Optimal Robust Positioning Using Factor Graph

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BIOGRAPHY

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ABSTRACT

Smartphone GNSS positioning accuracy is often limited by the use of low-cost antennas and chips, resulting in significant impact of signal degradations due to non-line-of-sight (NLOS) and multipath effects. Traditional techniques, such as extended Kalman filtering (EKF), often struggle to maintain accuracy, especially in urban settings. However, factor graph optimization (FGO) has emerged as a promising solution, demonstrating robust navigation capabilities in urban environments by leveraging multi-epoch GNSS measurements to estimate user positions concurrently. Therefore, we will implement an optimal robust positioning solution using FGO framework in this study. The integration of the Inertial Measurement Unit (IMU) and Global Navigation Satellite System (GNSS) measurements is achieved through an approach that employs the factor graph model. Unlike traditional filtering algorithms, the factor graph framework leverages all preceding measurements for state estimation. This study conducts experiments utilizing the Google Smartphone Decimeter Challenge 2023 dataset as its basis. This research aims to develop an optimal and resilient positioning algorithm by integrating GNSS and IMU through a loosely coupled approach leveraging FGO formulation with directly combining position data from GNSS receivers with the IMU measurements. This method of integration is easily implementable in both hardware and software. We compare this to the use of Kalman Filter/RTS for the same integration process. The Kalman Filter/RTS results gave an accuracy of 2.603 m while the accuracy of FGO implementation was 1.661 m. This research achieved an approximately 56.71% increase in accuracy with the implemented FGO approach.

Keywords - Factor graph optimization; GNSS positioning; IMU measurements; PNT.

I. INTRODUCTION

Global Navigation Satellite System (GNSS) is the most commonly used method for Positioning, Navigation and Timing (PNT) in outdoor environments. However, it is limited and has well-known vulnerabilities when exposed to constrained environments such as urban canyons and indoor environments due to phenomena such as multipath, Non-Line-of-Sight (NLOS) reception as well as signal blockage. Some of these errors and vulnerabilities can be addressed using techniques that help monitor the GNSS measurement quality, such as Fault Detection and Exclusion (FDE). Some GNSS system errors can be directly removed by applying more accurate corrections, such as in Precise Point Positioning (PPP).

GNSS measurements are often disrupted by external factors such as multipath, but integrating GNSS with IMU is advantageous because the strengths and weaknesses of each technology complement one another, facilitating accurate solutions even in challenging environments. Several techniques are being used to improve position estimation robustness when integrating GNSS with Inertial Measurement Units (IMU). Combining these technologies is usually accomplished using extended Kalman filtering (EKF), however, some other studies have applied factor graph optimization (FGO) for the positioning estimation [1], [2], [3]. However, in [4], FGO was used to improve the position estimation without integration with IMU.

This study aims to create an optimal and robust positioning algorithm for integrating GNSS and IMU using an approach based on FGO framework. We utilized loosely coupled integration with a FGO-based integration methods to combine GNSS and IMU into a single system. We used the acceleration and gyroscope data from the IMU for inertial navigation calculations to determine position data, which was then integrated with GNSS information.

II. RELATED RESEARCH

Precise point positioning (PPP) using carrier phase measurements has been extensively researched in the GNSS field. In [5], Watson implements and compares PPP using carrier phase measurements with graph optimization against a Kalman filter-based approach. The integration of RTK position estimation with IMU and other sensors has been investigated in [6, 7], demonstrating improved positioning accuracy over least-squares-based methods. GNSS position estimation is highly influenced by the operating environment of the GNSS receiver, such as open sky or urban canyons. In [4], Suzuki considers the data collection area during position estimation to account for these environmental effects. In [2], the paper proposes an ARIMA auxiliary model to predict GNSS measurements during interruptions, filling data gaps. Additionally, INS and GNSS measurements are loosely coupled using a factor graph model, which utilizes all previous measurements for state estimation. This method provides higher precision and continuous navigation results, even with GNSS data interruptions, compared to traditional KF/EKF GNSS/INS integration.

