ACTA
WASAENSIA

No. 54
Business Administration 20
Accounting and Finance

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Measuring the Long-Run Profitability of the Firm

A Simulation Evaluation of the Financial Statement Based IRR Estimation Methods

UNIVERSITAS WASAENSIS
VAASA 1997

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ABSTRACT

Salmi, Timo and Ilkka Virtanen (1997). Measuring the long-run profitability of the firm: A simulation evaluation of the financial statement based IRR estimation methods. *Acta Wasaensia* No. 54, 66 p.

Four methods, estimating the firm's long-term profitability as the internal rate of return (IRR) of the firm's capital investments, are revisited and evaluated using simulated financial statements. The methods of Kay, Ijiri-Salamon, Ruuhela and the averaged accountant's rate of return (ARR) are analyzed. It is observed that the methods are disrupted by large deviations between the firm's growth and profitability, but are in most cases insensitive to cyclical fluctuations and to major capital investment shocks. Kay's method fares marginally best in numerical performance, and it is theoretically very well founded, with the average ARR method close by. The Ijiri-Salamon method fares reasonably well numerically, but its error is unpredictable. Theoretically, it is the most ad-hoc of the methods. Ruuhela's method has a strong theoretical background, but when its strict assumption of steady state growth is violated, numerically it fares the worst. In the literature's long-standing dispute about the validity of ARR as a proxy for the IRR the simulation results strongly support the school of thought siding with the validity. The conclusion of the research is to recommend the average ARR method in financial analysis practice. It is in a class of its own in pragmatic applicability being based on well-established accounting practice.

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Key words: Long-term profitability, accountant's rate of return, internal rate of return, return on investment, IRR estimation methods, Kay, Ijiri, Salamon, Ruuhela, simulation.

JEL: M40, G31, C88, C15

Acknowledgments: Our thanks are due to professor Erkki K. Laitinen and professor Reijo Ruuhela for their useful comments. Financial support for our research project from the foundation of Jenny ja Antti Wihurin Rahasto is gratefully acknowledged.

1. INTRODUCTION

1.1. Background

The firm's ability to find and implement successful capital investment opportunities decides its long-run profitability and financial position. There is no doubt that the questions of profitability measurement and the valuation of the firm's financial assets are the most important questions in financial accounting research. The question of a theoretically sound and pragmatic profitability measurement is of crucial importance not only to the firm but also to an economy's overall welfare. The allocation of resources in an economy is directly affected by the validity and reliability of the decision makers' measures of the firms' performance (profitability) and financial position. For example, in loan and credit decisions the creditors are not only interested in the applicant's short-term situation but in the firm's long-term ability to generate income.

The firm's profitability is crucially reflected in the financial statements of the firm. The stakeholders of the firm need the profitability information for their decision making both for the short and for the long run. In the economics literature the internal rate of return (IRR) is the widely used theoretical long-run profitability concept. A recent survey by Pike (1996) in the area capital budgeting confirms that IRR is a well-established measure also among practitioners. Furthermore, the investment theory of finance recognizes IRR as a profitability measure, albeit under restrictive assumptions.

Strictly speaking the theory of finance states that, for example, under capital rationing only the net present value method is uniquely consistent with maximizing the value of the stockholders' wealth. See any good text-book of finance such as Copeland and Weston (1979), Levy and Sarnat (1986), Brealey and Myers (1991) for a discussion. However, under ordinary practical conditions of investment opportunities in the same size categories and conventional cash-flow patterns the internal rate of return method can in most cases be expected to give conforming evaluation for the capital investment evaluation. In this paper we accept IRR as the valid long-run profitability measure for the firm. The focus of the paper is on the theoretical consistency and numerical accuracy of the methods presented in earlier literature for estimating the IRR from financial statements.

The accountant traditionally measures profitability as the ratio between the firm's annual income and the book value of its assets. This ratio is often called the accountant's rate of return (ARR) in literature. Other common terms for it are the return on the capital invested (ROI) and the book yield. This measure looks at profitability after the fact. The economist has a different definition of income. It is based on the changes in the market value of the firm defined as its discounted future cash flows. The economist's definition is based on expectations about the future. The internal rate of return (IRR) is consistent with the economist's concept of income. The internal rate of return also is prominent in the capital investment theory.

One traditional way of looking at the firm is to regard it as a series of capital investments. As discussed, the IRR of the capital investments making up the firm is the well-accepted, theoretically valid measure of the firm's profitability. The problem with this theoretical notion is, however, that the IRR of the firms is not readily measurable in actual business and financial analysis practice, while the annual values of the ARR are calculated routinely for business firms. There is a considerable body of literature that discusses the possibility of analytically deriving or empirically estimating the firm's IRR. Since the mid 1960's there is a long-standing controversy, both conceptual and technical, whether it is possible successfully to estimate the firm's IRR. The discussion is too extensive to review in the presentation at hand. For the references see the review article by Salmi and Martikainen (1994), Butler, Holland and Tippet (1994) and Stark (1994).

The approaches in literature to the IRR estimation can be classified into several, partly overlapping categories. The first approach is trying to establish a link from ARR to IRR. This approach is exemplified by Kay (1976) and later by Peasnell (1982a, 1982b). Kay's method has been evaluated for example by Whittington (1979), Salmi and Luoma (1981), Brief and Lawson (1992) and Salmi and Virtanen (1995). A second approach is to derive the IRR by utilizing an auxiliary estimate such as CRR (the cash recovery rate). This approach has been suggested by Ijiri (1979 and 1980), extended and tested by Salamon (1982) and Gordon and Hamer (1988). The Ijiri-Salamon method has been further tested by Shinnar, Dressler, Feng and Avidan (1989) and Stark, Thomas and Watson (1992). A third approach seeks to establish the IRR directly from the published financial statements. This category is represented by Ruuhela (1972) and its mathematically streamlined rederivation in Salmi (1982). The assumptions of Ruuhela’s model and the consequences of relaxing them have theoretically been considered by Tamminen (1976). Another direct IRR estimation method has been presented in Finland by Laitinen (1980). Furthermore, Kay (1976; 455) presented how his IRR estimate could be improved if the ratio of the accountant's valuation of the firms assets and the economist's valuation of the firms assets were available. Steele (1986) suggested the use of market values from the stock market to represent the economist's valuation of the firm's assets needed in Kay's correction. Lawson (1980) presented an approach based on cash flows and market values. Artto (1980) advocates a cash-flow-based profitability estimation.

Which of the various methods put forward in literature should one select? For the business practitioner, as well as for an academic researcher, facing the number of the various long-run profitability estimation methods, and the theoretical controversy of their correctness, the question becomes the following. What methods are reliable and applicable for evaluating the long-run profitability of a business firm? In other words which method or methods work both in practice and in theory? In particular, might it be, after all, that the practice of calculating a straight-forward average of the annual ARRs would be at par with the more theoretical IRR estimation methods?

1.2. Overview of Research Problem and Methodology

The discussion in literature on the possibility of a sound estimation of the firm's IRR has been inconclusive. The controversy has concerned both the generality of the theoretical derivations and the empirical applicability. It is our view that the various methods are best evaluated in their empirical context. But even the empirical investigation has not been unproblematic. The following difficulty has arisen. The empirical estimates of the IRR given by the various methods have been compared only relative to each other in the earlier literature. Thus, the earlier empirical approach has not resolved the absolute reliability of the methods compared. The true IRRs of the firms under observation are needed as benchmarks for an objective reliability evaluation. Unfortunately, the true IRRs cannot be known when actual financial statement data are used. This dilemma can be solved by using a simulation approach. A simulation approach with a preset IRR facilitates an objective evaluation of the ability of the various methods to estimate the firm's true IRR.

Our paper evaluates the three financial-statement-based methods by Kay, Ijiri-Salamon and Ruuhela. In addition, we compare these IRR estimation methods to the simple practice of using the average of the annual accountant's rate of returns as the estimate of the firm's IRR. The market-value-based methods of Lawson and Steele are excluded in the present paper, since their evaluation is not readily amenable to our simulation approach. An attempt at a consistent simulation of the stock market values of the firms is beyond the present scope.

The IRR estimation methods of Kay, Ijiri-Salamon and Ruuhela all are mathematically non-trivial. They are not straight-forward to apply in practice on actual financial data. The practitioner's obvious alternative would be to use the averaged accountant's rate of return as a surrogate of the IRR estimate. However, in earlier literature there are reservations on using the average ARR as the estimate. The reservations can be traced as far back as to Vatter (1966). Later e.g. Fisher and McGowan (1983; 82) stated that "accounting rates of return provide almost no information about economic rates of return". On the other hand, as pointed out by Pike (1996; 83-84) in connection with capital budgeting, the technically simple methods such as the payback period and the average ARR has been condoned by several authors starting from Weingartner (1969).

We intend to revisit the question of the usefulness of the average ARR as an ex-post long-term profitability measure, since it has not been unequivocally demonstrated that the average ARR method would necessarily be markedly inferior to the more complicated IRR estimation methods presented in literature. Given the obvious fact that the business firms continuously use accounting measurement we would find it rather surprising if ARR would not be a useful concept also for the firm's long-run profitability (IRR) measurement. Hence we will consider the average ARR method together with the more complicated methods in this paper.

In the literature on IRR estimation some general assumptions have become conventions. We use these same conventions. An important, established convention in the long-run profitability research is to consider the firm as a series of repetitive capital investments.

Stating this research convention in Salamon's (1982; 294) words "... the firm is a collection of projects that have the same useful life, same cash-flow pattern, and same IRR". See, however, the critique of this standard assumption by Kelly and Tippet (1991). The assumption of the constant cash-flow pattern has usually been presented as a necessary, technical simplification of the business reality. However, this restriction is not an unrealistic, technical assumption. It can be posed that the assumption is in line with observing often long periods of stable business culture in individual firms. The business culture of the firm is above all created by its CEO-level management and their ability to generate and utilize capital investment opportunities.

Another strong convention is the firm's access to the financial markets freely to obtain the funding for the capital investments. In other words the implied capital markets in this area of research conventionally are perfect and complete. There is no capital rationing. Therefore, the question of financing of the simulated capital investments need not be considered in this paper.

1.3. Problem Statement

In the current paper we are interested, in general terms, in evaluating the accuracy of the selected long-term profitability estimation methods under different economic circumstances, under different capital investment payback profiles and under different accounting decisions on depreciation. More specifically, the following research questions will be considered.

In the earlier research a constant growth approach to the capital investments has been fairly common. This restriction has meant the absence of business cycles and noise. A priori one would expect that the cycles can have a drastic effect on the ability of the methods to estimate the correct IRR. We relax the steady-state restriction. Therefore, our first research question is:

1. Are the methods sensitive to business cycles in the capital investment activities? Are the methods sensitive to ordinary irregularities in the capital investments?

Second, an outside stakeholder has to base the profitability estimates on the financial data provided by the firm. In the financial statement data the capital investments and their cash flows are totally mixed. It is not possible to know the contribution pattern of the capital investments based on the external data. The question of the effect of the different contribution patterns arises as in Salamon (1982) and Gordon and Hamer (1988). Hence, our second research question is:

2. Are the methods sensitive to the underlying, alternative cash contribution patterns and life-span of the firm's capital investments?

Third, it has been put forward in the earlier literature that there are some particular instances where the profitability estimates given by the accountant's rate of return theoretically become close or equivalent to the underlying, true profitability of the capital investments making up the firm. These include the case where growth equals profitability as presented by Solomon (1966; 115) and the case where the theoretical annuity method of depreciation is postulated as presented in e.g. Salmi and Luoma (1981; 28) and Peasnell (1982a; 364). The annuity depreciation is the economist's depreciation in defining the concept of economic income discussed e.g. in Bromwich (1992; 31-51). Hence, our third research question is:

3. Are the methods sensitive to disparities between the firms growth and profitability?

Fourth, in accounting practice the choice between the depreciation methods such as the prevalent straight-line and the declining-balance methods affects the reported annual income figure. Our fourth question is:

4. Are the methods sensitive to the depreciation choice that the firm has used in producing its financial statements?

Fifth, the IRR estimation methods are largely based on the idea of regular development uninterrupted by structural changes or other major one-time events causing exceptional capital investment peaks. Our fifth question relates to this aspect:

5. Are the methods sensitive to major capital investment shocks?

An economic time series is made up by several constituents. These are the growth trend, the business cycle, the seasonal variation and the noise. Furthermore, there can be regular or irregular shocks. The growth trend and the business cycle are relevant in this paper. Seasonal variations are intra-year. Thus they do not arise in our research questions. It is true that the economic activities of the firm are continuous in nature. However, the financial data used for the profitability estimation in the methods under observation use discontinuous observations from the annual statements.

This study contributes to the existing knowledge in the following, related ways. First, it lends new evidence in the long-standing theoretical controversy of the ability of the ARR to serve as a useful proxy of the IRR. Second, it improves on the simulation approach to evaluate various IRR estimation methods by closely emulating the flow-patterns of real-life business firms. Third, it comes up with a theoretical and empirical comparison of four major IRR estimation methods presented in earlier literature. Finally, it arrives at a well-founded practical recommendation (usage of averaged ARR) for the evaluation of the long-run profitability of business firms.

2. SIMULATION EVALUATION APPROACH

This chapter presents our simulation model. First, we present the simulation engine that generates the capital investment time series. Second, we present the generation of the cash inflows from the capital investments in terms of alternative contribution distributions. Third, we present the alternative depreciation methods the simulated firm may apply.

2.1. Generation of the Capital Investment Time Series

As discussed in the introduction the firm can be considered a series of cash outflows to capital investments and the cash inflows generated by these capital investments. The earlier discussion of the methods has been based on the implicit assumption of constant, exponential growth of the capital investments that make up the firm. In a previous simulation approach to analyze Kay's method Salmi and Luoma (1981) also assumed capital investments obeying constant, exponential growth. Their engine to generate the capital investments was the standard exponential growth model

(1) $g_t = g_0 (1+k)^t.$

where

g_0 = initial level of capital investments,
g_t = capital investments in year t,
k = growth rate.

Assuming a constant growth is a major simplification of the reality of capital investment decisions in business firms. To evaluate the reliability of the IRR estimation methods under observation it is of paramount importance to know whether the methods are sensitive to business cycles, noise and disruptive irregularities in the capital investment activities. To tackle these questions we extend the Salmi-Luoma simulation engine to generate the capital investments with the possibility of business cycles, noise and shocks. We use

(2) $g_t = g_0 (1+k)^t\{1 + A \sin[(2\pi/C) + \varphi]\}(1 + \sigma z)(1 + \delta_{t\tau} S).$

For the indexing of the years t in the simulation engine we denote

T = length of the simulation period,
n = length of the observation period (number of years under observation for the profitability estimation).

In the simulation the index t must run all the way from year 1 to year T. The simulated firm is founded at the beginning of year 1. The transient, initial period from year 1 to year T-n represents the stage needed to reach a going-concern phase. The evaluation of the selected long-run profitability estimation methods is best conducted only after the going-concern phase has been reached. Thus the actual observation period for the evaluation of the profitability estimation methods is from T-n+1 to T. For brevity, the indexing is not presented in the formulas.

The first part in Formula (2) of the simulation engine is equivalent to the constant exponential growth Formula (1) used earlier in the simulation by Salmi and Luoma (1981). We have for the trend component the same g_0, g_t and k definitions as in Formula (1).

In our extension we first incorporate a sinusoidal business-cycle component to the engine. For this augmented cyclical fluctuation component we denote

A = amplitude of the cycle,
C = length of the cycle,
φ = technical phase adjustment for the cycle.

In the above, the term φ is purely a technical phase adjustment needed in the engine. It is needed to slightly shift the continuous sine curve so that its maximum and minimum values coincide with the discrete observations. For example, for an average length of six years of real-life business cycles the value of φ becomes $\pi/6$.

Seasonal variations do not arise. This is because our simulation engine is discrete with one-year intervals. In the terms of real-life business practice this is tantamount to using annual financial statements instead of the potential quarterly reports.

Next, we incorporate a random component. We use white noise as the random component and denote

σ = the standard deviation of the random fluctuation in the capital expenditures,
z = random variable following the (0,1)-normal distribution.

For the shock (disruption) component we have

S = capital investment shock coefficient,
t = the year of the capital investment shock (t = ∞ for no shock in the simulation),
δ = Kronecker's delta, $\delta_{t\tau} = 1$ when $t = \tau$, and 0 otherwise.

All the new components, which are augmented into Formula (1) to arrive at the generalized capital investments generation engine presented in Formula (2), are multiplicative. In other words, the components are defined relative to the trend-level of the capital investments. This means, for example, that in the terms of statistics the standard deviation of the random fluctuation in the capital expenditures is heteroscedastic in nature. Likewise, the relative amplitude of the business cycles stays constant while the absolute magnitude of the business cycles increases over time.

Compared with the constant-growth approach, the inclusion of the business cycles and noise components make the simulation engine realistic. This is attested by the fact that in simulation the extended engine produces financial time series which resemble the time series profiles observed on actual business firms. See e.g. the sample of the time series drawn in Salmi et al. (1984; 46-48).

2.2. Cash Inflows Produced by the Capital Investments

The capital investments g_t induce later, corresponding cash inflows. The relationship between the initial outlay of a capital investment and its cash inflows can be expressed in terms of a contribution distribution. Denote by b_i an individual, relative cash-inflow contribution from a capital investment that has been made i years back. This term is called the contribution coefficient. The contribution distribution is naturally made up by the individual contribution coefficients for the life-span of the capital investment. The mathematical formulation below is based on e.g. Ruuhela (1972). The cash flow profiles in Ijiri (1979), Salamon (1982) and Gordon and Hamer (1988) represent the same idea of contributions induced by the capital investments of the firm.

First consider the contributions (the cash inflows) from a single capital investment made at time point t = 0, illustrated by Figure 1.

