UNIVERSITY OF VAASA
FACULTY OF BUSINESS STUDIES
DEPARTMENT OF ACCOUNTING AND FINANCE

Hanh Vo

VOLATILITY AND RETURN RELATIONSHIP
A case of Nordic stock market

Master Thesis in Accounting and Finance

Master's Degree Programme in Finance

VAASA 2018

UNIVERSITY OF VAASA

Faculty of Business Studies

Author:	Vo Thi My Hanh
Supervisor:	Anupam Dutta
Topic of the Thesis:	Volatility and return relationship – a case of Nordic stock market
Degree:	Master of Science in Economics and Business Administration
Department:	Accounting and Finance
Master's Program:	Finance
Year of Admission:	2014
Year of Completion of Thesis:	2018 **Pages: 57**

ABSTRACT

This study investigates the relation of risk-return of four Nordic stock market's indices – OMX Copenhagen, OMX Helsinki, OMX Stockholm and Oslo Exchange All Shares from January 1996 to February 2017. EGARCH and GARCH model are used to model conditional volatility. OMX Copenhagen shows a reliable negative risk-return relation through the estimated period in both daily and monthly frequency. There is no positive risk return correlation found in this study. This study indicates that there is strong covariation in risk premium and unexpected volatility in four Nordic stock market's indices. The leverage effect testing shows that the lower return would induce the higher realized volatility change.

Key words: Conditional volatility, Excess return, GARCH, EGARCH.

TABLE OF CONTENTS **page**

LIST OF FIGURES AND TABLES

FIGURES

TABLES

1. INTRODUCTION

Volatility and return are the important concepts in finance. Investors use volatility of stock as one of their important reference to decide their expected return of that stock which affects to its price. Thus, the topic of return and volatility relationship attracts quite a lot of attentions among scientists.

1.1 Purpose and Contribution of the Study

There is significant amount of studies about the relationship of volatility and stock excess returns but those are investigated mainly in American stock market and a few extended to other stock markets like Ang et al (2008) studies in 23 other developed markets or Chuang, Liu, and Susmel (2012) examine in 10 Asian countries. It is necessary to expand the study more to other stock markets. Since the result of risk-return relationship is controversial and research of Nordic stock market about risk-return is rare, this study aims to inspect the relationship of volatility and stock market return in Nordic countries' stock market which includes Finland, Norway, Sweden, Denmark. The investigation proceeds in stock portfolio level using indices in both daily and monthly frequency. Since the and the research about Nordic stock market about risk-return is rare, it is interesting to investigate.

Highlight studies of the relationship between volatility and returns is gathered and discussed in this study. Their methodologies as well as their results also mentioned. The focus of the study to investigate the explanatory power of conditional volatility on excess returns. Furthermore, the volatility is broken down into expected volatility and unexpected volatility. The leverage effect is tested to see whether they exist in Nordic equity market. The aim of this research is to provide additional aspect into the stock market volatility and excess returns for four Nordic stock market's indices.

1.2 Thesis Structure

This dissertation consists six chapters. Chapter one is the introduction part which discusses about the scope of the study as well as its contribution. The following chapter is the literature review. I will go through some important studies about the volatility-return relationship. Both methods and important results are mentioned so that we have a better view about the what have been done and how the methodologies have been improved. In third chapter, the theoretical background is clarified. This part helps to define some basic terms and important theories that are used in this study. The empirical part proceeds in chapter four, five and six. Fourth chapter describes data and methodology used in this study. Chapter five and six present empirical results and thesis' summary respectively.

2. THEORETICAL BACKGROUND

2.1 Volatility definition

It is useful to briefly explain the term of volatility before going further to how to model volatility. Volatility is the degree of discrepancy of all outcomes of an uncertain variable. Statistically volatility of a stock is measured as standard deviation of its returns. Standard deviation is degree of variation of a series moving from its mean value.

Sometimes, variance or natural logarithm of standard deviation are also used to measure volatility. Historical volatility is derived from the time series of historical prices. On the other hand, implied volatility is imbedded in the market price of a market traded options.

2.2 Theories about volatility and return

Volatility and stock return relationship is an appealing topic which always have significant amount of researcher's attention. At first, scientists raised the phenomenon and anomalies they observed in stock market. Through time, these anomaly is demonstrated and tested in different time periods and markets to see whether it is repeated in different time periods and it exists in different markets. This part is going through briefly some popular theories, puzzle, phenomenon that are documented about risk and return.

2.2.1 Risk return trade-off

Risk – return trade-off is one of the principle theory of financial economics. The theory suggests that expected return of the asset should be higher if the investor expose higher risk asset. The potential rise of return is always accompanied with the increase in risk. This is the trade-off that investors face while making investing decision. Investors require a larger risk premium for a higher risk suggest a positive risk return relation. Volatility is one way of quantifying risk.

In line with risk return trade-off theory, French, Schwert and Stambaugh (1987) finds the positive relationship between risk and predictable volatility. Ghysels, Santa-Clara and Valkanov (2005) demonstrate the existence of risk-return trade-off in American stock market. However, Glosten, Jagannathan and Runkle (1993) argues that higher risk premium may not be required for higher risk asset because the investor may want to invest in riskier assets during the less risky period which increases the price of risky asset and lower the risk premium.

Bali and Peng (2006) finds evidence of risk return trade off from high frequency data. They find the significant positive relation between the conditional mean and conditional volatility of market returns at the daily level

2.2.2 Leverage effect and volatility feedback

Originally, leverage refers to how much debt the company use to finance its assets. It is an investment strategy that the company borrow capital to increase the potential return. In stock market, stock price falling leads to the decrease of the firm's equity market value. So that the leverage of firm also rises which increases the risk of the firm. Black (1967) pointed out that changes in stock price is often negative related to the changes in volatility. Stock price fall leads to a decrease of return and an increase of volatility. In other words, leverage effect suggests a negative relationship between stock returns and realized volatility. Yet there are many studies propose the leverage effect has insignificant effect in stock market. Figlewski and Wang (2000) points out that there is the leverage effect however the coefficient is less than a half of -1.0 and it is highly asymmetrical between up and down market. The effect just lasts for a few months. It is interesting that no significant effect on volatility is found when leverage changes because of a change in outstanding debt or shares, only when stock prices change which raise a wonder that whether leverage effect has anything linked to financial leverage. Likewise, Aït-Sahalia, Fan, Li (2013) using high frequency data and finds the nearly zero correlation between the daily returns and daily volatility changes.

The volatility feedback effect proposes the same correlation with leverage effect but reverses the causality. Volatility feedback implies that surge in volatility leads to future negative returns emphasized by Pindyck (1984). It is found that volatility is negative correlated with current and lagged returns and this relationship lasts for several days.

2.2.4 Return reversal

Return reversal is the change of return in the opposite direction of the current trend. It documents the overreaction of investor in short term period which is opposite with momentum anomaly. Short term return reversal effect establishes a trading strategy that investors buy recent losers and sell recent winners with the hope that the trend will be inverse. Jegadeesh (1990) documents profits of about 2% per month between 1934 and 1987 by buying and selling stocks based on their prior-month returns and holds them for one month. Likewise, Da, Liu and Schaumburg (2011) shows this strategy generates the significant excess returns. They suggest that short-term return reversal is persistent and determined by investor sentiment on the short-side and liquidity shocks on the long-side. Additionally, Avramov et al. (2006) find that the standard reversal strategy profits mainly derive from small, high turnover, and illiquid stocks. Huang et al (2007) demonstrate that it is important to include the return reversal effect while study volatility-return relation as the omission of the previous month's stock returns can lead to a negatively biased estimate of the relation.

2.3 LITERATURE REVIEW

There is a significant amount of research about the relationship between idiosyncratic volatility and stock returns. However, a lot of contradict results were pointed out. Besides, the methodologies conducted, and the measures of volatility vary in each study and through time. Until now, the volatility and return true relation is still debatable among researchers. This literature review part goes through some highlight studies about the stock returns and volatility relationship. The target of this part to give the better view of what has been done and their problems which is needed to dig deeper. Furthermore, the innovation of the research methodologies is also mentioned in this part.

2.3.1 Contradiction of volatility-return relationship research results

According to risk return tradeoff theory, the return of an investment is based on how risky of the investment is. The riskier of the asset, the higher return should be rewarded. The theory suggests that the positive relationship should be found between risk and return. There are a lot of empirical literature that has tried to demonstrate this relationship. However, the results are conflicting. Even though studies were conducted in the same stock market, the results came out significantly different. In general, most of the studies find the significant relationship between stock returns and volatility but their results are mixed. Some studies find it negative while others find it positive. Some of those find it not significant. The reason of this contradiction could be because of the proxy used for volatility as is not directly observable. Some researches use lag, expected or conditional volatility which could draw the significant different results of the volatility-return relationship status. Fink et al (2012) already proves that by changing volatility measure can draw a significant different result. Secondly, they are the methodologies to conduct the research as well as the time horizons. Thus, it is critical to examine carefully those factors to see whether it is a real contrast results among studies or it is simple because they use the different definitions and measurements for their researching subjects. The different use of methodology might lead to different results and the economical explanation behind.

Many theoretical asset pricing papers suggest a positive relationship between risk and volatility. French, Schwert and Stambaugh (1987) shows that excess holding period returns is positive related to predictable level of volatility of Standard & Poor composite portfolio and CRSP value weighted portfolio from 1928 to 1984. The strong negative relation between the unpredictable volatility and adjusted returns is also documented in this study. Using the asymmetric GARCH-M Model, Campbell and Hentschel (1992) observe a positive relation between excess return and conditional variance.

Glosten, Jagannathan and Runkle (1993) agree with risk-return theory, however, they point out that the theory is just applied for a given point of time not across time. So that whether investors require a larger risk premium on average for a riskier investment during time remains an open question. The study claims that the standard GARCH-M model assumptions are not reasonable so that they modified the model by relaxing some assumptions of GARCH-M model. It allows the negative and positive unanticipated returns to have different impact on conditional variance as well as allow it to be explained by more variables such as risk-free rate, seasonal variables. The result of standard GARCH-M model comes out insignificantly positive relation but then turning to significantly negative by using the modified GARCH-M model. Furthermore, they find the volatility exhibit seasonality in October and January as well as the conditional volatility non-persistence. Similarly, Nelson (1991) points out that GARCH model has some major drawbacks which make it does not fit to model the conditional volatility. He introduces EGARCH and finds the negative relation between excess returns and conditional volatility.

While Chan, Karolyi, and Stulz (1992) as well as Goyal and Santa-Clara (2003) find no significant statistical relation of market variance and market return in American stock market. However, a link between idiosyncratic equity risk and market returns is recognized in Goyal and Santa-Clara (2003) as they show that one-month lag of average stock variance is positively related to stock market return even when controlling for business cycle.

In other perspective, other authors use different measure of risk such as implied volatility or they inspect different component of volatility. Banerjee, Doran and Peterson (2007) use implied volatility is a measure of market risk. They document a positive relationship between VIX and portfolio future return. Whereas Adrian and Rosenberg (2008) break volatility down into long and short-term components. Each component is measured by first order autoregressive process. They find significant negative relationship between cross-sectional returns and both components of market volatility. Besides, the short-term volatility is suggested as a measure of tightness of financial constraints while long-term component is interpreted as a sign of business cycle risk. However, they find the positive risk-return relationship when investigating total volatility. Furthermore, Chua et al (2010) argues that the reason of the conflicting results in risk return relation is that the use of total volatility as well as using realized return as a proxy for expected return. Thus, he breaks down both returns and volatility in expected and unexpected components. The study suggests the result that component unexpected volatility and expected volatility are positive related to unexpected return an expected return respectively.

In recent studies, idiosyncratic volatility attracts considerable attention from researchers. Starting with a theory proposed by Merton (1987), he said that investors often hold the undiversified portfolio including stocks that they are familiar with according to investor recognition hypothesis. Thus, he builds extension for CAPM model which capture the idiosyncratic risk which investors demand to compensate for. His theory predicts a positive relationship between returns and idiosyncratic risk, but it is lack of empirical result to support the theory. Boehme et al (2009) attempts to support Merton (1987) prediction and find that stocks that have low institutional holding and limited short selling experience the positive relationship between firm specific risk and return. The standard deviation of weekly excess raw returns is use as the proxy for firm specific risk in this study. However, Miller (1977) points out that risk and the divergence of investor opinions go together. The higher of opinion divergence is the steeper of the demand curve is which leads to the higher price of security the investors must pay. It explains why ex post return is lower for riskier security. Nevertheless, Fama and Macbeth (1973) decline the predicting power of idiosyncratic risk in the asset pricing model. Malkiel and Xu (2006) argue that most of investors fail to hold market portfolios such as those several actively managed mutual funds and pension funds and even some index funds, namely Wilshire 5000 or Russell 3000, so they believe that higher idiosyncratic volatility stock should be subject to additional risk premiums. This study replicates the frameworks of Fama and Macbeth (1973) and Fama and French (1992) to demonstrate that idiosyncratic risk is a pricing factor of returns. Furthermore, idiosyncratic risk variable proxied by the idiosyncratic risk hedging portfolio is inserted into CAPM and Fama-French three factor models. They find out that the new factor is statistically significant as well as it enhances the predicting power of those two models.

Furthermore, the study of Ang et all (2006) documents the significantly negative relationship between the aggregate volatility with average returns in American stock market which has gotten many attentions. They create the innovation measure of aggregate market volatility from the change of VIX index and the multi-factor model is used to conduct the study. Their research also studies the cross-sectional relationship between the past idiosyncratic volatility and stock returns in firm level by forming the different quintile portfolios based on idiosyncratic volatility. They find the significantly difference of average returns between the highest idiosyncratic volatility portfolio and the lowest one which is -1.06% per month. The result is averse to many theories earlier which is explained that those past studies failed to directly classify those portfolios based on their idiosyncratic volatility. They also claim that their results are robust when controlling for other firm specific risk such as size, value, liquidity, momentum, volume, dispersion of analyst forecast as well as in different stages of business cycle. In the later research published in 2009, they extend the study in 23 developed markets which also cover Nordic stock market except Norway and come up with similar results especially in G7 countries. It points out that "Stocks with recent past high idiosyncratic volatility have low future average returns around the world".

