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**BETTING AGAINST BETA**  
**CASE OMX HELSINKI**

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**Pages: 88****ABSTRACT**

One of the most famous theories in finance is the Capital Asset Pricing Model – a theory which is shown not to hold in empirical tests. The failure of the model and the abnormal performance of low-beta assets relative to high-beta assets is widely documented. Defensive securities have had higher returns than aggressive securities historically, in different markets and even asset classes, as shown by many studies. The phenomenon is opposite to the prediction of the CAPM, which expects that higher systematic risk would be rewarded by higher returns. Some authors have named the phenomenon as “the greatest anomaly in finance”.

Why have low-beta and low-volatility securities been superior to their high-beta and high-volatility alternatives historically? Can this phenomenon reasonably be expected to persist in the future? Does betting-against-beta, a strategy that takes a long position in low-beta stocks and a short position in high-beta stocks, provide positive excess returns in the OMX Helsinki stock market? This paper attempts to answer this question by reviewing previous literature and performing an empirical analysis using monthly stock data from the Finnish stock market, ranging from December 2001 to December 2017. In the relatively small and remote stock market of Finland, low-beta equities seem to have particularly strong returns over the period of study. The betting-against-beta strategy also performs convincingly and has positive excess returns, but there are also some caveats regarding the feasibility to execute the strategy in the studied market.

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**KEYWORDS:** Betting against beta, defensive equity, low volatility, low beta, capital asset pricing model



## 1. INTRODUCTION

The post-financial crisis period has been profitable for investors who have placed their money in stocks. Since the crisis, stock markets have headed up – the S&P 500 index has more than quadrupled in value by March 2019, after hitting its bottom ten years earlier in March 2009. However, history has shown that bull markets eventually come to an end. This provides a dilemma for the investor – when is it the right time to sell or buy protection against market downturns?

Market timing is difficult, as risks are not always visible. Also, it can be costly to react to looming risks early, as asset prices might keep rising for long before those risks realize or become known in the overall market. The ever-looming risk of sharp market declines might cause demand for strategies that provide constant protection against stock market downturns. A concerned investor would desire an investment strategy that not only provides positive returns during bull markets, but also provides protection during bear markets.

Low beta strategies are a potential solution to the presented need. Different low volatility and low beta strategies have been studied in financial research since when Black, Jensen, and Scholes (1972) found empirical evidence of assets' risk-return relationships that contradicted with the predictions of the CAPM. Recently, these strategies have gained new interest from academics. Blitz & Van Vliet (2007) study the “low volatility effect” and show that it is possible to achieve lower risk without sacrificing returns. Baker, Bradley & Wurgler (2011) study the same phenomenon and name it as the “low volatility anomaly”.

An old wisdom in the financial markets is that risk and return move in conjunction. This implies that investors are required to take on risk, if they wish to gain higher returns on their investments than the risk-free investment (Bodie & Kane, 2014). However, many studies during the past decades (such as Black, et al. (1972) and Frazzini & Pedersen, 2014) have shown that this relationship is to some extent erroneous – higher risk and higher returns do not always comove. These kinds of results are interesting and have many implications for investors, who seek for better risk-adjusted returns. Causative of the academic research on the subject, investors have noticed the ‘low-volatility anomaly’. Novy-Marx (2014) notes that money has flown to defensive equity funds recently, while

low volatility and low beta strategies have already been popular with insurance companies, pension funds and other institutional investors.

One of the most well-known models in modern financial theory, the capital asset pricing model (CAPM), provides a prediction of the risk-return relationship of an asset. The model estimates the returns of a security based on the risk-free rate, market risk premium, and the beta of the security. Beta measures the systematic risk of a security, which is the tendency of the price changes of the security to correlate with price changes in the market, or benchmark portfolio. Stocks with betas higher than one are said to be aggressive, and stocks with betas lower than one are said to be defensive. (Bodie & Kane, 2014).

According to the model, aggressive stocks should provide higher returns than defensive stocks – the CAPM assumes a linearly upward sloping line when systematic risk and expected returns are plotted on the x and y axes respectively – when exposure to the market is greater, expected returns should be greater as well (assuming that the market has positive returns). However, empirical tests have suggested otherwise: Black, Jensen, and Scholes (1972) find that excess returns on the market the portfolio for high beta securities had negative intercepts while low beta securities had positive intercepts. Both findings are significant, and thus contrary to the predictions of the CAPM, as these findings suggest that the security market line is flat relative to the upward sloping line predicted by the CAPM.

Low beta strategies attempt to take advantage of the shortcomings of the capital asset pricing model. The CAPM assumes that investors invest in the portfolio with the highest expected return per risk, and that they can leverage their portfolios to fit their preferred level of risk. However, real-life investors face funding constraints – due to margin requirements and leverage constraints, investors such as pension funds, individuals and mutual funds tend to overweight risky assets instead of using leverage to invest in lower risk assets. These restrictions imply lower risk premiums for risky assets and inversely higher ones for less risky assets, thus lowering the expected returns for high-beta assets and vice versa for low-beta assets. (Frazzini & Pedersen, 2014).

Frazzini and Pedersen (2014) find that portfolios with high-beta assets have lower alphas and Sharpe ratios relative to portfolios with low-beta assets, which again signals relative flatness of the security market line. The evidence is robust as the tests are conducted in 19 international equity markets and within different asset classes.

The studies and their findings discussed previously imply relative attractiveness for investing in low-beta, or defensive securities. As investments are often evaluated by their risk-to-return relationship, higher returns with lower variability are desired by every investor.

### 1.1. Purpose of the study

The purpose of this study is to empirically investigate whether a betting-against-beta strategy provides positive excess returns over the risk-free interest rate, and to compare the returns of low-beta stocks against aggressive stocks and broad market index returns. The study focuses on equities listed on the OMX Helsinki Stock Exchange. Financial institutions, such as pension funds, are a significant player in the Finnish markets, and it is interesting to examine whether high-beta equities underperform, as they are observed to do by Frazzini & Pedersen (2014) – the study explains the underperformance with the funding constraints that these institutions face, which in theory causes them to overweight high-beta stocks and suppresses the returns for these stocks. This is examined more closely later in this paper.

Novy-Marx (2014) notes that there has been a significant inflow of funds to defensive equity strategies after the financial crisis. The infamous crisis has made investors to seek for alternative, less risky investment strategies, as traditional asset allocation did not provide protection during the crisis when cross-asset correlations rose (Szado, 2009). Therefore, the subject of low-beta investing is timely, and as there is not much research on the topic from outside the U.S., it is reasonable to study the subject using data from the European markets, and more specifically Finland.

### 1.2. Research hypotheses

The aim of the study is to find out, whether the betting against beta strategy provides positive excess returns or not, and to compare its performance against the market index during the entire period of study and in different conditions. Also, the performance of the two components of BAB, the low-beta and high-beta portfolios, are compared against each other. The primary interest is on the BAB factor and its performance. Based on previous research (Frazzini & Pedersen, 2014) it is expected that BAB delivers

significantly positive excess returns, and a positive regression intercept in a CAPM regression model. The main hypothesis for this study is the following:

*H<sub>1</sub>*: The betting against beta strategy provides positive excess returns in the OMX Helsinki stock market.

The main hypothesis is to be tested by calculating the performance of the betting against beta strategy for the period of study, ranging from December 2001 to December 2017. In order for the null hypothesis to be rejected, the factor must provide returns higher than the risk-free rate of interest over the period of study.

Another point of interest is to compare the performance of betting against the beta and the market index during different volatility environments. The period of study is divided in low- and high-volatility periods based on the level of uncertainty each month. VIX is used as a proxy for uncertainty. The second hypothesis takes its form as below. The BAB factor is expected to beat the market index in absolute returns during, high-volatility periods.

*H<sub>2</sub>*: Betting against beta overperforms the market index in absolute returns during market downturns and periods of increased volatility.

### 1.3. Intended contribution

Previous research has been done on the subject, which is also introduced in detail in the literature review section. What this paper aims to contribute, is to study the performance of the betting against beta strategy in a small stock market with relatively recent data, and to see whether the findings of Black et al. (1972) apply in modern day financial markets – do low-beta equities provide better risk-adjusted returns than expected by the CAPM, and to what extent? The intention is to recreate the BAB factor of Frazzini & Pedersen (2014), and to test it in the OMX Helsinki market.

As the low-beta strategies have been studied quite extensively, and betting-against-beta is also studied to some extent earlier, other aspects than pure returns are considered in this study as well. In addition to the pure comparison of returns and return-to-risk performance for BAB and the market, the two strategies are also tested and compared against each other during different market environments, such as periods of high volatility

and bear markets. Also, the components of the BAB factor, the low- and high-beta portfolios are studied to obtain a more complete understanding of the drivers of BAB returns.

The objective is to provide a thorough analysis of the betting against beta strategy and its results in the OMX Helsinki stock market. This paper contributes to the group of papers made on low-beta investing and betting against beta by extending the period of study until the end of 2017. Also, to the best knowledge of the author, other papers that examine the BAB factor in the Finnish stock market are not made. The approach of Frazzini & Pedersen (2014) is largely followed in this study, but slight adjustments in methodology are made to customize the methods to fit the relatively small and partially illiquid OMX Helsinki stock market. In any case, the aim is to produce an empirical study on BAB and to obtain results that are comparable to the Frazzini & Pedersen (2014) study.



## 2. LITERATURE REVIEW

In this section, previous academic literature on defensive equity investing will be discussed. Firstly, the studies providing the groundwork for betting against beta strategies are introduced. The empirical failure of the CAPM is already documented in the early 1970's by several authors – these studies will be presented as a foundation and complemented by more recent studies, which extend and affirm the results.

There is no clear consensus of what has driven the overperformance of low-beta and low-volatility assets historically, although there are almost as many theories and suggestions as there are studies on the subject. This literature review section aims to provide, if not straight-forward answers, at least suggestions on why betting against beta has been profitable in the past, and why it may well be profitable in the future as well.

### 2.1. Betting against beta

Black, Jensen, and Scholes (1972), perform empirical tests of the CAPM model, and find conflict between observed results and the predictions of the model. In empirical testing, the actual, observed security market line is found to deviate from what is expected. Slope intercepts are observed to be significantly different from zero, while slope coefficients are found to be either steeper or flatter than predicted by the CAPM, depending on the studied sub-period. The findings of the two latter sub-periods indicate that aggressive stocks seem to underperform in terms of Sharpe ratios, while defensive stocks tend to overperform relative to the expectations of the traditional model. This finding has spurred the establishment of many low beta or “betting against beta” funds, and it is also one of the main motivators for this paper.

Black et. al (1972) create ten portfolios with different levels of systematic risk (beta), using monthly stock market data ranging from 1931 to 1965. They plot average monthly excess returns on portfolios against systematic risk to obtain regression intercepts ( $\hat{\gamma}_0$ ) and slope coefficients ( $\hat{\gamma}_1$ ). Theoretically, if the CAPM were to hold, the regression intercept  $\hat{\gamma}_0$  should be zero. However, the observed values for the intercepts  $t(\hat{\gamma}_0)$  are statistically different from zero. Also, regression slope coefficients ( $\hat{\gamma}_1$ ) are found to deviate significantly from the expected, or theoretical, slope coefficients.

During one of the four ten-year sub-periods in the Black et al. (1972) study, the inclination of the regression slope is not only flatter than expected – it is facing downwards. This violates a basic premise of the CAPM: When market returns are positive, increasing systematic risk should increase returns, not decrease them. During this 10-year period, beta and excess returns on assets had a negative relationship, which is unexpected and inspires further research.

Friend and Blume (1970) study asset returns with some one-parameter investment performance measures that emerged soon after the discovery of the security market line and the CAPM. These measures include those of Sharpe (1964), Treynor (1965) and Jensen (1968). The authors perform empirical tests on the relationship between asset returns against risk, where asset performance is tracked by each of the three metrics. The authors find that Jensen's performance measure (popularly known as Jensen's alpha) had a significant negative relationship with systematic risk (beta) during the period 1960-1968. This conflicts with the CAPM theory, according to which assets with higher systematic risk should be compensated by higher returns. These results are in line with the subsequent findings of Black et. al (1972), as the negative relation between Jensen's alpha and beta essentially means that "CAPM excess returns" for an asset shrink when beta increases, and vice versa.

Frazzini and Pedersen (2014) provide explanations for the excess performance of low beta strategies and introduce a betting against beta (BAB) factor. The authors argue that due to various types of leverage constraints that investors face, high-beta assets are overpriced. This pushes the prices of those assets higher and thereby shrinks their future return potential. Leverage constraints apply to individual investors as well as many institutional investors, such as pension funds, that need to invest in riskier (high-beta) assets to gain higher returns, whereas those without such constraints are able to invest in assets with lower beta using leverage to increase their exposure. (Frazzini and Pedersen, 2014).

As Black et al. (1972) earlier, Frazzini and Pedersen (2014) also observe a relatively flat security market line. They even show that the phenomenon does not only apply to US markets, but to different markets and asset classes worldwide. This finding provides groundwork for their betting against beta factor, which is constructed to be long in low-beta assets, and short in high-beta assets. This factor is market neutral, as the long side (low beta) has been leveraged to a beta of one and the short side (high beta) has been de-levered to a beta of one. The returns on the BAB factor are impressive in different asset

classes and geographically, thereby supporting the authors' theory of low-beta (high-beta) assets being underpriced (overpriced). Table 1 shows some interesting results from Frazzini and Pedersen (2014); stocks are ranked based on their beta and assigned to decile portfolios. P1 has the stocks with the lowest beta, and P10 the stocks with the highest beta. BAB is the betting against beta factor, which is long on low-beta stocks and short on high-beta stocks. Bolded values in the table indicate statistical significance at the 5% level.

**Table 1.** Beta-sorted portfolio returns and betting against beta (BAB) factor results: International stocks, period 1984-2012. Frazzini & Pedersen (2014).

Portfolio	Excess return	CAPM alpha	3-factor alpha	Realized beta	Volatility (annualized)	Sharpe ratio (annualized)
P1 (low)	<b>0.63</b>	<b>0.45</b>	<b>0.28</b>	0.66	14.97	0.50
P2	<b>0.67</b>	<b>0.47</b>	<b>0.30</b>	0.75	16.27	0.50
P3	<b>0.69</b>	<b>0.48</b>	<b>0.29</b>	0.78	17.04	0.48
P4	<b>0.58</b>	<b>0.36</b>	0.16	0.85	17.57	0.40
P5	<b>0.67</b>	<b>0.44</b>	0.22	0.87	18.08	0.44
P6	0.63	<b>0.39</b>	0.11	0.92	19.42	0.39
P7	0.54	0.28	0.01	0.98	20.42	0.32
P8	0.59	0.32	-0.03	1.03	22.05	0.32
P9	0.44	0.15	-0.23	1.09	23.91	0.22
P10 (high)	0.30	0.00	-0.50	1.16	27.12	0.13
BAB	<b>0.64</b>	<b>0.64</b>	<b>0.65</b>	-0.02	8.07	0.95

Frazzini & Pedersen (2014) find leverage constraints to be one explanation for the “low-beta anomaly”. They suggest that investors who are leverage constrained, i.e. unable to use or restricted in using leverage, invest excessively in high-beta assets to gain higher expected returns than the market portfolio. This pushes up the prices of high-beta assets, lowering their future returns. Several studies have documented the relationship between BAB factor returns and leverage constraints (for example Adrian et al. (2014), Boguth & Simutin (2015), Malkhozov et al. (2017) and others).

Adrian et al. (2014) create a leverage factor that is intended to measure funding constraints. The authors find that the leverage factor correlates positively with the returns of the BAB strategy. The interpretation of this is that as funding constraints tighten and

leverage among investors decreases, the returns of low beta assets also decrease as investing in them requires leverage. In these conditions, high-beta assets outperform low-beta assets and the return of the BAB factor is negative. Conversely, when funding is available, leverage increases and low beta stocks outperform their high beta peers, according to the results. Adrian et al. (2014) study U.S. stocks for the period 1968-2009 sorting stocks to decile portfolios based on their ten-year betas and find that the lowest-decile portfolio (low beta) outperforms the highest-decile (high beta) portfolio by 7% per annum.

Boguth and Simutin (2015) proxy the tightness of leverage constraints by the average beta of actively managed mutual funds. According to the study, mutual funds face constraints in borrowing and thus the beta of their investments is expected to reflect the level of their desire for borrowing, and therefore it serves as a proxy for leverage constraint tightness (LCT). The authors observe that when LCT decreases (increases), i.e. when funding constraints loosen (tighten), BAB returns are relatively high (low). Also, Boguth and Simutin (2015) find that the level of LCT predicts BAB returns – after times of non-binding funding constraints (LCT is low), BAB returns are observed to be only half of those returns observed after times of tight leverage constraints (high LCT). These results are found to be even stronger when observing BAB returns over a longer horizon. In overall, the findings of Boguth and Simutin (2015) are in line with the theories of Frazzini and Pedersen (2014) and provide support for the role of leverage constraints as an important driver of BAB factor returns.

Malkhozov et al. (2017) study the effect of capital constraints on global asset returns and observe that the level of illiquidity has significant effects on asset prices. The authors observe that increasing global illiquidity, i.e. decreasing global liquidity, increases the intercept and lowers the slope coefficient of the average global security market line. Malkhozov et al. (2017) explain that this happens because investors that are capital-constrained prefer investing in stocks with higher beta to obtain a higher exposure to the global market factor. This kind of behavior bids up the prices of high-beta stocks and shrinks their future returns – a theory in line with Frazzini and Pedersen (2014), who also attribute the behavior of leverage constrained investors as one cause for the relative underperformance of high-beta stocks versus low-beta stocks. In cross-sectional comparisons, Malkhozov et al (2017) observe that betting-against-beta strategies perform better in countries that have a high level of illiquidity (i.e. low liquidity). This is expected after the observation that the SML is flatter in these countries than in countries with high liquidity.

Karceski (2002) studies mutual funds and argues that the returns-chasing behavior by mutual fund investors is one considerable reason for the failure of the CAPM to price assets. According to the study, mutual funds experience significant inflows of cash from clients during and after good market returns. Funds constantly compete to beat their competitors, since the best-performing ones attract the largest proportion of investments. Karceski (2002) explains that this makes outperforming competitors during bull markets the primary concern of portfolio managers – who then attempt to accomplish it by increasing market exposure by over-allocating funds in high-beta stocks as they tend to provide higher returns than low-beta stocks during bull market conditions.

Karceski (2002) finds similar reasons that contribute to the underperformance of high beta as the earlier studies, however, the perspective of mutual fund investing behavior is new. The study shows that mutual funds overweigh aggressive stocks relative to the overall market. The author explains that the fund's reward for outperforming competitors during bull markets is more important than the same during neutral or bear markets, due to the returns-chasing behavior by investors. This again causes the expected returns for high-beta stocks to shrink, as many other papers introduced earlier have explained. Thus, mutual fund behavior may contribute positively to BAB strategy returns, as the strategy benefits when low beta stocks perform better than high beta stocks.

A recent paper by Cederburg & Doherty (2016) study a conditional capital-asset pricing model in attempt to solve the 'beta anomaly'. The authors comment that previous research on the subject of 'betting against the beta' has focused only on 'unconditional' CAPM alphas. They argue that if portfolio betas change systematically as market volatility and the market risk premium change, unconditional alphas are biased estimates of real portfolio alphas. Cederburg & Doherty (2016) are critical towards the results obtained for low-beta investing and betting-against-beta strategies, and argue that the statistically significant return differences obtained in previous literature for high- and low-beta portfolios can be attributed to biases in performance measures that are unconditional, i.e. measures that do not take factors such as changes in market volatility into account. The authors show that the results of beta-sorted investing strategies become statistically insignificant and lesser in economic magnitude, as conditional alphas are considered. According to Cederburg & Doherty (2016), this is because the conditional alphas consider systematic trends in stock betas and market weights, as well as the time-variance of the distribution of betas.

