Nooa Nyman

Option Pricing During Turbulent Markets

Stochastic and Computational Approach

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Author: Nooa Nyman
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ABSTRACT:
This thesis examines the impact of modern market standards on the pricing of options. Market turbulence leads to high volatility and short interest rate fluctuations. Traditional option pricing models are unable to accurately predict option prices in this challenging market environment. Therefore, new models are needed that are able to take into account these unconventional "market disturbances".

In particular, this paper will focus on interest rate and volatility fluctuations and the phenomena they cause. It does so by presenting the theoretical background to the topic and by examining relevant, recent literature on the subject.

This thesis provides an understanding of the challenging world of option pricing that is essential for institutional investors and risk management professionals. It also provides a basic understanding of computational power in pricing models and stochastic valuation processes.

KEYWORDS: Option, stochastic, pricing, volatility, interest-rate, Black-Scholes

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TIIVISTELMÄ:

Tämä kandidaatin tutkielma tutkii modernien markkinastandardien vaikutusta optioiden hinnoitteluun. Markkinoiden turbulenssi johtaa voimakkaaseen volatiliteetin ja lyhyen korkotason vaihteluun. Alkuperäiset optioiden hinnoittelumallit eivät kykene ennustamaan optioiden hintoja tarkasti tässä haastavassa markkinaympäristössä. Tämän vuoksi tarvitaan uusia malleja, jotka pystyvät ottamaan myös nämä epätavanomaiset "markkinahäiriöt huomioon".

Tässä kandissa keskitytään etenkin korkojen ja volatiliteetin vaihteluun ja niiden aiheuttamien ilmiöiden tutkiskeluun. Se tekee niin esittelemällä aiheen teoreettisen taustan ja tutkimalla relevanttia, uutta kirjallisuutta aiheeseen liittyen.

Tutkielman tehtävä on tarjota ymmärrystä liittyen haasteelliseen optioiden hinnoittelun maailmaan, minkä ymmärtäminen institutionaalisille sijoittajille ja riskien hallinnan ammattilaisille on välttämätöntä. Työ tarjoaa myös pohjatason ymmärryksen tietokoneellista tehoa käyttäville hinnoittelumalleille sekä stokastisille valuaatio prosesseille.

AVAINSANAT: Option, stochastic, pricing, volatility, interest-rate, Black-Scholes

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1 Introduction

One of the cornerstones of modern finance has been the pricing of derivative securities, especially options, since their formal introduction to the markets. These instruments used widely for both hedging and speculative purposes, offer unparalleled flexibility but also present intricate challenges. As financial markets evolve, marked by heightened volatility and frequent economic shocks, the option pricing mechanism demands continual refinement to ensure accuracy and robustness. Frequent fluctuations in interest rates and persistent volatility clusters have become defining traits of today's markets rather than rare exceptions. These trends emphasize the importance of developing models that accurately capture real-world complexities while providing practical value for financial professionals.

The global derivatives market, which boasts a notional value exceeding $667 trillion in 2023 (International Swaps and Derivatives Association, 2024), far surpasses the value of its underlying assets. This significant disparity can be attributed to the leverage inherent in derivatives, making them both a powerful financial instrument and a potential source of systemic risk. Options, a key category of derivatives, play a crucial role in the financial ecosystem by providing mechanisms to manage price uncertainty across a variety of underlying assets, including equities and commodities(Vo et al., 2019). However, mispricing of these instruments can result in substantial financial losses and systemic instability, underscoring the urgent need for reliable and robust valuation methods.

For decades, traditional option pricing models, particularly the Black-Scholes framework, have been the foundation of financial theory and practice. With its mathematical formulation, this model revolutionized the approach to derivatives pricing by introducing concepts like risk-free portfolios and put-call parity (Black & Scholes, 1973). However, despite its widespread application, the Black-Scholes model relies on several simplifying assumptions, such as constant volatility and interest rates, which are frequently inconsistent with real-world market conditions. The inability of these models to capture the nuances of volatile and turbulent markets, where frequent interest rate movements may occur or the rates might drop even to negative numbers, has led to significant mispricings, particularly during periods of economic uncertainty (Hull & White, 1990)

The limitations of traditional models have catalyzed the development of advanced stochastic and computational frameworks designed to overcome these limitations. Stochastic interest rate models, such as those introduced by Cox et al. (1985) incorporate the mean-reverting behavior of interest rates, providing a dynamic perspective that better reflects observed market behavior. Similarly, stochastic volatility models, like the Heston model, account for phenomena such as volatility clustering and the volatility smile, providing a more nuanced view of pricing dynamics (Heston, 1993). Together, these innovations, along with advancements in computational methods, have transformed the landscape of option pricing, facilitating more accurate valuations even in the most challenging market conditions (Bormetti et al., 2020).

Understanding the effects of short-term volatility and interest rate fluctuations on option pricing is essential for traders, portfolio managers, and policymakers alike. Market participants are increasingly dependent on adaptable models to navigate the complexities of financial markets, where swift changes can render traditional frameworks inadequate. This thesis tries to show connections between market turbulence, interest rate fluctuations, and option pricing. Drawing on an extensive body of literature, it explores how modern stochastic and computational approaches can improve pricing accuracy, offering valuable insights for both academics and practitioners.

As financial markets continue to evolve in complexity, there will be a growing demand for efficient, accurate, and robust pricing models. This thesis aims to give its reader an understanding of theoretical advancements and real-world applications, emphasizing the transformative impact of incorporating stochastic and computational methods into option pricing.

Comprehending the intricacies of option pricing extends beyond academic interest; it has significant real-world implications for optimizing portfolios, managing risks, and maintaining market stability and liquidity. When options are mispriced, it can result in major financial losses, disrupt market efficiency, and elevate systemic risk. As derivatives markets expand and exert greater influence, the need for precise and reliable pricing models has never been more critical. By addressing the challenges posed by today's market conditions, this work seeks to lay the groundwork for future research and innovation in the field.

1.1 Purpose of Study

The primary object and purpose of this study is to investigate the effect that short-term volatility and interest-rate changes can have on the pricing of options. It also offers alternative pricing models during turbulent market conditions. This thesis aims to offer a comprehensive understanding of how short-term market turbulence may affect the valuation process behind the options' market value.

Options play a pivotal role in financial markets, serving as essential tools for both risk management and speculative activities. However, the valuation of these instruments has always remained one of the key elements in finance. Traditional models, such as the Black-Scholes framework, often fall short under volatile or rapidly changing market conditions, particularly due to their reliance on assumptions like constant interest rates and volatility. This study addresses these gaps by examining how modern stochastic and computational methods can enhance pricing accuracy during such scenarios.

The motivation for this thesis lies in recognizing that traditional pricing models are increasingly inadequate for capturing the complexities of modern financial markets. With growing instances of negative short-term interest rates and heightened market turbulence, also compared with increasingly volatile underlying assets, the need for adaptable and accurate pricing models has become more pressing. These considerations lead to the following hypotheses:

H1: Pricing option through stochastic interest-rate process achieves greater accuracy, especially during volatile times, where even negative short-term rates can be seen.

H2: Modern option pricing models demonstrate greater accuracy in pricing options by accounting for stochastic volatility, especially during periods of market turbulence.

H3: Computational models reach a far greater accuracy compared to traditional models in situations where market changes are rapid

This study is built upon existing literature in the fields of stochastic modeling, computational finance, and option pricing theory. Stochastic interest-rate models, such as the Hull and White concept and CIR framework, address the shortcomings of constant-rate assumptions, offering a dynamic perspective that aligns with real-world market behaviors, also paving the way for newer approaches and models that reach even greater accuracy under certain market circumstances. Similarly, stochastic volatility models capture the clustering and smile effects often observed in markets, providing more realistic pricing insights. Lastly, the integration of computational approaches, including machine learning algorithms and fractional calculus, underscores the potential for improved precision and adaptability in option pricing.