In the other hand, Huang, Liu, Rhee and Zhang (2007) emphasized that the negative relationship disappears when the past month return is controlled. Maio, P. F. (2013) also finds that three month moving average of the standard deviation has negative correlation with stock returns. Furthermore, Fu (2009) indicates a significantly positive relation between idiosyncratic volatilities and expected returns and proves that Ang et al studies are biased because of return reversal of small stocks. Furthermore, this study finds the first order of idiosyncratic volatility experiences high autocorrelation. Thus, the lag idiosyncratic volatility used in Ang et al (2006) should not be an appropriate proxy of expected idiosyncratic volatility which makes the findings Ang et al (2006) not accurate. Fu even replicates models of Ang et al (2006) and draws the similar results. To avoid the autocorrelation problem, Fu uses the EGARCH (1,1) to estimate the expected idiosyncratic volatility and finds the statistically positive relationship. The zero-investment strategy suggests that high volatility stock earns one percent per month higher than the lower one. Using the same method as Fu (2009), Brockman and Schutte (2007) finds the similar result in international data. Yet, Fink et al (2012) finds no relation between idiosyncratic volatility which is measured from the available information to trader and expected return so that there is no abnormal return could be exploited from this. However, the forward looking idiosyncratic volatility is positive related to stock returns consistent with Fu (2009). They also claim that Fu (2009)'s idiosyncratic volatility measure includes forward looking information into their volatility estimations which might experience the look-ahead bias.

In the scope of this study, the risk-return relation is investigated in index portfolio level of four Nordic stock markets – Finland, Sweden, Norway and Denmark.

2.3.2 The innovation in estimating volatility methodology

One of the crucial points that leads to the mixed results of the volatility and return relation is the method to measure those variables. Fink (2012) shows that the change of volatility measurement might create completely different result. In the early studies, the simplest measure of volatility are the standard deviation and variance which estimate the dispersion of return from its mean value. Haugen and Hines (1975) use standard deviation as a measure of risk and find a significant negative correlation between monthly standard deviation and mean returns. In addition, Boehme et al (2009) uses weekly standard deviation of excess raw return as a proxy of idiosyncratic risk as well as Maio (2013) using moving average of the standard deviation.

Subsequently, there are more sophisticated models are invented to model volatility. One of those is Autoregressive conditional heteroskedasticity (ARCH). It firstly introduced by Engle, Robert F. (1982) to estimate the Variance of United Kingdom Inflation. Afterward, this model is widely used in financial time series modeling that exhibit time-varying volatility clustering such as stock returns. Chua, Goh and Zhang (2007) adopt AR (2) to measure volatility in their study. Variance and mean are estimated jointly in ARCH-in-mean model assuming that investors update their estimates of the mean and conditional variance of returns using the recent news in last period.

Evolving from ARCH, the generalized autoregressive conditional heteroskedasticity model (GARCH) model is a straightforward way to estimate conditional volatility. French, Schwert and Stambaugh (1987) uses GARCH (1,2) to model the market volatility. Additionally, Bollerslev, Engle and Wooldrigdge (1988) use a multivariate

GARCH to model time varying risk premiums. However, the standard GARCH assumes a symmetric response of volatility to returns which is demonstrated is not correct.

To solve that problem, Exponential GARCH (EGARCH) model is introduced to capture the asymmetry of conditional volatility by Nelson (1991) and Engle and Ng (1993). Besides, there are no parameter value constraint to avoid negative variance as GARCH. Pagan and Schwert (1990) find that EGARCH model is the best in overall to model the monthly US stock returns. EGARCH is one of effective way to deal with the occur of asymmetric effect. Adopting EGARCH model, Fu (2009), Spiegel and Wang (2006), Eiling (2006) find the positive relationship of risk and return in firm level in US data. Glosten, Jagannathan and Runkle (1993) using EGARCH to confirm the sign of between mean return and volatility and demonstrate that EGARCH-M is more powerful compared to GARCH-M in modeling volatility.

On the other hand, Ang et all (2006) uses implied volatility (VIX) as a measure of risk. Engle and Mustafa (1992) model implied volatility using the market price of stock options.

3. METHODOLOGY AND DEVELOPMENT OF HYPOTHESIS

This part defines the methodology used in this study. First part describes how to model risk premium and volatility which are main variables. The introduction of ARCH models used to model volatility is mentioned and some tests and reason are clarified why ARCH model is used to model expected volatility. Following part is the development hypothesis and models testing the risk premium and predicted volatility, unpredicted volatility as well as the leverage effect.

3.1 Risk premium

The returns of indices are measured as changes of natural logarithm of daily returns or monthly returns. Interbank rates are used as risk free rates which are converted to one day holding period returns and monthly holding period returns by dividing by 365 and 12 respectively. The models use excess holding period returns which is calculated as the percentage change in index minus the risk-free rate for the correspondent period.

$$R_t = 100 * \ln(\frac{I_t}{I_{t-1}})$$
$$Risk\ premium = R_t - R_{ft}$$

I_t = index price for day t or month t

R_{ft} = risk-free rate for day t or moth t

3.2 Volatility modeling

This part presents how volatility is modelled in this study. It is common the employ ARCH type model to measure the conditional volatility. GARCH and EGARCH is described more in detail and how they are applied to model the Nordic stock index's volatility. Furthermore, two tests of heteroscedasticity and stationary for data sets are mentioned.

3.2.1 GARCH model

Engle (1982) shows how to simultaneously model the mean and the variance of a series with the change of volatility. Autoregressive Conditional Heteroskedasticity (ARCH) models was first introduced by Engle (1982) which used to model and forecast conditional variances. The variance of the dependent variable is modeled as a function of past values of the dependent variable and exogenous variables. Bollerslev (1986) extend Engle's work to generalized ARCH model (GARCH model) which allowed for both autoregressive and moving average components in the heteroskedastic variance. ARCH and GARCH models are widely used to model volatility and stock market returns. The risk premium relies on the expected return and the variance of that return according to asset pricing model. The conditional mean and volatility of stock returns are assumed to be predictable using past available information at a given point in time, such as past returns and past volatility measures in GARCH models. The GARCH (p,q) process is given as below.

$$y_t = \alpha + \beta x_t' + \varepsilon_t$$

$$\varepsilon_t = z_t \sigma_t;\ z_t \sim N(0;1)$$

$$\sigma_t^2 = \omega + \sum_{i=1}^{p} b_i \sigma_{t-i}^2 + \sum_{i=1}^{q} c_i \varepsilon_{t-i}^2$$

where

ω, b_i, $c_i \geq 0$ and $\sum_{i=1}^{p} b_i + \sum_{i=1}^{q} c_i \leq 1$

p = the order of the GARCH terms σ^2

q = the order of ARCH term ε^2

In this study, the GARCH (1,2)-MA(1) is used to estimate the conditional volatility. The mean equation comprises first moving average MA(1) to capture the non-synchronous trading effect following French et al (1987)'s model. σ_{it}^2 is the conditional variance of R_{it} based on the information set up to time t-1

$$R_{it} - R_{fit} = \alpha + \varepsilon_{it} - \theta\varepsilon_{i,t-1}$$
$$\sigma_{it}^2 = a + b\sigma_{i,t-1}^2 + c_1\varepsilon_{t-1}^2 + c_2\varepsilon_{t-2}^2$$

R_{it} = return of index in country i at time t

R_{fit} = risk free rate in country i at time t

σ_{it}^2 = conditional variance in country i at time t

ε_{it} = error term i at time t

3.2.2 Exponential GARCH (EGARCH)

Nelson (1991) pointed out that the negative correlation between current returns and predictable volatility by GARCH model of Black (1976) is biased because of the model assumption as well as the restriction of GARCH model parameters may affect the estimated coefficients restricting the dynamic of conditional variance process. Volatility tends to rise when excess return is higher than expected return and decrease when the excess return is lower than expected return. It means that volatility inclines differently between the negative and positive unexpected volatility. However, the conditional variance from GARCH model is calculated based on the magnitude of the error term but not the sign of unanticipated excess returns. EGARCH is built in the way that conditional variance responds asymmetrically to positive and negative residuals. In EGARCH model, the log of variance is computed instead of variance which make sure the variance is always positive regardless the non-positive coefficients. It helps to remove the restrictions of GARCH model. The specification of EGARCH (p,q) is as below:

$$y_t = \alpha + \beta x_t' + \varepsilon_t$$
$$\log\sigma_t^2 = a + \sum_{i=1}^{p} b_i log\sigma_{t-i}^2 + \sum_{i=1}^{q} c_i g(\frac{\varepsilon_{t-i}}{\sigma_{t-i}})$$
$$g(\frac{\varepsilon_t}{\sigma_t}) = \gamma_1\frac{\varepsilon_t}{\sigma_t} + \gamma_2(\left|\frac{\varepsilon_t}{\sigma_t}\right| - E\left(\left|\frac{\varepsilon_t}{\sigma_t}\right|\right);\ \varepsilon_t \sim N(0;1)$$

where

p = the order of the GARCH terms σ^2

q = the order of ARCH term ε^2

EGARCH (1,2)-MA(1) is used in the study.

$$R_{it} - R_{fit} = \alpha + \varepsilon_{it} - \theta\varepsilon_{i,t-1}$$

$$\log \sigma_t^2 = a + b_1 log\sigma_{t-1}^2 + c_1 \left|\frac{\varepsilon_{t-1}}{\sigma_{t-1}}\right| + c_2 \left|\frac{\varepsilon_{t-2}}{\sigma_{t-2}}\right| + c_3 \frac{\varepsilon_{t-1}}{\sqrt{\sigma_{t-1}^2}} + c_4 \frac{\varepsilon_{t-2}}{\sqrt{\sigma_{t-2}^2}}$$

R_{it} = return of index in country i at time t

R_{fit} = risk free rate in country i at time t

σ_{it}^2 = conditional variance in country i at time t

ε_{it} = error term

3.2.3 Heteroscedasticity - ARCH effect test

A time series has autoregressive conditional heteroscedastic effects (ARCH effect) if it includes conditional heteroscedasticity or autocorrelation in its squared residual term. Before applying any ARCH type models, it is suggested to examine the residuals for the evidence of heteroscedasticity to make sure that this type of model is appropriate for the data used in the study. The Lagrange Multiplier (LM) test for ARCH effects proposed by Engle (1982) is applied to test the presence of heteroscedasticity in residual of index's excess return. First, the residuals are obtained from the ordinary least squares regression of the conditional mean equation as below where c is the constant and ε_t is the residual term at time t.

$$R_{it} - R_{fit} = c + \varepsilon_{it}$$

Then the regression of the squared error term is run against the constant β_0 and three lagged squared residuals

$$\varepsilon_t^2 = \beta_0 + \beta_1\varepsilon_{t-1}^2 + \beta_2\varepsilon_{t-2}^2 + \beta_3\varepsilon_{t-3}^2 + v_t$$

The null hypothesis is that there is no ARCH effect.

$$H_0: \beta_1 = \beta_2 = \beta_3 = 0$$

The ARCH LM test statistic is computed as the number of observations times the R-squared statistic from the regression according to Engle (1982). If the LM test statistic is greater than the Chi-square table value, we reject the null hypothesis and conclude there is an ARCH effect in the model otherwise we do not reject the null hypothesis.

3.2.4 Stationary - Unit root test

Stationary is an assumption underlying many probability theories so that it is necessary to check whether the estimated time series is stationary. If the series is non-stationary, it is impossible to obtain meaningful sample statistics such as means, variances, and correlations with other variables which are useful information of future behavior.

Stationary process is a stochastic process whose joint probability distribution does not change when shifted in time which means that mean, variance, autocorrelation of the series should be constant over time. A non-stationary series usually contain trend in mean. In other word, there is the presence of unit root which makes the series to have no tendency to return to long-run deterministic path as well as the variance of the series is time dependent. Unit root test provides a simple method for testing whether a series is non-stationary. This study employs Augmented Dickey-Fuller unit root (ADF) test proposed by Dickey and Fuller (1979) and KPSS test by Kwaitkowski, Phillips, Schmidt and Shin (1992) to verify the stationarity.

The ADF test which includes the drift term and the lagged changes is showed below. The dependent variable y is the excess return of stock index. The null hypothesis is that the series has a unit root.

$$\Delta y_t = a_0 + \gamma y_{t-1} + \sum_{i=2}^{p} \beta_i \Delta y_{t-i+1} + \varepsilon_t$$

KPSS test is used to complement the ADF test. Kwaitkowski, Phillips, Schmidt and Shin (1992) propose the test of the null hypothesis that an observable series is stationary around a deterministic trend. This test uses the Lagrange multiplier test of the hypothesis that the random walk has zero variance. The KPSS statistic is calculated from the residuals of the OLS regression.

3.3 Risk premiums and volatility relation

Merton (1980) raised the hypothesis of market risk premium and volatility relation. He predicted a positive relation without any empirical evidence. In his hypothesis, he used contemporaneous rather than predictable measure of volatility. Later, French et al (1986) clarified this hypothesis, on the other hand, using ex ant measures of volatility. They also included both ex ant volatility and the unexpected change of volatility in their study. They do not find any significant relationship with predicted volatility. However, there is significant negative relationship between risk premium and unpredicted component of volatility.

In this study, the similar methodology of French et al (1986) is applied for Nordic stock market's indices. The expected risk premium is regressed on predictable standard deviation and variance which is measured from GARCH(1,2)-MA(1) and EGARCH(1,2)-MA(1). A Weighted least squares regression of the excess monthly returns of indices is ran against the constant a and conditional volatility extracting from ARCH type process. Predicted standard deviation $\hat{\sigma}_{it}$ is used to standardize each observation. The relationship is tested in two forms of volatility i.e. standard deviation, variance.

$$R_{it} - R_{fit} = \alpha + \beta\hat{\sigma}_{it} + \varepsilon_{it}$$
$$R_{it} - R_{fit} = \alpha + \beta\hat{\sigma}_{it}^2 + \varepsilon_{it}$$

R_{it} = return on a stock market portfolio at time t in country i

R_{fit} = risk free rate at time t in country i

$\hat{\sigma}_{it}$; $\hat{\sigma}_{it}^2$ = conditional standard variance, variance at time t in country i

ε_{it} = error term

Alternatively, the GARH-in-mean model (GARCH-M) is utilized to estimate the volatility-return relation directly in Nordic stock market. Engle and Woldridge (1985) propose GARCH-in-means which links the conditional variance to the conditional mean of return to study the relationship between market risk and expected return. Later, French, Schewet and Stambaugh (1987) use GARCH-M to estimate the ex-ante relation between the conditional mean and conditional volatility. The advantage of this model is that it predicts the variance and estimate the relation of variance and return at the same time. To be precise, this study employs GARCH-M (1,2)-MA(1) and EGARCH-M(1,2)-MA(1) to estimate correlation between conditional variances, standard deviation and risk premium of Nordic stock indices. The conditional standard deviation and variance are respectively included in the mean equation. The result of ARCH in mean model is expected like earlier models as the methodology is practically the same. The GARCH in mean and EGARCH in mean process is specified by the following equations.