From Black et al. (1972) to Frazzini & Pedersen (2014), there is a lot of evidence for the success of betting-against-beta strategies. However, the recent paper by Cederburg & Doherty (2016) criticizes the results and argues that they may be, at least partly, due to biases in the used methodology. It is interesting to see whether additional research provides support for the claims of Cederburg & Doherty (2016) in the near future.

## 2.2. Defensive investing strategies

There are many different approaches defensive equity investing. However, they all share the same trait of aiming to achieve attractive risk-adjusted returns by over-weighting securities with less risk and under-weighting riskier securities. Risk in this context generally refers to the level of volatility or market exposure (beta) of the security (Novy-Marx, 2014).

Stocks with low beta or volatility may be called defensive, while stocks with higher betas and volatilities may be called aggressive or risky. The focus on defensive equity papers is often either on beta (see Frazzini & Pedersen, 2014), or volatility (see Ang et. al (2006), Blitz, & Van Vliet (2007), and others). Some papers even study how low beta and low volatility work together (see Baker, Bradley, and Wurgler (2011)).

Defensive investing strategies have been a popular research topic recently, as the turmoil of the financial crisis made investors look for less riskier equity investing options (Frazzini, Friedman, & Kim, 2012). Also, academic research has widely shown that low beta and low volatility have provided attractive returns on both risk-adjusted and absolute return metrics (see Frazzini & Pedersen (2014), Novy-Marx (2014) and others). During the 40 years preceding the financial crisis in 2008, low volatility portfolios overperformed high volatility alternatives and provided high average returns with little downside risk (Baker et al. (2011)).

Baker et al. (2011) argue that the long-run success of low beta and low volatility strategies, which is contrary to the basic supposition that taking higher risk is rewarded with higher returns, might be the “greatest anomaly in finance”. The authors study U.S. stock returns from the 1968-2008 period and find that low risk, whether defined as low beta or low volatility, constantly outperforms high risk during the period of study. Baker et al. (2011) attempt to explain the “low risk anomaly” by behavioral factors and limits to arbitrage, which inhibits sophisticated investors from exploiting the tendency of low

risk stocks to overperform and vice versa. Behavioral aspects are introduced and explained more closely in the following sub-chapter.

Regarding limits to arbitrage, they may make it impossible to take advantage of and thus make the return difference between low- and high-risk stocks disappear. As Asness, Frazzini and Pedersen (2012) note, pension funds and most of the mutual funds often have a restricted capacity of taking leverage. Baker et al. (2011) also observe that the stocks with the highest volatilities are often small and illiquid stocks, which may be difficult or expensive to trade in large quantities, which also may make it difficult or expensive to short them. The study also suggests that institutional investors are less likely to exploit the low volatility anomaly due to *benchmarking*, which causes portfolio managers to often strictly follow a specific index, thus preventing them from exploiting the anomaly.

One interesting paper on the subject, although criticized is Ang et. al (2006). The study finds a strong negative relation between lagged idiosyncratic volatility and average future returns. Idiosyncratic volatility is here defined as the residual of the Fama & French (1993) three-factor model. This finding is rather surprising, as several studies (Merton (1987), Barberis and Huang (2001) and others) suggest that higher idiosyncratic, or firm-specific volatility would cause stock prices to include additional risk premia to compensate the investors for the risk. Ang et. al (2006) describe the findings in the paper as “something of a puzzle”.

Ang et al. (2006) perform empirical tests to find differences in average returns for stocks with different sensitivities to changes in aggregate volatility. Changes in aggregate volatility are proxied by changes in the VIX index. The authors find that the portfolio with the lowest sensitivity to changes in volatility significantly overperforms the portfolio with the highest aggregate volatility innovation sensitivity, as far as by 1.04% in monthly average returns. The results persist after controlling for both the market factor and the Fama and French (1993) three factor model. This result is contrary to many earlier studies that report positive relations to returns for idiosyncratic volatility (Lintner (1965), Lehmann (1990) and Malkiel & Zhu (2002)). In a more recent paper, Ang, Hodrick, Xing and Xhang (2009) extend the geographical scope of the study and find that the same “puzzle” exists globally – stocks with high idiosyncratic volatility in the recent past are found to have significantly lower returns than those stocks that have had a low idiosyncratic volatility recently.

Bali and Cakici (2008) also study the relationship between idiosyncratic volatility and cross-section of expected returns and reject the findings of Ang et al. (2006). They find that the discernibility of a relation between stock-specific volatility and returns is much affected by which methods are used in the handling of the data, such as using specific breakpoints to sort the stocks into volatility portfolios or using a certain data frequency to estimate idiosyncratic volatility. Therefore, Bali and Cakici (2008) reject the existence of a robust and significant relation between idiosyncratic volatility and future returns. Fu (2009) also directs criticism towards Ang et al. (2006) by arguing that a relation between idiosyncratic risk and expected return should not be implied, as idiosyncratic risk is time-varying. Fu (2009) finds opposite results to Ang et al. (2006) and claims that the findings in the controversial paper are mostly explained by return reversal of a group of small stocks that affects the return figures of the high idiosyncratic volatility portfolio.

Jordan and Riley (2015) study performances of mutual funds and observe that volatility significantly predicts future returns of mutual funds. The authors find that low volatility funds gain annual Fama-French (1993) alphas of more than 5% higher than portfolios of high volatility funds. The paper rules out size, lower costs and manager skills as explanators for the outperformance. Jordan and Riley (2015) also show that the volatility of total fund returns is driving the effect instead of idiosyncratic volatility. The conclusion of the study is that portfolio volatility is a powerful indicator of future returns, and the authors attribute the low volatility effect either as an important pricing factor or a large and persistent market inefficiency.

Novy-Marx (2014) studies defensive equity strategies in depth and attempts to isolate the factors that contribute to their relatively strong historical performances. The paper finds that defensive stocks have beaten the most aggressive stocks on a 50-year period, and that volatility-based defensive strategies have delivered significant alphas when evaluating with the Fama and French three-factor model – a result in line with Jordan and Riley (2015).

Table 2 shows the results of DMA (defensive minus aggressive) strategies in Novy-Marx (2014) to demonstrate the low volatility effect combined with size. The ten deciles, or portfolios, are created by sorting companies based on their size. The first decile contains the smallest firms, and the tenth decile the largest firms. The DMA portfolios are created by selling (buying) the 30% of stocks with the highest (lowest) expected volatilities within each decile. The strategy thus shorts high volatility stocks and takes a long position in



defensive stocks. As the table shows, the smallest decile portfolios have the highest monthly excess returns. The results are significant at the one percent significance level.

**Table 2.** Performance of DMA portfolios by size deciles (Novy-Marx, 2014). D stands for decile, and R for return.

<b>D</b>	<i>1</i>	<i>2</i>	<i>3</i>	<i>4</i>	<i>5</i>	<i>6</i>	<i>7</i>	<i>8</i>	<i>9</i>	<i>10</i>
<b>R</b>	1.18	0.82	0.72	0.55	0.47	0.41	0.36	0.19	0.31	0.13

In the analysis of style characteristics of defensive equities, Novy-Marx (2014) finds that defensive strategies tilt towards large size and high profitability. Several different specifications of high profitability, such as gross profitability in Novy-Marx (2013) and book-level operating profitability in Fama and French (2014) have significant negative correlations with volatility. The negative relation between firm size and volatility is intuitively easy to understand. Also, when controlling for size and profitability, Novy-Marx (2014) observes that value correlates negatively with volatility. This adds one more characteristic to the typical high-volatility stock; not only is it small and unprofitable, but it also tends to carry a high valuation.

Novy-Marx (2014) concludes that the high three-factor model alphas of low volatility strategies are largely explained by the exclusion of unprofitable small growth firms in defensive strategies, which the three-factor model is shown not to be able to price (Fama and French 1993, 2014). This can partly be observed in Table 2; the performances of the lowest decile portfolios benefit from the bad performance of small, unprofitable growth stocks. As these stocks typically belong to the high volatility category, the DMA strategy takes a short position in these stocks.

Dutt & Humphery-Jenner (2013) document the ‘low volatility effect’ in both developed and emerging markets, outside the broadly studied U.S. market. They observe, in accordance to earlier studies such as Blitz and van Vliet (2007), Baker et al. (2011) and others, that low volatility stocks significantly outperform high volatility stocks. The authors provide two main explanations for the phenomena; limits to arbitrage due to benchmarking (see Baker et al. 2011) and operating profitability (see Novy-Marx, 2014). They note that firms with low volatility tend to have strong operating performance. This is explained to contribute to stock returns among other things through increased cash

flows, which allow the profitable companies to pursue expansion opportunities and thus increase expected returns on the stock. The findings are observed to be robust even after controlling for faint trading activity (low liquidity) and transaction costs – this finding is especially promising regarding this paper, as a significant amount of stocks in the OMX Helsinki stock exchange have a relatively low trading activity.

Blitz, Falkenstein and van Vliet (2014) analyze the proposed explanations for the tendency of low volatility stocks to outperform high volatility stocks, named as the “low volatility effect”. The authors approach the issue from the perspective of breaking down the problems with CAPM assumptions. Firstly, the assumption of no constraints on taking leverage or short selling is rather unfounded in the actual markets (Blitz et al. 2014). Soon after the emergence of the CAPM, Brennan (1971) shows that restrictions to lending and borrowing may lower the security market line. These leverage restrictions include among others margin requirements, tax rules and bankruptcy legislations (Blitz et al. 2014).

The intuition of the CAPM is, that investors invest in the one efficient portfolio with leverage or de-levered, based on their level of risk aversion. However, as investors face constraints to borrowing, they cannot invest in lower beta assets using leverage and thus have to increase their exposure to equity risk premium by tilting towards high beta stocks. As also noted by Frazzini and Pedersen (2014), this generates excess demand for high beta securities in relation to low beta securities, and may cause the flatness or even inversion of the security market line. (Blitz et al. 2014).

Especially institutional investors such as mutual and pension funds may be unable to allocate a large portion of their portfolio to low volatility stocks due to regulatory constraints. If investors aren't allowed to freely allocate to equities or if they face solvency or capital buffers, they may need to invest in the high volatility segment of the equity market to effectively increase their equity exposure. (Blitz et al. 2014).

### 2.3. Behavioral aspects

There are also behavioral aspects that suggest “overpurchasing” of high-beta assets. A recent paper by Bali, Brown, Murray and Tang (2017) suggests that the beta anomaly, which is the tendency of high (low) beta stocks to provide low (high) abnormal returns, and first documented by Black, Fischer, and Scholes (1972), is partly explained by a phenomenon called “lottery demand”. Lottery demand is demand generated by investors,

who hope to make quick and large profits by investing in securities with high probabilities of short-term gains. These stocks are most often highly sensitive to market movements; thus, they have high betas. This disproportionate price pressure for lottery-demand-based securities is reflected as an increase in the prices of high-beta stocks, which decreases their future return-potential.

Further evidence of the lottery effect is presented in Kumar (2009), which examines the behavioral aspect of the possible high-beta asset overpricing by studying gambling behavior in the stock market. The study shows that some individual investors tend to prefer stocks with lottery-like features. These kinds of stocks have high betas or volatilities and may possibly double or triple in value in a short time, but also bare a significant risk of losing value.

Ilmanen (2012) also arguments that there is a well-documented preference for positive skewness among investors, i.e. demand for enhancing the right tail (the possibility for extreme positive returns) and thus assets or instruments with ‘lottery-like’ characteristics tend to be overpriced relative to actuarially neutral prices. Ilmanen (2012) thus states that low volatility investment strategies should benefit from under-weighting the most speculative and lottery-like investments.

Bali, Cakici and Whitelaw (2011) also present some evidence of investors’ preference for lottery-like returns. The authors study the performance of quantile portfolios where stocks are sorted in quantiles based on their highest daily return during the previous month (MAX). Then, the performance of the portfolios for the following month are examined. As a result, significant monthly return differences of over 1 % are observed between the portfolios with the highest and the lowest daily returns during the previous month. Bali et al. (2017) interpret these results as an indication of investors’ willingness to pay more for stocks that have had extreme positive returns recently – as a result, the prices of these stocks are pushed higher, and thus their future return potential is weakened.

Baker et al. (2011) also note the lottery demand effect, but in addition present two more behavioral factors. First one of these is *representativeness*. In a classic paper focusing on heuristics in human behavior, Kahneman and Tversky (1974), representativeness is presented as the tendency of people to misjudge probabilities of uncertain events based on how much the event resembles something that is familiar to them. In the context of stock picking this could mean, say, an investor is confident about the future gains of a penny stock in the technology sector, as he recalls that the Apple stock was once a penny

stock as well. This way, the heuristic weakens the ability of the investor to make rational decisions and makes her biased. According to Kahneman and Tversky (1974), characteristic to representativeness is that the affected person is insensitive to prior probability of outcomes, sample size and has misconceptions of chance; an example of this could be people falsely believing that the sequence 1, 2, 3, 4, 5, 6, 7 appearing in a lottery is more unlikely than a sequence that seems more random, such as 3, 11, 14, 2, 29, 36, 8. The probability of occurrence for these sequences is naturally the same.

But how does representativeness relate to stock investing and especially to the dominant performance of defensive strategies? Baker et al. (2011) explain this with a theory where investors think of which investments would have been great in the past, such as buying Microsoft shares in their initial offering. Then they look at similar, speculative stocks that Microsoft shares were at the time, and are biased by the representativeness effect – how these stocks remind them of one of the most successful software companies of all time, an investment that would have been the passageway to riches. Meanwhile, the investor forgets the fact that most of the small software companies with a short history aren't very successful, nor are they profitable investments in the long term. To conclude, as a result of the representativeness bias the investor tends to overpay for risky, or aggressive stocks.

The third behavioral bias contributing to stock preferences, according to Baker et al. (2007), is *overconfidence*. Lichtenstein, Fischhoff and Phillips (1982) provide a meta-analysis of studies focusing on calibration, that is, how accurate people are estimating the extent to which they are correct. For example, people may be asked a series of questions, and after each answer they may be asked to assign a probability that their answer is correct. If the actual rate at which the individual is correct in his answers is close to the probabilities that he estimated himself, he can be said to be well calibrated (Fischhoff, Slovic and Lichtenstein, 1979). However, Lichtenstein et al. (1982) find that most often, people overestimate their success rate of being correct, thus, they are overconfident. The authors point that being well calibrated is essential for the decision-making ability, yet they find little evidence of underconfidence, whereas its opposite is evident in many areas.

In the context of equity investing, the effect of the overconfidence bias is rather simple to understand. Investors are overconfident about their ability to predict price intervals for the stock price in the future, and often these confidence intervals are too narrow. The investor might also disagree about the consensus predictions of volatile stocks and trust his own ability to price the stock correctly. Thus, the investor may be willing to pay higher

price for a risky stock as he strongly believes in his own assessment of its return potential (Baker et al. 2007).

Baker et al. (2007) cleverly point out, that one more assumption is required so that overconfidence can be predicted to cause overpurchasing of volatile stocks. This brings an additional behavioral trait to the discussion: *Optimism* must exceed *pessimism* so that the bias, volatile stocks being overpriced relative to less volatile stocks, may happen. Miller (1977) observes that when there is little selling in a market, the demand for a stock is mainly created by those who are most optimistic about its future returns. This means that the prices are often set by optimists, which then leads to securities having higher prices, and thus producing lower future returns (Baker et al. 2007).

#### 2.4. Who is on the losing side?

Regarding the attempts to explain the overperformance of low-beta assets, the theory that constantly emerges from the literature is that the prices of high-beta assets are pushed higher by leverage constrained investors who attempt to increase their market exposure. As leverage constrained investors are not, due to different reasons, able to borrow money to leverage their holdings in low-beta securities, they must increase the beta of their portfolios by buying assets with high betas (Frazzini & Pedersen, 2014). As the prices of assets with high-beta increase due to demand for higher market exposure, their future return potential decreases. Karceski (2002) supports this theory by arguing that mutual funds “over-allocate” funds to high-beta assets while trying to beat their competitors in returns during bullish markets. Mutual funds often have restrictions on borrowing (Boguth & Simutin, 2015), and thus they must allocate funds to high-beta assets instead of increasing market exposure by leveraging low-beta assets. Malkhozov et al. (2017) document that betting-against-beta strategies perform better in conditions of low liquidity – again, the proposed explanation is that capital-constrained investors prefer high-beta securities, as the BAB strategy profits from the relative underperformance of high-beta when compared to low-beta.

All of the previously presented studies propose that the returns of high-beta assets are weakened by investors who ‘over-invest’ in these securities and thus bid up their prices. As the prices of these securities become higher in the present, they have less ‘return potential’ in the future. Either way, this means that some investors are purchasing high-beta assets and are thus suffering in the form of lower returns on their investments, when

compared to those who invest in low-beta securities. Therefore, it could be stated that these investors are paying for the success of the investors on the ‘other side’ of beta. If this is the case, then who are these investors on ‘the losing side’?

As tightness of funding and leverage constraints is a well-documented explanation for the excessive purchasing of high-beta securities (Karceski (2002), Frazzini & Pedersen (2014), Blitz et al. (2014) and others), the investors that seem to belong in this category are those that are unable, or unwilling, to invest in low-beta assets with leverage. Blitz et al. (2014) note that some institutional investors such as pension funds and mutual funds are not allowed to freely allocate large proportions of their funds in low-beta securities, and they may face regulatory constraints regarding leverage usage. In addition to regulatory reasons that make funds *unable* to perform certain activities, funds may also be *unwilling* to invest in low-beta, as is shown by Karceski (2002).

In addition to institutional investors, smaller operators such as individual investors are likely to face funding constraints as well, as they might not have access to cheap funding. However, there are other, more extensively studied factors that might indicate that many individual investors are on the other side of the ‘low-beta anomaly’ – Kumar (2009), Bali et al. (2011, 2017) and Ilmanen (2012), among others discuss “lottery demand” and its implications for investor behavior. Lottery demand refers to the tendency of investors to prefer securities that have a possibility of high returns over a relatively short-term; Ilmanen (2012) formulates the phenomenon as investors’ preference for enhancing the right tail of the return distribution, i.e. the possibility for extreme gains in value. Other behavioral factors that may cause investors to over-allocate to high beta are *overconfidence* and *representativeness*, which Baker et al. (2007, 2011) propose as partial explanations for the overpurchasing of risky assets.

As shown in this section, there is plenty of evidence that leverage constrained investors are likely to suffer from the underperformance of aggressive assets – especially during market downturns, as hypothesis two expects. These investors include some institutional investors as well as individual investors, although the effect of institutional investors is arguably economically more important than the effect of individuals. Meanwhile, individual investors seem to also contribute to the suppression of the high-beta security returns through their irrational behavior. To answer the question – who is on the losing side – it could be concluded that some professional investors (mutual funds) are there as well as individual investors who seek to gamble in stock market.

### 3. ASSET PRICING THEORY

One of the main functions of finance is to price assets. During several decades, theories on asset pricing have formulated. In this section, two important theories that are most relevant to this study are presented.

#### 3.1. Capital Asset Pricing Model

The Capital Asset Pricing Model (CAPM) is arguably one of the most widely known asset pricing models in the field of finance. The CAPM presents the risk and return of an asset as a linear function where risk is measured by beta, which measures how the asset's returns comove with market returns. The higher the beta coefficient, the higher the systematic or market risk is for the asset. According to the theory, the slope of the line should equal the market risk premium; when market risk increases, investors require more compensation for the increased risk (risk premium) and expected returns are higher. The slope, or security market line (SML) is expected to be upward-sloping, as assets with higher betas should have higher risk premiums.

The CAPM is the most important and relevant asset pricing model regarding this paper. Ironically, this paper focuses on betting against beta, an investing strategy which may as well be formulated as "betting against the CAPM". As Black et al. (1972) observed, the model does not hold in empirical tests and based on this finding, low beta investing and ultimately betting against beta as a concept has emerged. Despite its flaws, the CAPM is still an important theory, and a closer examination of it is essential as the concept of 'market beta' is at the very core of this paper.