Understanding the impact of short-term volatility and interest-rate changes on option pricing is critical for market participants, including traders, portfolio managers, and policymakers. By evaluating the interplay between market turbulence and pricing models, this thesis aims to contribute valuable insights to the development of more robust, efficient, and accurate valuation methodologies. These insights will help professionals navigate the complexities of modern financial markets and better manage risk and opportunities.

1.2 Structure of Study

This thesis is structured to prove a comprehensive examination of option pricing throughout the context of fluctuating interest rates and changing volatility during short periods. The first chapter lays the foundation for the thesis by introducing the topic, justifying the purpose of why this thesis is written, and presenting the three hypotheses that will be studied throughout the work. The second, third, fourth, and fifth chapters aim to give a reader a theoretical foundation about the subject. The basics of financial hedging are discussed alongside a brief overview of derivatives before diving more deeply into the basic concepts of derivative pricing where terms such as time value, arbitrage, and risk-neutral valuation are presented. Further refining understanding from derivatives to options fundamentals about option pricing and related terminology is presented before diving more deeply into the fundamentals of option pricing and the Black-Scholes model. The sixth and seventh chapters offer a thorough literature review, drawing from a wide range of relevant and recent literature discussing the topic of this thesis, to highlight the current state of knowledge on the topic, and to credibly address the claims of hypotheses. The eighth and final chapter of the thesis concludes everything together, summarizing the key findings, discussing their implications and, reflect on possibilities of future research.

2 Financial Hedging

Financial hedging is a key strategy in modern finance, seeking to limit exposure to all financial risks. Hedging, by definition, involves taking opposing positions to protect against negative financial outcomes, thus enabling firms and individuals to manage uncertainty effectively. Financial hedging has increased in importance with the surging complexity and globalization of financial markets, becoming an integral part of risk management by corporations, financial institutions, and individual investors.

Hedging primarily serves as a mechanism for reducing risk, and addressing diverse sources of financial uncertainty such as price volatility, fluctuations in interest rates, exchange rate risks, and credit risks. For multinational corporations, managing currency exposure represents a vital aspect of hedging, as variations in exchange rates can substantially influence cash flows and overall profitability. Likewise, financial institutions employ hedging strategies to mitigate risks linked to interest rate changes and credit exposures, thereby maintaining operational stability (Bliss et al., 2018). The significance of financial hedging extends beyond individual organizations, playing a crucial role in promoting the overall stability of financial markets.

The principles of hedging extend to diverse instruments and strategies, ranging from operational adjustments to sophisticated financial contracts. While derivatives have become the dominant tools for hedging, they are complemented by other strategies such as natural hedging, aligning revenue and cost streams in the same currency, and insurance products. This diversity highlights the adaptability of hedging strategies to the specific needs and risk profiles of different market participants.

Financial institutions, particularly banks, hold a central position in enabling hedging activities. Acting as intermediaries within derivative markets, they offer access to tailored hedging instruments along with specialized expertise in risk management. Interestingly, their exposure to market risks is frequently hedged. For example, (Bliss et al., 2018) highlight the necessity of aligning hedging strategies with an organization's risk management objectives to prevent undue financial strain. Banks' use of derivatives, including swaps and options, demonstrates their dependence on advanced tools to effectively hedge against interest rates and credit risks.

The benefits of financial hedging are well-documented in academic literature. Bartram (2019) highlights that firms engaging in hedging activities experience reduced cash flow volatility and enhanced financial predictability. These benefits are particularly pronounced during periods of economic stress, where hedging can stabilize operations and maintain access to capital markets. For example, during the global financial crisis of 2008, firms with robust hedging programs were better equipped to navigate the heightened volatility and liquidity constraints.

Hedging carries significant implications for shareholder value. By minimizing the volatility of cash flows and earnings, firms can reduce their cost of capital and improve their overall valuation. This is particularly relevant in industries highly exposed to commodity price fluctuations or foreign exchange risks, where effective hedging strategies can offer a substantial competitive edge.

However, financial hedging is not without its challenges. The complexity of hedging instruments, especially derivatives, necessitates a thorough understanding of financial markets and risk dynamics. Mismanagement or improper use of these tools can result in considerable financial losses, as illustrated by notable failures such as the collapse of Barings Bank. Furthermore, the use of over-the-counter (OTC) derivatives introduces counterparty risk and poses potential systemic challenges, highlighting the importance of regulatory oversight and comprehensive risk management practices.

Another challenge lies in aligning hedging strategies with a firm's overall financial objectives. Bartram (2019) notes that firms must carefully assess the trade-offs between the costs and benefits of hedging, ensuring that these activities add value rather than simply mitigating risk. Furthermore, the dynamic nature of financial markets requires continuous monitoring and adjustment of hedging positions to remain effective.

Financial hedging allows firms and institutions to manage risks and maintain operational stability in an increasingly uncertain economic environment. Although derivatives are integral to hedging strategies, the broader scope of financial hedging includes a variety of tools and practices designed to meet the unique requirements of market participants. The success of hedging initiatives relies not only on the selection of appropriate instruments but also on the alignment of these strategies with the broader objectives of financial stability and value creation. As financial markets continue to evolve, the advancement of innovative hedging practices and the establishment of robust risk management frameworks will be crucial for addressing emerging challenges and ensuring economic resilience.

3 Derivatives and Hedging

Derivatives for risk management are most used by institutional giants to achieve their financial goals and protect their assets (Rampini et al., 2020). During the late stages of the 20th century, the derivatives market experienced its most rapid growth, via numerous new derivative instruments and underlying assets to hedge and speculate the complexity of the market had started to get out of hand. This complexity is one of the reasons that led to risk management failure during the financial crisis. Rampini et al. (2020) state that even after those events, there is still a lack of understanding of the basic risk management processes in financial institutions and practices of risk management in banks.

Since the market of derivatives started growing, it has reached an astonishing 667 trillion USD by the end of 2023. The market is dominated by interest-rate derivative securities.

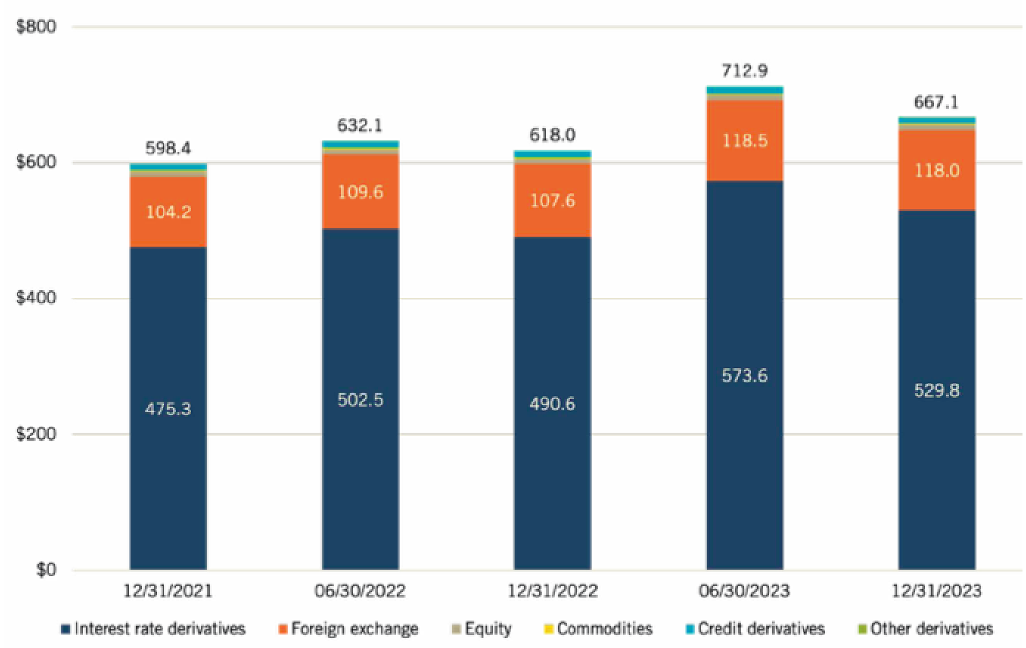

Table 1 Global OTC Derivatives Notional Outstanding (US$ trillions) (International Swaps and Derivatives Association, 2023)

A derivative is a financial instrument whose price can be derived from an underlying asset value. Many assets like commodities such as oil or stocks can be hedged with derivatives against price risk. Derivatives can be traded either in exchange or over-the-counter markets (Vo et al., 2019). Different derivative exchanges can be found in most financially significant areas which are usually delimited geographically. Over-the-counter (OTC) trading is separated from the exchange.