GARCH-M (1,2)-MA(1) process

$$R_{it} - R_{fit} = \alpha + \beta\sigma_{it} + \varepsilon_{it} - \theta\varepsilon_{i,t-1} \qquad (1a)$$

$$R_{it} - R_{fit} = \alpha + \beta\sigma_{it}^2 + \varepsilon_{it} - \theta\varepsilon_{i,t-1} \qquad (1b)$$

$$\sigma_{it}^2 = a + b\sigma_{i,t-1}^2 + c_1\varepsilon_{t-1}^2 + c_2\varepsilon_{t-2}^2 \qquad (2)$$

EGARCH-M(1,2)-MA(1)

$$R_{it} - R_{fit} = \alpha + \beta\sigma_{it} + \varepsilon_{it} - \theta\varepsilon_{i,t-1}$$

$$R_{it} - R_{fit} = \alpha + \beta\sigma_{it}^2 + \varepsilon_{it} - \theta\varepsilon_{i,t-1}$$

$$\log\sigma_t^2 = a + b_1 log\sigma_{t-1}^2 + c_1\left|\frac{\varepsilon_{t-1}}{\sigma_{t-1}}\right| + c_2\left|\frac{\varepsilon_{t-2}}{\sigma_{t-2}}\right| + c_3\frac{\varepsilon_{t-1}}{\sqrt{\sigma_{t-1}^2}} + c_4\frac{\varepsilon_{t-2}}{\sqrt{\sigma_{t-2}^2}}$$

R_{it} = return of index in country i at time t

R_{fit} = risk free rate in country i at time t

σ_{it}^2 = conditional variance in country i at time t

ε_{it} = error term i at time t

If the correlation β is greater than 0 and statistically significant, conditional volatility is positively correlate to predict excess stock returns. If β is significantly less than 0, it is said that there is a negative relationship between risk premium and predictable volatility. If β is equal to 0, it means that there is no relation between the predicted volatility and risk premium in Nordic stock market.

3.4 Unexpected volatility

In this part, the volatility is broken down into two components which are expected volatility and unexpected volatility. The monthly historical volatility is calculated from the daily historical returns only within the month which makes the volatility estimate is more precise and non-overlapping sample of returns compared to rolling twelve-month estimators (French et al, 1986). To avoid the autocorrelation, monthly variance is estimated as a sum of the squared daily excess holding period returns plus twice the sum of the products of adjacent excess holding period returns.

$$\sigma_{mt}^2 = \sum_{i=1}^{N_t} R_{it}^2 + 2\sum_{i=1}^{N_t-1} R_{it}R_{i+1,t}$$

R_{it} = daily holding period return in month t

N_t = number of trading days within month t

The predictable component is forecasted using the fitted value from GARCH (1,2)-MA (1) model. The GARCH model is run for historical monthly holding period returns against the constant α and the first order of moving average.

$$R_{it} = \alpha + \varepsilon_{it} - \theta\varepsilon_{i,t-1} \qquad (3)$$
$$\sigma_{it}^2 = a + b\sigma_{i,t-1}^2 + c_1\varepsilon_{t-1}^2 + c_2\varepsilon_{t-2}^2 \quad (4)$$

The unexpected variance (standard deviation) is calculated from historical value minus the predictable variance (standard deviation) from GARCH process $\hat{\sigma}_{it}^{pu}$=$\sigma_{it}^{p} - \hat{\sigma}_{it}^{p}$. The monthly excess holding period return is regressed on expected and unexpected components. The model clarifies deeper the relationship of return and volatility. There are two separate regressions for variance and standard deviation.

$$\left(R_{it} - R_{fit}\right) = \alpha + \beta\hat{\sigma}_{it}^{p} + \gamma\sigma_{it}^{pu} + \varepsilon_{it} \qquad (6)$$

$\hat{\sigma}_{it}^{p}$ = predictable volatility of stock index or the fitted value from GARCH process
$\hat{\sigma}_{it}^{pu}$ = unpredictable part of volatility or the prediction error for the volatility of stock returns

p=1 is standard deviation's regressions.

$$\left(R_{it} - R_{fit}\right) = \alpha + \beta\hat{\sigma}_{it} + \gamma\sigma_{it}^{u} + \varepsilon_{it} \qquad (6a)$$

p=2 is variance's regressions.

$$\left(R_{it} - R_{fit}\right) = \alpha + \beta\hat{\sigma}_{it}^{2} + \gamma\sigma_{it}^{2u} + \varepsilon_{it} \qquad (6b)$$

The regressions are estimated using weighted least squares (WLS) where the predicted standard deviation of the Nordic stock market's indices $\hat{\sigma}_{it}$ is used to standardize each observation. Bollerslev-Wooldridge (1992) is applied to robust standard errors & covariance.

3.4 The leverage effect with returns

In this section, the leverage effect is analyzed in term of the relationship between the return on stock market index and historical volatility. The test is applied for four stock market indices – OMX Copenhagen, OMX Helsinki, OMX Stockholm and Oslo Exchange All Shares. This study does the simple regression replicating the one of French, Schewert and Stambaugh (1987). The percentage change in historical standard deviation of Nordic stock market' indices are regressed against the continuously compounded return of Nordic stock market's indices.

$$\ln(\sigma_{mt}/\sigma_{mt-1}) = \alpha_0 + \beta_0 \ln(1 + R_{mt}/100) + \varepsilon_t \quad (7)$$

σ_{mt} = estimated standard deviation at month t and the previous month.

σ_{mt-1} = estimated standard deviation at month t-1.

R_{mt} = the estimated return on market index at month t.

ε_{it} = error term i at time t

If the stock market volatility is constant and the absolute value of coefficient β_0 equal to 1, it means that the change of index's volatility is fully explained by the change of stock index. If the value less than 1, it means that the change of stock index is not fully incorporate to volatility change. β_0 less than 0 suggests that the volatility rises when the returns go down. The elasticity between the proportion change in standard deviation on the proportion change in stock price should be between 0 and -1.0. Black (1976) finds the estimated elasticity which is reliably less than -1.0 using the sample of thirty stocks. Similarly, French et al (1986) finds elasticity of -1.89 (statically significant) for the S&P composite portfolio from 1953 to 1984 supporting the conclusion that leverage is not the sole explanation for the negative relation between stock returns and unexpected volatility.

4. DATA & DESCRIPTIVE STATISTIC

4.1 Data

The stock market indices of Finland, Sweden, Denmark and Norway are used to conduct the study. OMX Helsinki, OMX Stockholm, OMX Copenhagen and Oslo Exchange All Share index are denoted for Finland, Sweden, Denmark and Norway respectively. The monthly interbank rate is used for the corresponding risk-free rate in each country. Stock index and interbank rate are downloaded from Data stream. The study covers the period from 1 January 1996 to 28 February 2017. Table 1 explains the data is used in this study.

Table 1. *Data used in the study*

Country	**Stock index**	**Interbank rate**
Finland	OMX Helsinki (OMXH)	FIN1M
Sweden	OMX Stockholm (OMXS)	SIBOR1M
Denmark	OMX Copenhagen (OMXC)	CIBOR1M
Norway	OSLO Exchange All Share (OEAS)	NOR1M

Figure 1 illustrates the monthly logarithmic excess holding period returns of four Nordic stocks indices. It is seen that there was extreme volatility around the Financial crisis 2007 as well as the Dot com buble of 1997-2001 in Nordic stock markets. Furthermore, figure 2 plots the monthly realized standard deviation of OMX Copenhagen, OMX Helsinki, OMX Stockholm and Oslo Exchange All Share. Each monthly standard deviation is calculated based on its returns within the month only.

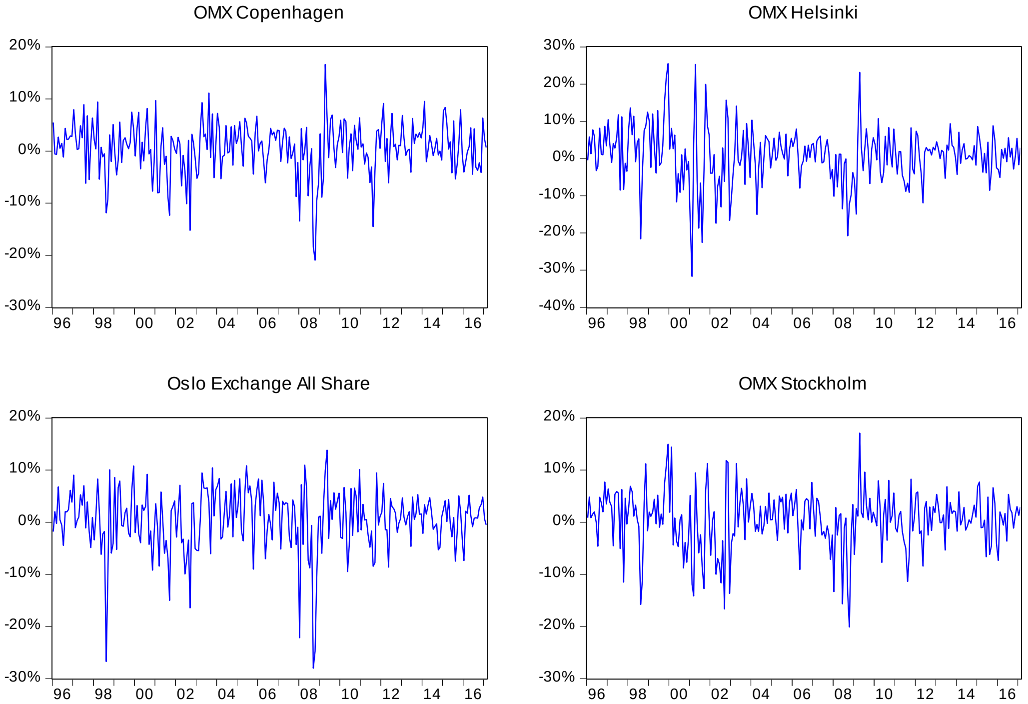

Figure 1. *Monthly logarithmic excess holding period return from Jan 1996 to Feb 2017 of Nordic stock indices.*

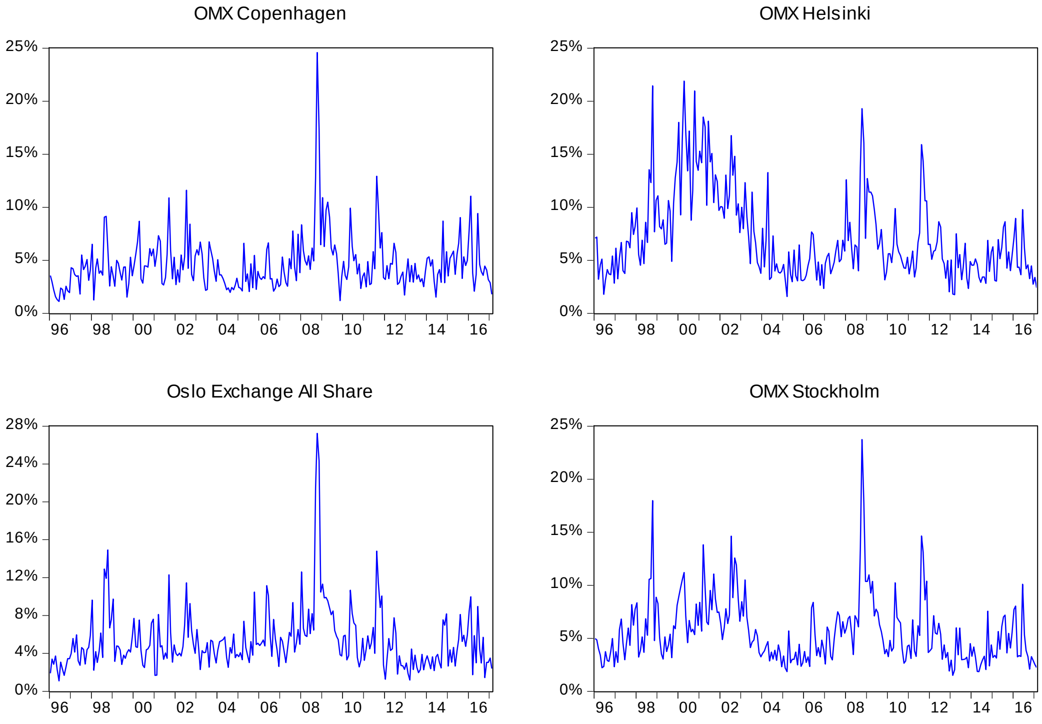

Figure 2. *Monthly Realized standard deviation of Nordic stock index's returns. σ_{mt} is the monthly standard deviation of month t estimated from daily returns Rit within the month t from Jan 1996 to Feb 2017*

$$\sigma_{mt}^2 = \sum_{i=1}^{N_t} R_{it}^2 + 2\sum_{i=1}^{N_t-1} R_{it}R_{i+1,t}$$

4.2 Descriptive Statistic

Table 2 reports the summary descriptive statistics of index daily and monthly logarithmic returns from January 1996 to February 2017. The studied period includes 5522 observations of daily returns per market as well as 253 observations of monthly returns per market. The statistic of daily market return and monthly market returns are exhibited in panel A and panel B respectively. In panel A, it is showed that the average daily return around 0.031% per day for OMX Stockholm index, 0.036% per day for OMX Copenhagen index and Oslo Exchange All Share index. OMX Helsinki index has the highest average daily return with 0.043% per day, following the highest daily standard deviation 1.76%. Similarly, panel B also provides that OMX Helsinki has the highest average monthly return 0.946%, following by OMX Copenhagen and Oslo Exchange All Share index. Four markets display a negative skewness and high kurtosis in both daily and monthly return series. Negative skewness indicates that large negative returns happen more frequently than large positive returns. OMX Stockholm index’s return has the lowest skewness in daily frequency at -0.02% per day while Oslo Exchange All Share shows the highest skewness of -0,6%. In monthly frequency, Oslo index also experiences the large skewness of -1.33% per month. All the examined returns series appear to have kurtosis exceeding 3 with heavy tails which make the return distribution non-normal. In addition, the kurtosis appears smaller in monthly frequency compared to the them in monthly frequency in all four markets.

