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Optimal Sizing and Siting of Electric Vehicle Charging Stations in Distribution Networks with Robust Optimizing Model

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***Abstract*— Optimal planning of power distribution systems with local resources is crucial to meet energy demand and avoid disruptions in energy supply for consumers. This requires the system operators to manage available resources and utilize suitable risk management tools to control and study uncertainties and their potential consequences. This paper proposes an uncertainty-based optimization framework based on the robust optimization and scenario methodology for optimal sizing and siting of electrical vehicle charging stations (EVCSs). The proposed model seeks to take advantage of the flexibility introduced by EVCSs and gain financial profit for the operator of the power distribution system through reducing power losses and offering services to electricity markets. To handle the uncertainties posed by different resources, two risk measures are employed simultaneously. The uncertainty originating from the state of charge (SOC) of electric vehicles (EVs) is addressed through stochastic programming, while the robust optimization method (ROM) enables the operator of the power distribution system to be informed of the consequences of uncertainty in electricity load. Therefore, appropriate strategies can be taken to tackle the uncertainties while keeping the system operation stable and gaining financial profit. Thus, three strategies are studied in the proposed model as follows: risk-neutral, risk-averse, and risk-taker. In addition, the non-linear terms in power flow modeling were linearized through a set of linear functions which transforms the proposed model to a MILP problem. The IEEE 33-bus test system under different levels of load uncertainty and considering the uncertainty in SOC of EVs is utilized to ensure the effectiveness of the proposed model. The results highlight the efficiency of the proposed model in considering uncertainties and taking advantage of the consideration of different risk attitudes by the decision-maker that ROM provides for the optimal operation of the power distribution system.**

***Index Terms*— Electric vehicles charging stations (EVCSs), power distribution systems, uncertainty management, robust optimization method (ROM), scenario method.**

NOMENCLATURE

Indices

t	Year index
h	Level of demand
i, j	Bus indices in power the distribution system
sc	Scenario index
$disp$	Dispatch index in the spinning reserve service
Ω	Set of buses
Ω_G	Set of buses connected to the upstream grid
Ω_{CS}	Set of buses connecting the distribution system and EVCS
K	Set of scenarios

Decision Variables

$\delta_{j,t,h}^{sc}$	Voltage angle
C_{INV}	Total cost of installing EVCSs ($)
C_{RS}	Cost of providing spinning reserve service ($)
C_{CH}	Cost of providing charging service ($)
C_{DCH}	Cost of providing discharging service ($)
$N_i^{CS,sc}$	The capacity of installed EVCS
PR	Total profit of system operator ($)
PR_{RS}	Profit gained by offering spinning reserve ($)
PR_{CH}	Profit gained by charging EVs ($)
PR_{DCH}	Profit gained by discharging EVs ($)
PR_{LS}	Profit gained by reducing power losses ($)
$P_{i,t,h}^{LS,NP,sc}$	The active power loss of the system without EVCSs (kW)
$P_{i,t,h}^{LS,P,sc}$	The active power loss of the system with EVCSs (kW)
$P_{i,t,h}^{CS,sc}$	Charge/discharge power of EVCSs (kW)
$P_{t,i,h}^{RS,sc}$	The capacity available for spinning reserve (kW)
${P_{i,j,t,h}^{FL,sc}}^2$	Active power flow in the distribution system (kW)
$P_{t,h,i}^{G,sc}$	Imported active power from upstream grid (kW)
$Q_{t,h,i}^{G,sc}$	Imported reactive power from grid (kVAr)
${Q_{i,j,t,h}^{FL,sc}}^2$	Reactive power flow in the power distribution system (kVAr)
$V_{i,t,h}^{sc}$	Voltage magnetite on each bus
Z	Dual variable of ROM optimization problem
$q_{t,h}^{0,sc}$	Dual variable of ROM optimization problem

Parameters

φ_{conv}	Vehicle-to-grid equipment efficiency of EVCSs
$\lambda_{t,h}$	Price of electricity sold to the customer ($/kWh)
$\lambda_{t,h,pur}^{grid}$	Price of electricity purchased from wholesale electricity market ($/kWh)
$\lambda_{t,h,pur}^{EV}$	Price of electricity purchased from EV owners ($/kWh)
λ_t^{cap}	Capacity price ($/kW per hour)
λ_t^{ans}	Price of electricity for contracted reserve energy ($/kWh)
ξ	Annual load growth
t_{disp}	Dispatch duration in spinning reserve service
ψ_h	The time duration of demand level h (hour)

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μ_h Demand level factor each h
ψ_{plug} Plugged-in duration of EVs for spinning reserve
$B_{i,j}$ Susceptance value of distribution system admittance
C_{Deg} Degradation cost of the battery of EVs in EVCSs
C_{ac} Investment cost of EVCSs including the costs of charger units, charger installation, and construction ($)
$d_{t,h}^{sc}$ Load increase level in ROM
IfR Rate of inflation
ItR Rate of interest
E_{disp}^{sc} Total energy demand requested for spinning reserve (kWh)
$G_{i,j}$ Conductance value of distribution system admittance
H Scheduling horizon
N_{disp} Maximum number of dispatches for spinning reserve service
$Lco_{t,h}^{sc}$ Load coefficient in ROM formulation
$P_{i,base}^{D}$ Base active load of the distribution system (kW)
$P_{i,t,h}^{D}$ System active load (kW)
$Q_{i,base}^{D}$ Base reactive load of the distribution system (kVAr)
$Q_{i,t,h}^{D}$ System reactive load (kVAr)
$r_{i,j}$ The resistance value of lines in the power distribution system
${S^{G}}^2$ Transformer's capacity in the substation bus (kVA)
${S_{i,j}^{FL}}^2$ Line capacity of the distribution system (kVA)
T Planning horizon
$x_{i,j}$ Reactance value of lines in the power distribution system
Γ Budget of uncertainty

Binary variables

U_i^{sc} Binary variable to allocate EVCSs in the power distribution system

Abbreviations

EVCS Electric vehicle charging station
MILP Mixed-integer linear programing
ROM Robust optimization method
DRP Demand response program
PV Photovoltaic
SOC State of charge
RC Residential complex

I. INTRODUCTION

ELECTRICAL distribution networks are responsible for delivering reliable electricity to customers. However, the regular operation of these networks might be challenged by various factors. Line interruption, uncertainties, power losses, and voltage drops are some of the evident examples of the challenging issues in these networks. Electrical vehicle charging stations (EVCSs) are new facilities in distribution networks that can help the operators to operate the system while gaining financial profit. Optimal siting and sizing of EVCSs can efficiently overcome the existing problems in power distribution systems. Additionally, considering the possible uncertainties within the planning and scheduling of EVCSs can ensure the optimal performance of such systems in different potential conditions.