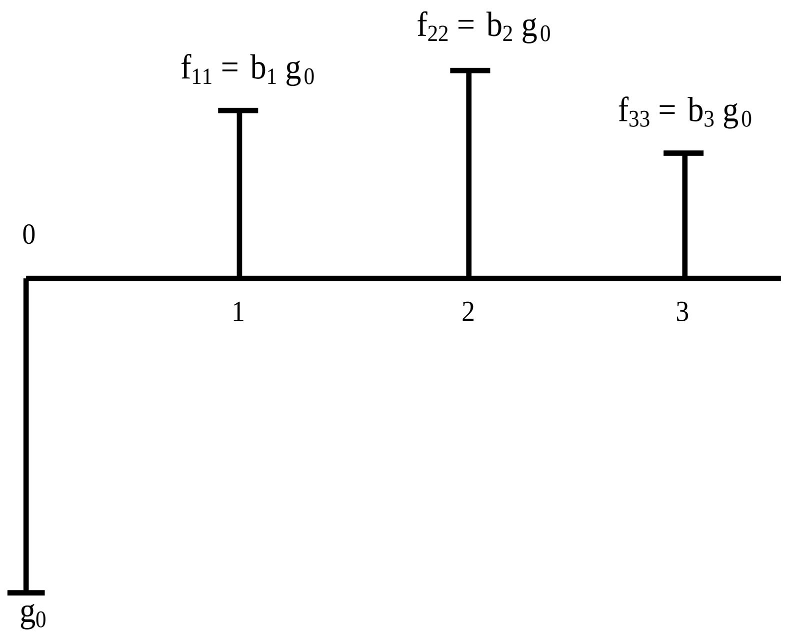

Figure 1. Structure of a single capital investment project

In a more general denotation, a contribution (i.e. the cash inflow) in year t from a capital investment made in year t-i is given by

(3) $f_{t\,i} = b_i \; g_{t-1}$,i = 1,..., min(N,t),

where

$f_{t\,i}$ = absolute contribution in year t from capital investment i years back,
b_i = relative contribution from capital investment i years back,
N = life-span of a capital investment project (the same for all capital investments).

Under the regular going-concern phase, which is to be used for evaluating the profitability estimation methods, the index i runs from 1 to N. However, during the transient initial period before the year N, i.e. t < N, there can be contributions only from t years back. Hence the term min(N,t).

The total contribution in any year t (i.e. all the cash inflows in that year) is cumulated from the contributions from the capital investments made in the earlier years. Hence we have

(4) $$f_t = \sum_{i=1}^{\min(N,t)} f_{ti} = \sum_{i=1}^{\min(N,t)} b_i \; g_{t-i}$$

which defines

f_t = cash inflow in year t.

The accumulation of the contributions (the cash inflows) in year t from all the capital investments in the previous years is illustrated by Figure 2.

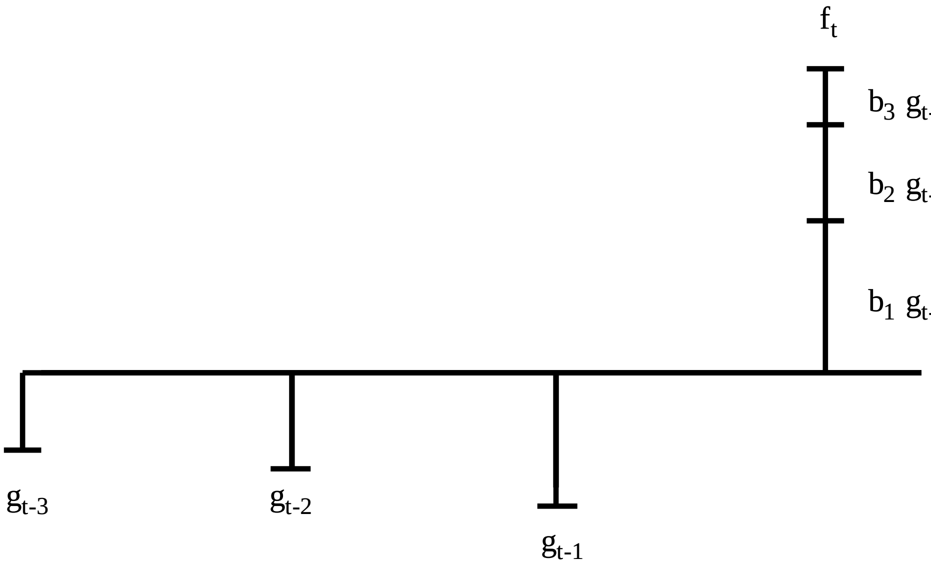

Figure 2. Composition of an annual cash inflow

A comment on the mathematically discontinuous nature of the simulation model is in order at this stage. As is familiar from capital investment literature, the capital investment model involves a discretization of what basically are partly continuous events. An initial outlay made at time t = 0 is assumed to produce its corresponding contributions at times t = 1,...,N. Likewise, the depreciations for a capital expenditure made at time t = 0 will take place at t = 1,...,N. The same pattern is repeated for all capital investments for the simulation period. Our simulation model considers all the events as discrete as is common in capital investment models.

In line with the standard treatment in literature the contribution distribution b_i in our simulation engine is the same for all the capital investments. In other words, the profitability of the capital investments remains the same over the period under observation. Furthermore, constant returns of scale on the capital investments are assumed, as is the custom in growth models. When the firm grows, there are no economics of scale. See e.g. the standard reference Levhari and Shrinivasan (1969; 153).

A contribution distribution b_i fixes the internal rate of return. As was noted, the contribution distribution is assumed unchanged for all the consecutive capital investments even though the level of the capital investment outlays varies over the business cycles as defined by Formula (2). Hence the internal rate of return, i.e. the profitability of the simulated firm, is defined by the cash flows of any individual, simulated capital investment. The internal rate of return corresponding to a given contribution distribution is defined by equating the initial outlay with the sum of the future, discounted cash inflows:

$$(5) \qquad g_t = \sum_{i=1}^{N} f_{t+i,i} \ (1+r)^{-i} = \sum_{i=1}^{N} b_i \, g_t \ (1+r)^{-i} = g_t \sum_{i=1}^{N} b_i \, (1+r)^{-i}$$

which is readily reduced to

$$(6) \qquad \sum_{i=1}^{N} b_i \ (1+r)^{-i} = 1.$$

The r given by Formula (6) is the true internal rate of return of the simulated firm that is the benchmark in evaluating the various IRR estimation methods. It should be noted that Formula (6) is not suggested to be used as another estimation method for the long-run profitability of the firm from actual business data. Such a direct estimation would not be practical, nor maybe even possible, because the literature does not currently have adequate means readily to identify the contribution distribution of the capital investments making up the firm.

In the simulation evaluation the internal rate of return r which corresponds to a chosen contribution distribution b_i can be readily assessed from Formula (6) using the numerical analysis methods such as the bisection method. For the bisection method see any standard text-book of numerical analysis such as Conte (1965; 39-43). The discussion of the specification of the alternative contribution distributions will be postponed till the next section.

It is a well-known fact that under non-conventional cash flows (more than one sign alteration) there can be multiple or no real roots for the internal rate or return r in Equation (6). See e.g. Teichroew, Robichek and Montalbano (1965). This problem does not arise in our simulations. A conventional cash-flow contribution pattern will be used.

Profitability defined as the IRR in our simulation is assessed from the contributions of the capital investments only. The financing issue does not come to the fore. This separation of capital investments from financing is in line with the classic results of Modigliani and Miller. For a discussion on this issue, see for example Yli-Olli (1980). This separation also is in line with the standard usage of IRR in connection with the capital investment decision. In making the decision, the decision maker compares the IRR of the capital investment project prior interest to the cost of capital. Including the interest (i.e. the cost of financing) in the cash estimates for the project's flows would be double accounting as pointed out by any good textbook on capital investments.

The question of financing and its costs do not arise in our simulations as long as it can be safely assumed that the firm remains sufficiently profitable to be able to obtain new capital as the need arises. Hence chronically declining activities (divestments) or infeasible combinations of growth and profitability will not be considered in our research, since in actual business practice this would in the long-run cause restrictions or even a cessation of the availability of capital to the firm. For a discussion of feasible growth/profitability combinations see Suvas (1994).

2.3. Contribution Distribution

As discussed in the previous section the true internal rate of return r determined by Formula (6) is a function of the contribution distribution b_i introduced in Formula (3). The true form of the contribution distribution is not generally known for real-life business firms. In order to assess the effect the different, potential contribution patterns of the firm's capital investments we will perform our simulations with three different contribution patterns from the capital investments. Figure 3 illustrates three different types of potential contribution distribution, a neutral, a typical growth-maturity-decline life-cycle pattern and a steadily declining case.

The three distributions we choose are the uniform contribution distribution for the neutral case, the negative binomial contribution distribution for the growth-maturity-decline case and a linearly declining distribution (Anton distribution) for the steady decline.

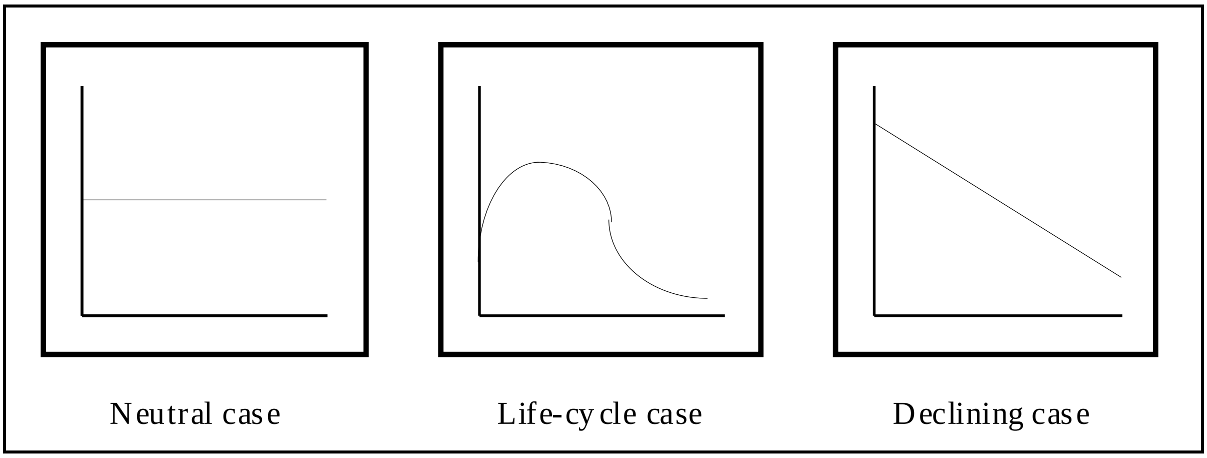

Figure 3. Three contribution pattern alternatives

The uniform contribution distribution is defined by the annuity factor

$$(7) \qquad b_i = \left[\sum_{j=1}^{N} (1+r)^{-j} \right]^{-1}, \quad i = 1,\ldots,N.$$

The uniform contribution distribution for the life-span of the investments is an obviously neutral choice. It produces the same level of contribution each year throughout the entire life-span of the capital investment. In the simulation the numerical values of the contribution coefficients which lead to the preselected true profitability r are given directly by substituting the numerical value r into Formula (7).

The typical life-cycle of a product includes an early growth phase, maturity, and decline. The negative binomial distribution corresponds to this cycle. For our simulation purposes it has the further advantage of being different from the uniform contribution distribution in two important respects. It is not constant and it is not symmetrical.

The general definition for the negative binomial distribution is given by

$$(8) \qquad P_j = \binom{j-1}{m-1} q^m (1-q)^{j-m}, \qquad j = m, m+1, \ldots$$

where q is a shape parameter and m is a location parameter. For our simulation we choose q = 0.15 and m = 2 which leads to a typical life-cycle profile. For the definition and the properties of the negative binomial distribution see Fisz (1967; 167).

For our purposes, two technical adjustments to the generic negative binomial distribution are needed. First, the distribution is cut from the right at the life-span instead of letting it continue to infinity. Second, the distribution is shifted to the left to coincide with the capital investments' life-span. Hence we have as our negative binomial contribution coefficients

(9) $$b_i = s\,(i+1)\,q^2\,(1-q)^i, \qquad i = 1,\ldots,N$$

where s is a scaling factor inducing the desired level of true profitability. In the simulation the numerical values of the contribution coefficients which lead to the preselected true profitability r are found by finding the value of s which fulfills Formula (6). It is given by

(10) $$s = \left\{ q^2 \sum_{i=1}^{N} (1+i) \left(\frac{1-q}{1+r} \right)^i \right\}^{-1} .$$

The Anton distribution presented in Anton (1956) is defined as

(11) $$b_i = \frac{1 + (N - i + 1)r}{N}, \qquad i = 1,\ldots,N.$$

This is a linearly declining contribution distribution with convenient theoretical properties. It has been shown (see Salomon (1971; 168 footnote and Appendix 2 of the current paper)) that if the contributions from the firm's capital investments follow the Anton distribution the theoretical annuity depreciation (to be discussed in a later section) and the practical straight-line depreciation coincide and hence lead exactly to the same reported profit for the firm. (This is tantamount to the accountant's and the economist's concepts of income agreeing under these special circumstances.)

2.4. Depreciation Methods

To complete the simulation model we need the formulas for alternative depreciation methods in order to have the annual profit and book value figures. First consider the accounting relationships between these concepts.

The profit p_t is defined by the cash inflow f_t less depreciation d_t as

(12) $$p_t = f_t - d_t.$$

The book value v_t of the firm at the end of year t is defined by book value at the beginning of the year plus the capital investments g_t less the depreciation d_t. Hence

(13) $$v_t = v_{t-1} + g_t - d_t.$$

In our simulation model the book value of the firm involves only the capital expenditures and the depreciation. For simplicity cash, inventories and other assets are not modeled separately.

Next consider depreciation. The firm's choice of the depreciation method is central to profit measurement and asset valuation both in accounting theory and practice. We build into our simulation model the possibility of three alternative depreciation methods to be employed by the simulated firm in its financial statements. The alternatives are the straight-line depreciation method, the double declining-balance method and the theoretical annuity depreciation method.

An important feature of the current research approach is to be able to evaluate how well the different IRR estimation methods perform under realistic conditions. The first two of the alternative depreciation methods for the simulated firm are prevalent in actual accounting practice. The idea of straight-line depreciation method is that it allocates the costs evenly based on the passage of time over the expected life-span of the asset. Decreasing charge depreciation methods are based on the idea of equipment being more efficient in their early life. We choose double-declining-balance method as a representative of the decreasing charge methods because it is by definition (the doubled rate) related to the corresponding straight-line method. Annuity depreciation is included as one of the alternatives for quite a different reason. It is purely a theoretical concept. It is included to verify whether the simulation and the profitability estimation algorithms produce the correct internal rate of return under annuity depreciation as predicted by theory.

Straight-line depreciation is calculated as

$$(14) \qquad d_t = \sum_{i=1}^{\min(N,t)} (1/N)\, g_{t-i} \,.$$

Double-declining-balance depreciation is a decreasing depreciation used in the U.S. practice. See Davidson and Weil (1977). Double-declining-balance depreciation method formula is

$$(15) \qquad d_t = \sum_{i=1}^{\min(N,t)} (2/N)\left[1 - (2/N)\right]^{i-1} g_{t-i} \,.$$

The above formula for the double-declining-balance depreciation forms an infinite geometric series. However, in accounting the capital investment expenditure is exhausted at the end of the life-span. We use the historical cost convention. Hence, all the remaining book value of the relevant investment is depreciated in the last year of the life-span.

The well-accepted definition for the annuity depreciation is that the profit (before interest and taxes) p_t is assessed as the interest on the initial capital stock v_{t-1} in year t. Thus

$$(16) \qquad p_t = r\, v_{t-1}$$

and hence from Formula (12) we get

(17) $d_t = f_t - r\, v_{t-1}$.

Annuity depreciation is a theoretical construct. As is evident from Formula (17), a circular reasoning is involved. It is necessary to know in advance the value of r (the internal rate of return) in order to be able to apply the annuity depreciation method. In other words, the profitability information is needed for estimating the profitability. In a simulation model, however, this is possible since the true internal rate r can be fixed in advance.

2.5. Accountant's vs. Economist's Profits and Annuity Depreciation

The construction of our simulation model was concluded in the previous section. However, annuity depreciation has a pivotal role in the relationship between accountant's and the economist's profit and valuation concepts. Hence the discussion started in the section on the problem statement is continued utilizing the notation introduced.

The accountant and the economist have different concepts of income. The accountant's profit and the accountant's rate of return are based on historical data. The accountant needs depreciation in defining annual profits. The accountant's rate of return is given by

(18) $$ARR_t = \frac{p_t}{v_{t-1}} = \frac{f_t - d_t}{v_{t-1}}.$$

The economist's income concept is independent of depreciation. It is based on future cash flows. The well-known economist's valuation of the firm is defined as the present value of the future net cash inflows:

(19) $$w_t = \sum_{i=t+1}^{\infty} \frac{f_i - g_i}{(1+r)^{i-t}}.$$

In accordance to the classic results, discussed in the section on the problem statement, IRR and ARR (appropriately weighted, if not constant) agree if the annuity method of depreciation is used for depreciating the book value of the firm's assets. This result is tantamount to proving that if the economist's valuation w_t and accountant's valuation v_t of the firm's assets agree, then IRR and ARR agree.

A second, relevant classic result is that if the steady state growth of the firm is equal to its internal rate of return, then ARR and IRR agree. See Solomon (1966; 115). For a discussion and a presentation of the proofs see for example Salmi and Luoma (1981).

3. REVIEW OF FOUR PROFITABILITY ESTIMATION METHODS

The previous chapter presented our simulation engine. The current chapter presents four IRR estimation methods from earlier literature to be analyzed and evaluated with our simulation approach. The methods are Kay's, Ijiri-Salamon's, Ruuhela's and the accounting-practice compliant average ARR method. Before any IRR estimation method can be applied on the simulated (or actual financial) statements, the IRR estimation method must be made operational for the financial data available. This fact is observed whenever necessary. We do not evaluate market-value-based methods in this paper. However, the market-value-based methods by Lawson and Steele are briefly discussed at the end of the chapter.