Table 2. *Descriptive Statistics of index returns from Jan 1996 to Feb 2017*

Country	**Denmark**	**Finland**	**Norway**	**Sweden**
Panel A: Daily market returns (%)				
Stock index	OMX Copenhagen	OMX Helsinki	Oslo Exchange All Share	OMX Stockholm
Mean	0.0364	0.0435	0.0363	0.0311
Median	0.0439	0.0423	0.0581	0.0372
Maximum	8.2013	14.5631	9.1864	9.8834
Minimum	-10.5826	-17.1718	-9.7088	-8.0720
Standard deviation	1.0777	1.7578	1.3317	1.3604
Skewness	-0.41098	-0.23333	-0.60282	-0.02950
Kurtosis	8.77885	9.58462	9.40947	7.42890
Jarque-Bera	7839	10026	9787	4514
Observations	5522	5522	5522	5522
Panel B: Monthly market returns (%)				
Stock index	OMX Copenhagen	OMX Helsinki	Oslo Exchange All Share	OMX Stockholm
Mean	0.7906	0.9461	0.7885	0.6764
Median	1.1953	1.4481	1.3698	1.2845
Maximum	16.8445	25.7507	14.0154	17.1685
Minimum	-20.4345	-31.3043	-27.3572	-19.7052
Standard deviation	4.9843	7.5536	5.9175	5.5876
Skewness	-0.8061	-0.3107	-1.3331	-0.5802
Kurtosis	5.1221	5.3562	7.3187	4.3512
Jarque-Bera	75	63	273	34
Observations	254	254	254	254

Table 3 illustrates the summary descriptive statistic of adjusted index's returns between January 1996 to February 2017. The excess holding period returns are obtained from the logarithmic holding period returns of index minus the corresponding risk-free returns. The statistic figures come out quite similar to holding period returns in table 2. The average of daily excess returns is raging from 0.024% to 0.038% while monthly excess return ranging from 0.462% to 0.769%. It is captured the negative skewness as well as high kurtosis in each studied market in both daily and monthly frequency.

Table 3. *Descriptive Statistic of adjusted index returns from Jan 1996 to Feb 2017*

Country	**Denmark**	**Finland**	**Norway**	**Sweden**
Panel A: Daily adjusted market returns (%)				
Stock index	OMX Copenhagen	OMX Helsinki	Oslo Exchange All Share	OMX Stockholm
Mean	0.0297	0.0377	0.0261	0.0241
Median	0.0362	0.0352	0.0471	0.0312
Maximum	8.1848	14.5497	9.1753	9.8723
Minimum	-10.5991	-17.1840	-9.7252	-8.0795
Standard deviation	1.0779	1.7579	1.3319	1.3606
Skewness	-0.4138	-0.2374	-0.6062	-0.0323
Kurtosis	8.7821	9.5843	9.4118	7.4253
Jarque-Bera	7850	10027	9797	4507
Observations	5522	5522	5522	5522
Panel B: Monthly adjusted market returns (%)				
Stock index	OMX Copenhagen	OMX Helsinki	Oslo Exchange All Share	OMX Stockholm
Mean	0.5878	0.7693	0.4787	0.4624
Median	1.0803	1.1760	1.0700	0.9690
Maximum	16.6427	25.4897	13.8446	17.1081
Minimum	-20.9366	-31.6995	-28.0005	-20.0894
Std. Dev.	5.0116	7.5669	5.9611	5.6075
Skewness	-0.8312	-0.3405	-1.3667	-0.6083
Kurtosis	5.1896	5.3756	7.4079	4.3888
Jarque-Bera	80	65	285	36
Observations	254	254	254	254

4.3 Heteroscedasticity – ARCH test

As discussed above, ARCH test is recommended before applying any ARCH type model to test the presence of heteroscedasticity in the data sets. In this study, ARCH type models such as GARCH and GARCH-M are applied for monthly index returns, monthly and daily excess returns. So, it is necessary to test the present of heteroscedasticity for those data sets. The test result for ARCH test is presented in table 4. LM Statistics and F-statistics for joint hypothesizes whose p-values are less than 0,1% are statically significant in all estimated data sets of each market. Thus, the null hypothesizes are declined. There is an ARCH effect in all data sets. The assumption of homoscedasticity is inappropriate. The ARCH type model could be applied for monthly index returns, monthly and daily excess returns.

Table 4. *ARCH test*

Ordinary least squares regression of the returns of indices (adjusted return of indices) against the constant c and the residual term ε_t at time t. Then the regression of the squared error term is run against the constant β_0 and three lagged squared residuals

Adjusted return $R_{it} - R_{fit} = c + \varepsilon_{it}$

Return $R_{it} = c + \varepsilon_{it}$

$$\varepsilon_t^2 = \beta_0 + \beta_1\varepsilon_{t-1}^2 + \beta_2\varepsilon_{t-2}^2 + \beta_3\varepsilon_{t-3}^2 + v_t$$

Monthly index returns				
	LM-Statistic	Prob. Chi-Square(3)	F-statistic	Prob. F(3,247)
OMX Copenhagen	20,1617	0,0002	7,1911	0,0001
OMX Helsinki	30,4960	0,0000	11,3868	0,0000
OMX Stockholm	16,0386	0,0003	6,8044	0,0002
Oslo Exchange All Share	19,1602	0,0011	5,6201	0,0010
	β_0	β_1	β_2	β_3
OMX Copenhagen	18,5411	0,2827	0,0060	-0,0243
t-statistic	4,8076	4,4443	0,0901	-0,3816
OMX Helsinki	34,5839	0,0979	0,3266	-0,0252
t-statistic	3,9518	1,5392	5,4033	-0,3958
OMX Stockholm	25,4982	0,2794	-0,0326	0,0408
t-statistic	3,9968	4,3945	-0,4946	0,6421
Oslo Exchange All Share	20,1752	0,1690	0,0385	0,1544
t-statistic	4,3676	2,6890	0,6041	2,4555
Montly adjusted retruns				
	LM-Statistic	Prob. Chi-Square(3)	F-statistic	Prob. F(3,247)
OMX Copenhagen	20,1617	0,0002	7,1911	0,0001
OMX Helsinki	30,4960	0,0000	11,3868	0,0000
OMX Stockholm	16,0386	0,0011	5,6201	0,0010
Oslo Exchange All Share	19,1602	0,0003	6,8044	0,0002
	β_0	β_1	β_2	β_3
OMX Copenhagen	18,5411	0,2827	0,0060	-0,0243
t-statistic	4,8076	4,4443	0,0901	-0,3816
OMX Helsinki	34,5839	0,0979	0,3266	-0,0252
t-statistic	3,9518	1,5392	5,4033	-0,3958
OMX Stockholm	20,1752	0,1690	0,0385	0,1544
t-statistic	4,3676	2,6890	0,6041	2,4555

Oslo Exchange All Share	25,4982	0,2794	-0,0326	0,0408
t-statistic	3,9968	4,3945	-0,4946	0,6421
Daily adjusted returns				
	LM-Statistic	Prob. Chi-Square(3)	F-statistic	Prob. F(3,5515)
OMX Copenhagen	825,4763	0,0002	7,1911	0,0001
OMX Helsinki	278,5537	0,0000	11,3868	0,0000
OMX Stockholm	543,3388	0,0011	5,6201	0,0010
Oslo Exchange All Share	933,4552	0,0003	6,8044	0,0002
	β_0	β_1	β_2	β_3
OMX Copenhagen	18,5411	0,2827	0,0060	-0,0243
t-statistic	4,8076	4,4443	0,0901	-0,3816
OMX Helsinki	34,5839	0,0979	0,3266	-0,0252
t-statistic	3,9518	1,5392	5,4033	-0,3958
OMX Stockholm	20,1752	0,1690	0,0385	0,1544
t-statistic	4,3676	2,6890	0,6041	2,4555
Oslo Exchange All Share	25,4982	0,2794	-0,0326	0,0408
t-statistic	3,9968	4,3945	-0,4946	0,6421

4.4 Stationary - Unit root test

ADF test and KPSS test are carried out to verify the unit root and stationarity of monthly index's return, excess monthly and daily index's returns series for each market – Denmark, Finland, Sweden and Norway. Table 5 reports results of the unit-root analysis. The ADF t-statistics are lower -12 for all data sets significantly exceeding the test critical value of -3,4561 for 1% significance level which suggests that the null hypothesis is rejected. Thus, the monthly index return, adjusted monthly and daily index's returns series do not exhibit unit root.

The KPSS test results provide consistent findings with those of the ADF that null hypotheses that the data set is stationary cannot be declined. The findings of the ADF and KPSS tests indicate that monthly index's return, adjusted monthly and daily index's return series are stationary and do not have unit root.

Table 5. *Stationary and Unit root test*
Stationary and Unit root test for Monthly index's returns and Excess monthly and daily index's returns from 1 January 1996 to 28 February 2017. The test critical value of ADF test is from MacKinnon (1996) one-sided p-values and KPSS test is from Kwiatkowski-Phillips-Schmidt-Shin (1992)

Monthly index return	**ADF test t-Statistic**	**KPSS test LM-Statistic**
OMX Copenhagen	-13,5932	0,0666
OMX Helsinki	-12,4641	0,2424
OMX Stockholm	-13,6818	0,0859
Oslo Exchange All Share	-12,8586	0,0482
Excess monthly index's return		
OMX Copenhagen	-13,4689	0,0647
OMX Helsinki	-12,4309	0,2072
OMX Stockholm	-13,6002	0,0724
Oslo Exchange All Share	-12,7231	0,0492
Excess daily index's return		
OMX Copenhagen	-70,0332	0,0979
OMX Helsinki	-73,3066	0,2495
Oslo Exchange All Share	-73,8798	0,1050
OMX Stockholm	-73,4385	0,0663
Test critical values		
for ADF test	1% level	-3,4561
	5% level	-2,8728
	10% level	-2,5728
for KPSS test	1% level	0,7390
	5% level	0,4630
	10% level	0,3470

5. EMPIRICAL RESULTS

This part discusses the results of the studied models providing the empirical conclusion about the relationship of volatility and returns of Nordic stock's indices. It includes three parts. First part presents the empirical results about the relation between excess return and conditional volatility extracting from GARCH and EGARCH process in daily and monthly frequency. Besides, the results from GARCH-M and EGARCH-M model where the volatility is included in the mean equations are provided. Third part shows how the monthly unexpected volatility component affect the excess returns. Following, the quick test result about the presence of leverage effect is discovered in Denmark, Finland, Sweden and Norway stock's indices.

5.1 Risk premium and return relation

5.1.1 Weighted Least Square regressions

The one step ahead forecast monthly and daily variance is computed from the established GARCH and EGARCH process showed in table A1, A2, A3, A4 respectively in the appendix. From the established model, the conditional variance series used as the input of later regression are extracted as the fitted value of the regression. Later, the Weighted Least Squares regressions of the excess monthly returns of indices are run against the constant and one step ahead forecast volatility from GARCH and EGARCH process to estimate the risk-return relation of Nordic stock indices. Conditional standard deviation $\widehat{\sigma}_{it}$ series is used to standardize each observation.

Table 6 describes the result of WLS regressions in daily frequency. Negative correlation is seen in all estimated regressions. However, not all of relationship are statistically significant. In general, R^2 and t-statistics grow in EGARCH predicted conditional volatility compared to those of GARCH predicted model. OMX Copenhagen shows the statistically strong negative relationship at 1% significance. The intercept β is -0.12 for GARCH standard deviation and -0.05 for GARCH variance of OMX Copenhagen. It means that when investor expected standard deviation, variance of OMX Copenhagen changes 1% will lead to the change of excess return -0.12% and -0.05% respectively.

EGARCH volatility of OMX Copenhagen show the similar result that there is a reliable negative relationship between expected volatility and return. The R^2 of OMX Copenhagen is also highest compared the R^2 of other index's regressions. Besides, daily EGARCH conditional standard deviations and variance of Oslo Exchange All Share show statistically negative correlation with excess returns at 5% significance. OMX Helsinki's daily EGARCH conditional standard deviation has 10% statistically significantly negative correlation with returns.

The monthly frequency's results are presented in table 7. It is seen that there is negative relation between monthly excess holding period return and predictable volatility for all estimated stock market indices. Yet, only OMX Copenhagen correlation coefficients are statistical significant at 1% confidence level in both conditional GARCH and EGARCH volatility's regressions. EGARCH conditional standard deviation and variance of OMX Stockholm shows the statistically significant relationship with monthly excess return at 5% level of significance. The correlation coefficients are -0.57 and -0.05 for conditional standard deviation and variance respectively.

Table 6. Weighted least squares regression of the excess daily returns of indices from January 1996 to February 2017

Weighted least squares regression of the excess daily returns of indices against the constant a and estimated volatility extracting from GARCH and EGARCH model.
Weighted least squares regression of the excess daily returns of indices against the constant a and estimated volatility extracting from GARCH and EGARCH model. Predicted standard deviation $\widehat{\sigma_{it}}$ is used to standardize each observation.

$$\left(R_{it} - R_{fit}\right) = \alpha + \beta\hat{\sigma}_{it}^{p} + \varepsilon_{it} \qquad (5)$$

R_{it} *= return of index in country i at time t*
R_{fit} *= interbank rate in country i at time t used as risk-free rate*
$\hat{\sigma}_{it}^{p}$ *= is the predictable volatility of stock index extracting from GARCH and EGARCH models*
ε_{it} *= error term i at time t*
White's (1980) correction is applied for heteroskedasticity correction.