Significant interest has been in integrating electric vehicles (EVs) and power distribution systems in recent years [1]. EVCSs have been studied from different viewpoints, briefly in this section. Due to the economic benefits of integrating EVs into power distribution systems, the sizing and siting of such energy units have gained great attention [2]. The authors in [3] proposed an optimization framework for sizing EV charging stations considering the quality of service for the EV owners. A genetic algorithm is implemented in [4] to find the optimal location for EV parking lots while considering the travel costs of EV owners. A multi-criteria decision-making method is implemented in [5]to find the most proper place for an EV parking lot from a sustainable perspective. In [6], a bi-layer Pareto multi-objective optimization problem is presented for the sizing and siting of an EV parking garage. The goal of the proposed model is to maximize the profits of EV parking garage investors while minimizing the losses and voltage deviations for the distribution system operator. In [7], a combined model formulation is presented for sizing and allocating renewable energy resources, the EV parking lot, and distributed energy storage systems in distribution systems, where the Markov Chain Monte Carlo model is employed to model the uncertainties of renewable generation and charging demand of EVs. In [8], EV parking lots have been allocated and sized to reduce the operation costs of the distribution system and charging PEV. In [9], the genetic algorithm is implemented for optimal sizing and siting of EV parking lots with optimal scheduling in power systems.

Electrification of the transportation system led to a detailed study of the different types of EVs aimed for utilization in public transportation [10]. For instance, there are a number of models that consider the sizing and siting of electric bus charging stations [11], [12]. The authors in [11] introduced a two-stage stochastic model considering the uncertainty posed by renewables. Furthermore, the study proposes a step-wise solution approach to determine the optimal locations for charging stations, with a particular focus on integrating energy storage systems tested on a transit network in Beijing, China. In another study [12], an optimization model was presented for electric bus charging station locations. On the station configurations, seasonality is also taken into account. Seasonality reflects the impact of air temperature on electric bus battery performance. The model was applied to a sub-transit network in Beijing, China.

To schedule a smart distribution company, including renewable energy sources, EVs, and parking lots, a new bi-level framework has been proposed in [13]. By minimizing the cost of power purchased from the wholesale market in the upper level, the profit of the parking lot owner is maximized in the lower level. In the model, by setting the scenarios, the uncertainty in the output power of photovoltaic (PV) units and wind turbines, the initial state of charge (SOC) of EVs, and the duration of the presence of EVs in the parking lot are simultaneously taken into account. In[14], a new bi-level model for the optimal scheduling of a distribution company is presented. In this work, the authors considered not only the technical terms but also the environmental terms in the objective function. At each level, the profits of the owners of the parking lot and the distribution company are maximized. The financial profit of an EV parking lot is maximized through a market mechanism based on the Knapsack Algorithm in [15]. In this study, different scenarios are considered for the level of energy trade and the number of EVs. In [16], using a new techno-economic modal, the optimal operation of a smart distribution company is studied in which the benefit of the owner of the parking lot and renewable energy resources is maximized subject to various uncertainties of EVs and renewable energy resources.

Power loss reduction is also an essential factor in integrating EVs into distribution systems. An algorithm, namely the quantum binary lightning search algorithm, is introduced to optimally allocate EV charging electric vehicle stations to minimize power losses while improving voltage quality in [17]. The genetic algorithm and the particle swarm optimization algorithm are employed in [18] to reduce the distribution system power losses while improving the EV parking lot's availability from economic perspectives. To maximize the load factor during the daily operation of an EV parking lot under the demand response program, i.e., the peak load mitigation, the authors in [19] utilized an EV parking lot energy management strategy based on a linear programming framework.

Within the proposed risk-constrained approaches for distribution network operation, various strategies are presented to take into account uncertainties. To study short-term operational scheduling of the distribution network in the presence of market price uncertainty, a robust optimization model (ROM) is implemented in [20]. It should be noted that the scenario method had been employed for modeling the current existing uncertainties. Besides, a flexible local microgrid owner encouraged the PEV owners to participate in DRP to bring profit for both sides. The optimal participation of EV aggregators in the energy market considering the price uncertainty at different levels is investigated through a robust optimization method in [15] to maximize the profit of EV aggregators. In [21], by employing a probabilistic approach based on the point estimate method, optimal siting and sizing of EV parking lots are studied under the uncertainty of driver's behavior. Intelligent energy management and charging scheduling model is introduced for PEV'S charging station and management system in [22], whereby deploying battery control as well as communication facilities, and convenient energy management services are provided to the drivers.

The optimal operation of an energy system integrated into a residential complex is studied in [23], where DRP is developed to reduce the cost of RC. It should be noted that valuable researchers are developing EV charging stations for other applications, such as reliability improvement and peer-to-peer energy trading that are not discussed here considering the scope of the proposed model.

It should be noted that the ancillary services' goal is to assist the maintenance of the reliability and security of the electricity supply. To always maintain the balance between demand and generation, regulation of the frequency, in particular, necessitates storing a specific amount of active power in reserve. Thus, a broad definition of the reserve is the amount of generation capacity that is available for use in producing active power over a specific time period but has not yet been committed to the generation of energy during this time. In reality, a variety of reserve services are needed to respond to a range of occurrences over a range of time frames. While the phrase "spinning reserve" is frequently used in literature, numerous definitions exist for this particular function. On the other hand, since EVs are usually parked at the EVCSs, it would be an excellent opportunity for the distribution system operator to participate in the spinning reserve market and gain financial profit. In reference [24], a multi-stage stochastic linear programming model has been developed to reduce the anticipated total energy expenses over a finite time horizon in public EV parking lots. The local transformer capacity restrictions, the maximum battery acceptance rate for electric vehicles, and user demands are three sources of uncertainty the model took into account when predicting future demand. These sources are subject to some operational constraints. A finite scenario tree has then been used to approximate the model. Even for a modest number of phases, the model was computationally intractable. To match the time-dependent charging conditions, the well-known decomposition method Stochastic Dual Dynamic Programming (SDDP) has been modified. The process has been run once in the offline mode and applied the findings for the online version because it takes several hours to achieve a high-quality solution.

Reference [25] suggested a day-ahead smart charging technique from the viewpoints of EV owners, governments, and distribution system operators (DSOs). By planning the active and reactive power of EV parking lots integrated with solar (PV) systems and determining the best network architecture, this method aims to reduce the operating cost of microgrids, the degrading cost of EV batteries, and the emission cost. The authors in [25] used data-driven techniques based on generative adversarial networks to address the uncertainties and tested the strategy on a real reconfigurable microgrid. The main drawback of this work is not considering the uncertainties. Moreover, this paper is mainly related to the operation of EVPL rather than planning it. Hence the optimal allocation and sizing of the EVPL needs to be studied.

In [26], a day-ahead co-optimization algorithm has been suggested to lessen the negative impacts that parking lots of EVs have on the power grid. By managing active and reactive powers concurrently, the suggested algorithm aimed to reduce the cost of energy losses as well as transformer operational costs. Additionally, the suggested method took into account the impact of harmonics, which are generated by the chargers of EVs in the PEVs. Additionally, a technique that included the transformer's purchase price, loading costs, and losses costs was used to calculate the operating costs of the transformer. The method also enhances the distribution network's power quality characteristics, such as voltage and power factor, by controlling reactive power. Not considering uncertainties is the main shortcoming of this paper, where the authors didn't analyze the impact of the uncertainty on the final outcome of the optimization framework. Moreover, this work studied the operation of the EVs disregarding the optimal sizing and siting of the EV parking lots. The implemented method in [27] allowed the selection of a subset of existing EV parking lots where the charging equipment will be placed. The authors created a collection of ideal answers for numerous predetermined restrictive and somewhat incompatible criteria by combining a genetic algorithm with fuzzy logic, Pareto front analysis, and other techniques. A medium-sized city in southern Poland has been used to study the method. However, the optimal size for the EV parking lots is not addressed, and the existing sources of uncertainty are not taken into account.