3.1. Kay's Method

3.1.1. Presentation of the Method

Kay (1976) presented an iterative method for estimating the IRR. Kay's original presentation used continuous notation. From the accounting point of view, however, a discrete version of Kay's results is needed to make the method applicable on simulated (or real-life) financial statements of a business firm. For Kay's method we have from Kay (1976; 451), Salmi and Luoma (1981; 25) and Peasnell (1982a; 371) the discrete version for IRR estimation

$$\hat{r} = \frac{\sum_{t=2}^{n} p_t (1+\hat{r})^{-t}}{\sum_{t=2}^{n} v_{t-1} (1+\hat{r})^{-t}} \tag{20}$$

As is recalled, the years in our data-generating simulation engine run from 1 to T while the actual observation period is from T-n-1 to T. For notational simplicity the indexing of the years of the observation period has been adjusted in Formula (20) to run from 1 to n. In this notation, the annual accountant's profit (operating income) p_t and the book values of the firm's assets v_t at the end of each year are now observed for years 1 to n. Therefore the first v_{t-1} available is for year t = 2. This fact is duly reflected in the summation notation in Formula (20).

Kay's iterative method is easily coded as a computer program to solve Kay's IRR estimate given the profit and book value observations from the financial statements. For the conditions of convergence of the IRR iteration procedure see Steele (1986; 2-5). The actual programs coded for this paper are Turbo Pascal 7.01 programs for an MS-DOS PC. (The programs are made publicly available on the World Wide Web at the following address <URL:http://www.uwasa.fi/~ts/smuc/prog/smucprog.html>.)

3.1.2. Discussion of the Method

Rewrite Formula (20) as

$$(21) \qquad \hat{r} = \sum_{t=2}^{n} \frac{v_{t-1}(1+\hat{r})^{-t}}{\sum_{t=2}^{n} v_{t-1}(1+\hat{r})^{-t}} \frac{p_t}{v_{t-1}}$$

It is immediately obvious that Kay's IRR estimate is a weighted average of the accountant's rates of return over the observation period, which would have been

$$(22) \qquad \overline{ARR} = \frac{1}{n-1} \sum_{t=2}^{n} \frac{p_t}{v_{t-1}}$$

The factors $v_{t-1}(1+\hat{r})^{-t}$ act as Kay's weights. Thus Kay's method implicates a strong link between the economist's and the accountant's rate of return concepts.

The following question naturally arises and will be tackled in our simulation evaluation. Is the more complicated IRR estimation Formula (20) decidedly better than the straight-forward Formula (22) which furthermore is based on well-established accounting concepts?

The link between the accountant's and the economist's valuation concepts are very evident also in Kay's (1976; 455) derivation. Kay presents the following relationship between the IRR estimate ($\hat{r}$), the true (economist's) internal rate of return (r), growth-rate (k), the accountant's book value (v) and the economist's valuation of the firm (w)

$$(23) \qquad \hat{r} = k + (r - k)(w/v).$$

In the above the economist's value of the firm w is the present value of the future net cash flows (c.f. Formula (19)). The accountant's book value is based on the historical accounting data of the capital investments and depreciation (c.f. Formula (13)). If the two valuations agree, then also the accountant's and the economist's rates of return agree. The first corollary of this fact is that if the (theoretical) annuity depreciation could be used, then Kay's method is expected to give the exactly correct IRR estimate ($\hat{r} = r$). See e.g. Salmi and Luoma (1981; Appendix III) for the proof. The second corollary, in line with Solomon (1966; 115), is that if the growth and the profitability agree (r = k) then, again, Kay's method is expected to give the exactly correct IRR estimate ($\hat{r} = r$).

3.2. Ijiri-Salamon Method

As was seen in the previous section Kay's method can be interpreted as a method that seeks the link between the IRR and the ARR. Another route is taken in the Ijiri-Salamon method.

Ijiri (1979) presented what Salamon (1982) interpreted and expanded as an IRR estimation method based on the concept of the cash recovery rate, CRR. Ijiri (1979; 259) derived the following relationship between CRR and IRR

$$\text{(24)} \qquad CRR = \frac{r}{1 - (1+r)^{-N}} .$$

When the CRR is known, the corresponding value of IRR can be readily solved by numerical iteration from Formula (24) using e.g. the bisection method. The IRR estimation problem thus becomes a CRR estimation problem. The central idea of the Ijiri-Salamon method is using this surrogate because CRR is easier than IRR to estimate from the financial statements.

The cash recovery rate CRR can be defined as the ratio between the cash inflows from capital investments and the outstanding gross capital investments. Ijiri (1980; 55) presents the calculation of an annual CRR from published financial statements as

$$\text{(25)} \qquad CRR_t = \text{Cash Recoveries / Gross Assets}$$

where

Cash Recoveries = (Funds from Operations)
+ (Proceeds from Disposal of Long-Term Assets)
+ (Decrease in Total Current Assets)
+ (Interest Expense)

and

Gross Assets = (Total Assets) + (Accumulated Depreciation),
averaged between beginning and ending balances.

In our simulation evaluation the cash recoveries are simply equivalent to f_t. The gross assets must be discussed in more detail. The total assets are given directly by the book value v_{t-1}. First, when the total assets have been defined the accumulated depreciation must be assessed to get the gross assets. Second, the beginning instead of the average book values are used in our study.

In financial statement analysis practice the accumulated depreciation is typically obtained by canceling backwards the depreciations for a suitable span of years. In analysis practice the choice of the backwards span tends to be somewhat arbitrary. However, it is mathematically obvious that given the average life-span of the capital investments and a constant level annual depreciations, the accumulated depreciation will be given by accumulating the depreciations from half the average life-span. While this result concerns the straight-line depreciation, the choice will be used as the best approximation for all the depreciation profiles.

Furthermore, Ijiri's approach requires an estimate of the life-span N of the firm's capital investments. This means a potential source of further estimation errors in the method. In simulation the true life-span of the capital investments is known accurately. Hence the effect of the accuracy of estimating the life-span of the capital investments can be examined for Ijiri-Salamon method in our simulation approach. Note that this potential source of error is not present in Kay's method.

Next consider the different conventions in calculating the book values in financial statement analysis. Instead of the often suggested averaging between the annual beginning and ending book values we use the beginning values v_{t-1}. This leads to more accurate results when a discrete instead of a continuous approach is used. This choice is in line with the treatment of Kay's method in Salmi and Luoma (1981) and Peasnell (1982a).

The estimates of the annual cash recovery rates CRR_t are calculated from

(26) $$CRR_t = \frac{f_t}{V_{t-1}} ,$$

where V_t denotes the gross assets at the end of year t calculated from

(27) $$V_t = v_t + \sum_{i=0}^{\frac{N}{2}-1} d_{t-i} .$$

In (27) the life-span N is assumed an even integer for notational simplicity.

The calculated CRR_t values are averaged and the average is substituted as CRR into Formula (24) in line with Ijiri (1980). Ijiri's IRR estimate can then be iterated from Formula (24).

Since we are using a simulation approach with a fully known engine to generate the observations, we also have the option to calculate the exact accumulated depreciation. This enables us to differentiate between the sources of the error in the IRR estimate. The components of the error are the error due to Ijiri-Salamon's method and the error due to the approximation of the accumulated depreciation.

3.3. Ruuhela's Method

The third method to be included in our analysis is the IRR estimation component of Ruuhela's "Growth, Profitability and Financing" model. As we have seen in the above, Kay's method is based on a relationship between the ARR and the IRR and Ijiri-Salamon method on the relationship between the CRR and IRR. Ruuhela's method can be considered to fall into a category of direct estimation of the IRR from the financial statements without the intermediate ARR or CRR concepts.

The method was first presented in Ruuhela (1972) and mathematically streamlined by Salmi (1982). The method was restructured in Ruuhela et al. (1982). The explicit estimation of the firm's growth and the assumption of a stable business-culture period are characteristic of Ruuhela's approach.

Ruuhela's IRR estimate is given by

$$\hat{r} = k \frac{N a_{N,k} - F}{(N a_{N,k} - 1) F} \quad (28)$$

where k is growth-rate trend of the capital expenditures, $a_{N,k}$ is the annuity factor

$$a_{N,k} = \frac{k (1+k)^N}{(1+k)^N - 1} \quad (29)$$

and F is defined as the capital investment ratio

$$F = \frac{g_t}{f_t}. \quad (30)$$

Ruuhela's method assumes a constant, exponential growth of the capital-investment g_t and the cash-inflow f_t time series of the firm. The quotient F of the two time series thus is constant in the method. Ruuhela's method also assumes that the capital investments contribute in accordance to the Anton distribution.

In applying Ruuhela's method an estimate of the common growth rate of the firm's time series is needed. Most often an OLS estimate of the growth-trend of the firm's funds from operations corresponding to f_t is used as the estimate. Given the OLS estimate $\hat{k}$ of the growth trend the capital investment ratio is estimated from

$$\hat{F} = \frac{\sum_{t=1}^{n} g_t (1+\hat{k})^{-t}}{\sum_{t=1}^{n} f_t (1+\hat{k})^{-t}}. \quad (31)$$

3.4. Discussion of the Model-Oriented IRR Estimation Methods

Consider the conceptual backgrounds of the three methods presented so far. The IRR estimation formulas of Kay's and Ijiri-Salamon methods draw on the relationship between an income statement variable (a flow variable) and a balance sheet variable (a stock variable). As is seen from Formula (20) Kay's method involves the accounting profit p_t and the book value of the firm v_t. Conceptually, Kay's presentation leans heavily on exploring the relationship between the economist's and the accountant's rate of profit.

The Ijiri-Salamon method involves the cash inflows f_t , the gross assets V_t , i.e. the book value of the assets undepreciated, and the life-span N of the firm's capital investments. This is readily seen from Formulas (26) and (20). The concept of the cash recovery rate is central in the method.

Ruuhela's method is directly based on the conventional internal rate of return model of capital investments. Ruuhela's method consequently directly involves the two relevant flow variables the cash outflows to the capital investments g_t and the annual cash inflows f_t and the concept of discounting in the form of the annuity factor $a_{N,k}$. No stock-concept variable is involved. The role of the growth variable k comes from the fact that the consecutive capital investments that produce the corresponding, lagged cash inflows, typically grow in a going concern. Ruuhela stresses that the profitability of the firm is a long-term concept based on business culture of the firm to be able to generate and utilize capital investment opportunities. According to Ruuhela firms usually experience long phases of stable business culture when the long-run profitability stays on a rather fixed level. Profitability can be measured for such stable intervals. In corporate life a change or a discontinuity in business culture often coincides with a change of the top-level management. At such junctures the long-run profitability typically changes and should be estimated anew. Ruuhela prefers to call the profitability of such a stable period the profitability of the business culture rather than the profitability of the legal entity, the firm. In our simulation testing the business culture is taken as unchanged.

3.5. Averaged Accountant's Rate of Return Method

The fourth and last method included into our analysis is based on straight-forward accounting practice. Much of the discussion, ever since Vatter (1966), in the ARR vs. IRR debate has centered around the question whether or not the ARR is a good approximation of the IRR. Instead of re-entering the deductive debate we seek a resolution to this question by including the averaged ARR in our simulation and comparison. The inclusion of the average ARR method is prompted by the fact that accounting practitioners routinely use and are comfortable with the concept of annual profits and return on investment. Employing averaged ARR as the IRR estimate can be considered a direct extension of this business practice.

The average ARR is calculated as the arithmetic average of the accountant’s annual rate of return from Formula (22). Technically, an average can be calculated as an arithmetic average or a value-weighted average. We use the former for two reasons. First, the arithmetic average is in line with business practice. Second, an average with a large fairly stable denominator is very little affected by the choice of the averaging method. Only in the case of major shocks some differences might exist. The beginning book values are used in the denominator instead of the annual averages in line with our treatment of Kay's and Ijiri-Salamon methods.

Our advance hypothesis is that the average ARR method will not be inferior to the other methods. Our hypothesis is based on the concept of economic Darwinism. Quoting Watts and Zimmerman (1986; 195) "Competition among firms implies that operating procedures ... that are used systematically by surviving organizations are efficient."

3.6. Discussion of Market-Based Methods

The methods discussed so far use pure accounting data from the income statement and the balance sheet. The internal rate of return, however, is based on future cash flows in line with the economist's income concepts and valuation of assets. The question arises if IRR estimation methods based on market values rather than book values should be used. There are several papers putting forward implicit or explicit suggestions of an estimation of the IRR involving the market values of the firm's stock.

Reconsider Kay's method. Formula (23) can be interpreted as a suggestion by Kay to adjust the accounting-based IRR estimate with the market value of the firm w_t to arrive at the internal rate of return which would agree with the economist's rate of return.

Lawson (1980) presented a method for estimating the equity, debt and entity rates of return for the firm. To estimate IRR, his cash-flow based method equates the discounted operating cash flow less net capital investment less tax payments less/plus liquidity change to the discounted sum of initial and the ending (market-based) value of the firm.

Steele (1986; 8) suggested in his paper evaluating the derivations in Salmi (1982) and Peasnell (1982a, 1982b) an alternative version of Kay's Formula (20) to include market values into the estimation of the firm's IRR.

Theoretically the idea of basing the IRR estimates on stock prices is sound, because the prices reflect the economist's valuation of the firm's future income in line with the internal rate of return concept. However, there are some serious problems with the practicality of this theoretically well-founded approach. First, firms are not necessarily traded on stock exchanges so genuine market values would not be readily available for a considerable number of firms. Second, it is a well-known fact that stock prices are more volatile than accounting earnings. This indicates a potential, temporal instability in what should be stable long-run profitability estimates. Third, it is not easy to assess whether or not the accounting function of business firms would agree on a measure of income based on market values instead of deep-rooted accounting conventions.

Despite the practical reservations stated in the above, the evaluation of the market-based methods of IRR estimation would be highly interesting. However, we do not pursue this avenue in the paper at hand. The line of enquiry is not readily amenable to the present simulation model. In particular, the problem is establishing a reliable procedure that would give the market values of the simulated firm which would be exactly compatible with the true internal rate of return. Extrapolating the time series into infinity in the simulation is not a viable answer. The results would be too volatile for the simulation evaluation. Furthermore, an extrapolation to infinity would be unrealistic. The business culture of a firm is not preserved to the infinity with an unchanged long-term profitability.

However, it can be noted that there is some recent research information about the IRR estimates arrived at by the accounting-based vs. market based methods. An unpublished master's thesis prepared under our supervision tentatively indicates that the IRR estimates from a sample of real-life business firms derived from Ijiri-Salamon method are much more closely related to the estimates from the accounting-based Ruuhela's method than the from the market-based Lawson's method.

4. EVALUATION OF THE ESTIMATION METHODS

4.1. Simulation Design and Data Description

To tackle the research questions posed we use the research design delineated by Figure 4. The financial data is generated for the different parameter combinations listed in Table 1. The IRR estimates are obtained for the chosen methods under these different parameter combinations. The obtained IRR estimates are then compared with the true internal rate of return for which the data was generated.

Table 1. The variation of the parameters in the simulation runs

Parameter	Symbol	Values
First initial investment	g_0	100.00
Growth rate	k	0.08
True internal rate of return	r	0.04, 0.08, 0.12, 0.16
Amplitude	A	0.00 0.50 1.00
Cycle length	C	6 years
Technical phase adjustment	φ	$\pi/6$
Noise	σ	0, 0.20
Shock timing	τ	None, early, late
Shock coefficient	S	0, 5.309, 17.924
Life-span of investments	N	16, 20, 24
Length of observation period	n	13 (years 22-34)
Contribution distribution		Uniform, negative binomial, Anton
Depreciation method		Straight-line, declining, annuity

Our first research question concerns the effect of the business cycles on the robustness of the four IRR estimation methods. For our simulation it is realistic to assume that the long-run average length of a business cycle is six years (C = 6 in Formula (2)). In the simulation the length of the observation period is set at 13 years covering two full business cycles. Three alternative amplitudes of the cycles are used in our simulations. For no cycles we set A = 0, for medium cycles we set A = 0.50 and for strong cycles A = 1.00. With an amplitude A = 0 there are no business cycles in the capital investments, only the trend and the noise. With A = 1.00 the capital expenditures double from the trend and fall to zero in six year cycles. The amplitude A = 0.50 is between the two. Where the results are found to be insensitive to the cycles, the amplitude is fixed at the average case in the exposition of the results.

The IRR estimation results for the methods under observation will be presented for the different combinations of the essential parameters based on one instance of each

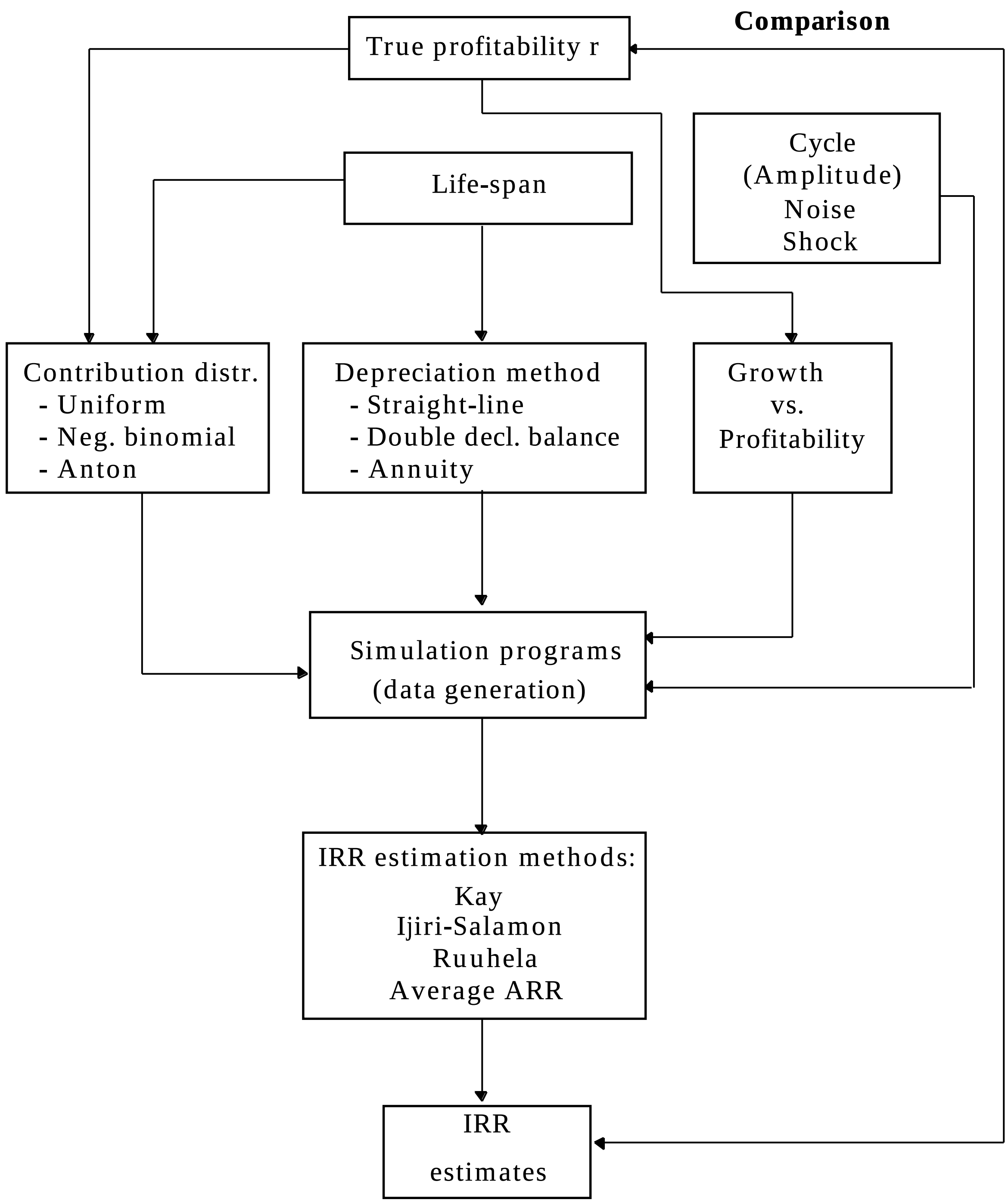

Figure 4. Structure of the simulation design

combination. One of the components is the random fluctuation in the cyclical level of the capital investments, i.e. the noise term $1+\sigma z$ in the capital-investment generating Formula (2). We choose a moderate noise level of σ = 20% to arrive at a realistic capital investment time series. Only one realization (for each parameter combination) of the randomized time series is picked in our simulation. Stated in terms of statistics and operations research our approach is not a Monte Carlo simulation that would repeat the same parameter combinations with the random term varied. However, to assess the effect due to the noise we conduct for comparison the simulations without the random term (σ = 0%). This approach has the advantage of avoiding an exponential amount of further computations without a significant loss of generality.