Daily adjusted returns	**GARCH Volatility**			**EGARCH Volatility**		
	α	**β**	**R^2**	**α**	**β**	**R^2**
OMX Copenhagen						
Standard deviation	0,1542	-0,1279	0,14 %	0,1951	-0,1713	0,29 %
t-statistic	4,0684	-2,8292		5,4131	-3,9316	
Variance	0,0851	-0,0475	0,10 %	0,1095	-0,0715	0,22 %
t-statistic	4,7769	-2,4248		5,9372	-3,3288	
OMX Helsinki						
Standard deviation	0,1094	-0,0461	0,03 %	0,1376	-0,0656	0,06 %
t-statistic	2,4782	-1,2662		3,2273	-1,8434	
Variance	0,0748	-0,0117	0,02 %	0,0890	-0,0166	0,05 %
t-statistic	3,3826	-1,1581		4,0425	-1,5868	
OMX Stockholm						
Standard deviation	0,0838	-0,0511	0,03 %	0,0742	-0,0426	0,02 %
t-statistic	1,9973	-1,2398		1,8708	-1,0692	
Variance	0,0477	-0,0125	0,01 %	0,0475	-0,0130	0,01 %
t-statistic	2,3555	-0,8728		2,3416	-0,8452	
Oslo Exchange All Share						
Standard deviation	0,1280	-0,0883	0,07 %	0,1810	-0,1361	0,20 %
t-statistic	2,9580	-2,0076		4,3476	-3,1700	
Variance	0,0685	-0,0239	0,05 %	0,0945	-0,0403	0,13 %
t-statistic	3,4833	-1,5933		4,7084	-2,4834	

Table 7. Weighted least squares regression of the excess monthly returns of indices from January 1996 to February 2017

Weighted least squares regression of the excess monthly returns of indices against the constant a and estimated volatility extracting from GARCH and EGARCH model. Predicted standard deviation $\widehat{\sigma_{it}}$ is used to standardize each observation.

$$\left(R_{it} - R_{fit}\right) = \alpha + \beta\hat{\sigma}_{it}^{p} + \varepsilon_{it} \qquad (5)$$

R_{it} = return of index in country i at time t
R_{fit} = interbank rate in country i at time t used as risk-free rate
$\hat{\sigma}_{it}^{p}$ = is the predictable volatility of stock index extracting from GARCH and EGARCH model
ε_{it} = error term i at time t
White's (1980) correction is applied for heteroskedasticity correction.

Monthly adjusted returns	GARCH Volatility			EGARCH Volatility		
	α	β	R_2	α	β	R_2
OMX Copenhagen						
Standard deviation	4,9638	-0,9011	2,74 %	3,4999	-0,6060	1,78 %
t-statistic	3,2998	-2,7427		2,7358	-2,0748	
Variance	2,5038	-0,0762	2,34 %	1,9882	-0,0562	1,81 %
t-statistic	3,6636	-2,5774		3,6015	-2,2451	
OMX Helsinki						
Standard deviation	2,5603	-0,2614	0,70 %	2,7851	-0,2922	0,82 %
t-statistic	2,3167	-1,3504		2,3451	-1,4271	
Variance	1,6373	-0,0152	0,53 %	1,7924	-0,0185	0,69 %
t-statistic	3,0022	-1,2169		3,0374	-1,3470	
OMX Stockholm						
Standard deviation	2,4365	-0,3667	0,85 %	3,5348	-0,5723	2,13 %
t-statistic	2,1501	-1,5601		3,0834	-2,4006	
Variance	1,4236	-0,0301	0,76 %	2,0434	-0,0506	2,03 %
t-statistic	2,5356	-1,4823		3,5108	-2,3436	
Oslo Exchange All Share						
Standard deviation	1,3882	-0,1559	0,14 %	2,1678	-0,2973	0,77 %
t-statistic	1,1715	-0,6813		2,2017	-1,4258	
Variance	0,8970	-0,0109	0,13 %	1,4557	-0,0282	0,88 %
t-statistic	1,6980	-0,7104		2,9473	-1,5794	

5.1.2 ARCH-in-mean Model

ARCH in mean model is used to estimate the relation of conditional volatility and excess return directly by including the ARCH term in the mean regression. The empirical results from GARCH in mean model and EGARCH in mean model are explained in daily and monthly frequency. The t-statistic obtained from Bollerslev-Wooldridge (1992) to robust standard errors & covariance.

Table 8 and 9 shows the result in daily frequency of GARCH-M and EGARCH-M models respectively. The coefficient β shows the volatility and return relation in these regressions. It is observed that there is no reliable β found in GARCH-M model. Yet, OMX Copenhagen’s and Oslo Exchange All Shares’ adjusted returns have statistically significant negative relationship with conditional standard deviations at 10% and 1% level respectively in EGARCH-M model.

Table 8. GARCH-M models for daily excess holding period returns

GARCH-M(1,2)-MA(1) models for daily excess holding period returns of Nordic stock market's indices from January 1996 to February 2017

$$R_{it} - R_{fit} = \alpha + \beta\sigma_{it} + \varepsilon_{it} - \theta\varepsilon_{i,t-1} \qquad (1a)$$
$$R_{it} - R_{fit} = \alpha + \beta\sigma_{it}^2 + \varepsilon_{it} - \theta\varepsilon_{i,t-1} \qquad (1b)$$
$$\sigma_{it}^2 = a + b\sigma_{i,t-1}^2 + c_1\varepsilon_{t-1}^2 + c_2\varepsilon_{t-2}^2 \qquad (2)$$

R_{it} = return of index in country i at time t
R_{fit} = interbank rate in country i at time t used as risk-free rate
σ_{it}^2 = conditional variance in country i at time t
ε_{it} = error term i at time t
Heteroskedasticity consistent covariance (Bollerslev-Wooldridge) is used to robust standard errors & covariance

Daily returns	**α**	**β**	**θ**	**a**	**b**	**c_1**	**c_2**
OMX Copenhagen							
Standard deviation	0,1191	-0,0520	0,0627	0,0239	0,8697	-0,0093	0,1199
t-statistic	3,1832	-1,1586	4,1953	5,4556	63,9672	-0,3974	5,5130
Variance	0,0879	-0,0148	0,0629	0,0238	0,8700	-0,0096	0,1198
t-statistic	4,9099	-0,7520	4,2046	5,4778	64,1900	-0,4108	5,5239
OMX Helsinki							
Standard deviation	0,1003	-0,0106	0,0338	0,0107	0,9409	-0,0339	0,0907
t-statistic	2,2386	-0,2804	2,1983	2,2828	80,6405	-1,1951	3,7146
Variance	0,0919	-0,0023	0,0338	0,0107	0,9408	-0,0338	0,0907
t-statistic	4,0721	-0,2183	2,2029	2,2843	80,6420	-1,1923	3,7121
OMX Stockholm							
Standard deviation	0,0535	0,0239	-0,0038	0,0237	0,8829	0,0403	0,0661
t-statistic	1,2223	0,5566	-0,2608	4,5768	72,3477	1,7550	3,4218
Variance	0,0646	0,0115	-0,0038	0,0238	0,8828	0,0401	0,0663
t-statistic	3,1167	0,7668	-0,2631	4,5780	72,2916	1,7487	3,4302
Oslo Exchange All Share							
Standard deviation	0,0982	-0,0154	0,0151	0,0294	0,8738	-0,0312	0,1405
t-statistic	2,3586	-0,3638	1,0212	5,4851	66,0616	-1,1951	5,2004
Variance	0,0865	-0,0028	0,0153	0,0294	0,8738	-0,0314	0,1406
t-statistic	4,5432	-0,1901	1,0293	5,4911	66,1615	-1,2050	5,2120

Table 9. EGARCH-M models for daily excess holding period returns

EGARCH-M(1,2)-MA(1) models for daily excess holding period returns of Nordic stock market's indices from January 1996 to February 2017

$$R_{it} - R_{fit} = \alpha + \beta\sigma_{it} + \varepsilon_{it} - \theta\varepsilon_{i,t-1}$$

$$R_{it} - R_{fit} = \alpha + \beta\sigma_{it}^2 + \varepsilon_{it} - \theta\varepsilon_{i,t-1}$$

$$\log\sigma_t^2 = a + b_1 log\sigma_{t-1}^2 + c_1\left|\frac{\varepsilon_{t-1}}{\sigma_{t-1}}\right| + c_2\left|\frac{\varepsilon_{t-2}}{\sigma_{t-2}}\right| + c_3\frac{\varepsilon_{t-1}}{\sqrt{\sigma_{t-1}^2}} + c_4\frac{\varepsilon_{t-2}}{\sqrt{\sigma_{t-2}^2}}$$

R_{it} = return of index in country i at time t
R_{fit} = interbank rate in country i at time t used as risk-free rate
σ_{it}^2 = conditional variance in country i at time t
ε_{it} = error term
Heteroskedasticity consistent covariance (Bollerslev-Wooldridge) is used to robust standard errors & covariance

Daily returns	α	β	θ	a	b_1	c_1	c_2	c_3	c_4
OMX Copenhagen									
Standard deviation	0,1226	-0,0787	0,0555	-0,1422	0,9727	0,2039	-0,0218	-0,0753	0,0128
t-statistic	3,4131	-1,7840	3,5525	-7,5897	168,14	4,7884	-0,4991	-2,7051	0,4915
Variance	0,0781	-0,0249	0,0560	-0,1416	0,9725	0,2033	-0,0223	-0,0739	0,0125
t-statistic	4,1502	-1,1092	3,6088	-7,5535	171,23	4,7891	-0,5154	-2,6570	0,4793
OMX Helsinki									
Standard deviation	0,1029	-0,0373	0,0303	-0,0835	0,9944	0,1005	0,0156	-0,1710	0,1337
t-statistic	2,3044	-1,0239	1,9887	-5,9499	432,56	2,2104	0,3152	-5,9678	4,7895
Variance	0,0706	-0,0059	0,0305	-0,0834	0,9941	0,1008	0,0153	-0,1702	0,1331
t-statistic	2,9102	-0,5342	1,9954	-5,9437	430,96	2,2128	0,3085	-5,9361	4,7606
OMX Stockholm									
Standard deviation	0,0481	-0,0094	-0,0126	-0,1167	0,9803	0,0291	0,1292	-0,1921	0,1066
t-statistic	1,1047	-0,2197	-0,8534	-7,8765	260,53	0,6955	3,0039	-7,9522	4,7550
Variance	0,0283	0,0104	-0,0095	-0,1169	0,9789	0,0283	0,1304	-0,1921	0,1064
t-statistic	1,2540	0,6127	-0,6372	-7,8859	257,44	0,6747	3,0397	-7,9182	4,7419
Oslo Exchange All Share									
Standard deviation	0,1664	-0,1164	0,0047	-0,1317	0,9802	0,1514	0,0252	-0,1883	0,1209
t-statistic	3,9204	-2,6944	0,3232	-9,2781	249,38	3,6246	0,6471	-7,4699	4,9160
Variance	0,0814	-0,0241	0,0085	-0,1311	0,9790	0,1541	0,0217	-0,1854	0,1201
t-statistic	3,8924	-1,4662	0,5855	-9,2627	245,07	3,6831	0,5562	-7,2988	4,8449

Table 10 and table 11 shows the result of ARCH-M type models in monthly frequency. The conditional standard deviation and variance of OMX Copenhagen again negatively relates to excess returns at 10% significant in GARCH-M and EGARCH-M. Besides the conditional standard deviation and variance of OMX Stockholm show negative relationship with excess returns at 10% significant in EGARCH-M. The same movement is seen in OMX Helsinki and Oslo Exchange All Share index. However, they are not statistically reliable.

Table 10. GARCH-M models for monthly excess holding period returns

GARCH-M(1,2)-MA(1) models for monthly excess holding period returns of Nordic stock market's indices from January 1996 to February 2017

$$R_{it} - R_{fit} = \alpha + \beta\sigma_{it} + \varepsilon_{it} - \theta\varepsilon_{i,t-1} \qquad (1a)$$
$$R_{it} - R_{fit} = \alpha + \beta\sigma_{it}^2 + \varepsilon_{it} - \theta\varepsilon_{i,t-1} \qquad (1b)$$
$$\sigma_{it}^2 = a + b\sigma_{i,t-1}^2 + c_1\varepsilon_{t-1}^2 + c_2\varepsilon_{t-2}^2 \qquad (2)$$

R_{it} = return of index in country i at time t
R_{fit} = interbank rate in country i at time t used as risk-free rate
σ_{it}^2 = conditional variance in country i at time t
ε_{it} = error term i at time t
Heteroskedasticity consistent covariance (Bollerslev-Wooldridge) is used to robust standard errors & covariance

Monthly returns	α	β	θ	a	b	c_1	c_2
OMX Copenhagen							
Standard deviation	4,4545	-0,7330	0,0652	2,8120	0,7288	0,0361	0,1280
t-statistic	2,5870	-1,9186	0,9819	1,3221	4,6074	0,2762	1,3042
Variance	2,5704	-0,0668	0,0659	2,8629	0,7371	0,0194	0,1324
t-statistic	3,2408	-1,7964	1,0037	1,3012	4,6308	0,1570	1,3910
OMX Helsinki							
Standard deviation	2,3494	-0,2245	0,1860	1,3556	0,8081	0,0627	0,1158
t-statistic	2,1137	-1,1478	2,8546	1,0410	10,6045	0,3961	0,8755
Variance	1,5931	-0,0136	0,1902	1,3551	0,8110	0,0627	0,1121
t-statistic	2,7826	-1,0763	2,9342	1,0385	10,8006	0,3997	0,8541
OMX Stockholm							
Standard deviation	2,0126	-0,2238	0,0755	1,9304	0,7892	-0,0082	0,1642
t-statistic	1,8482	-1,0014	1,1155	1,6792	8,7124	-0,0764	1,7408
Variance	1,4406	-0,0199	0,0739	1,9616	0,7880	-0,0147	0,1710
t-statistic	2,6363	-1,0062	1,1024	1,6987	8,6176	-0,1369	1,8226
Oslo Exchange All Share							
Standard deviation	0,8512	0,0378	0,1452	3,6525	0,6688	0,2132	0,0521
t-statistic	0,8901	0,2089	2,2563	1,5550	6,9003	1,5142	0,6713
Variance	1,0121	0,0008	0,1461	3,6086	0,6754	0,2062	0,0517
t-statistic	2,2409	0,0661	2,2767	1,5330	7,0062	1,4905	0,6662

