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Choosing factors in the German stock universe

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ABSTRACT:
This study implements the choosing factors approach on the German stock universe and performs a scientific replication of Fama and French's (2015) study. Using a unique German CDAX time-series data set from January 2007 to December 2020, this study computes 4x4 value-weight portfolios that consider a relatively smaller sample size than in the United States. The average adjusted R squares for the Fama-French three-, five-, and six-factor asset pricing models are 51%, 53%, and 54%, respectively. Regarding the Wald test, none of the nested baseform Fama-French asset pricing models can explain the cross-section of portfolio returns double sorted on size and value, profitability, investment, or momentum. Furthermore, spanning regressions expose that size, value, profitability, or investment factors do not matter in Germany, whereas the market factor and momentum expand the mean-variance frontier.

KEYWORDS: Asset pricing models, German markets, Fama-French, Mean-variance frontier, Three-factor model, Five-factor model, Six-factor model.

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TIIVISTELMÄ:

Tämä tutkimus soveltaa riskifaktoreiden valitsemistapaa Saksan osakeuniversumissa ja suorittaa tieteellisen kopion Faman ja Frenchin (2015) tutkimuksesta. Tämä tutkimus käyttää ainutlaatuista saksalaista CDAX aikasarjadataa tammikuusta 2007 joulukuuhun 2020 laskien 4x4 markkina-arvopainotetut portfoliot, sillä näiden portfolioiden otoskoko on suhteellisesti pienempi kuin Yhdysvalloissa. Keskiarvoiset selittävyysasteet Fama-French kolmen, viiden ja kuuden faktorin malleille ovat 51%, 53% ja 54%. Wald testin mukaan yksikään samanlaisia faktoreita sisältävistä Fama-French malleista ei pysty selittämään portfolioiden tuottoja, jotka on tuplalajiteltu koon ja arvon, kannattavuuden, investointimäärän tai momentumin mukaan. Lisäksi ”spanning” regressiot kertovat, että osakkeiden koko, arvo, kannattavuus tai investointitekijät eivät ole merkityksellisiä Saksan kontekstissa, kun taas markkinafaktori ja momentum pystyvät selittämään tuottoja Saksan osakeuniversumissa.

Avainsanat: Osakkeiden hinnoittelumallit, Saksan markkinat, Fama-French, Tehokkuusrintama, Kolmen faktorin malli, Viiden faktorin malli, Kuuden faktorin malli.

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1 Introduction

The popularity of asset pricing models has increased steadily through 60 years since the introduction of the Capital Asset Pricing Model (CAPM) by Sharpe (1964), Lintner (1965), and Mossin (1966). Even though the CAPM is still the most extensively used method for stock pricing, financial literature has documented other variables which cannot be explained by the CAPM. For the first instance, these variables include past stock returns (DeBondt & Thaler, 1985; DeBondt, 1987; Jegadeesh & Titman, 1993; 2001), size (Banz, 1981; Reinganum, 1981), earnings-price (Basu, 1977; 1983), leverage (Bhandari, 1988) and book-to-market (Capaul, Rowley & Sharpe, 1993; Chan, Hamao & Lakonishok, 1991; Fama & French, 1992; Rosenberg, Reid & Lanstein, 1985). These essential results have driven experts to expand the CAPM formula with different anomalies proposed as potential risk-factors in asset-pricing, which should capture the most variation in average equity returns.

Fama and French (1992) found out that two easily measured variables, size, and book-to-market equity, can seizure most of the variation in average stock returns and that most of the other variables' explanatory power vanished. Thereby, Fama and French (1993) present a three-factor model and argue that it is more capable in pricing future stock returns than the CAPM since size reflects to a risk effect and small-cap stocks outperform big-cap stocks. Moreover, high book-to-market stocks tend to earn significant positive excess returns whilst low book-to-market stocks tend to earn significant negative excess returns, and that book-to-market ratio is highly correlated with the future performance.

About fifteen years later, Novy-Marx (2013) discovered a proxy for predicted profitability that is substantially associated with the average return. Aharoni, Grundy, and Zeng (2013) document a weaker but statistically dependable relationship between investment factor and average return. Titman, Wei, and Xie (2004) also claim that the three-factor model is not capturing all expected returns by missing the variation in average returns related to profitability and investment factors. In 2015, Fama and French upgrade the three-factor model by adding profitability and investment factors, creating the five-factor model. The authors derived these new risk-factors from the dividend discount model (Miller & Modigliani, 1961), which makes three strong statements of expected stock returns: 1) whenever book-to-market ratio is higher, also the expected return is higher; 2) whenever expected profitability is higher, also the expected return is higher; 3) whenever the investment level or equivalent higher growth in book equity implies a lower expected return. However, after strong criticism and acknowledged fact that the five-factor model misses to price the momentum premium, Fama and French (2018) follow Carhart (1997) and include momentum factor into their model to satisfy insistent popular demand but worry to add factors that seem to have empirical robustness but do not reflect any theoretical support.

1.1 Motivation of the study

Motivated in finding the risk-factors that expand the mean-variance frontier, the purpose of this study is to do a scientific replication of Fama and French's (2015) asset pricing study by using different sorts, time-period, and stocks. Therefore, the Fama-French asset pricing models are tested in a German context to build on the previous empirical studies. This study computes its own value-weighted portfolios, slightly different from Fama and French (2015; 2017; 2018) as well as Grobys and Kolari's (2021) studies to distinguish between new findings and similarities of the previous studies in contrast to the German stock universe. Additionally, this study critically analyzes literature about the five-factor model's performance and biases as well as unites the latest findings of choosing factors empirical evidence since the financial literature lacks studies covering the six-factor model. Recently, Dirkx and Peter (2018) implemented the five-factor model for the German stock market and found that the international validity of the profitability and investment factors does not hold in the German stock universe. Ziegler, Schroeder, Schulz, and Stehle (2007) applied the three-factor model to the German stock universe and concluded that the three-factor model is more capable in capturing the cross-section of average stock returns in the US than in Germany. Finally, Hanauer, Kaserer, and Rapp (2011) used the Carhart four-factor model to the German stock universe and demonstrated that the four-factor model improves the explanatory power compared to the three-factor model.

Consequently, the German stock market has not been a target of such choosing factors study before. As such, this study brings an interesting point of view into the debate about the Fama-French risk-factors, as Germany represents alone most of the whole European equity markets and is the largest economy in Europe and the 4th largest in the world (measured by nominal GDP). Moreover, international studies such as Fama and French (2017) as well as Grobys and Kolari (2021) consider Europe as a whole and do not distinguish at a country-specific level.

1.2 Hypotheses and the research question of the study

This study is motivated to investigate a total of four hypotheses. These hypotheses measure whether different model variations increase the mean-variance frontier in the German stock universe. Consequently, the research question of this study is as follows: Which Fama-French risk-factors expand the mean-variance frontier in the German stock universe?

H_1: The Fama-French three-factor model captures the average return in the German stock universe with statistically significant results.

H_2: The Fama-French five-factor model captures the average return in the German stock universe with statistically significant results.

H_3: The Fama-French six-factor model captures the average return in the German stock universe with statistically significant results.

In 2015, Fama and French show that value does not matter in the United States, thus this study is interested in determining the value factor’s role in Germany. In contrast, Grobys and Kolari (2021) report that the value factor increases the mean-variance frontier in Europe. Thus, the fourth hypothesis in this study is as follows:

H_4: HML is a redundant factor in the German stock universe.

1.3 Structure of the study

This study is structured as follows: Chapter 2 gives a general introduction to the formation of financial theories including e.g., the modern portfolio theory, capital asset pricing model, efficiency market hypothesis, anomalies, and the Fama-French factor models. Chapter 3 describes the performance of the Fama-French five-factor model in the US and all over the world. Chapter 4 continues to investigate the Fama-French five-factor model's biases and failures. Chapter 5 concentrates on Fama-French asset pricing models' data and methodology. Chapter 6 highlights regression and comparison results and discusses the empirical findings. Finally, Chapter 7 shortly finishes the study by summarizing the most important results and observations.

2 Literature review

Practically, there were no quantitative methods to optimate investment portfolios before the mid-1900s. Investment decisions were made based on suppositions and speculations on how stock market returns will behave. Indeed, there were no measurable or mathematical models which could constitute strategic frameworks for stock and portfolio investments. Financial professionals acknowledged that to do rational investments, the investment portfolios must be diversified by buying different financial assets and thus include some insurance against possible risks. In fact, Henry Markowitz arises as a well-known professor for his pioneering work in Modern Portfolio Theory management.

2.1 Modern portfolio theory

The modern portfolio theory by Markowitz (1952 states that investors should focus on selecting portfolios using mean-variance analysis instead of individual securities. In addition, investors are wary about taking risks, so they choose a less risky portfolio to a riskier one given the same level of return. Next to this, they are maximizing returns in the risk-return-ratio. The expected returns on the portfolio are calculated as the weighted average of expected returns on all the financial instruments held in the portfolio. The formula for portfolio expected returns is written as follows,

$$E(R_p) = \sum_{i=1}^{n} w_i \, r_i \,, \qquad where \sum_{i=1}^{n} w_i = 1 \tag{1}$$

and w is the portfolio weight of each security. In turn, the portfolio risk is illustrated as the portfolio variance, and in the case of two different stocks it can be calculated as,

$$\sigma_p^2 = w_a^2\sigma_a^2 + w_b^2\sigma_b^2 + 2w_a w_b \sigma_a \sigma_b p_{ab} \tag{2}$$

where w_a is the portfolio weight of the first security, w_b is the weight of the second security, $\sigma_a$is the standard deviation of the first security, σ_b is the standard deviation of the second security and p_{ab} is the correlation coefficient between the two stocks, that is interpreted as $|p| \leq 1$. It symbolizes the values range between -1 and 1. A correlation coefficient of 1 indicates that the returns of a and b evolve in the same direction, which is called a perfect positive correlation. On the other hand, a perfect negative correlation means that a and b move in opposite directions, while a zero correlation implies no relationship at all. Consequently, by manipulating the structure of the portfolio, investors can influence the correlation coefficient by reducing or increasing it, which affects the combination variance. This is a very central affair that Markowitz (1952) discloses in his fundamental work. Indeed, the modern portfolio theory laid the groundwork for financial modeling.

According to Markowitz (1952), there exist optimal portfolios that offer the best relationship of risk-return by utilizing the right correlation coefficient relationship. A rational investor should select a portfolio from this specific set with the right combination of risk and return. This is graphically illustrated in the Figure 1 on the following page, where $E(r)$ is the expected return and σ is the standard deviation. The positively sloped curve that plots the expected return and risk is known as the efficient frontier. It determines the risk-and-return trade-off and localizes the efficient portfolios, the global minimum variance portfolio, and inefficient portfolios.

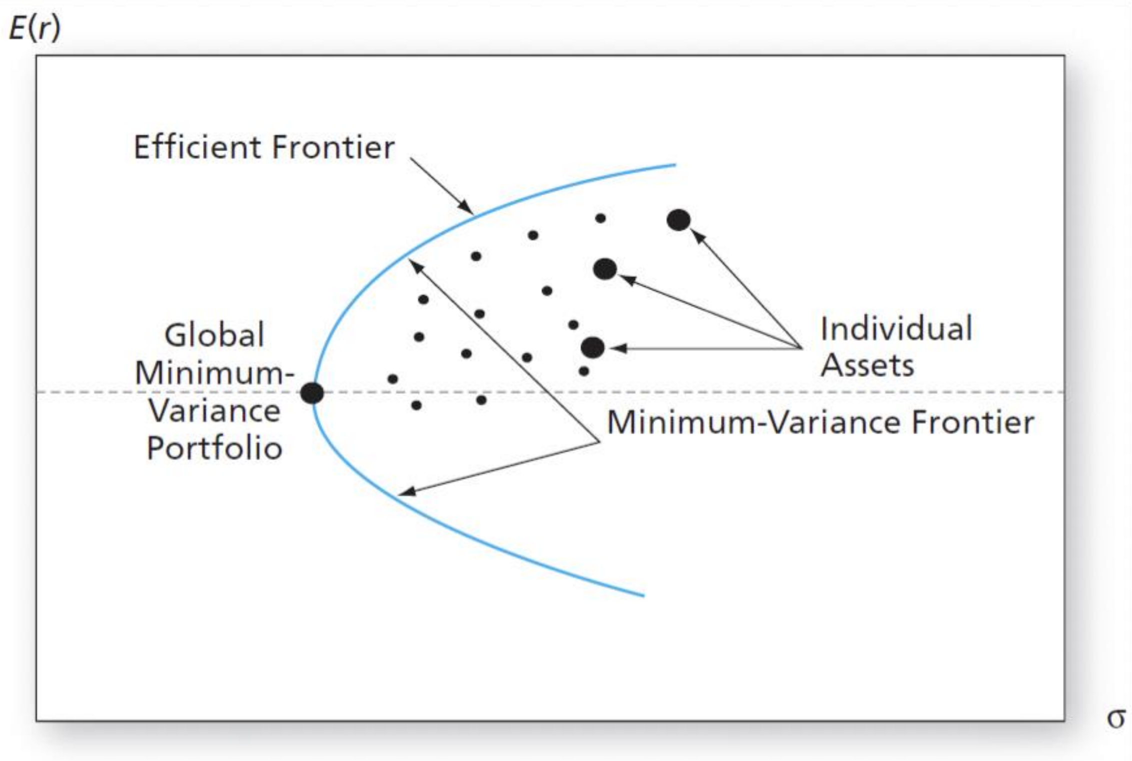

Figure 1. Efficient frontier and investment opportunities (Bodie, Kane & Marcus, 2011: 209).