Derivatives can be used in two different ways – either to hedge against an unwanted risk or to speculate by taking apposition in anticipation of a market movement (Rao, 2012). By speculating the S&P500 index, for example, the trader predicts which direction the index will move in the future. According to Rao (2012), there are several instruments derivatives trading is carried out, them being; Futures contracts, option contracts, index futures, index options, commodity derivatives, and swaps. The structures of different types of derivatives might be challenging to understand and pre-usage they must be understood completely otherwise they might cause big harm. Effective risk management with derivatives requires thorough knowledge of the instruments otherwise it can lead to significant financial losses as seen in the cases like Barings Bank collapse (Rao, 2012).

Derivatives offer great protection against volatility risk. On Average derivative users gained cash flow volatility decreased by 30% and total risk reduction by 20% compared to non-users (Bartram et al., 2011). Systematic risk is a type of risk that concerns only a single sector, according to Bartram et al., (2011) derivative users had a 6% reduction in their market betas compared to non-users. This suggests that firms using derivatives can stabilize their financial outcomes by reducing exposure to market fluctuations.

3.1 Basic Concepts of Derivative Pricing

To understand more about the pricing of derivatives, it is vital to grasp several fundamental principles that serve as building blocks for much more complex models. Time value of money, arbitrage, and risk-neutral valuation serve as a basis when it comes to pricing options for example. This section will briefly introduce their simplified forms and applications in derivative pricing.

3.1.1 Time Value of Money and Present Value

The key idea behind the time value of the money is, that a dollar today is worth more than the same dollar in the future due to its potential to earn interest or investment return over time. This theory is expected in various financial studies, especially in ones that involve valuation and future cash flows. Funds available today can be invested to yield a return, whereas future funds cannot capitalize on current investment opportunities. This concept is particularly relevant in option pricing, where future payouts are discounted to reflect their value in present terms (Black & Scholes, 1973).

Present value reflects the worth of future cash flows in today's terms, accounting for the risk-free rate or an appropriate discount rate based on the risk profile of the cash flow source (Cox et al., 1979). This principle is widely applied in corporate finance, derivatives, and capital budgeting, where the timing of cash flows is critical to investment decision-making (Merton, 1973).

In present value calculations, an appropriate interest rate is crucial as it impacts the valuation of future cash flows. Higher discount rates reduce the present value, reflecting the increased risk or opportunity cost of the funds tied up in the investment. Conversely, lower discount rates increase the present value, making future cash flows more valuable in today's terms. In options pricing, for example, the risk-free interest rate is commonly used as the discount rate, as it reflects the minimum expected return in a risk-neutral environment (Black & Scholes, 1973). This use of risk-free rates in derivative pricing aligns with the no-arbitrage principle, ensuring that derivative prices remain consistent with the expected returns of comparable investments (Samuelson, 1973).

Empirical studies have demonstrated the practical implications of the time value of money across various asset classes. Fama and French (1989) illustrate how the time value of money affects the valuation of equities, as companies with expected cash flows further in the future tend to have higher risk premiums and lower present values. Similarly, the concept of present value is integral to capital asset pricing models (CAPM), which quantify the relationship between expected returns and risk for different assets (Sharpe, 1964). In corporate finance, present value calculations are essential for determining the viability of long-term projects, as managers assess whether the future benefits of a project justify the present investment cost (Graham & Harvey, 2001).

3.1.2 Arbitrage Principles

Arbitrage refers to the activity of earning risk-free returns by exploiting differences in prices between markets or instruments. The no-arbitrage principle simply states that an efficient market cannot have identical assets or cash flows priced differently because such a setup would lead to an opportunity to earn a profit without risk. The no-arbitrage principle has played a very important role in financial theory, especially in derivative pricing, because it ensures related asset prices are consistent. In effect, arbitrage drives as a self-correcting mechanism that drives prices toward equilibrium (Fama, 1970).

The no-arbitrage principle plays a pivotal role in modern asset pricing models, especially in the Black-Scholes model it being one of the key assumptions. Black & Scholes (1973) created the model under the assumption of the no-arbitrage principle, allowing them to create a replicating portfolio of the option and the underlying asset, which mimics the option’s payoff. Cox et al. (1979) also used a no-arbitrage condition approach in their binomial option pricing model, which values options through a stepwise replication strategy.

Arbitrage is often perceived as identical assets, but it can also apply to synthetic positions. A synthetic position means replicating assets’ payoff by combining other assets. This is possible by buying two different instruments that will provide the same payoff under the same conditions. For example, the aforementioned synthetic position can be created by buying a long future contract on oil and simultaneously shorting the commodity index that moves in the same direction as oil prices. This position now replicates actual oil price movements without owning the commodity itself. If this synthetic replication is not priced equivalently to the actual position under no-arbitrage, arbitrageurs can exploit the price differential (Stoll, 1969).

Fama (1970) states that in efficient markets the price "fully reflects" the available information, largely due to arbitrage actions that eliminate any mispricing. However, in the real world, there are limitations to arbitrage actions. Shleifer & Vishny (1997) highlight the risks and costs involved in arbitrage activities, such as market risk and transaction costs which can prevent arbitrageurs from fully exploiting price discrepancies. These limitations mean that while arbitrage tends to correct prices, it may not always eliminate mispricing, especially under conditions of market stress.

3.1.3 Risk Neutral Valuation

Risk-neutral valuation is a modern financial pricing model that is used for derivative pricing. It offers a simplified framework on how the fair value of financial assets is valued under uncertainty. "The risk-neutral world" is one of the key assumptions in the Black-Scholes model which means that the expected return of the underlying asset is the risk-free rate (Black & Scholes, 1973). While this implication doesn't represent real-world investment behavior it is a powerful mathematical tool that simplifies option pricing.

In risk-neutral valuation, the expected cash flows are discounted at the risk-free rate of return, as opposed to a risk-adjusted discount rate. This approach is based on the very important observation that, in a world with a so-called risk-neutral measure, all assets have an expected return equal to the risk-free rate. The idea behind this method relies on the fact that the pricing of a derivative depends only on the distribution of future outcomes and the absence of arbitrage, and not on the particular risk preferences (Harrison & Kreps, 1979). At least one martingale measure needs to exist in the securities market model for it to be viable.