To provide day-ahead peak-shaving and valley-filling for power systems with different peak hours in daily operation, a structure has been developed for EV parking lots in [28]. In this paper, EV parking lots are used for peak-shaving and valley-filling for power systems with different peak hours in daily operation without considering the optimal allocation and size of the parking sites. A hybrid robust-stochastic model has been introduced in [29] for optimal day-ahead scheduling of the plug-in EVs parking lots with renewable generation and considering the inherent uncertainties like the arrival and departure of plug-in electric vehicles s and to concern the worst-case of PV generation to maximum possible profit. The uncertainties posed by the behavior of the EV owners are neglected while it significantly impacts the strategy shown in the simulation results. With the aim of optimal planning of EV charging stations along with capacitors, quantum-behaved Gaussian mutational DA has been introduced as an approach in

[30] to maintain voltage deviation and decrease power loss. The sources of uncertainty posed by the effective aspects of the solution of the optimal planning of the EV charging stations are not assumed.

In order to optimal scheduling management of the EV parking lot and formulate optimal decentralized charging control strategies for EVs, a linear quadratic (LQ) Mean Field Game (MFG) theory with a significant player method has been presented in [31]. The main objective considered as minimizing the cost of EV parking lots through implementing the Nash Certainty Equivalence (NCE) of related optimization problems without demonstrating the optimal siting and sizing of the EV's parking lots and the effects of the uncertainty of the taken management strategy. A taxonomy table is presented below to illustrate a comparison of the proposed method with similar works in this area, i.e., Table I.

Although valuable models regarding the development of EVCSs in power system operation are presented in the literature, a mathematical-based model is still lacking for power distribution systems to take advantage of such stations under uncertain environments to provide power and energy services while ensuring the possible financial profit for the operator of power operating systems. This necessitates developing a precise optimization model along with appropriate uncertainty management tools to ensure optimal operation of power distribution systems while controlling the uncertainties by offering preattentional strategies for the possible uncertainty levels. The research gaps for proposing the ROM-based model in the paper can be listed as follows:

- A mathematical-based model is still lacking for power distribution systems to take advantage of EVCSs under uncertain environments to provide power and energy services. This requires siting and sizing EVCSs under uncertain power distribution systems environments.

- An uncertainty modeling methodology to prepare an operating strategy for a power distribution system while managing uncertainty. ROM can address this, as this method would inform the system operator of the uncertainty and would let the operator take risk-averse, risk-neutral, and risk-seeking operating strategies in uncertain environments.

TABLE I

COMPARISON BETWEEN THE PROPOSED MODEL AND PREVIOUS STUDIES

Ref.	Siting	Sizing	Uncertainty management	Objective function
[26]	×	×	×	Min cost of energy losses and operational costs
[28]	×	×	×	Min the electricity price
[29]	×	×	✓	Max the profit
[31]	×	×	×	Min of EVs charging cost
[25]	×	×	✓	Min operation cost of microgrids, degradation cost of EV batteries, and emission cost
[27]	✓	×	×	Min negative environmental impact of transport
[24]	×	✓	✓	Min the anticipated total energy expenses
[30]	✓	✓	×	Stabilize voltage and reduce power loss
This work	✓	✓	✓	Max the total profit from reducing power losses, offering charge/discharge, and reserve services minus the cost of installing EVCSs

Therefore, within the context of uncertainty management, this paper develops an optimization framework based on ROM and scenario-based methods for optimal sizing and siting of EVCSs in power distribution systems. The proposed model deploys a scenario method to consider the uncertainties in SOC of EVs in EVCSs and utilizes ROM to inform the power distribution system operator of the consequences of uncertainties in electricity loads; thus, by offering appropriate strategies, it helps the power distribution system operator to tackle the uncertainties while keeping the system operation stable and gaining financial profit. In this paper, the non-linear power flow constraints in the power distribution system are linearized through a set of linear models. Thus, the proposed optimization framework is modeled as a mixed-integer linear programming (MILP) problem and solved by a CPLEX solver using a general algebraic modeling system (GAMS) package, ensuring the optimal solution for the studied problem. Optimal sizing and siting of EVCSs in power distribution systems under the uncertainty of the load are studied on the IEEE 33-bus test power distribution system. In this regard, the contributions of the proposed model can be listed as follows:

- Developing a comprehensive optimization framework for sizing and siting EVCSs in power distribution systems under the uncertainties in electricity load and SOC of EVs through implementing a scenario-based methodology for modeling the uncertainties in SOC of EVs and ROM for handling the load uncertainty.
- Addressing different uncertainty levels in electricity load management within power distribution systems. The model facilitates the determination of optimal configurations for the placement and sizing of EVCSs through taking risk-averse, risk-neutral, and risk-seeking operating strategies introduced by the ROM.
- Reducing the power loss within the distributed power system. This is achieved by incorporating linearized power flow constraints into the optimization framework. The model's representation as a MILP problem ensures the attainment of global optimal solution for the proposed model with the aim of maximizing the profit the operator through different involving aspects such as charging/discharging, reserve and power loss reduction services.

The rest of the proposed paper is structured as follows: The studied problem is formulated in Section II. ROM and scenario methods are briefly explained in Section III. Simulations are conducted, and the results are discussed in Section IV. Finally, the conclusions are presented in Section V.

II. PROPOSED MODEL

In this section, the optimal siting and sizing of EVCSs in power distribution systems under the uncertainty in electricity load is mathematically formulated. The proposed model seeks to maximize the total profit from offering charge, discharge, and reserve services as well as reducing the power loss in the power distribution system subject to the uncertainties in load and SOC of EVs as well as the technical constraints in the operation of the power distribution system and EVCSs. The schematic of the proposed model is illustrated in Fig. 1.

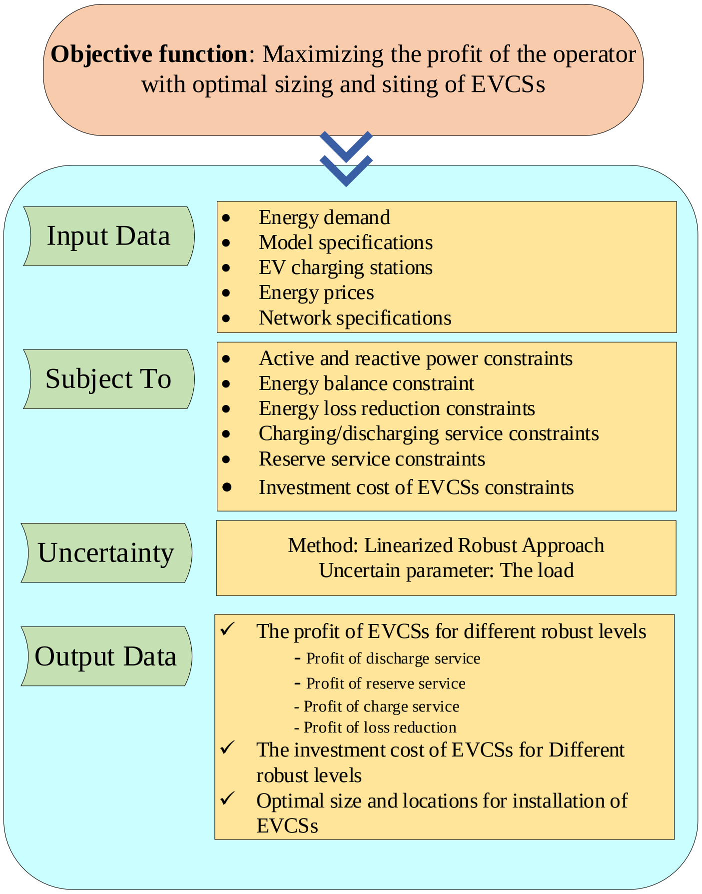

Fig. 1. The proposed optimization framework

A. *Objective function*

The objective function of the proposed model expressed in (1) is to maximize the total profit from reducing power losses and offering charge, discharge, and reserve services minus the cost of installing EVCSs [32].