In the same decade, Tobin (1958) notes that it is irrational to hold non-interest-bearing assets and complements Markowitz's thesis by including the risk-free interest rate into the analysis. The standard deviation of the risk-free interest rate is assumed to be zero. In addition, there is no correlation of returns with a risky portfolio. Combining the risk-free interest rate and risky stocks in the portfolio will lead to a situation, where a standard deviation depends on the weights of different kinds of financial instruments. Adapting the weight of the risk-free interest rate leads to portfolios with lower or higher standard deviations, and so on to lower or higher expected returns. These alternative strategies allow investors to form portfolios, which can be moved along the tangent line, underperforming, or outperforming those on the efficient frontier.

2.2 The efficient market hypothesis

Besides the modern portfolio theory, the efficient market hypothesis, henceforth called the EMH has been one of the main interests of researchers for a long time. Kendall

(1953) found that stock prices were evolving randomly, it seemed like the stock market is controlled by irregular market psychology without any logical rules. Further research and economists came to reserve Kendall's (1953) findings and point out that random price movements illustrated rather a properly functioning or efficient market instead of an irrational one. For example, Samuelson (1965) claimed that stock prices are already reflecting all available information and that returns evolve randomly. Consequently, randomness of price variation, and unpredictability can be simply explained by the competition between investors in the market.

A common assumption to deal with is that efficient markets are rational and free of friction. According to Fama (1970a) the definition of an efficient market is that stock prices perfectly reflect all available information. Appropriately investors are not able to achieve superior returns based on historical movements, because current stock prices already reflect all historical information, and all possibly arbitrage profits are eliminated. Consequently, the EMH has strong implications for security analysis, for example, there are investors who use a lot of resources to analyze company value chains, financial statements, and human capital, etc. Therefore, the question may be presented that what is the purpose of this kind of analysis made by investors. Grossman and Stiglitz (1980) argue that when arbitrage is expensive, it is illogical for markets to be in equilibrium and completely arbitraged on a regular basis. However, Gromb and Vayanos (2002) state that arbitrageur's presence should benefit all the investors through liquidity.

Fama (1970a) classifies the EMH into three divergent versions. These versions differ by the level of all available information. The weak-form hypothesis claims that stock prices must reflect all information from the trading volume, short interests, and past performance, etc. Thus, using historical prices, trend or technical analysis is pointless to predict future returns. The semi-strong-form hypothesis states that all publicly available information is totally reflected to present-day stock price. Such information consists of patents held, annual report data, earnings forecasts, quality of the management, share repurchases, etc. in addition to historical prices. Furthermore, fundamental analysis is useless. The strong-form hypothesis imposes that stock prices must reflect all information relevant to the company. This also includes the information that is considered as insider information. As a result, future price changes are completely unpredictable based on any available data.

2.2.1 Anomalies are the absences of efficiency

In efficient markets stock prices fully reflect all available information. An essential fact, however, is that the EMH does not require that investors to be fully rational, so mispricing opportunities exist. An individual investor can act randomly but the whole market is always right. After all, the right stock price is a consensus of investors' sentiment of a company's value. In other words, after anomalies are recognized and analyzed, they often seem to vanish or attenuate because an enormous group of investors exploits the arbitrage opportunities. Indeed, Chordia, Subrahmanyam, and Tong (2014) investigate stock data samples before and after 1993. They argue that abnormal returns caused by anomalies largely disappear, or at least weakened in the post-1993 period, but with one exception of the book-to-market effect. Furthermore, Hou, Xue, and Zhang (2018) reinvestigate 452 different anomaly variables that have been reported in the financial literature. The authors argue that 65% of the reinvestigated anomalies cannot be replicated and are not significant. They also demonstrate that the capital markets are more efficient than previously detected and assumed.

Fama and French (2008a) state that abnormal risk-adjusted returns which cannot be captured by the chosen pricing model are considered anomalies. Multifactor models, for instance, aim to capture anomaly variables through their factors in order to explain average stock returns. Before moving to factor models, however, it is purposeful to orientate to the CAPM which is the backbone of asset pricing models.

2.3 Capital asset pricing model

As announced in the first chapter, the CAPM was built separately by Sharpe (1964), Lintner (1965), and Mossin (1966). The CAPM is one of the centerpieces of modern financial economics, and relays on Markowitz's (1952) theory, and Tobin's (1958) findings of tangency portfolio. Therefore, the CAPM includes several underlying assumptions. First, investors can borrow and lend at a risk-free rate of interest indefinitely. Second, investors have similar expectations, which is consistent with the assumption that all meaningful information is publicly available. Third, markets are in equilibrium. Fourth, all investors have the same investing alternatives available to them. Finally, investors set aside the same amount of time to hold their investments.

Based on these assumptions, the efficient frontier of risky assets loses its position as the best investment opportunity for investors. As presented in the Figure 2 on the following page, investors with the same projected return and risk estimations will all detect their portfolios on the tangency line connecting the risk-free interest rate and the frontier. Rational and homogenous investors will always invest in the optimal portfolio of risky assets since it has the highest Sharpe Ratio, which is the point T, the mean-variance efficiency.

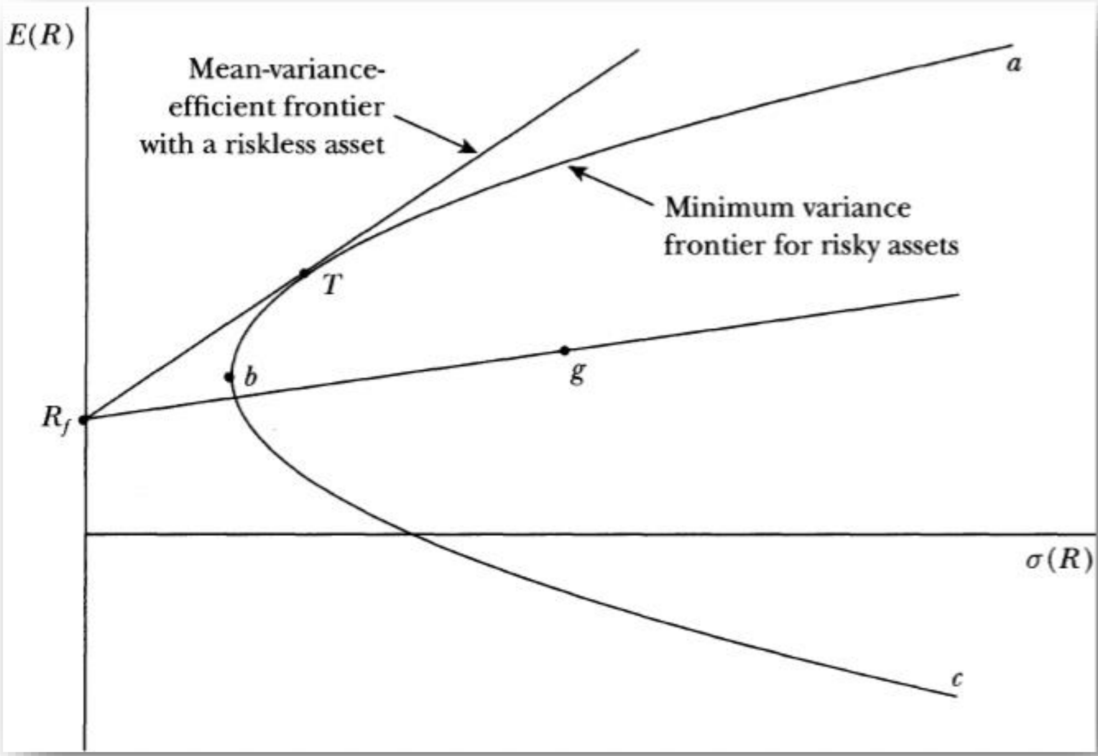

Figure 2. Optimal tangency portfolio (Fama and French, 2004).

Figure 2 describes the optimal portfolio opportunity and geometrically identifies the CAPM's formula, where $E(R)$ is the expected return, R_f is the risk-free interest rate (usually assumed to be a short-term T-bill rate), $\sigma(R)$ is the standard deviation, and portfolios that combine risk-free lending or borrowing with some risky portfolio g plot along a straight line from R_f through g. In the light of the assumptions mentioned above, all investors hold the same risky tangency portfolio T, which is the mean-variance optimal portfolio. According to the CAPM, there is a linear relationship between the expected return and market risk when there is an equilibrium. Therefore, the CAPM formula can be written as follows,

$$E(R_i) = R_f + \beta\left[E(R_M) - R_f\right] \quad (3)$$

where $E(R_i)$ is the expected return of security or portfolio i, R_f is the risk-free interest rate, $E(R_M)$ is the expected market return, and β can be composed as,

$$\beta = \frac{COV\ (R_i R_M)}{\sigma^2(R_M)} \tag{4}$$

where R_i is the stock return, R_M is the market portfolio return, $\sigma^2(R_M)$ is the variance of the return of the market portfolio, and $COV\ (R_i R_M)$ is the covariance between the return of the market portfolio and the return of the security. β itself is the exposure against the systematic factor, or systematic risk, of an individual security in comparison to the unsystematic risk of the whole market.

The CAPM is basically a single-factor model that uses the market factor and a specific exposure β, as its only explanatory factor for the expected stock or portfolio returns. The expected return is proportional to β, if there is a strong positive relationship between stock return and the return on the market, β will be high, consequently, the CAPM formula predicts higher expected return as it tends to be riskier and vice versa. The CAPM's hypothesizes that unsystematic risk is eliminated through effective diversification, as a repercussion it is not recompensed with greater expected returns. In the case of CAPM the efficient portfolios lie on a straight line, which is called Security Market Line (SML), which has R_f as its intercept and tangent thought T, as presented in the Figure 2 above. In conclusion, only systematic risk will be rewarded with a risk premium. Nonetheless, the CAPM has been widely criticized by researchers as a too straight and simplified version for risky asset pricing.

2.3.1 Theoretical criticism and CAPM extensions

Like many scientific models, the CAPM has its drawbacks. The unrealistic assumptions and lack of performance of the CAPM have been bothering academics for quite a long time. According to Fama and French (1993) the CAPM's major weaknesses and concerns are incapability to explain the observed market returns of small stocks and the timeline. Furthermore, the CAPM's problems include difficulty for estimating β, finding the best index to represent market portfolio, and choosing the right risk-free interest rate. Even Sharpe (1964) himself grants that the CAPM assumptions are deeply restrictive and unrealistic assumptions. Therefore, different new theoretical models were developed extending the original CAPM or proposing alternative assumptions that relax some of these questionable assumptions. Fama (1970b) takes an intertemporal approach to the original CAPM, arguing that if preferences and future investment opportunity sets are not state-dependent, or constant, then intertemporal utility maximization can be considered as if investors have a single period utility function. On the other hand, the assumptions behind this case are restrictive.

In 1973, Merton developed the Intertemporal Capital Asset Pricing Model, henceforth referred as the ICAPM. According to Merton (1973) the ICAPM is deduced from portfolio selection behavior by investors who are trying to maximize the expected utility of lifetime consumption. Furthermore, the uncertainty of future investment opportunities has an impact on today's demands. Consequently, investors hedge risky positions for example avoiding highly correlated assets. The other additional source of risk is inflation that affects the price of consumption goods. Investors may want to sacrifice some expected return in order to purchase financial assets that have higher return if cost of living changes negatively. The ICAPM includes multiple variables and can be considered as a linear multi-factor model. Another relevant model was introduced by Lucas (1978) and Breeden (1979), known as the Consumption Based Capital Asset Pricing Model, henceforth called the CCAPM. This model is based on the theorem that when rational investors optimize, the return of assets is linearly linked to the growth rate in aggregate consumption if the parameters of the linear relationship are constant over the years.

According to empirical studies, the security market line (SML) is flatter than projected by the original CAPM (Black, Jensen & Scholes 1972; Black 1972; Fama & MacBeth 1973). This discovery drove researchers to investigate other variables that could be more efficient in explaining the fraction of excess returns not explained by the CAPM approach. Despite this, Roll (1977) argues that the CAPM can only be evaluated if the whole market portfolio is recognized.

Banz (1981) and Reinganum (1981) present that size of the stock is a remarkable factor in explaining the cross-section of expected stock returns. They found that security's β affects to its average returns. Consequently, small stocks tend to exhibit higher earnings and large stocks tend to exhibit lower earnings. In addition, a more recent study by Baker, Bradley, and Wurgler (2011) demonstrates that stocks with high β have underperformed compared to stocks with lower β in the US stock markets since January 1968.

However, Hong and Sraer (2016) argue that stocks with high β are more often overpriced, therefore exhibiting lower earnings. Basu (1983) shows that earnings price, size, and β are together capable of explaining the cross-section of average stock returns. On the other hand, other researchers have argued that different macroeconomic variables control abnormal stock returns (Chen, Roll & Ross, 1986; Roll & Ross, 1980). According to Bhandari (1988) securities debt/equity ratio and returns are in a positive relationship. However, his findings are disagreed by the CAPM, which assumes that the risk created by the debt/equity ratio is already captured by β.