Our second research question concerns the effect of contribution patterns and the life-span estimates of the capital investments. As was discussed in an earlier section, the underlying contribution pattern of the capital investment process of a real-life business firm cannot be readily, if at all, unraveled from the firm's financial statements. Thus the generic contribution distribution of the firm is not known. Consequently, we simulate the effects of three potential contribution distributions (c.f. Figure 3). The "neutral" uniform contribution distribution, the "growth-maturity-decline" negative binomial contribution distribution and the "steady-decline" Anton contribution distribution are selected. The life-span of the capital investments in the simulation will be set at 20 years. The contribution coefficients for the uniform contribution distribution from Formula (7) for alternative profitabilities become 0.0735 for r = 4%, 0.1018 for 8%, 0.1339 for 12% and 0.1686 for 16%. The negative binomial contribution distribution coefficients from Formulas (9) and (10) are delineated by Figure 5 for an 12% example-level of profitability. Likewise, from Formula (11) the corresponding contribution coefficients for the Anton distribution for the 12% profitability level decline linearly from 0.170 to 0.056.

The life-span of the capital investments affects the numerical values of the chosen contribution distribution and the annual depreciation figures. The life-span of the capital investments is known in the simulation (we have chosen a typical 20 years), but it cannot be accurately known in applications on real-life business firms. This is one of the potential sources of inaccuracy in the IRR estimation methods. The Ijiri-Salamon method and Ruuhela's method require an estimate of the life-span as part of the IRR estimation procedure while Kay's and the average ARR methods do not. The effect of misestimating the life-span in the two susceptible methods will be considered in the analysis section by comparing the IRR estimates with a 20-year life-span to the results with a 16-year and a 24-year life-span.

Our third research question concerns the effect of a disparity between the firm's growth and profitability. As was discussed the earlier literature poses that a growth and profitability equality has a special meaning in the relationship between IRR and ARR. We fix a growth rate of k = 8% in the simulation. The simulated data is generated to produce true profitability figures of r = 4%, 8%, 12% and 16%. The true rates are at and on both sides of the growth rate. Here the relation between the profitability and growth is crucial rather than the absolute levels. Therefore, either growth or profitability could have been fixed for a

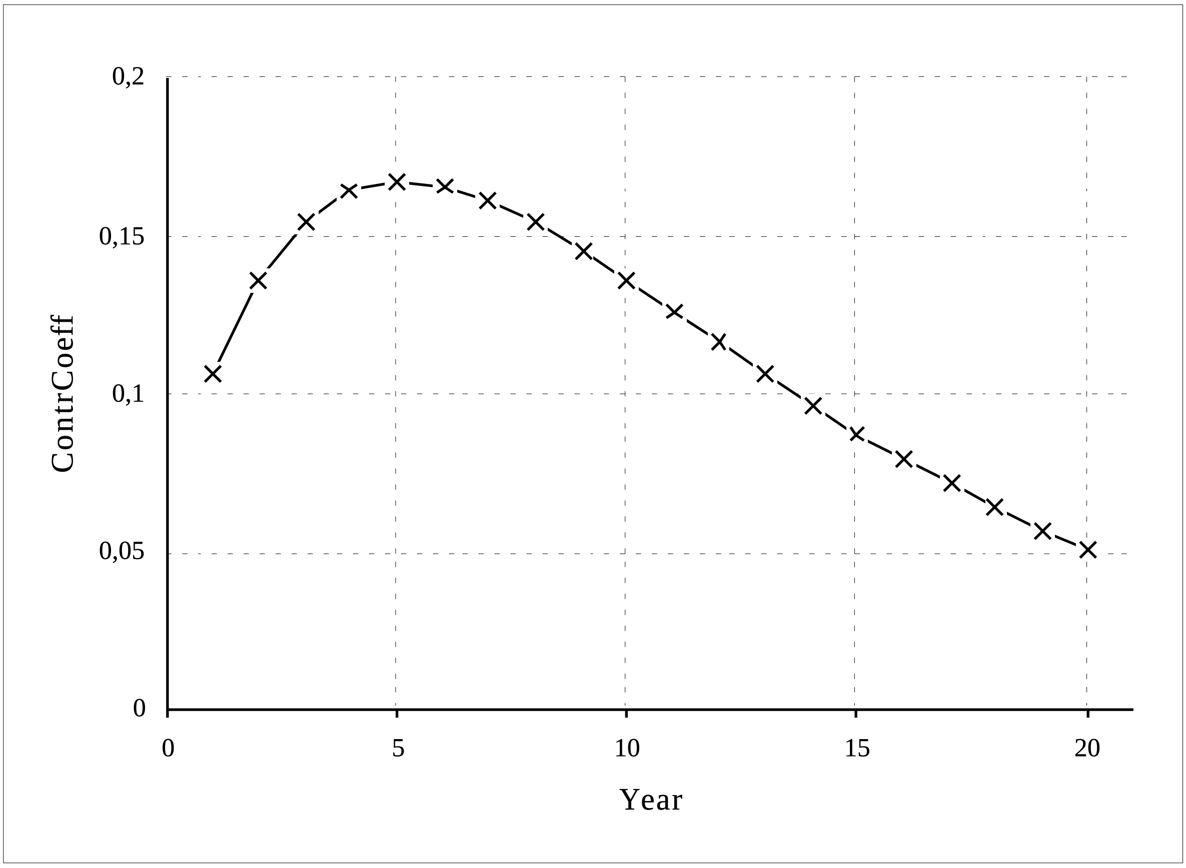

Figure 5. Negative binomial contribution distribution for profitability of 12%

meaningful simulation and the other varied. We have chosen to fix the growth rate and vary profitability to achieve the cases of low profitability (4%) compared to growth, equal rates (8%) and high profitabilities (12% and 16%). The selected combinations are intended to tally with common growth-profitability combinations of real-life business firms. Figures 6 and 7 present the growth vs. profitability combinations for a sample of 87 U.S. and 244 Finnish firms between 1969-88 and 1965-94 respectively. The data are based on unpublished master's theses written at the University of Vaasa using one of the methods, Ruuhela's method.

Our fourth question concerns the sensitivity of the methods to the depreciation choice that the firm has used in preparing its financial statements. The simulated time series are produced for three different depreciation methods to evaluate their effect on the results. The first two methods are the straight-line depreciation and double-declining-balance depreciation based on the common accounting practice. The third method to be used in the analysis is the theoretical annuity depreciation. The assumed 20-year life-span of the simulated capital investments means that the annual rate of depreciation in generating the simulated data is 5% in the straight-line method and 10% in the double-declining-balance method. The figures for the theoretical annuity method of depreciation are a function of the true profitability as is seen in Formula (17).

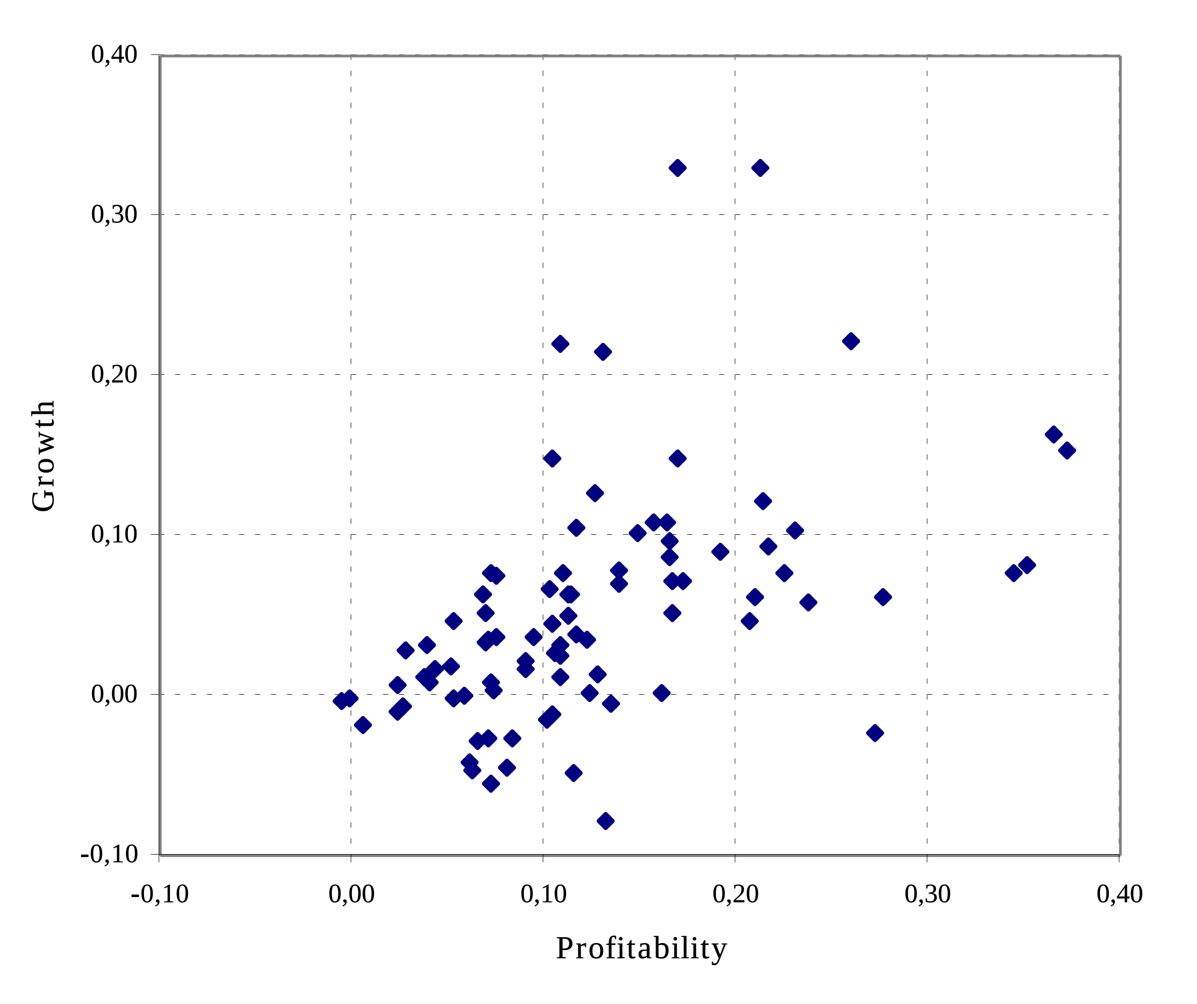

Figure 6. Growth vs. profitability; U.S. observations

Our last question involves the effect of major irregularities in the level of the capital investments. The robustness of a profitability estimation method can be tested by including capital investment shocks in the model. In business terms such a shock is usually related to a major deviation from the level of capital investment pattern. Experiments are made with different magnitudes and timing of a one-time shock. The shock alternatives simulated are a five-fold shock and a seventeen-fold shock relative to the normal capital investment level in the third or in the ninth year.

Table 2 gives an example of one realization of the time series from the simulated financial statements. The observation period in Table 2 is 13 years from the simulated year 22 to 34 (the lines not denoted by the *). The realization presented in Table 2 is for the case of the negative binomial contribution distribution with a true profitability of 12%, a growth trend of 8%, medium amplitude (A = 0.50) of business cycles, with noise (σ = 0.20), no shock, a life-span of 20 years of the capital investments, and a double-declining-balance depreciation of 10%.

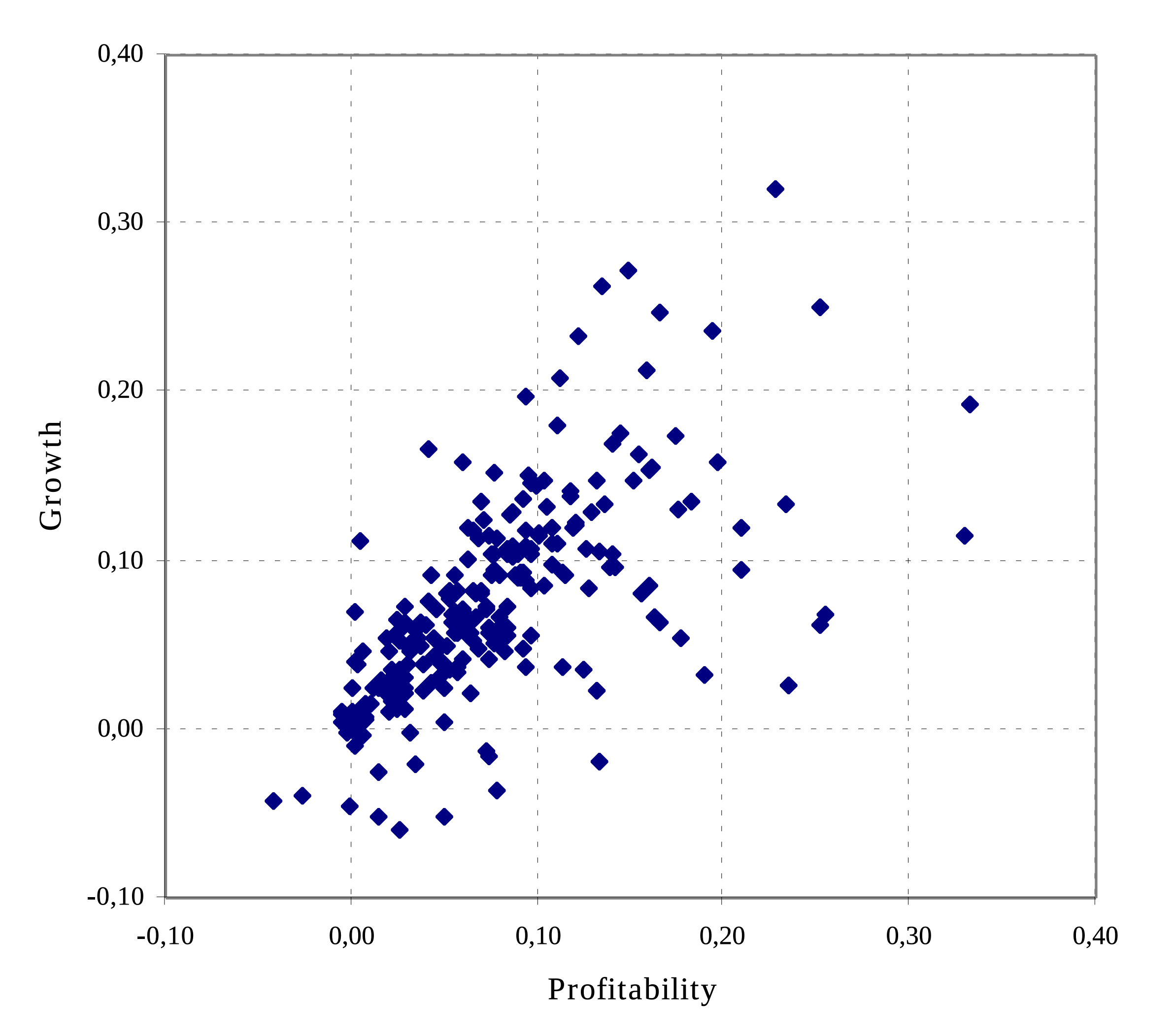

Figure 7. Growth vs. profitability; Finnish observations

The data are presented graphically in Figure 8. Because of their different scale the book values have not been included in Figure 8. The figure can be visually compared with the corresponding time series of actual business firms. Contrary to the more rigid, steadily growing series of earlier research, the series produced by our simulation model and parameters are realistic in terms of factual observations. This contention is readily corroborated by the empirical time series data gathered in the course of several research projects at University of Vaasa, such as Ruuhela et al. (1982).

As in real-life business firms the simulated time series of the capital investments show a wide fluctuation while the derivative series are much smoother. This results from the fact that the capital investments produce the corresponding cash inflows over a long, lagged period and that similarly the depreciation is extended over the life-span of the capital investments. Furthermore, despite the fluctuations the underlying growth-trend for the firm is a constant in the simulation.