$$Max(PR_{LS} + PR_{CH} + PR_{DCH} + PR_{RS} - C_{INV}) \tag{1}$$

The profits that can be obtained by the operator of the power distribution system through offering charge, discharge, and reserve services, as well as the cost of installing EVCSs in (1), are formulated in the following subsections:

1) *Profit from reducing power loss*

By sitting and sizing EVCSs in power distribution systems, the injected power by the charging stations reduces the power loss in the system. Therefore, the total revenue from reducing power loss can be defined as the difference between the power loss value before and after the installation of EVCSs as follows:

$$R_{LS} = \sum_{t=1}^{T}\sum_{h=1}^{H}\sum_{sc\in\square} \lambda_{t,h}(P_{i,t,h}^{LS,NP,sc} - P_{i,t,h}^{LS,P,sc})\left(\frac{1+IfR}{1+ItR}\right)^{t} \tag{2}$$

2) *Profit from offering charging service*

At the end of every day, many EVs are usually out of charge as the battery is utilized for driving purposes within the day. Due to the differences between the price of electricity at night and other time periods, the power distribution system operator desires to supply the electricity from the wholesale market with lower wholesale electricity market prices and charge the EVs, which provides an excellent opportunity for the operator of the power distribution system to get financial revenues. This process is mathematically formulated in (3)-(5). In more detail, the revenue of charging EVs in EVCSs is presented in (4), and the cost of supplying charging power from the wholesale market is presented in (5), where the cost associated with the degradation of EV battery is considered as a fixed input value per each EV being charged.

$$PR_{CH} = R_{CH} - C_{CH}, \tag{3}$$

$$R_{CH} = \sum_{t=1}^{T}\sum_{h=1}^{H}\sum_{i\in\Omega_{CS}}\sum_{sc\in\square} \lambda_{t,h}P_{i,t,h}^{CS,sc}\,\psi_h\left(\frac{1+IfR}{1+ItR}\right)^{t} \tag{4}$$

$$C_{CH} = \sum_{t=1}^{T}\sum_{h=1}^{H}\sum_{i\in\Omega_{CS}}\sum_{sc\in\square}\left(\frac{\lambda_{t,h,pur}^{grid}}{\varphi_{conv}} + C_{Deg}\right)P_{i,t,h}^{CS,sc}\,\psi_h\left(\frac{1+IfR}{1+ItR}\right)^{t} \tag{5}$$

3) *Profit from offering discharging service*

In peak time periods, expensive generators on the generation side are operated to supply power system requirements, which leads to higher wholesale market electricity prices. In such a case, the development of EVCSs would be an excellent opportunity for the operator of the distribution system to supply local power to the distribution system through discharging EVs in the EVCSs. The total financial profit that the operator of the power distribution system can gain through offering discharging services of EVCSs is formulated in (6)-(8), where the total revenue from injecting the discharge power of EVCSs into the power distribution system is modeled in (7). The cost of the power purchased from EVCSs is modeled in (8).

$$PR_{DCH} = R_{DCH} - C_{DCH}, \tag{6}$$

$$R_{DCH} = \sum_{t=1}^{T}\sum_{h=1}^{H}\sum_{i\in\Omega_{CS}}\sum_{sc\in\square} \lambda_{t,h}P_{i,t,h}^{CS,sc}\,\psi_h\left(\frac{1+IfR}{1+ItR}\right)^{t} \tag{7}$$

$$C_{DCH} = \sum_{t=1}^{T}\sum_{h=1}^{H}\sum_{i\in\Omega_{CS}}\sum_{sc\in\square}\left(\frac{\lambda_{t,h,pur}^{EV}}{\varphi_{conv}} + C_{Deg}\right)P_{i,t,h}^{CS,sc}\psi_h\left(\frac{1+IfR}{1+ItR}\right)^{t} \tag{8}$$

4) *Profit from offering reserve service*

Because EVs are usually parked at the EVCSs, it would be an excellent opportunity for the operator of the distribution system to participate in the spinning reserve market and gain financial profit. In this regard, the total profit, revenue, and cost of offering reserve service of EVCSs into the spinning reserve market are modeled in (9)-(11), where the revenue from capacity and energy payments is presented in (10). The cost of service taken from EVCSs is presented in (11).

$$PR_{RS} = R_{RS} - C_{RS}, \tag{9}$$

$$R_{RS} = \sum_{t=1}^{T}\sum_{h=1}^{H}\sum_{i\in\Omega_{CS}}\sum_{sc\in\square}\left(\left(\lambda_t^{cap}P_{t,i,h}^{RS,sc}\psi_{plug}\right) + \left(\lambda_t^{ans}E_{disp}^{sc}\right)\right)\left(\frac{1+IfR}{1+ItR}\right)^{t} \tag{10}$$

$$C_{RS} = \sum_{t=1}^{T}\sum_{h=1}^{H}\sum_{i\in\Omega_{CS}}\sum_{sc\in Z}\left(\frac{\lambda_{t,h,pur}^{EV}}{\varphi_{conv}} + C_{Deg}\right)\left(\sum_{disp=1}^{N_{disp}} P_{disp}^{sc}t_{disp}\right)\left(\frac{1+IfR}{1+ItR}\right)^{t} \tag{11}$$

5) Investment cost of EVCSs

The installation cost of EVCSs is modeled in (12), where procurement, preparation, and installation of equipment and cost associated with them in each installed EVCS are taken into account.

$$C_{INV} = \sum_{i\in\Omega_{CS}} \sum_{sc\in?} C_{ac} N_i^{CS,sc} \tag{12}$$

B. Network Constraints

In this part, the power flow constraints in power distribution system operation are formulated via a set of linear functions introduced in (13)-(19)[33]. The methodology introduced in [33] utilizes approximations around the expected values of voltage magnified and angle to come up with a set of linear power flow constraints. After applying linearization, the active and reactive load balance constraints in the distribution system are presented in (13)-(14), where the injected power from the upstream network along with EVCSs should supply active and reactive loads in the power distribution system.

$$\sum_{i\in\Omega_G} P_{t,h,i}^{G,sc} + \sum_{i\in\Omega_{CS}} P_{i,t,h}^{CS,sc} - P_{i,t,h}^{D} = \sum_{j\in\Omega_i} \begin{pmatrix} -G_{i,j} + B_{i,j}\delta_{i,t,h}^{sc} - B_{i,j}\delta_{j,t,h}^{sc} \\ + G_{i,j}V_{i,t,h}^{sc} + G_{i,j}V_{j,t,h}^{sc} \end{pmatrix} \tag{13}$$

$$\sum_{i\in\Omega_G} Q_{t,h,i}^{G,sc} - Q_{i,t,h}^{D} = \sum_{j\in\Omega_i} \begin{pmatrix} -G_{i,j} - B_{i,j}\delta_{i,t,h}^{sc} + B_{i,j}\delta_{j,t,h}^{sc} \\ + G_{i,j}V_{i,t,h}^{sc} + G_{i,j}V_{j,t,h}^{sc} \end{pmatrix} \tag{14}$$

