		
RS_RV7	0.257	0.421	0.947	0.603	0.211	0.353	0.908	0.597	0.333	0.535	1	
RS_RV30	0.18	0.248	0.552	0.972	0.152	0.21	0.519	0.95	0.217	0.301	0.603	1

Ethereum

Variable	**PK_RT**	**PK_RV1**	**PK_RV7**	**PK_RV30**	**GK_RT**	**GK_RV1**	**GK_RV7**	**GK_RV30**	**RS_RT**	**RS_RV1**	**RS_RV7**	**RS_RV30**
PK_RT	1											
PK_RV1	0.396	1										
PK_RV7	0.313	0.525	1									
PK_RV30	0.215	0.299	0.594	1								
GK_RT	0.985	0.332	0.269	0.189	1							
GK_RV1	0.398	0.985	0.481	0.273	0.332	1						
GK_RV7	0.312	0.535	0.993	0.578	0.268	0.5	1					
GK_RV30	0.214	0.304	0.596	0.996	0.188	0.281	0.585	1				
RS_RT	0.819	0.478	0.369	0.235	0.71	0.487	0.367	0.234	1			
RS_RV1	0.298	0.819	0.534	0.302	0.257	0.71	0.517	0.298	0.333	1		
RS_RV7	0.286	0.453	0.935	0.581	0.245	0.391	0.888	0.569	0.34	0.544	1	
RS_RV30	0.207	0.272	0.562	0.966	0.183	0.24	0.533	0.94	0.224	0.305	0.594	1

BNB

Variable	PK_RT	PK_RV1	PK_RV7	PK_RV30	GK_RT	GK_RV1	GK_RV7	GK_RV30	RS_RT	RS_RV1	RS_RV7	RS_RV30
PK_RT	1											
PK_RV1	0.502	1										
PK_RV7	0.393	0.579	1									
PK_RV30	0.269	0.34	0.617	1								
GK_RT	0.984	0.429	0.346	0.243	1							
GK_RV1	0.52	0.984	0.532	0.314	0.442	1						
GK_RV7	0.403	0.597	0.996	0.606	0.353	0.554	1					
GK_RV30	0.272	0.347	0.622	0.999	0.246	0.322	0.612	1				
RS_RT	0.811	0.6	0.431	0.273	0.695	0.637	0.45	0.279	1			
RS_RV1	0.314	0.811	0.579	0.329	0.277	0.695	0.581	0.33	0.34	1		
RS_RV7	0.331	0.489	0.963	0.616	0.295	0.429	0.937	0.614	0.345	0.547	1	
RS_RV30	0.245	0.306	0.591	0.986	0.222	0.275	0.572	0.976	0.243	0.319	0.612	1

Tether

Variable	PK_RT	PK_RV1	PK_RV7	PK_RV30	GK_RT	GK_RV1	GK_RV7	GK_RV30	RS_RT	RS_RV1	RS_RV7	RS_RV30
PK_RT	1											
PK_RV1	0.227	1										
PK_RV7	0.142	0.435	1									
PK_RV30	0.07	0.201	0.502	1								
GK_RT	1	0.226	0.145	0.072	1							
GK_RV1	0.243	1	0.438	0.203	0.242	1						
GK_RV7	0.147	0.436	1	0.504	0.15	0.439	1					
GK_RV30	0.073	0.202	0.502	1	0.076	0.204	0.504	1				
RS_RT	0.995	0.222	0.138	0.066	0.994	0.238	0.143	0.069	1			

RS_RV1	0.157	0.995	0.43	0.196	0.157	0.994	0.431	0.196	0.153	1		
RS_RV7	0.119	0.427	0.998	0.497	0.122	0.429	0.997	0.497	0.116	0.426	1	
RS_RV30	0.058	0.197	0.497	0.998	0.06	0.199	0.499	0.997	0.053	0.194	0.495	1

XRP

Variable	**PK_RT**	**PK_RV1**	**PK_RV7**	**PK_RV30**	**GK_RT**	**GK_RV1**	**GK_RV7**	**GK_RV30**	**RS_RT**	**RS_RV1**	**RS_RV7**	**RS_RV30**
PK_RT	1											
PK_RV1	0.291	1										
PK_RV7	0.277	0.509	1									
PK_RV30	0.241	0.325	0.639	1								
GK_RT	0.989	0.265	0.25	0.22	1							
GK_RV1	0.294	0.989	0.481	0.303	0.267	1						
GK_RV7	0.284	0.52	0.995	0.618	0.257	0.497	1					
GK_RV30	0.245	0.33	0.646	0.997	0.223	0.31	0.628	1				
RS_RT	0.887	0.284	0.28	0.239	0.81	0.286	0.286	0.243	1			
RS_RV1	0.222	0.887	0.484	0.314	0.202	0.81	0.477	0.313	0.215	1		
RS_RV7	0.229	0.451	0.948	0.641	0.206	0.409	0.913	0.637	0.233	0.486	1	
RS_RV30	0.219	0.301	0.602	0.973	0.2	0.275	0.572	0.955	0.217	0.31	0.64	1

4 Result

4.1 Summary Statistics

Table 1 is shown below with descriptive statistics of range-based volatility measures estimated for five major cryptocurrencies: BTC (Bitcoin), ETH (Ethereum), BNB (Binance Coin), USDT (Tether USD), and XRP (Ripple). This table includes three types of range-based volatility measures: Parkinson (PK), Garman-Klass (GK), and Rogers-Satchell (RS), with four different measurement periods, i.e., RT (daily return), RV1 (one-day realized variance), RV7 (seven-day realized variance), and RV30 (thirty-day realized variance). There are 2,162 daily observations for BTC, ETH, and BNB, while USDT and XRP have 2,161 observations.