Table 2. Example of simulated observations, negative binomial contribution distribution, declining balance depreciation, growth 8%, IRR 12%, amplitude 50%, noise 20%, no shock

Year t	Capital expenditure g_t	Cash inflows f_t	Declining depreciation d_t	Operating income p_t	Book value v_t
* 0	100.00	0.00	0.00	0.00	100.00
* 1	151.36	10.69	10.00	0.69	241.36
:	:	:	:	:	:
* 21	271.45	642.50	317.62	324.87	2946.09
22	318.86	681.22	315.54	365.68	2949.41
23	451.70	713.39	304.34	409.05	3096.77
24	604.02	757.26	316.11	441.15	3384.69
25	1048.43	816.45	351.73	464.72	4081.39
26	1116.67	922.53	431.57	490.96	4766.49
27	927.32	1045.47	506.52	538.94	5187.29
28	440.87	1155.13	542.92	612.21	5085.24
29	801.76	1213.39	529.79	683.60	5357.22
30	1158.04	1293.30	546.28	747.02	5968.98
31	1969.10	1411.90	616.13	795.77	7321.95
32	1138.25	1619.83	758.03	861.80	7702.17
33	996.62	1762.03	814.36	947.66	7884.42
34	771.56	1871.29	822.48	1048.80	7833.51

Capital investment shocks are simulated to test the robustness of the IRR estimation methods. When shocks are included the noise is excluded (see $1+\sigma z$ and $1+\delta_{t\tau}S$ in Formula (2)). This is done in order not to confuse the effect of the irregularities caused by the ordinary noise and the shocks with each other. The time-series data with the one-time shock of a five-fold order relative to the normal capital investment level is presented in Figure 9.

The Ijiri-Salamon method needs an estimate of the gross book value of assets as is seen in Formula (26). This figure is not routinely available on the balance sheet of a business firm. For obtaining the gross book value an estimate of the cumulative depreciation is needed as is seen in Formula (27). Table 3 displays the cumulative depreciation and the gross book value for the data in Table 2. The numbers are calculated for our error analysis in two different ways. The first two columns are calculated with the exact cumulative depreciation. In a simulation approach this is possible since the engine producing the financial data is known accurately. The two last columns are calculated in line with what could be done with actual data from business firms.

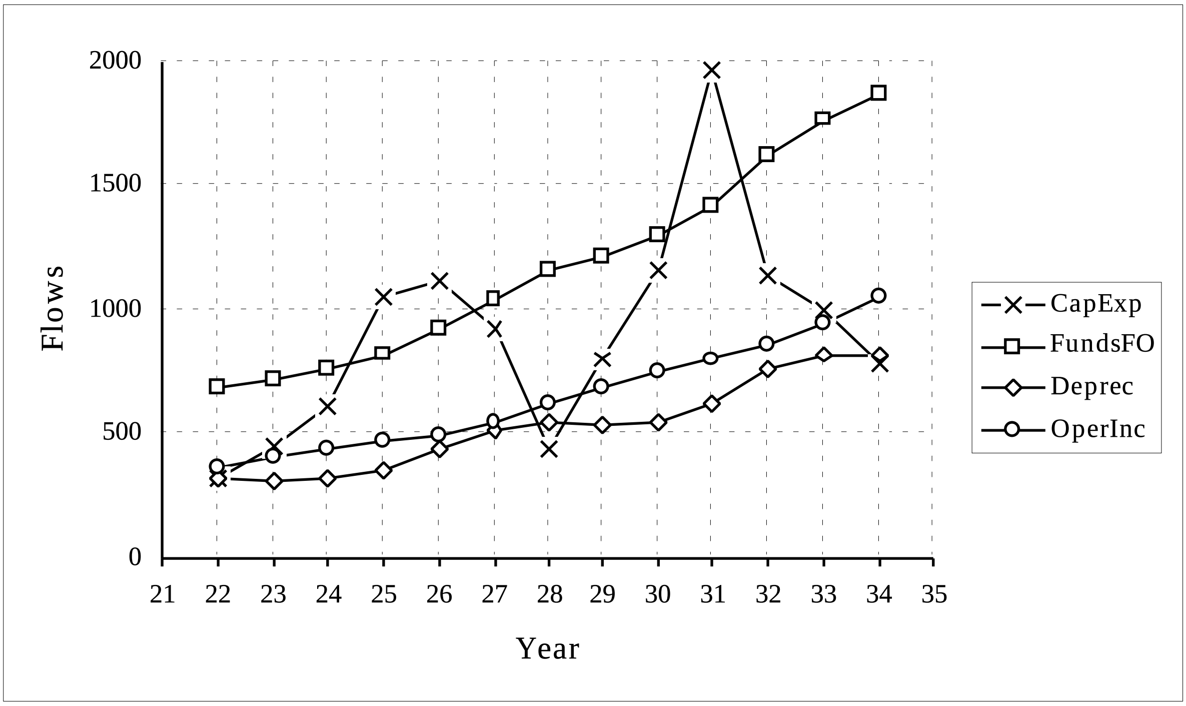

Figure 8. Visualization of simulated observations; the case of negative binomial contribution distribution, declining balance depreciation, growth 8%, IRR 12%, amplitude 50%, noise 20%, no shock

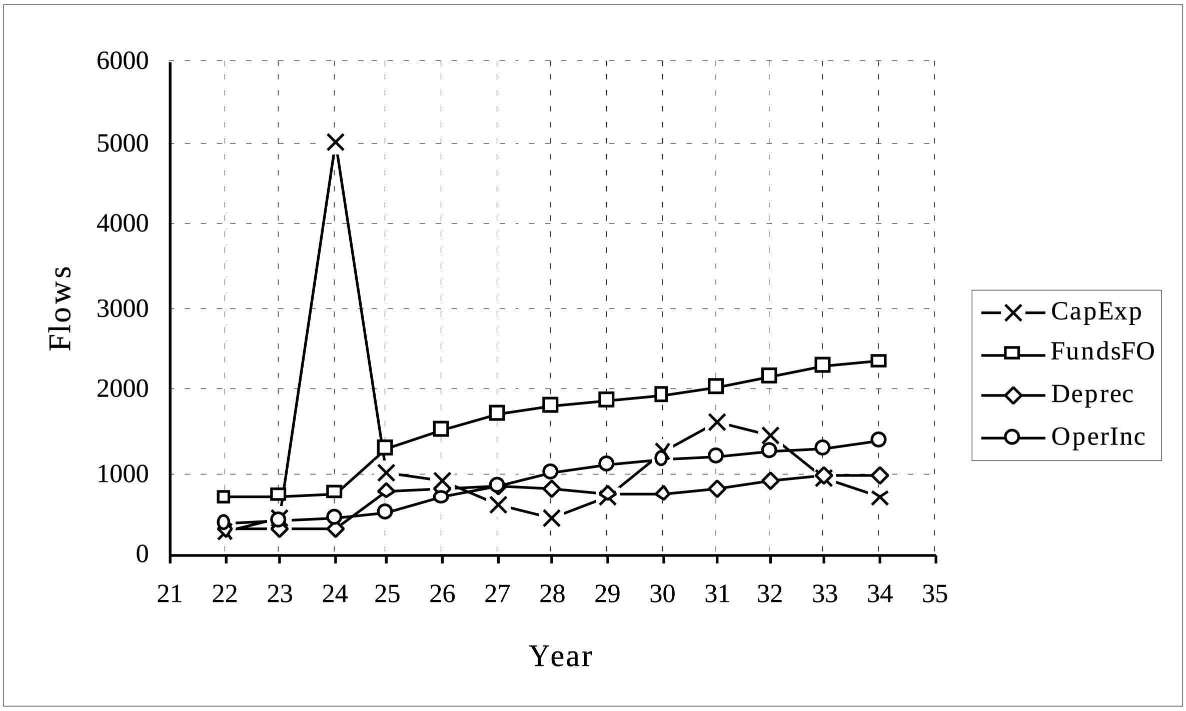

Figure 9. Visualization of simulated observations; the case of negative binomial contribution distribution, declining balance depreciation, growth 8%, IRR 12%, amplitude 50%, no noise, early five-fold shock

Table 3. Accumulated, declining balance depreciation and gross book values; accurate and estimated figures

Year t	Accurate cumul. depreciation D_t	Accurate gross book value V_t	Estim. cumul. depreciation $\hat{D}_t$	Estim. gross book value $\hat{V}_t$
* 0	0.00	100.00	..	..
* 1	10.00	251.36	..	..
:	:	:	:	:
* 21	2223.87	5169.97	..	..
22	2367.22	5316.64	..	..
23	2594.22	5691.00	..	..
24	2857.41	6242.11	..	..
25	3100.06	7181.46	..	..
26	3338.88	8105.37	..	..
27	3599.66	8786.95	..	..
28	3943.57	9028.81	..	..
29	4298.44	9655.66	..	..
30	4757.84	10726.82	..	..
31	5215.78	12537.74	4460.96	11782.92
32	5761.26	13463.43	4903.45	12605.63
33	6212.51	14096.93	5413.47	13297.90
34	6754.98	14588.49	5919.85	13753.36

4.2. Evaluation of Kay's Method

4.2.1. Effect of Regular Business Cycles

We begin the evaluations by assessing the effect of business cycles on Kay's method. The IRR estimates by Kay's method are presented in Table 4 for the three different levels of amplitudes in the business cycle. The results are presented in Table 4 for the negative binomial contribution distribution which is the most general of the alternative distributions. To see the pure effect of the cyclical component we first omit the random noise term. The results are presented for the four different growth-profitability combinations and the three different depreciation methods "Str" straight-line depreciation, "Decl" double-declining-balance depreciation and "Ann" annuity depreciation.

It is readily seen in the table that the effect of the business cycles is marginal for Kay's IRR estimation method. In the worst case with the strong cycles (A = 1.00) the difference between the IRR estimates 18.9% and 18.6% (16% true profitability and double-declining-

Table 4. Estimation of IRR with Kay's method, negative binomial contribution distribution, growth rate k = 8%, no noise, no shock

Cycle amplitude		A = 0.00			A = 0.50			A = 1.00		
Depreciation		Str	Decl	Ann	Str	Decl	Ann	Str	Decl	Ann
True r	4%	4.1	3.5	4.0	4.1	3.4	4.0	4.1	3.4	4.0
	8%	8.0	8.0	8.0	8.0	8.0	8.0	8.0	8.0	8.0
	12%	12.3	13.1	12.0	12.3	13.2	12.0	12.4	13.2	12.0
	16%	17.0	18.6	16.0	17.1	18.8	16.0	17.1	18.9	16.0

balance depreciation) is only 0.3%. The presented result is for the negative binomial contribution distribution. The results for the other two contribution distributions, the uniform distribution and the Anton distribution, indicate a similar insensitivity. (The additional tables are not displayed for brevity.) Hence we can safely conclude that Kay's IRR estimation method is not affected by regular business cycles. This being the case the rest of the analysis of Kay's method can be conducted without a loss of generality using the medium cycle strength (A = 0.50).

4.2.2. Overall Accuracy of the Kay's IRR estimates

We can now analyze the total error in Kay's IRR estimates. Table 5 presents the results for Kay's IRR estimation method under medium business cycles. The noise component is included at this phase. The results are condensed into a single table for the three contribution distributions.

Table 5. Estimation of IRR with Kay's method, growth rate k = 8%, amplitude A = 50%, noise = 20%, no shock

Contr. distribution		Uniform			Neg. binomial			Anton		
Depreciation		Str	Decl	Ann	Str	Decl	Ann	Str	Decl	Ann
True r	4%	3.5	2.6	4.0	4.2	3.3	4.0	4.0	3.1	4.0
	8%	7.8	7.6	8.0	8.0	7.8	8.0	8.0	7.8	8.0
	12%	12.6	13.3	12.0	12.3	12.9	12.0	12.0	12.5	12.0
	16%	17.9	19.5	16.0	17.0	18.4	16.0	16.0	17.3	16.0

The general impression conveyed by Table 5 is that the level of Kay's IRR estimates is fairly well in line with the true profitability. In particular, when the firm's growth and profitability are near each other, Kay's method performs excellently.

There are, however, situations where Kay's method performs poorly. The biggest absolute discrepancy in Table 5 in an estimate ($\hat{r}$ = 19.5% vs. r = 16%) takes place when the true internal rate of return deviates most from growth, the capital investments contribute according to the uniform distribution and the firm uses the double-declining-balance method. Kay's IRR estimate is off by 3.5% (by a fifth in relative terms). Likewise, at the true profitability of 4% the IRR estimate, with the uniform contribution distribution and the double-declining-balance depreciation, is off by a third (2.6% vs. 4.0%). These are marked deviations.

It is not easy to evaluate how serious the observed errors are from the point of view of decision making. It depends on whether the alternative methods give better estimates. Most importantly, the seriousness of a deviation would depend on what would be the consequences of the management of the firm having erroneous profitability information. Predicting such consequences in quantitative terms is a very involved question and is outside the scope of our research.

4.2.3. Effect of Noise

In Table 4 it was observed that the effect of the business cycles on the estimation error is marginal. To asses the effect of the noise component Table 6 presents Kay's IRR estimates without the noise for a comparison with Table 5.

Table 6. Estimation of IRR with Kay's method, growth rate k = 8%, amplitude A = 50%, no noise, no shock

Contr. distribution		Uniform			Neg. binomial			Anton		
Depreciation		Str	Decl	Ann	Str	Decl	Ann	Str	Decl	Ann
True r	4%	3.6	2.8	4.0	4.1	3.4	4.0	4.0	3.3	4.0
	8%	8.0	8.0	8.0	8.0	8.0	8.0	8.0	8.0	8.0
	12%	12.9	13.9	12.0	12.3	13.2	12.0	12.0	12.7	12.0
	16%	18.3	20.3	16.0	17.1	18.8	16.0	16.0	17.5	16.0

While the noise term in the capital investment level seems to have more effect on the Kay's IRR estimation than the regular cycles the effect of noise is rather mild. At most the IRR estimate changes from 20.3% to 19.5% (in the case of the 16% true profitability, uniform contribution distribution and double-declining-balance depreciation). The magnitude of the difference is 0.8% compared to a total error of 4.3%. We conclude that noise is not a main source of the estimation errors. Hence the analysis can be founded on the results in Table 5.

At this juncture a general word of caution is in order. It goes without saying that generalizing from the conclusions based on simulation rather than analytical deduction always should be considered with a fair amount of caution.

4.2.4. Effect of Contribution Patterns, Growth-Profitability Relationship and Firm's Depreciation Choice

Our second research question concerns the effect of the type of capital investments available to the firm. Consider Table 5 anew for effect of the alternative contribution patterns. As pointed out earlier, the shape of the contribution distribution of the capital investments is not readily known for real-life firms. Therefore it is of interest to test whether the IRR estimation results are sensitive to this factor. It is seen that under capital investment opportunities that contribute in accordance with the negative binomial distribution, or the Anton distribution, the results are more accurate than under the non-declining uniform contribution distribution.

Our third research question concerns the effect of the discrepancy between growth (k) and profitability (r). It is obvious from the results that a discrepancy between growth and profitability levels is the crucial source of error in the Kay's IRR estimates. It is also noted that when $r > k$ Kay's IRR systematically overestimates the true profitability (the special case of the straight-line depreciation under the Anton contribution distribution will be discussed in a later section). Thus it appears that Kay's method gives even too optimistic IRR estimates to firms with good profitability. For $r < k$ the direction on the estimation error depends on the contribution distribution, the depreciation combination and the irregularities in the capital investments (the noise). Thus it would seem that it is not possible to make any predictions whether Kay's estimates for firms with low profitabilities are optimistic or pessimistic.

Our fourth research question concerns the effect of the depreciation method choice that the firm makes. The effect of the firm's accounting choice appears highly important to the accuracy of the IRR estimates. The error in the estimates in Table 5 is about half or less when the firm applies the straight-line depreciation method instead of the double-declining-balance method. This observation raises interesting accounting issues about the depreciation method choice.

4.2.5. Effect of Major Capital Investment Shocks

Our fifth research question concerns the effect of major capital investment shocks. Figure 9 delineates an example time-series data. Table 7 gives Kay's IRR estimates under a third year shock ("early shock"). Table 8 is for a ninth year shock ("late shock"). To isolate the effect of the shocks, noise has been excluded.

Kay's IRR estimation method seems to be reasonably robust to the capital investment shocks even if there is some disruption in the estimates. The effect of the shock seems to be to decrease the IRR estimates, the more the bigger and later the shock appears. The observed behavior is easy to explain. The one-time investment shock becomes dominating, and its effects are much outside the period under observation.

Table 7. Estimation of IRR with Kay's method, negative binomial contribution distribution, growth rate k = 8%, amplitude A = 50%, no noise, early shock (τ = 24)

Shock factor		S = 0.00			S = 5.309			S = 17.924		
Depreciation		Str	Decl	Ann	Str	Decl	Ann	Str	Decl	Ann
True r	4%	4.1	3.4	4.0	4.3	3.3	4.0	4.5	3.2	4.0
	8%	8.0	8.0	8.0	8.1	7.6	8.0	8.1	7.3	8.0
	12%	12.3	13.2	12.0	12.2	12.3	12.0	12.0	11.6	12.0
	16%	17.1	18.8	16.0	16.5	17.3	16.0	16.2	16.1	16.0

Table 8. Estimation of IRR with Kay's method, negative binomial contribution distribution, growth rate k = 8%, amplitude A = 50%, no noise, late shock (τ = 30)

Shock factor		S = 0.00			S = 5.309			S = 17.924		
Depreciation		Str	Decl	Ann	Str	Decl	Ann	Str	Decl	Ann
True r	4%	4.1	3.4	4.0	4.0	2.4	4.0	3.8	1.3	4.0
	8%	8.0	8.0	8.0	7.6	6.7	8.0	7.2	5.2	8.0
	12%	12.3	13.2	12.0	11.8	11.6	12.0	11.0	9.6	12.0
	16%	17.1	18.8	16.0	16.4	17.1	16.0	15.3	14.6	16.0

For high profitabilities relative to growth the shock compensates for the error caused by the growth-profitability discrepancy. For low profitabilities the error from the growth-profitability discrepancy is even aggravated. It can be noted, however, that logically it is not equally likely that major capital investment shocks will appear in corporations with profitability problems than in firms with good profitability prospects. Furthermore, as will be observed in the next section, the introduction of major capital investment shocks will cause deviations from the theoretically expected results.

4.2.6. Theoretical Considerations

There are several theoretical assertions about the relationship between the internal rate of return and the accountants rate of return under the specific growth rates, depreciation methods and contribution distributions presented in earlier literature. Next we consider these assertions, under the more general conditions of business cycles and noise, utilizing our simulation results.