The voltage magnitude on each bus in the power distribution system is limited in (15).

$$V^{min} \le V_{i,t,h}^{sc} \le V^{max}, \tag{15}$$

The active and reactive powers taken from the wholesale market are constrained in (16).

$${P_{t,h,i}^{G,sc}}^2 + {Q_{t,h,i}^{G,sc}}^2 \le {S^G}^2, \tag{16}$$

Finally, the active and reactive powers flowing between buses *i* and *j* in the power distribution system are constrained in (17).

$${P_{i,j,t,h}^{FL,sc}}^2 + {Q_{i,j,t,h}^{FL,sc}}^2 \le {S_{i,j}^{FL}}^2, \tag{17}$$

It should be noted that active and reactive loads in the distribution system are considered to be growing at a rate of ξ [32]. In this regard, the load profiles for each year are expressed as:

$$P_{i,t,h}^{D} = P_{i,base}^{D}\mu_h(1+\xi)^t, \tag{18}$$

$$Q_{i,t,h}^{D} = Q_{i,base}^{D}\mu_h(1+\xi)^t. \tag{19}$$

III. Uncertainty modeling

A. Robust Optimization Method

In this part, the ROM utilized for uncertainty modeling of the electricity load in the distribution system is formulated. ROM is one of the powerful uncertainty management tools that can provide operational strategies for the operation of energy systems [34]. The power distribution system operator in the studied model in this paper can take advantage of ROM to get informed about the possible consequences of load uncertainty, and thus, by making proper decisions, it can control the uncertainty while gaining financial profit through the operation of EVCSs. This critical feature makes ROM a powerful tool that can model uncertainty while offering strategies to tackle them and keep the system's operation safe. It has a mathematical basis and considers all aspects of uncertainty so that for any level of uncertainty, an appropriate operational strategy is given to the operator of the system to manage the system operation while satisfying the planning/ scheduling expectations. In order to express the ROM, a simple optimization formulation is given in this part as follows:

$$Min \sum_{h=1}^{H} e_h x_h, \tag{20}$$

s.t. (21)

$$\sum_{h=1}^{H} a_{mt} x_h \le b_m, \tag{22}$$

$$x_h \ge 0, \tag{23}$$

$$x_h \in \{0,1\}. \tag{24}$$

In the simple optimization problem formulation, (20) shows the objective function that consists of e_h and x_h. The decision variables are denoted by x_h and the correlated coefficients are denoted by e_h. It should be mentioned that the coefficients can be chosen from a range from e_h to $e_h + d_h$. The deviance from the rated coefficient is denoted by d_h. After the assumption of d_h, the problem formulation can be changed to a new form as written below:

$$Min \sum_{h=1}^{H} e_h x_H + \ Max \sum_{h=1}^{H} d_h |x_h|, \tag{25}$$

s.t.

Constraints (22) – (23). (26)

The duality equilibrium theory is applied to the robust optimization formulation to find the optimum solution in (27) – (32). According to this theory, both primary and its correlated duality problem formulation reach the same optimum point.

$$Min \sum_{h=1}^{H} e_h x_h + z_0\Gamma_0 + \sum_{h=1}^{H} q_{oh}, \tag{27}$$

s.t.

Constraints (22) – (23), (26)

$$z_0 + q_{oh} \ge d_h y_h, \tag{28}$$

$$q_{oh} \ge 0, \tag{29}$$

$$y_h \ge 0, \tag{30}$$

$$z_0 \ge 0, \tag{31}$$

$$y_h \ge x_h. \tag{32}$$

In the dual objective function, an integer parameter, i.e., $utilized, select\ a$ value between 0 to $|J_0| = \{h|d_h > 0\}$. The deviations from the objective function impose a cost if $\Gamma_0 = J_0$. It should be mentioned that in the above mathematical formulation, z_0 and q_{oh} are dual variables of the primary problem. Based on the explanation given about the robust optimization model, the studied model in (1)-(19) is converted into a minimization problem, and the terms and variables of ROM are added to the mentioned problem. Hence, the objective function of the proposed model is formulated as follows:

$$Min\left(-[PR_{LS} + PR_{CH} + PR_{DCH} + PR_{RS} - C_{INV}] + z_0\Gamma_0 + \sum_{t=1}^{T}\sum_{h=1}^{H}\sum_{sc=1}^{NS} q_{t,oh}^{sc}\right) \tag{33}$$

This optimization problem in (33) should be solved subject to a set of constraints of the studied model in (2)-(19) and the constraints associated with modeling ROM in (34)-(39).

Constraints (2)-(19), (34)

$$z_0 + q_{t,oh}^{sc} \ge d_{t,h}^{sc} y_{t,h}^{sc}, \tag{35}$$

$$q_{t,oh}^{sc} \ge 0, \tag{36}$$

$$y_{t,h}^{sc} \ge 0, \tag{37}$$

$$z_0 \geq 0, \quad (38)$$

$$y_{t,h}^{sc} \geq Lco_{t,h}^{sc}. \quad (39)$$

More details regarding the development of ROM can be found in [34] [35].

B. Scenario-based methodology

In order to model uncertainty in the SOC of EVs affecting the services offered by EVCSs, scenario methodology is implemented. In this approach, 1000 scenarios with a standard deviation of 10% are generated. Later, to keep the computational burden of the optimization problem at an acceptable range, a scenario-reduction approach introduced in [36] is utilized and the number of scenarios is reduced to a total of 10 scenarios, which probabilities are presented in Table II.

Hence, the proposed optimization model for sizing and siting EVCSs in the power distribution networks is formulated for each scenario in (33)-(39), and the objective function is maximized to get the most expected financial profit for the operator of the power distribution system.

IV. Numerical Studies

In this section, the proposed model for sizing and siting EVCSs in a power distribution system under the uncertainties in load and SOC of EVs is studied regarding the 33-bus IEEE test system, as shown in Fig. 2.

A. Input data

In this section, utilized parameters and input data for the simulation of the studied problem are presented. More specifically, the power distribution system technical data is presented in Table III.

The share of each bus in the power distribution system from the system demand is calculated based on the share of each bus from the total demand of the system and presented in Table IV [37].

Please note that the scheduling horizon in the proposed model is divided into 4 time periods $h = [h_1, h_2, h_3, h_4]$ and the price data are accordingly presented. The distribution system base active and reactive loads are considered to be 3715 kW and 2300 kVAr, and the load coefficients for time periods h_1-h_4 are considered to be respectively 1, 0.94, 0.86, and 0.6. It is noteworthy that the EVs are considered to be available for the charge, discharge, and spinning reserve services during time periods h_1-h_3.