Rosenberg, Reid, and Lanstein (1985) show that in the US, there is a positive link between book-to-market equity and stocks. Chan, Hamao, and Lakonishok (1991) find that stocks' book-to-market equity plays a key role in explaining cross-section stock returns on Japanese stocks. A decade later in 1992, Fama and French converge with these observations. Empirical studies have also found evidence of past stock returns impact to abnormal yields, which cannot be explained by the CAPM. DeBondt and Thaler (1985) argue that stock returns over three to five years can explain future returns. They propose that stocks that have underperformed the market in the previous three to five years will outperform the market in the future and vice versa. Two years later DeBondt (1987) finds similar results in his own study. Jegadeesh and Titman (1993;

2001) report that stocks with strong past performance over 3 to 12 months continue outperforming stocks with weak past performance. This market anomaly is also known as a momentum effect.

These findings together represent that there are multiple possible variables besides the CAPM market factor and a specific exposure β that can explain the cross-section variation of average stock returns. One of the fundamental multifactor models is the Arbitrage Pricing Theory, which aims to explain these variables by various factors.

2.3.1.1 Arbitrage pricing theory

Stephen Ross developed the Arbitrate Pricing Theory, now on referred as the APT, in 1976. Like the CAPM, the APT aims to predict stock market's returns by merging expected returns and risk, although it takes an alternative procedure to the security market line. According to Ross (1976) the APT builds on three key propositions: First, security returns can be explained by a multifactor model. Second, unsystematic risk can be diversified away by increasing uncorrelated stocks in a portfolio. Third, a properly functioning financial markets abolishes arbitrage opportunities. However, the APT theory indicates that arbitrage opportunities can exist momentarily but will be wiped out by the adjustment of the prices. Considering these propositions, the expected return can be calculated as a linear function of the number of macroeconomic variables it is sensitive to. However, the theory does not definite how large the sample of variables is, or what these variables are. Thus, the APT formula may be expressed as,

$$E(R_i) = R_f + \beta_{i1}\lambda_1 + \beta_{i2}\lambda_2 + \ldots \beta_{in}\lambda_n \tag{5}$$

where according to Ross (1976), $E(R_i)$ is the security or portfolio's expected return i, R_f is the risk-free interest rate, each β_{ij} demonstrates the sensitiveness of security or portfolio's return to risk factor j, λ_j is the premium for factor j, and n corresponds to the number of explaining possible factors.

Differencing between the CAPM and the APT, the latter is an extremely appealing model. The APT hypothesis allows mispricing for individual securities and therefore solely suits to properly diversified portfolios, contrary to the CAPM. Based on its function as a multifactor model, the APT can be extended into disparate models. Moreover, the APT does not require a market portfolio to estimate the relationship between return and β. While the APT is more flexible and relaxes some of the unrealistic CAPM assumptions, it is more complex. Paradoxically, the CAPM gains its reputation and popularity from its simplicity as a mathematical model to determine expected stock returns. Indeed, for the same reasons as it has been criticized. Further research made by Fama and French (1992) found that two easily measured variables, which can be added to asset pricing formula without making it too complex.

2.4 The three-factor model

In the light of empirical evidence that the CAPM does not fully resolve the cross-section of average stock returns and observations of anomaly variables, Fama and French (1992) determine how β, size, earnings yield, leverage, and book-to-market are able to resolve the cross-section of average stock returns. In 1993, Fama and French suggest that a three-factor model could offer a more effective explanation for stock returns according to their previous study (1992) size and book-to-market variables can capture most of the variation of average stock returns, while other variables lose their explanatory power, or get dominated by these two more effective variables. Indeed, in their paper, Fama and French (1993) demonstrate that size, assessed by market capitalization, and value, assessed by book-to-market ratio are the definite proxies that can be used to capture risk exposure beyond the CAPM β. The authors proposed that size and value factors can be determined as SMB (small minus big) and HML (high minus low). Consequently, the three-factor model of explaining average stock returns is as follows,

$$R_{it} - R_{Ft} = \alpha_i + \beta_i(R_{Mt} - R_{Ft}) + s_i SMB_t + h_i HML_t + \epsilon_{it} \quad (6)$$

where according to Fama and French (1993), R_{it} is the return of security or portfolio i for period t, R_f is the risk-free interest rate, R_{Mt} is the return on the value-weight market portfolio, SMB_t is the return on a properly diversified portfolio of small stocks minus the return on a properly diversified portfolio of big stocks, HML_t is the difference between the returns on properly diversified portfolios of high and low book-to-market stocks, and ϵ_{it} is a zero-mean residual. The error term is created when the regression model does not fully exhibit the relationship between the independent variables and the dependent variables. Thereby, if the relationship is incomplete, the error term represents the amount at which the regression equation may deviate during empirical analysis. Additionally, treating the parameters in three-factor model as true values rather than estimates, if the factor exposures β_i, s_i, and h_i capture all the variation in expected returns, the intercept α_i is zero for all securities and portfolios i.

Fama and French (1993) conduct regression tests with excess returns of 25 portfolios sorted by size and book-to-market on the three-factor model and discover that the model outperforms the CAPM by absorbing anomalies. Indeed, the largest portion of the regression intercepts are close to zero. Further regression tests on five portfolios constructed on earning yield and the other five constructed on dividend yield were also performed in order to check the robustness of the three-factor model's ability to capture the cross-section of average stock returns. The outcome of the regressions is proportional to results with size and book-to-market factors, but a little bit weaker. However, still bringing the intercepts near zero.

The empirical failure of the CAPM and the performance of the three-factor model motivated researchers to investigate why it works through various studies. They have found three potential explanations. First, like Merton's (1971) the ICAPM and Rosses (1976) the APT, the three-factor model adds alternative risk factors, SMB and HML in order to explain the expected stock returns. Various studies demonstrate that the performance of the three-factor model in explaining anomalies in the stock market is a crucial evidence that these variables are proxies for underlying risk (Fama & French, 1995; 1996; Davies, Fama & French, 2000). According to Liew and Vassalou (2000), the SMB and HML factors are able to predict future gross domestic product in some countries. In turn, Lettau and Ludvigson (2001) argue that the factors SMB and HML are related to the consumption wealth ratio. Second, data-snooping is making the three-factor model so explanatory (Lo & MacKinlay, 1990; Black, 1993a; 1993b; Kothari, Shanken and Sloan, 1995). Third, biases of behavioral finance and the absence of market efficiency might cause these anomalies, that can be explained more specifically by the factors SMB and HML (Lakonishok, Shleifer & Vishny, 1994; Daniel & Titman, 1997; LaPorta, Lakonishok, Shleifer & Vishny, 1997; Daniel, Titman & Wei, 2001; Skinner & Sloan, 2002; Teo & Woo, 2004). However, Fama and French (1996) announced that the three-factor model is not able to explain medium-term momentum.

While the three-factor model has been efficiently capable to capture the cross-section average stock returns. Researchers have found other variables that also have explanatory power in stock returns. In 2006, Fama and French investigated expected stock returns from valuation theory's perspective.

2.5 The five-factor model

In the wake of valuation theory, expected stock returns are linked to the following variables: the book-to-market ratio, expected profitability and expected investment (Fama & French, 2006). This important discovery can be demonstrated with the dividend discount model which represents that the market value of a company can be considered as the current value of all future dividends. The formula itself is as follows,

$$M_t = \sum_{\tau=1}^{\infty} \frac{E(D_{t+\tau})}{(1+r)^{\tau}} \tag{7}$$

where M_t is the stock price at time t, $E(D_{t+\tau})$ is the future dividend per share for period $t+\tau$, and r is the average expected stock return in the long-term. Adapting the findings of Miller and Modigliani (1961) the dividend discount model is rebuilt as follows,

$$M_t = \sum_{\tau=1}^{\infty} \frac{E(Y_{t+\tau} - \Delta B_{t+\tau})}{(1+r)^{\tau}} \tag{8}$$

where, $Y_{t+\tau}$ is the total equity earnings for period $t+\tau$, and $\Delta B_{t+\tau}$ is the change in total book equity, and lastly dividing by time t book equity results in,

$$\frac{M_t}{B_t} = \sum_{\tau=1}^{\infty} \frac{\frac{E(Y_{t+\tau} - \Delta B_{t+\tau})}{(1+r)^{\tau}}}{B_t} \tag{9}$$

which makes three strong statements of expected stock returns. First, if all the terms in formula (9) are constants except M_t and r, a lower of value M_t generates a higher expected return. This is also considered as a higher book-to-market equity ratio. Second, assuming for a turn that M_t/B_t and $\Delta B_{t+\tau}$ are constants, higher expected profitability generates higher expected returns and vice versa. Third, given M_t/B_t and $\mathrm{Y}_{t+\tau}$, higher expected investment or equivalently higher growth in book equity generates lower expected returns. Basically, the formula (9) tells us that the M_t/B_t is a noisy proxy for expected return because capitalization also correlates to expected earnings and investment. Empirical evidence found by Fama, and French (2006) tend to support these predictions above.

Piotroski (2000) and Griffin and Lemmon (2002) confirm that company strength, which is a proxy of expected net cash flows, is positively related to average returns as the findings of the formula (9). There is also much more evidence, besides already announced, that high book-to-market generates higher average stock returns. Haugen and Baker (1996) and Cohen, Gompers, and Vuolteenaho (2002) confirm that controlling for book-to-market equity, average stock returns are positively linked to profitability. Correspondingly, Fairfield, Whisenant, and Yohn (2003) and Titman, Wei, and Xie

(2004) and Cooper, Huseyin, and Schill (2008) find a negative relationship between average stock returns and investment. In addition, Sloan (1996) demonstrates that accruals are negatively linked to profitability as result higher accruals indicate lower stock returns. Researchers were also interested in analytical forecasts. Abarbanell and Bushee (1998), Frankel and Lee (1998), Dechow, Hutton, and Sloan (1999), investigate earnings forecasts in order to determine investment's impact to the relationship between net cash flows and share price. The results establish that higher expected net cash flows lead to higher stock returns. However, Fama and French (2006) state that the empirical test results from profitability and investment factors add bit or none to the explanation of stock returns beyond the book-to-market factor's explanatory power. Despite this, Novy-Marx (2013) and Aharoni, Grundy, and Zeng (2013) conduct their own studies and came to disagree with this outcome.

Novy-Marx (2013) accomplished in identifying profitability as a proxy that has approximately the same capacity as book-to-market as an explanation of the cross-sectional average stock returns. The author states that profitability should be measured by the ratio of a company's gross profits to its assets. Novy-Marx (2013) then uses different methods than Fama and French (2006) whose method used current earnings as the proxy for profitability. Novy-Marx (2013) justifies his perspective by arguing that gross profit is the optimal accounting measure of economic performance since it is unaffected by expenses like research and development or human capital development. Furthermore, Novy-Marx (2013) found a negative link between profitability and book-to-market that is a remarkable tool to use in portfolio management.

According to the valuation model, the relationship between returns and expected investment should be negative. However, Fama and French (2006) found a positive relationship that is insignificant. Fama and French (2008b) argue that the narrow success of the valuation formula is because book-to-market captures data about both expected cash flows and discount rates. Nevertheless, Aharoni, Grundy, and Zeng (2013) argue that Fama and French (2006) failed in their test because they used per-share level measures of expected investment and expected profitability, and changes in the number of shares are devastating for the valuation formula. Indeed, the valuation formula may not work properly in pre-share analysis, so the investment factor occurs small and insignificant. On the other hand, applying the company level analysis Aharoni, Grundy and Zeng (2013) find a positive relationship between expected profitability and returns and, absolutely, a negative relationship between expected investment level and returns as the valuation model predicts. Besides Novy-Marx (2013), the authors report findings that profitability has a greater effect on companies that have low book-to-market ratio than companies that have higher book-to-market ratio.

Perhaps inspired by the discount valuation model and a new perspective of their 2006 paper's empirical documentations and the criticism of Titman, Wei, and Xie (2004) and more recent findings of Novy-Marx (2013) and Aharoni, Grundy, and Zeng (2013), Fama and French decided to scrutinize the three-factor model and other possible variables more carefully.

In 2015, Fama and French (2015) introduced the five-factor model that is aimed at capturing the size, value, profitability, and investment variables in average stock returns. The authors argue that the five-factor model is more capable in explaining stock returns than the three-factor model because of the empirical evidence that profitability and investment factors are strongly related to stock returns and the three-factor model misses much of the variation in average returns related to these two variables (Fama and French, 2015). Mimicking Aharoni, Grundy, and Zeng (2013), Fama and French (2015) use operating profitability as a variable of the company's profitability and the change in total assets as a variable of investment. The authors proposed that profitability and investment factors can be determined as RMW (robust minus weak) and CMA (conservative minus aggressive). Consequently, the regression model of explaining average stock returns is as follows,

$$R_{it} - R_{Ft} = \alpha_i + \beta_i(R_{Mt} - R_{Ft}) + s_i SMB_t + h_i HML_t + r_i RMW_t + c_i CMA_t + \epsilon_{it} \quad (10)$$

where according to Fama and French (2015), RMW_t is the difference between the returns on properly diversified portfolios of stocks with robust and weak profitability, and CMA_t is the difference between the returns on properly diversified portfolios of the stocks of low and high investment companies, which are called conservative and aggressive. Other variables are defined in the same way as in the three-factor model. If the exposures to factors β_i, s_i, h_i, r_i, and c_i, capture all variation in expected returns, and the intercept α_i is zero for all securities and portfolios i.

2.6 The six-factor model

Due to existing empirical evidence, criticism, and their own (2016; 2018) findings (which are discussed further in chapters 3 and 4), Fama and French (2018) incorporated a long-missed momentum factor to the five-factor model as follows,

$$\begin{aligned} R_{it} - R_{Ft} = \alpha_i + \beta_i(R_{Mt} - R_{Ft}) + s_i SMB_t + h_i HML_t + r_i RMW_t + c_i CMA_t \\ + m_i MOM_t + \epsilon_{it} \end{aligned} \tag{11}$$

where according to Fama and French (2018), MOM_t is the difference between returns on properly diversified portfolios of stocks that are considered as winners and losers with respect to past performance. Moreover, it is updated monthly rather than annually. All the other factors follow the same construction principles as in the three- and five-factor models.