Supporting the previous ideas another study by Cox et al. (1979) adopts the discrete-time binomial model to price options, providing an intuitive framework for understanding how expected cash flows are discounted using the risk-free rate under a risk-neutral measure. Both Cox et al. (1979) and Harrison and Kreps (1979) emphasize that option pricing relies only on future outcomes' distribution and no-arbitrage conditions.

$$p = \frac{r - d}{u - d} \tag{1}$$

Equation 1 Risk-neutral probability (Cox et al., 1979)

Where:

r = Risk-free rate

d = Down factor in the binomial tree (stock price goes down)

u = Up factor in the binomial tree (stock price goes up)

$$C = \frac{[pC_u + (1 - p)C_d]}{r} \tag{2}$$

Equation 2 Expected payoff under the risk-neutral measure (Cox et al., 1979)

Where:

C = The value of the call option

C_u = Options payoff in the upstate

C_d = Options payoff in the downstate

r = Risk-free rate

p = The risk-neutral probability

Based on these fundamentals, empirical studies have shown how risk-neutral valuation works in real-life applications. A studied case where implied risk-neutral measures derived from options markets can be used to deduce market expectations about future volatility and macroeconomic uncertainty (Carr & Wu, 2004). These practical uses underline the continuing relevance of risk-neutral valuation, not only as a theoretical framework but also as an effective tool for decision-making in financial markets.

4 Options - Theoretical background

To be able to understand this study better, it is first important to understand what type of a derivative option is. An option gives its holder a right to buy (call-option) or to sell (put-option) a specified quantity of an underlying asset, within a specified period of time (Black & Scholes, 1973). American options can be exercised at any given time up to the date that the option expires, European options can only be exercised on a specific future date. The underlying assets for options can vary a lot from commodities to currencies. For mostly in this study we will be using the simplest form of option defied by Black and Scholes (1973) which gives its holder a right to buy a single share of common stock, "a call-option".

4.1 Option Pricing Fundamentals

Option pricing theory is part of a very general class of economic problems and is relevant in all areas of finance (Cox et al., 1979). The fundamental model for option pricing was created in 1973 when Fischer Black and Myron Scholes created a so-called Black-Scholes model which was extended that same year by Robert Merton in several important ways. The Black-Scholes model was widely recognized as being the first standard model for pricing options. Since its publishment, this model has been a foundation for numerous subsequent academic studies. Even though this model has revolutionized the theory of option pricing it has faced a lot of critique. As stated by Teneng (2011) most of the limitations in the model relate to fundamental aspects of the market and therefore it is important to try to come up with a model that considers these assumptions better than the Black-Scholes model.

The intrinsic value of the option refers to the value the option would have if it was exercised immediately. For call options this means the difference between the current stock price and the strike price, assuming that the stock price is higher than the strike price. Otherwise, the intrinsic value of the option would be 0 and it is said to be "out of the money". For example, if the current stock price is 40 euros and the exercise price is

20 euros, the option has an intrinsic value of 20 euros (in-the-money). For the put option intrinsic value is the difference between the strike price and the current stock price, if the strike price is higher. So the intrinsic value tells the “moneyness” of the option whether it is in-the-money, at-the-money, or out-of-the-money.

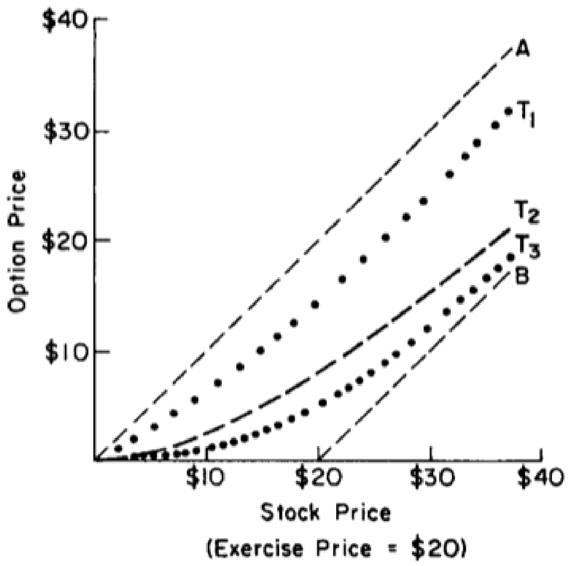

Table 2 The Relation Between Option Value and Stock Price Black and Scholes (1973)

Where:

A = Maximum value of the option

B = Minimum value of the option

T_1 = Longer maturity of the option

T_2 = Medium maturity of the option

T_3 = Shorter maturity of the option

As we can see from the figure when the maturity of the option is the longest, it has the most time to benefit from potential favorable movements in the underlying asset's price. When the maturity is shorter the time value of the option starts to decline, reducing the overall option value. When the maturity is very short the option's value closely aligns with its intrinsic value. For options that are deep in-the-money or out-of-the-money, the impact of time value is negligible.

The extrinsic value of the option, also known as “time value”, is a portion of the options price that is attributable to different factors than the intrinsic value. These factors are prevailing interest rates, the volatility of an underlying asset, or the maturity. Black and Scholes (1973) mention that usually, the value of the options declines as its maturity date gets closer. When maturity is far in the future the extrinsic value of the option is higher because of the greater uncertainty regarding the future movements of the underlying asset’s price.

The volatility of an underlying asset has also a key impact on the option pricing fundamentals. The realized volatility, or historical volatility, of an underlying asset’s price, gives investors an understanding of how sensitive the underlying asset is to price changes. However, it might not predict the future’s volatility accurately. Implied volatility however measures the market’s expectations on how much the price of the underlying asset will fluctuate in the future. In the state of an efficient options market, the implied volatility should be an efficient forecast of future volatility (Christensen & Prabhala, 1998). According to Christensen and Prabhala (1998), the calculation formula for implied volatility is derived from the Black-Scholes model and inferred from the market price of an option.

The spot price of the option represents the real-time equilibrium price determined by supply and demand dynamics in the market. It determines the underlying asset's current value, which directly impacts the option's intrinsic value (Abadie & Chamorro, 2017). The spot price is not tied to an option's payoff condition, according to Abadie & Chamorro (2017), it reflects the value of the underlying asset at a specific moment, independent of any option contracts. The spot revenue refers to the income generated from transactions executed at the spot price in spot markets (Abadie & Chamorro, 2017). This term is more commonly used in commodities options for example. Spot revenue pertains to immediate transactions rather than those executed under long-term or futures contracts. Spot revenue differs from financial derivatives concepts like intrinsic value because it focuses on realized income from current transactions rather than the theoretical or potential value of an option.

5 Black-Scholes Model

The Black-Scholes model has been recognized as the first major advancement in option pricing theory, laying the groundwork for how options are valued today in financial markets. The model also enabled large-scale trading with options in the bourse. The idea behind the model is that with the help of mathematical principles under certain assumptions it tries to capture the relationship between key factors affecting options price. According to (Black & Scholes, 1973) the key behind the theory is that it is possible to create a risk-free portfolio momentarily by owning a single share of stock and a put option. The payoff of this portfolio is the risk-free rate. With this knowledge, the price of the option, based on market arbitrage-free pricing, can be derived.

Black & Scholes (1973) highlighted the key assumptions in their model which apply to both the option and the stock;

A) The short-term interest rate is known and is constant through time.
B) The stock price follows a random walk in continuous time with a variance rate proportional to the square of the stock price. Thus the distribution of possible stock prices at the end of any finite interval is log-normal. The variance rate of the return on the stock is constant.
C) The stock pays no dividends or other distributions.
D) The option is “European”.
E) There are no transaction costs in buying or selling the stock or the option.
F) It is possible to borrow any fraction of the price of a security to buy it or to hold it, at the short-term interest rate.
G) There are no penalties for short selling. A seller who does not own a security will simply accept the price of the security from a buyer and will agree to settle with the buyer on some future date by paying him an amount equal to the price of the security on that date.

One of the main assumptions of the Black-Scholes model is that the price of the underlying stock follows a so-called geometric Brownian motion. The expected return and volatility of the stock are assumed to be constant. This leads to a lognormally distributed stock price at the option expiration date.