Based on the analysis of existing research, the main differences between the factor graph algorithm and the EKF algorithm are as follows:

1. The factor graph algorithm connects all historical information through the INS factor, leveraging the time correlation of measurements to resist outliers. This enhances the positioning accuracy and robustness of the system. In contrast, the EKF algorithm only considers the current measurement and the previous estimated value, lacking high robustness.
2. The factor graph can integrate non-synchronized sensor information, which is crucial for multi-source system expansion. Conversely, the EKF requires synchronized information; otherwise, the system needs reconfiguration.

3. In terms of real-time navigation, the EKF algorithm outperforms the factor graph algorithm because global optimization in factor graphs is more time-consuming. Consequently, improving the real-time performance of factor graphs is an emerging research focus.

III. MATERIALS, METHODOLOGIES, AND MODELS

This work is part of the Google Smartphone Decimeter Challenge (GSDC) 2023-2024. The goal of this competition is to determine the limit of smartphone GNSS positioning accuracy: that could be down to the decimeter or even centimeter level. In our work, we will make use of the post-processed kinematic (PPK) precise positioning techniques to process the GNSS datasets in an attempt to achieve sub-meter-level accuracy. GNSS/IMU-based localization is then applied to get the position estimation.

The approach we adopted for integrating GNSS/IMU within FGO is outlined as follows:

1. Data collection, and analysis

The data used is from the Google Smartphone Decimeter Challenge 2023 [8]. The data consist of a pool of raw GNSS measurements from Android smartphones along with high accuracy ground truth: 196 different drives over tens of different routes, with raw GNSS measurements including carrier phase and IMU data. The goal of this challenge is to determine the limit of smartphone GNSS positioning accuracy - performance down to the decimeter or even centimeter level.

2. GNSS data processing using PPK technique

The GNSS data is processed using the post-process kinematic (PPK) technique. In the PPK technique, carrier phase data is used to compute the user position. As PPK requires differential corrections, raw observation data for the appropriate dates and times were downloaded from a nearby CORS station on the NOAA National Geodetic Survey website using a base observation. The satellite broadcast navigation data (BRDM files) for each data set was also downloaded from the International GNSS Service website, including the clock navigation data, and satellite precise orbits. A python script is used to implement RTKLIBS programs [9] to post-process the GNSS data using the PPK technique.

3. Factor graph GNSS/IMU-based localization

The Inertial Measurement Unit (IMU) is frequently used in robotics, vehicles, and, of course, also in mobile phones. An IMU is usually made up of an accelerometer, gyroscope, and also a magnetometer. A tactical grade IMU is fully calibrated, whereas low-cost ones typically come with factory calibrations stored in the IMU registers. The calibration only covers the scaling factors of each axis. The accelerometer measures the specific force that can be derived into the acceleration of a movement, the gyroscope measures the angular speed of the sensor, and the magnetometer measures the Earth's local magnetic field direction and magnitude. The integration of GNSS/IMU using factor graph optimization has recently attracted attention as an alternative to the extended Kalman filter (EKF) for GNSS-IMU integration [1].

The recently introduced formulation of Factor Graph Optimization (FGO) by Indelman et al. [10] has introduced a fresh perspective on multisensor integration, as highlighted by Chen & Gao [11], and Pfeifer & Protzel [12]. This approach is structured as a probabilistic graphical model, featuring nodes representing various system states ($x_i \in \mathbf{X}$) and factors ($f_i \in \mathbf{F}$) corresponding to measurements. The flow chart of the method proposed in this paper is shown in Figure 1

FGO provides a flexible framework for integrating various sensor data measurements ($z_i \in \mathbf{Z}$) through the factorization of the probability distribution, representing the joint probability distribution of all states and measurements as a product of factors, each relating a small subset of variables [10],

$$f(\mathbf{X}) = \prod_i f_i(\mathbf{X}_i), \tag{1}$$

where $\mathbf{X}_i$ be the set of all variables x_i that are connected to the factor f_i by an edge. In terms of non-linear least squares optimization, each factor encapsulates an error function aimed at minimization. The optimal estimate $\mathbf{X}^*$ is determined by minimizing the overall error across the entire factor graph,

$$\mathbf{X}^* = \arg\min_{\mathbf{X}} \sum_i \|e_i(\mathbf{X}_i, \mathbf{Z}_i)\|^2_{\Sigma_i}, \tag{2}$$

where Σ is the known covariance of the measurement vector.