Table 11. EGARCH-M models for monthly excess holding period returns

EGARCH-M(1,2)-MA(1) models for monthly excess holding period returns of Nordic stock market's indices from January 1996 to February 2017

$$R_{it} - R_{fit} = \alpha + \beta\sigma_{it} + \varepsilon_{it} - \theta\varepsilon_{i,t-1}$$

$$R_{it} - R_{fit} = \alpha + \beta\sigma_{it}^2 + \varepsilon_{it} - \theta\varepsilon_{i,t-1}$$

$$\log\sigma_t^2 = a + b_1 log\sigma_{t-1}^2 + c_1\left|\frac{\varepsilon_{t-1}}{\sigma_{t-1}}\right| + c_2\left|\frac{\varepsilon_{t-2}}{\sigma_{t-2}}\right| + c_3\frac{\varepsilon_{t-1}}{\sqrt{\sigma_{t-1}^2}} + c_4\frac{\varepsilon_{t-2}}{\sqrt{\sigma_{t-2}^2}}$$

R_{it} = return of index in country i at time t
R_{fit} = interbank rate in country i at time t used as risk-free rate
σ_{it}^2 = conditional variance in country i at time t
ε_{it} = error term
Heteroskedasticity consistent covariance (Bollerslev-Wooldridge) is used to robust standard errors & covariance

Monthly returns	α	β	θ	a	b_1	c_1	c_2	c_3	c_4
OMX Copenhagen									
Standard deviation	3,2315	-0,5491	0,0037	1,9770	0,2883	0,2468	-0,0017	-0,1404	-0,3732
t-statistic	2,1837	-1,7026	0,0551	3,2421	1,3110	1,3404	-0,0114	-1,5601	-3,8678
Variance	2,0742	-0,0612	-0,0180	2,0079	0,2819	0,2523	-0,0234	-0,1657	-0,3594
t-statistic	2,7367	-1,8351	-0,2582	3,3046	1,2620	1,4881	-0,1702	-1,9464	-3,6636
OMX Helsinki									
Standard deviation	2,3004	-0,2253	0,1990	-0,0907	0,9591	0,2454	0,0754	-0,0431	0,0100
t-statistic	1,8379	-1,0365	2,8099	-0,8700	31,0071	1,1743	0,3594	-0,3387	0,0942
Variance	1,5403	-0,0146	0,1988	-0,0904	0,9594	0,2422	0,0769	-0,0536	0,0219
t-statistic	2,4002	-0,9925	2,8311	-0,8707	30,9486	1,1844	0,3738	-0,4263	0,2110
OMX Stockholm									
Standard deviation	2,9793	-0,4578	0,0444	0,1065	0,9099	0,2242	0,0254	-0,1834	0,0868
t-statistic	2,4536	-1,8115	0,4783	0,7933	18,5940	1,2983	0,1439	-1,4585	0,7029
Variance	1,9300	-0,0459	0,0309	0,0996	0,9112	0,2461	0,0075	-0,1851	0,0917
t-statistic	3,0927	-1,9100	0,3184	0,7450	18,4819	1,4948	0,0448	-1,4328	0,7497
Oslo Exchange All Share									
Standard deviation	0,6631	-0,0194	0,1502	0,1082	0,9215	-0,2402	0,4392	-0,3336	0,2129
t-statistic	0,6045	-0,0886	2,3125	0,7668	25,0735	-1,3823	2,4531	-3,4997	3,1794
Variance	0,7158	-0,0057	0,1438	0,1067	0,9216	-0,2350	0,4357	-0,3324	0,2072
t-statistic	1,3186	-0,3390	2,3064	0,7542	24,7993	-1,3577	2,4392	-3,5189	3,1383

The results of WLS regressions are quite varied in different frequency and different volatility modeling. Only OMX Copenhagen shows stable result proving the negative relation between the conditional volatility and returns over the estimated period. There are more reliable negative correlation coefficients found when using the EGARCH conditional volatility measurement. It is found that there is the negative risk – return correlation in Oslo Exchange All Share in daily frequency but not in monthly frequency. In the other hand, OMX Stockholm show the reliable negative relation using EGARCH volatility in monthly frequency but not in daily frequency. In ARCH-M type model, the result is also not stable except OMX Copenhagen index which still shows the statistically significant negative relationship between conditional volatility and holding period returns. Negative risk return relation is found in monthly conditional EGARCH volatility of OMX Stockholm. There is no positive relation found in all estimated models.

The negative correlation means that the higher conditional volatility would lead to a lower excess holding period return which is in line with Glosten, Jagannathan and Runkle (1993), Nelson (1991), Ang et all (2006). Glosten et al (1993) argues that that time periods which are relatively riskier could coincide with time periods when investors are better able to bear particular types of risk. However, it is contradicted with French et al (1987) finding the positive relationship in S&P composite portfolio as well as Merton (1987) prediction even though the similar methodology applied in this study. Glosten et al (1993) points out that most of the positive correlation is found from GARCH-M model and negative correlation is found from EGARCH model. However, no positive correlation is observed in four studied indices in GARCH-M model.

5.3 Unexpected volatility

In this part, the volatility is broken down into expected volatility and unexpected volatility. The MA(1)-GARCH(1,2) process is run. The one step ahead forecast variance is computed from the established GARCH process showed in table 12. The relation of monthly excess holding period return and unexpected volatility is presented in table 13.

Table 12. GARCH model for monthly holding period return

Generalized autoregressive conditional heteroskedasticity (GARCH) models for monthly holding period returns of Nordic stock market's indices from January 1996 to February 2017

$$R_{it} = \alpha + \varepsilon_{it} - \theta\varepsilon_{i,t-1} \qquad (1)$$
$$\sigma_{it}^2 = a + b\sigma_{i,t-1}^2 + c_1\varepsilon_{t-1}^2 + c_2\varepsilon_{t-2}^2 \quad (2)$$

R_{it} *= return of index in country i at time t*
σ_{it}^2 *= conditional variance in country i at time t*
ε_{it} *= error term i at time t*
Heteroskedasticity consistent covariance (Bollerslev-Wooldridge) is used to robust standard errors & covariance

Monthly return	**α**	**θ**	**a**	**b**	**c_1**	**c_2**
OMX Copenhagen	1,3162	0,0949	2,5783	0,1095	0,0233	0,7665
t-statistic	4,7776	1,3988	1,3219	1,0046	0,1750	5,8314
OMX Helsinki	1,2829	0,1960	1,3660	0,1040	0,0725	0,8093
t-statistic	3,1384	2,9830	1,0230	0,8074	0,4682	10,7075
OMX Stockholm	1,1399	0,0818	2,0921	0,1591	0,0019	0,7784
t-statistic	3,9200	1,2134	1,6570	1,6328	0,0168	7,8617
Oslo Exchange All Share	1,1183	0,1613	3,1943	0,0431	0,1183	0,7586
t-statistic	3,6007	2,4835	1,3909	0,5494	1,1061	9,5046

The unexpected variance (standard deviation) is calculated from historical value minus the predictable variance (standard deviation) from GARCH process $\hat{\sigma}_{it}^{pu}$=$\sigma_{it}^{p} - \hat{\sigma}_{it}^{p}$. The monthly excess holding period return is regressed on expected and unexpected components. Table 17 shows that the adjusted R^2 increases significantly when unexpected component of volatility is included in the regression. It provides a strong evidence of negative relation between risk premium and unexpected volatility. The coefficients of standard deviation are high which are around -1.2 to -0.9 while the coefficients of variance are around -0.07 to -0.03. They are all statistically significant at 1% confidence interval. The significant level of OMX Copenhagen predictable volatility correlation decreases from 1% to 5%. The result shows a contrast with the conclusion form French et al (1986) that the negative correlation with unexpected volatility would induce the positive correlation with expected volatility in case of OMX Copenhagen index. It is seen that risk premium of OMX Copenhagen index has reliable negative correlation with both expected and unexpected variance and standard deviation.

Table 13. Monthly Unexpected volatility

Weighted least squares regression of the excess monthly returns of indices against the constant a and estimated volatility extracting from GARCH model. Predicted standard deviation $\hat{\sigma}_{it}$ is used to standardize each observation.

$$\left(R_{it} - R_{fit}\right) = \alpha + \beta\hat{\sigma}_{it}^{p} + \gamma\sigma_{it}^{pu} + \varepsilon_{it} \qquad (6)$$

R_{it} = *return of index in country i at time t*
R_{fit} = *interbank rate in country i at time t used as risk-free rate*
$\hat{\sigma}_{it}^{p}$ = *the predictable volatility of stock index extracting from GARCH model*
$\hat{\sigma}_{it}^{pu}$ = *the unpredictable part of volatility (the prediction error for the volatility of stock returns)*
ε_{it} = *error term i at time t*

Montly returns	**α**	**β**	γ	**Adjusted R_2**
		OMX Copenhagen		
Standard deviation	4,9002	-0,9381	-0,9217	18,20 %
t-statistic	3,3465	-2,9637	-6,0003	
Variance	2,1843	-0,0559	-0,0643	19,63 %
t-statistic	3,1961	-1,9027	-4,6442	
		OMX Helsinki		
Standard deviation	2,9202	-0,2935	-0,7429	9,84 %
t-statistic	2,6305	-1,5014	-4,5478	
Variance	1,7589	-0,0090	-0,0390	8,87 %
t-statistic	3,2599	-0,7146	-3,7657	
		OMX Stockholm		
Standard deviation	1,7807	-0,2159	-1,0486	19,44 %
t-statistic	1,5360	-0,9090	-7,0109	
Variance	1,0741	-0,0023	-0,0616	17,50 %
t-statistic	1,9522	-0,1148	-5,8722	
		Oslo Exchange All Share		
Standard deviation	1,2138	-0,2127	-1,2011	29,38 %
t-statistic	0,9716	-0,8990	-7,1782	
Variance	0,6496	0,0020	-0,0690	30,40 %
t-statistic	1,0745	0,1046	-5,7062	

5.4 Leverage effect

The idea is to test the relationship between continuously compounded return and the change of realized volatility. The empirical result of the leverage effect testing in Nordic stock market is presented in table 14. The elasticity in four tested indices are reliably negative. In detail, OMX Copenhagen have the coefficient of -1.7, OMX Stockholm and Oslo Exchange All share have coefficients of -2.4 which are less than -1.0 and statistically significant at 1% level which is in line with previous findings. OMX Helsinki index elasticity is -0.8 reliably greater than -1. It is concluded that the change stock market volatility is explained by the its contemporaneous holding period return.

Table 14. Leverage effect

Ordinary least squares regression of the percentage change in standard deviation of Nordic stock market' indices are regressed against the continuously compounded return of Nordic stock market's indices.

$$ln(\sigma_{mt}/\sigma_{mt-1}) = \alpha_0 + \beta_0\, ln(1 + R_{mt}/100) + \varepsilon_t \quad (7)$$

σ_{mt} *= standard deviation at month t and the previous month*
σ_{mt-1} *= standard deviation at month t-1*
R_{mt} *= return on market index at month t*
ε_{it} *= error term i at time t*
White's (1980) correction is applied for heteroskedasticity correction.

Montly returns	**α**	**β**	**Adjusted R^2**
OMX Copenhagen	0,0083	-1,7150	2,68 %
t-statistic	0,2659	-2,6033	
OMX Helsinki	0,0010	-0,8002	1,57 %
t-statistic	0,0365	-2,3358	
OMX Stockholm	0,0092	-2,4213	10,39 %
t-statistic	0,3664	-5,3870	
Oslo Exchange All Share	0,0154	-2,3572	7,48 %
t-statistic	0,4861	-4,6849	

6. CONCLUSION

This study investigates the risk-return relationship of four Nordic stock market's indices – OMX Copenhagen, OMX Helsinki, OMX Stockholm and Oslo Exchange All Shares. The volatility is predicted by ARCH-type models as well as it is broken out into predictable volatility and unpredictable volatility. Some necessary tests for the suitability of ARCH-type model for the four indices are carried out and show that it is suitable to apply ARCH-type model to predict conditional volatility. The research is carried out during the period between January 1996 and February 2017 in both daily and monthly frequency. The main observation is that OMX Copenhagen shows a reliable negative risk-return relation through the estimated period. It is found some reliable negative relationship in Oslo Exchange All Share and OMX Stockholm. However, the relation is not persistent. There is no positive risk return correlation found in this study.

While the relation between the expected volatility and realized return is not clear except OMX Copenhagen, the investigation indicates that a strong evidence of negative relation between risk premium and unexpected volatility is found in Nordic stock market's indices. Besides the leverage effect testing shows that the lower return would induce the higher realized volatility change.

REFERENCES

Ang, A., R. Hodrick, Y. Xing and X. Zhang, 2006, The cross-section of volatility and expected returns, Journal of Finance 61, 259-300.

Ang, A., R. Hodrick,Y. Xing, and X. Zhang, 2009, High idiosyncratic volatility and lowreturns: International and further U.S. evidence, Journal of Financial Economics 91, 1–23.

Evdokia Xekalaki, Stavros Degiannakis (2010). ARCH Models for Financial Applications. Wiley.

Baker, M., Brendan, B., & Jeffrey, W. (2011). Benchmarks as limits to arbitrage: Understanding the low volatility anomaly. Financial Analysts Journal, 67(1), 1–15.

Bali, T., Peng, L., 2006. Is there a risk-return tradeoff? Evidence from high-frequency data. Journal of Applied Econometrics 21, 1169–1198.

Blitz, D., Falkenstein, E. G., and Van Vliet, P. (2013), “Explanations for the Volatility Effect: An Overview Based on the CAPM Assumptions”, Working Paper.

Bollerslev, T., Litvinova, J., Tauchen, G., 2006. Leverage and volatility feedback effects in high-frequency data. Journal of Financial Econometrics 4, 353–384.