Black and Scholes (1973) call these the ideal conditions, however, in the real economy, the assumptions of the model face several challenges. For example, the short-term interest rate may not be constant and may experience sharp movements during the securities' maturity. Dividends may be paid on an underlying asset during the maturity. Also, on almost every trading platform trading fees are charged when buying instruments. Under these "ideal conditions," the value of the option will depend only on the price of the stock and time and on variables that are taken to be known constants (Black & Scholes, 1973).

The Black-Scholes formula provides a closed-form solution for pricing European call options. The equation shows how the price of a call option depends on certain variables such as the current stock price, time of expiration, risk-free interest rate, and volatility. The normal distribution functions represent the probability that the option will end up in-the-money.

The formula for the Black-Scholes model is following:

$$w(x,t) = xN(d_1) - ce^{r(t-t^*)}N(d_2) \quad (3)$$

Equation 3 Option pricing model (Black & Scholes, 1973)

Where:

$x =$ current stock price

$c =$ strike price

r = risk-free interest rate

t^* = time of expiration

$N(d_1)$ / $N(d_2)$ = cumulative normal distribution function

$$d_1 = \frac{\ln\frac{x}{c} + \left(r + \frac{1}{2}v^2\right)(t^* - t)}{v\sqrt{t^* - t}}$$

$$d_2 = \frac{\ln\frac{x}{c} + \left(r - \frac{1}{2}v^2\right)(t^* - t)}{v\sqrt{t^* - t}} \tag{4}$$

Equation 4 Normal distribution functions (Black & Scholes, 1973)**.**

Where:

v = volatility of the stock price, which represents the expected variability in the stock return

5.1.1 Put-Call Parity

The put-call parity is a central theory in the work of Black and Scholes. According to Black & Scholes (1973), it reflects the price interdependence of call- and put options with the same strike price and expiration date. A simplified form of the theory is that the price of a call option plus the present value of the strike price equals the price of a put option plus the current value of the underlying asset.

The formula for put-call parity is following:

$$C + PV(K) = P + S \tag{5}$$

Equation 5 Put-Call parity (Black & Scholes, 1973)

Where:

C = Price of the call-option

P = Price of the put-option

S = Spot price of the underlying asset

$PV(K)$ = Present value of the strike price discounted at the risk-free rate

6 Limitations of Older Models

It is challenging to reflect the complexities of real-world financial markets, where interest rates vary due to macroeconomic changes, monetary policies, and market fluctuations. These shortcomings have been widely recognized, motivating the creation of advanced pricing models. Interest rate fluctuations can significantly affect option pricing, especially for long-term options where the time value of money is critical. During volatile or extreme rate environments, like periods of zero or negative rates, the inadequacy of constant interest rate assumptions is somewhat problematic (Hull & White, 1990). In such scenarios, the Black-Scholes model fails to accurately represent the risk-free discounting process, resulting in option mispricing.

Additionally, the constant rate assumption overlooks potential correlations between interest rates and other market variables, such as stock prices or volatility. Stochastic interest rate models, including the Hull-White and Cox-Ingersoll-Ross (CIR) models, address this issue by treating interest rates as a stochastic process. These models capture mean-reversion behaviors and rate volatility, offering a more realistic approach to option pricing (Cox et al., 1985; Hull & White, 1990). Empirical evidence suggests that stochastic rate models enhance pricing accuracy, particularly in environments where interest rate dynamics heavily influence the term structure of financial instruments (Broadie & Kaya, 2006).

Another significant limitation of the constant interest rate assumption is its inability to capture abrupt changes in monetary policy or shifts in economic regimes. Contemporary approaches, such as regime-switching models (Zhu et al., 2019), address this by allowing interest rates to transition between different states, effectively reflecting changes in market conditions or policy decisions. These models enhance the adaptability and realism of option pricing frameworks, making them better suited to real-world applications.

While the computational simplicity of the Black-Scholes model remains a primary reason for its widespread use, advancements in computational techniques, like Monte Carlo simulations, have mitigated the challenges associated with more complex models. These methods facilitate the integration of stochastic interest rates by simulating multiple rate paths, significantly enhancing the robustness of pricing models in dynamic interest rate environments (Broadie & Kaya, 2006). This progress underscores the feasibility of employing advanced models to account for the complexities of real-world interest rate dynamics.

7 Modern Hedging Strategies with Options

The complexity of financial markets requires the assumptions in modern hedging strategies to go beyond the traditional assumptions of older strategies. Modern advancements in deep learning, algorithms, and sophisticated delta adjustments have opened an endless amount of opportunities for various strategies with different styles of options. These strategies are adaptable and they are tailored to the complexities of contemporary markets.

Marzban et al. (2023) introduced an actor-critic reinforcement learning (ACRL) algorithm specifically tailored for dynamic expectile risk measures. Reinforced learning (RL) is a form of machine learning where the "agent" inspects the environment, can understand it, and performs according to it. Unlike static risk measures, dynamic expectile measures provide consistency in hedging strategies over time, even in unpredictable and incomplete markets. This is made possible through off-policy learning, which efficiently leverages historical data using replay buffers, addressing the computational challenges of traditional dynamic programming. The ACRL algorithm improves the precision of hedging strategies for both vanilla and exotic options, such as basket options, especially in high-dimensional settings where conventional approaches often fall short (Marzban et al., 2023).

The use of deep learning in option pricing and hedging has significantly advanced the field. Na & Wan (2023) introduced a deep recurrent neural network (RNN) framework tailored for high-dimensional American options. Their approach leverages two RNNs: one dedicated to learning continuation prices and another for estimating deltas across the entire spacetime domain. This dual-network structure effectively reduces computational costs and memory usage, overcoming the curse of dimensionality that challenges traditional methods. By incorporating path-wise derivatives, the framework calculates deltas efficiently, supporting accurate and dynamic hedging strategies. According to Na & Wan (2023), their model outperforms conventional regression-based techniques and finite difference methods, particularly in handling complex, multi-asset portfolios. The ability to compute prices and deltas in real time makes this method especially valuable on trading floors, where swift decision-making is essential (Na & Wan, 2023)

Cryptocurrencies form a choice in hedging markets. Alexander & Imeraj (2023) introduce a dynamic delta hedging of Bitcoin options, leveraging smile-implied and smile-adjusted deltas. These adjustments account for the volatility smile, which reflects the market's price-volatility correlation. Smile-adjusted deltas represent a notable improvement over the Black-Scholes delta, especially for assets like Bitcoin that display significant volatility dynamics. By utilizing perpetual contracts as hedging tools, Alexander & Imeraj (2023) demonstrated a reduction in hedging error variance of up to 30% for out-of-the-money puts and an average of 15% for short-term out-of-the-money calls. These findings have substantial practical implications, as perpetual contracts help minimize basis risk and provide traders with a more efficient alternative to standard futures contracts. The study highlights the critical need to customize hedging strategies based on the unique characteristics of specific assets, particularly in emerging markets such as cryptocurrencies.

The impact of these strategies is extensive. By incorporating reinforcement learning and neural networks, strategies can be designed to adapt effectively and operate with computational efficiency (Marzban et al., 2023; Na & Wan, 2023). Smile-adjusted deltas enhance traditional delta hedging, making it possible to manage risks more accurately in assets with unique volatility patterns (Alexander & Imeraj, 2023). These advancements collectively address the shortcomings of classical models, equipping professionals with powerful tools to handle the complexities of today's financial markets. As markets evolve, adopting these innovative methods into standard practices will be essential for ensuring resilience and achieving strong risk management results.

7.1 Modern Pricing Models

Limitations with the constant interest rate assumption have catalyzed the development of advanced option pricing models. These modern frameworks now take into consideration dynamic market conditions by considering stochastic interest rates, volatility, and computational techniques. The following subsections describe some of the key modern models that address these challenges in enhancing accuracy and applicability in option pricing.