Solomon (1966; 115) posed that when the growth rate and the true internal rate of return are equal, the accountant's rate of return also becomes the same. Consequently, it is theoretically to be expected that if the growth and profitability are exactly equal, Kay's method should give exactly the correct IRR estimate because it is built on the relationship between the IRR and ARR. The equality would be expected to hold over all the contribution distributions and over all the depreciation methods.

Formula (2) generates the capital investments. It added several components to the constant growth model. Consider the presented theoretical contention with the added components. Table 6 confirms that the expected equality holds (within the used numerical precision) not only in the case of constant, exponential growth but also in the case with the business cycles added. However, when irregularities are introduced in terms of the noise (cf. Table 5), the expected theoretical result no more fully holds. The deviation is not marked numerically, but theoretically the assertion breaks. As is natural, the disruptive effect of the one-time capital investments shocks is more marked than that of the noise.

Analytically, the accountant's rate of return and the internal rate of return are equal when the annuity method of depreciation is used (see e.g. Salmi and Luoma, 1981; 28 and Peasnell, 1982a; 364). The simulation results for Kay's method are in agreement with this contention for all the observed combinations of growth vs. profitability and for all contribution distributions even with the irregularities introduced upon the growth-trend and the business cycles in terms of the noise and the capital investments shocks. See the columns marked "Ann" in Tables 5, 7 and 8. The theoretical results about the annuity depreciation are very strong. They are in line with discussions and results in literature about accountant's and the economist's concepts of income.

It is a well-known result that the theoretical annuity depreciation method and the business practice straight-line depreciation method yield the same depreciation if the contribution distribution for the capital investments is the Anton distribution. See Solomon (1971; 168 footnote) for references. Consequently, for the Anton contribution distribution the simulations should produce the same IRR estimate for the straight-line depreciation as it does for the annuity depreciation. Also this theoretical contention is corroborated by the simulation. Compare the columns marked "Ann" and "Str" below "Anton" in Table 5. This result holds even if major capital investment shocks are introduced. (The numerical tables for the Anton distribution with the major investment shocks are not displayed for brevity.)

4.2.7. Conclusions about Kay's Method

The main findings about Kay's IRR estimation method are the following. Under ordinary circumstances Kay's method performs quite well. However, the deviation of Kay's IRR estimates from the true internal rate of return can be considerable if the growth rate of the firm and its profitability are not near each other. This is the main source of error in Kay's method.

Kay's method seems to lead to systematically overoptimistic profitability estimates when the firm's true profitability exceeds the firm's growth considerably. If the true profitability is below the firm's growth the nature of Kay's IRR estimate is ambiguous.

The magnitude of the error caused by a growth-profitability gap is jointly dependent on the contribution pattern of the capital investments, the firm's depreciation choice and the noise in the capital investment time series.

Kay's method is not affected by regular business-cycle fluctuations in the capital investment time series, but it is mildly affected by noise. Kay's method is reasonably robust to major capital investments shocks. The irrelevance of business cycles and the mild effect of noise on the accuracy of the estimates are important advantages in Kay's method.

Kay's method has a firm theoretical background in the theory of accountant's and economist's profit concepts. This fact is reflected in always getting exactly the expected IRR estimates under the theoretical annuity depreciation and getting fairly accurate IRR estimates under the equality of growth and profitability. Furthermore, if the capital investments contribute in accordance to the Anton distribution, the estimates under the firm applying a straight-line depreciation are accurate.

4.3. Evaluation of Ijiri-Salamon Method

4.3.1. Exposition of the IRR Estimates with Ijiri-Salamon Method

The cash-recovery-rate-based Ijiri-Salamon IRR estimation method differs from Kay's method in two respects in the data that it needs. An estimate of the life-span of the firm's capital investments is needed. Furthermore, an estimate of the gross book value of the firm's assets is needed. (The gross assets V_t are the net assets v_t plus the accumulated depreciation. Cf. Formula (25).) This fact introduces two additional, potential sources of error to the method: a misestimation of the life-span of the capital investments and a misestimation of the gross book value. In evaluating Ijiri-Salamon method we can utilize the fact that in simulation the life-span (N = 20) and the true accumulated depreciation, and hence the gross book value of the firm's assets are known precisely. An example of the accurate accumulated depreciation D_t and the accurate gross book value V_t was presented in Table 3 in describing the data of the simulation.

Tables 9 to 11 present the IRR estimates with Ijiri-Salamon method with noise. These tables for the three different contribution distributions include the results for three alternative estimates of the capital investments' life-span E(N). The IRR estimates are presented assuming a correctly estimated life-span of 20 years, an underestimate 16 years, and an overestimate 24 years. In other words for the life-span estimates being off the mark by a fourth. Table 12 presents the IRR estimates for comparison without the noise. Table 13 presents the estimates in the case of early, realistic shock. For brevity, only the cases with the negative binomial distribution are displayed by Tables 12 and 13. The full set of the tables can, however, be readily reproduced for verification since the relevant computer source codes have been made available to the interested reader from the World Wide Web: <URL:http://www.uwasa.fi/~ts/smuc/prog/smucprog.html>.

The IRR estimation results are presented assuming that the firm either employs the straight-line depreciation ("Str") or the double-declining-balance depreciation ("Decl"). The accumulated depreciation must be estimated from the financial statements. In accounting practice, the accumulated depreciation figure usually is an approximation based on a time series of recent financial statements. We use the estimate given by Formula (27). An example of the gross book value figures can be seen in the last column of Table 3. The accumulated depreciation can also be calculated accurately in the simulation approach. Ijiri-Salamon's IRR estimates with accurate accumulated depreciation is presented in the "Accu" column of the tables. This particular information facilitates a decomposition analysis of the error sources in the IRR estimates.

Table 9. Estimation of IRR with Ijiri-Salamon method, uniform contribution distribution, growth rate k = 8%, amplitude A = 50%, noise = 20%, no shock

Estimated life-span		16 years			20 years			24 years		
Depreciation		Str	Decl	Accu	Str	Decl	Accu	Str	Decl	Accu
True r	4%	2.8	3.5	2.0	4.2	4.7	4.0	4.9	5.3	5.2
	8%	7.4	8.2	6.4	8.3	6.7	8.0	8.6	9.0	8.9
	12%	11.9	12.8	10.8	12.3	13.0	12.0	12.3	12.8	12.6
	16%	16.4	17.5	15.1	16.4	17.2	16.0	16.0	16.7	16.4

Table 10. Estimation of IRR with Ijiri-Salamon method, negative binomial contribution distribution, growth rate k = 8%, amplitude A = 50%, noise = 20%, no shock

Estimated life-span		16 years			20 years			24 years		
Depreciation		Str	Decl	Accu	Str	Decl	Accu	Str	Decl	Accu
True r	4%	3.5	4.2	2.7	4.8	5.3	4.6	5.6	6.0	5.8
	8%	7.6	8.4	6.6	8.4	9.0	8.1	8.9	9.3	9.1
	12%	11.6	12.5	10.5	12.0	12.6	11.6	12.2	12.7	12.5
	16%	15.6	16.7	14.4	15.6	16.4	15.2	15.5	16.1	15.9

Table 11. Estimation of IRR with Ijiri-Salamon method, Anton contribution distribution, growth rate k = 8%, amplitude A = 50, noise = 20%, no shock

Estimated life-span		16 years			20 years			24 years		
Depreciation		Str	Decl	Accu	Str	Decl	Accu	Str	Decl	Accu
True r	4%	3.3	4.0	2.5	4.7	5.2	4.5	5.3	5.7	5.6
	8%	7.5	8.4	6.6	8.5	9.1	8.2	8.7	9.1	9.0
	12%	11.3	12.3	10.3	11.9	12.6	11.6	11.8	12.3	12.1
	16%	14.8	15.9	13.6	15.0	15.8	14.7	14.7	15.3	15.0

Table 12. Estimation of IRR with Ijiri-Salamon method, negative binomial contribution distribution, growth rate k = 8%, amplitude A = 0.50, no noise, no shock

Estimated life-span		16 years			20 years			24 years		
Depreciation		Str	Decl	Accu	Str	Decl	Accu	Str	Decl	Accu
True r	4%	3.4	4.2	2.5	4.6	5.2	4.4	5.4	5.8	5.7
	8%	7.4	8.3	6.4	8.2	8.8	7.9	8.6	9.1	9.0
	12%	11.4	12.4	10.2	11.7	12.4	11.4	11.9	12.4	12.3
	16%	15.4	16.6	14.0	15.3	16.1	14.9	15.2	15.9	15.7

Table 13. Estimation of IRR with Ijiri-Salamon method, binomial contribution distribution, growth rate k = 8%, amplitude A = 0.50, no noise, early realistic shock (τ = 24, S = 5.309)

Estimated life-span		16 years			20 years			24 years		
Depreciation		Str	Decl	Accu	Str	Decl	Accu	Str	Decl	Accu
True r	4%	3.8	4.4	3.1	4.9	5.3	4.8	5.6	5.9	5.8
	8%	7.9	8.6	7.1	8.5	9.0	8.3	8.8	9.2	9.1
	12%	12.0	12.8	11.0	12.1	12.7	11.9	12.1	12.5	12.4
	16%	16.0	17.0	15.0	15.8	16.4	15.5	15.5	16.0	15.9

4.3.2. Effect of Various Factors on Ijiri-Salamon IRR Estimates

As is recalled, the first of our research questions concerns the effect of the business cycles on the IRR profitability estimation methods. As for Kay's method our simulations for Ijiri-Salamon method indicate that the method is not sensitive to cycles. For brevity, the numerical IRR estimation results for the different cycle amplitudes are not displayed. Therefore, the cycle amplitude was fixed at A = 0.50 in the tables presented in the previous section. As for Kay's method, the effect of noise is rather mild as can be seen comparing the representative Tables 10 and 12.

Our fifth research question concerns the effect of investment shocks on the profitability estimates given by the various methods. Our simulations indicate that like Kay's method the Ijiri-Salamon method is reasonably robust to capital investment shocks. Compare Tables 12 and 13 for an example effect of the capital investment shock. In fact, an investigation of the two tables shows that the effect of misestimating the life-span of the capital investments is mostly more marked a source of the IRR estimation error than the effect of the capital investment shocks. A comparison of Tables 12 and 13 with the pair of Tables 6 and 7 for Kay's method indicates that while the effect of the capital investment shocks is not destructive on the methods, its effect on Ijiri-Salamon method is more unpredictable.

Overall, Ijiri-Salamon method fares on the average in the simulations comparably to Kay's method. The worst cases in the regular Tables 9 to 11 appear when the profitability is low compared to the growth. Ijiri-Salamon IRR estimate at worst is 50% off the mark in relative terms. However, in the Ijiri-Salamon method there is no clear pattern to the errors. Unlike in Kay's method there are no cases where the error would disappear. Furthermore, there is no clear pattern to the direction and the magnitude of the error.

As has been discussed, the realization of the theoretical assertions concerning the growth-profitability equality conditions, the annuity depreciation and Anton distribution could be checked. However, these assertions do not cover the relationship between the cash recovery rate and the internal rate of return. This state of matters also is clearly reflected in the simulation results as a lack of similar theoretical regularities as were observed in the results for Kay's IRR estimation method. This can be considered a disadvantage.

4.3.3. Decomposition of the Ijiri-Salamon Method Estimation Error

The simulation results for Ijiri-Salamon method seem at rough par with Kay's method. However, a decomposition of the sources of the overall error exposes a more critical picture of the potential quality of the IRR estimates by Ijiri-Salamon method. The total error in Ijiri-Salamon's IRR estimates is made up by several components, which individually can be larger in absolute terms than the total error, but the components of the error compensate each other in the presented simulations. Table 14 gives one example of the decomposition of the total error into three components. The error decomposed is for the IRR estimates listed in Table 10 for the columns of the double-declining-balance depreciation.

Table 14. Decomposition of the estimation error in Ijiri-Salamon method. An example with negative binomial distribution, declining balance depreciation, growth k = 8%, amplitude A = 50%, noise = 20%, no shock

Est. life-span		16 years				20 years				24 years			
Source of error		For-mula	Life-span estim	Cumu depr calc	Total error	For-mula	Life-span estim	Cumu depr calc	Total error	For-mula	Life-span estim	Cumu depr calc	Total error
True r	4%	0.4	-1.9	1.7	0.2	0.5	0	0.8	1.3	0.6	1.3	0.1	2.0
	8%	0.0	-1.5	1.9	0.4	0.1	0	0.9	1.0	0.1	1.1	0.1	1.3
	12%	-0.5	-1.2	2.2	0.5	-0.4	0	1.0	0.6	-0.3	0.9	0.1	0.7
	16%	-1.0	-0.9	2.6	0.7	-0.8	0	1.2	0.4	-0.9	0.8	0.2	0.1

The total error is made up of the following three components. If the user of Ijiri-Salamon method knew exactly the true life-span of the capital investments and were able to calculate the accumulated depreciation figures accurately, all the error would be attributable to the method's formal derivation. This error is listed in Table 14 in the column "Formula". However, the focus of interest is on deriving the estimates for real-life business firms. Hence the life-span of the capital investments cannot be readily known accurately. The column "Life-span estim" displays how much of the total error is due to errors in estimating the life-span. Furthermore, obtaining the accumulated depreciation from a time series of published financial statements is not trivial and involves approximations in actual accounting practice. The column "Cumu depr calc" reflects the resultant error. The column "Total error" gives the total error, which is equivalent to the error in Table 10 between the estimated IRR and the true internal rate of return.

4.3.4. Conclusions about Ijiri-Salamon Method

The main findings about Ijiri-Salamon IRR estimation method are the following. Like Kay's method the Ijiri-Salamon method performs quite well in estimating the long-run profitability of the firm. However, the error of the Ijiri-Salamon method is less predictable and thus more risky than in Kay's method, because of the many sources of the error. In Kay's method the main source of error is a discrepancy between growth and profitability. In the Ijiri-Salamon method it is not possible to pinpoint the main source of error because of their complicated interaction.

Ijiri-Salamon method is unaffected by regular business-cycle fluctuations, but it is mildly affected by noise. The method is reasonably robust to one-time capital investments shocks. The other sources of errors dominate the shocks.

Ijiri-Salamon method lacks similar theoretical results as are characteristic of Kay's method. The mathematical derivation of the method is sound. But the method is not based on the linkage between the income determination in accounting and economics. Hence, there are no theoretical expectations for the method's behavior under special circumstances. To sum up, the method fares comparatively well in practice but fares less well in the theoretical background.

4.4. Evaluation of Ruuhela's Method

Also Ruuhela's IRR estimation method differs from Kay's in the financial statement data that it uses. Like Ijiri-Salamon method an estimate of the life-span of the capital investments is needed. Furthermore, Ruuhela's method needs the estimate of the growth rate of the firm. On the other hand, and very importantly, Ruuhela's method does not need the time series of depreciation. Ruuhela's IRR estimation method is independent of the depreciation method that the firm chooses.

4.4.1. Effect of Regular Business Cycles

We begin the simulation evaluation of Ruuhela's IRR estimation method by considering our first research question which concerns the effect of business cycles. Tables 15 and 16 present the IRR estimates for the Anton contribution distribution. The derivation of Ruuhela's method assumes the Anton contribution distribution. The noise component is omitted in this section. We make these two choices in order to minimize the number of concurrent issues that need to be taken into account at this stage. The estimates in the tables are displayed for the different growth vs. profitability combinations, the alternative cycle amplitudes and the alternative estimates of the life-span of the investments.

Ruuhela's method needs an estimate of the firm's growth rate. This growth rate is estimated in Ruuhela's method by OLS regression from the time series of the funds from operations corresponding to f_t , i.e. the simulated cash inflows. The OLS-estimated growth rates are given within the parentheses in Table 15. For comparison, Table 16 presents Ruuhela's IRR estimates with exactly the correct growth (k = 8%).

Table 15. Estimation of IRR (and growth) with Ruuhela's method, Anton contribution distribution, true growth rate k = 8, true life-span N = 20, no noise, no shock

Cycle amplitude		A = 0.00			A = 0.50			A = 1.00		
Estimated life-span		16	20	24	16	20	24	16	20	24
True r	4%	3.4	4.0	4.4	4.3	4.8	5.1	5.2	5.6	5.9
		(8.0)	(8.0)	(8.0)	(8.4)	(8.4)	(8.4)	(8.7)	(8.7)	(8.7)
	8%	8.0	8.0	8.0	8.0	8.0	8.0	8.0	8.0	8.0
		(8.0)	(8.0)	(8.0)	(8.4)	(8.4)	(8.4)	(8.8)	(8.8)	(8.8)
	12%	12.3	13.1	12.0	12.3	13.2	12.0	12.4	13.2	12.0
		(8.0)	(8.0)	(8.0)	(8.4)	(8.4)	(8.4)	(8.8)	(8.8)	(8.8)
	16%	17.0	18.6	16.0	17.1	18.8	16.0	17.1	18.9	16.0
		(8.0)	(8.0)	(8.0)	(8.4)	(8.4)	(8.4)	(8.8)	(8.8)	(8.8)

Table 16. Comparison estimation of IRR with Ruuhela's method using the true growth k = 8%, Anton contribution distribution, true life-span N = 20, no noise, no shock

Cycle amplitude		A = 0.00			A = 0.50			A = 1.00		
Estimated life-span		16	20	24	16	20	24	16	20	24
True r	4%	3.4	4.0	4.4	4.0	4.5	4.8	4.6	5.0	5.3
	8%	8.0	8.0	8.0	8.7	8.7	8.6	9.6	9.4	9.2
	12%	12.6	12.0	11.6	13.5	12.8	12.4	14.5	13.7	13.2
	16%	17.1	16.0	15.3	18.2	17.0	16.2	19.5	18.1	17.1

It is readily seen that unlike in Kay's and Ijiri-Salamon methods Ruuhela's method is sensitive to the business cycles. It is also seen in Table 15 that when there are no cycles (A = 0.00), when the life-span estimate is equal to the true life-span (20 years) of the capital investments and when the capital investments contribute according to the Anton distribution that Ruuhela's method produces exactly the correct IRR estimates. Like Kay's method Ruuhela's method has under its own assumptions a direct linkage to the income (and depreciation) theory. Furthermore, it is obvious both from the formulas of Ruuhela's method (especially Formula (30)) and the empirical results presented (see Table 16, columns with cycles for the 20-year life-span estimate) that Ruuhela's constant-growth assumption is crucial for his method. The business cycles cause a deviation under even perfect growth estimates and correctly estimated life-spans of the capital investments. Given the methods assumptions of constant-growth and its observed sensitivity to business cycles it is not surprising that the worst cases in the tables appear with strong business cycles (A = 1.00) and with misestimated life-spans.