In the speculative cryptocurrencies, the one with the largest mean daily realized variance using the Garman-Klass method is Ripple (XRP) with 0.005592, then Binance Coin (BNB) with 0.003593 and Ethereum (ETH) with 0.003399. Since Bitcoin is already established and well-held by many investors, it has the smallest mean daily variance when compared to other speculative cryptocurrencies (GK: 0.001949). This is in line with the existing empirical evidence that shows more liquidity and institutional trading result in lower volatility levels. The mean variance values for speculative cryptocurrencies suggest a much higher level of risk compared to the normal market conditions (Baur et al., 2018; Makarov & Schoar, 2020).

USDT or Tether USD, which is a USD-backed stablecoin, demonstrates an extremely low mean variance through all three estimation methods at all horizons with daily means around 10^{-6} in both Parkinson and Garman-Klass frameworks as the evidence of its construction as an asset with the lowest volatility. In comparison to the highly volatile speculative crypto assets, the stable nature of the price series of the USDT makes it a valid boundary case benchmark for this analysis.

All the three measures yield highly positively skewed distributions among the speculative assets with skewness statistics well above 10 for daily realized variance, along with considerably large values of excess kurtosis, in keeping with the findings from studies on cryptocurrencies that show heavy tails among the returns distributions (Gkillas & Katsiampa, 2018). Weekly (RV7) and monthly (RV30) aggregates tend to exhibit lower values of skewness and kurtosis relative to daily measures, owing to the effect of window averaging. Garman-Klass estimates of variance appear to yield higher averages than Parkinson variance estimates for all assets at all timescales because the former includes both open-to-close range as well as high-low range of prices within a day.

The correlation matrix provided in Tables 2a through 2e shows that there exists an extremely high correlation amongst the three volatility estimators when the aggregating periods are similar in each asset with PK-GK being above 0.98 for all the speculative cryptocurrencies in the daily period. The high collinearity amongst the contemporaneous periods shows that the three estimators capture the same volatility signal, but differ in efficiency properties through their forecast performance instead of capturing different levels. However, cross-period correlations are lower when the aggregating periods are not similar, with a correlation between PK_RV1 and PK_RV30 in Bitcoin volatility being 0.284, indicating that there exists different information in the daily and monthly volatility measures regarding persistence.

4.1.1 Basic Fitting of the HAR Model

The results from Table 3 are provided for USDT. For this price-stable instrument, the HAR model yields significantly weaker and less stable results. While the estimates for the intercept and lagged variables in USDT are much smaller by several orders of magnitude, it reflects low or zero underlying volatilities for this stablecoin. The daily and weekly lagged variables are significant for the PK and RS estimations, whereas the monthly lagged variable is uniformly insignificant for all estimators considered in this study. The estimated model with the GK procedure for USDT shows peculiar results as the t-value associated with the daily variable is close to zero, while the coefficient itself has a positive value of 0.2180, which might be explained by numerical instability of the model due to the constant opening prices observed in the USDT data.

Table 3. HAR Model Estimation Results Bitcoin (BTC) (2020–2025)

Dependent variable: RVt+1

Variable	PK	GK	RS
Intercept	0.0004***	0.0008***	0.0004***
	(4.6294)	(4.6664)	(4.5808)
t−1	0.3600***	0.2896***	0.2052***
	(15.9592)	(12.638)	(8.6841)
t−7	0.1528***	0.1524***	0.3489***
	(3.2463)	(2.9905)	(7.2153)
t−30	0.1106*	0.1232	0.0842
	(1.6535)	(1.6397)	(1.283)
Model Statistics			
Observations	2162	2162	2162
F-statistic	160.661	102.217	118.396
R^2	0.1826	0.1244	0.1413
Adjusted R^2	0.1814	0.1232	0.1401

*Notes: N = 2162. t-statistics in parentheses. PK = Parkinson, GK = Garman-Klass, RS = Rogers-Satchell. Significance: *** p < 0.01, ** p < 0.05, * p < 0.10.*

Table 4. HAR Model Estimation Results Ethereum (ETH) (2020–2025)

Dependent variable: RVt+1

Variable	PK	GK	RS
Intercept	0.0007***	0.0013***	0.0007***
	(4.5828)	(4.6018)	(4.6309)
t−1	0.3210***	0.2639***	0.2118***
	(13.9616)	(11.3556)	(8.9355)

t−7	0.1955***	0.1930***	0.3364***
	(4.1821)	(3.8548)	(7.1617)
t−30	0.1362**	0.1512**	0.1047
	(2.096)	(2.1031)	(1.6346)
Model Statistics			
Observations	2162	2162	2162
F-statistic	151.325	103.483	124.959
R^2	0.1738	0.1258	0.148
Adjusted R^2	0.1727	0.1246	0.1468