Brandt, M., Kang, Q., 2004. On the relationship between the conditional mean and volatility of stock returns: A latent VAR approach. Journal of Financial Economics 72, 217–257

Brandt, M.W., Brav, A., Graham, J.R., 2005. The idiosyncratic volatility puzzle: time trend or speculative episodes

Brockman, P., Schutte, M., 2007. Is idiosyncratic volatility priced? The international evidence. Unpublished working paper. University of Missouri – Columbia.

Campbell and Ludger Hentschel, 1992, No news is good news: An asymmetric model of changing volatility in stock returns, Journal of Financial Economics 31, 281-318

Campbell, J. Y. (1996). Understanding risk and return. Journal of Political Economy, 104(2), 298–345.

Campbell, John Y., Martin Lettau, Burton G. Malkiel, and Yexiao Xu, 2001, Have individual stocks become more volatile? An empirical exploration of idiosyncratic risk, Journal of Finance, 56 (2001), pp. 1–43

Canarella G, Miller S, Pollard S. NAFTA Stock Markets: Dynamic Return and Volatility Linkages [e-book]. New York: Nova Science Publishers, Inc; 2010. Available from: eBook Collection (EBSCOhost), Ipswich, MA. Accessed July 4, 2017.

Chiang, T. C., & Doong, S. C. (2001). Empirical analysis of stock returns and volatility: Evidence from seven Asian stock markets based on TAR-GARCH model. Review of Quantitative Finance and Accounting, 17(3), 301–318.

Chua, C., Goh, J., Zhang, Z., 2007. Idiosyncratic volatility matters for the cross-section of returns – in more ways than one! Unpublished working paper. Singapore Management University, Singapore.

Conditional Heteroskedasticity in Asset Returns: A New Approach, Daniel B. Nelson Econometrica, Vol. 59, No. 2. (Mar., 1991), pp. 347-370. Available online at http://links.jstor.org/sici?sici=00129682%28199103%2959%3A2%3C347%3ACHIARA%3E2.0.CO%3B2-V

Corhay, A. and T. Rad. 1994. 'Statistical Properties of Daily Returns: Evidence from European Stock Markets', Journal of Business Finance and Accounting, 21(2): 271–82.

Cutler, D. M. (1989). Stock market volatility, cross section volatility and stock returns (Working paper). MIT.

Da, Zhi and Liu, Qianqiu and Schaumburg, Ernst, Decomposing Short-Term Return Reversal (August 25, 2011). Available at SSRN: https://ssrn.com/abstract=1551025 or http://dx.doi.org/10.2139/ssrn.1551025

De Santis, G., & Imrohoroglu, S. (1997). Stock returns and volatility in emerging financial markets. Journal of International Money and Finance, 16(4), 561–579.

Dimitrios, D., & Theodore, S. (2011). The relationship between stock returns and volatility in the seventeen largest international stock markets: A semi-parametric approach. Modern Economy, 2(1), 1–8.

Engle, Robert F. "Autoregressive Conditional Heteroskedasticity with Estimates of the Variance of United Kingdom Inflation." Econometrica. Vol. 50, 1982, pp. 987–1007

Engle, Robert F. (1982). "Autoregressive Conditional Heteroscedasticity with Estimates of the Variance of United Kingdom Inflation". Econometrica. 50 (4): 987–1007. JSTOR 1912773.

Eric Ghysels, Pedro Santa-Clara, Rossen Valkanov, 2005, There is a risk-return trade-off after all, Journal of Financial Economics, Volume 76, Issue 3, 2005, Pages 509-548, ISSN 0304-405X

Fama, E., French, K., 1993. Common risk factors in the returns on stocks and bonds. Journal of Financial Economics 33, 3-56.

Figlewski, Stephen and Wang, Xiaozu, Is the 'Leverage Effect' a Leverage Effect? (November 2000). Available at SSRN: https://ssrn.com/abstract=256109 or http://dx.doi.org/10.2139/ssrn.256109

Fu, F., 2010, "On the Robustness of the Positive Relation Between Expected Idiosyncratic Volatility and Return," Singapore Management University Working Paper

FU, Fangjian. Idiosyncratic Risk and the Cross-Section of Expected Stock Returns, 2009, Journal of Financial Economics. 91, (1), 24-37. Research Collection Lee Kong Chian School of Business.

Giot, P., 2005. Relationships between implied volatility indexes and stock index returns. Journal of Portfolio Management 26, 12–17.

Glosten, L. R., Jagannathan R., & Runkle D. E. (1993). On the relation between the expected value and the volatility of the nominal excess return on stocks. Journal of Finance,

Glosten, Laurence R., Ravi Jagannathan, and David E. Runkle, 1993, On the relation between the expected value and the volatility of the nominal excess return on stocks, Journal of Finance 48(5), 1779–1801.

Goyal, A., and P. Santa-Clara, 2003, Idiosyncratic risk matters! Journal of Finance 58, 975–1007.

Goyal, Amid, and Pedro Santa-Clara (2003), ‘Idiosyncratic Risk Matters!’ Journal of Finance, Vol. 58, pp. 975-1007.

Guo, H., Savickas, R., 2007. Aggregate idiosyncratic volatility in G7 countries. Review of Financial Studies, forthcoming

Guo, H., Whitelaw, R., 2006. Uncovering the risk–return relationship in the stock market. Journal of Finance 61, 1433–1463

Hamilton, James and Gang Lin, 1996, Stock market volatility and the business cycle, Journal of Applied Econometrics, 11, 573□593.

Huang, Wei and Liu, Qianqiu and Rhee, S. Ghon and Zhang, Liang, Return Reversals, Idiosyncratic Risk, and Expected Returns (January 11, 2009). Review of Financial Studies, Forthcoming. Available at SSRN: https://ssrn.com/abstract=1326118

Huang,W., Liu, Q., Rhee, G., Zhang, L., 2006. Another look at idiosyncratic risk and expected returns. Unpublished working paper, University of Hawaii at Manoa.

Johnson, T., 2004, Forecast dispersion and the cross section of expected returns, Journal of Finance 59, 1957–78.

Kenneth R. French, G.William Schwert and Robert F.Stambaugh (1987). Expected stock returns and volatility. Journal of Financial Economics, 19(1), 3–29.

Kwiatkowski, D.; Phillips, P. C. B.; Schmidt, P.; Shin, Y. (1992). "Testing the null hypothesis of stationarity against the alternative of a unit root". Journal of Econometrics. 54 (1–3): 159–178

Lee, C. F., Chen, G., & Rui, O. (2001). Stock returns and volatility on China’s stock markets. Journal of Financial Research, 24(4), 523–543.

Levy, H., 1978. Equilibrium in an imperfect market: a constraint on the number of securities in the portfolio. American Economic Review 68, 643-658.

Malkiel, Burton G., and Yexiao Xu, 1997, Risk and return revisited, Journal of Portfolio Management 23, 9-14.

Malkiel, Burton G., and Yexiao Xu, 2002, Idiosyncratic risk and security returns, Working paper, University of Texas at Dallas.

Merton, R.C., 1973. An intertemporal capital asset pricing model. Econometrica 41 (5), 867–887.

Miller, E., 1977. Risk, uncertainty, and divergence of opinion. Journal of Finance 32, 1151–1168.

Porterba, Jame M and Lawrence H Summers, 1986, The Persistence of volatility and stock market fluctuations

Rahman, M. A. (2007). The information content of crosssectional volatility for future market volatility: Evidence from Australian equity returns. Frontiers in Finance and Economics, 4(1), 91–124.

Rakesh Kumar and Raj S. Dhankar, 2010, Empirical Analysis of Conditional Heteroskedasticity in Time Series of Stock Returns and Asymmetric Effect on Volatility, Global Business Review, 11, (1), 21-33

Robert A. Haugen and A. James Heins, 1975 "Risk and the Rate of Return on Financial Assets, Some Old Wine in New Bottles," Journal of Financial and Quantitative Analysis, Vol. X, No. 5, 775-784.

Sanjay Sehgal and Vidisha Garg. Cross-sectional volatility and stock returns: evidence for emerging markets

Schwert, G. W. (1989). Why does stock market volatility change over time? Journal of Finance, 44(5), 1115–115448(5), 1779–1801.

Schwert, G., 1989. Why does stock market volatility change over time? Journal of Finance 44, 1115-1153.

Shalen, C. T. (1993). Volume, volatility, and the dispersion of beliefs. Review of Financial Studies, 6, 405–434.

Shin, J. (2005). Stock returns and volatility in emerging stock markets. International Journal of Business and Economics, 4(1), 31–43.

Wen-I Chuang, Hsiang-Hsi Liu and Rauli Susmel, 2012, The bivariate GARCH approach to investigating the relation between stock returns, trading volume, and return volatility, Global Finance Journal, 23, (1), 1-15

Yacine Aït-Sahalia, Jianqing Fan, Yingying Li, The leverage effect puzzle: Disentangling sources of bias at high frequency, Journal of Financial Economics, Volume 109, Issue 1, 2013, Pages 224-249, ISSN 0304-405X,

APPENDIX

Table A1. *GARCH(1,2)-MA(1) process for monthly adjusted holding period returns of Nordic stock market's indices from January 1996 to February 2017*

$$R_{it} - R_{fit} = \alpha + \varepsilon_{it} - \theta\varepsilon_{i,t-1}$$
$$\sigma_{it}^2 = a + b\sigma_{i,t-1}^2 + c_1\varepsilon_{t-1}^2 + c_2\varepsilon_{t-2}^2$$

R_{it} = return of index in country i at time t
R_{fit} = interbank rate in country i at time t used as risk-free rate
σ_{it}^2 = conditional variance in country i at time t
ε_{it} = error term
Heteroskedasticity consistent covariance (Bollerslev-Wooldridge) is used to robust standard errors & covariance

Monthly adjusted returns	**α**	**θ**	**a**	**b**	**c_1**	**c_2**
OMX Copenhagen	1,1528	0,0960	2,5942	0,1133	0,0255	0,7614
t-statistic	4,1921	1,4182	1,3481	1,0462	0,1899	5,8079
OMX Helsinki	1,1619	0,1938	1,3897	0,1055	0,0730	0,8073
t-statistic	2,8235	2,9635	1,0370	0,8129	0,4724	10,7160
OMX Stockholm	0,9714	0,0810	2,0517	0,1611	-0,0011	0,7813
t-statistic	3,3635	1,2045	1,7150	1,6614	-0,0101	8,1286
Oslo Exchange All Share	1,0330	0,1461	3,5975	0,0518	0,2043	0,6771
t-statistic	3,4795	2,2777	1,5284	0,6675	1,4816	7,0259

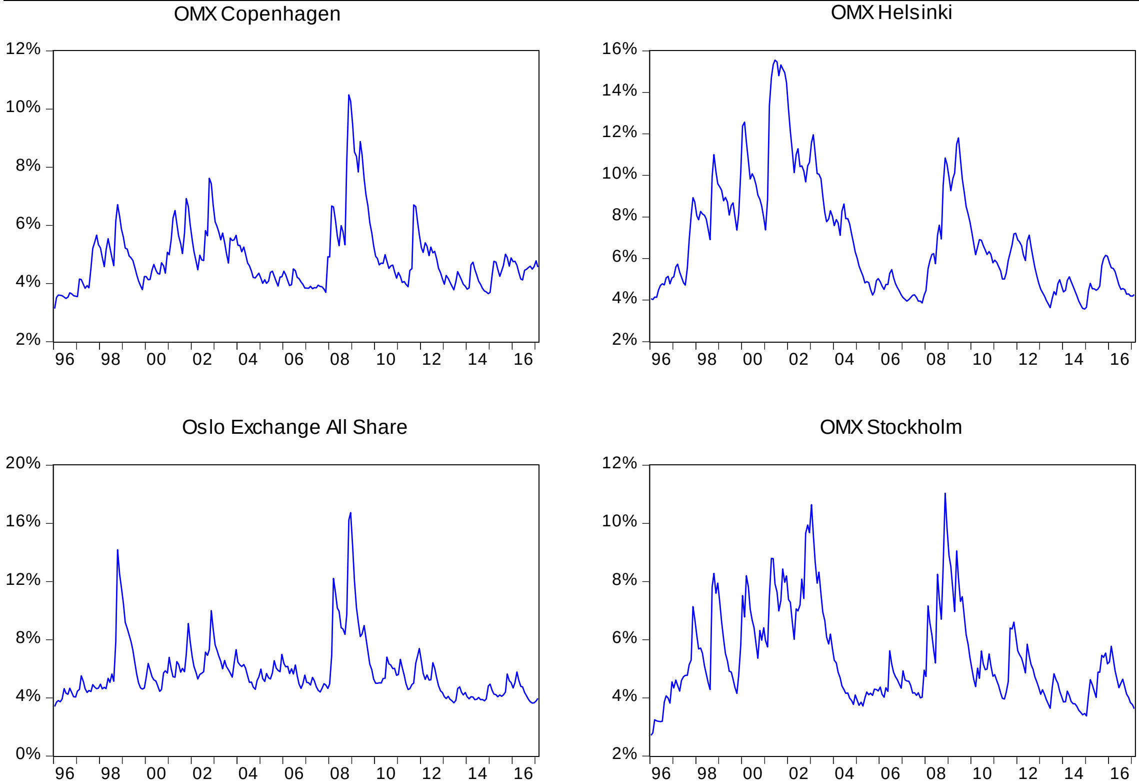

Figure 3. *Monthly GARCH Conditional Variance*

Table A2. *EARCH-M(1,2)-MA(1) process for monthly excess holding period returns of Nordic stock market's indices from January 1996 to February 2017*

$$R_{it} - R_{fit} = \alpha + \varepsilon_{it} - \theta\varepsilon_{i,t-1}$$

$$\log\sigma_t^2 = a + b_1 log\sigma_{t-1}^2 + c_1\left|\frac{\varepsilon_{t-1}}{\sigma_{t-1}}\right| + c_2\left|\frac{\varepsilon_{t-2}}{\sigma_{t-2}}\right| + c_3\frac{\varepsilon_{t-1}}{\sqrt{\sigma_{t-1}^2}} + c_4\frac{\varepsilon_{t-2}}{\sqrt{\sigma_{t-2}^2}}$$