TABLE II

Reduced Scenarios And Their Associated Probabilities

Scenario	Probability	Scenario	Probability
1	0.092	6	0.144
2	0.128	7	0.085
3	0.1	8	0.148
4	0.127	9	0.023
5	0.07	10	0.083

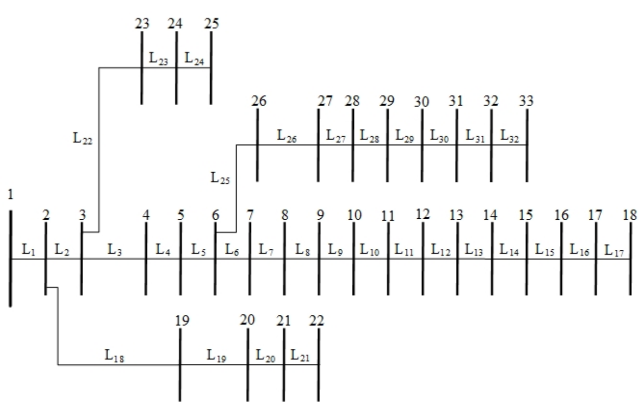

Fig. 2. Studied IEEE 33-bus test system

TABLE III

Power Distribution System Data

Branch number	Section from	Section to	R (Ω)	X (Ω)	Active load (kW)	Reactive load (kVAr)
1	1	2	0.0922	0.047	100	60
2	2	3	0.493	0.2511	90	40
3	3	4	0.366	0.1864	120	80
4	4	5	0.3811	0.1941	60	30
5	5	6	0.819	0.707	60	20
6	6	7	0.1872	0.6188	200	100
7	7	8	0.7114	0.2351	200	100
8	8	9	1.03	0.74	60	20
9	9	10	1.044	0.74	60	20
10	10	11	0.1966	0.065	45	30
11	11	12	0.3744	0.1298	60	35
12	12	13	1.468	1.155	60	35
13	13	14	0.5416	0.7129	120	80
14	14	15	0.591	0.526	60	10
15	15	16	0.7463	0.545	60	20
16	16	17	1.289	1.721	60	20
17	17	18	0.732	0.574	90	40
18	2	19	0.164	0.1565	90	40
19	19	20	1.5042	1.3554	90	40
20	20	21	0.4095	0.4784	90	40
21	21	22	0.7089	0.9373	90	40
22	3	23	0.4512	0.3083	90	50
23	23	24	0.898	0.7091	420	200
24	24	25	0.896	0.7011	420	200
25	6	26	0.203	0.1034	60	25
26	26	27	0.2842	0.1447	60	25
27	27	28	1.059	0.9337	60	20
28	28	29	0.8042	0.7006	120	70
29	29	30	0.5075	0.2585	200	600
30	30	31	0.9744	0.963	150	70
31	31	32	0.3105	0.3619	210	100
32	32	33	0. 341	0.5302	60	40

TABLE IV

Share of Buses From The System Demand

Bus Number	Share (%)	Bus Number	Share (%)
2	2.69	18	2.42
3	2.42	19	2.42
4	3.23	20	2.42
5	1.61	21	2.42
6	1.61	22	2.42
7	5.38	23	2.42
8	5.38	24	11.30
9	1.61	25	11.30
10	1.61	26	1.61
11	1.21	27	1.61
12	1.61	28	1.61
13	1.61	29	3.23
14	3.23	30	5.38
15	1.61	31	4.03
16	1.61	32	5.65
17	1.61	33	1.61

Technical parameters of EVCSs, as well as other sets of data regarding the energy prices, are presented in Table V. It should be noted that the subscripts regarding the initial SOC charge and discharge are for indication of min-level, mid-level, and max-level of the loads [32]. It is worthwhile to mention that if the EV owners do not choose the most adjacent charging station, the obtained final solution would not be the optimal. Therefore, the effect of the passengers' cost can be mentioned that is considered in an indirect way on the objective function. However, for a more accurate and dedicated analysis of the cost of EV owners choosing the charging station, different decision-makers should be considered like as the operator and EV owners. In this way, the behaviors of the EV ow-

Table V
EVCSS AND ENERGY PRICE DATA

Parameters	Value
Initial SOC (discharging)	$SOC_{d_1} = 0.8$, $SOC_{d_2} = 0.85$, $SOC_{d_3} = 0.9$
Discharging Vehicles	$n_{d1} = 25\%, n_{d_2} = 25\%, n_{d_3} = 50\%$
Initial SOC (charging)	$SOC_{c_1} = 0.1, SOC_{c_2} = 0.2, SOC_{c_3} = 0.3$
Charging Vehicles	$n_{c_1} = 40\%, n_{c_2} = 40\%, n_{c_3} = 20\%$
φ_{conv}	0.90
$\lambda_{t,h}$	(0.22, 0.13, 0.11, 0.8) ($/kWh)
$\lambda_{t,h,pur}^{grid}$	(0.2, 0.115, 0.95, 0.7) ($/kWh)
$\lambda_{t,h,pur}^{EV}$	(0.18, 0.16) ($/kWh)
λ_t^{cap}	0.01 ($/kWh)
λ_t^{ans}	0.2 ($/kWh)
C_{ac}	3000 ($)
C_{Deg}	0.03 ($/kWh)
ItR	5 (%)
IfR	4 (%)

ners can be modeled and studied in detail through the implementation of game theory approaches. This can form one of the directions of future work in analyzing the model from various viewpoints such as network operators, EV owners, and etc.

Using ROM, the studied problem in the proposed model is solved for different uncertainty levels of load, i.e., min, mid, and max load levels. The simulations for each load level are done in several iterations, which means that the simulations for the minimum, expected, and maximum load levels are carried out in iterations 1-3, respectively.

The studied model for optimal sizing and siting of EVCSs in power distribution systems under the uncertainties in load and SOC of EVs is solved through the commercial solver. The studied problem is solved with the optimality gap of 0.05% in GAMS, and the solution time for iterations 1-3 are 6.773, 15.288, and 18.805 seconds respectively.

B. Results Discussion

The proposed optimization model based on ROM for sizing and siting of EVCSs in the power distribution systems under the uncertainties in load and SOC of EVs is solved, and the results for iterations 1-3 with min, mid, and max load levels are presented in different scenarios. For instance, the size of EVCSs to be installed in scenario 1 is depicted in Fig. 3. According to this Fig, due to uncertainty, when the load value increases (max load level), EVCSs with larger capacities are installed in different locations in the power distribution system to gain financial profit for the operation of the system. These results are obtained by taking a risk-seeking strategy by the operator of the system toward the load uncertainty in this case, where the increase of load is considered as an opportunity for the power system operator to take advantage of EVCSs and minimize the system power loss as much as possible. In such a condition, with more significant investments in the installation of EVCSs, the services offered by these stations increase. Therefore the system operator earns more profit from the planning of EVCSs.

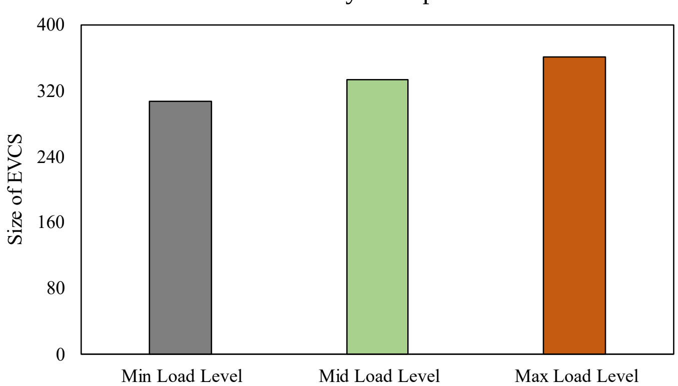

Fig. 3. The optimal size of installed EVCSs

On the other hand, when the load value decreases due to uncertainty, the system operator prefers to take a risk-averse strategy and therefore invests less in the installation of EVCSs. In this case, the operators prefer to avoid over-installing EVCSs with bigger sizes, and consequently, the size of installed EVCSs decreases. All these strategies are provided by ROM, where possible positive and negative consequences of load uncertainty are taken into account optimal strategies are given to the power distribution system operator to deploy when supplying loads.