This study has introduced different factor models and culminated to the six-factor model. When one considers asset pricing models, the six-factor model seems to be the best option in order to understand expected stock returns. However, an important question arises, how well the variants of the six-factor model actually can explain the average stock returns, and does it really increase the mean-variance frontier. The following chapters investigate previous financial literature considering the Fama-French models' factors by using the five-factor model as a base model. This approach attempts to reveal whether additional factors (RMW & CMA) add statistically significant explanatory power and on which magnitude the predecessor models miss the momentum premia.

3 Empirical performance of the five-factor model

This chapter investigates the performance of the Fama-French five-factor model. Before moving forward, it is important to know that accumulated evidence on the ability of factor models to capture the cross-section of returns is commonly based on CRSP and Compustat samples that start in July 1963. In addition, the backbone studies made by Fama, and French are investigated a little bit more specifically in order to understand the factor constructing better. Furthermore, the five-factor model's performance will be compared, for instance, to the three-factor model, the CAPM, and other factor models.

3.1 Managing anomalies

Fama and French (2015) conduct the first tests on the five-factor model. Their study focuses on the US sample including all NYSE, AMEX, and NASDAQ stocks with the exception of foreign listings and non-equity stocks. The time-period is July 1963 to December 2013. Fama and French (2015) decide to use a value-weight portfolio as the proxy for the market return and one-month U.S. treasury bill rate as the risk-free interest rate in their tests. The authors use NYSE breakpoints to allocate stocks into specific groups and form left-hand-side (that indicates the bid price, henceforth called the LHS), portfolios.

Constructing the SMB, HML, RMW, and CMA factors Fama and French (2015) start by sorting stocks into two market capitalization groups and three book-to-market, operating profitability, and investment groups. Profitability and investment factors are defined as the value factor. The size breakpoint is the NYSE median market capitalization, and the book-to-market, operational profitability, and investment breakpoints are the 30^{th} and 70^{th} NYSE percentiles. Consequently, forming six value-weight portfolios from 2x3 sorted region. Hence, creating the RMW and CMA factors the 2x3 sort method produces extra factors called as SMB_{OP} and SMB_{Inv}. The SMB factor from the three 2x3 sorts is determined as the average of $SMB_{B/M}$, SMB_{OP}, and SMB_{Inv}. Consequently, Fama and French (2015) state that the SMB is the average of the returns on the nine small security portfolios of the three 2x3 sorts minus the average of the return on the nine big security portfolios. The authors also construct tests on 2x2 sorts and 2x2x2x2 sorts on size, value, operating profitability, and investment.

Fama and French (2015) build the LHS regression portfolios in order to gain more crucial evidence about the model's performance as an explanation of returns. In the light of Novy-Marx's (2013) and Aharoni, Grundy, and Zeng's (2013) documentations, Fama and French (2015) perform tests with three sets of 25 (5x5) value-weight portfolios and three sets of 32 (2x4x4) value-weight portfolios. The first mentioned is sorted by pairs into size and book-to-market, size and operating profitability, size and investment, using NYSE quintile breakpoints as a distributor. On the other hand, the latter one is sorted on by size and two other variables as follows, size/book-to-market/operating profitability and size/book-to-market/investment and size/operating profitability/investment, using the NYSE median as size breakpoint and NYSE quartiles for operating profitability and investment.

A question arises, does the five-factor model shrink anomalies and how well it performed against its rivals and predecessors? – At last, the first regression results of the model are shown. Fama and French (2015) utilize the GRS statistics of Gibbons, Ross, and Shanken (1989) to combine asset pricing models and determine the five-factor model's capability in capturing expected stock returns. The GRS, however, rejects the five-factor model's capability at capturing size, book-to-market, profitability, and investment patterns. Nevertheless, Fama and French (2015) estimate that the five-factor model explains 71% – 94% of the cross-section variance of expected stock returns for the examined portfolios. Indeed, the authors find that the five-factor model outperforms the three-factor model and the CAPM but performs as well as the four-factor model that excludes the HML factor. They also point out that the five-factor model's explaining power is not respective to the factor construction. In further research, Fama and French (2016) argue that the effect of anomalies decreases in the five-factor model since anomalous returns do not vanish but become less anomalous, and the returns associated with various anomaly variables distribute factor exposures, implying that they are in large part the same aspect. The authors continue their statement by noting that there exist two exceptions, accruals, and momentum, however, still explaining average equity returns more efficiently than the original three-factor model.

Indeed, in a more recent study, Fama and French (2016) continue investigate the five-factor model's effectiveness at shrinking anomalies. The authors mimic the path of Lewellen, Nagel, and Shanken (2010) and look at anomalies not directly captured by the five-factor model e.g. momentum (Jegadeesh & Titman, 1993), volatility (Ang, Hodrick, Xing & Zhang, 2006), accruals (Sloan, 1996), net share issues (Ikenberry, Lakonishok & Vermaelen, 1995; Loughran & Ritter, 1995), and of course the security market line flatness against the CAPM predictions (Black, Jensen & Scholes, 1972; Black, 1972; Fama & MacBeth, 1973). The LHS portfolios are constructed from the same stock exchanges as in their previous study, but the data is gathered from July 1963 to December 2014 (Fama & French, 2015).

Fama and French (2016) compare the five-factor model's performance against the three-factor model and three four-factor models that combine the SMB, HML, RMW, CMA, and MOM factors, the latter is known as a momentum factor. Once again, the authors use reliable GRS statistics to summarize the results from various tests. The GRS test rejects all the examined factor models. However, Fama and French (2016) state that they are most interested in identifying the model with the best explanatory power for average returns, even if it is not perfect. Furthermore, it seems like that the MOM factor has quite a little effect on five-factor model's performance because the sorts do not generate portfolios with meaningful momentum effects. Moreover, the five-factor approach, according to Kozak, Nagel, and Santosh (2018), fails to seizure much of the cross-section of average stock returns.

Additionally, Fama and French (2016) demonstrate that the five-factor model performs badly on portfolios constructed on momentum because regression intercepts are scattered. Looking at the results of comparing the models, Fama and French (2016) detect again that the four-factor model that drops the HML factor performs mostly equitably as the five-factor model. However, they want to keep the HML factor in the model and define the $HMLO$ (orthogonal HML) factor from the analysis data in order to estimate the best asset pricing model, but it provides almost nothing help.

As a conclusion, Fama and French (2016) demonstrate that the five-factor model's positive exposures to the RMW and CMA factors help explain the high average stock returns considering low market β, low stock return volatility, and share repurchases. Oppositely, negative RMW and CMA slopes support to capture the low average stock returns considering high market β, high stock return volatility, and large share issues. Indeed, the authors state that the five-factor model typically outperforms the three-factor model.

The five-factor model has also been tested in a more recent study by Weber (2018). The author measures cash flow duration at the company level using financial report data and stocks listed on NYSE, AMEX, and NASDAQ from July 1963 to June 2013. The equities are divided into deciles based on cash flow duration, with a return differential of more than 1% per month between low and high duration stocks. Weber (2018) demonstrates that low duration stocks produce 1.45% of mean excess return per month and high duration stocks have a return of 0.32% per month. He finds that classical risk factors fail at explaining these returns and that the five-factor model earns a highly statistically significant alpha of 0.48%. Furthermore, Hou, Mo, Xue, and Zhang (2019) argue, that q-factor and q^5 model (which are derived from Tobin's q theory) outperform the five-factor model by explaining factor premiums.

In their 2018 paper, Fama and French attempt to develop insights about the maximum squared Sharpe ratio (MSSR) for factor models as a measure for ranking asset pricing models. Harvey, Liu, and Zhu (2015) demonstrate that there exist over 316 (estimate low) different anomalies that are possible factor variables. Consequently, the factor choosing process is challenging. Thereby, Fama and French (2018) conduct tests with, for instance, the CAPM, three-factor model, five-factor model, and six-factor model that adds a momentum factor. The used sample is NYSE, AMEX, and NASDAQ listed US stocks from July 1963 to June 2016. When summarizing the results, Fama and French (2018) notice that the intercepts are strong in RMW and CMA factor regressions and that the six-factor model wins. The authors, however, are critical at expanding the factor models because it could be the beginning of "a dark age" of data snooping.

3.2 Worldwide evidence

The first international empirical tests were driven by Fama and French (2017). The authors examine the five-factor model's performance abroad, across different markets besides the US. Moreover, other researchers have conducted tests in various stock markets all over the world. As opposite, the six-factor model has not yet been investigated on such a large scale and country-specific tests occur quite rarely.

Fama and French (2017) examine the five-factor model's global and local versions for the period July 1990 to October 2015. The stock data sample is gathered from Bloomberg, supplemented by Datastream and Worldscope. The authors construct the LHS regression portfolios from 23 developed markets, sorted into four regions: North America, Japan, Asia Pacific, Europe. In addition, Global portfolios are also tested by combining regions. Generally, the testing methodology is like previously with only a few changes (Fama and French, 2015). Indeed, Fama and French (2017) construct right-hand-side (that indicates the ask price, henceforth called the RHS), portfolios in order to keep results comparable.

The RHS portfolios are formed from 2x3 sorts on size, book-to-market, and operational profitability, or investment. The size breakpoints are the 10^{th} and 90^{th} percentiles of market capitalization for the region. The book-to-market, operating profitability, and investment (the rest of the RHS factors) breakpoints are the 30th and 70th percentiles as in the 2015 study. Correspondingly, the LHS portfolios are constructed in the wake of the previous study (Fama and French, 2015). Indeed, Fama and French (2017) use three sets of 25 (5x5 sorts) value-weight portfolios and three sets of 32 portfolios (2x4x4 sorts) value-weight portfolios. The authors, however, make an exception with breakpoints when sorting stocks to the portfolios. 25 portfolio size breaks are constructed by using the 3rd, 7th, 13th, and 25th percentiles of the region's aggregate market capitalization. This methodology corresponds approximately to the average market capitalizations for the NYSE quintile breakpoints. The other variables are organized by using quintile breakpoints for big stocks that are the top 90% of market capitalization in every region. In the same manner, 32 portfolios are sorted by using the 10th and 90th percentiles of the market capitalization of a region. Finally, other variables are sorted into four groups by using quartiles.

As assumed, Fama and French (2017) use the GRS statistics to determine the five-factor model's capability at capturing average returns for a region. The three-, four-, and five-factor models are compared with GRS statistic and alpha metrics. The results from international tests are fascinating because the Japanese sample stands out in a crowd. Indeed, the GRS test rejects all models that are used with North America, Europe, and Asia Pacific samples. Correspondingly, all models with the Japanese sample pass the GRS test. The authors report that in Japan, there is a considerable positive relationship between book-to-market and average returns. The average stock returns, however, are only barely dependent on profitability or investment. Consequently, the intercepts ratios are dependent on whether a sort involves book-to-market.

In their international paper, Fama and French (2017) conduct factor spanning regressions, which are used in order to determine the contribution of a factor to a model. The reason behind this is that the authors want to know which factor is redundant. The results differ widely depending on region and are at least interesting when compared to their 2015 and 2016 studies. For instance, all five factors are illustrative and important for explaining average returns in North America in the between 1990 – 2015. Whereas Europe and Asia Pacific are depending on the Mkt (i.e., market premium), HML, and RMW, with an assist from CMA in Asia Pacific. On the other hand, the CMA improves average return descriptions only a little bit in Europe during 1990 – 2015 and is therefore considered as a redundant factor. Furthermore, Japan, as mentioned above, is counting on the HML with perhaps an assist from the RMW, and like in Europe, the CMA is redundant for Japan at the sample period. A fascinating result is that the three-, four-, (that excludes the CMA), and five-factor model can seemingly price the returns in Japan. Indeed, the HML is the most explaining factor and the RMW is significant only when the CMA is included to the regression. However, the SMB is considered redundant at least between 1990 and 2015 everywhere except in North America. Additionally, the HML is not a redundant factor for describing 1990 – 2015 average returns in the US but with a longer sample period 1963 – 2013 it is a redundant variable (Fama and French, 2015).

The global five-factor model that uses combinations of the four region samples performs poorly in order to explain regional results. Fama and French (2017) comment that the international models may fail because markets are globally nonintegrated or the whole model is incorrectly structured. So, if the global five-factor model's explanatory power collapses, then the global three-factor model is a failure as well.

Fama and French (2017) find that the region-based five-factor model generally outperforms the three- and four-factor models in all metrics and that the four-factor (that drops the HML or RMW) model shows stronger performance against the three-factor model. Moreover, the three-factor model seems to be extremely weak when profitability is one of the sorting variables. Conversely, the five-factor model mostly absorbs the value, profitability, and investment patterns in average returns. This is in the line with Fama and French's (2015; 2016) previous studies.

Since 2015, other studies have also been interested in the five-factor model's explanatory power. Chiah, Chia, Zhong, and Li (2016) use an extensive Australian sample over the 1982 – 2013 period in order to determine the five-factor model's performance in the Australian stock universe. Briefly, the authors demonstrate that the five-factor model outperforms the three-factor model and Carhart's (1997) four-factor model by capturing profitability and investment patterns and shrinking anomalies with the best results. Moreover, unlike Fama and French (2015), the authors demonstrate that the HML factor is not redundant for the Australian stock universe. The reason behind this could be that there exist low correlations among the HML, RMW, and CMA factors.