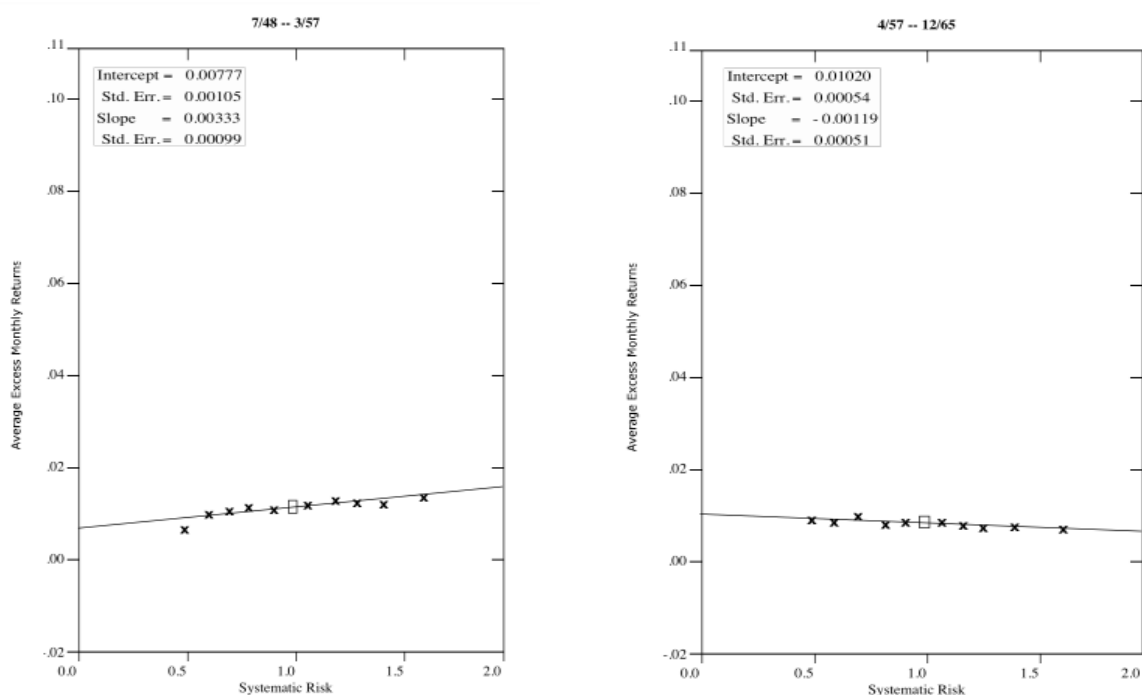
Several studies have contributed to the formation of the CAPM. The most influential papers are arguably those of Treynor (1965), Sharpe (1964), Lintner (1965a), and Mossin (1966), who all have played their part in formation of the renowned model. Also, Markowitz (1952) provides a groundwork for the theory in his ground-breaking paper on portfolio theory and diversification. The capital asset pricing model is presented below (as in Black, 1972).

$$(1) \quad E(\tilde{R}_i) = R_f + \beta_i[E(\tilde{R}_M) - R_f]$$

In the formula,  $\tilde{R}_i$  is the total return for the asset  $i$  on a given period,  $R_f$  is the risk-free rate,  $\tilde{R}_M$  is the return for the market portfolio and  $\beta_i$  is the market sensitivity, or beta of asset  $i$ . The beta is the slope coefficient of the regression line when returns of asset  $i$  ( $\tilde{R}_i$ ) are plotted against returns of the market portfolio ( $\tilde{R}_M$ ). The beta coefficient can also be obtained by dividing the covariance of the asset's and the market's returns by the variance of the market portfolio's returns (Black 1972):

$$(2) \quad \beta_i = \frac{\text{Cov}(\tilde{R}_i, \tilde{R}_M)}{\text{Var}(\tilde{R}_M)}$$

Figure 1 below shows the results from the empirical testing of the CAPM from Black et al (1972) for two different periods. In the figures average monthly excess returns are plotted on the y-axis and systematic risk is plotted on the x-axis. The figure on the left shows the results for the period July 1948 to March 1957; the slope is 'flatter' than could be expected by the CAPM. The figure on the right shows results for the period April 1957 to December 1965; the slope coefficient is surprisingly negative. These results demonstrate the empirical failure of the CAPM.



**Figure 1.** Figures 4 and 5 from Black et al. (1972). Average excess monthly returns versus systematic risk for periods 1/1931-9/1939 and 4/1957-12/1965.



The CAPM theory requires certain assumptions to hold for equation 1. These assumptions are listed below (as in Black, (1972)):

- a) All investors have a common joint probability distribution for the returns of all available assets. Thus, they have the same opinion or view about the possibilities of various prices for the assets at the end of the period.
- b) The expected returns for the assets are normally distributed.
- c) All investors choose a portfolio that maximizes their utility of wealth at the end of the period; the utility function increases at a decreasing rate as the end-of-period wealth increases. Also, all investors are expected to be risk-averse.
- d) All investors may take a long or short position without any limitations in size or in the choice of asset, including the risk-free asset. All investors may borrow or lend without limitations at the risk-free rate of interest.

### 3.2. Efficient Market Hypothesis

According to Fama (1970), the most important role of the capital markets is to allocate capital resources effectively to facilitate ownership, production and investment. The capital markets function ideally, when prices reflect all available information regarding a security. When prices fully reflect available information constantly, the market is called “efficient”. (Fama 1970).

The theory of efficient markets is relevant to this paper as past price information should not indicate information about future returns (Fama, 1970). If past prices do indeed contain information of future returns, the hypothesis of efficient markets is violated even at its weakest form. As the BAB strategy is constructed based on historical information of prices (magnitude and direction of price movements in relation to market, i.e. beta), statistical evidence on the profitability of BAB might indicate the failure of the efficiency hypothesis. Although such conclusions probably cannot be made from the empirical analysis in this paper, the theory is considered relevant to for this study, and is thus presented here.

The efficient market hypothesis (EMH) may be categorized to three different forms, where the information sets used in the tests are different. The three categories of hypothesis (as presented in Jensen, (1978)) are the following:

- (1) The weak form of the EMH, where the information set is considered as the past price history at time  $t$ .
- (2) The semi-strong form of the EMH, where the information set is all information that is publicly available to anyone at time  $t$ .
- (3) The strong form of the EMH, where the information set includes all information known to anyone at time  $t$  – this includes both public and ‘insider’ information.

Fama (1970) tests the hypothesis of efficient markets with these three different levels or subsets of information. In the weak form tests the subset of information considered is the historical price or return sequence data of the security – if the EMH were to hold in its weak form, historical prices should not reflect any information about future prices. In the semi-strong form, the scope of information considered is expanded to “all obviously publicly available information”. Thus, any news or public announcements regarding a stock should immediately reflect in its price for the EMH to hold in its semi-strong form. The strong form tests take a step further: they are concerned with whether some investors have ‘inside’ information – whether some market participants possess information others are not able to access. Thus, for the EMH to hold in its strong form, stock prices should without any delay react to all new information, whether the information is public or not. (Fama 1970).

The model of efficient markets assumes that at any point of time, all available information is reflected in the prices of securities. Fama (1970) points out, that even though data seems to match the hypothesis quite well, the null hypothesis of prices including all available information is an “extreme” one and is not expected to be entirely true in every situation. The division of the hypothesis to weak, semi-strong and strong forms makes it easier to notice at which point the hypothesis of perfect market efficiency fails (Fama 1970).

Weak form tests in Fama (1970) support the efficient market hypothesis. Although some “inefficiencies” are observed, such as consistently positive serial correlations, Fama (1970) comments that even the smallest trading costs would diminish away the expected profits from any attempt to turn these “inefficiencies” into profits. Even though some dependencies are observed etc. in weekly stock returns (Cootner 1962), no evidence of their usability as a basis for a profitable trading strategy is presented. Thus, Fama (1970) concludes that the markets seem to withstand the weak form tests, and past price information does not seem to convey any useful information of future returns.

Semi-strong form tests have been performed around some of the most ‘important’ public announcements regarding publicly listed companies. If the EMH were to hold in its second-strictest form, stock prices should immediately adjust according to latest news such as earnings announcements. Fama, Fisher, Jensen & Roll (1969) study how markets react to announcements of stock splits, which have historically been associated with increased dividends paid out by the company executing the split. The authors note that on average the markets react to the announcements and the associated expected future dividend rates very rapidly, which supports the EMH in its semi-strong form. Ball & Brown (1968) examine accounting numbers from annual earnings reports and find that markets adapt to new information promptly and stock prices adjust accordingly – the semi-strong form hypothesis also has empirical support.

Fama (1970) emphasizes that the strong-form hypothesis is foremost a benchmark against which deviations from an efficient market can be compared. Fama (1970) notes that some deviations from the strong-form hypothesis have been documented: Unattainable to most investors, some individuals at security exchanges can access information of limit orders that have not been executed and may benefit from this information in trading (Niederhoffer & Osborne, 1966). In addition, Scholes (1969) finds, rather expectedly, that insiders in corporations possess information about their firms that others do not. However, Fama (1970) argues that as no other deviations are documented, the model serves as a good estimation of reality for most investors.

The efficient market theory by Fama (1970) has naturally received some criticism. Grossman & Stiglitz (1980) argue that prices cannot fully reflect available information, since there would then be no compensation for those who spent resources to obtain the information. Therefore, there would be contradiction between the incentives to acquire useful information and the efficiency of the markets in spreading information. This means that a precondition for the efficient market hypothesis should be that trading costs and information acquisition costs would always be zero – an assumption rather unrealistic. (Grossmann & Stiglitz, 1980).

Another, less strict and economically more realistic version of the hypothesis, assumes that prices reflect information to such extent that the marginal costs of acting on information do not exceed the marginal benefits of such actions (Jensen, 1978). In a later study, Fama (1991) states that even though costs of information and trading cause ambiguity, there is a more serious problem with testing for market efficiency – the joint-hypothesis problem. It means that it is not possible to be certain whether the observed

results are due to market inefficiency, a bad model or the way the model is implemented (Fama, 1991).

The topic is revisited in a more recent paper, Fama (1991). The author analyses research that has been completed during the 20-year span after Fama (1970). According to Fama (1991), better availability of stock data has facilitated especially event studies during this period. Fama (1991) notes, that because of extensive studying on the subject, there are studies that have found anomalies which indicate that the stock markets would not be efficient – however, he argues that with few exceptions, the evidence supports the EMH.

### 3.3. Fama-French three-factor model

Fama & French (1992b) introduce a three-factor model to explain stock market return variation. The authors identify five common risk factors for securities, of which three are stock market factors and two are related to bond markets. The stock market factors are the overall stock market factor, the size factor and the value factor. The bond-market factors are related to default risk and bond maturity. Fama & French (1992b) find that these factors are capable of explaining average returns on equities and bonds.

Fama & French (1992b) note that empirical studies on cross-sections of stock returns have found that the relationship with the market betas or asset-pricing models is rather weak or merely observable. Other variables, or factors, have shown explanatory power in the cross-section of stocks instead. These factors include size, leverage, price-to-earnings, book-to-market and others (Fama & French, 1992b). The authors attempt to answer to this presented lack of reliable models with their three-factor asset pricing model.

The size effect was documented by Banz (1981), who finds that company size improves the explanatory power of market betas; the author observes that stocks with low (high) market equity have too high (low) returns given their betas. Banz (1981) observes a strong negative relationship between firm size and average returns. Fama & French (1992b) introduce the SMB (small minus big) factor, which is intended to mimic the performance difference between ‘small-stock’ and ‘big-stock’ portfolios – this factor reflects the size-related risk factor in the cross-section of stock returns.

The grounding for the value factor is provided mainly by Rosenberg, Reid & Lanstein (1985), who observe a positive relationship between stock returns and the book-to-market

equity ratios of firms. This means that stocks with relatively high book values relative to market equity values have had higher returns when compared to those stocks with relatively high market valuations relative to book values. Fama & French (1992b) introduce the HML (high minus low) factor, which reflects the return difference between portfolios with high book-to-market ratios and portfolios with low book-to-market ratios. This factor is intended to mimic the book-to-market-related risk factor in the cross-section of stock returns. The HML is also known as the 'value', or 'value versus growth' factor.

In empirical tests, Fama & French (1992a) find that stocks with small market equities earn higher returns than the stock portfolios comprised of stocks with higher market equity. When betas are included in the study, small stocks are observed to provide even more superior returns to large stocks when combined with low beta. The returns are shown to increase linearly, when moving from high beta towards low beta, and from large size to small size. Small stocks with low betas have the highest returns, and large stocks with high betas the lowest returns.

As for the 'value effect', Fama & French (1992a) rank stocks in twelve portfolios based on their book-to-market ratios. The finding is that returns increase linearly when moving from the decile with lowest B/M ratio to the decile with the highest B/M ratio. The portfolio with lowest book values relative to the market valuation earns only a monthly return of 0.3% on average, whereas the portfolio with the highest book value relative to market valuation earns as much as 1.8% per month. Fama & French (1992a) note that the book-to-market effect is stronger than the size effect.

Size and value are combined in Fama & French (1992a) by dividing each size decile into ten "sub-deciles", or portfolios based on their ranked B/M values for the stocks in each portfolio. Returns are observed to increase within each size decile, when the B/M ratio increases. Within value (B/M) deciles, returns are observed to decrease when size increases. The difference in returns between the portfolio with the highest B/M and the portfolio with the lowest B/M is found to be 0.99% per month on average within the size deciles. Similarly, the difference between the lowest- and highest size portfolios within B/M deciles is 0.58 monthly percentages. Thus, the smallest stocks with the highest book-to-market ratios are the most profitable. Fama & French (1992a) note, that book-to-market strongly captures variation in equity returns when size is controlled for, and vice-versa for the size factor when book-to-market equity is controlled for.

Fama & French (1992a) observe a negative correlation for the cross-sections of size and 'value' for individual stocks. They explain this finding by the fact that small firms tend to have poorer prospects on average than large firms, which results in lower stock prices and thus higher book-to-market ratios, as the denominator is smaller. Larger firms for their part tend to have better prospects for the future, which results in higher stock prices and lower B/M values, as the denominator increases relative to the nominator. The authors comment the large firms also have lower average returns due to the higher valuation.

Fama & French (1992b) conclude that the model with three stock market factors, RM-RF (market), SMB (size) and HML (value) succeed well in explaining cross-sectional average stock returns and time-series variation. The authors comment that the residuals from three-factor regressions manage isolating idiosyncratic components of returns better than a model without the SMB and HML factors.

The three-factor model will be used in the empirical section of this paper as the betting-against-beta factor returns are regressed against the three factors. It is of interest to examine the slope coefficients on the various factors as well as the regression intercept and to test whether they are similar to those obtained in the reference study, Frazzini & Pedersen (2014).

#### 4. PORTFOLIO PERFORMANCE MEASUREMENTS

In order to measure the performance of any investment strategy, measurements are naturally needed. During the span of its lifetime, financial literature has provided various measurements that make it possible to compare the performances of portfolios and investment strategies against each other. In this section, some measures useful for evaluating the performance of BAB and the market index are presented.

The objective in any kind of investing is to gain profit for the invested capital. The goal of investing is summed up well in the ground-breaking paper, Markowitz (1952), where the desirable thing in investing is stated to be greater returns, while greater variance of returns is seen undesirable. Thus, the measurements used to evaluate investments should take both the return- and the risk perspectives into account. There are many measures and ratios that do this, of which the most known is probably the Sharpe ratio. This and many other measures will be introduced later in this section.

Markowitz (1952) proposes that variance in returns would be something to avoid for the investors. But is variance really that bad? Large negative swings in the returns of the investments are undesirable, but similar turns in the opposite direction cannot be seen as unwanted by any means – still they reflect to the variance metrics of the investment. Thus, the conclusion is that there is good and bad variance in investing. Some measurements account for this and recognize “downside risk” (Simpson 2015). One measure for such risk is the Sortino ratio, which is also to be introduced later.

On the other hand, while there is desired and undesired volatility, variance in general might affect security prices negatively, if risk-averse investors demand compensation for uncertainty in returns. Instead of measuring average returns, risk-adjusted metrics should be used to perform a meaningful analysis of investment performance (Bodie et al. 2014: 837). This is a widely accepted concept in the world of investing, and at the core of this paper is the analysis whether securities with certain characteristics provide those risk-adjusted returns that are expected from them in relation to securities with other types of characteristics. In this context, these characteristics are mainly the beta and the volatility of the securities, and the expectation is based on traditional models such as the capital asset pricing model and closely related concept of the security market line. Next, a presentation of portfolio performance metrics is provided.

#### 4.1. Sharpe ratio

The ‘reward-to-variability ratio’ (R/V), nowadays widely known as the Sharpe ratio, is introduced in Sharpe (1966). It measures the investments return on volatility, by negating the risk-free rate from the average portfolio return, and divides this with the standard deviation observed during the period of measurement (Bodie et al. 2014). Thus, the ratio shows the reward received (return) per unit of variability (volatility). The higher the ratio, the higher are the returns relative to variation. Therefore, when comparing securities or portfolios, higher Sharpe ratios indicate better risk-adjusted performance.

$$(2) \quad S_P = \frac{R_p - R_f}{\sigma_p}$$

where:  $S_P$  = Sharpe ratio  
 $R_P$  = Rate of return on the portfolio  
 $R_f$  = The risk-free rate of interest  
 $\sigma_p$  = Standard deviation of the portfolio

#### 4.2. Jensen’s alpha

Introduced in Jensen (1969), Jensen’s alpha is a measure of excess returns. It is closely related to, and relies heavily on the CAPM. The ratio shows how much the average return of the portfolio exceeds the return which is predicted by the CAPM, when the portfolio’s beta and the average rate of return of the market are given (Bodie et al. 2014: 840).

When the Jensen’s alpha is positive, the portfolio has exceeded those returns it was predicted by the CAPM to gain. Again, the higher the ratio, the better the portfolio has performed. Unlike the previous metric, Sharpe ratio, this measure does not consider pure variation of a portfolio’s returns, but rather the performance in relation to the CAPM. According to the faulty-proven CAPM (Jensen et al. 1972) stocks with higher betas should earn higher returns. As discussed earlier, the relationship has proven to be weaker than expected or even opposite. This must be considered as well when applying Jensen’s alpha to measure performance – it is based on a model that is something of a failure. However, it is very useful in this paper as security returns are analyzed in relation to the CAPM expectations. Jensen’s alpha is often simply referred to as the ‘alpha’ of the portfolio.



$$(3) \quad \alpha_p = R_p - [R_f + \beta_p (R_m - R_f)]$$

where  $R_p$  = Rate of return on the portfolio  
 $R_f$  = The risk-free rate of interest  
 $\beta_p$  = Beta of the portfolio (systematic risk)  
 $R_m$  = Average market return

#### 4.3. Treynor's measure

Treynor's measure was presented in Treynor (1965), and it was designed to the purpose of ranking investment management companies. The measure closely resembles the Sharpe ratio, the only difference being the denominator where this measure uses the beta of the investment/portfolio instead of volatility.

$$(4) \quad \text{Treynor's measure} = \frac{\bar{r}_p - \bar{r}_f}{\beta_p}$$

The above formula shows the calculation of the Treynor's measure.

#### 4.4. Sortino ratio

Sortino ratio is a portfolio performance measurement tool introduced in Sortino and Price (1994). It is a modification of Sharpe ratio, and a useful tool when portfolio returns are asymmetric and there is negative skewness. Instead of standard deviation as in Sharpe ratio, this measurement uses semistandard deviation in the denominator of the formula. Semistandard deviation only takes positive values for returns below a specified value  $t$ . It is therefore sensitive to skewness in the data as well as to the probability of negative returns, unlike standard deviation which weighs extreme positive and negative outcomes equally. Thus, Sortino ratio allows one to only measure "bad" volatility, or downside risk of returns instead of only looking at aggregate volatility of the investment. Positive deviations of the specified value are ignored in this measure, as they are not considered as bad volatility. Pedersen & Satchell (2002) present a formula for calculation of the ratio:

$$(5) \quad S = \frac{\bar{r}_p - t}{\theta_{rp}(t)}$$

Where  $\bar{r}_p$  denotes portfolio return and  $t$  the target return -  $t$  is often defined as the rate of return on a long-dated bond or Treasury bill.  $\theta_{rp}(t)$  denotes semi standard deviation. (Pedersen & Satchell (2002).