7.1.1 Stochastic Interest-rate Models

The stochastic interest-rate models incorporate the dynamic interest rate behavior into the derivative valuation. In these models, the interest rate is treated as a random process that evolves over time. This responds better when aligning pricing methodologies with the reality of fluctuating rates. Particularly when pricing options with long maturity the Stochastic interest-rate models tend to perform better compared to older models like the Black-Scholes model.

The CIR model is a foundational base for stochastic interest-rate models. According to Cox et al. (1979), the model ensures non-negative rates which makes it particularly applicable to pricing short-term interest rates and for them sensitive options. The modern adaptations usually tend to combine the CIR model with a stochastic volatility framework making it possible to understand the combination of fluctuating interest rates and market volatility's effect on the option's price.

Modern models that use mainly computational power can unite complex theories together to create more accurate results in pricing. In one of these models, the negative interest rate is taken into consideration whether it can improve the option pricing and implied volatility forecasting (Recchioni et al., 2017). As known many of the foundational pricing models use constant interest rate assumptions, however after the financial crisis the key regions of the world economy have seen big fluctuations in short-term government bond yields, also negative values have been seen. This raised the question of whether the negative interest rates should be taken into consideration when pricing options. To tackle this problem Recchioni et al. (2017) proposed a few modifications, one being the usage of the stochastic interest rate and that the process could allow for negative values.

By combining the framework of two existing models (Heston-model and CIR-model) Recchioni et al. (2017) account for the simultaneous impact of volatility and interest rate fluctuations. A key feature of the study is the correlation between the stochastic processes, which indicate the correlation between the stock price and the volatility, and also a partial correlation between the stock price, interest rate, and volatility to incorporate their relationships. The model is capable of producing accurate pricing for S&P 500 index options and thus performs better than the models with constant interest rate assumptions.

Using the same foundations like CIR-model and Heston framework Recchioni & Sun (2016) ended up finding similar correlations between the stock price and interest rate as well as a partial correlation between the stock price, volatility, and interest rate in the study. They use a hybrid Heston model which is based on the idea of a stochastic interest rate whereas the older versions of the same model use the volatility factor as an interest rate (Recchioni & Sun, 2016). The hybrid model is being tested on three different occasions, the most remarkable result being it successfully outperforms its predecessor the Heston model in interpreting the call option prices. According to Recchioni & Sun (2016), the two-stage calibration provides evidence that the stochastic interest rate plays a significant role as a volatility factor in option pricing.

7.1.2 Stochastic Volatility Models

Stochastic volatility models have gained prominence for addressing the inadequacies of constant volatility assumptions in traditional option pricing frameworks. These models dynamically account for changing market conditions, capturing the complex interplay between asset prices and their volatility over time. By incorporating stochastic volatility, they aim to reflect observed market phenomena such as volatility clustering and the volatility smile, which are left unexplained by simpler models.

A foundational study for many of the newer stochastic volatility models is the Heston model, which extends the Black-Scholes model by assuming volatility follows a mean-reverting square-root process (Heston, 1993). This model is suitable for tracing stock options' prices under stochastic volatility. As Heston (1993) states the model can cause almost any kind of bias to option prices, but in particular the biases are linked with the dynamics of the spot price and the distribution of spot returns. Also, the model shows that empirical evidence is very strong in the Black-Scholes (1973) case. This is reflected in the fact that most of the stochastic volatility models provide identical option prices for at-the-money options compared to the Black-Scholes model which shows a strong theoretical basis because options are usually traded near the money. Nevertheless Hestons model has proved to be not only good for pricing options but also to solve other option-related problems.

One interesting model created in Heston's groundwork is a stochastic-local volatility model (SLV). By connecting stochastic models and local volatility the model improves pricing and hedging of options (Felpel et al., 2023). The SLV uses advanced numerical techniques to optimize the calibration against the market data, reaching high accuracy in pricing exotic derivatives. According to Felpel et al. (2023) during scenarios of high market turbulence, the SLV model outperforms pure stochastic models or local volatility models in capturing the market phenomena making it a useful tool for professionals in varying volatility regimes. The model aligns with market-implied volatility, demonstrating its accuracy and calibration capabilities.

Implied volatility comparison for a maturity of 0.083 years, demonstrating the SLV-nSABR model's ability to closely replicate market-implied volatilities.

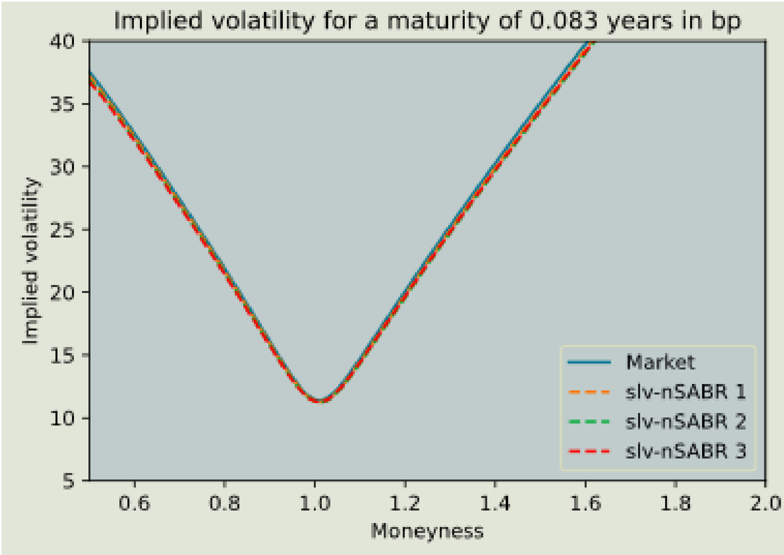

Table 3 Projected variance and implied volatility evaluated for one-month maturity (Felpel et al., 2023)

Another study incorporates realized measures of volatility derived from high-frequency data to enhance the accuracy of option pricing. High-frequency volatility measures are integrated into a stochastic framework, to enhance the option pricing performance across a variety of market conditions (Bormetti et al., 2020). The empirical test is made using European options for the S&P 500 index from approximately 13 years of option price data (Bormetti et al., 2020). According to both Felpel et al. (2023) and Bormetti et al. (2020) these models work very well during highly volatile periods in the economy. Both of the models use similar techniques of volatility clustering and time-varying dynamics.

The figure demonstrates the superior calibration accuracy of stochastic volatility models, particularly SV-LHARG, in replicating market-observed volatility patterns. The in-sample period from, January 1st, 1999 to December 31st, 2008.

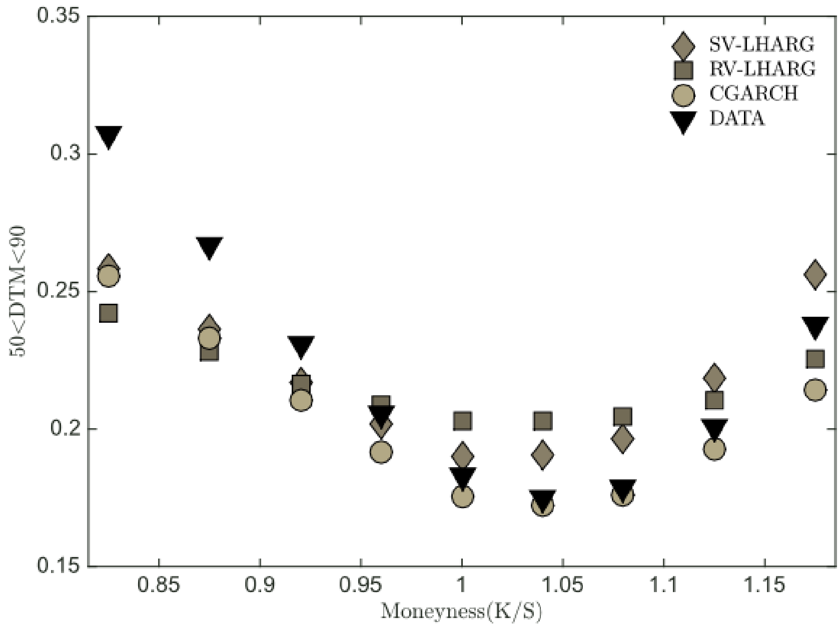

Table 4 Comparison of implied volatility smiles for various models (SV-LHARG, RV-LHARG, CGARCH) against market data across moneyness levels (Bormetti et al., 2020).