4.4.2. Effect of Noise

To observe the effect of noise on Ruuhela's method we present the IRR estimates of Table 15 anew in Table 17 this time with the noise component included.

Table 17. Estimation of IRR (and growth) with Ruuhela's method, Anton contribution distribution, true growth rate k = 8%, true life-span N = 20, noise = 20%, no shock

Cycle amplitude		A = 0.00			A = 0.50			A = 1.00		
Estimated life-span		16	20	24	16	20	24	16	20	24
True r	4%	3.5 (9.1)	4.1 (9.1)	4.6 (9.1)	4.4 (9.3)	4.9 (9.3)	5.3 (9.3)	5.4 (9.6)	5.9 (9.6)	6.2 (9.6)
	8%	8.1 (9.1)	8.2 (9.1)	8.3 (9.1)	9.3 (9.4)	9.3 (9.4)	9.3 (9.4)	10.7 (9.6)	10.5 (9.6)	10.5 (9.6)
	12%	12.7 (9.1)	12.3 (9.1)	12.0 (9.1)	14.2 (9.4)	13.9 (9.4)	13.3 (9.4)	15.9 (9.6)	15.2 (9.6)	14.7 (9.6)
	16%	17.2 (9.1)	16.3 (9.1)	15.7 (9.1)	19.1 (9.4)	18.0 (9.4)	17.3 (9.4)	21.2 (9.7)	19.9 (9.7)	19.0 (9.7)

Two observations can be made by comparing Tables 15 and 17. First the noise obviously affects the growth OLS estimates. On the other hand the profitability estimates do not change much. Hence the sensitivity of Ruuhela's method to noise alone is mild, but noise aggravates the effect of cyclical fluctuations.. This corroborates the importance of the effect of the cyclical component on Ruuhela's IRR estimation results.

4.4.3. Effect of Contribution Patterns and Growth-Profitability Relationship and Other Factors

Our second research question concerns the effect of the cash contribution patterns of the capital investments available to the firm. Our third question concerns the effect of disparities between the firms growth rate and profitability. Tables 18 and 19 respectively give the IRR estimates for the different growth-profitability combinations under the uniform contribution distribution and the negative binomial distribution. Table 17 in the previous section contains the IRR estimates under the Anton contribution distribution for the firm's capital investments.

Table 18. Estimation of IRR (and growth) with Ruuhela's method, uniform contribution distribution, true growth rate k = 8%, true life-span N = 20, noise = 20%, no shock

Cycle amplitude		A = 0.00			A = 0.50			A = 1.00		
Estimated life-span		16	20	24	16	20	24	16	20	24
True r	4%	3.0 (9.1)	3.7 (9.1)	4.1 (9.1)	3.8 (9.3)	4.4 (9.3)	4.8 (9.3)	4.7 (9.5)	5.3 (9.5)	5.6 (9.5)
	8%	7.8 (9.1)	8.0 (9.1)	8.1 (9.1)	9.0 (9.3)	9.0 (9.3)	9.0 (9.3)	10.3 (9.5)	10.2 (9.5)	10.1 (9.5)
	12%	13.4 (9.1)	12.9 (9.1)	12.5 (9.1)	14.8 (9.3)	14.2 (9.3)	13.8 (9.3)	16.5 (9.5)	15.7 (9.5)	15.2 (9.5)
	16%	19.4 (9.1)	18.2 (9.1)	17.4 (9.1)	21.2 (9.3)	19.9 (9.3)	18.9 (9.3)	23.3 (9.5)	21.8 (9.5)	20.7 (9.5)

The contribution pattern of the capital investments has an effect, but the effect is a joint effect with the other parameters of the IRR estimation situation. As discussed, in the case of Ruuhela's method the Anton distribution has a special role since it is used as an assumption in the derivation of the method. This is also seen in the tables. The best IRR estimates are gained under the Anton contribution distribution.

The comparison of the tables for the case when growth equals profitability produces near-correct but not perfect estimates under no business cycles. A discrepancy between growth and profitability has a considerable effect on the quality of Ruuhela's IRR estimates. The effect of the fluctuations in the capital investments caused by business cycles is overriding in Ruuhela's method. With the increase of the cyclical fluctuations the growth vs. profitability equality loses its effect in Ruuhela's method.

Table 19. Estimation of IRR (and growth) with Ruuhela's method, negative binomial contribution distribution, true growth rate k = 8%, true life-span N = 20, noise = 20%, no shock

Cycle amplitude		A = 0.00			A = 0.50			A = 1.00		
Estimated life-span		16	20	24	16	20	24	16	20	24
True r	4%	3.8 (9.2)	4.4 (9.2)	4.8 (9.2)	4.6 (9.2)	5.1 (9.2)	5.4 (9.2)	5.5 (9.3)	5.9 (9.3)	6.2 (9.3)
	8%	8.1 (9.2)	8.3 (9.2)	8.3 (9.2)	9.2 (9.2)	9.2 (9.2)	9.2 (9.2)	10.4 (9.3)	10.3 (9.3)	10.2 (9.3)
	12%	13.1 (9.2)	12.6 (9.2)	12.3 (9.2)	14.4 (9.2)	13.8 (9.2)	13.4 (9.2)	15.9 (9.3)	15.2 (9.3)	14.7 (9.3)
	16%	18.4 (9.2)	17.3 (9.2)	16.6 (9.2)	20.0 (9.2)	18.8 (9.2)	18.0 (9.2)	21.9 (9.3)	20.5 (9.3)	19.5 (9.3)

Our fourth question concerns the effect of the firm's choice of the depreciation method on the quality of the IRR estimates. In Ruuhela's method this question does not rise since the method is independent of the firm's depreciation choices.

4.4.4. Effect of Major Capital Investment Shocks

Our last research question concerns the effect of major capital investment shocks on the reliability of the IRR estimation methods. Table 20 gives the OLS growth estimates $\hat{k}$, Ruuhela's IRR estimates with the estimated growth and the IRR estimates with the true growth (k = 8%).

Table 20. Estimation of IRR (and growth) with Ruuhela's method, negative binomial contribution distribution, true growth rate k = 8%, true life-span N = 20, realistic shock S = 5.309

Shock timing		Early			Late		
Estimate		$\hat{k}$	$\hat{r}\,(\hat{k})$	$\hat{r}$ (8%)	$\hat{k}$	$\hat{r}\,(\hat{k})$	$\hat{r}$ (8%)
True r	4%	10.9	5.2	3.5	13.3	5.4	1.8
	8%	10.9	9.3	7.2	13.3	9.5	4.9
	12%	10.9	13.8	11.3	13.3	14.0	8.4
	16%	10.9	18.8	15.8	13.3	18.9	12.2

It is readily seen from the table that with the introduction of major capital investment shocks the OLS growth estimation procedure is derailed. In conclusion, if there are major capital investment shocks, Ruuhela's method should not be applied on a the time period including such a structure-changing shock. (At the very least another method of growth estimation, like LAD estimation should be considered.) This observation is in line with Ruuhela's own observations about IRR estimation being valid only for periods of stable business culture.

4.4.5 Analysis of the Estimation Error in Ruuhela's Method

Tables 21 and 22 decompose the IRR estimation error in Tables 19 and 17, respectively, into its components for Ruuhela's method. The results are presented for our benchmark contribution distributions, the negative binomial distribution, and for Anton distribution which features in the derivation of Ruuhela's method. The components of the total error are attributable to deviation in the OLS growth estimate ("Grwt estim") and the error in the capital investments' life-span estimate ("Life-span estim"). The third component is the remainder of the total error. The remainder is attributed to the IRR estimation formula ("Formula"). The errors can either strengthen or dampen each other.

Table 21. Decomposition of the estimation error in Ruuhela's method. An example with negative binomial distribution, growth rate k = 8%, amplitude A = 50%, noise = 20%, no shock

Est. life-span		16 years				20 years				24 years			
Source of error		For-mula	Grwt estim	Life-span estim	Total error	For-mula	Grwt estim	Life-span estim	Total error	For-mula	Grwt estim	Life-span estim	Total error
True r	4%	0.1	1.0	-0.5	0.6	0.1	1.0	0	1.1	0.1	1.0	0.3	1.4
	8%	0.0	1.2	0.0	1.2	0.0	1.2	0	1.2	0.0	1.2	0.0	1.2
	12%	0.3	1.5	0.6	2.4	0.3	1.5	0	1.8	0.3	1.5	-0.4	1.4
	16%	1.0	1.8	1.2	4.0	1.0	1.8	0	2.8	1.0	1.8	-0.8	2.0

Table 22. Decomposition of the estimation error in Ruuhela's method. An example with Anton contribution distribution, growth rate k = 8%, amplitude A = 50%, noise = 20%, no shock

Est. life-span		16 years				20 years				24 years			
Source of error		For-mula	Grwt estim	Life-span estim	Total error	For-mula	Grwt estim	Life-span estim	Total error	For-mula	Grwt estim	Life-span estim	Total error
True r	4%	-0.1	1.0	-0.5	0.4	-0.1	1.0	0	0.9	-0.1	1.0	0.4	1.3
	8%	0.0	1.3	0.0	1.3	0.0	1.3	0	1.3	0.0	1.3	0.0	1.3
	12%	0.0	1.6	0.6	2.2	0.0	1.6	0	1.6	0.0	1.6	-0.3	1.3
	16%	0.1	1.9	1.1	3.1	0.1	1.9	0	2.0	0.1	1.9	-0.7	1.3

4.4.6. Conclusions about Ruuhela's Method

The main findings about Ruuhela's IRR estimation method are the following. Like Kay's method Ruuhela's method has a strong theoretical background in the linkage to the income determination theories of accounting and economics. The formal requirements of Ruuhela's method are more restrictive than Kay's. The constant-growth assumption is essential in Ruuhela's method. It explains the method's considerable sensitivity to business cycles and noise. Shocks should be excluded. They usually are involved with a change of business culture. An assumption of a unique IRR would be contested in applying Ruuhela's approach under the circumstances.

A disparity between the firm's growth and profitability generally increased the deviation of Ruuhela's IRR estimate from the true IRR. This feature is common with Kay's method.

The quality of the growth estimate affects Ruuhela's IRR estimate. The effect, however, is a joint effect with the other potential sources of error.

Ruuhela's method is independent of the depreciation method that the firm uses. Thus the accounting choices of the firm with regard to depreciation policies do not affect Ruuhela's IRR estimation method unlike the other methods.

4.5. Averaged Accountant's Rate of Return Method

The last method to be analyzed in this paper is the method of using the average ARR as the IRR estimate. The long-standing debate about the relevance of the averaged accountant's rate of return as a surrogate of the economist's theoretical profitability comes down to the question whether the average ARR is a good approximation of the firm's IRR, or whether the more complicated methods are the only avenue to a proper long-term profitability estimation (or if any are). The accountant's way of evaluating annual profits is dominant in business practice. Hence the soundness of extending the ARR concept to long-term profitability estimation is of paramount practical importance and interest.

4.5.1. Closeness of the Average ARR Method to Kay's Method

As for Kay's method the effect of cycles is negligible for the average ARR method. Thus the results of the simulation analysis are not presented for all the cycle alternatives. Table 23 gives the IRR estimates using the average ARR method in the case of medium level of business cycles (A = 0.50).

Table 23. Estimation of IRR with the average ARR method, growth k = 8%, amplitude A = 50%, noise = 20%, no shock

Contr. distribution		Uniform			Neg. binomial			Anton		
Depreciation		Str	Decl	Ann	Str	Decl	Ann	Str	Decl	Ann
True r	4%	3.6	2.6	4.0	4.2	3.4	4.0	4.0	3.2	4.0
	8%	7.8	7.6	8.0	8.0	7.9	8.0	8.0	7.9	8.0
	12%	12.6	13.3	12.0	12.3	12.9	12.0	12.0	12.6	12.0
	16%	17.9	19.5	16.0	17.0	18.4	16.0	16.0	17.2	16.0

The IRR estimates produced by the average ARR method in Table 23 are strikingly similar to the simulation results with Kay's method in Table 5. The maximum difference in the estimates is only 0.1 per cent in absolute terms. This closeness is not an unexpected result, since Kay's method in the format in Formula (21) can be interpreted as an iterative weighted-average ARR method. Only if major investment shocks are introduced the average ARR method gives estimates that are markedly different from Kay's estimates. This can be seen by comparing Table 24 for the average ARR method and Table 7 for Kay's method for an early shock. A similar comparison be done for a late shock in Tables 8 and 25. The second entry in each cell gives the deviation between the IRR estimates from the average ARR method and Kay's method.

The tables confirm that under ordinary cyclical conditions the average ARR method and Kay's method give virtually equivalent results. In practical long-run profit evaluation terms of the accountant there is no numerical difference between the two methods. Only with the

Table 24. Estimation of IRR with the average ARR method, negative binomial contribution distribution, growth rate k = 8%, amplitude A = 0.50, no noise, early shock (τ = 24); deviations from Kay's estimate on the second rows

Shock factor		S = 0.00			S = 5.309			S = 17.924		
Depreciation		Str	Decl	Ann	Str	Decl	Ann	Str	Decl	Ann
True r	4%	4.1	3.5	4.0	4.4	3.5	4.0	4.6	3.9	4.0
		0.0	0.1	0.0	0.1	0.2	0.0	0.1	0.7	0.0
	8%	8.0	8.1	8.0	8.2	8.0	8.0	8.4	8.5	8.0
		0.0	0.1	0.0	0.1	0.4	0.0	0.3	1.2	0.0
	12%	12.3	13.2	12.0	12.4	13.1	12.0	12.7	13.6	12.0
		0.0	0.0	0.0	0.2	0.8	0.0	0.7	2.0	0.0
	16%	17.1	18.7	16.0	17.1	18.6	16.0	17.4	19.2	16.0
		0.0	-0.1	0.0	0.6	1.3	0.0	1.2	3.1	0.0

Table 25. Estimation of IRR with the average ARR method, negative binomial contribution distribution, growth rate k = 8%, amplitude A = 0.50, no noise, late shock (τ = 30); deviations from Kay's estimate on the second rows

Shock factor		S = 0.00			S = 5.309			S = 17.924		
Depreciation		Str	Decl	Ann	Str	Decl	Ann	Str	Decl	Ann
True r	4%	4.1	3.5	4.0	4.1	3.0	4.0	4.1	2.7	4.0
		0.0	0.1	0.0	0.1	0.6	0.0	0.3	1.4	0.0
	8%	8.0	8.1	8.0	7.8	7.3	8.0	7.7	7.0	8.0
		0.0	0.1	0.0	0.2	0.6	0.0	0.5	1.8	0.0
	12%	12.3	13.2	12.0	12.0	12.2	12.0	11.8	11.7	12.0
		0.0	0.0	0.0	0.2	0.6	0.0	0.8	2.1	0.0
	16%	17.1	18.7	16.0	16.6	17.5	16.0	16.3	16.9	16.0
		0.0	-0.1	0.0	0.2	0.4	0.0	1.0	2.3	0.0

excessive seventeen-fold capital investment shocks the picture of the equivalence between the two methods changes. The methods start deviating markedly for the disparate growth-profitability combinations. Neither method, Kay's nor the average ARR, consistently outperforms the other when shocks are present. For example, Kay's method fares better for an early seventeen-fold shock in the case high profitabilities, but the situation is reversed for the late shock or low profitabilities.

4.5.2. Theoretical Considerations and Conclusion

Given the close kinship between Kay's method and the average ARR method it is interesting to observe which of the theoretical contentions still hold in the simulation for the average ARR method.

The first theoretical contention discussed in connection with Kay's method was Solomon's position that when the growth rate and the true internal rate of return are equal, the accountant's rate of return also becomes the same. For Kay's method no numerical deviation from this equivalence is observed assuming perfectly regular cycles, no noise and no shocks (see Table 6). For the average ARR method the same observation is made when there are no cyclical fluctuations, no noise and no shocks. However, with the cyclical fluctuations, but no noise in Table 26 the relationship no longer holds accurately. The deviation, however, is marginal. (The maximum deviation 0.1 occurs in the table in the case of negative binomial contribution distribution and double-declining-balance depreciation).

Table 26. Estimation of IRR with the average ARR method, growth rate k = 8%, amplitude A = 0.50, no noise, no shock

Contr. distribution		Uniform			Neg. binomial			Anton		
Depreciation		Str	Decl	Ann	Str	Decl	Ann	Str	Decl	Ann
True r	4%	3.6	2.9	4.0	4.1	3.5	4.0	4.0	3.3	4.0
	8%	8.0	8.0	8.0	8.0	8.1	8.0	8.0	8.0	8.0
	12%	13.0	13.9	12.0	12.3	13.2	12.0	12.0	12.7	12.0
	16%	18.3	20.2	16.0	17.1	18.7	16.0	16.0	17.4	16.0

As will be recalled, the next theoretical contention is about the equivalence of the IRR and the ARR under the theoretical annuity depreciation method. The validity of this contention is very strong. In our simulations it holds throughout both for Kay's method and the average ARR method, even under disparate growth-profitability combinations and major capital investment shocks as can be observed from Tables 7, 8, 24 and 25.

If the contributions from the capital investments follow the Anton distribution, the straight-line depreciation method results remain equivalent to the annuity depreciation results. Looking at Table 23 this result is seen to hold even under the ordinary conditions of business cycles and noise. However, with major capital investment shocks this theoretical contention ceases to hold both for Kay's and the average ARR methods.

To conclude about the average ARR method, the simulated IRR results are virtually equivalent to the results with Kay's method with the exception of the effect of excessive capital investment shocks. Therefore much the same numerical conclusions apply which already were discussed in connection of evaluating Kay's method. They are not repeated. The general conclusion about the business-practice based average ARR method is, however, very important. The average ARR method mostly performs as well (or as badly) as any of the sophisticated IRR estimation methods analyzed in our research project. Considering this fact and the average ARR method's practical appeal it is safe to say that for a practitioner it comes out best of the methods analyzed in this paper. The importance of the other IRR estimation methods, especially that of Kay's and Ruuhela's, lies in their merits for the theory of accounting.