R_{it} = return of index in country i at time t
R_{fit} = interbank rate in country i at time t used as risk-free rate
σ_{it}^2 = conditional variance in country i at time t
ε_{it} = error term i at time t
Heteroskedasticity consistent covariance (Bollerslev-Wooldridge) is used to robust standard errors & covariance

Monthly return	**α**	**θ**	**a**	**b**	**c₁**	**c₂**	**c₃**	**c₄**
OMX Copenhagen	0,9223	0,0299	2,2271	0,2103	0,1981	0,0257	-0,1579	-0,3856
t-statistic	3,2170	0,4430	3,4260	0,9247	0,9837	0,1546	-1,6193	-3,6306
OMX Helsinki	1,0707	0,2166	-0,0838	0,9570	0,2183	0,1028	-0,0572	0,0288
t-statistic	2,3805	3,0048	-0,7131	28,0699	1,0584	0,5005	-0,4474	0,2740
OMX Stockholm	0,8854	0,1025	0,1558	0,8933	0,1921	0,0626	-0,1841	0,0889
t-statistic	2,7136	1,4174	0,9715	16,8602	0,8924	0,2855	-1,3788	0,6724
Oslo Exchange All Share	0,5718	0,1532	0,1108	0,9208	-0,2414	0,4400	-0,3339	0,2139
t-statistic	1,5127	2,6099	0,7648	24,0533	-1,3940	2,4804	-3,4876	3,1768

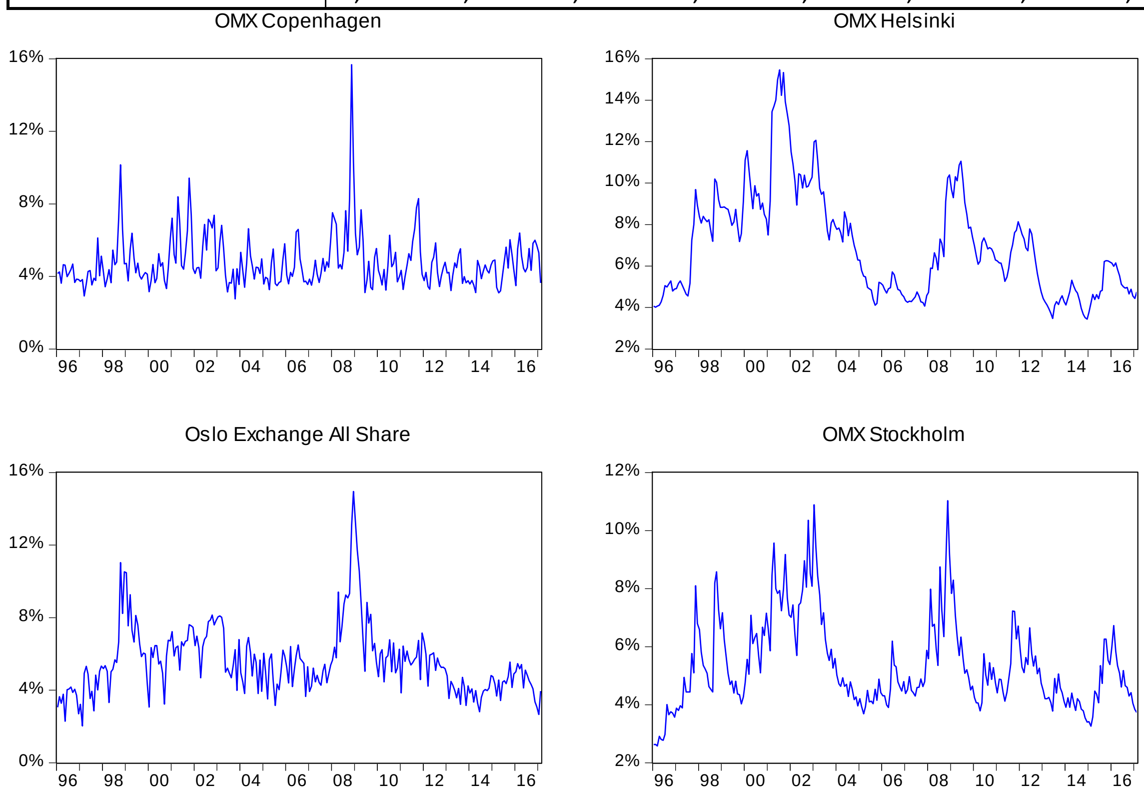

Figure 4. *Monthly EGARCH Conditional standard deviation*

Table A3. *GARCH(1,2)-MA(1) process for daily adjusted holding period returns of Nordic stock market's indices from January 1996 to February 2017*

$$R_{it} - R_{fit} = \alpha + \varepsilon_{it} - \theta\varepsilon_{i,t-1}$$
$$\sigma_{it}^2 = a + b\sigma_{i,t-1}^2 + c_1\varepsilon_{t-1}^2 + c_2\varepsilon_{t-2}^2$$

R_{it} = return of index in country i at time t
R_{fit} = interbank rate in country i at time t used as risk-free rate
σ_{it}^2 = conditional variance in country i at time t
ε_{it} = error term i at time t
Heteroskedasticity consistent covariance (Bollerslev-Wooldridge) is used to robust standard errors & covariance

Daily return	**α**	**θ**	**a**	**b**	**c_1**	**c_2**
OMX Copenhagen	0,0775	0,0631	0,0239	0,1199	-0,0095	0,8697
t-statistic	6,7039	4,2157	5,5110	5,5392	-0,4077	64,1375
OMX Helsinki	0,0885	0,0339	0,0107	0,0908	-0,0340	0,9408
t-statistic	5,2124	2,1989	2,2871	3,7174	-1,1966	80,6251
OMX Stockholm	0,0765	-0,0040	0,0237	0,0661	0,0402	0,8829
t-statistic	5,6331	-0,2774	4,5637	3,4116	1,7472	72,3989
Oslo Exchange All Share	0,0837	0,0153	0,0294	0,1408	-0,0316	0,8739
t-statistic	6,6274	1,0330	5,4923	5,2264	-1,2155	66,3228

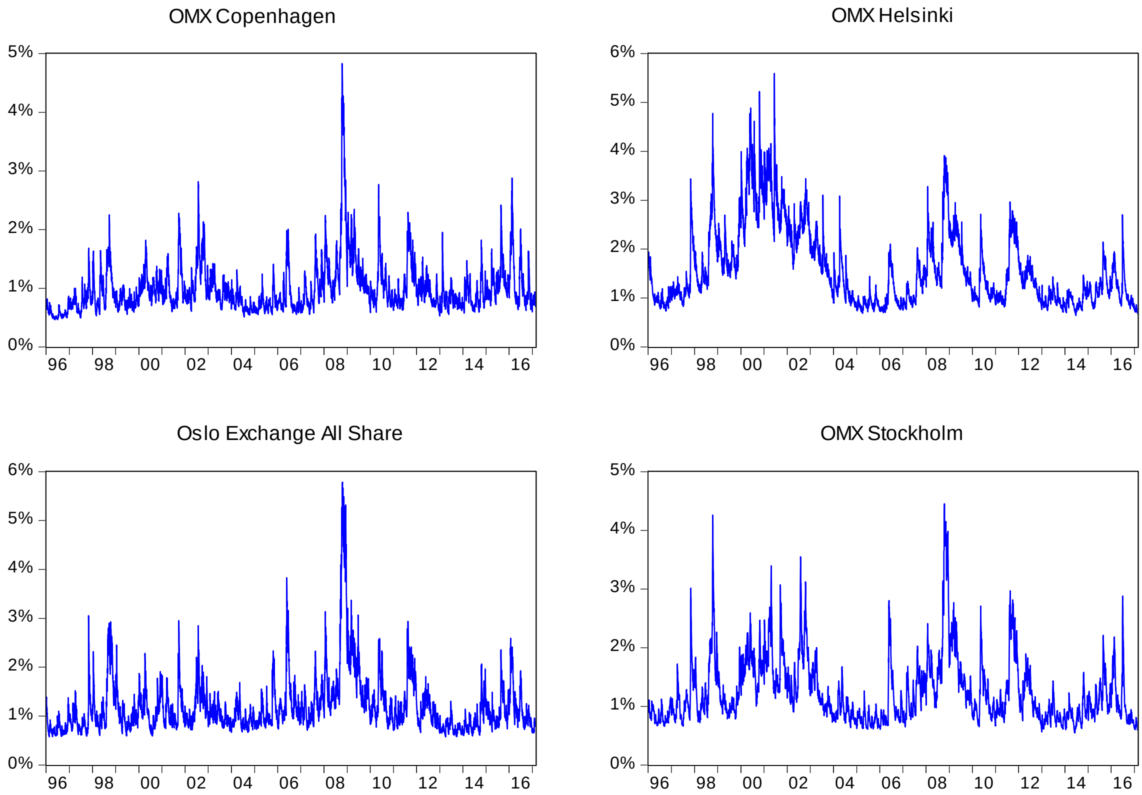

Figure 5. *Daily GARCH Conditional standard deviation*

***Table A4**. EARCH-M(1,2)-MA(1) process for daily excess holding period returns of Nordic stock market's indices from January 1996 to February 2017*

$$R_{it} - R_{fit} = \alpha + \varepsilon_{it} - \theta\varepsilon_{i,t-1}$$

$$\log\sigma_t^2 = a + b_1 log\sigma_{t-1}^2 + c_1\left|\frac{\varepsilon_{t-1}}{\sigma_{t-1}}\right| + c_2\left|\frac{\varepsilon_{t-2}}{\sigma_{t-2}}\right| + c_3\frac{\varepsilon_{t-1}}{\sqrt{\sigma_{t-1}^2}} + c_4\frac{\varepsilon_{t-2}}{\sqrt{\sigma_{t-2}^2}}$$

R_{it} = return of index in country i at time t
R_{fit} = interbank rate in country i at time t used as risk-free rate
σ_{it}^2 = conditional variance in country i at time t
ε_{it} = error term
Heteroskedasticity consistent covariance (Bollerslev-Wooldridge) is used to robust standard errors & covariance

Daily return	**α**	**θ**	**a**	**b**	**c₁**	**c₂**	**c3**	**c4**
OMX Copenhagen	0,0615	0,0568	-0,1415	0,2020	-0,0217	-0,0740	0,0128	0,9715
t-statistic	5,2805	3,7387	-7,5786	4,7849	-0,5086	-2,6799	0,4935	177,7852
OMX Helsinki	0,0625	0,0319	-0,0833	0,1009	0,0152	-0,1698	0,1328	0,9938
t-statistic	3,5563	2,1099	-5,9052	2,2120	0,3066	-5,9071	4,7467	450,6067
OMX Stockholm	0,0393	-0,0120	-0,1168	0,0290	0,1295	-0,1921	0,1067	0,9800
t-statistic	2,8601	-0,8388	-7,9019	0,6910	3,0124	-7,9498	4,7585	285,0669
Oslo Exchange All Share	0,0598	0,0120	-0,1303	0,1548	0,0197	-0,1848	0,1200	0,9778
t-statistic	4,4146	0,8416	-9,2769	3,7100	0,5068	-7,2375	4,8272	249,8833

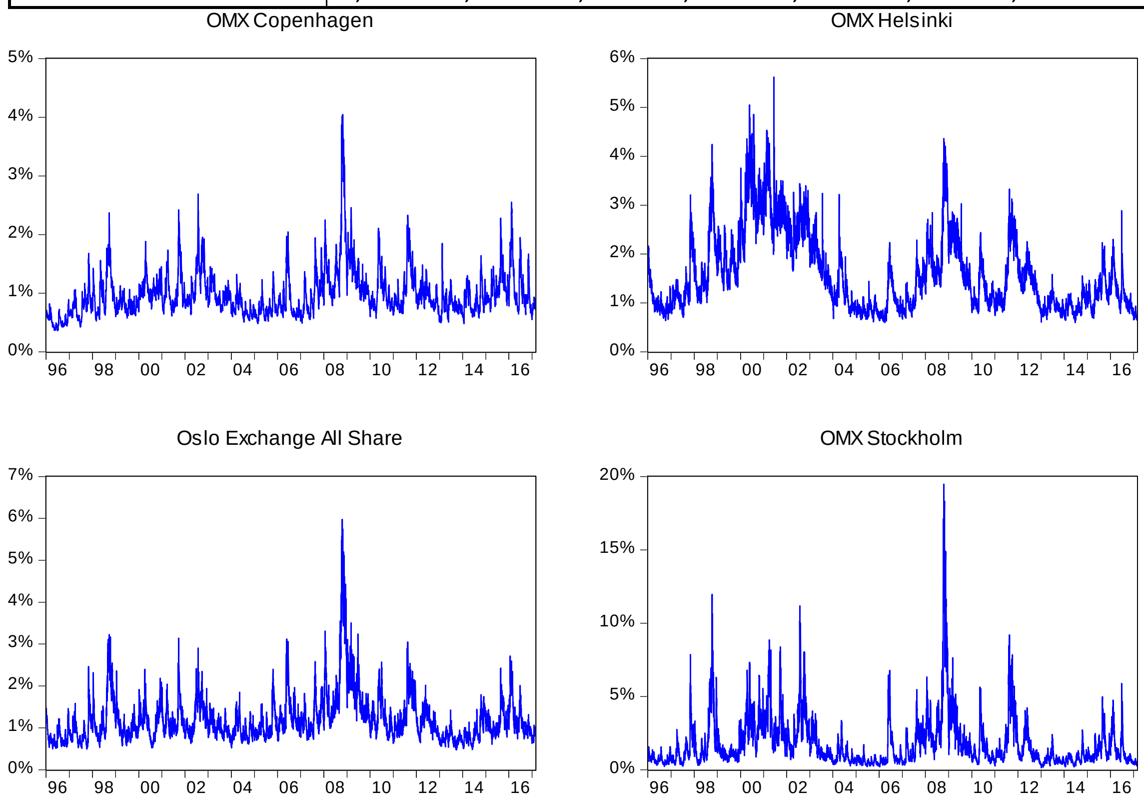

***Figure 6.** Daily EGARCH Conditional standard deviation*