According to Fig. 3, the size of EVCSs installed in the power distribution system for the mid, mid, and max load levels are 307, 334, and 361, respectively. These results are all related to the strategies ROM introduced to the power distribution system operator. As mentioned, in addition to the optimal size for the installation of EVCSs, optimal locations for their installation for different load uncertainty levels are also provided. In more detail, the optimal locations for installation of EVCSs in the power distribution system are buses 6, 7, 8, 9, and 10 for the min load level case, buses 5, 6, 7, 26, and 27 for the mid load level case, and buses 5, 23, 24, 25, and 29 for the max load level case. To assess the way power distribution system operation is impacted under risk-averse and risk-seeking strategies, the total revenue of the power distribution system operator for different load uncertainty levels in scenario 1 is presented in Fig. 4.

According to Fig. 4, the load reduction significantly impacts power distribution systems operation, where the total profit of the system operator drops from $109,437 to $100,283. This reduction achieved through taking risk-averse strategy would make system operation robust enough toward load uncertainty. On the other side, by taking a risk-seeking strategy, system profit increases from $109,437 to $118,132 when the load value rises. As it can be seen, by implementing ROM, the power distribution system operator can assess every possible consequence of uncertainty, and therefore, by making appropriate decisions, it can manage system operation in uncertainty-based environments. For more details on the way staking risk-averse and risk-seeking strategies impact power distribution system operation, the profit of the system from offering discharge, reserve, and charge services, as well as reducing power losses in scenario 1 is also presented in Fig. 4.

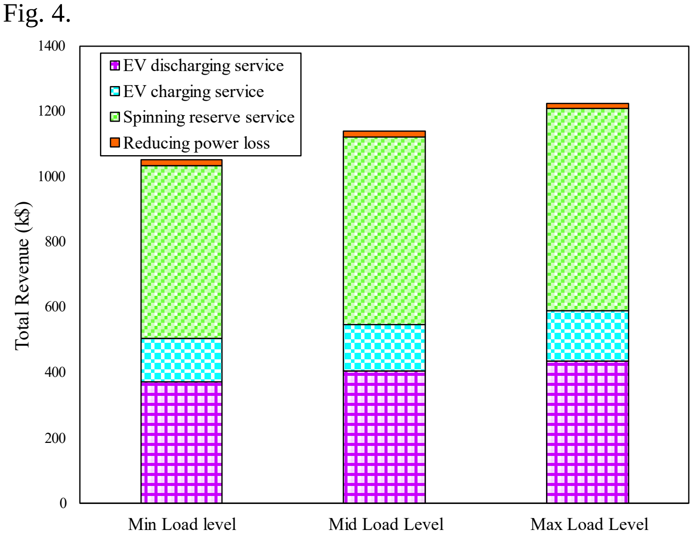

Fig. 4. Total revenue of power distribution system operator in scenario 1

Regarding the results for profit from offering discharge service, it should be noted that a risk-seeking strategy is taken against the load uncertainty; the total profit of the system from offering discharge service in scenario 1 increases from \$383,244 to \$414, 146, which shows an 8.06% increase in profit. This is mainly because with increasing load value, the service (discharge power) offered by these stations becomes more preferable than upstream grid power, so the system operator decides to take advantage of these stations to supply distribution system requirements. On the other side, when the load value decreases, the total service offered by these stations decreases, which consequently reduces the financial profit for the power distribution system operator from \$383,244 to \$351,881.

Another detail shown in Fig. 4 is the total profit from offering reserve service in scenario 1. According to this figure, by taking a risk-averse strategy, the offered reserve service from EVCSs drops; therefore, the total profit from mentioned services reduces 8.91%. On the other side, taking advantage of possible opportunities in load uncertainty by a taking risk-seeking strategy, the total profit of the system from offering reserve service increases by 8.06%. A similar analysis is also accurate for the total profit of the system from offering charging services.

Moreover, As given in Fig. 4, the total profit of the system from offering charging service in min load level reduces by 8.18%. This reduction is mainly because when the load value of the system decreases, the system operator does not risk building EVCSs with more enormous capacities, which is logical considering the investment costs of EVCSs. Conversely, when the load value increases, the power distribution system operator tends to risk more. By installing EVCSs with more enormous capacities, it earns more profit by offering charging services to the EVs in the EVCSs.

The total profit of the system operator from power loss reduction is also presented in Fig. 4. As shown, the operator's revenue from this factor is lower than the other factors. Meanwhile, when the load value decreases, EVCSs seem to have less impact on reducing system power loss. This is mainly due to the reduction of load; the total power loss value of the system reduces in both cases of not installing/installing EVCSs, and therefore the impact of ECVSs on further reduction of power losses decreases in the min load level case, reducing the total profit of system 9.31.% On the other side, with the increase in load value in the max load level case, EVCSs play a significant role in reducing power loss value in the power distribution system, and therefore, by reducing power loss value, stations contribute to 7.32% more profit.

Finally, the installation cost of EVCSs in scenario 1 is presented in Fig. 5. As shown in this figure and considering the results discussed above, by taking a risk-averse strategy in the min load level case, the system operator attempts to install EVCSs with smaller capacities in order to control the load uncertainty and its possible risks, which leads to minor investment cost of EVCSs in this case. On the other side, by taking a risk-seeking strategy in the max load level case, the operator decides to install ECVSs with more enormous capacities to offer more services and therefore earn more profit. This leads to higher investment costs of EVCSs in this case, as depicted.

The results discussed above are also valid for the rest of the scenarios in the studied model. For instance, the size of EVCSs and the total profit of power distribution system operator scenario 5 are shown in Figs. 6 and 7. As can be seen, Similar strategies are taken against the load uncertainty by the system operator to supply the system requirement while controlling the risk.

To further evaluate this issue, the total expected profit of the power distribution system operator for the min, mid, and max load levels is presented in Table VI.