#### 4.5. Leland's Alpha

Leland's Alpha is presented in Leland (1999). It resembles Jensen's alpha, but instead of using the beta measure of the portfolio  $\beta_p$ , a modified risk factor  $B_p$  (Leland Beta) is used instead. The main difference between the risk factors ( $\beta_p$ ,  $B_p$ ) is that  $B_p$  considers the skewness, kurtosis and other characteristics that describe the shape of the return distributions. Leland (1999) heavily criticizes the CAPM, since the model strongly assumes that all portfolio returns are normally distributed, which is, per Leland (1999) most often not the case.

Leland's model requires independently and identically distributed returns for the market portfolio at each moment of time, and that the markets are "perfect" and therefore there are no transaction costs. The formula for Leland's Alpha is presented below. (Leland, 1999).

$$(6) \quad \alpha_p = E(r_p) - \beta_p [E(r_{mkt}) - r_f] - r_f$$

where  $E(r_p) = r_f + \beta_p [E(r_{mkt}) - r_f]$

$\alpha_p$	=	Leland's Alpha
$E(r_p)$	=	Expected rate of return for portfolio
$\beta_p$	=	Leland's Beta
$E(r_{mkt})$	=	Expected market return

## 5. DATA AND METHODOLOGY

The data used for the empirical part of this paper includes monthly stock data from the Nasdaq Helsinki stock exchange, as well as monthly data for the market index and risk-free interest rates. Additionally, data for the VIX index is obtained. This chapter will present the data and data sources used and will make ground for the later chapter that introduces and analyses the results of the empirical tests.

### 5.1. Data

Data sources used include University of Vaasa databases, Nasdaq OMX Nordic, Bank of Finland and the Chicago Board of Exchange. In the stock data set, only stocks that are listed on the main market in Helsinki are included; stocks that are listed in alternative markets such as Nasdaq First North are not inside the scope of this study, as the focus is on the more liquid main exchange. The ‘market’ index used in this study is the OMX Helsinki CAP Total Return Index (OMXH CAP GI), for which monthly data is also collected. The index is market capped. Besides serving as a return benchmark, it is used as reference against which the betas for the stocks are calculated. The index considers paid dividends as returns that are included in the index, and it thus differs from the commonly used price index (OMXH PI), which only considers stock prices and thus excludes dividends. Therefore, OMXH CAP GI is a more relevant benchmark than the price index, as the stock data takes dividends into account as well.

Risk-free interest rates are yields for the benchmark five-year Finnish government bonds – a relevant ‘risk-free’ investing alternative for the stocks in the Nasdaq Helsinki stock exchange. This data is obtained from Bank of Finland. VIX index data is collected to serve as a proxy of market ‘nervousness’. The intention is to examine the performance of BAB and the low- and high beta portfolios separately during periods of different investor sentiment. VIX, also known as the “investor fear gauge” (Whaley, 2000) serves this purpose by showing higher values when investors expect higher volatility, reflecting option implied volatility over the next 30 days (Dash & Moran, 2005).

Nasdaq Helsinki is used as the market of study, because to the author’s best knowledge, there are not many studies that test betting against beta or low beta strategies on that market. Although Frazzini and Pedersen (2014) study the performance of the ‘betting

against beta' factor using international equity data, including Finland, the focus of the study is not on Finnish stocks – the results regarding Finland are presented in quite condensed form without further analysis or discussion. Furthermore, the data used in this paper extends the period of study longer, until the end of 2017 (Frazzini and Pedersen (2014) only study data until 2012).

The period of study ranges from December 2001 to December 2017, thus there are 193 monthly periods for which the portfolios are formed, and the BAB performance calculated. The data consists of 114 stocks traded on the main list of the Nasdaq Helsinki stock exchange. The requirement for a stock to be included in the data set is, that there is uninterrupted monthly price data for the stock for at least 36 months during the examination period – this is required to calculate the beta for a stock. The methodology for the beta calculation is presented in the next sub-chapter.

According to Nasdaq, the OMX Helsinki CAP index contains all the shares listed on the Helsinki Stock Exchange. It is thus designed to reflect changes and the overall status in the market, and it mirrors the performance of an index strategy where dividends are re-invested. As of the end of 2018, the largest sector weights are on the financial and industrial sectors, both with a share of larger than a fifth. Technology and basic materials are the next-largest sectors with weights of approximately 12% each. Based on the most important (largest) sectors, the Nasdaq Helsinki market is rather cyclical – however, this should not be a cause of trouble regarding the study, as stocks can still be divided to 'defensive' and 'aggressive' sides according to their three-year betas and the corresponding median betas of the whole set of stocks.

**Table 3.** Descriptive statistics of OMX Helsinki CAP Total Return index monthly return series. The data set ranges from 12/2001 to 12/2017. Data source: Datastream.

	OMXH CAP GI
Mean	0.00917
Median	0.01477
Maximum	0.26587
Minimum	-0.15824
Std. Dev. (annualized)	0.18037
Skewness	0.11600
Kurtosis	2.91239
Observations	193

The average monthly return for the benchmark index is 0.9%, while the largest observed return is 26.6% and the lowest -15.8%. The examination period includes one significant stock market crash: the financial crisis of late 2008 and early 2009. Also, stock markets especially in Europe suffered from the ‘Eurozone crisis’ around 2011. Besides this, stock markets have been relatively stable in terms of volatility and have provided positive returns, as reflected in the mean and median monthly return numbers in the table above.

Table 4 below shows the descriptive statistics for the return sets of low and high beta stocks separately. Low beta stocks are stocks with 36-month betas lower than the median and high beta stocks are stocks with betas higher than median, correspondingly. These stocks are later assigned to portfolios to form the ‘short’ and ‘long’ sides of the BAB strategy. Stocks are assigned to low-beta and high-beta portfolios each month, according to their betas from the preceding 36-month period.

**Table 4.** Descriptive statistics for the monthly return series of high- and low beta stock portfolios during the period 12/2001 – 12/2017. Stocks in the left-hand column have betas below median, and stocks in the right-hand column have betas higher than median.

	Low beta stocks	High beta stocks
Mean	0.0064	0.0040
Median	0.0058	0.0064
Maximum	0.0917	0.3012
Minimum	-0.0909	-0.2931
Std. Dev. (annualized)	0.1181	0.2262
Skewness	-0.1824	-0.2095
Kurtosis	0.1945	3.8032
Correlation	0.8231	
Observations	193	193

The table above shows some of the key characteristics of the two subsets of stocks. Median monthly returns for both sets are close to 0.6% per month, but the lower mean return figure for high beta stocks (0.40% vs 0.64% for low beta) indicates that the high-beta stocks may have experienced more declines versus inclines relative to the low beta alternatives. The volatility figures (standard deviation) show that the low beta group is less volatile with annualized volatility approximately only a half of the high beta group’s figure. Also, the two sets are highly positively correlated, as could be expected as well –

stocks with different betas could have a perfect positive correlation, but the other could still have higher volatility. Both high and low beta stocks tend to comove with the market, but high beta stocks are more sensitive to it.

## 5.2. Methodology

The methodology used in this paper is almost similar to that in Frazzini and Pedersen (2014), which ranks stocks monthly based on their beta, and takes a short (long) position in high-beta (low-beta) stocks. However, there are some differences in the implementation of the basic idea. Frazzini and Pedersen (2014) weigh stocks inside the long and short portfolios in such manner that stocks with high beta have the largest weight in the high beta portfolio (short side), and stocks with the lowest betas have the largest weighting on the low beta portfolio (long side). As the amount of stocks included in the portfolios in this paper is only 62 at minimum (in the beginning of the period of study), the weighing-approach of Frazzini & Pedersen (2014) would assign excessive weights on those individual stocks with the highest and lowest beta coefficients. Therefore, the stocks inside the high- and low beta portfolios are equally weighted in the empirical analysis.

The aim of this paper is to compare the performance of low-beta stocks versus high-beta stocks and to examine the performance of a “betting against beta” strategy on various metrics. Before the statistical analysis, the BAB strategy and the portfolios need to be constructed. The next sub-section focuses on the formation of the long and short side portfolios and explains the details and background for the strategy. The sequential sub-chapter presents and explains the methods used in the empirical analysis.

### 5.2.1. Formation of betting against beta portfolios

Apart from few exceptions and differences in details, the BAB strategy is formatted in the same manner as in Frazzini & Pedersen (2014). Stock betas are calculated using monthly observations for a 36-month period. Therefore, each stock at time  $t$  has a beta estimation based on the beta calculated from the preceding period of 36 months. Each month, stocks are sorted to long- and short portfolios based on their betas during the previous three-year period; stocks with betas below the median beta are assigned to the long side portfolio. Conversely, stocks with betas higher-than median are assigned to the short side portfolio. This way, one half of the stocks at each time is either in the long portfolio or in the short portfolio.

The performance of these long- and short side portfolios is calculated for each month using the monthly return data. After each month, the portfolio is rebalanced as explained earlier. The return of the BAB strategy equals the return of the long portfolio minus the return of the short portfolio – the specific calculation method is introduced shortly.

The portfolio is also balanced in such manner that it has the estimated (ex-ante) beta of zero, thus the BAB portfolio is beta-neutral. This is achieved by weighing both the low and the high beta portfolios with one divided by the beta of each portfolio – the beta used here for each side is the average beta of the portfolio before the performance calculation period, in  $t-1$  (ex-ante). The same approach is used in Frazzini & Pedersen (2014) as well. The realized betas (ex-post) may, and most likely will, differ from the estimated betas, but still the average betas stay relatively close to zero until the portfolios are rebalanced again after the one-month period.

Frazzini & Pedersen (2014) even weigh stocks inside the long and short portfolios so that stocks with the highest (lowest) beta in the short portfolio have the largest (smallest) weight, and vice versa in the long portfolio. This amplifies the effect of stocks that are in the far ends of the volatility spectrum. As noted earlier, the amount of equities on the OMX Helsinki main list is relatively small. For this reason, this approach is not seen as suitable for this study, as individual stocks could have an unproportionate effect on the portfolio return, if it were used.

### 5.2.2. Calculating the betting against beta strategy performance

The methodology used to calculate the return for the betting against beta strategy, similar to Frazzini & Pedersen (2014), is presented below:

$$(7) \quad r_t^{BAB} = \frac{1}{\beta_{t-1}^L} (r_t^L - r^f) - \frac{1}{\beta_{t-1}^H} (r_t^H - r^f)$$

The total return of the strategy is expressed in excess return over the risk-free rate. The portfolio is “self-financing”; the low beta portfolio (long side) is financed by “short-selling” the risk-free rate and the proceeds for short-selling the high beta portfolio (short side) are invested in the risk-free rate (Frazzini & Pedersen, 2014). Like discussed previously, both the short and the long side are balanced so that the BAB portfolio has an ex-ante beta of zero – the low beta portfolio is leveraged to a beta of one, and the high-beta is de-leveraged to have an ex-ante beta of one, respectively. For example, if the low-

beta portfolio has an estimated beta of 0.7, 1.43 euros is invested in it instead of one ( $1/0.7 = 1.43$ ) to ‘scale’ the beta to one. The same is done for the high-beta portfolio in order to scale its beta to one: If the beta of the high-beta portfolio is 1.43, 0.7 euros is invested in it ( $1/1.43 = 0.7$ ) This is the same approach as in Frazzini & Pedersen (2014), who note that as the BAB factor has a beta of zero, it is market neutral. Also, they state that BAB is similar to HML and SMB factors in such way that it is a factor which is the difference between excess returns.

Frazzini & Pedersen (2014) predict that the expected excess return for BAB is positive, and that it increases as funding constraints tighten – this prediction is shown more formally in equation (9) below.

$$(8) \quad E_t(r_{t+1}^{BAB}) = \frac{\beta_t^H - \beta_t^L}{\beta_t^L \beta_t^H} \psi_t \geq 0$$

Where  $\psi$  measures funding constraint tightness (funding constraint tightness is out of the scope of consideration in this paper) at time  $t$ . This prediction is in line with the findings in Adrian et. al (2014) and Malkhozov (2017), that both discover the relationship between leverage constraints (or funding constraints) and BAB returns.

Another prediction in Frazzini & Pedersen (2014) is that BAB returns increase as the beta spread in time  $t-1$  (ex-ante) increases. Beta spread is the difference between the betas of the high-beta (short position) and the low-beta (long position) portfolios. This prediction is tested in the empirical section as well, and is shown more formally below in formula (9):

The expected excess return of BAB increases, as the ex-ante beta spread

$$(9) \quad (\beta_t^H - \beta_t^L) / (\beta_t^L \beta_t^H) \quad \text{increases.}$$



## 6. EMPIRICAL RESULTS

Based on earlier literature, it is expected that a low beta strategy will provide higher risk-adjusted returns when compared to high beta strategies (Baker et al. (2011), Frazzini & Pedersen (2014), and others). Based on the same findings, the low beta portfolio is also expected to beat the high beta portfolio in absolute returns. Thus, the return of the betting-against-beta portfolio is expected to be positive (first hypothesis). The BAB strategy is expected to provide competitive returns at least during market downturns, as it profits from the bad performance of the most market sensitive stocks, and simultaneously holds the less-sensitive stocks on the long side. It is expected that the zero-cost BAB strategy overperforms the market during market downturns (second hypothesis). As the betting-against-beta is a ‘zero-cost’ strategy, it can be considered a success if it provides significantly positive excess returns. As the stock markets have cumulatively gained value during the examination period, the market index most likely beats BAB in absolute returns over the entire period – however, it is interesting to examine how BAB has performed in risk-adjusted returns.

### 6.1. Performance of the high- and low-beta portfolios

In this section, the returns of the low-beta and the high-beta portfolios are examined throughout the period. At this point, both portfolios are considered from a long position perspective. The BAB strategy performance with a long position in the low-beta portfolio and a short position in the high-beta portfolio is examined in the following chapter – the purpose of this chapter is to provide a grounding for the next chapter by analyzing the two main components of the BAB; the low- and high-beta portfolios.

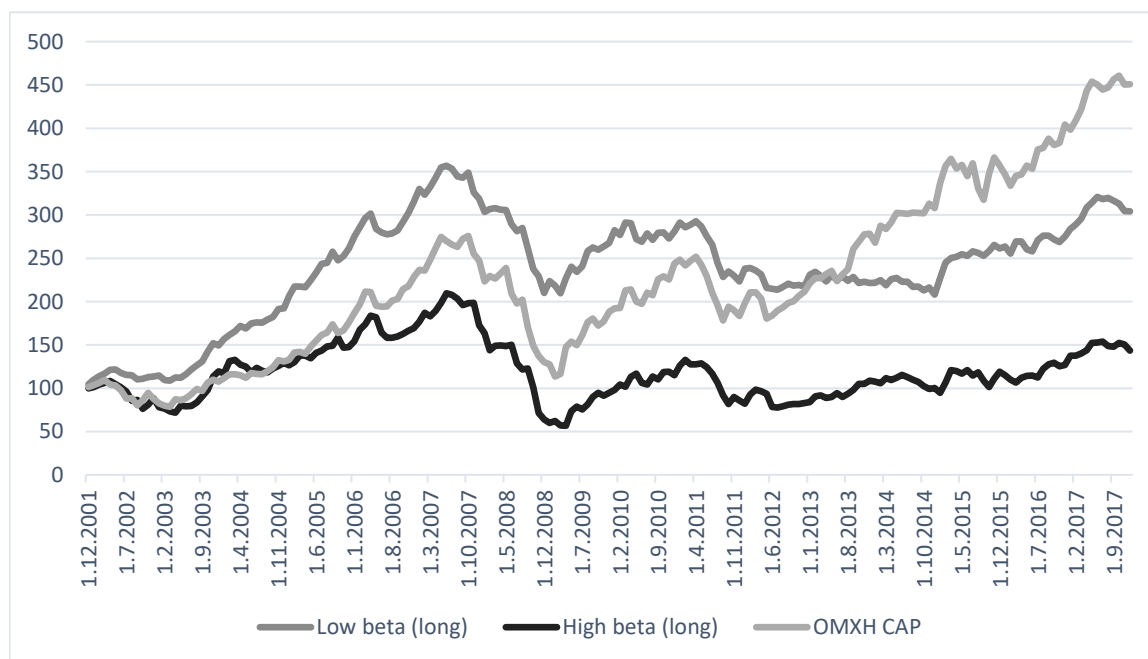
The examination period begins soon after the “tech bubble” crash in the beginning of the 2000’s. In December 2001, when the first monthly performance for the portfolios is calculated, the market index had not yet reached its bottom. Hence it is not surprising that the high-beta portfolio performs poorly in the beginning of the period of study, while the low-beta portfolio is stable. After the first two or three years the stock markets started a five-year climb, and both portfolios gained value along with the market. In 2008, ‘the music stopped’ as the financial crisis melted asset values. The crisis hit the high-beta portfolio hard, erasing all of its gains from the beginning of the period. The low beta portfolio also suffered losses, but due to its strong performance in the preceding years its

overall performance was still clearly positive. At the bottom of the financial crisis in March of 2009, the low-beta portfolio had gained a total of 109.5% (10.7% p.a.) in value from the beginning of the period, despite the harsh decline due to the crisis. Meanwhile, the high-beta portfolio had fallen by 43.1% (-7.4% p.a.) in value.

The low beta portfolio lost relatively less in value during the bear market, and also it had gained more in value relative to the high-beta portfolio during the preceding bull markets. This preliminary result is rather promising, and it is also somewhat expected according to the literature covering low beta investing (Blitz et. al (2007), Baker et. al (2011), Novy-Marx (2014) and others). The difference in the performance is remarkable.

After the financial crisis, the era of “quantitative easing” began. Apart from the eurozone crisis in 2011-2012, the global stock markets, as well as Helsinki, climbed steadily until the end of 2017, where the examination period ends. Figure 1 below visualizes the performance of the low- and high-beta portfolios and compares them against the market index. It is assumed that an equal amount of money is invested in each portfolio (long and short) in the beginning, and that no further investments are made. The holdings in each portfolio may change monthly as the betas of stocks change – as a stock obtains an ex-ante beta higher (lower) than the median, it is assigned in the high-beta (low-beta) portfolio.

**Figure 2.** Performance of low-beta and high-beta portfolios versus OMXH CAP GI during 12/2001-12/2017.



In Figure 2 (above) the graph that is on the top until 2013 represents the low-beta portfolio, into which the stocks with betas lower-than-median are selected each month. The graph that ends up lowest, represents the high-beta portfolio, which contains the rest of the stocks, that is the stocks with betas higher-than-median. The graph that lies between these two until 2013, and is on the highest level in the end, is the market portfolio (OMXH CAP). What is interesting in this chart is the performance of the low-beta portfolio prior to the financial crisis – the portfolio more than triples its value before 2008. Also, the performance of the market index after the eurozone crisis is interesting – in 2013, it seems to outperform both the low-beta and the high-beta portfolios, which seems counter-intuitive as the market could be expected to be an average of the two. This unexpected move in the market index may be due to the slight discrepancy between the contents of the market index and low- and high beta portfolios, as explained earlier. However, besides of this brief period, the returns are aligned.

Even though the main purpose of this study is to focus on the BAB strategy, it is both interesting and relevant to pay closer attention to the two main components of the strategy – the low-beta and the high-beta portfolios. It is reasonable to examine the performance and characteristics of the high- and low-beta portfolios before the two portfolios are combined together and the BAB factor is formed. A distinct analysis of the components of BAB is necessary to build a more complete understanding of the strategy and the drivers of its performance.