*Notes: N = 2162. t-statistics in parentheses. PK = Parkinson, GK = Garman-Klass, RS = Rogers-Satchell. Significance: *** p < 0.01, ** p < 0.05, * p < 0.10.*

Table 5. HAR Model Estimation Results Binance Coin (BNB) (2020–2025)

Dependent variable: RVt+1

Variable	**PK**	**GK**	**RS**
Intercept	0.0006***	0.0011***	0.0007***
	(3.6773)	(3.6446)	(3.8581)
t−1	0.4139***	0.3566***	0.2168***
	(18.332)	(15.5563)	(9.1609)
t−7	0.1853***	0.1962***	0.3193***
	(4.4413)	(4.4155)	(6.774)
t−30	0.1235**	0.1370**	0.1407**
	(2.3075)	(2.347)	(2.2328)
Model Statistics			
Observations	2162	2162	2162
F-statistic	265.141	196.168	130.33
R^2	0.2693	0.2143	0.1534
Adjusted R^2	0.2683	0.2132	0.1522

*Notes: N = 2162. t-statistics in parentheses. PK = Parkinson, GK = Garman-Klass, RS = Rogers-Satchell. Significance: *** p < 0.01, ** p < 0.05, * p < 0.10.*

Table 6. HAR Model Estimation Results Ripple (XRP) (2020–2025)

Dependent variable: RVt+1

Variable	**PK**	**GK**	**RS**
Intercept	0.0010***	0.0019***	0.0012***
	(3.5903)	(3.6687)	(3.5316)
t−1	0.2027***	0.1854***	0.1342***
	(8.6072)	(7.8677)	(5.6668)
t−7	0.1868***	0.1839***	0.1774***
	(3.6127)	(3.4848)	(3.1771)
t−30	0.2976***	0.2932***	0.3436***
	(4.128)	(3.9078)	(4.3068)
Model Statistics Observations	2161	2161	2161
F-statistic	92.7715	78.2628	58.9172
R^2	0.1143	0.0982	0.0757
Adjusted R^2	0.1131	0.0969	0.0745

*Notes: N = 2161. t-statistics in parentheses. PK = Parkinson, GK = Garman-Klass, RS = Rogers-Satchell. Significance: *** p < 0.01, ** p < 0.05, * p < 0.10.*

Table 7. HAR Model Estimation Results Tether USD (USDT) (2020–2025)

Dependent variable: RVt+1

Variable	PK	GK	RS
Intercept	0.000001**	0.000002**	0.000003**
	(2.0612)	(2.0782)	(1.9896)
t−1	0.204070***	0.218015***	0.126424***
	(8.7702)	(9.3846)	(5.3838)
t−7	0.106427*	0.105196**	0.136492**
	(1.9566)	(1.9597)	(2.3677)
t−30	0.012883	0.020035	-0.009426
	(0.1352)	(0.2149)	(-0.0915)

Model Statistics

Observations	2161	2161	2161
F-statistic	40.971	46.576	19.521
R^2	0.0539	0.0608	0.0264

*Notes: N = 2161. t-statistics in parentheses. PK = Parkinson, GK = Garman-Klass, RS = Rogers-Satchell. Significance: *** p < 0.01, ** p < 0.05, * p < 0.10.*

4.1.2 Model Fit: R^2 Across Estimators and Assets

According to Table 3-7, the Parkinson (1980) estimator consistently yields the greatest R^2 for four of the five assets, with values spanning from 11.43% for Ripple to 26.93% for Binance Coin. Tether USD is the sole example where GK somewhat Surpasses PK, while both levels remain low due to the asset's near-zero volatility.

The Parkinson estimator's superiority in R^2 is remarkable, considering the Garman-Klass estimator is theoretically more efficient, boasting an efficiency of 7.4 compared to Parkinson's 4.9, as it utilizes open, high, low, and close prices instead of solely the high-low range (Garman & Klass, 1980; Molnár, 2012). This research empirically demonstrates that the theoretical advantage does not result in improved in-sample fit. This apparent reversal may indicate structural characteristics of cryptocurrency marketplaces that diminish the efficiency benefit of the GK estimator. In contrast to conventional equity markets, cryptocurrency markets function incessantly across disparate, non-integrated exchanges devoid of centralized opening auctions (Makarov & Schoar, 2020), resulting in an environment where microstructure noise is recognized to distort volatility estimates (Bandi & Russell, 2008; Hansen & Lunde, 2006). Given that GK integrates opening prices while Parkinson exclusively utilizes the high-low range, the latter may exhibit reduced susceptibility to distortions inside the cryptocurrency context. This explanation aligns with Shu and Zhang (2006), who demonstrated that opening price surges induce systematic bias specifically in the GK estimator. The Parkinson estimator thus demonstrates greater robustness to this noise, yielding a clearer signal for the HAR model.