4.6. Comparison of the Results

In comparing the different methods for estimating the internal rate of return of the firm's capital investments the following aspects are relevant: numerical performance, theoretical foundations and practical applicability. In this section we summarize the results in general terms.

First, consider numerical performance. In our simulations the relevant parameters are given such values as should put them in a realistic range with regard to actual business firms. Within the observed range none of the methods unequivocally outperforms the others in the simulation. The deviations in Kay's and the average ARR method are more regular and predictable than the deviations in Ijiri-Salamon and Ruuhela's methods. The number of potential sources of errors in Ijiri-Salamon and Ruuhela's method is greater than the other two methods. Since the errors of these methods partly compensate for each other, the resulting total error, while less predictable, is no worse for Ijiri-Salamon method than for the other methods. Ruuhela's method is the most dependent of the methods on its internal assumptions. Under its restrictive assumptions it works perfectly, but in a general situation it also produces the worst of the overestimation errors if there are strong business cycles and if the firm's profitability exceeds its growth considerably.

No common, generalizable pattern of errors emerged for the observed, different parameter combinations, with one tentative exception. Kay's method, Ruuhela's method and the average ARR method all have a tendency to overestimate rather than underestimate the true profitability when the firm's profitability exceeds its profitability considerably.

In the simulations of the present paper each of the boxes in the different tables can be considered "equally weighted". One potential direction of further research would be to adopt a numerical index to compare the numerical performance of the methods with each other. For this purpose it would be necessary to estimate from factual business observations the relative frequencies of the different combinations of the key parameters. (Some indication of the relative frequencies of the different cases are provided by the data in Figures 6 and 7.) In simulation a Monte-Carlo approach could be considered.

Second, consider the methods' theoretical robustness in the light of the simulation results. Kay's method came out as the theoretically most generic, with the average ARR method very close by. The ARR equality to IRR when the growth rate and the IRR agree, the theoretical annuity depreciation method's IRR-conformance, and the posed relationship of the annuity and straight-line depreciation methods under Anton contribution distribution all were confirmed in the simulations with Kay's method. Ruuhela's method is theoretically very sound, but its constant-growth and Anton contribution distribution assumptions make it empirically more vulnerable than Kay's and the average ARR method. Ijiri-Salamon method does not conform empirically to any of the expected theoretical propositions. This fact casts serious doubts on the theoretical validity of the method despite its relative reliability in the numerical simulation. The conclusion for the Ijiri-Salamon method is that it can be regarded as an elaborate, good rule of thumb. The other methods have deep roots within income theories of accounting and economics.

Last, consider practical applicability. In this area the average ARR method has the outstanding merit of being directly based on established accounting practice of performance measurement. It would be trivial to use computers to calculate Kay's IRR elaborate weighted-average estimates in business practice. However, the marginal improvement compared to the average ARR method does not compensate the obvious disadvantages of having to "sell" an iterative method to the users of financial information over the suggestion of using an average return on investment (ROI = ARR) for long-term profitability measurement. Ijiri-Salamon and Ruuhela's method are at a considerable disadvantage compared to the average ARR method since they require a fairly involved estimation process. In this light, for the practitioner it is our recommendation to choose for long-term profitability estimation the average ARR method over the more sophisticated IRR estimation methods. Knowing and understanding the analyzed, more sophisticated methods is not wasted, however. On the contrary, the practitioner should be aware of and familiar with the foundations of the methods s/he applies in order to make sound decisions.

5. SUMMARY AND CONCLUSIONS

This research analyzes four internal rate of return (IRR) estimation methods from literature for assessing the long-term profitability of a business firm from its published financial statements. The IRR estimation methods considered are Kay's, the Ijiri-Salamon, Ruuhela's and the average ARR methods. A realistic simulation approach is developed to evaluate and compare the methods. A simulation approach with a known internal rate of return makes it possible to study the ability of the various methods to estimate the firm's true IRR. The research contributes by evaluating the performance of selected IRR estimation methods under more general conditions than the earlier literature. This is facilitated by including cyclical fluctuations, noise and the possibility of major capital investment shocks into the simulated financial data. Most importantly the research contributes in literature's long-standing dispute about the validity of accountant's rate of return ARR as a proxy for the IRR.

Five research questions are posed concerning Kay's, Ijiri-Salamon, Ruuhela's and the average ARR methods. The questions cover how the methods are affected by business cycles and irregularities in the capital investments, the methods' sensitivity to capital investments' payback patterns, their sensitivity to disparity between growth and profitability, and their sensitivity to the accounting choices made by the firms.

First, the effect of business cycles and ordinary noise around the growth-trend of the firm's capital investments is of interest in evaluating the performance of the IRR estimation methods. The simulation model includes capital investment cycles in generating the simulated financial data. It is observed that three of the four methods are insensitive to cyclical fluctuations. The exception is Ruuhela's method which relies heavily on its constant-growth assumption. In the case of Kay's, Ijiri-Salamon and the average ARR method the insensitivity to business cycles is an important result because it confirms the applicability of the methods beyond the common steady-state assumptions. Furthermore, it is observed that ordinary noise in the capital investment time-series does not have a marked effect on the IRR estimates.

Second, the sensitivity of the IRR estimation methods to the capital investment's payback patterns is of interest. The true pattern of contributions from the firm's capital investments is not known for actual business firms. Therefore, alternative contribution distributions are considered. It is observed that all the methods can be sensitive to the contribution distribution. The effect of the shape of the contribution distribution on the IRR estimates is interactively dependent on the depreciation methods applied by the firm and the relationship between growth and profitability. The conclusion is that contribution distribution of the firm's capital investments can have an effect of the quality of the IRR estimates given by the analyzed IRR estimation methods. Furthermore, contrary to the other two IRR estimation methods, Ijiri-Salamon and Ruuhela's methods require an estimate of the life-span of the firm's capital investments. The reliability of the IRR estimates by Ijiri-Salamon and Ruuhela's method depends on the quality of the life-span estimate.

Third, it is to expected from theory that a disparity between the firm's growth rate and its long-term profitability affects the quality of the IRR estimates. It is observed that the reliability of the IRR estimates of all the methods is very sensitive to the relationship between the underlying true profitability and the firm's growth rate. In accordance to the simulation results the discrepancy between the true growth and profitability is the dominating source of the error in the IRR estimates in all the methods analyzed. In addition, the other sources of errors in the IRR estimates interact with the growth-profitability discrepancy. The errors can be aggravated by the discrepancy. This indicates that for better IRR estimation methods a correction for growth-profitability discrepancy should be an integral part.

Fourth, the depreciation method applied by the firm in its financial statements can affect the IRR estimation result in concert with the contribution distribution of the capital investments. Also this effect is strongly related to the growth-profitability discrepancy. For example, for Kay's and the average ARR method a worst case of the interactive effect appears under the following circumstances: The firm grows fast, it has low profitability and the firm applies an accelerated depreciation method in a situation where the contribution from the capital investments happens to follow the uniform distribution. In this respect Ruuhela's method has an advantage over the other methods since it is unaffected by the firm's depreciation choice.

Fifth, the simulations mostly indicate an unexpectedly good tolerance of the analyzed IRR estimation methods to major capital investment shocks. Ruuhela's method is the exception in this respect since its growth estimation is disrupted by such shocks. However, as discussed, in corporate practice a major capital investment shock is likely to coincide with a change in business culture. It is the literature's standard assumption of a constant IRR for the firm that comes to doubt under such circumstances.

To conclude, the simulation comparison of the selected IRR estimation methods shows that none of the analyzed sophisticated methods performs consistently better than the average ARR method. Thus, considering the various facets discussed in this paper, the accounting-practice-based average ARR method can be recommended as the best choice for the long-term profitability estimation. However, none of the methods, including the average ARR, is an unbiased estimator of the firm’s IRR. For fast growing firms with low profitability and for slow-growth firms with good profitability the long-term profitability estimates should be interpreted with much caution. On the other hand, the average ARR method can be safely used when a firm has comparable growth and profitability even when there are ordinary fluctuations and noise in the capital investment intensity.

REFERENCES

Artto, E. (1980), *Profitability and Cash Stream Analyses*. Helsinki School of Economics, D-44, Helsinki.

Brealey, R.A. and S.C. Myers (1991), *Principles of Corporate Finance*, 4th ed. McGraw-Hill, Inc., New York, N.Y.

Brief, R.P. and R.A. Lawson (1992), The role of the accounting rate of return in financial statement analysis, *Accounting Review* 67:2, 411-426.

Butler, D., K. Holland and M. Tippett (1994), Economic and accounting (book) rates of return: application of a statistical model, *Accounting and Business Research* 24:96, 303-318.

Conte, S.D. (1965), *Elementary Numerical Analysis; An Algorithmic Approach*. Kogakusha Company, Ltd., Tokyo.

Copeland, T.E. and J.F. Weston (1979). *Financial Theory and Corporate Policy*. Addison-Wesley Publishing Company, Reading, Mass.

Fisher, F.M. and J.J. McGowan (1983), On the misuse of accounting rates of return to infer monopoly profits, *American Economic Review* 73:1, 82-97.

Fisz, M. (1967). *Probability Theory and Mathematical Statistics*. 3rd ed. John Wiley & Sons, Inc., New York, N.Y.

Gordon, L.A. and M.M. Hamer (1988), Rates of return and cash flow profiles: an extension, *The Accounting Review* 63:3, 514-521.

Ijiri, Y. (1979), Convergence of cash recovery rate, in: *Quantitative Planning and Controlling. Essays in Honor of William Wager Cooper on the Occasion of His 65th Birthday* (ed. Y. Ijiri and A.B. Whinston). Academic Press, New York, N.Y.

Ijiri, Y. (1980), Recovery rate and cash flow accounting, *Financial Executive* 1980:March, 54-60.

Kay, J.A. (1976), Accountants, too, could be happy in a golden age: the accountants rate of profit and the internal rate of return, *Oxford Economic Papers (New Series)* 28:3, 447-460.

Kelly, G. and M. Tippett (1991), Economic and accounting rates of return: a statistical model, *Accounting and Business Research* 21:84, 321-329.

Laitinen, E.K. (1980), *Financial ratios and the basic economic factors of the firm: a steady state approach*, Jyväskylä Studies in Computer Science, Economics and Statistics 1, Jyväskylä.

Lawson, G.H. (1980), The measurement of corporate profitability on a cash-flow Basis, *The International Journal of Accounting Education and Research* 16:1, 11-46.

Levhari, D. and T. Shrinivasan (1969), Optimal savings under uncertainty, *Review of economic studies* 36:2, 153-163.

Levy, H. and M. Sarnat (1986), *Capital Investments and Financial Decision*, 3rd ed. Prentice-Hall Inc., Englewood Cliffs, N. J.

Peasnell, K.V. (1982a), Some formal connections between economic values and yields and accounting numbers, *Journal of Business Finance and Accounting* 9:3, 361-381.

Peasnell, K.V. (1982b), Estimating the internal rate of return from accounting profit rates, *The Investment Analyst* 1982:April, 26-31.

Pike, R. (1996), A longitudinal survey on capital budgeting practices, *Journal of Business Finance and Accounting* 23:1, 79-92.

Ruuhela, R. (1972), *Yrityksen kasvu ja kannattavuus* (in Finnish, English summary: A capital investment model of the growth and profitability of the firm), Acta Academiae Helsingiensis, Series A:8, Helsinki.

Ruuhela, R., T. Salmi, M. Luoma and A. Laakkonen (1982), Direct estimation of the internal rate of return from published financial statements, *The Finnish Journal of Business Economics* 31:4, 329-345. Also available from World Wide Web: <URL:http://www.uwasa.fi/~ts/dire/dire.html>

Salamon, G.L. (1982), Cash recovery rates and measures of firm profitability, *Accounting Review* 57:2, 292-302.

Salmi, T. (1982), Estimating the internal rate of return from published financial statements, *Journal of Business Finance and Accounting* 9:1, 63-74.

Salmi, T. and M. Luoma (1981), Deriving the internal rate of return from accountant's rate of profit: analysis and empirical estimation, *The Finnish Journal of Business Economics* 30:1, 20-45. Also available from World Wide Web: <URL:http://www.uwasa.fi/ ~ts/jkay/jkay.html>.

Salmi, T. and T. Martikainen (1994), A review of the theoretical and empirical basis of financial ratio analysis, *The Finnish Journal of Business Economics* 43:4, 426-448. Also available from World Wide Web: <URL:http://www. uwasa.fi/~ts/ejre/>.

Salmi, T., R. Ruuhela, A. Laakkonen, R. Dahlstedt and M. Luoma (1984), Extracting and analyzing the time series for profitability measurement from published financial statements: with results on publicly traded Finnish metal industry firms, Part III, *The Finnish Journal of Business Economics* 33:1, 23-48.

Salmi, T. and I. Virtanen (1995), *Deriving the Internal Rate of Return from the Accountant's Rate of Return; A Simulation Testbench*, Proceedings of the University of Vaasa. Research Papers 201. Also available from World Wide Web: <URL:http://www.uwasa.fi/~ts/simu/>.

Shinnar, R., O. Dressler, C.A. Feng and A.I. Avidan (1989), Estimation of the economic rate of return for industrial companies, *Journal of Business* 62:3, 417-445.

Stark, A.W. (1994), Some analytics of why conditional IRRs can contain growth rate related measurement error, *Journal of Business Finance and Accounting* 21:2, 219-229.

Stark, A.W., H.M. Thomas and I.D. Watson (1992), On the practical importance of systematic error in conditional IRRs, *Journal of Business Finance and Accounting* 19:3, 407-424.

Steele, A. (1986), A note on estimating the internal rate of return from published financial statements, *Journal of Business Finance and Accounting* 13:1, 1-13.

Suvas, A. (1994), Profitability, growth and the prediction of corporate failure, *The Finnish Journal of Business Economics* 43:4, 449-468.

Tamminen, R. (1976), *A theoretical study in the profitability of the firm*, Acta Wasaensia, No 5, Vaasa.

Teichroew, D., A.A. Robichek and M. Montalbano (1965), An analysis of criteria for investment and financing decision under certainty, *Management Science* 11:3, 395-403.

Vatter, W.J. (1966), Income models, book yield and the rate of return, *Accounting Review* 41:4, 681-698.

Watts, R.L. and J.L. Zimmerman (1986), *Positive Accounting Theory*, Prentice-Hall Inc., Englewood Cliffs, N.J.

Weingartner, H.M. (1969), Some new views on the payback period and capital budgeting decisions, *Management Science* 15:2, 131-140.

Whittington, G. (1979), On the use of the accounting rate of return in empirical research, *Accounting and Business Research* 9, 201-208.

Yli-Olli, P. (1980), *Investment and financing behaviour of Finnish industrial firms*, Acta Wasaensia, No 12, Vaasa.

APPENDIX 1. List of Symbols

i, j, t	=	auxiliary indexes
g_0	=	initial level of capital investments
g_t	=	capital investments in year t
k	=	growth rate of the capital investments
T	=	length of the simulation period
n	=	length of the observation period (number of years under observation for the profitability estimation)
A	=	amplitude of the cycle
C	=	length of the cycle
f	=	technical phase adjustment for the cycle
σ	=	the standard deviation of the random fluctuation in the capital expenditures
z	=	random variable following the (0,1)-normal distribution
S	=	capital investment shock coefficient
τ	=	the year of the capital investment shock ($\tau = \infty$) for no shock in the simulation)
δ	=	Kronecker's delta, $\delta_{t\tau} = 1$ when $t = \tau$, and 0 otherwise
f_{ti}	=	absolute contribution (cash-inflow) in year t from capital investment i years back
b_i	=	relative contribution from capital investment i years back
N	=	life-span of a capital investment project (the same for all capital investments)
f_t	=	cash inflow in year t
r	=	true internal rate of the simulated firm
$\hat{r}$	=	estimate of the internal rate of return
q	=	shape parameter for negative binomial distribution
m	=	location parameter for negative binomial distribution
P_j	=	negative binomial distribution
s	=	scaling factor
p_t	=	accounting profit in year t
d_t	=	depreciation in year t
v_t	=	book value of the firm at the end of year t
w_t	=	market value of the firm at the end of year t
CRR_t	=	cash recovery rate in year t
V_t	=	gross assets at the end of year t
D_t	=	accumulated depreciation
$\hat{V}_t$	=	estimate of the gross assets at the end of year t
$\hat{D}_t$	=	estimate of the accumulated depreciation

E(N)	=	Estimate of the life-span of the capital investments
$a_{N,k}$	=	annuity factor for N years at a rate of k
F	=	capital investment ratio
$\hat{F}$	=	estimate of the (constant) capital investment ratio
$\hat{k}$	=	OLS estimate of the firm's growth from f_t
ARR_t	=	accountant's rate of return in year t
$\overline{ARR}$	=	average of the accountant's rate of return over a period

APPENDIX 2. Annuity depreciation under Anton contribution distribution

Assuming Anton contribution distribution from Formula (12), Formula (18) defines the annuity depreciation of a single capital investment g as

$$\text{(A2.1)} \qquad d_t = \frac{1 + (N-t+1)r}{N} g - r\, v_{t-1}, \quad t = 1,...,N.$$

For t = 1 we have, considering that $v_0 = g$,

$$\text{(A2.2)} \qquad d_1 = (1/N + r)\, g - r\, g = (1/N)\, g.$$

Thus the annuity depreciation d_1 is equal the straight-line depreciation (1/N)g. Likewise, for t = 2 we have

$$\text{(A2.3)} \qquad d_2 = \{1/N + [(N-1)/N]\, r\}\, g - r\, [g - d_1]$$

$$= \{1/N + [1 - 1/N]\, r\}\, g - r\, [1 - 1/N]\, g = (1/N)g.$$

Repeating the process the general d_t becomes

$$\text{(A2.4)} \qquad d_t = \frac{1 + (N-t+1)r}{N} g - r \left(g - \sum_{i=1}^{t-1} d_i \right)$$

$$= \frac{1 + (N-t+1)r}{N} g - r \left(g - \frac{t-1}{N} g \right) = (1/N)\, g$$

which, again, is equal the straight-line depreciation.