Huynh (2018) also compares the performance of the three-factor and the five-factor models to explain anomalies in the Australian stock market. However, Huynh (2018) takes a different path than Chiah, Chia, Zhong, and Li (2016) by extending his work to 16 anomalies, including several not previously examined in Australia. In short, Huynh (2018) notes that profitability and investment patterns are captured by the five-factor model and the number of unexplained anomalies decreases under the five-factor model. Not surprisingly, the GRS test rejects the Fama-French models. Furthermore, Huynh (2018) admits that the HML factor is important in explaining risk outside the US and that the five-factor model is not capable to fully capture expected returns in Australia between 1990 – 2013 sample period.

Kubota and Takehara (2018) explore the five-factor model's performance in the Japanese stock universe during the sample period 1978 – 2014. Summarily, the authors find that the historical averages of the RMW and CMA factors are not large and are statistically insignificant. Moreover, it seems like both the RMW betas and CMA factor betas are only slightly correlated with the cross-sectional variations of average stock returns. Finally, they argue that the RMW and CMA coefficients are insignificant when the estimating technique is a generalized method of moments (GMM) with the Hansen-Jagannathan distance measure. Furthermore, the authors argue that the five-factor model is not capable to explain average stock returns in the Japanese stock universe and the HML is not redundant for the Japanese data sample. These all findings are contrary to the US evidence conducted by Fama and French (2015). Indeed, Kubota and Takehara (2018) demonstrate that the GRS tests rejects all models and the four- and five-factor models are inferior to the three-factor model.

Racicot and Rentz (2016) test the five-factor model's capabilities in the light of robust instruments by the GMM test in the same manner as Kubota and Takehara (2018). They collect data from Kenneth French's website from January 1986 to December 2014. In brief, the explaining power of the five-factor model appears to be impressive for all the 12 Fama and French's different industries the authors examined. However, the GMM test results are devastating for the five-factor model. It seems that the model's explanatory power is sensitive to the data sample. Furthermore, Richey (2017) investigates the five-factor model's performance against the CAPM, the three-, and four-factor model. The data sample is gathered from vice stocks (companies that manufacture and sell products like alcohol, tobacco, gaming goods and services, and military equipment, etc.) over the period 1996 – 2016 in the US and the measuring method is Jensen's α. Richey (2017) finds significant α for the CAPM, the three-, and four-factor model. Surprisingly, α's significance level substantially decreases when both the RMW and CMA are added to the model. However, with the five-factor model, the abnormal stock returns vanish. Moreover, Dhaoui and Bensalah (2017) confirm that the empirical findings support the five-factor model's validity in capturing average stock returns.

In a couple of decades, China has become one of the world's largest economies. However, Shanghai Stock Exchange (SHSE) and Shenzhen Stock Exchange (SZSE) were not founded until 1990 and 1991. Consequently, China started the stock exchange in the modern world. Therefore, researchers get enthusiastic to conduct tests with the Chinese data sample. Unlike Fama and French (2017), Guo, Zhang, Zhang, and Zhang (2017) run empirical regressions with the five-factor model in the Chinese stock universe between July 1995 and June 2014. Indeed, the results are interesting. The authors observe robust size, value, and profitability patterns in average stock returns, but the investment pattern is detected as weak. The five-factor model outperforms the three-factor model because of the RMW's explanatory power of describing average stock returns is so strong with the Chinese sample. On the other hand, the CMA factor is considered redundant during 07/1995 – 06/2015 and 07/1997 – 12/2013. However, the five-factor model manages to pass the GRS test for many of the portfolios that the authors test. Furthermore, looking at the GRS p-values in the same manner as Fama and French (2016) did for the Japanese data sample, we can see that the p-values are larger than 5% for the three-, four-, and five-factor models. Thus, the asset pricing models can price the average stock returns.

In the most recent study, Grobys and Kolari (2021) extend Fama and French's (2018) study on choosing factors to international stock markets. They propose a new methodology and use a block bootstrap approach that pays respect to factor dependencies and test nested and non-nested asset pricing models for North America, Europe, Asia excluding Japan, and Japan. However, testing non-nested models is beyond the scope of this thesis so we focus solely on their findings considering nested models. Grobys and Kolari (2021) found that the six-factor model proposed by Fama and French (2018) produces the highest maximum squared Sharpe ratio for most of the regions, an exception is Asia excluding Japan. Furthermore, the authors show that SMB is a redundant factor across their data sample, whereas HML is not a redundant factor in Europe, Asia excluding Japan, and Japan. Regarding Germany, Dirkx and Peter (2018) found that RMW and CMA factors do not matter in the German equity markets and that the average adjusted R^2 increases only slightly compared to the three-factor model. These findings suggest that factor redundancy is a sample-specific issue, respective to Fama and French's (2015; 2017; 2018) studies.

In summary, Fama-French asset pricing models have been empirically tested in various stock exchanges under alternative time-period, market integration, portfolio formation, etc. Majority of the empirical studies tend to agree the dominance of the five-factor model over the CAPM and three-factor in capturing average stock returns. However, the five-factor model gets rejected by the GRS test in most of the studies but there exist a couple of exceptions with Chinese and Japanese samples in Guo, Zhang, Zhang, and Zhang's (2017) and Fama and French's (2017) studies. Moreover, the five-factor model succeeds to capture profitability and investment patterns in various tests. On the other hand, the model's failures are also related to the RMW, CMA, and the absence of a momentum factor as shown by Fama and French (2018) as well as Grobys and Kolari (2021). Furthermore, the redundancy of a factor seems to be relative to the data sample. The next chapter will enter into the criticism and failures of the five-factor model.

4 Empirical dilemmas of the five-factor model

So far, we have discussed the performance of the five-factor model. This chapter looks at the biases and failures of the five-factor model. While the model has been quite successful in capturing average stock returns through its variables: SMB, HML, RMW, and CMA the researchers have criticized it, or at least suggested alternative and modified factors to the model or even, whole new factor models, as seen in the previous chapter. Nevertheless, Hou, Mo, Xue, and Zhang (2019) argue that all the modern factor models are closely related.

4.1 HML as an explaining factor

Fama and French (2015) state that the four-factor model that drops the HML factor is as successful as the five-factor model in capturing average stock returns. It seems like the HML factor return is being captured by other factors in the model. Indeed, the authors argue that the HML is redundant for interpreting average stock returns, at least in the US stock universe for July 1963 – December 2013. In other words, the HML factor does not expand the mean-variance frontier. However, Wahal (2019) demonstrates that, unlike the post-1963 period, the HML is not redundant in the five-factor model prior to 1963. Indeed, the five-factor model survives all the tests before 1963. Moreover, in their international tests Fama and French (2017) state that the HML factor is significant for 1990 – 2015 average stock returns in all tested stock universes.

The HML factor regressions made by Fama and French (2015) show us that the majority of the average return is essentially absorbed by the slopes for the RMW and CMA. The CMA factor slopes are robustly positive, supporting the fact that high-value companies invest only a bit. Correspondingly, also the RMW factor slopes are robustly positive, which indicates that controlling for other factors, value stocks act much like stocks with robust profitability, even if completely value stocks are not that profitable. Furthermore, Fama and French (2016) show that the four-factor model that drops the

HML factor explains average stock returns as efficiently as the five-factor model when the year 2014 is added to the US sample.

Furthermore, in their newer study (2018) Fama and French repeat themselves by showing that the HML does not contribute to returns. Barillas and Shanken (2018) tend to support this statement. Further, the authors demonstrate that the HML is not a redundant factor if it is updated monthly instead of yearly. Wahal (2019), however, demonstrates that his findings could say that the CMA factor is, in turn, redundant prior-1963 but admits that it depends on how one views the samples. On the other hand, Weber (2018) does not comment that the HML is redundant. Hou, Mo, Xue, and Zhang (2019) state that although the HML is a separate factor from the CMA it is still redundant in explaining average stock returns. The authors explain that there exists an economical relationship between investment and value, thus the HML should be deeply correlated with the CMA.

However, Fama and French (2015) argue that if investors hold a portfolio twisted toward size, value, profitability, and investment premiums, the five-factor model is the right model. Indeed, keeping the redundant factors in the model cause no harm. However, adjusted R^2 can decrease when more explanatory variables with no explanatory power are included in the regression.

4.2 Criticism, limitations, and findings

Fama and French (2006; 2015; 2016; 2017; 2018) have always stated that the five-factor model is motivated by the valuation theory. However, strong criticism from Hou, Mo, Xue, and Zhang (2019) challenge the groundwork of the five-factor model. Indeed, they claim that the valuation theory cannot justify the five-factor model since the linkages between book-to-market, investment, and profitability with the internal rate of return (IRR) do not always translate to the one-period-ahead expected return. In fact, Fama and French (2017) admit that the discount model ignores important things. For instance, the dividend discount model does not explain why the premiums in average stock returns associated with book-to-market, profitability, and investment are not captured by the CAPM. Moreover, the discount model cannot confirm whether the predicted average stock returns patterns are results of rational or irrational pricing.

One of the first empirical findings by Fama and French (2015) is that the five-factor model is incapable of capturing small stocks' low average return. Indeed, small stocks tend to have negative exposures to the RMW and CMA factors. The negative CMA exposures are always linked to companies that invest a lot, and negative exposures to RMW do not mostly correspond to low profitability. These stock returns behave much like companies that invest aggressively despite low profitability. This fact is in line with Fama and French's (2016; 2017) studies.

As stated in chapter 3, Fama and French (2016) found evidence that accruals and momentum are also variables that cause concerns to the five-factor model at shrinking anomalies. The dilemma appears to be that sorts on accruals generate average stock returns that the five-factor model cannot explain. Indeed, empirical testing reveal that portfolios in the smallest size quintile have negative RMW slopes but correspondingly do not obtain the forecasted low average stock returns. This empirical finding is supported by Hou, Xue, and Zhang (2015). On the other hand, according to Fama and French (2016), the problem caused by momentum among small stocks cannot be explained by adding a momentum factor. Nevertheless, Fama and French (2018) argue that the six-factor model (updated with a momentum factor) improves the performance in capturing stock returns at least during July 1963 – June 2016 in the US but the authors, however, are skeptical to extend their model. Furthermore, Fama and French (2018) state that through time, various patterns in average stock returns are noticed and become possible candidates for inclusion in multifactor models. But there exists a danger among data-snooping.

Fama and French (2016) demonstrate that time variation in the regression slopes is a possible problem because all slopes are estimated as constants. In addition, the five-factor model and empirical tests assume that market frictions, like transaction costs and taxes do not exist.

Fama and French (2016; 2017) state that the successes and dilemmas of the five-factor model are related to patterns in the slopes for the RMW and CMA factors which seems to be reasonable. However, Hou Mo, Xue, and Zhang (2019) disagree by arguing that using past investment as a proxy for the expected investment is the main dilemma of the five-factor model. The authors justify their arguments by invoking to the lack of economics literature of micro-level investment data and conducting their own mathematical tests. Furthermore, the authors state that the link between the expected investment and the expected return is likely positive, not negative. Consequently, they argue that the investment CAPM (Zhang, 2017) is the only theoretical framework that favors accounting variables in forecasting returns. However, Chiah, Chai, Zhong, and Li (2016) find that the HML holds its explanatory power in the Australian sample and therefore supports the valuation theory framework.

Nevertheless, Chiah, Chai, Zhong, and Li (2016) call for a better model and improvement because the five-factor model cannot totally explain the time-series variations in portfolio returns. Their conclusion reflects with Fama and French's own (2015) findings. Moreover, Huynh (2018) also asks for a better model because of the five-factor model's empirical failures in the GRS tests. Kubota and Takehara (2018) demonstrate that the five-factor model fails badly in Japanese data during 1978 – 2014 because the RMW and CMA cannot explain the cross-sectional average stock returns. Racicot and Rentz (2016) find that the five-factor model is effective in explaining stock returns when using a standard economic estimator (e.g., OLS) but the explanatory power vanishes when using a more sophisticated method (e.g., GMM).

Richey (2017) points out that when investigating the vice stock returns in the US the SMB loses its significance when the RMW and CMA are added to the original three-factor model. Dhaoui and Bensalah (2017) demonstrate that the updated five-factor model holds its explanatory power only with the standard macroeconomic approach. The authors mean by this that investors are fully rational and only economical goals motivate them. Furthermore, Dhaoui and Bensalah (2017) argue that their revised model which incorporates two additional factors, momentum and investor sentiments and emotions accomplishes to explain small stock returns better than the standard five-factor model in the US. Nevertheless, Guo, Zhang, Zhang, and Zhang (2017) show that the Fama-French five-factor model passes the GRS test in the Chinese stock universe but still cannot fully explain the average returns of small stocks.

To sum up, there seem to be some biases and theoretical problems in the five-factor model. Empirical studies, for instance, strongly argue for the momentum factor or at least state that the five-factor model is not able to capture the widely known momentum effect. Furthermore, small stocks occur as a magnificent problem for the model. Motivated by Fama and French (2018), Grobys and Kolari (2021) as well as Hanauer, Kaserer, and Rapp's (2011) previous empirical evidence considering the momentum effect, the next chapter introduces data and methodology that are utilized in exploring which Fama-French risk-factors increase the mean-variance frontier in a German context.

5 Data and methodology

This chapter concentrates on the data sample, variable and factor construction, descriptive table, factor correlations as well as factor spanning tests methodology.