**Table 5.** Performance of low- and high-beta portfolios and the market index (OMXH CAP GI) during 12/2001-12/2017.

	Low beta portfolio	High beta portfolio	OMXH GI
Return (p.a.)	0.076	0.048	0.110
Volatility	0.118	0.226	0.180
Beta	0.488	1.158	1.000
Sharpe ratio	0.430	0.007	0.430
Sortino ratio	0.684	0.009	0.633
Jensen's Alpha	0.013	-0.088	0.000
Treynor ratio	0.104	0.001	0.077

Table 5 above shows the performance of the two portfolios, that will later become components of the BAB strategy. The table presents in numbers the same information, that Figure 2 visualizes. In calculation of the Sharpe and Sortino ratios, the risk-free rate is the average yield for five-year Finnish government bonds. In Sortino ratio calculations, the minimum accepted return (MAR) is the risk-free rate and returns higher than this are

thus excluded when the ‘semistandard deviation’ is calculated. The returns of the low beta portfolio are impressive, as it outperforms its ‘aggressive’ peer in absolute returns, and competes well with the market portfolio on risk-adjusted return metrics – Sharpe ratios are equal for the market and the low-beta portfolio, but the latter has a higher Sortino ratio indicating stronger ‘downside risk’ adjusted performance. Especially notable in the table is the low volatility of the defensive portfolio, which positively affects both the Sharpe and the Sortino ratios. Jensen’s alpha is positive for the low-beta portfolio and negative for the high-beta portfolio – this indicates that the former has returned more than expected by the CAPM, and the latter has returned less than the model would have predicted. Low-beta has an impressive Treynor ratio as well, while the corresponding figure for high-beta is merely positive. This ratio indicates excess return over the risk-free rate relative to each unit of market risk (beta). The risk-free rate is 2.12% on average, which is almost equals the average return on the high-beta portfolio, hence the low Treynor ratio of 0.001.

Next, a step closer to the complete BAB factor is taken. In the following examination, any rescaling of betas is not done yet – both the low-beta and the short-beta portfolios have equal weightings. The return difference between the two portfolios should give some indication of BAB performance.

**Figure 3.** Performance of the low-beta and high-beta portfolios, with a short position taken in the high-beta portfolio during the period 12/2001-12/2017.

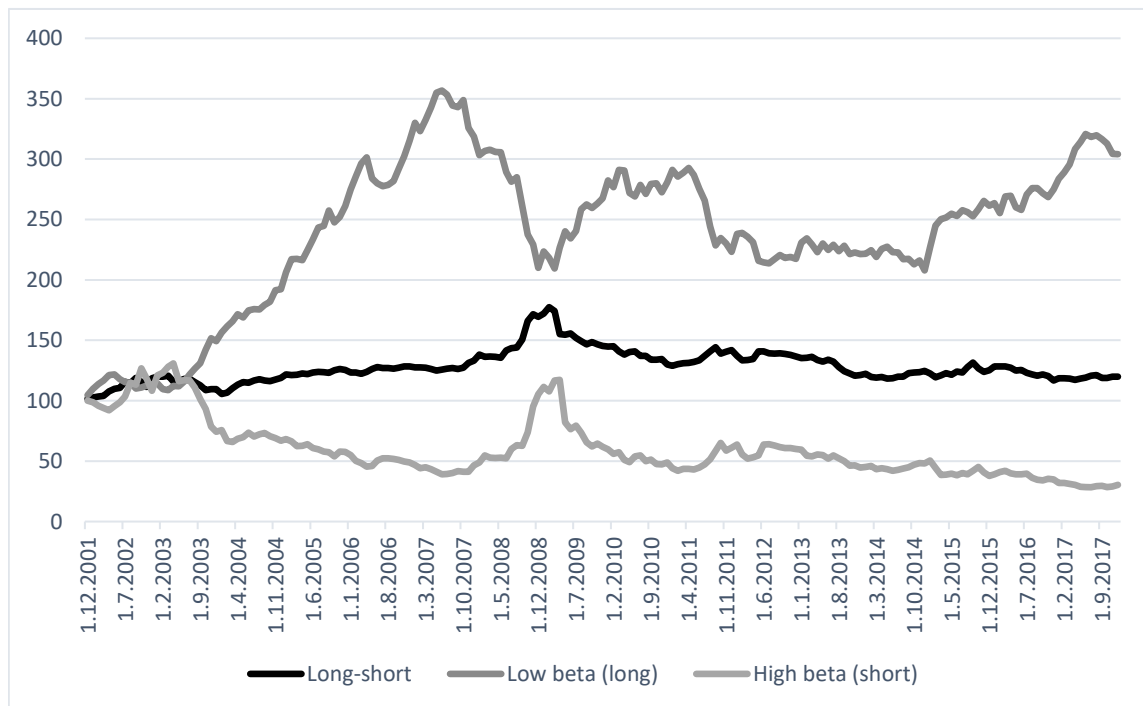


Figure 3 shows the performance for the long (low-beta) and the short (high-beta) portfolios with the same assumption as earlier: An equal amount is invested in both portfolios in the beginning. The low-beta curve is similar to that in Figure 2, but curve representing the high-beta portfolio is “inverted”, as a short position is taken in it. The dark curve presents the total return of investing 50% in low-beta and 50% in high-beta in the beginning.

As it can be observed from the graph, the long-short strategy has steady gains until the financial crisis which is can be noticed as a steep decline in the low-beta portfolio and a rapid increase in value of the high-beta portfolio. After the crisis, the strategy begins a steady decline which continues until the end of the period. This was mostly a period of low volatility and climbing stock prices, which may not be ideal conditions for betting-against-beta – whether or not there is a relationship between VIX and the BAB, is examined later. However, these initial results are promising regarding the expectations for the capability of BAB to provide positive total returns during the period.

It is important to note, that the graph shows a pure difference in the performance of a long position taken in low-beta stocks and a short position taken in high-beta stocks. Therefore, it does not quite yet represent the BAB strategy, in which the portfolio beta is rebalanced monthly, and the construction of the factor is otherwise different as well. However, the long-short curve can be viewed as a preliminary indicator of the BAB strategy performance – the effect of beta rescaling will be analyzed later as the BAB factor is ‘completed’.

Table 6 below shows the results for regressing low- and high-beta portfolio returns against the benchmark index returns. The regression models (OLS) used are similar to the CAPM regressions in Frazzini & Pedersen (2014). The models are the following:

$$(10) \text{ Low beta regression: } R_{L\beta,t} = \alpha_{L\beta} + \beta_i R_{OMXH,t} + e_{L\beta,t}$$

$$(11) \text{ High beta regression: } R_{H\beta,t} = \alpha_{H\beta} + \beta_i R_{OMXH,t} + e_{H\beta,t}$$

In Table 6, statistical significance is indicated by asterisk’s in the following manner: significance at the 1% level = \*\*\*, significance at the 5% level = \*\*, significance at the 10% level = \*.

**Table 6.** Results of simple OLS regression of low-beta- and high-beta monthly returns on monthly market (OMXH CAP GI) returns for the period December 2001 – December 2017.

Variable	Low beta, $R_{L\beta}$		High beta, $R_{H\beta}$	
	Coefficient	t-statistic	Coefficient	t-statistic
A	0.0019	1.1287	-0.0066***	-3.5837
$R_{OMXH}$	0.4877***	15.4347	1.1576***	33.1679
Standard error	0.0228		0.0252	
Adjusted $R^2$	0.5527		0.8513	
F-statistic	238.23		1100.11	
F-stat. prob.	0.0000		0.0000	

The results show, that low beta has a positive intercept (alpha) against the market – however, it is not statistically significant. The slope coefficient (beta) is low and positive, as should be expected from a low-beta portfolio. This figure is significant at the 1% significance level. As could be predicted based on Black et al. (1972) and later studies on the faultiness of CAPM, the high-beta portfolio has a negative alpha coefficient when regressed against the market. The coefficient is also statistically significant at all conventional significance levels. Beta for the high-beta portfolio is higher than 1 as could be expected, and it is also significant.

A note regarding the analyzed data set must be made: all the stocks that are included in this study (in the low- and high beta portfolios and in the construction of BAB) are part of the market index, but it does not perfectly work the other way – the market index may also include stocks at various times that are not part of the sample of stocks considered in this study. Some stocks are left out due to insufficient observations, as each stock included in the analysis is required to have at least 36 months of price data between the period 12/1998-12/2017 in order to be accepted in the sample. This is because three years of consecutive returns are required for each stock to calculate its beta. Also, all stocks that are part of the sample of stocks were listed in the exchange in the end of December 2017 – thus, those stocks that have previously been part of the index but were removed due to various reasons are not included. Therefore, the stock sample might also suffer from ‘survivorship bias’ to some extent.

The initial results of the distinct comparison of both sides of the to-be betting-against-beta strategy are promising with regard to the BAB factor. As expected from the review of previous literature, the ‘low beta anomaly’ seems to, prior to further analysis and

robustness checks, exist in the Helsinki market as well – the regression coefficient for low beta was positive, although not significant, and correspondingly negative for the high-beta portfolio, while being statistically significant. In the next sub-chapter, we move forward to analyze the results of the ‘complete’ BAB strategy – BAB with beta rescaling.

## 6.2. Betting-against-beta portfolio performance

In this chapter, the results of the betting-against-beta strategy are analyzed. Firstly, the descriptive statistics of the monthly BAB factor data set are shown, and the factor is compared against the market portfolio. BAB returns are first shown without rebalancing the betas, that is before the portfolio is made beta-neutral. After that, the rebalancing is included in the examination and the results of the beta neutral BAB factor are introduced. Later, the return drivers of the strategy are broken down, and its performance during different times and volatility environments are examined. Also, an attempt to present the key problems and shortcomings of the approach is made.

As explained in closer detail in the methodology section, some assumptions are made regarding the implementation of the strategy. It is assumed that money can be both borrowed and lent at the risk-free rate. Also, no costs for borrowing or trading the securities are considered. Money is lent at the risk-free rate and invested in the low-beta portfolio. On the other side of the strategy, the high-beta portfolio is “sold short” and the proceeds invested in the risk-free rate. Therefore, the return of the BAB strategy each month, when the beta rebalancing is not considered, is equal to the excess returns for the low-beta portfolio minus the returns for the short-beta portfolio. The return is presented in excess returns over the risk-free rate. When the beta rebalancing is also considered in later examination, the return of BAB is calculated similarly with the exception that excess returns of both sides (long and short portfolios) are multiplied with one divided by the beta of each portfolio – this way, the factor is ‘beta-neutralized’, as each side (long and short) is rescaled to have a beta of one.

### 6.2.1 BAB without beta rescaling

In this section, results of the betting-against-beta strategy without the ‘beta-rebalancing’ are presented. The difference between the returns of low- and high-beta portfolios was shown earlier, and it differs from the BAB factor without rebalancing in the way that BAB is not the average return of two portfolios – it is a ‘self-financing, zero-cost strategy’

(Frazzini and Pedersen, 2014), and thus the return is calculated by subtracting the excess returns of the high-beta portfolio from the excess returns of the low-beta portfolio. Another difference is that, unlike BAB, the earlier shown comparison between the low-beta (long) and high-beta (short) portfolio returns did not consider excess returns, but raw returns. Descriptive statistics from the monthly BAB returns prior to beta rebalancing are shown in table 7.

BAB returns prior to rescaling are positive on average, and less volatile than market index returns. However, the market index has notably higher mean monthly returns. The correlation between the two is negative and surprisingly high, which is not expected as BAB has at this stage an equally weighted position on both halves of the sample of stocks; long position is low beta and short position in high beta. The explanation for the highly negative correlation could originate from the high-beta portfolio, as it is more sensitive to the market and a short position is taken in it.

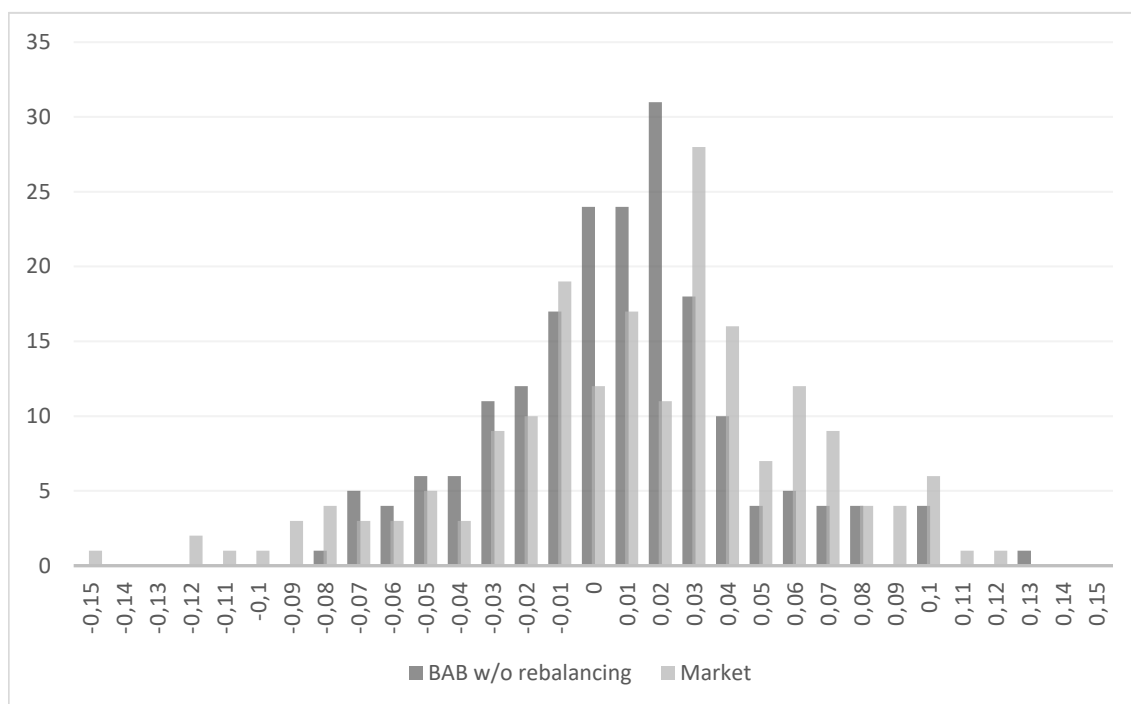
It should be noted, that no final conclusions should not be drawn from the comparison of BAB versus the market, as BAB is a zero-cost strategy. Therefore, its success should not be judged solely against market returns. However, the comparison against market index results is presented for illustrative purposes, and to highlight the difference in characteristics between the two. Any leveraging is not yet used at this stage, which also makes the comparison of BAB and the market index more meaningful. What is rather surprising, is the negative correlation of -0.83 between the market and BAB. The correlation figure is expected to move closer to zero after the portfolio betas are ‘rescaled’.

**Table 7.** Descriptive statistics of monthly excess returns on BAB without beta-rescaling vs monthly excess returns on the market index during 12/1999-12/2017.

	BAB w/o rescaling	OMXH GI
Mean	0.23%	0.92%
Median	0.28%	1.48%
Max	20.22%	26.59%
Min	-21.84%	-15.82%
Standard dev.	14.53%	18.04%
Skewness	-0.0295	0.1160
Kurtosis	5.8659	2.9124
Observations	193	193
Correlation		-0.83



Figure 4 graphs the return distribution for the market index (OMXH CAP GI) and the BAB without beta rebalancing. As was noted from Table 7 that presents descriptive statistics for the two data sets, BAB has a high kurtosis when compared to the market index – a larger proportion of observations is clustered close to the center. BAB has negative skewness, but the market index returns are, perhaps surprisingly, positively skewed. After removing extreme outliers, most of the ‘extreme’ return observations belong to the market index. The market has had positive returns of 3% per month or higher more often than BAB but has also suffered losses higher than 9% per month several times, while the BAB has mostly avoided such collapses.



**Figure 4.** Distribution of returns for the market index and the BAB factor prior to rescaling, December 2001 – December 2017.

The descriptive statistics indicate that BAB would, at least before beta rebalancing, lag the market index in absolute returns. However, as BAB is intended to be a zero-cost, market neutral strategy, it is not expected that it would earn similar returns than the market during a period when stock markets have mostly been ‘bullish’. According to one of the hypotheses for this study, BAB is only expected to beat the market in absolute returns during bear markets or in a high market volatility environment. During times of rising and less volatile stock markets, providing statistically and economically significant excess returns over the risk-free rate could be considered as a success for the BAB strategy.

The first hypothesis for this study is that BAB would provide positive risk-adjusted returns in the Helsinki stock market. At this stage, rescaling of betas is not done yet, and thus the BAB factor is a ‘raw’ version of the Frazzini & Pedersen (2014) BAB factor. As rescaling is expected to improve BAB returns, the chances of observing better risk-to-return metrics for BAB relative to the market are likely to be lower before the rebalancing is done. The expectation is such, because rebalancing the portfolio betas involves leveraging the low-beta portfolio which, according to the results from the earlier section, performs strongly relative to the high-beta portfolio during the period.

Table 8 shows some key performance evaluation measures for the BAB factor before rescaling of portfolio betas. The same statistics are also shown for the market index. Average returns are arithmetic monthly average returns, and volatility figures are annualized. As the table shows, the beta figure for BAB is negative – this indicates that the relationship between the factor prior to rebalancing and the market would be negative.

**Table 8.** Performance measurement statistics for BAB prior to beta-rebalancing vs market index during 12/1999-12/2017.

	BAB w/o rebalancing	OMXH GI
Average return (monthly)	0.0023	0.0092
Average return p.a. (geometric)	0.0176	0.0987
Volatility	0.1453	0.1804
Beta	-0.6698	1.0000
Sharpe ratio	0.1214	0.4297
Sortino ratio	0.1696	0.6332
Jensen’s alpha	0.0696	0.0000
Treynor’s measure	-0.0263	0.0775

The beta coefficient for BAB could be expected to be closer to zero, even though the strategy is not ‘rebalanced’ to have a beta of zero at this stage. To some extent, this is mostly likely due to the same explanation that was provided earlier: The high-beta portfolio is, by definition, more sensitive to the market than the low-beta portfolio and the “inverted” exposure (short position) to it makes the beta negative. Also, the betas used to allocate the stocks to the two portfolios each month are ‘ex-ante’ betas – they are calculated from the period of the preceding 36-months. Therefore, the betas used to allocate the stocks in portfolios are estimates, and the realized betas can only be observed after each period (ex-post) and they are likely to differ from the estimated (ex-ante) betas. However, it is unlikely that the betas of stocks would change dramatically after one month

(ex-post), as monthly observations from a period of three years can be considered sufficient for a robust evaluation.

The market index performs distinctly better on both of the risk-adjusted return metrics considered. The Sharpe ratio, which measures excess returns per unit of variability, is 0.12 per year for BAB. Sortino ratio, that only considers ‘downside risk’ by excluding positive return variation from the calculation of volatility, is 0.17 for BAB. The corresponding figures for the market index are 0.43 and 0.63. As noted earlier, BAB is a return difference strategy, supposed to be market neutral and thus meant to provide positive returns regardless of the direction of the general market. Considering the development of the stock markets globally during the period of study, it is not surprising that the market has performed better than BAB, even with a wide margin. However, the positive return-to-risk figures for BAB are promising, although no final conclusions can be made before the scaling of portfolio betas is done at a later stage.

The last two performance metrics in the table are interesting. Jensen’s alpha, that is closely related to the CAPM and is intended to measure excess returns, shows a positive reading for BAB. The ratio shows how much the average return of the portfolio exceeds the return predicted by the CAPM (Bodie et al. 2014: 840); for this reason, Jensen’s alpha for the market index is zero – the market index has a beta of one and is thus expected by the CAPM to return exactly as much as the market. According to Jensen et al. (1972), when the measure is positive, the portfolio has exceeded those returns predicted by the capital asset pricing model. The positive Jensen’s alpha for BAB is mostly due to the fact that the beta of BAB is negative, and thus the CAPM predicts negative returns (if the risk-free rate is not considered) for the factor. In the end, the positive Jensen’s alpha figure might tell more about the faultiness of the CAPM than it tells about the positive performance of the BAB. However, as the faultiness of CAPM is at center stage in this paper, this measurement is not only interesting but also very relevant.