The Rogers-Satchell estimator consistently produces the lowest R^2 for all speculative assets, indicating that drift correction offers minimal additional forecasting value beyond what the high-low range currently encompasses. R^2 values, in absolute terms, span from roughly 7% to 27%, significantly lower than the approximately 46% documented by Corsi (2009) for foreign exchange markets and by Sapkota (2022) for Bitcoin utilizing sentiment-augmented HAR parameters. The comparatively modest values indicate the intrinsic challenge of predicting cryptocurrency volatility, which is influenced by frequent structural breaks and significant price fluctuations not accounted for by historical volatility metrics alone (Chaim & Laurini, 2018; Ftiti et al., 2023).

4.1.3 Cross-Asset Comparison of HAR Model Performance

Market structures, level of liquidity, and investors of the five cryptocurrencies differ, and this difference manifests itself in the different impact of volatility from the previous period on the volatility for the following periods in each of the cases under the HAR model. According to all three estimations, Binance Coin exhibits the largest in-sample R^2 value. Moreover, the PK model specification gives the highest R^2 of 26.93% among all five currencies. It may be interpreted as the fact that volatility of BNB is more predictable from the previous periods than volatility of any other currency considered.

The two cryptocurrencies Bitcoin and Ethereum demonstrate similar fits in terms of HAR models, having adjusted R^2 for PK estimations at 18.14% and 17.27% respectively. The reason why these two assets possess similar predictability features lies in the similarities of their features as the two oldest and the most institutionalized cryptocurrencies available in our dataset. Indeed, the presence of multiple types of agents who trade these coins on various horizons corresponds well with the Heterogeneous Agents concept used in the development of HAR model (Müller et al., 1997). Ripple shows the smallest R^2 value among all speculative cryptocurrencies, namely, PK estimation gives 11.43% only due to the fact that its price dynamics are more unstable and more dependent on events rather than lagged volatilities.

The case with Tether USD shows a very different scenario compared to other speculative cryptos. The relatively low R^2 ratios (from 2.64% up to 6.08%), close to zero coefficients for intercepts, as well as lower and less significant coefficients of volatility lags in general serve to indicate that volatility clustering and persistence captured through the HAR model design can hardly describe the dynamics of volatility in an asset which is not subject to speculations. However, all five F-statistics are significant at the 1% confidence level, proving that even the models whose parameters differ significantly from each other in terms of significance are still valid in their general design. Therefore, based on the empirical analysis conducted in this chapter, the Parkinson (1980) measure can be regarded as providing the best in-sample fit among all realised volatility estimators in the studied speculative cryptocurrencies.

4.2 Conclusion

This study utilized the HAR-RV model, integrating three range-based volatility estimators Parkinson (1980), Garman and Klass (1980), and Rogers and Satchell (1991), across five principal cryptocurrency marketplaces from January 2020 to December 2025. The empirical results indicate that lagged volatility components possess statistically significant predictive capability for all analyzed assets, validating the relevance of the HAR framework in forecasting cryptocurrency volatility. Of the three estimators, the Parkinson (1980) measure consistently demonstrated the greatest in-sample explanatory power. This outcome is paradoxical considering the theoretical advantages of the Garman-Klass estimator and is ascribed to the lack of centralized opening auctions in continuously traded cryptocurrency markets, which introduces microstructure noise that disproportionately impacts estimators that utilize opening price data. The Rogers-Satchell estimator produced the least effective in-sample fit among the five assets, indicating that drift adjustment offers no further forecasting advantage in this scenario. The extent of volatility persistence significantly differed among assets, indicating variations in market microstructure and investor makeup, hence necessitating asset-specific model parameters for precise volatility forecasting.

Practically speaking, the results provide concrete recommendations that can be put into immediate use by traders and risk managers in the cryptocurrency world. Because Parkinson variance estimation requires only the daily high-low prices, which are readily accessible on all major exchanges, the trader will not require any kind of special data collection setup in order to construct a useful HAR-RV model of volatility forecasting. This system can be applied practically to sizing positions, placing stop losses, and trading into and out of volatile environments based on anticipated volatility levels. HAR models derived using Parkinson estimates serve as a sound foundation for valuing cryptocurrency options and spotting mismatches between implied and actual volatilities for derivatives dealers. Risk managers working with investment portfolios can make use of such forecasts when computing Value at Risk and Expected Shortfall, with consideration being made to the distinct volatility persistence properties found herein, such as those related to the comparatively higher persistence of XRP relative to BTC and ETH on long horizons.

This work has certain limitations that should be recognized and indicate potential avenues for future investigation. The range-based estimators utilized fail to consider price leaps or discontinuities, which are common in cryptocurrency markets and may result in an upward bias in daily volatility estimates without corrections for microstructure noise. The sample is confined to five large-capitalization cryptocurrencies chosen based on market capitalization, which restricts the applicability of the findings to smaller, less liquid digital assets with unique microstructural traits. Third, although the six-year sample period is extensive, it fails to comprehensively represent HAR model behavior throughout all volatility regimes, especially during the significant market dislocations experienced in the 2018 and 2022 bitcoin market declines.