5.1 Sample description

The data comprises of monthly CDAX returns from July 2007 to December 2020, covering a 162-month period, having the first accounting data observations in the end of 2005, respectively the last portfolio rebalance is in June 2020. The data is derived from Thomson Reuters' database. The methodology follows Fama and French's (2015; 2017; 2018) methodologies, thus not including financials nor negative book-to-market stocks to the final data sample. The sample selection process requires also to exclude companies for which profitability, investment measures, market capitalization, and return data at the time t cannot be computed based on Fama and French's (2015) study. The sorted portfolios are recomputed at the end of June every year. Following these procedures, the average number of stocks corresponds to 276, while the number of analyzed stocks never drops below 210. Unlike Fama and French's (2015; 2017; 2018) and the US studies the sample size is relatively smaller, therefore the value-weighted portfolios are computed by 4x4 (16) basis instead of 5x5 (25). In the European context, the risk-free is not based on the one-month T-bill rate but on the one-month EURIBOR rates and stock data currency is in Euros.

5.2 Variable and factor construction

The construction of the risk factors mainly follows Fama and French's (2015) methodology and builds dependent 2x3 sorts as well as calculates the value-weight return on the market portfolio of all 358 sample stocks, after applying the necessary filters, minus the one-month EURIBOR rate. The median market capitalization of the sample stocks serves as the breakpoint for size. Correspondingly, the 30th and 70th quantiles of the sample stocks are used as breakpoints for sorting book-to-market, profitability, and investment variables. The following equations construct the Fama-French variables at the end of June each year when portfolios are formed by using all accounting data at the time t-1.

$$B/M_t = \frac{Total\ Equity_{t-1}}{Market\ Capitalization_{t-1}} \tag{12}$$

$$OP_t = \frac{Revenue_{t-1} - COGS_{t-1} - SG\&A_{t-1} - Interest\ Expenses_{t-1}}{Total\ Equity_{t-1}} \tag{13}$$

$$Inv_t = \frac{Total\ Assets_{t-1} - Total\ Assets_{t-2}}{Total\ Asses_{t-2}} \tag{14}$$

where, $COGS$ is Cost of Goods Sold. The amount includes the cost of the materials and labor used to create the goods but excludes indirect expenses, such as distribution costs. $SG\&A$ refers to Selling, General, and Administrative costs. The amount stands for the sum of all direct and indirect selling, general and administrative costs of a company, but is not assigned to a specific product, and thereby not included in the Cost of Goods Sold section.

Dependent sort allocation divides stocks into two size groups and three groups based on book-to-market, operating performance, and investment variables. Considering the more comprehensive factor construction below, sorts based on size are classified as small (S), and big (B). Regarding book-to-market, stocks are specified as high (H), neutral (N), and low (L). In contrast, operating profitability-based stocks are specified as robust (R), neutral (N), and weak (W). Stocks that are based on their investment character are specified as conservative (C), neutral (N), and aggressive (A). Finally, momentum follows stocks that have performed well in the past (W), neutral performers (N) and losers (L). The final SMB is computed by equally weighting additional sub-size factors $SMB_{B/M}$, SMB_{OP}, and SMB_{Inv}. The final risk-factors are computed in the following equations:

$$SMB_{B/M} = \frac{S_H + S_N + S_L}{3} - \frac{B_H + B_N + B_L}{3} \tag{15}$$

$$SMB_{OP} = \frac{S_R + S_N + S_W}{3} - \frac{B_R + B_N + B_W}{3} \tag{16}$$

$$SMB_{Inv} = \frac{S_C + S_N + S_A}{3} - \frac{B_C + B_N + B_A}{3} \tag{17}$$

$$SMB_{MOM} = \frac{S_C + S_N + S_A}{3} - \frac{B_W + B_N + B_L}{3} \tag{18}$$

$$SMB = \frac{SMB_{\frac{B}{M}} + SMB_{OP} + SMB_{Inv}}{3} \tag{19}$$

$$HML = \frac{S_H + B_H}{2} - \frac{S_L + B_L}{2} \tag{20}$$

$$RMW = \frac{S_R + B_R}{2} - \frac{S_W + B_W}{2} \tag{21}$$

$$CMA = \frac{S_C + B_C}{2} - \frac{S_A + B_A}{2} \tag{22}$$

$$MOM = \frac{S_W + B_W}{2} - \frac{S_L + B_L}{2} \tag{23}$$

Size-valuation (Size-B/M), size-profitability (Size-OP), size-investment (Size-Inv), and size-momentum (Size-Mom) double-sorted portfolios are used to estimate the Fama-French factor models, as previously stated. The portfolios are designed in the style of Fama and French (2015), but with a reduced sample size (4x4).

Table 1. Average number of stocks in value-weighted (4x4) double-sorted portfolios

Panel A: Size-B/M Portfolios

	Low	2	3	High
Small	16	16	16	17
2	16	16	16	18
3	16	16	16	17
Big	16	17	16	18

Panel C: Size-Inv Portfolios

	Low	2	3	High
Small	17	17	17	18
2	17	17	17	18
3	17	17	17	18
Big	17	18	17	19

Panel B: Size-OP Portfolios

	Low	2	3	High
Small	17	17	17	18
2	17	17	17	19
3	17	17	17	18
Big	17	18	17	19

Panel D: Size-Mom Portfolios

	Low	2	3	High
Small	17	17	17	19
2	17	18	17	19
3	17	17	17	19
Big	17	18	18	19

Value-weighted double-sorted portfolios are constructed similar but finer way as the risk factors. Stocks are divided into four size groups (Small to Big) and four B/M groups (Low to High) every June when the German stock universe is rebalanced. Instead of median market capitalization as a breakpoint for size each of these value-weighted portfolios use sample quartiles of size and B/M as breakpoints. OP, Inv, and Mom replace B/M, thus they are constructed similarly. This methodology generates 16 value-weighted size-valuation, 16 value-weighted size-profitability, 16 value-weighted size-investment, and 16 value-weighted size-momentum double-sorted portfolios.

Table 2. Average excess returns, t-statistics, and standard deviations of regression portfolios

Panel A: Size-B/M Portfolios

	Low	2	3	High
Small	0.21	0.22	0.73	-0.05
	(0.35)	(0.49)	(1.65)	(-0.14)
2	0.40	0.58	2.19	0.74
	(0.76)	(1.10)	(1.31)	(1.54)
3	0.41	0.58	0.65	0.30
	(0.72)	(1.15)	(1.47)	(0.62)
Big	0.25	0.03	-0.11	0.64
	(0.59)	(0.08)	(-0.21)	(1.17)

Panel E: Size-B/M Portfolios

	Low	2	3	High
Small	7.53	5.72	5.64	4.94
2	6.64	6.72	21.27	6.10
3	7.30	6.44	5.58	6.30
Big	5.35	5.35	6.61	6.94

Panel B: Size-OP Portfolios

Panel F: Size-OP Portfolios

	Low	2	3	High
Small	0.33	-0.09	0.82	0.14
	(0.50)	(-0.19)	(1.86)	(0.29)
2	0.25	0.31	0.59	1.74
	(0.54)	(0.66)	(1.29)	(1.18)
3	0.11	0.49	0.35	0.52
	(0.25)	(0.97)	(0.66)	(0.96)
Big	-0.08	-0.04	0.00	0.00
	(-0.19)	(-0.08)	(-0.01)	(0.01)

	Low	2	3	High
Small	9.02	6.00	5.62	6.36
2	5.89	6.02	5.79	18.84
3	5.81	6.42	6.81	6.93
Big	5.32	6.41	5.59	5.84

Panel C: Size-Inv Portfolios

	Low	2	3	High
Small	0.79	0.07	0.60	0.01
	(1.47)	(0.17)	(1.25)	(0.01)
2	0.14	0.27	2.19	0.56
	(0.32)	(0.61)	(1.21)	(1.07)
3	0.46	0.27	0.63	0.31
	(0.94)	(0.48)	(1.47)	(0.52)
Big	0.06	-0.02	0.00	0.11
	(0.13)	(-0.05)	(0.01)	(0.24)

Panel G: Size-Inv Portfolios

	Low	2	3	High
Small	6.88	5.06	6.11	6.68
2	5.67	5.65	22.97	6.68
3	6.20	6.99	5.49	7.68
Big	5.51	5.43	6.63	5.92

Panel D: Size-Mom Portfolios

	Low	2	3	High
Small	0.75	3.56	0.57	1.32
	(1.04)	(1.21)	(1.29)	**(2.46)**
2	2.83	-0.15	0.28	1.13
	(1.19)	(-0.35)	(0.73)	**(2.63)**
3	0.98	0.14	-0.06	0.63
	(0.97)	(0.30)	(-0.14)	(1.58)
Big	-0.08	-0.48	0.24	0.21
	(-0.11)	(-1.16)	(0.64)	(0.47)

Panel H: Size-Mom Portfolios

	Low	2	3	High
Small	9.23	37.54	5.64	6.83
2	30.15	5.38	4.89	5.49
3	12.77	5.83	5.11	5.11
Big	9.09	5.32	4.85	5.64

Table 2 shows time-series averages of excess returns, t-statistics, and standard deviations of 64 value-weighted portfolios formed on size, valuation, operating profitability, investment, and momentum double-sorted portfolios between July 2007 and December 2020. Size-Mom portfolios have the highest average excess return of 0.74% per month while Size-OP portfolios have the lowest average excess return of 0.34% per month. The high return of Size-Mom portfolios is mostly due to stocks that have small and mid-size market capitalization.

Table 3. Descriptive statistics and factor correlations

Variable	Mean	Standard deviation	t-Statistics
Rf	0.06	0.14	**4.95**
MRF	0.35	5.71	1.78
SMB	0.52	4.82	1.38
HML	0.09	2.97	0.38
RMW	0.53	6.27	1.08
CMA	-0.09	2.83	-0.41
MOM	0.75	4.09	**2.35**

Factor Correlations

	MRF	**SMB**	**HML**	**RMW**	**CMA**	**MOM**
MRF	1	-0.06	-0.01	0.08	-0.24	-0.50
SMB	-0.06	1	-0.02	0.68	0.05	-0.04
HML	-0.01	-0.02	1	0.09	0.31	-0.09
RMW	0.08	0.68	0.09	1	0.16	-0.07
CMA	-0.24	0.05	0.31	0.16	1	0.17
MOM	-0.50	-0.04	-0.09	-0.07	0.17	1

As said, risk-factors are calculated by 2x3 sort allocation. In other words, all factors are first sorted by using size into two groups then by using separately value, profitability, investment, and momentum variables into three groups. Table 3 introduces the constructed factors and the average returns, t-statistics, standard deviations, and correlations among other risk-factors. Among the six factors, MOM has the highest average return of 0.75% per month (t-stat 2.35), indicating a strong momentum effect in the sample. Although there are not many high correlation coefficients among the risk-factors, RMW and SMB have clearly a positive correlation of 0.68. Moreover, MRF and MOM have a negative correlation of -0.5.

5.3 Factor-spanning tests

In the wake of the previous empirical findings and discussion of a factor redundancy like the HML, factor spanning tests are performed in order to determine any potential redundant factors. This is done by using the OLS method and regressing a particular factor return on a constant c and the other factors that are left on the right-hand side. The methodology is identical to Fama and French (2018) as well as Grobys and Kolari's

(2021) right-hand-side (RHS) approach. If the intercept occurs as a non-zero, the factor increases the mean-variance frontier in that sample period. Indeed, this methodology evaluates the maximum squared Sharpe ratio based on the spanning regressions as follows:

$$R_{it} - R_{Ft} = \alpha_i + s_i SMB_t + h_i HML_t + r_i RMW_t + c_i CMA_t + m_i MOM_t + \epsilon_{it} \quad (24)$$

$$SMB_t = \alpha_i + \beta_i(R_{Mt} - R_{Ft}) + h_i HML_t + r_i RMW_t + c_i CMA_t + m_i MOM_t + \epsilon_{it} \quad (25)$$

$$HML_t = \alpha_i + \beta_i(R_{Mt} - R_{Ft}) + s_i SMB_t + r_i RMW_t + c_i CMA_t + m_i MOM_t + \epsilon_{it} \quad (26)$$

$$RMW_t = \alpha_i + \beta_i(R_{Mt} - R_{Ft}) + s_i SMB_t + h_i HML_t + c_i CMA_t + m_i MOM_t + \epsilon_{it} \quad (27)$$

$$CMA_t = \alpha_i + \beta_i(R_{Mt} - R_{Ft}) + s_i SMB_t + h_i HML_t + r_i RMW_t + m_i MOM_t + \epsilon_{it} \quad (28)$$

$$MOM_t = \alpha_i + \beta_i(R_{Mt} - R_{Ft}) + s_i SMB_t + h_i HML_t + r_i RMW_t + c_i CMA_t + \epsilon_{it} \quad (29)$$

6 Empirical regression results

The results of the empirical analysis are reported in this chapter. The three-, five-, and six-factor intercepts, factor exposures, t-statistics, and adjusted R^2 are introduced for each value-weighted portfolio allocation. These 64 portfolios are formed on 4 sets of 16 smaller portfolios that follow the sorting allocation, which was described in the previous chapter. Size-B/M, Size-OP, Size-Inv, and Size-Mom portfolios are separately constructed and regressed against the risk-factors using seemingly unrelated regression and unstructured data with 162 observations.