In FGO for loosely coupled GNSS/IMU integration, we assume that measurements follow a Gaussian distribution. Considering two types of factors used in the loosely coupled GNSS/IMU integration with FGO, the optimal state set $\mathbf{X}^*$ is obtained by

solving the following optimization problem:

$$\mathbf{X}^* = \arg\min_{\mathbf{X}} \sum_i \|e_i^{\text{GNSS}}\|^2_{\Sigma_i^{\text{GNSS}}} + \sum_i \|e_i^{\text{IMU}}\|^2_{\Sigma_i^{\text{IMU}}}, \tag{3}$$

where $\|e\|_\Sigma = e \cdot \Sigma^{-1} \cdot e$ is the Mahalanobis norm of e weighted by covariance Σ, $e_i = z_i - h(x_i)$ is the difference between the received measurement and the measurement expected for the true state, and Σ^{GNSS} and Σ^{IMU} are the covariance matrix for the GNSS and IMU factors, respectively. To solve optimization problem (3), a batch solver like Gauss-Newton [13], Levenberg-Marquardt [14, 15], or an incremental one like provided by iSAM2 [16] can be used.

4. Levenberg–Marquardt (LM) algorithm

The Levenberg-Marquardt algorithm combines gradient descent and Gauss-Newton numerical minimization algorithms. Gradient descent reduces the sum of squared errors by adjusting coefficients in the direction of steepest descent. Meanwhile, Gauss-Newton reduces these errors by approximating the least squares function as locally quadratic in coefficients and finding its minimum. The Levenberg-Marquardt algorithm behaves akin to gradient descent when coefficients are far from optimal, and shifts towards Gauss-Newton when coefficients are close to their optimal values [17].

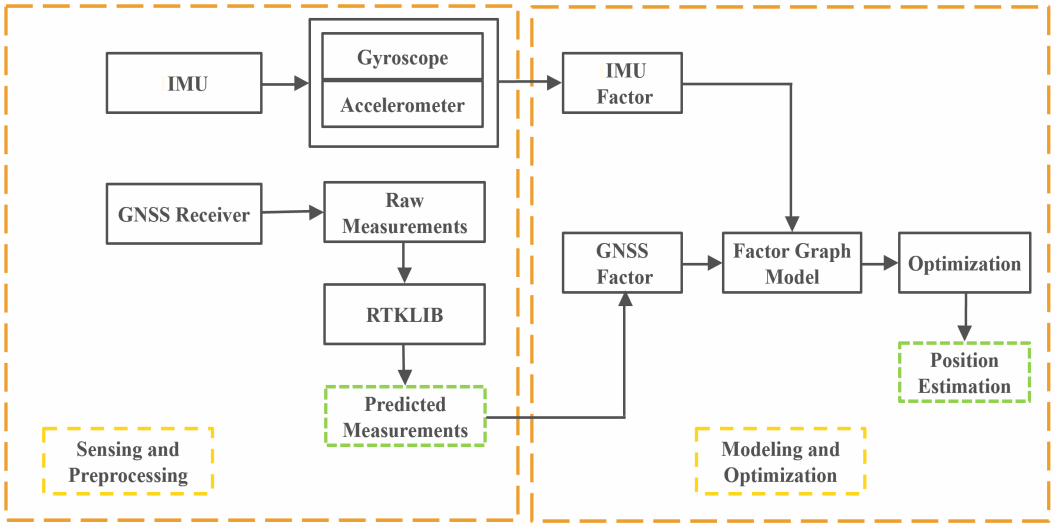


Figure 1: Flowchart of the proposed FGO-based GNSS/IMU integration.