The last performance metric is Treynor’s measure, also known as the Treynor ratio. It shows a negative figure for BAB – however, this does not indicate the performance is bad, as the negative sign could be erroneously interpreted. The ratio is calculated similarly to the Sharpe ratio, the only difference being in the denominator, where Treynor ratio has beta of the security instead of its volatility. Therefore, as the beta for BAB is negative and its excess returns positive, the result is a negative Treynor’s measure. The ratio suggests that the non-scaled BAB factor has generated a positive excess return by ‘betting against the market’.

The same OLS regression analysis that is done for the low- and high-beta portfolios earlier is also performed for the BAB factor. BAB returns without rebalancing are regressed against the market index using the simple model below:

$$(12) \text{ BAB (raw) vs market regression: } R_{BAB,t} = \alpha_{BAB} + \beta_i R_{OMXH,t} + e_{BAB,t}$$

**Table 9.** Results of a simple OLS regression of BAB returns prior to rebalancing versus market (OMXH CAP GI) returns for the period December 2001 – December 2017.

Variable	BAB without scaling of betas, $R_{BAB}$	
	Coefficient	t-statistic
A	0.0085***	4.9606
$R_{OMXH}$	-0.6698***	-20.6725
Standard error	0.0234	
Adjusted $R^2$	0.6895	
F-statistic	427.35	
F-stat. prob.	0.0000***	

In table 9, statistical significance is indicated by asterisk's in the following manner: significance at the 1% level = \*\*\*, significance at the 5% level = \*\*, significance at the 10% level = \*. BAB is observed to have a strongly significant positive coefficient when it is regressed against market returns. This is in line with the earlier presented table 8 which shows performance figures for the strategy – the table shows that BAB has a positive Jensen's alpha. The slope coefficient, or beta, of -0.67 was already shown in earlier examination. It is also significant at the 1% significance level. Adjusted R-squared is 0.69, which indicates that the model explains roughly two thirds of the variation in returns around their mean. The F-statistic is highly significant, thus the null hypothesis of all of the regression coefficients being equal to zero is rejected.

### 6.2.2. BAB with beta rescaling

In this section, results for the 'complete' BAB factor are presented and analyzed – the rescaling of portfolio betas is done for each period in such manner, that the beta of the BAB strategy will be as close to zero as possible at the beginning of each period (ex-ante). This is the approach used in Frazzini & Pedersen (2014) – however, some small modifications, which cause the methodology to slightly differ from the reference study, are made.

The methodology used to ‘rescale’ the portfolio betas to obtain the market neutral ‘zero beta’ BAB factor is explained in the methodology section. The formula used here is the same with two exceptions. Firstly, as the stocks in the market index are not exactly the same as in the sample of stocks used, the betas of the portfolios are not rebalanced to one, but rather to the average beta of the whole sample of stocks each month. Second, the weighing of the portfolio returns is limited in the following manner: the rescaling factors, that are obtained by dividing number one with the ex-ante beta of each portfolio, are limited to a maximum value of 2.00 on the low-beta side, and a minimum of 0.50 on the high-beta side. This means, that the low-beta portfolio is leveraged by a factor of two at most, whereas the high-beta portfolio is de-leveraged by a factor of 0.50 at most.

These adjustments to the methodology are done because of some very low beta figures for the low-beta portfolio in the beginning of the examination period. In the beginning of the period of study, there are only 31 stocks in each portfolio, which is not quite a representative sample of all stocks in the index – hence the slight distortion of betas in the beginning. In order to leverage, or scale, the low beta portfolio to have a beta equal to the average of all stocks, the leverage coefficient should be more than five during some of the first months. Capping the scaling coefficients to 2 for the low-beta portfolio and 0.5 for the high-beta portfolio respectively is seen reasonable to avoid excessive scaling coefficients.

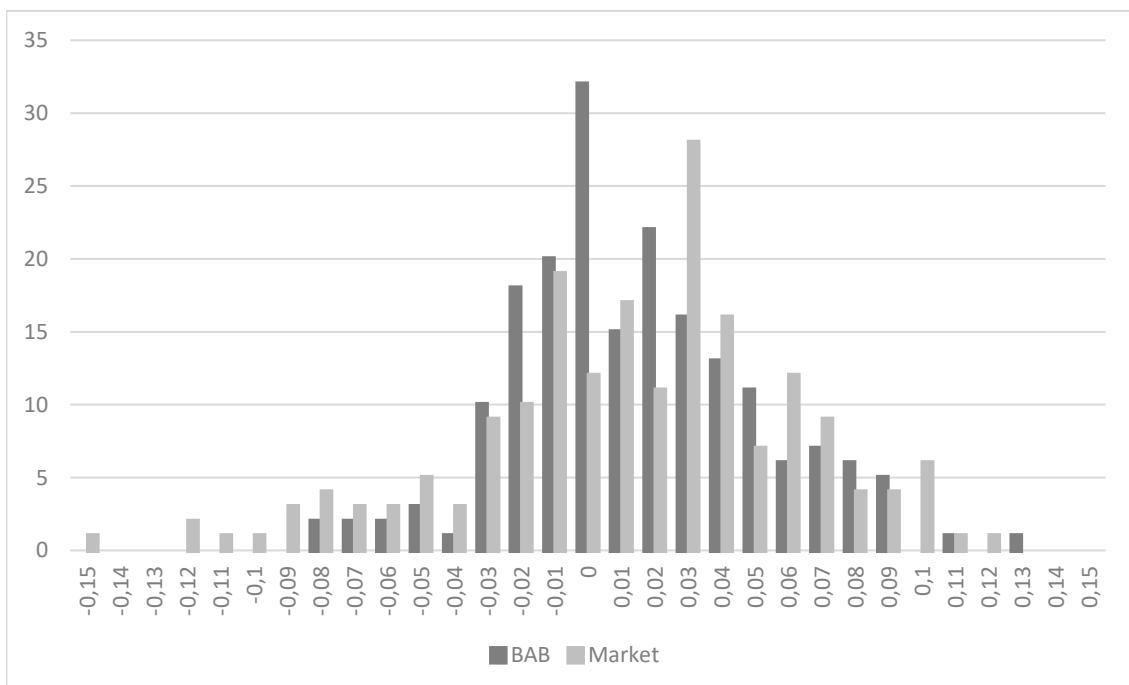
The rescaling of betas has a positive effect on BAB returns. This is not surprising, as the scaling involves leveraging the low beta portfolio, which was earlier observed to outperform both the high-beta portfolio and the market in returns.

**Table 10.** Descriptive statistics of BAB returns after beta-rebalancing, and same figures for the OMXH CAP GI and BAB before rebalancing, for period 12/1999-12/2017. The data set consists of monthly observations. Standard deviation is annualized.

	BAB before rescaling	<b>BAB after rescaling</b>	Market index (OMXH CAP GI)
Mean	0.23%	0.90%	0.92%
Median	0.28%	0.41%	1.48%
Max	20.22%	12.63%	26.59%
Min	-21.84%	-8.77%	-15.83%
Standard dev.	14.53%	12.56%	18.04%
Skewness	-0.0295	0.2780	0.1158
Kurtosis	5.8659	0.3111	2.9120
Observations	193	193	193
Correl. vs market	-0.83	-0.06	

While the low-beta portfolio is leveraged, the other component of the BAB strategy, the high-beta portfolio, is for its part de-leveraged. As was shown in the earlier comparison of the returns of the two portfolios, taking a short position on the high-beta portfolio provides cumulative losses for the strategy. Therefore, having a smaller weight on that portfolio by de-leveraging the short position, naturally improves the returns of the strategy. The descriptive statistics of the ‘complete’ BAB strategy are shown in Table 10. Same figures for the market index and the ‘raw’ BAB factor (prior to rescaling of betas) are also presented for comparison.

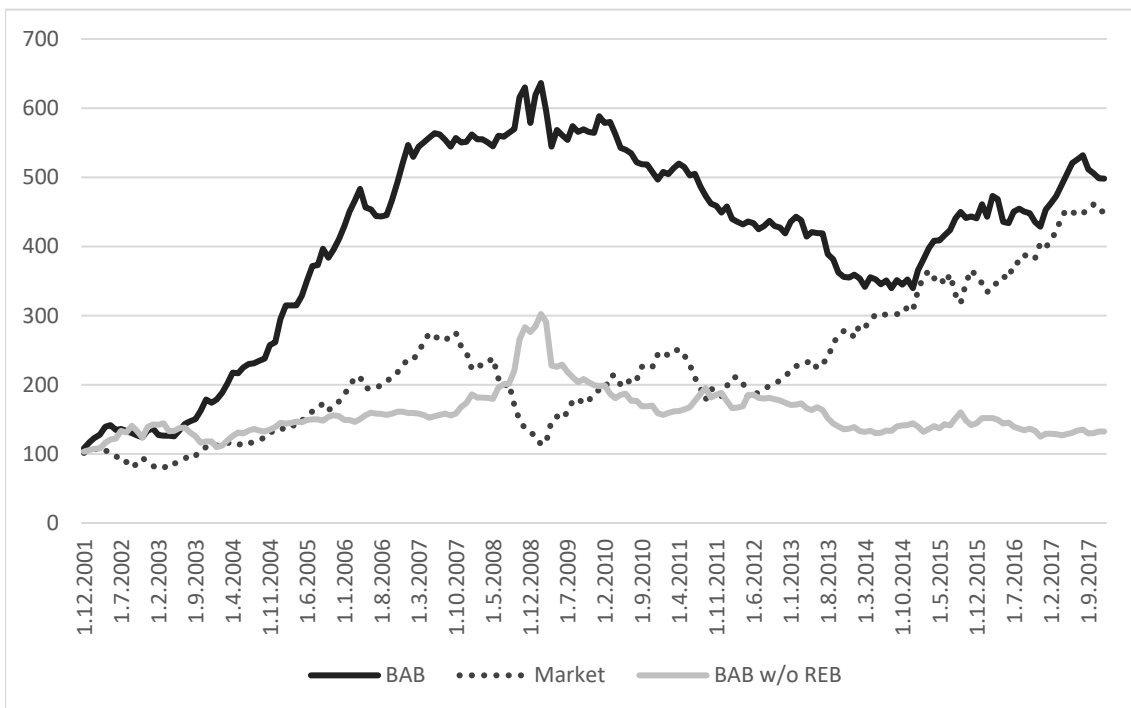
The data in Table 10 shows that BAB performance seems to have been improved significantly by rescaling the betas of the portfolios. Both mean and median returns are higher, and standard deviation is lower. BAB has almost equally high mean monthly returns, but lower median returns, when compared against the market. Rescaling seems to make the volatility of BAB returns even smaller. The difference between BAB after rescaling and BAB before rescaling is distinct on all descriptive statistics. The extreme returns (min & max) for BAB are diminished after rescaling. BAB returns are close to normally distributed, as the kurtosis is only 0.31. Correlation with the market is -0.06 after rebalancing, as it was -0.83 before rebalancing. The rebalancing has thus adjusted the BAB factor towards being ‘market neutral’, as it was intended.



**Figure 5.** Distribution of the monthly returns for the rescaled BAB factor and the market index (OMXH CAP GI) for the period 12/2001-12/2017.

Figure 5 shows the distribution of the monthly returns for the rescaled BAB factor and the market index. The difference in skewness is observable from the graph. Most of the BAB returns are between -0.5 and 0.5 per cent, which is the highest-rising bar on the graph. A significant proportion of BAB returns are also slightly negative, within the -3.5% to -0.5% region. However, there is only a handful of observations below this range for BAB, while there is a larger number of monthly market index returns that are worse than -3.5%. The same can be observed on the positive side of the mean – BAB returns are more concentrated in the middle. Extreme outliers are removed from the graph, and only monthly observations between -15% and 15% are shown.

As noted earlier, the rebalancing of the BAB factor involves certain assumptions. Among these is the assumption that money can be borrowed at the risk-free rate. The long side of the portfolio can thus be leveraged at a low cost – the average annual risk-free rate during the whole period was 2.16%, and only 0.72% for the second half of the period. The assumption of being able to borrow at the risk-free rate is an important driver for BAB returns.



**Figure 6.** Performance of the BAB strategy both with and without rebalancing of betas, and the performance of the market index for the period of December 2001 – December 2017.

The improved performance of the rebalanced BAB factor is demonstrated visually in Figure 6. The performances of the three strategies (BAB after rescaling of betas, BAB before rescaling and the market index) are compared in the graph. The performances are indexed to 100 in the beginning. The patterns of BAB and BAB before rescaling are quite similar, with the difference that the leveraged BAB factor surges during the pre-financial crisis period. The ‘BAB index’ reaches 636 at the stock market bottom in February 2009, which means it returned more than 500% up to that point from the beginning of the period of study. This is more than twice the return of BAB without rebalancing, and multiple times the return of the market index as it suffered losses while the BAB gained in value in 2008-2009. After the financial crisis, BAB has negative cumulative returns until the end of 2014, after which it gains in value but ends up lower than its peak during the financial crisis. In the end, the stock market had a total return of approximately 351% during the entire period, whereas BAB gained 398% in value. BAB without rebalancing would have only provided a total return of 32%. Prior to closer examination of performance, it seems BAB performs even better than expected, and that rescaling the portfolio betas – leveraging the low-beta portfolio and de-leveraging the high-beta portfolio – makes all the difference for the betting-against-beta factor performance.

Table 11 shows performance figures for BAB. The results are compared against the corresponding figures for the market index and BAB prior to the rescaling. The ‘completed’ BAB factor has arithmetic average monthly returns of 0.9%. Average (geometric) annual return is 10.6%, which is 0.7% higher than the corresponding figure for the market portfolio. Meanwhile, annualized volatility is only 12.6%. Even though slight modifications are done to the methodology used in Frazzini & Pedersen (2014) to fit this paper better, the rebalancing seems to have worked as the realized beta for the BAB factor after rebalancing is -0.03. The goal of the rebalancing is to make the strategy a ‘zero-beta’, or ‘market neutral’ strategy by adjusting ex-ante betas to zero. This is achieved successfully, as the ‘ex-post’, or realized beta is close to zero. Frazzini & Pedersen (2014) obtain a realized beta coefficient of -0.06 for their BAB factor.

The combination of higher returns and lower volatility is seen in the high Sharpe ratio for the BAB factor. The figure is 0.67, while the market index and BAB before rebalancing obtain values of 0.43 and 0.12, respectively. BAB seems to provide clearly more return on risk based on this metric. The difference to the market and the factor prior to rebalancing on return-to-risk is even more distinct when using the Sortino ratio for comparison. BAB has a Sortino ratio of 1.26, while the corresponding figures for the market index and BAB prior to scaling are 0.21 and 0.17, respectively. Therefore, it seems



that the BAB strategy has relatively higher returns relative to risk, when only ‘downside risk’ is considered.

**Table 11.** Performance measurement statistics for BAB compared to the earlier presented BAB prior to beta-rescaling and the market index. Period of study is 12/1999-12/2017. Returns are excess returns. Annual returns are geometric averages – volatility, Sharpe ratio and other ratios are annualized.

	BAB w/o rebalancing	<b>BAB</b>	OMXH GI
Monthly return	0.0023	0.0090	0.0092
Annual return	0.0176	0.1056	0.0987
Volatility	0.1453	0.1256	0.1804
Beta	-0.6698	-0.0316	1.0000
Sharpe ratio	0.1214	0.6716	0.4297
Sortino ratio	0.1696	1.2621	0.6332
Jensen’s alpha	0.0696	0.0859	0.0000
Treynor’s measure	-0.0263	-2.6683	0.0775

Jensen’s alpha is positive, and higher than it was before rescaling. This indicates that BAB has overperformed relative to the expectations of the CAPM, a model which assumes that the returns of securities are a function of their market risk and the excess returns of the market. Treynor’s measure shows a high value, as the beta of BAB is very low. The negative sign for the measure is due to the negative beta figure, and it does not indicate bad performance – it just says that the beta coefficient is negative.

The Treynor’s measure, just like Jensen’s alpha, is somewhat distorted by the minimal beta of BAB, and thus it does not provide much useful information. However, the two measures, especially Jensen’s alpha, do demonstrate the failure of the CAPM to some extent – according to CAPM, obtaining positive returns by taking a long position in the low-beta half of stocks and a short position in the high-beta half of stocks should not happen as long as market returns are positive. As market returns are positive during the period, there is some indication towards the failure of CAPM, which is proved already decades earlier by Black et al. (1972) and others.

A similar regression that is done for BAB prior to rescaling is done for the rescaled BAB factor as well. The model is the same linear OLS regression that is used earlier. The regression model may also be called a “CAPM regression” model, as the returns on BAB and the market index are excess returns – thus, the risk-free rate has been subtracted from the returns. The model is the following:

$$(13) \text{ BAB vs market index: } R_{BAB,t} = \alpha_{BAB} + \beta_i R_{OMXH,t} + e_{BAB,t}$$

**Table 12.** Results from the BAB factor vs. market index (OMXH CAP GI) regression. Monthly excess returns are used in the regression, and the period is December 2001 – December 2017.

Variable	BAB ( $R_{BAB}$ )	
	Coefficient	t-statistic
$A$	0.0092***	3.4983
$R_{OMXH}$	-0.0316	-0.7206
Standard error	0.0363	
Adjusted $R^2$	-0.0025	
F-statistic	0.5193	
F-stat. prob.	0.4720	

The constant, or alpha, is positive (0.92%) and statistically significant at the 1% significance level. For comparison, Frazzini & Pedersen (2014) obtain a CAPM alpha of 0.73% for their BAB factor using U.S. equities. The slope coefficient (beta) is -0.03, but it is not significant. The explanatory power of the model is poor, as the adjusted r-squared is very low. The interpretation is that the model does not succeed in explaining BAB returns. However, as market index returns are the only explanatory variable in the model this is not surprising, as the BAB factor is intended to be market neutral. Instead of attempting to explain the variation of BAB returns, the purpose of the model is rather to measure the alpha of BAB relative to CAPM.

To conclude this section, rescaling the betas of the long and short portfolios before each one-month period seems to significantly improve the returns of the BAB factor. The rebalancing is done using the methodology in the reference study Frazzini & Pedersen (2014), however with slight adjustments as explained. Rebalancing, or scaling, means that the low-beta portfolio is leveraged to such extent, that its beta equals the average beta for the whole sample of stocks. Respectively, the high-beta portfolio is de-leveraged to also have a beta equal to the average beta of all stocks. The performance of BAB at each period is then calculated by subtracting the excess returns of the high-beta portfolio from the excess returns of the low-beta portfolio. As both portfolios are rebalanced to have the same beta (average beta of all stocks) the strategy is market neutral – it has a beta of (approximately) zero as the long and the short side have equal betas with opposite signs, and thus cancel each other out.