Table 4. Regressions for 16 value-weight Size-B/M portfolios

Panel A: The Fama-French three-factor model

	α					t(α)			
	Low	**2**	**3**	**High**		**Low**	**2**	**3**	**High**
Small	-0.14	-0.13	0.47	-0.33	**Small**	-0.29	-0.38	1.42	-1.07
2	0.06	0.23	-0.04	0.31	**2**	0.17	0.64	-0.04	0.98
3	0.07	0.25	0.37	-0.04	**3**	0.17	0.79	1.22	-0.11
Big	0.07	-0.09	-0.31	0.51	**Big**	0.27	-0.36	-1.02	1.59
	β					**t(β)**			
Small	0.78	0.67	0.67	0.52	**Small**	**9.48**	**11.61**	**11.70**	**9.66**
2	0.87	0.84	0.67	0.78	**2**	**15.22**	**13.61**	**4.22**	**14.09**
3	0.94	0.88	0.71	0.84	**3**	**13.89**	**15.82**	**13.23**	**14.82**
Big	0.71	0.73	0.91	0.77	**Big**	**15.83**	**17.45**	**17.45**	**13.93**
	s					**t(s)**			
Small	0.20	0.19	0.07	0.18	**Small**	**2.05**	**2.85**	0.98	**2.77**
2	0.15	0.19	3.54	0.25	**2**	**2.26**	**2.56**	**18.63**	**3.80**
3	0.04	0.05	0.05	0.09	**3**	0.54	0.82	0.83	1.31
Big	-0.08	-0.25	-0.22	-0.42	**Big**	-1.44	**-5.05**	**-3.61**	**-6.36**
	h					**t(h)**			
Small	-0.36	0.11	-0.04	0.01	**Small**	**-2.28**	1.02	-0.41	0.09
2	-0.47	-0.42	1.71	0.29	**2**	**-4.29**	**-3.55**	**5.56**	**2.71**
3	-0.04	-0.05	0.00	0.03	**3**	-0.31	-0.51	0.01	0.29
Big	-0.33	-0.04	-0.02	0.89	**Big**	**-3.83**	-0.52	-0.22	**8.32**
	Adj. R^2								
Small	0.36	0.45	0.45	0.37					
2	0.60	0.55	0.70	0.56					
3	0.54	0.60	0.51	0.57					

Big	0.62	0.67	0.66	0.66

Panel B: The Fama-French five-factor model

α	Low	2	3	High
Small	-0.13	-0.11	0.46	-0.32
2	0.04	0.21	-0.02	0.31
3	0.04	0.25	0.38	-0.05
Big	0.08	-0.08	-0.31	0.50
β				
Small	0.85	0.71	0.70	0.55
2	0.88	0.83	0.41	0.81
3	0.92	0.88	0.73	0.84
Big	0.68	0.71	0.87	0.72
s				
Small	0.60	0.32	0.26	0.33
2	0.34	0.26	1.70	0.44
3	0.05	0.08	0.18	0.21
Big	-0.31	-0.41	-0.50	-0.69
h				
Small	-0.28	0.09	0.01	0.03
2	-0.40	-0.35	1.24	0.32
3	0.03	-0.04	0.02	0.11
Big	-0.41	-0.09	-0.08	0.85
r				
Small	-0.45	-0.15	-0.22	-0.17
2	-0.20	-0.07	2.04	-0.22
3	0.00	-0.03	-0.14	-0.13
Big	0.26	0.18	0.31	0.30
c				
Small	0.07	0.19	-0.02	0.05
2	-0.10	-0.18	0.07	0.04
3	-0.24	-0.03	0.06	-0.17
Big	0.06	0.03	-0.05	-0.10
Adj. R^2				
Small	0.43	0.46	0.47	0.38
2	0.62	0.55	0.88	0.58
3	0.54	0.60	0.52	0.58
Big	0.67	0.69	0.70	0.69

t(α)	Low	2	3	High
Small	-0.30	-0.34	1.45	-1.07
2	0.14	0.60	-0.04	1.02
3	0.11	0.78	1.25	-0.16
Big	0.31	-0.36	-1.10	1.65
t(β)				
Small	**10.37**	**11.87**	**11.86**	**9.85**
2	**15.07**	**12.83**	**3.90**	**14.35**
3	**12.85**	**15.00**	**13.15**	**14.26**
Big	**15.46**	**16.67**	**16.79**	**13.01**
t(s)				
Small	**4.72**	**3.43**	**2.86**	**3.78**
2	**3.64**	**2.55**	**10.41**	**5.01**
3	0.41	0.89	**2.03**	**2.29**
Big	**-4.50**	**-6.21**	**-6.22**	**-7.90**
t(h)				
Small	-1.78	0.77	0.09	0.30
2	**-3.51**	**-2.82**	**6.17**	**2.97**
3	0.21	-0.35	0.14	0.99
Big	**-4.76**	-1.10	-0.78	**7.98**
t(r)				
Small	**-4.50**	**-2.00**	**-3.02**	**-2.52**
2	**-2.76**	-0.93	**15.96**	**-3.15**
3	0.05	-0.43	**-2.06**	-1.82
Big	**4.80**	**3.47**	**4.92**	**4.42**
t(c)				
Small	0.40	1.49	-0.19	0.43
2	-0.82	-1.31	0.30	0.35
3	-1.58	-0.20	0.49	-1.38
Big	0.61	0.32	-0.42	-0.88

Panel C: The Fama-French six-factor model

α **t(α)**

	Low	2	3	High
Small	-0.01	-0.06	0.46	-0.29
2	0.10	0.37	0.03	0.36
3	0.24	0.34	0.48	-0.11
Big	0.14	-0.11	-0.15	0.50
		β		
Small	0.80	0.69	0.70	0.53
2	0.86	0.77	0.39	0.79
3	0.84	0.84	0.69	0.86
Big	0.66	0.72	0.81	0.72
		s		
Small	0.59	0.32	0.26	0.33
2	0.33	0.24	1.69	0.44
3	0.03	0.07	0.17	0.22
Big	-0.32	-0.41	-0.52	-0.69
		h		
Small	-0.30	0.08	0.01	0.03
2	-0.41	-0.38	1.23	0.31
3	-0.01	-0.06	0.00	0.12
Big	-0.42	-0.08	0.81	0.72
		r		
Small	-0.45	-0.15	-0.22	-0.17
2	-0.20	-0.07	2.04	-0.28
3	0.01	-0.03	-0.14	-0.13
Big	0.26	0.18	0.32	0.30
		c		
Small	0.09	0.20	-0.02	0.06
2	-0.09	-0.16	0.07	0.05
3	-0.21	-0.01	0.07	-0.18
Big	0.07	0.02	-0.02	-0.10
		m		
Small	-0.13	-0.06	0.00	-0.04
2	-0.06	-0.17	-0.05	-0.05
3	-0.21	-0.10	-0.11	0.06
Big	-0.07	0.03	-0.17	-0.01
		Adj. R^2		
Small	0.43	0.46	0.47	0.38
2	0.62	0.55	0.88	0.58
3	0.54	0.60	0.52	0.58
Big	0.67	0.69	0.71	0.69

	Low	2	3	High
Small	-0.02	-0.16	1.40	-0.92
2	0.30	1.02	0.05	1.14
3	0.61	1.05	1.56	-0.32
Big	0.58	-0.47	-0.54	1.61
		t(β)		
Small	**8.65**	**10.15**	**10.42**	**8.45**
2	**12.97**	**10.58**	**3.27**	**12.35**
3	**10.50**	**12.71**	**11.00**	**12.86**
Big	**13.14**	**14.91**	**13.97**	**11.41**
		t(s)		
Small	**4.63**	**3.37**	**2.85**	**3.73**
2	**3.58**	**2.42**	**10.35**	**4.95**
3	0.26	0.80	1.93	**2.34**
Big	**-4.59**	**-6.15**	**-6.46**	**-7.88**
		t(h)		
Small	-1.91	0.68	0.09	0.24
2	**-3.56**	**-3.05**	**6.06**	**2.86**
3	-0.05	-0.51	-0.04	1.07
Big	**-4.87**	-1.03	**13.97**	**11.41**
		t(r)		
Small	**-4.49**	-1.99	**-3.02**	**-2.51**
2	**-2.75**	-0.89	**15.97**	**-3.13**
3	0.10	-0.40	**-2.04**	-1.84
Big	**4.84**	**3.46**	**5.04**	**4.42**
		t(c)		
Small	0.50	1.54	-0.19	0.48
2	-0.76	-1.16	0.34	0.41
3	-1.40	-0.10	0.61	-1.43
Big	0.71	0.27	-0.22	-0.87
		t(m)		
Small	-1.04	-0.62	-0.01	-0.46
2	-0.63	-1.69	-0.34	-0.60
3	-1.95	-1.13	-1.33	0.63
Big	-1.06	0.49	**-2.11**	-0.06

Table 4 presents results for the Fama-French three-, five-, and six-factor models on Size-B/M double-sorted portfolios. Panel A shows estimated intercepts, factor exposures, and adjusted R^2 for the three-factor model. Panel B presents corresponding values for the five-factor model. Panel C includes the results for the six-factor model. Stocks are sorted into four size groups (Small to Big) and four valuation groups (Low to High). Instead of median market capitalization as a breakpoint for size each of these value-weighted portfolios use sample quartiles of size and valuation as breakpoints. Bolded t-statistics indicate significance at the 5% level.

The three-, five-, and six-factor model regressions do not find significant α_i, values for portfolios double sorted on size and value at the 5% significance level. The coefficients on the market factor β_i range from 0.39 to 0.94, and do not indicate any decreasing or increasing observations concerning size and value sorted portfolios. The coefficients for s_i mainly decrease with increasing size, showing statistical significance in most of the cases for each asset pricing model. The factor exposures for h_i are mostly positive for all models, an exception is the three-factor model. Strong test statistic results focus mainly on the second lowest size quartile and for the highest value quartiles. The results indicate that the value factor becomes more important with rising book-to-market value. The coefficients for r_i are mostly negative for the five- and six-factor models but turn positive and significant for the highest size quartiles, indicating a stronger profitability effect on stocks that have larger market capitalization. Moreover, the coefficients for c_i are equally divided into positive and negative for both models but do not show any statistical significance at the 5% level. Finally, the coefficients for m_i are mostly negative, showing statistical significance only for the highest quartile, irrespective of size. The average adjusted R^2 is largest for each asset pricing models' highest size quartile, whereas adjusted R^2 values range from 0.36 to 0.70 for the three-factor model, 0.38 to 0.88 for the five-factor model, and 0.43 to 0.88 for the six-factor model.

Table 5. Regressions for 16 value-weight Size-OP portfolios

Panel A: The Fama-French three-factor model

α	Low	2	3	High	t(α)	Low	2	3	High
Small	-0.10	-0.41	0.50	-0.20	**Small**	-0.19	-1.12	1.53	-0.49
2	-0.17	-0.06	0.24	-0.30	**2**	-0.56	-0.19	0.80	-0.37
3	-0.24	0.19	0.01	0.19	**3**	-0.87	0.52	0.04	0.50
Big	-0.26	-0.23	-0.16	-0.14	**Big**	-0.97	-0.80	-0.58	-0.57
β					**t(β)**				
Small	0.92	0.65	0.64	0.66	**Small**	**10.24**	**10.18**	**11.10**	**9.43**
2	0.78	0.72	0.76	0.82	**2**	**15.19**	**12.13**	**14.65**	**5.96**
3	0.81	0.78	0.92	0.88	**3**	**16.96**	**12.32**	**15.22**	**13.31**
Big	0.68	0.83	0.75	0.82	**Big**	**14.69**	**16.67**	**15.90**	**19.14**
s					**t(s)**				
Small	0.22	0.14	0.21	0.23	**Small**	**2.03**	1.90	**3.12**	**2.75**
2	0.29	0.25	0.16	3.17	**2**	**4.80**	**3.50**	**2.67**	**19.30**
3	0.13	0.06	0.03	0.05	**3**	**2.25**	0.83	0.41	0.62
Big	-0.17	-0.30	-0.20	-0.25	**Big**	**-3.05**	**-5.01**	**-3.58**	**-4.87**
h					**t(h)**				
Small	-0.04	0.24	-0.18	-0.09	**Small**	-0.23	1.95	-1.60	-0.66
2	-0.05	-0.05	-0.04	1.12	**2**	-0.55	-0.41	-0.44	**4.21**
3	0.02	-0.04	0.04	-0.01	**3**	0.25	-0.30	0.32	-0.08
Big	0.31	0.57	-0.05	-0.14	**Big**	**3.43**	**5.96**	-0.55	-1.69
Adj. R^2									
Small	0.39	0.39	0.44	0.36					
2	0.60	0.48	0.57	0.71					
3	0.63	0.47	0.58	0.51					
Big	0.59	0.68	0.62	0.71					

Panel B: The Fama-French five-factor model

α	Low	2	3	High	t(α)	Low	2	3	High
Small	-0.12	-0.41	0.51	-0.18	**Small**	-0.26	-1.15	1.60	-0.47
2	-0.16	-0.09	0.23	-0.29	**2**	-0.62	-0.25	0.77	-0.66
3	-0.26	0.18	0.00	0.17	**3**	-0.96	0.50	0.00	0.46
Big	-0.25	-0.23	-0.16	-0.14	**Big**	-0.96	-0.85	-0.63	-0.59
β					**t(β)**				
Small	0.97	0.69	0.68	0.69	**Small**	**11.00**	**10.67**	**11.49**	**9.41**
2	0.84	0.70	0.76	0.57	**2**	**17.33**	**11.26**	**14.00**	**6.93**
3	0.80	0.78	0.91	0.86	**3**	**16.22**	**11.75**	**14.48**	**12.35**
Big	0.68	0.79	0.71	0.80	**Big**	**13.88**	**15.86**	**15.39**	**18.55**
s					**t(s)**				
Small	0.70	0.39	0.40	0.33	**Small**	**5.06**	**3.86**	**4.32**	**2.87**
2	0.63	0.29	0.25	1.49	**2**	**8.35**	**2.96**	**2.98**	**11.60**