Unlike the traditional EKF-based integration, the FGO captures the posterior probability of states over time, integrating both past measurements and system updates to optimize the entire state set. Historical information plays a crucial role in FGO, where all measurements and states are encoded into a factor graph, and the sensor fusion problem is iteratively tackled through optimization using non-linear optimization method. Consequently, errors arising from linearization steps are minimized. Additionally, FGO adeptly handles delayed measurements by seamlessly incorporating them as supplementary factors into the FGO upon reception. It should be noted that we performed all experiments with the GTSAM framework [18], in which we used the Levenberg-Marquardt algorithm to solve the problem (3). In addition, we used the IMU preintegration scheme as presented in [19].

IV. RESULTS

Accurate position estimation is important in several GNSS applications. Recent studies have focused on the comparison of using EKF-based and FGO-based methods for GNSS/IMU integration, with the focus on the potential of the latter. This research shows that the latter approach is more reliable as it helps to reduce the chance of a decrease in position accuracy due to missing GNSS data points and its ability to capture the posterior probability of states over time, integrating both past measurements and system updates to optimize the entire state set. Filling up the missing GNSS data can also enhance combined navigation, ensuring uninterrupted position estimation even during GNSS device degradation or outages.

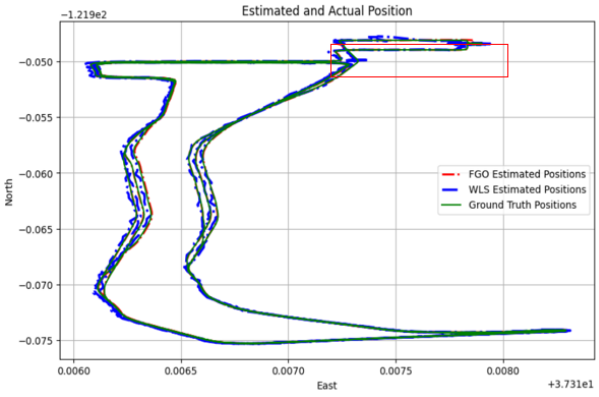
When comparing the WLS position estimation with the implemented FGO-based GNSS/IMU integration using the Vincenty distance from the ground truth in the training dataset, we observed a significant improvement in the accuracy score. In table

1, we present the accuracy score comparison between the WLS baseline score and the FGO-based GNSS/IMU integration for some selected phones and routes. It was however noticed that at some drive routes, the Samsung phones performed very poorly compared to other phones. The best position estimation accuracy when comparing phones were the Pixel phones and the Pixel6pro to be specific. However, when estimating the percentage increase in accuracy, the Mi8 phone has a percentage increase in accuracy of approximately 65.46%.

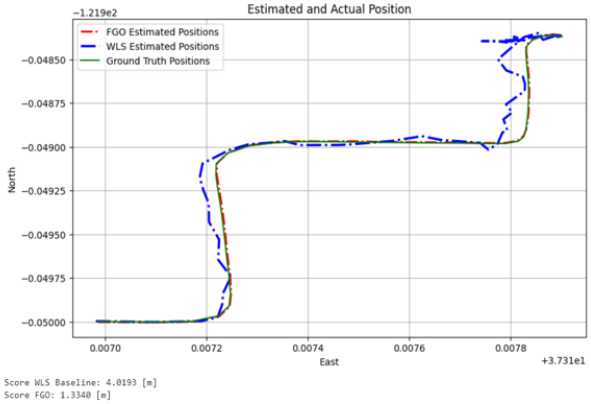
Table 1: Position estimation accuracy

Phone	WLS Baseline [m]	FGO [m]
Pixel4	2.94	1.47
Pixel4xl	3.64	1.36
Pixel5	5.03	1.48
Pixel6pro	3.85	0.99
Pixel7pro	5.39	1.46
Samsung	4.43	1.96
Mi8	5.99	2.07