Finally, the results for optimum sizing and siting of the EVCSs are depicted in Fig. 8. According to this figure, it can be seen that the number of EVs in the risk-averse and risk-neutral conditions are similar in the studied test system for siting the largest EVCs. Where most of the EVs are placed in buses 5-8. Therefore, the best option for investment for building the largest EVCS is choosing the location of one of these buses. Besides that, two other EVCSs can be built in buses 10-12 and buses 20-22 for risk-averse

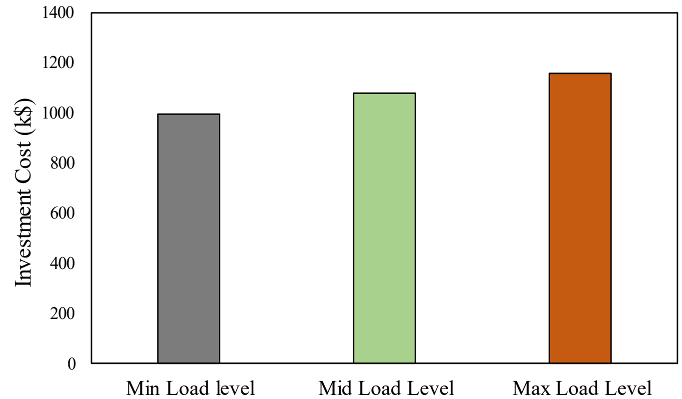

Fig. 5. Total investment cost of EVCSs in scenario 1

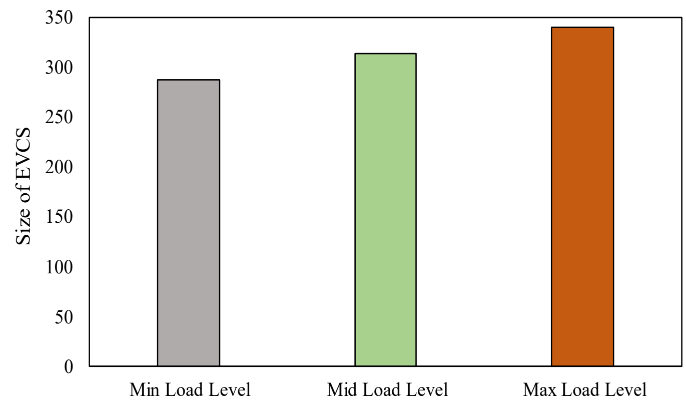

Fig. 6. Optimal size of installed EVCSs in scenario 5

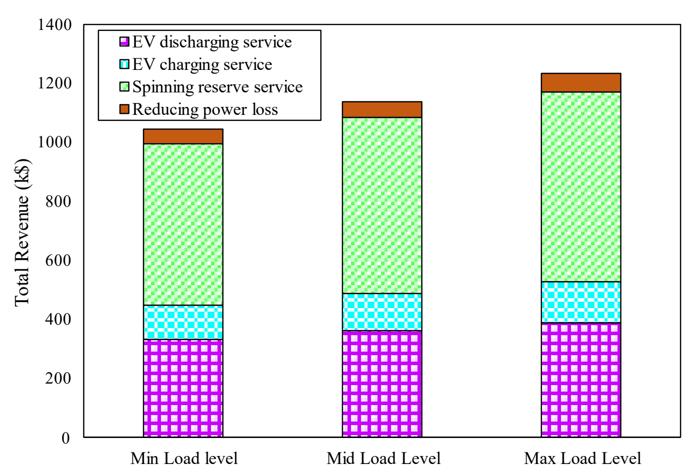

Fig. 7. Total Revenue of power distribution system operator in scenario 5

TABLE VI
EXPECTED PROFIT OF POWER DISTRIBUTION SYSTEM OPERATOR

#	Value ($)		
	Min Load Level	**Mid Load Level**	**Max Load Level**
Total expected profit	940365.77	1024181.71	1487999.02
Total expected profit from discharge service	372684.22	403738.38	434412.92
Profit expected profit from reserve service	529932.36	574572.39	618654.23
Profit expected profit from charge service	132645.85	143698.65	154616.34
Profit expected profit from loss reduction	15271.84	16772.30	17925.77
The total investment cost	995959.44	1078948.41	1160922.91

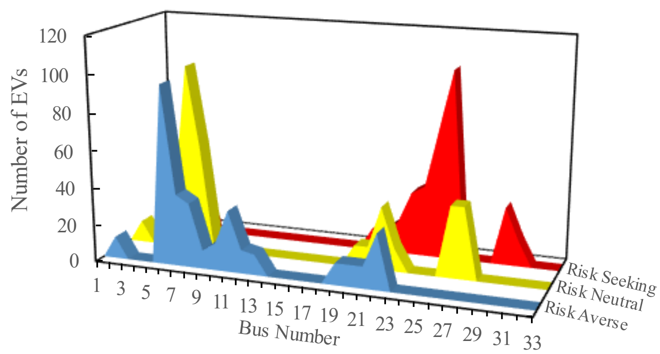

Fig. 8. The size and site of the EVCSs in different conditions

conditions. At the same time, the two smaller EVCSs for risk-neutral conditions are proper to be built among the buses 20-23 and 26-27. However, the risk-seeking operator would instead build the biggest EVCS in buses 23-25, as most of the EVs are being placed in these buses and siting the smaller ones in buses 4-5 and buses 29-30. Therefore, optimal planning of EVCSs through ROM leads to optimal operational strategies in all scenarios, and therefore, appropriate decisions can be taken by the system operator to supply system requirements while maintaining the expected profit. The strategy introduced by ROM helps distribution system operators invest properly in developing EVCSs and reach the expected planning/scheduling objectives.

V. CONCLUSION

This paper presented a ROM-based optimization framework for optimal sizing and siting of EVCSs in power distribution systems. The uncertainties posed by the load and SOC of EVs are handled through different risk measures simultaneously. Hence, in order to model the load uncertainty in the proposed framework, the ROM was employed, and to model the uncertainty of SOC of EVs, a stochastic programming approach was implemented. The stochastic approach enables us to model the variability and randomness in the SOC of EVs. Numerous scenarios were generated and then reduced using scenario-reduction methods to mitigate the computational burdens of the studied problem. The model was analyzed and studied based on several risk attitudes including risk-averse, risk-neutral, and risk-seeking strategies which help the system operator to make appropriate decisions in the presence of uncertainties. Moreover, the non-linear terms in power flow modeling were linearized through a set of linear functions, and therefore the studied problem was modeled as a MILP problem which makes it easier to reach the optimal solution through the available commercial solvers. The IEEE 33-bus test system was employed to investigate the feasibility of the proposed model, and the results for different uncertainty levels of load were analyzed.

By implementing the proposed optimization framework, appropriate operational strategies were obtained to be taken by the operator of power distribution while supplying system requirements. According to the results, the load reduction significantly impacts power distribution systems operation, where the total profit of the system operator drops seriously. Therefore, after developing EVCSs, their role in reducing power losses in the system became evident. Hence, the system operator decided to take a risk-averse strategy and invest less in the planning and construction of EVCSs. Therefore, the number of installed EVCSs, in this case, decreased, which directly impacted the revenues from offering charge, discharge, and reserve services. Conversely, with the increase in the load due to uncertainty, the total profit of the distribution system operator increased. This was mainly due to the significant reduction of power loss value by EVCSs that motivated distribution system operators to take a risk-seeking strategy and invest more in the planning and building of EVCSs. Thus, it could earn more profit not only from the reduction of system power loss value but also by offering more charge, discharge, and reserve services to the clients. It was noteworthy that under different scenarios of EV's SOC, the total profit of system operators from developing EVCSs varies, which depicted the critical impact of arriving SOC of EVs into EVCSs in power distribution systems. To sum up, the proposed framework in this paper successfully captured the consequences of uncertainties in the load and SOC of EVs and helped the power distribution system operator to make appropriate decisions to manage the system operation and supply the loads while controlling the uncertainties. As the future direction of this work, we can consider studying the model from the perspectives of other players, such as EV owners, transportation authorities, or environmental agencies. Analyzing the model from multiple stakeholders' viewpoints would provide a more well-rounded assessment of the system's performance and consider a broader range of interests and objectives.

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