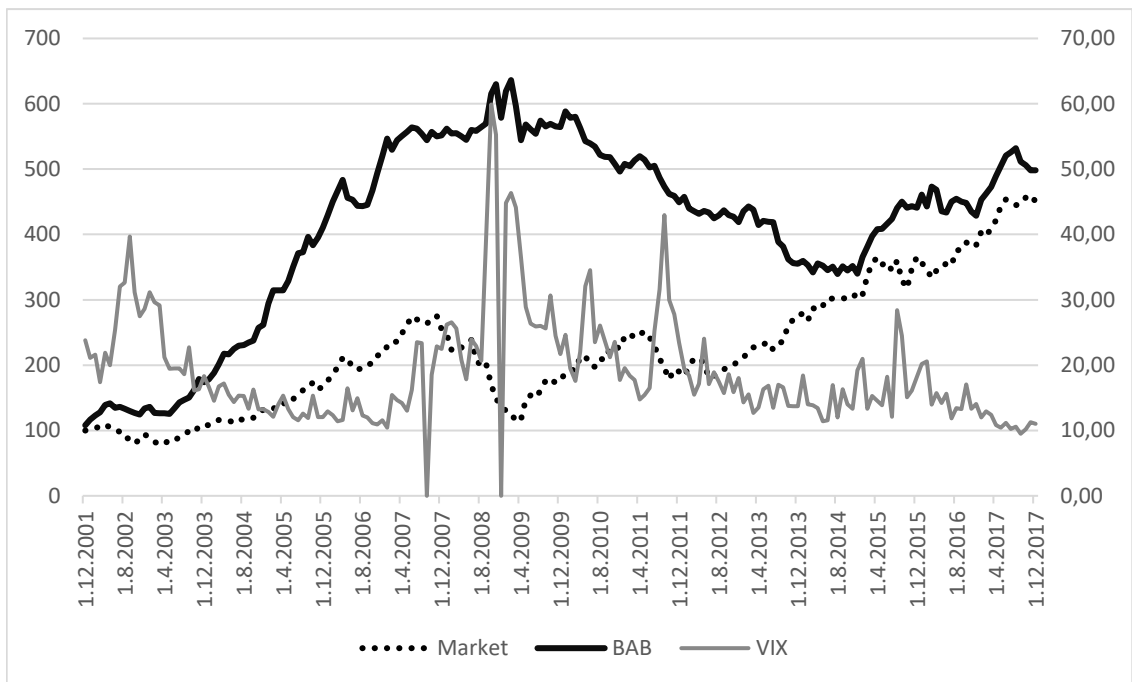
Prior to rescaling BAB has only slightly positive returns, and it lags the market index in both absolute and risk-adjusted return metrics. Considering the fact that BAB is a zero-cost strategy justifies the lag to the market index, and as long as the factor provides excess returns over the risk-free rate it can be considered somewhat successful. However, when the BAB factor is ‘completed’, and beta scaling is done for the portfolios, BAB outperforms the market index on all measures, even on absolute returns where it provides an annual return of 10.6% during the period of study. Return-to-risk metrics are even more convincing, which indicates that in addition to high returns during the period, BAB also has had convincing returns relative to risk.

### 6.2.3. BAB performance and market volatility

In this section, the relationship between BAB performance and market volatility is examined. One of the hypotheses for this paper is, that BAB outperforms the market index during times of risen uncertainty in the markets. As BAB was already shown to do better than the market index in the whole period, the focus shifts on whether the factor performs especially well during times of high volatility. In other words, the intention is to examine whether BAB performs better during times of high market uncertainty and volatility than it does during times of lower volatility.

The VIX index is used as proxy for market uncertainty. The index is calculated by the Chicago Board Options Exchange (CBOE), and it is designed to measure market expectations of short-term volatility – it reflects the expected 30-day volatility implied by S&P 500 option prices (Dash & Moran, 2005). The index is known to spike during periods of increased market uncertainty or even panic, and Whaley (2000) has named VIX as the “investor fear gauge”. Because of its traits, VIX is the natural choice to serve as an indicator of market uncertainty.

By definition, stocks with higher betas comove more with the market than low beta stocks. Hence, it could be expected that the BAB benefits from peaks in market uncertainty – when uncertainty increases, the stock market usually loses value. During these conditions, high-beta stocks decline more aggressively than low-beta stocks, thus the short position in high-beta stocks should be profitable for BAB. Naturally, low-beta stocks are likely to suffer from market declines as well, but by definition they will not decline to the extent that high-beta stocks do. Therefore, the long position in the low-beta stocks should not create higher losses than the short side generates profits, and the strategy as a whole is expected to be profitable in volatile market conditions.



**Figure 7.** The development of the VIX, the market index and the BAB factor during the period December 2001 – December 2017.

In figure 7, BAB and the market index are indexed at 100 in the beginning and their values at each point of time are shown on the left side of the graph. The values for the VIX are shown on the right-hand side of the graph. Dash & Moran (2005) point out that sharp declines in the stock market are marked by sharp movements upward in the VIX. This can also be observed from the graph – the market index and the VIX often tend to move in opposite directions. In fact, the correlation between the two during the entire period is -0.43.

There is one clear highpoint for the VIX, where its value rises above 60. This spike is timed in the fall 2008, when the financial crisis was at its worst and Lehman Brothers went bankrupt. Actually, the highest value for VIX was over 89.23, but as month-end values are considered here this is not shown. The graph shows that BAB increases at the time when VIX rises to its highest, which is promising regarding the ability of BAB to perform during difficult times in the market. However, based on the graph, this does not seem to happen during all upward movements of the VIX, and further examination is required to reveal a possible relationship. The table below shows some summary statistics for the VIX.

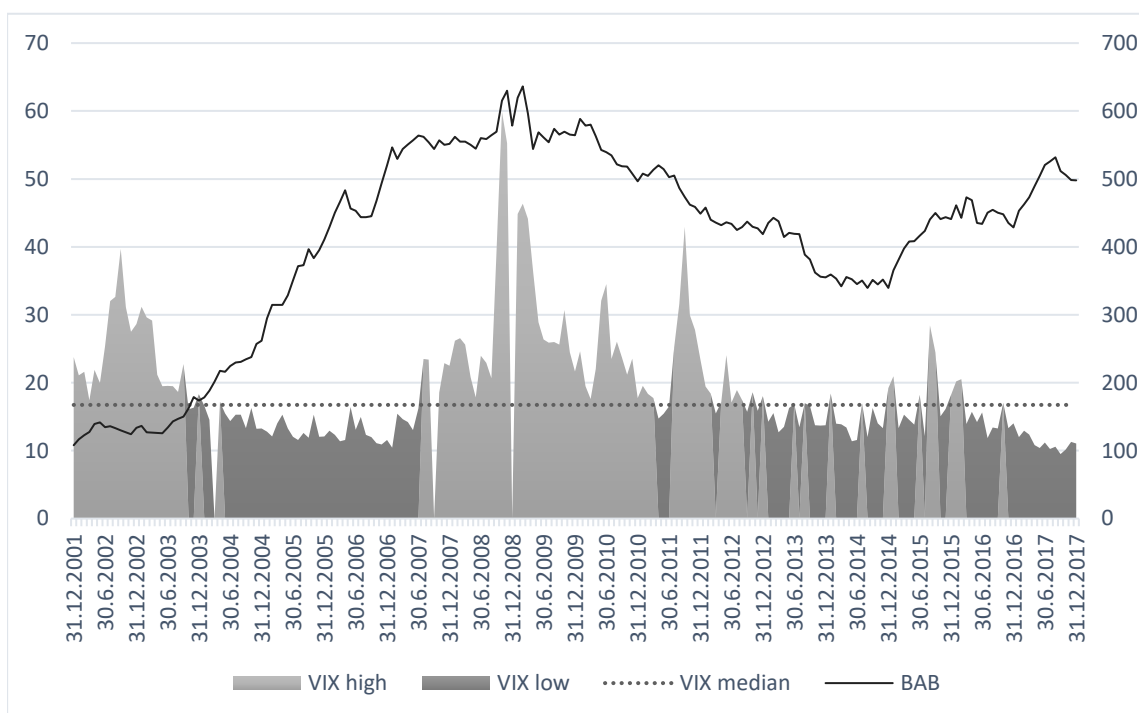
**Table 13.** Descriptive statistics for month-end observations in the VIX. The examination period is from 12/2001 to 12/2017. Data from [www.cboe.com](http://www.cboe.com).

	VIX
Mean	19.22
Median	16.74
Maximum	59.89
Minimum	9.51
Correlation with market	-0.44
Correlation with BAB	0.11
Observations	193

Giot (2005) shows, that negative stock index returns are associated with rising levels in VIX, and vice versa. Table 13 does not contradict with this finding – there is a negative correlation between VIX and the market index. However, the relationship of BAB and the VIX is not as clear and thus requires closer examination.

Firstly, some time periods when VIX is ‘higher than normal’ are chosen from the whole period to represent volatile market environments. Thus, the 16-year period of study is divided to volatile and less volatile periods. Then the intention is to show whether or not BAB performs better during the volatile times than it does during less volatile times. The period ranging from December 2001 to December 2017 is divided to equally many periods of low VIX and high VIX. The division is made by labeling each one-month period either as high VIX or low VIX, depending on whether the month-end value of VIX for that month is lower or higher than the median value for VIX during the entire period. Using this methodology, 97 months are labeled as high-VIX periods and 96 months as low-VIX periods.

Volatility is known for its tendency to cluster, which means that volatility is often increased for longer periods at once rather than being high one day and low the next day. As VIX represents expected volatility, it has the same trait. Figure X graphs the VIX and shows the periods of low VIX and high VIX. The clustering feature of the ‘fear-index’ can be observed from the graph. The longest streak of consecutive high-VIX periods is 45 months, which ranges from July 2007 to March 2011. The corresponding figure for low-VIX periods is 38 months, ranging from May 2004 to June 2007. In addition to these periods, there is one period for both low-VIX and high-VIX where the VIX is either higher or lower than the median for more than 12 months in a row.



**Figure 8.** The development of VIX divided to periods of lower-than median and higher-than median VIX, and the development of BAB during the period 12/2001-12/2017.

In figure 8, BAB performance is shown on the scale on the right, and the level of the VIX is shown on the left. The seemingly largest gain for BAB seems to be timed on the 38-month long period of low VIX between 2004 and 2007.

**Table 14.** Performance of BAB (monthly returns) during low-VIX and high-VIX periods.

	BAB (entire period)	BAB (low VIX)	BAB (high VIX)
Mean	0.0090	0.0149**	0.0031**
Median	0.0041	0.0123	-0.0034
Volatility (p.a.)	0.1256	0.1221	0.1263
Beta	-0.0316	0.3043	-0.1486
Sharpe ratio	0.6716	1.3297	0.0946
Sortino ratio	1.2621	2.7047	0.1674
Jensen's alpha	0.0859	0.1472	0.0194
Treynor's measure	-2.6683	0.5337	-0.0804

Table 14 breaks down the performance of BAB during both the high- and low-VIX periods. Corresponding performance figures for BAB for the whole period from December 2001 to December 2017 are also shown for comparison; the first column contains the observations from the entire period, whereas the latter two columns each

contain a half of the monthly observations for BAB, allocated between the columns based on the level of VIX in the observation month. According to Table 14, BAB seems to underperform by a large margin during times of increased volatility – the average monthly returns are significantly higher during low-VIX periods. These results are opposite to those that were expected. The difference in average returns between low-VIX and high-VIX is as much as 1.18% per month, which is significant at the 5% level. BAB was expected to perform better in high-VIX conditions – however, the explanation for the observed results is relatively simple. The returns on the low-beta portfolio are very strong especially during the low-volatility period prior to the financial crisis, as was observed earlier. Also, the rescaling of portfolio betas involves leveraging the low-beta portfolio and deleveraging the high-beta portfolio, which strengthens the effect of the low-beta portfolio and weakens the effect of the high-beta portfolio on BAB returns. As the low-beta side is leveraged, the returns on BAB are stronger in a situation where low-beta stocks increase in value, compared to a situation where the profit comes mainly from the decreasing values of shorted high-beta stocks.

Besides monthly average returns, BAB performs better during low volatility conditions than it does during high volatility conditions on every other metric as well. In high-VIX conditions, the median monthly return for BAB is even negative (-0.34%) whereas it is 1.23% positive in low-VIX conditions. Volatility is almost equal during both conditions, and thus there is a large difference on risk-adjusted return metrics: BAB (high-VIX) has a Sharpe ratio of 0.09 and Sortino ratio of 0.17, while the respective figures for BAB (low-VIX) are 1.33 and 2.70. The difference in these return-on-risk measures comes from the difference in monthly average returns. The realized beta (ex-post) for BAB is 0.41 during low-VIX and -0.08 during high-VIX. This indicates that BAB somewhat follows the direction of the market in a low volatility environment, but this relationship disappears during times of high volatility.

As the BAB can be shown to provide significantly higher monthly returns during times of low-volatility, the prediction of improved performance for the factor in high-volatility conditions can be rejected. Hypothesis number two for this paper is partly open, however. The hypothesis states that BAB will outperform the market index in absolute returns during bear markets and periods of increased volatility. In fact, this hypothesis includes two sub-hypotheses, as all periods of higher-than-normal volatility cannot be counted as “bear markets”. Therefore, these two questions are compared separately. First, BAB performance against market portfolio is compared during increased volatility conditions. The same approach that was used to compare BAB performance during different volatility

conditions is used here as well – the whole 193-month period of study is separated in two based on the level of VIX at each month. Table 15 compares the performance of the BAB factor against the market index during periods of low ( $VIX_{LOW}$ ) and high ( $VIX_{HIGH}$ ) volatility.

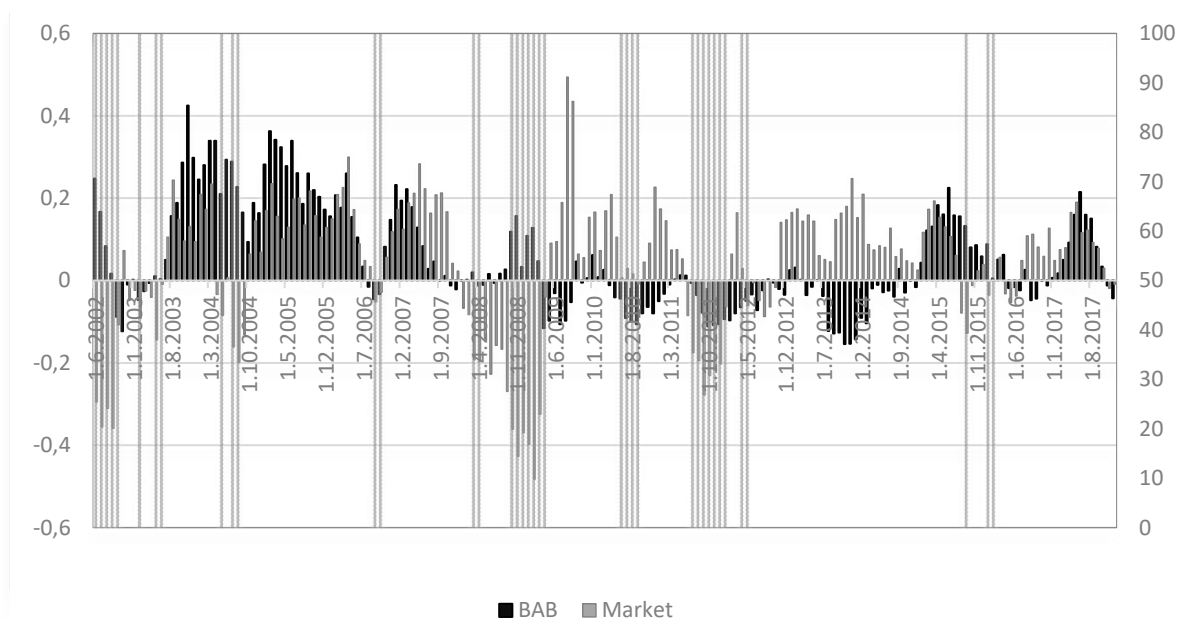
**Table 15.** Performance of BAB versus market index (OMXH CAP GI) during low-VIX and high-VIX periods. Period 12/2001-12/2017.

	$VIX_{HIGH}$		$VIX_{LOW}$	
	OMXH CAP GI	BAB	OMXH CAP GI	BAB
Mean	-0.0037	0.0031	0.0222	0.0149
Median	-0.0125	-0.0034	0.0243	0.0123
Volatility (p.a.)	0.2201	0.1263	0.1126	0.1221
Beta	1.0000	-0.0845	1.0000	0.4096
Sharpe ratio	-0.3184	0.0946	2.2126	1.3297
Sortino ratio	-0.6024	0.1674	6.2890	2.7047
Jensen's alpha	0.0000	0.0167	0.0000	0.1419
Treynor's measure	-0.0701	-0.1414	0.2492	0.3963

Mean and median in the table are monthly figures, all other measures are annual. The significance of the difference between the returns in different volatility conditions is tested by a t-test; the mean return of the market index in high-VIX conditions is compared against its mean return in low-VIX conditions. The t-statistic is -3.81, and the p-value is 0.0001 for the two-tailed test. Thus, the result is statistically significant at all conventional significance levels – the returns are significantly higher during low VIX conditions. Overall, the performance difference between  $VIX_{HIGH}$  and  $VIX_{LOW}$  conditions is more eminent for the market index. As the market index is a 'long-only' strategy, it tends to suffer when uncertainty increases. This is not the case for BAB, which is half invested in a long position and half invested in a short position. The volatility of the market portfolio is close to that of the BAB strategy during  $VIX_{LOW}$ , but it is twice as high during  $VIX_{HIGH}$  on average, which demonstrates the sensitivity of stocks to uncertainty. The difference in mean returns and variance of returns during the two different environments reflect in the Sharpe and Sortino ratios of the market portfolio. During  $VIX_{HIGH}$ , the values are deeply in the negative region, and during  $VIX_{LOW}$  the values are remarkably high. As the market naturally has a beta of one, Treynor's measure purely reflects the average annual excess returns over the risk-free rate. The difference in Treynor ratios sum up the relationship between increased VIX and the market returns quite well.



The prediction of BAB performing better than the market index during increased volatility cannot be proven statistically to hold – during  $VIX_{HIGH}$ , BAB earns 0.69% more per month than OMXH CAP GI. However, the difference is not even significant at the 10% level. The situation is the same regarding median returns as well. Even though the mean test does not provide significant figures, BAB can be seen to outperform the market on all performance metrics used. The lack of significance is most likely due to the small sample size. The first part of hypothesis two is hereby left partly unanswered, and next the analysis shifts to examining whether BAB outperforms OMXH GI during bear markets.



**Figure 9.** The cumulative 6-month returns of BAB and the market index at each point of time and the bear market periods highlighted.

In order to perform the analysis, “bear markets” need to be identified. Here, a ‘bear market’ is defined as any six-month period during which the market index has negative returns on four or more months. With this definition, a total of 39 different six-month-periods between December 2001 and December 2017 are classified as ‘bear market’ periods.

By definition, bear market periods are dominated by negative market index returns. Each data point on the graph shows the total return from the period of the preceding six months.

At the end of the financial crisis, the market index had accumulated six-month returns of -43.9%. During the same period, BAB gained 12.8%. The worst six-month period for BAB during the entire period of study was the second half of 2013, when it accumulated a return of -15.4%, when the market rallied in a low-volatility environment. Table 16 shows a summary of the performance of BAB and the market in bear market conditions.

**Table 16.** Performance of BAB and the market index in ‘bear market’ conditions, monthly returns. Volatility figures are annualized.

	OMXH CAP GI	BAB
Average return	-0.0140	-0.0049
Median return	-0.0210	-0.0052
Max	0.2659	0.0801
Min	-0.1248	-0.0877
Volatility	0.2612	0.1330
Lowest cumulative return	-0.4392	-0.1155
Highest cumulative return	0.0372	0.2270
Correlation		-0.3115
Observations	36	36

Average returns are calculated as arithmetic average monthly returns from the months when the ‘bear market’ condition (negative performance for the market index in four months out of the previous six) is satisfied. Like the market, BAB also has negative average and median returns during the 36 observation months. However, BAB losses are smaller (-0.49%) than those of the market (-1.40%) on average. Max and min rows show highest and largest single month- returns. The market index rebounds in early 2009 after the financial crisis, when it gained 26.12% in one month to end the current ‘bear market’ period. The negative correlation of the market and BAB seems to increase during bear markets – the table shows a correlation of -0.31. The correlation for the entire period between BAB and the market index is -0.06.

Even though BAB seems to clearly perform better than OMXH CAP GI in bear markets, the difference between the average returns of the market index and BAB is not statistically significant. However, average returns may be somewhat distorted by large returns in individual months, such as the single-month return of over 26% for the market index. Medians are not affected as much by such ‘outliers’, and thus it is reasonable to examine the difference between median returns using a median test. The result of the median test is significant at the 10% significance level. Again, the small sample size makes it difficult to obtain significant values, even though roughly two-thirds of monthly BAB return

observations are above the median, and the corresponding figure for the market is one third. This means that even though BAB outperforms the market index during bear markets, no robust statistical evidence can be presented and thus hypothesis two cannot be convincingly shown to hold – the null hypothesis of no difference in performance during bear market conditions cannot be outright rejected.

The intention with the methodology used was to obtain a ‘market neutral’ BAB factor, which has been successful. In a simple regression where excess returns on BAB are regressed against excess market returns, the slope coefficient obtained is 0.04, which is close to zero and does not indicate any dependence between BAB and the market.