Looking ahead, there are many ways through which this model can be practically applied to trading operations. With growing prevalence of algorithmic or quantitative trading strategies for cryptocurrencies, the volatility regime detection system using the Parkinson-based HAR volatility model can be directly incorporated into the automation strategy, providing automatic adjustment of positions sizes based on regimes detected in real-time. Traders may also incorporate their volatility forecast derived from the Parkinson-based HAR volatility model into their trading algorithm and use this forecast along with sentiment generated from news feeds or social media as trade entry conditions. The approach serves as a valuable tool for building volatility surfaces and discovering periods of mispricing of implied volatility versus the expected volatility predicted by the HAR model, thus allowing opportunities for straddle or volatility arbitrage strategies for options traders. Lastly, as the market for derivatives for cryptocurrencies becomes more developed, fund managers and trading desks can make use of the asset-specific structures of volatilities demonstrated in this paper and incorporate volatility adjustments to their asset allocations, giving preference to BTC and ETH compared to XRP at different horizons of predictability.

References

Acerbi, C., & Tasche, D. (2002). On the coherence of expected shortfall. *Journal of banking & finance*, *26*(7), 1487-1503.

https://doi.org/10.1016/S0378-4266(02)00283-2

Ahmed, M. S., El-Masry, A. A., Al-Maghyereh, A. I., & Kumar, S. (2024). Cryptocurrency volatility: A review, synthesis, and research agenda. *Research in International Business and Finance*, *71*, 102472.

https://doi-org.proxy.uwasa.fi/10.1016/j.ribaf.2024.102472

Alizadeh, S., Brandt, M. W., & Diebold, F. X. (2002). Range-based estimation of stochastic volatility models. *the Journal of Finance*, *57*(3), 1047-1091.

https://doi-org.proxy.uwasa.fi/10.1111/1540-6261.00454

Andersen, T. G., & Bollerslev, T. (1998). Answering the skeptics: Yes, standard volatility models do provide accurate forecasts. *International economic review*, 885-905.

https://doi.org/10.2307/2527343

Andersen, T. G., Bollerslev, T., & Diebold, F. X. (2007). Roughing it up: Including jump components in the measurement, modeling, and forecasting of return volatility. *The review of economics and statistics*, *89*(4), 701-720.

https://doi.org/10.1162/rest.89.4.701

Andersen, T. G., Bollerslev, T., Diebold, F. X., & Labys, P. (2003). Modeling and forecasting realized volatility. *Econometrica*, *71*(2), 579-625.

https://doi.org/10.1111/1468-0262.00418

Andersen, T. G., Bollerslev, T., Diebold, F. X., & Labys, P. (2001). The distribution of realized exchange rate volatility. *Journal of the American statistical association*, *96*(453), 42-55.

https://doi.org/10.1198/016214501750332965

Artzner, P., Delbaen, F., Eber, J. M., & Heath, D. (1999). Coherent measures of risk. *Mathematical finance*, *9*(3), 203-228. https://doi.org/10.1111/1467-9965.00068

Baillie, R. T., Bollerslev, T., & Mikkelsen, H. O. (1996). Fractionally integrated generalized autoregressive conditional heteroskedasticity. *Journal of econometrics*, *74*(1), 3-30.

https://doi-org.proxy.uwasa.fi/10.1016/S0304-4076(95)01749-6

Bandi, F. M., & Russell, J. R. (2008). Microstructure noise, realized variance, and optimal sampling. *The Review of Economic Studies*, *75*(2), 339-369.

https://doi-org.proxy.uwasa.fi/10.1111/j.1467-937X.2008.00474.x

Bariviera, A. F. (2017). The inefficiency of Bitcoin revisited: A dynamic approach. *Economics Letters*, *161*, 1-4.

https://doi-org.proxy.uwasa.fi/10.1016/j.econlet.2017.09.013

Barndorff-Nielsen, O. E., & Shephard, N. (2002). Econometric analysis of realized volatility and its use in estimating stochastic volatility models. *Journal of the Royal Statistical Society Series B: Statistical Methodology*, *64*(2), 253-280.

https://doi-org.proxy.uwasa.fi/10.1111/1467-9868.00336

Baur, D. G., Hong, K., & Lee, A. D. (2018). Bitcoin: Medium of exchange or speculative assets?. *Journal of International Financial Markets, Institutions and Money*, *54*, 177-189.

https://doi.org/10.1016/j.intfin.2017.12.004

Bergsli, L. Ø., Lind, A. F., Molnár, P., & Polasik, M. (2022). Forecasting volatility of Bitcoin. *Research in International Business and Finance*, *59*, 101540.

https://doi.org/10.1016/j.ribaf.2021.101540

Black, F., & Scholes, M. (1973). The pricing of options and corporate liabilities. *Journal of political economy*, *81*(3), 637-654. https://doi.org/10.1086/260062

Böhme, R., Christin, N., Edelman, B., & Moore, T. (2015). Bitcoin: Economics, technology, and governance. *Journal of economic Perspectives*, *29*(2), 213-238.

https://www.aeaweb.org/articles?id=10.1257/jep.29.2.213

Bollerslev, T. (1986). Generalized autoregressive conditional heteroskedasticity. *Journal of econometrics*, *31*(3), 307-327. https://doi.org/10.1016/0304-4076(86)90063-1