The SV-LHARG model demonstrates superior performance in capturing implied volatility for in-the-money options (e.g., K/S < 1). As shown in Table 4, the model's predictions align closely with market data, outperforming RV-LHARG and CGARCH models, which exhibit greater deviations in this region. This accuracy underscores the model's capability to address the volatility clustering and steep gradients characteristic of in-the-money options. This can be very useful for hedging strategies as often precisely in-the-money options are used.

7.1.3 Computational Approaches for Modeling

As computational power and efficiency grow, it will most definitely be the driving force for financial modeling in the future. Already some computational techniques have become indispensable tools in modern option pricing. Using advanced numerical methods and machine learning algorithms the models can create a new framework for more accurate and adaptable pricing frameworks.

One significant advancement in computational option pricing is the use of numerical simulation techniques to solve complex pricing models. Zhang et al. (2018) introduced a time-space fractional option pricing model that incorporates memory effects and spatial dependencies, reflecting the complex behavior of asset prices. Their fast numerical simulation approach, which utilizes optimized algorithms, demonstrates significant improvements in computational speed and accuracy, making it practical for large-scale applications. This model captures market characteristics such as heavy tails and skewed distributions more effectively than integer-order models. The fractional model demonstrates superior performance for in-the-money options by reflecting the convective nature of market jumps. These advancements emphasize the role of computational efficiency in making sophisticated models accessible for real-world financial tasks (Zhang et al., 2018).

Machine learning has also emerged as a powerful tool in modern option pricing. Ivașcu (2021) demonstrated the superiority of deep learning algorithms over traditional models in pricing options under high volatility conditions. These deep learning algorithms explore the use of neural networks (NNs), support vector regression (SVR), and gradient-boosting algorithms like XGBoost and LightGBM (Ivașcu, 2021). These methods excel in capturing the nonlinear, multidimensional relationships inherent in option pricing. Comparative studies demonstrate that machine learning algorithms consistently outperform parametric models like Black-Scholes, particularly for contracts with varying moneyness and maturity. For instance, XGBoost achieves up to 17 times lower prediction error than Black-Scholes with implied volatility (Ivașcu, 2021). The flexibility and adaptability of machine learning models allow them to dynamically adjust to market changes, a capability classical models lack.

Another computational innovation is the use of finite difference methods and Fourier transforms for pricing options under stochastic volatility and jump-diffusion models. Garces & Cheang (2021) developed a numerical framework for pricing exchange options, accommodating scenarios with sudden price jumps and volatility clustering. These methods provide robust and accurate solutions, demonstrating the importance of computational approaches in addressing the limitations of traditional pricing models.

The adoption of fractional processes in option pricing further highlights the versatility of computational methods. Zhang et al. (2018) illustrated the application of fractional dynamics to account for long-range dependence and memory effects in financial markets. These processes, combined with advanced numerical algorithms, offer a more nuanced understanding of price movements and volatility dynamics, leading to improved pricing precision.

The unifying factor across these studies is the emphasis on addressing the shortcomings of classical models by leveraging computational innovations. While stochastic volatility models improve the representation of market dynamics, machine learning, and fractional models bring scalability and robustness to diverse scenarios. These advancements highlight a shift towards hybrid frameworks that combine the strengths of various methodologies to enhance option pricing accuracy.

Computational approaches have transformed option pricing, addressing the limitations of traditional models by incorporating advanced numerical methods, machine learning, and fractional dynamics. These methods not only enhance pricing accuracy but also enable the modeling of complex financial phenomena, ensuring their continued relevance in modern financial markets.

8 Conclusions

This thesis examines the pricing of options in an economic environment where interest rate fluctuations and volatility changes are sudden. It presents the theoretical background required to understand the topic and examines the current literature that explores the subject.

The thesis aims to show that the most commonly used option pricing models in the research framework, such as the Black-Scholes model, fail to accurately model option pricing under challenging economic conditions. The assumptions of the Black-Scholes model, such as constant volatility and interest rates, are rigid and do not reflect the complexity of today's financial markets. This rigidity leads to inaccuracies, especially during periods of market turbulence or unusual interest rate cycles such as negative rates. It is clear that modern models, like those using stochastic volatility and interest-rate frameworks, address these issues by introducing dynamic elements that better reflect real-world conditions. These models provide enhanced pricing accuracy against the older models during such economical times.

By combining the CIR framework with the Heston model, the stochastic interest-rate model can produce accurate pricing for options under periods of fluctuating or even negative rates. The two-stage calibration provides evidence that the stochastic interest rate plays a significant role as a volatility factor in option pricing. By dynamically adjusting to the changing conditions of financial markets the stochastic volatility models can produce more accurate pricing for options. The stochastic-local volatility model (SLV) connects stochastic models and local volatility to improve the pricing and hedging of options. Given its performance, the SLV model seems particularly suited for professionals dealing with high-volatility regimes—a significant step beyond what stochastic models offer. The SV-LHARG model demonstrates superior performance in capturing implied volatility for in-the-money options, which is highly useful for hedging purposes.

Undoubtedly, computational methods play a transformative role in modern option pricing—a finding strongly emphasized throughout this thesis. Techniques such as machine learning algorithms, fast numerical simulations, and fractional calculus offer substantial improvements in precision and scalability. These approaches outperform traditional models, particularly in scenarios with rapid market changes, by capturing the non-linear relationships and memory effects inherent in financial markets. The gradient boosting algorithm XGBoost achieves 17 times lower prediction error than Black-Scholes with implied volatility, while the application of fractional dynamics to account for long-range dependence and memory effects improves the pricing precision of options. The numerical simulation that uses optimized algorithm techniques captures market characteristics such as heavy tails and skewed distributions more effectively than integer-order models.

The practical implications of this thesis are multifaceted. For practitioners in financial markets, this research highlights the need to adopt advanced computational and stochastic models to enhance pricing accuracy and risk management strategies. Investors and traders can leverage these insights to make more informed decisions in volatile environments. Interestingly, the findings emphasize the critical role of adapting pricing models to evolving market conditions, ensuring they remain robust and reliable.

While this thesis provides valuable insights into the interplay between interest rate dynamics, market volatility, and option pricing, it also acknowledges certain limitations. The complexity of financial markets and the diverse range of factors influencing option pricing make isolating the precise effects of interest rates and volatility challenging. Although the findings point to robust evidence of the advantages offered by modern pricing models, the degree of improvement varies based on specific market conditions.

Future research could explore additional dimensions of this topic. For instance, empirical studies could investigate the performance of modern pricing models across different asset classes or during extreme market conditions. Sector-specific responses to interest rate changes and volatility could also provide valuable insights. Such research, requiring detailed data analysis and empirical testing, would be well-suited for advanced academic work, such as a master’s thesis.