### 6.3. Drivers of BAB performance

The BAB factor consists of two parts: the low-beta portfolio, in which a long position is taken, and the high-beta portfolio, in which a short position is taken. Thereby, BAB returns equal the returns of the long portfolio minus the returns on the short portfolio. Also, rebalancing of betas affects the returns of each portfolio by magnifying (low-beta portfolio) or decreasing (high-beta portfolio) the returns, as explained earlier. Therefore, the returns of the low-beta portfolio and the ‘inverted’ returns of the short-beta portfolio are the main drivers of BAB performance. Each portfolio naturally has underlying factors (stocks) that affect their performance – before discussing stock characteristics such as industry, value or growth, the effect of the low- (long) and high-beta (short) portfolios on BAB returns is examined.

BAB returns are regressed against the returns of each portfolio, using the following model:

$$(14) R_{BAB,t} = \alpha + \beta_L R_{LONG,t} + \beta_S R_{SHORT,t} + e_{i,t}$$

$R_{BAB}$  is the excess return over the risk-free rate for the BAB factor,  $R_{LONG}$  is the excess return for the long position in the low-beta portfolio and  $R_{SHORT}$  is the excess return for the short position in the high-beta portfolio. As BAB consists of the two portfolios, they should be able to explain the variation in BAB returns. It is important to note, however, that the BAB factor returns are affected by the rebalancing of betas, while the explanatory variables are excess returns for the portfolios without leveraging or de-leveraging.

The regression is performed solely for the purpose of studying the slope coefficients for BAB against the long and the short sides, as it is supposed to give some information about to which extent the performance of each portfolio is affecting the BAB factor. Table 17 shows the results for the regression. As previously, statistical significance is indicated by asterisks; three asterisks indicate statistical significance at the 1% level.

**Table 17.** The regression of BAB excess returns against the excess returns of the low- and high-beta portfolios. Period 12//2001-12/2017.

Variable	Coefficient	$R_{BAB}$	t-statistic
$C$	0.0059***		2.8611
$R_{LONG}$	0.9430***		10.8044
$R_{SHORT}$	0.2525***		6.2418
R-squared		0.3875	
Adjusted $R^2$		0.3810	
Observations		193	

The coefficients on both portfolios are highly significant, as could be expected since they are the two components of which the BAB factor is constructed. Also, both coefficients are positive; returns of BAB and the long portfolio naturally have a positive relationship, and the BAB and the short portfolio do as well as the short portfolio returns are shown as the result of shorting the high-beta portfolio. The coefficient is 0.94 for the long portfolio, which indicates that when the returns on the long portfolio increase by one unit, BAB returns increase by 0.94. The respective coefficient is only 0.25 for the high-beta portfolio. It seems that the returns of the long portfolio affect the returns of BAB to a larger extent. This is mainly due to the rescaling of portfolio betas, which involves leveraging the long position in the low-beta portfolio up to a coefficient of two, while the high-beta portfolio is de-leveraged to having a coefficient of 0.5 at lowest. If the returns of the low-beta portfolio are multiplied by two and the returns of the high-beta portfolio by 0.5, the relationship between BAB and the former is strengthened, and the relation with the latter weakened. This is shown in the regression coefficients.

The main return driver of BAB is the low-beta portfolio, in which the BAB strategy has a long position. This is because the returns of the portfolio are leveraged in the process of making the BAB factor beta neutral. Due to beta rebalancing, the return effect of the low-beta portfolio is amplified in contrary to the high-beta portfolio, which has its effect on BAB returns downsized. Generally, causality cannot be proven by even significant beta

coefficients, but as it the BAB factor is constructed of the low-beta and the high-beta portfolio, they can be stated to be the cause for the performance of BAB.

### 6.3.1. Return drivers of the components of BAB

The previous section discussed effect of the low- and the high-beta portfolios, which are the two components that form the BAB strategy. Due to the reasons explained in the previous section, the portfolio consisting of low-beta stocks at each time has a larger effect on the returns of BAB than the portfolio consisting of high-beta stocks. Low-beta stocks perform better during the period of study and leveraging them has a very positive impact on BAB returns. In this section, the stocks inside both portfolios are examined to find out if some industry sectors, or factors such as size or value appear in more frequently in either portfolio.

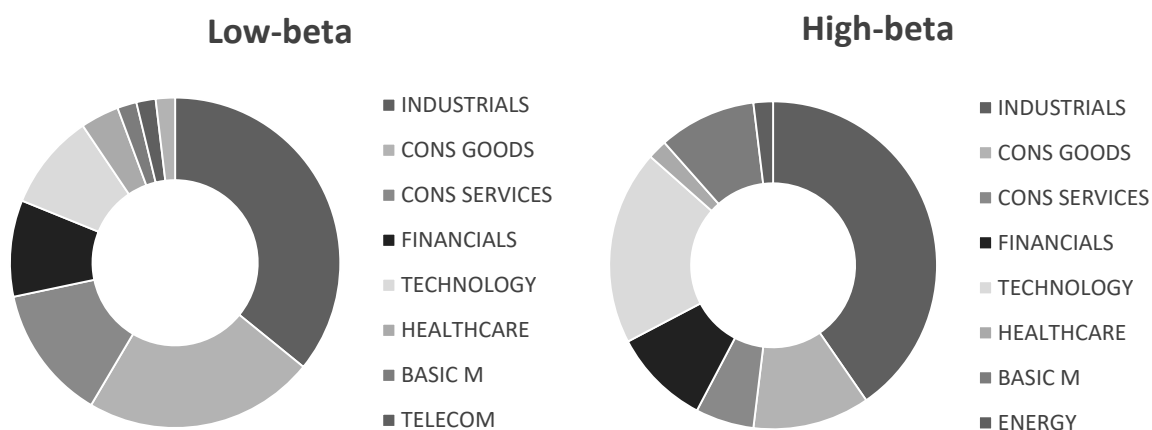
**Table 18.** The stocks most commonly appearing in each portfolio (low-beta and high-beta) during the period of 12/2001-12/2017.

Most often in <b>low-beta</b> portfolio			Most often in <b>high-beta</b> portfolio		
Stock	Count (LB)	Count (HB)	Stock	Count (HB)	Count (LB)
RAP1V	<b>192</b>	1	NOKIA	<b>193</b>	0
ILK2S	<b>190</b>	3	METSB	<b>193</b>	0
PKK1V	<b>189</b>	4	METSO	<b>191</b>	2
KSLAV	<b>185</b>	4	KCR	<b>184</b>	9
NLG1V	<b>185</b>	8	NRE1V	<b>183</b>	10
INVEST	<b>184</b>	9	STERV	<b>179</b>	14
APETIT	<b>182</b>	11	UPM	<b>176</b>	17
RAIVV	<b>178</b>	15	BITTI	<b>170</b>	23
VIK1V	<b>176</b>	17	IFA1V	<b>169</b>	20
ALBBV	<b>171</b>	22	YIT	<b>166</b>	27

Novy-Marx (2014) finds that the underperformance of high-beta stocks relative to low-beta stocks is mostly due to “small, unprofitable, and growth firms”, which are supposed to, at least partly, to explain the poor performance of aggressive stock portfolios. Novy-Marx (2014) does not have a say on different industries. Asness, Frazzini & Pedersen (2014) study ‘industry-neutral’ low beta investing and find that betting against beta has historically had positive returns even as an ‘industry-neutral’ bet. Thus, the authors refute the view that BAB would provide excess returns only because it invests in low-risk, but ultimately profitable, industries.

Table 18 shows the top ten most frequently appearing equities in each portfolio. The average amount of monthly observations for a single stock is 167, while the maximum is 193 (the length of the period). Most of the stocks are in both portfolios during the period of study, but a few appear on the other for almost the entire period. For example, Nokia and Metsä Board are in the high-beta portfolio every month, as they always have higher-than median betas. In the low-beta portfolio, Rapala and Ilkka-Yhtymä are included every time except for a handful of months.

**Figure 10.** The average sector allocations of the stocks in the low- and high beta portfolios.



The OMX Helsinki exchange is largely dominated by the industrial sector, which can also be seen in Figure 10. The stocks in the industrial sector are distributed evenly to both portfolios on average. The most distinct differences in the observation frequency of sectors are between consumer goods and services, and technology. On average, one third of the low-beta portfolio consists of stocks in the consumer goods or services sector, while these sectors only form less than one fifth of the high-beta portfolio on average. Technology stocks are twice as often represented in the high-beta portfolio than they are in the low-beta portfolio. Asness et. al (2014) conclude, that BAB is not driven by betting on different industries. Such an analysis is not possible to be made in this paper due to the small sample size for each sector. Thus, the analysis of sectors is not taken further from the summary table and charts.

### 6.3.2. Fama-French Three-Factor regression

Fama & French (1991) present three ‘common risk factors’ for the returns of stocks and bonds: The market, size and value factors. The study is popular in the field of financial study, and it is interesting to test the relationship between the factors and BAB returns. So far, only one of the three factors has been considered; the market factor. In this section, a “FF3” regression is performed to obtain the coefficients between BAB and the size and book-to-market (value) factors. It is also of interest to see if the market coefficient changes when two additional factors are included in the BAB versus market regression.

Performing a FF3 regression requires data sets for the dependent variable (BAB), risk-free rate, market factor, size (SMB) and value (HML). The first three ones are those already used in this study. Data on the two latter factors, SMB and HML, is obtained from the Kenneth French data library; the factors are for the European markets as a whole, not specifically for the Helsinki stock market. However, this is not expected to create any unbearable problems as OMX Helsinki correlates strongly with the European stock markets.

The regression model is similar to that used in Frazzini & Pedersen (2014). The model is the following:

$$(15) R_{BAB,t} = \alpha + \beta_i(MRKT - rf)_{i,t} + \beta_iSMB_{i,t} + \beta_iHML_{i,t} + e_{i,t}$$

**Table 19.** Fama-French 3-factor regression, as in formula (15). BAB and MRKT are excess returns. Coefficients are monthly percentages. Period 12//2001-12/2017.

	$R_{BAB}$	
Variable	Coefficient	t-statistic
$C$	0.0074***	2.8300
$MRKT$	0.0333	0.6421
$SMB$	0.0045***	3.3265
$HML$	0.0012	0.9318
R-squared		0.064
Adjusted $R^2$		0.049
Observations		193

Table 19 shows the results for the ‘FF3’ regression. The data set consists of monthly observations. BAB returns (BAB) are in excess returns over the risk-free rate. Statistical significance is indicated by asterisks (\*\*\*) = 1%, \*\* = 5%, \* = 10%).

The alpha coefficient is slightly lower when SMB and HML factors are added in the regression (0.74%), when compared to a regression with the MRKT as the only explanatory variable (0.87%). For comparison, Frazzini & Pedersen (2014) obtain a three-factor alpha of 0.73% (monthly) for U.S. stocks in their study. The alpha coefficient is significant, but the slope coefficient on the market factor is not. It is almost the same as the earlier presented coefficient from the BAB versus market index regression (beta). An interesting observation is that the small-minus-big factor is positive and significant. This could indicate that small companies are a return driver for the BAB factor. At least there is a positive relationship between the returns on the BAB factor and the overperformance of small companies over large companies.

The HML coefficient is positive as well, however, there is no statistical significance. If the t-statistic would be higher, it would imply that returns on the BAB factor increase by 0.12% per month when the HML factor increases by one per cent. As the p-value is large, such a conclusion cannot be made. Adjusted R-square for the regression model is low – the value of 0.06 indicates that the model does not explain BAB returns practically at all. The situation is the same as earlier when the BAB factor was regressed only against the market factor; after all, as BAB is intended to be market neutral, this is not surprising.

To conclude the results, BAB seems to have a positive relationship with the SMB factor. Even though the factor data used in the model is for the European markets and not purely for Finnish markets, the result can be considered relevant, as it is assumed that there is no substantial reason why the factors would not apply to the OMX Helsinki stock market.

#### 6.4. Problems and shortcomings

There are some issues regarding the data and the empirical analysis in this paper. Firstly, the sample of stocks is relatively small with only one market considered and an average of 167 stocks studied at each period. For comparison, Frazzini & Pedersen (2014) study equities for 19 different markets, each including 80-3200 stocks on average. The sample size in this study is sufficient to divide stocks into two portfolios and to study the betting against beta strategy, but it is too small to take the analysis much deeper – for example,



as there are only a few stocks in some of the different industry sectors, a meaningful statistical analysis between the industries cannot be made.

In addition, some adjustments were needed to be made with regard to methodology, due to the relatively small sample size. This is done to avoid the situation where individual stocks would affect the results too much. For example, Frazzini & Pedersen (2014) weigh the stocks inside the portfolios based on their betas, so that stocks with the lowest (highest) betas have the highest weights in the low-beta (high-beta) portfolio. This is not done in this study, as the number of stocks in each portfolio is low especially in the beginning of the period of study. Therefore, weighing stocks even inside the portfolios according to their betas would cause a situation where returns of individual stocks could considerably affect the performance of the whole factor, which is undesirable.

Regarding the empirical analysis and the betting against beta strategy itself, there are assumptions that most probably do not hold in the 'real' world. It is assumed that money can be borrowed at the risk-free rate, which is certainly not possible especially when the risk-free rate turns negative towards the end of the period. Also, it is assumed that securities can be borrowed for 'shorting' purposes and traded without any costs, which is unrealistic. In addition to these problems, there are some liquidity issues – many stocks in the OMX Helsinki exchange are thinly traded, which would practically make the execution of the BAB strategy in the market both difficult and expensive.

In spite of the presented problems, the study is still relevant in the author's point of view. Any academic paper or empirical research has its problems and shortcomings, including the Frazzini & Pedersen (2014) study on the same subject. Even though there are many assumptions regarding the formation of the betting against beta factor that may cause problems, the difference outperformance of defensive equities versus aggressive stocks is shown without any markable problems or unrealistic assumptions. Further research could be made especially on the sources of the overperformance of defensive equity – this study does not quite address that question. Also, characteristics of the defensive and the aggressive side of stocks could be analyzed in later research, to find possible relationships between BAB and value, size and other factors.

## 7. CONCLUSIONS

The abnormal performance of low-beta securities has attracted a significant amount of academic interest over the decades after Black et al. (1972) observed the empirical failure of the CAPM. During the 21<sup>st</sup> century, the interest has remained high and several influential papers on defensive investing such as Blitz et al. (2007) and Baker et al. (2011) have been published. After factor investing had become increasingly popular during the 2010's, Frazzini & Pedersen (2014) invented the BAB factor that “bets against beta” by shorting high-beta assets and taking a long position in low-beta assets.

This paper extends the study of the betting against beta strategy to the Finnish stock market. To the author's best knowledge, there are no other studies on the subject that exclusively focus on the OMX Helsinki stock exchange – Frazzini & Pedersen (2014) include Finland in their scope of study, but they merely scratch the surface by presenting only a few numbers. The results found in this paper are aligned with those in Frazzini & Pedersen (2014), even though slightly higher excess returns and Sharpe ratios are observed in this paper. This is potentially due to the later examination period including the ultra-low interest rate environment towards the end of the period, even though other factors are likely to play a part too. Also, there are slight differences in methodology, as explained previously – however, these differences are not likely to affect the results significantly.

The BAB factor is shown to be capable of providing positive excess returns in the OMX Helsinki market during the period of study. The performance is mainly driven by the stocks in the low-beta portfolio, which outperform their high-beta peers – this is not surprising considering previous research (Baker et al. (2011), Blitz et al. (2007), Frazzini & Pedersen (2014) and others). The difference in returns between the high- and low beta portfolios is amplified by the rescaling of portfolio betas, which involves leveraging the well-performing low-beta side and de-levering the high-beta side. This is done in attempt to make the factor market neutral, i.e. to have an ex-ante beta of zero. Before the rescaling of the portfolios, the BAB factor has a beta of -0.67. After the rescaling the realized beta for the BAB factor is -0.03, which indicates that the BAB factor is practically independent of the direction of the market.

Over the period of study, BAB provides average monthly excess returns of 0.9%, outperforming the market index by 0.18% in monthly excess returns. BAB has an annual

volatility of 12.6%, which is significantly lower than the respective figure for the market index (18.0%). Hence, the risk-adjusted return metrics (Sharpe ratio and Sortino ratio) for BAB are higher than those of the market. In fact, BAB outperforms the market on most of the performance metrics that are considered in this paper.

It must be noted that the performance of BAB increases significantly after rebalancing when the low- and high beta portfolios are both scaled to have betas equal to the sample average. Even prior to rescaling, the 'raw' BAB factor provides positive excess returns, but these are inferior to the market index. The 'raw' BAB factor is the same as the finalized BAB factor with the difference that the returns of the low- and high-beta portfolios are not multiplied by the scaling factor. The returns on the raw BAB factor are simply the excess returns of the low-beta portfolio minus the excess returns of the high-beta portfolio. The raw BAB factor earns a monthly return of 0.23% on average, and with its 1.8% annual returns it lags the market, which returns 9.9% per annum, by a margin of 8.1% per year. The performance lag to the market is smaller when return-to-risk measures are considered – the raw BAB factor has an annualized Sharpe ratio of 0.12, whereas the market has 0.43. Even though the raw BAB factor has lower returns than the market, it cannot be considered a failure; as it is zero-cost strategy that purely benefits from the return difference between the low-beta portfolio and the high-beta portfolio, comparison to the market index is not necessary. In fact, any positive excess returns could be considered a success for the strategy.

When the rescaling of portfolio betas is done, BAB returns improve significantly. The complete BAB factor earns annual returns of 10.6% per year, and the annualized Sharpe ratio of 0.67 exceeds the corresponding figure for the market index by 0.24. The BAB factor performs better than what was originally expected, mostly due to the rescaling of the low- and high-beta portfolios. The excess returns and regression alphas obtained for BAB are very much aligned with the results obtained in Frazzini & Pedersen (2014); the Fama-French three-factor model alpha is 0.74, which differs from the alpha obtained in the reference study by a mere 0.01 on monthly percentages.

The betting against beta strategy is successful in the market of study, but there are some practical caveats that require attention. It might not be feasible to execute the strategy in the OMX Helsinki market, as some shares even on the 'main list' have a very low liquidity, which can make trading difficult and costly, especially considering short positions. As no transaction costs are included in the calculation of BAB performance in this study, the actual results of performing the strategy in "real life" would be weaker to

an unknown extent. To test if the strategy earns positive risk-adjusted returns when transaction costs are included is out of the scope of this study. However, as the monthly excess return for BAB is 0.9% on average, it could be assumed that transaction costs would not diminish away the returns entirely.

In addition to examining performance over the entire period, the intention was to also examine how BAB performs relative to the market during high-volatility conditions and bear markets. As for bear markets, BAB is found to outperform the market index in absolute returns, when bear markets are defined as any month where monthly returns on the market have been negative for at least four out of six previous months. This is expected as the “other half” of BAB benefits from decreasing stock prices. The examination of performances during times of high uncertainty, as proxied by the VIX, does not provide surprises either. During periods of higher-than median VIX, the BAB factor outperforms the market index by a margin of 0.68% in average monthly returns. During these periods, BAB performance is slightly positive on mean returns but slightly negative on median returns. When VIX is lower-than median, the market index outperforms BAB by 0.73% per month in average returns, and by 1.21% per month in median returns. As the BAB factor is designed to be market neutral, it does provide protection during times of increased volatility and bear markets, but naturally lags the market when volatility is low and global stock markets are bullish.

In the end of the empirical research section, the components of BAB are broken down and the business sectors of the stocks included in both the high-beta and low-beta portfolios are analyzed. The average sector allocations of shares in each portfolio are shown, but they are not analyzed further. To examine the connection between industry sectors and BAB performance would be something to study in further research. Even though a Fama & French (1991) three-factor model regression is performed, some further analysis could be performed on how the value and size factors are related to BAB returns. Finally, further research could focus on limits of arbitrage and test how BAB returns vary when leverage constraints tighten and loosen.

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