Bollerslev, T., Chou, R. Y., & Kroner, K. F. (1992). ARCH modeling in finance: A review of the theory and empirical evidence. *Journal of econometrics*, *52*(1-2), 5-59.

https://doi.org/10.1016/0304-4076(92)90064-X

Bollerslev, T., Patton, A. J., & Quaedvlieg, R. (2016). Exploiting the errors: A simple approach for improved volatility forecasting. *Journal of econometrics*, *192*(1), 1-18.
https://doi-org.proxy.uwasa.fi/10.1016/j.jeconom.2015.10.007

Bouri, E., Gil-Alana, L. A., Gupta, R., & Roubaud, D. (2019). Modelling long memory volatility in the Bitcoin market: Evidence of persistence and structural breaks. *International Journal of Finance & Economics*, *24*(1), 412-426.
https://doi-org.proxy.uwasa.fi/10.1002/ijfe.1670

Bouri, E., Gupta, R., & Roubaud, D. (2019). Herding behaviour in cryptocurrencies. *Finance Research Letters*, *29*, 216-221.
https://doi-org.proxy.uwasa.fi/10.1016/j.frl.2018.07.008

Buncic, D., & Gisler, K. I. (2016). Global equity market volatility spillovers: A broader role for the United States. *International Journal of Forecasting*, *32*(4), 1317-1339.
https://doi-org.proxy.uwasa.fi/10.1016/j.ijforecast.2016.05.001

Buncic, D., & Gisler, K. I. (2017). The role of jumps and leverage in forecasting volatility in international equity markets. *Journal of International Money and Finance*, *79*, 1-19.
https://doi-org.proxy.uwasa.fi/10.1016/j.jimonfin.2017.09.001

Busch, T., Christensen, B. J., & Nielsen, M. Ø. (2011). The role of implied volatility in forecasting future realized volatility and jumps in foreign exchange, stock, and bond markets. *Journal of Econometrics*, *160*(1), 48-57.
https://doi.org/10.1016/j.jeconom.2010.03.014

Catania, L., & Grassi, S. (2022). Forecasting cryptocurrency volatility. *International Journal of Forecasting*, *38*(3), 878-894.
https://doi-org.proxy.uwasa.fi/10.1016/j.ijforecast.2021.06.005

Chaim, P., & Laurini, M. P. (2018). Volatility and return jumps in bitcoin. *Economics Letters*, *173*, 158-163. https://doi-org.proxy.uwasa.fi/10.1016/j.econlet.2018.10.011

Cheah, E. T., Mishra, T., Parhi, M., & Zhang, Z. (2018). Long memory interdependency and inefficiency in Bitcoin markets. *Economics Letters*, *167*, 18-25.

https://doi-org.proxy.uwasa.fi/10.1016/j.econlet.2018.02.010

Christoffersen, P. F. (1998). Evaluating interval forecasts. *International economic review*, 841-862. https://doi.org/10.2307/2527341

Ciaian, P., & Rajcaniova, M. (2018). Virtual relationships: Short-and long-run evidence from BitCoin and altcoin markets. *Journal of International Financial Markets, Institutions and Money*, *52*, 173-195. https://doi.org/10.1016/j.intfin.2017.11.001

Clark, P. K. (1973). A subordinated stochastic process model with finite variance for speculative prices. *Econometrica: journal of the Econometric Society*, 135-155.

https://doi.org/10.2307/1913889

Comte, F., & Renault, E. (1998). Long memory in continuous-time stochastic volatility models. *Mathematical finance*, *8*(4), 291-323.

https://doi.org/10.1111/1467-9965.00057

Cont, R. (2001). Empirical properties of asset returns: stylized facts and statistical issues. *Quantitative finance*, *1*(2), 223.

https://doi-org.proxy.uwasa.fi/10.1080/713665670

Corsi, F. (2009). A simple approximate long-memory model of realized volatility. *Journal of financial econometrics*, *7*(2), 174-196.

https://doi-org.proxy.uwasa.fi/10.1093/jjfinec/nbp001

Corsi, F., Pirino, D., & Reno, R. (2010). Threshold bipower variation and the impact of jumps on volatility forecasting. *Journal of Econometrics*, *159*(2), 276-288.

https://doi.org/10.1016/j.jeconom.2010.07.008

Diebold, F. X., & Mariano, R. S. (2002). Comparing predictive accuracy. *Journal of Business & economic statistics*, *20*(1), 134-144.

https://doi.org/10.1198/073500102753410444

Ding, Z., Granger, C. W., & Engle, R. F. (1993). A long memory property of stock market returns and a new model. *Journal of empirical finance*, *1*(1), 83-106.

https://doi.org/10.1016/0927-5398(93)90006-D

Dyhrberg, A. H. (2016). Bitcoin, gold and the dollar–A GARCH volatility analysis. *Finance research letters*, *16*, 85-92. https://doi-org.proxy.uwasa.fi/10.1016/j.frl.2015.10.008

Dyhrberg, A. H., Foley, S., & Svec, J. (2018). How investible is Bitcoin? Analyzing the liquidity and transaction costs of Bitcoin markets. *Economics Letters*, *171*, 140-143. https://doi-org.proxy.uwasa.fi/10.1016/j.econlet.2018.07.032