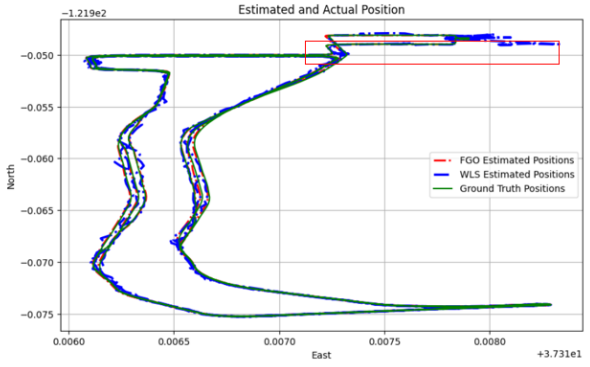
An interesting route 2020-12-10-22-52-us-ca-sjc-c in the train dataset had four phones on the same drive route to process. The figure 2 shows the plot of Mi8 and Pixel4 for ground-truth, WLS and FGO plots.



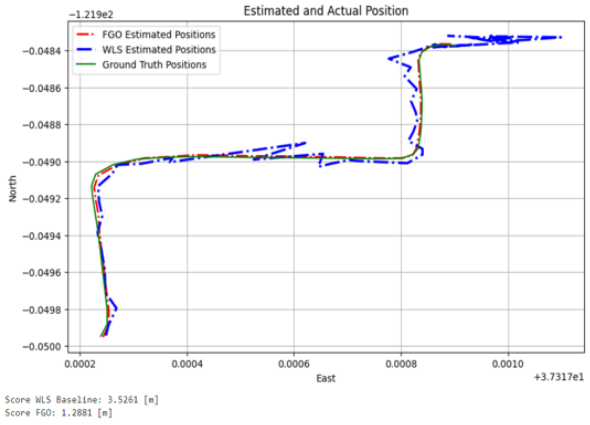
(a) Mi8



(b) Mi8 zoomed-in



(c) Pixel4



(d) Pixel4 zoomed-in

Figure 2: Comparison of positioning error between the selected phones baseline and proposed FGO-based method for particular driving path.

When the FGO-based method of integration was compared with extended Kalman filtering (KF-RTS) for the same integration process [20], the KF-RTS-based results gave an accuracy of 2.291 m while the accuracy of FGO-based implementation in this research was 1.977 m when evaluated on www.kaggle.com for the GSDC competition. There was an 15.88% increase in the accuracy.

V. CONCLUSION

The Global Navigation Satellite System (GNSS) is the preferred technology for outdoor positioning and navigation. However, it faces significant limitations and vulnerabilities in constrained environments such as urban canyons and indoor spaces, where phenomena like multipath, Non-Line-of-Sight (NLOS) reception, and signal blockage are prevalent. Techniques such as Fault Detection and Exclusion (FDE) and Precise Point Positioning (PPP) can mitigate some of these errors, but urban environments remain among the most challenging for accurate GNSS performance.

Integrating GNSS data with Inertial Measurement Unit (IMU) technology can significantly improve position estimation performance. While the Extended Kalman Filter (EKF) is a common approach for this integration, factor graph optimization (FGO) methods have also shown promise. Moreover, FGO has been independently utilized to enhance position estimation. This study aimed to develop an optimal and robust positioning algorithm for GNSS/IMU integration using a loosely coupled approach. This approach directly combines position data from GNSS receivers with IMU measurements, facilitating easy implementation in both hardware and software.

The contributions of this paper include the implementation of a factor graph optimization (FGO)-based method for integrating GNSS position estimation with IMU measurements using the LC method. By adopting this approach, we achieved improved position estimation accuracy compared to our previous studies that utilized the Kalman Filter/RTS for GNSS/IMU integration. The Kalman Filter/RTS results gave an accuracy of 2.603 m while the accuracy of FGO implementation was 1.661 m. This research achieved an approximately 56.71% increase in accuracy with the implemented FGO approach.

In our future study, we plan to do the following, development of robust FGO framework for tightly-coupled (TC) GNSS/INS integration, fusing new sensors to the same framework, reinforcement learning based robust FGO where we will perform optimized robust positioning using Factor Graph (FG) in combination with threat detection and apply adaptive positioning strategies using Reinforcement Learning (RL).

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