References

Abadie, L. M., & Chamorro, J. M. (2017). Valuation of Real Options in Crude Oil Production. *Energies (Basel)*, *10*(8), 1218. https://doi.org/10.3390/en10081218

Alexander, C., & Imeraj, A. (2023). Delta hedging Bitcoin options with a smile. *Quantitative Finance, 23(5),* 799-817. https://doi.org/10.1080/14697688.2023.2181205

Bartram, S. M. (2019). Corporate hedging and speculation with derivatives. *Journal of corporate finance (Amsterdam, Netherlands), 57,* 9-34. https://doi.org/10.1016/j.jcorpfin.2017.09.023

Bartram, S. M., Brown, G. W., & Conrad, J. (2011). The Effects of Derivatives on Firm Risk and Value. *Journal of Financial & Quantitative Analysis*, *46*(4), 967–999. https://doi.org/10.1017/S0022109011000275

Black, F., & Scholes, M. (1973). The Pricing of Options and Corporate Liabilities. *Journal of Political Economy*, *81*(3), 637–654. http://www.jstor.org/stable/1831029

Bliss, B. A., Clark, J. A., & DeLisle, R. J. (2018). Bank risk, financial stress, and bank derivative use. *The journal of futures markets, 38(7),* 804-821. https://doi.org/10.1002/fut.21902

Bormetti, G., Casarin, R., Corsi, F., & Livieri, G. (2020). A Stochastic Volatility Model With Realized Measures for Option Pricing. *Journal of Business & Economic Statistics*, *38*(4), 856–871. https://doi.org/10.1080/07350015.2019.1604371

Broadie, M., & Kaya, O. (2006). Exact Simulation of Stochastic Volatility and Other Affine Jump Diffusion Processes. *Operations Research*, *54*(2), 217–231. https://doi.org/10.1287/opre.1050.0247

Carr, P., & Wu, L. (2004). Time-changed Lévy processes and option pricing. *Journal of Financial Economics*, *71*(1), 113–141. https://doi.org/10.1016/S0304-405X(03)00171-5

Christensen, B. J., & Prabhala, N. R. (1998). The relation between implied and realized volatility. *Journal of Financial Economics*, *50*(2), 125–150. https://doi.org/10.1016/S0304-405X(98)00034-8

Cox, J. C., Ingersoll, J. E., & Ross, S. A. (1985). A Theory of the Term Structure of Interest Rates. *Econometrica*, *53*(2), 385–407. https://doi.org/10.2307/1911242

Cox, J. C., Ross, S. A., & Rubinstein, M. (1979). Option pricing: A simplified approach. *Journal of Financial Economics*, *7*(3), 229–263. https://doi.org/https://doi.org/10.1016/0304-405X(79)90015-1

Fama, E. F. (1970). Efficient Capital Markets: A Review of Theory and Empirical Work. *The Journal of Finance (New York)*, *25*(2), 383. https://doi.org/10.2307/2325486

Fama, E. F., & French, K. R. (1989). Business conditions and expected returns on stocks and bonds. *Journal of Financial Economics*, *25*(1), 23–49. https://doi.org/10.1016/0304-405X(89)90095-0

Felpel, M., Kienitz, J., & McWalter, T. A. (2023). Effective stochastic local volatility models. *Quantitative Finance*, *23*(12), 1731–1750. https://doi.org/10.1080/14697688.2023.2271514

Garces, L. P. D. M., & Cheang, G. H. L. (2021). A numerical approach to pricing exchange options under stochastic volatility and jump-diffusion dynamics. *Quantitative Finance*, *21*(12), 2025–2054. https://doi.org/10.1080/14697688.2021.1926534

Graham, J. R., & Harvey, C. R. (2001). The theory and practice of corporate finance: evidence from the field. *Journal of Financial Economics*, *60*(2), 187–243. https://doi.org/https://doi.org/10.1016/S0304-405X(01)00044-7

Harrison, J. M., & Kreps, D. M. (1979). Martingales and arbitrage in multiperiod securities markets. *Journal of Economic Theory*, *20*(3), 381–408. https://doi.org/10.1016/0022-0531(79)90043-7

Heston, S. L. (1993). A Closed-Form Solution for Options with Stochastic Volatility with Applications to Bond and Currency Options. *The Review of Financial Studies*, *6*(2), 327–343. https://doi.org/10.1093/rfs/6.2.327

Hull, J., & White, A. (1990). Pricing Interest-Rate-Derivative Securities. *The Review of Financial Studies*, *3*(4), 573–592. https://doi.org/10.1093/rfs/3.4.573

Ivașcu, C.-F. (2021). Option pricing using Machine Learning. *Expert Systems with Applications*, *163*, 113799. https://doi.org/10.1016/j.eswa.2020.113799

Marzban, S., Delage, E., & Li, J. Y. (2023). Deep reinforcement learning for option pricing and hedging under dynamic expectile risk measures. *Quantitative* *finance, 23(10),* 1411-1430. https://doi.org/10.1080/14697688.2023.2244531

Merton, R. (1973). The Theory of Rational Option Pricing. *Bell J Econ Manage Sci*, *4*, 141–183. https://doi.org/10.1142/9789812701022_0008

Na, A. S., & Wan, J. W. L. (2023). Efficient pricing and hedging of high-dimensional American options using deep recurrent networks. *Quantitative finance, 23(4),* 631-651. https://doi.org/10.1080/14697688.2023.2167666

Rampini, A. A., Viswanathan, S., & Vuillemey, G. (2020). Risk Management in Financial Institutions. *The Journal of Finance (New York)*, *75*(2), 591–637. https://doi.org/10.1111/jofi.12868

Rao, G. S. (2012). Derivatives in risk management. *International Journal of Advanced Research in Management and Social Sciences*, *1*(4), 55–60.

Recchioni, M. C., & Sun, Y. (2016). An explicitly solvable Heston model with stochastic interest rate. *European Journal of Operational Research*, *249*(1), 359–377. https://doi.org/10.1016/j.ejor.2015.09.035

Recchioni, M. C., Sun, Y., & Tedeschi, G. (2017). Can negative interest rates really affect option pricing? Empirical evidence from an explicitly solvable stochastic volatility model. *Quantitative Finance*, *17*(8), 1257–1275. https://doi.org/10.1080/14697688.2016.1272763

Samuelson, P. A. (1973). Proof That Properly Discounted Present Values of Assets Vibrate Randomly. *Bell Journal of Economics and Management Science*, *4*(2), 369–374. https://doi.org/10.2307/3003046

Sharpe, W. F. (1964). Capital Asset Prices: A Theory of Market Equilibrium under Conditions of Risk. *The Journal of Finance*, *19*(3), 425–442. https://doi.org/10.2307/2977928

Shleifer, A., & Vishny, R. W. (1997). The Limits of Arbitrage. *The Journal of Finance (New York)*, *52*(1), 35–55. https://doi.org/10.1111/j.1540-6261.1997.tb03807.x

Stoll, H. R. (1969). The Relationship between Put and Call Option Prices. *Journal of Finance*, *24*(5), 801–824. http://econpapers.repec.org/article/blajfinan/v_3a24_3ay_3a1969_3ai_3a5_3ap_3a801-24.htm

Teneng, D. (2011). Limitations of the Black-Scholes model. *Collection of Papers*, *1*, 143.

Vo, D. H., Huynh, S. Van, Vo, A. T., & Ha, D. T.-T. (2019). The Importance of the Financial Derivatives Markets to Economic Development in the World's Four Major Economies. *Journal of Risk and Financial Management*, *12*(1). https://doi.org/10.3390/jrfm12010035

Zhang, H., Liu, F., Chen, S., Anh, V., & Chen, J. (2018). Fast numerical simulation of a new time-space fractional option pricing model governing European call option. *Applied Mathematics and Computation*, *339*, 186–198. https://doi.org/10.1016/j.amc.2018.06.030

Zhu, D.-M., Lu, J., Ching, W.-K., & Siu, T.-K. (2019). Option Pricing Under a Stochastic Interest Rate and Volatility Model with Hidden Markovian Regime-Switching. *Computational Economics*, *53*(2), 555–586. https://doi.org/10.1007/s10614-017-9754-9