Engle, R. (2004). Risk and volatility: Econometric models and financial practice. *American economic review*, *94*(3), 405-420. https://pubs.aeaweb.org/doi/abs/10.1257/0002828041464597

Engle, R. F. (1982). Autoregressive conditional heteroscedasticity with estimates of the variance of United Kingdom inflation. *Econometrica: Journal of the econometric society*, 987-1007. https://doi.org/10.2307/1912773

Fakhfekh, M., & Jeribi, A. (2020). Volatility dynamics of crypto-currencies' returns: Evidence from asymmetric and long memory GARCH models. *Research in International Business and Finance*, *51*, 101075. https://doi.org/10.1016/j.ribaf.2019.101075

Fry, J., & Cheah, E. T. (2016). Negative bubbles and shocks in cryptocurrency markets. *International Review of Financial Analysis*, *47*, 343-352. https://doi.org/10.1016/j.irfa.2016.02.008

Ftiti, Z., Louhichi, W., & Ben Ameur, H. (2023). Cryptocurrency volatility forecasting: What can we learn from the first wave of the COVID-19 outbreak?. *Annals of Operations Research*, *330*(1), 665-690. https://link.springer.com/article/10.1007/s10479-021-04116-x

Fuertes, A. M., Izzeldin, M., & Kalotychou, E. (2009). On forecasting daily stock volatility: The role of intraday information and market conditions. *International Journal of Forecasting*, *25*(2), 259-281. https://doi.org/10.1016/j.ijforecast.2009.01.006

Garman, M. B., & Klass, M. J. (1980). On the estimation of security price volatilities from historical data. *Journal of business*, 67-78. https://www.jstor.org/stable/2352358

Ghysels, E., Santa-Clara, P., & Valkanov, R. (2005). There is a risk-return trade-off after all. *Journal of financial economics*, *76*(3), 509-548. https://doi-org.proxy.uwasa.fi/10.1016/j.jfineco.2004.03.008

Gkillas, K., & Katsiampa, P. (2018). An application of extreme value theory to cryptocurrencies. *Economics Letters*, *164*, 109-111.

https://doi-org.proxy.uwasa.fi/10.1016/j.econlet.2018.01.020

Glosten, L. R., Jagannathan, R., & Runkle, D. E. (1993). On the relation between the expected value and the volatility of the nominal excess return on stocks. *The journal of finance*, *48*(5), 1779-1801. https://doi.org/10.1111/j.1540-6261.1993.tb05128.x

Hansen, P. R., & Lunde, A. (2005). A forecast comparison of volatility models: does anything beat a GARCH (1, 1)?. *Journal of applied econometrics*, *20*(7), 873-889.

https://doi.org/10.1002/jae.800

Hansen, P. R., & Lunde, A. (2006). Realized variance and market microstructure noise. *Journal of Business & Economic Statistics*, *24*(2), 127-161.

https://doi.org/10.1198/073500106000000071

Hansen, P. R., Lunde, A., & Nason, J. M. (2011). The model confidence set. *Econometrica*, *79*(2), 453-497.

https://doi.org/10.3982/ECTA5771

Hasbrouck, J. (1991). Measuring the information content of stock trades. *The Journal of Finance*, *46*(1), 179-207.

https://doi-org.proxy.uwasa.fi/10.1111/j.1540-6261.1991.tb03749.x

Katsiampa, P. (2017). Volatility estimation for Bitcoin: A comparison of GARCH models. *Economics letters*, *158*, 3-6.

https://doi-org.proxy.uwasa.fi/10.1016/j.econlet.2017.06.023

Klein, T., Thu, H. P., & Walther, T. (2018). Bitcoin is not the New Gold–A comparison of volatility, correlation, and portfolio performance. *International Review of Financial Analysis*, *59*, 105-116. https://doi.org/10.1016/j.irfa.2018.07.010

Korkusuz, B., Kambouroudis, D., & McMillan, D. G. (2023). Do extreme range estimators improve realized volatility forecasts? Evidence from G7 stock markets. *Finance Research Letters*, *55*, 103992. https://doi.org/10.1016/j.frl.2023.103992

Köse, N., Yildirim, H., Ünal, E., & Lin, B. (2024). The Bitcoin price and Bitcoin price uncertainty: Evidence of Bitcoin price volatility. *Journal of Futures Markets*, *44*(4), 673-695. https://doi-org.proxy.uwasa.fi/10.1002/fut.22487

Koutmos, D. (2020). Market risk and Bitcoin returns. *Annals of Operations Research*, *294*(1), 453-477.
https://link.springer.com/article/10.1007/S10479-019-03255-6

Kumar, D., & Maheswaran, S. (2014). A new approach to model and forecast volatility based on extreme value of asset prices. *International Review of Economics & Finance*, *33*, 128-140.
https://doi-org.proxy.uwasa.fi/10.1016/j.iref.2014.04.001

Leong, M., & Kwok, S. (2023). The pricing of jump and diffusive risks in the cross-section of cryptocurrency returns. *Journal of Empirical Finance*, *74*, 101420.
https://doi-org.proxy.uwasa.fi/10.1016/j.jempfin.2023.101420

Lintner, J. (1975). The valuation of risk assets and the selection of risky investments in stock portfolios and capital budgets. In *Stochastic optimization models in finance* (pp. 131-155). Academic Press. https://doi.org/10.1016/B978-0-12-780850-5.50018-6

Liu, Y., & Tsyvinski, A. (2021). Risks and returns of cryptocurrency. *The Review of Financial Studies*, *34*(6), 2689-2727. https://doi.org/10.1093/rfs/hhaa113

Liu, Y., Tsyvinski, A., & Wu, X. (2022). Common risk factors in cryptocurrency. *The Journal of Finance*, *77*(2), 1133-1177. https://doi-org.proxy.uwasa.fi/10.1111/jofi.13119

Makarov, I., & Schoar, A. (2020). Trading and arbitrage in cryptocurrency markets. *Journal of Financial Economics*, *135*(2), 293-319.
https://doi.org/10.1016/j.jfineco.2019.07.001

Markowitz, H. (1952). Portfolio selection. *Journal of Finance, 7*(1), 77–91.
https://doi-org.proxy.uwasa.fi/10.1111/j.1540-6261.1952.tb01525.x

Merton, R. C. (1973). An intertemporal capital asset pricing model. *Econometrica: Journal of the Econometric Society*, 867-887. https://doi.org/10.2307/1913811

Mincer, J. A., & Zarnowitz, V. (1969). The evaluation of economic forecasts. In *Economic forecasts and expectations: Analysis of forecasting behavior and performance* (pp. 3-46). NBER. https://www.nber.org/system/files/chapters/c1214/c1214.pdf

Molnár, P. (2012). Properties of range-based volatility estimators. *International Review of Financial Analysis*, *23*, 20-29. https://doi.org/10.1016/j.irfa.2011.06.012

Müller, U. A., Dacorogna, M. M., Davé, R. D., Olsen, R. B., Pictet, O. V., & Von Weizsäcker, J. E. (1997). Volatilities of different time resolutions—analyzing the dynamics of market components. *Journal of Empirical Finance*, *4*(2-3), 213-239.

https://doi-org.proxy.uwasa.fi/10.1016/S0927-5398(97)00007-8

Nakamoto, S. (2008). *Bitcoin: A peer-to-peer electronic cash system*.

https://assets.pubpub.org/d8wct41f/31611263538139.pdf

Parkinson, M. (1980). The extreme value method for estimating the variance of the rate of return. *Journal of business*, 61-65. https://www.jstor.org/stable/2352357

Patton, A. J. (2011). Volatility forecast comparison using imperfect volatility proxies. *Journal of econometrics*, *160*(1), 246-256

https://doi.org/10.1016/j.jeconom.2010.03.034

Patton, A. J., & Sheppard, K. (2015). Good volatility, bad volatility: Signed jumps and the persistence of volatility. *Review of Economics and Statistics*, *97*(3), 683-697.

https://doi.org/10.1162/REST_a_00503

Pindyck, R. S. (1984). *Risk, inflation, and the stock market*. *American Economic Review, 74*(3), 335–351. https://www.jstor.org/stable/1804011

Poon, S. H., & Granger, C. W. J. (2003). Forecasting volatility in financial markets: A review. *Journal of economic literature*, *41*(2), 478-539.

https://www.aeaweb.org/articles?id=10.1257/002205103765762743

Qiu, Y., Wang, Z., Xie, T., & Zhang, X. (2021). Forecasting Bitcoin realized volatility by exploiting measurement error under model uncertainty. *Journal of Empirical Finance*, *62*, 179-201.

https://doi-org.proxy.uwasa.fi/10.1016/j.jempfin.2021.03.003

Rogers, L. C. G., & Satchell, S. E. (1991). Estimating variance from high, low and closing prices. *The Annals of Applied Probability*, 504-512.

https://www.jstor.org/stable/2959703

Sapkota, N. (2022). News-based sentiment and bitcoin volatility. *International Review of Financial Analysis*, *82*, 102183. https://doi.org/10.1016/j.irfa.2022.102183

Scaillet, O., Treccani, A., & Trevisan, C. (2020). High-frequency jump analysis of the bitcoin market. *Journal of Financial Econometrics*, *18*(2), 209-232.

https://doi.org/10.1093/jjfinec/nby013

Sharpe, W. F. (1964). Capital asset prices: A theory of market equilibrium under conditions of risk. *The journal of finance*, *19*(3), 425-442.

https://doi.org/10.1111/j.1540-6261.1964.tb02865.x

Shen, D., Urquhart, A., & Wang, P. (2020). Forecasting the volatility of Bitcoin: The importance of jumps and structural breaks. *European Financial Management*, *26*(5), 1294-1323.

https://doi-org.proxy.uwasa.fi/10.1111/eufm.12254

Shu, J., & Zhang, J. E. (2006). Testing range estimators of historical volatility. *Journal of Futures Markets: Futures, Options, and Other Derivative Products*, *26*(3), 297-313.

https://doi.org/10.1002/fut.20197

Urquhart, A. (2016). The inefficiency of Bitcoin. *Economics Letters*, *148*, 80-82.

https://doi-org.proxy.uwasa.fi/10.1016/j.econlet.2016.09.019