3	0.22	0.14	0.11	0.07	**3**	**2.86**	1.33	1.15	0.60
Big	-0.24	-0.53	-0.46	-0.45	**Big**	**-3.10**	**-6.80**	**-6.42**	**-6.74**
		h					**t(h)**		
Small	0.15	0.29	-0.16	-0.10	**Small**	0.88	**2.28**	-1.43	-0.72
2	0.01	0.03	0.01	0.72	**2**	0.14	0.25	0.14	**4.55**
3	0.11	0.01	0.09	0.05	**3**	1.13	0.04	0.78	0.40
Big	0.28	0.52	-0.11	-0.20	**Big**	**2.94**	**5.44**	-1.26	**-2.40**
		r					**t(r)**		
Small	-0.53	-0.28	-0.21	-0.12	**Small**	**-4.88**	**-3.50**	**-2.93**	-1.31
2	-0.38	-0.04	-0.10	1.86	**2**	**-6.42**	-0.51	-1.44	**18.56**
3	-0.10	-0.08	-0.09	-0.01	**3**	-1.62	-1.01	-1.16	-0.15
Big	0.08	0.26	0.29	0.23	**Big**	1.27	**4.30**	**5.19**	**4.31**
		c					**t(c)**		
Small	-0.24	0.05	0.11	0.13	**Small**	-1.32	0.36	0.89	0.86
2	0.06	-0.23	-0.12	-0.04	**2**	0.56	-1.75	-1.08	-0.24
3	-0.21	-0.08	-0.13	-0.21	**3**	**-2.03**	-0.57	-0.96	-1.41
Big	0.04	-0.04	-0.01	0.03	**Big**	0.43	-0.34	-0.11	0.33
		Adj. R^2							
Small	0.47	0.43	0.46	0.36					
2	0.67	0.48	0.57	0.91					
3	0.65	0.47	0.58	0.51					
Big	0.59	0.71	0.67	0.74					

Panel C: The Fama-French six-factor model

		α					**t(α)**		
	Low	**2**	**3**	**High**		**Low**	**2**	**3**	**High**
Small	-0.24	-0.31	0.65	-0.18	**Small**	-0.48	-0.84	**1.96**	-0.44
2	-0.03	0.07	0.22	-0.34	**2**	-0.12	0.21	0.72	-0.74
3	-0.20	0.15	0.11	0.29	**3**	-0.73	0.39	0.30	0.76
Big	-0.28	-0.11	-0.09	-0.03	**Big**	-1.04	-0.38	-0.35	-0.14
		β					**t(β)**		
Small	1.01	0.66	0.63	0.69	**Small**	**10.13**	**8.90**	**9.45**	**8.26**
2	0.79	0.64	0.76	0.58	**2**	**14.50**	**9.16**	**12.35**	**6.29**
3	0.78	0.80	0.87	0.81	**3**	**13.92**	**10.50**	**12.24**	**10.33**
Big	0.69	0.74	0.68	0.76	**Big**	**12.42**	**13.26**	**13.10**	**15.66**
		s					**t(s)**		
Small	0.71	0.39	0.39	0.33	**Small**	**5.13**	**3.78**	**4.21**	**2.86**
2	0.62	0.27	0.26	1.49	**2**	**2.80**	**2.83**	**2.98**	**11.60**
3	0.22	0.14	0.10	0.05	**3**	**2.80**	1.35	1.06	0.50
Big	-0.23	-0.54	-0.47	-0.46	**Big**	**-3.06**	**-6.98**	**-6.50**	**-6.91**
		h					**t(h)**		
Small	0.17	0.27	-0.19	-0.10	**Small**	1.00	**2.12**	-1.64	-0.72

2	-0.01	0.00	0.02	0.72	**2**	-0.13	0.01	0.15	**4.56**
3	0.10	0.01	0.08	0.03	**3**	1.01	0.09	0.62	0.23
Big	0.28	0.50	-0.12	-0.22	**Big**	**2.97**	**5.20**	-1.39	**-2.64**
		r					**t(r)**		
Small	-0.53	-0.27	-0.21	-0.12	**Small**	**-4.91**	**-3.48**	**-2.90**	**-1.31**
2	-0.38	-0.04	-0.10	1.86	**2**	**-6.44**	-0.47	-1.44	**18.56**
3	-0.10	-0.08	-0.09	-0.01	**3**	-1.61	-1.02	-1.14	-0.12
Big	0.08	0.27	0.29	0.23	**Big**	1.26	**4.38**	**5.23**	**4.39**
		c					**t(c)**		
Small	-0.26	0.06	0.13	0.13	**Small**	-1.40	0.46	1.04	0.86
2	0.08	-0.21	-0.12	-0.05	**2**	0.74	-1.59	-1.08	-0.28
3	-0.20	-0.09	-0.11	-0.19	**3**	-1.95	-0.60	-0.85	-1.30
Big	0.04	-0.02	0.00	0.04	**Big**	0.39	-0.17	-0.01	0.49
		m					**t(m)**		
Small	0.12	-0.11	-0.14	0.00	**Small**	0.91	-1.09	-1.55	-0.04
2	-0.14	-0.17	0.01	0.05	**2**	-1.91	-1.78	0.07	0.41
3	-0.06	0.04	-0.11	-0.13	**3**	-0.79	0.35	-1.15	-1.22
Big	0.03	-0.13	-0.07	-0.11	**Big**	0.42	-1.73	-1.00	-1.69
		Adj. R^2							
Small	0.47	0.43	0.47	0.35					
2	0.68	0.49	0.57	0.91					
3	0.65	0.47	0.58	0.51					
Big	0.59	0.71	0.67	0.74					

Table 5 presents results for the Fama-French three-, five-, and six-factor models on Size-OP double-sorted portfolios. Panel A shows estimated intercepts, factor exposures, and adjusted R^2 for the three-factor model. Panel B presents corresponding values for the five-factor model. Panel C includes the results for the six-factor model. Stocks are sorted into four size groups (Small to Big) and four operating profitability groups (Low to High). Instead of median market capitalization as a breakpoint for size each of these value-weighted portfolios use sample quartiles of size and operating performance as breakpoints. Bolded t-statistics indicate significance at the 5% level.

The three- and five-factor model regressions do not find significant α_i, values for portfolios double sorted on size and operating profitability at the 5% significance level. However, the six-factor model finds one problematic portfolio among the smallest size quartile without any sign of the reason on why this α_i occurs as significant. The factor exposures on the market factor β_i range from 0.57 to 1.01, and do not indicate any decreasing or increasing observations regarding size and operating profitability sorted portfolios, thus the results are similar to size and value sorted portfolios. The coefficients for s_i mainly decrease with increasing size, showing statistical significance in most of the cases for each asset pricing model, an exception occurs in each model's second-highest size quartile. The factor exposures for h_i are mostly negative for the three-factor model but turn positive for the five- and six-factor models, showing statistical significance among the biggest stocks. The coefficients for r_i are similar to size and value double-sorted portfolios, exhibiting positive and significant test statistics for the biggest stocks. Furthermore, the coefficients for c_i reveal no universal patterns and occur basically insignificant. Finally, the coefficients for m_i are insignificant. The average adjusted R^2 is largest for each asset pricing models' highest size quartile, whereas adjusted R^2 values range from 0.36 to 0.71 for the three-factor model, 0.36 to 0.91 for the five-factor model, and 0.35 to 0.91 for the six-factor model.

Table 6. Regressions for 16 value-weight Size-Inv portfolios

Panel A: The Fama-French three-factor model

	α					t(α)			
	Low	**2**	**3**	**High**		**Low**	**2**	**3**	**High**
Small	0.44	-0.20	0.24	-0.33	**Small**	1.00	-0.67	0.63	-0.80
2	-0.21	-0.08	-0.23	0.14	**2**	-0.67	-0.27	-0.24	0.41
3	0.18	-0.04	0.31	-0.10	**3**	0.54	-0.10	1.19	-0.25
Big	-0.13	-0.16	-0.22	0.00	**Big**	-0.51	-0.62	-0.71	-0.01
	β					**t(β)**			
Small	0.71	0.56	0.62	0.74	**Small**	**9.26**	**10.60**	**9.22**	**10.41**
2	0.70	0.73	0.80	0.88	**2**	**12.86**	**14.22**	**4.68**	**14.96**
3	0.80	0.81	0.76	1.02	**3**	**14.04**	**11.13**	**16.57**	**14.84**
Big	0.75	0.70	0.89	0.79	**Big**	**16.67**	**15.77**	**16.88**	**16.86**
	s					**t(s)**			
Small	0.21	0.15	0.25	0.16	**Small**	**2.32**	**2.33**	**3.15**	1.84
2	0.21	0.18	3.88	0.26	**2**	**3.28**	**2.86**	**19.01**	**3.68**
3	-0.01	0.05	0.11	0.10	**3**	-0.21	0.63	**2.01**	1.19
Big	-0.16	-0.25	-0.24	-0.29	**Big**	**-3.08**	**-4.76**	**-3.82**	**-5.19**
	h					**t(h)**			
Small	0.02	-0.02	0.13	-0.07	**Small**	0.12	-0.17	1.00	-0.51

2	-0.02	0.05	1.40	-0.20
3	0.07	-0.03	-0.03	0.06
Big	0.17	0.28	0.36	-0.13

2	-0.19	0.52	**4.24**	-1.75
3	0.61	-0.24	-0.37	0.49
Big	**1.96**	**3.28**	**3.59**	-1.40

Adj. R^2

Small	0.34	0.40	0.35	0.39
2	0.51	0.55	0.70	0.58
3	0.54	0.42	0.62	0.57
Big	0.64	0.64	0.66	0.66

Panel B: The Fama-French five-factor model

α

	Low	2	3	High
Small	0.46	-0.19	0.24	-0.35
2	-0.16	-0.08	-0.24	0.10
3	0.19	-0.06	0.30	-0.13
Big	-0.09	-0.15	-0.23	-0.04

t(α)

	Low	2	3	High
Small	1.08	-0.64	0.64	-0.89
2	-0.56	-0.28	-0.40	0.31
3	0.57	-0.14	1.14	-0.34
Big	-0.41	-0.63	-0.79	-0.17

β

	Low	2	3	High
Small	0.77	0.61	0.65	0.76
2	0.79	0.74	0.48	0.85
3	0.83	0.80	0.75	0.99
Big	0.77	0.68	0.84	0.72

t(β)

	Low	2	3	High
Small	**9.88**	**11.22**	**9.63**	**10.52**
2	**14.89**	**13.71**	**4.38**	**14.51**
3	**13.72**	**10.59**	**15.51**	**13.85**
Big	**18.31**	**15.31**	**16.00**	**15.53**

s

	Low	2	3	High
Small	0.45	0.32	0.53	0.42
2	0.38	0.25	1.86	0.41
3	0.05	0.17	0.14	0.15
Big	-0.36	-0.46	-0.51	-0.42

t(s)

	Low	2	3	High
Small	**3.67**	**3.74**	**4.95**	**3.75**
2	**4.56**	**2.97**	**10.85**	**4.43**
3	0.58	1.44	1.79	1.38
Big	**-5.49**	**-6.55**	**-6.27**	**-5.84**

h

	Low	2	3	High
Small	0.00	-0.03	0.21	0.05
2	-0.13	0.08	0.95	-0.03
3	0.06	0.05	0.02	0.18
Big	0.00	0.21	0.33	-0.03

t(h)

	Low	2	3	High
Small	0.01	-0.26	1.62	0.38
2	-1.30	0.72	**4.53**	-0.30
3	0.48	0.34	0.20	1.27
Big	0.04	**2.43**	**3.31**	-0.35

r

	Low	2	3	High
Small	-0.28	-0.20	-0.31	-0.29
2	-0.20	-0.09	2.24	-0.15
3	-0.08	-0.12	-0.02	-0.05
Big	0.21	0.23	0.31	0.16

t(r)

	Low	2	3	High
Small	**-2.87**	**-2.95**	**-3.68**	**-3.30**
2	**-3.11**	-1.29	**16.69**	**-2.13**
3	-1.07	-1.34	-0.40	-0.62
Big	**4.01**	**4.14**	**4.81**	**2.87**

c

	Low	2	3	High
Small	0.26	0.18	-0.05	-0.19
2	0.53	-0.02	-0.17	-0.44
3	0.10	-0.19	-0.16	-0.33
Big	0.41	0.08	-0.13	-0.44

t(c)

	Low	2	3	High
Small	1.58	1.57	-0.37	-1.28
2	**4.76**	-0.13	-0.75	**-3.56**
3	0.78	-1.17	-1.53	**-2.21**
Big	**4.59**	0.80	-1.14	**-4.53**

Adj. R^2

Small	0.37	0.43	0.40	0.44
2	0.57	0.55	0.89	0.63
3	0.54	0.43	0.62	0.58
Big	0.72	0.67	0.70	0.70

Panel C: The Fama-French six-factor model

	α					**t(α)**			
	Low	**2**	**3**	**High**		**Low**	**2**	**3**	**High**
Small	0.39	-0.16	0.14	-0.28	**Small**	0.90	-0.51	0.37	-0.69
2	-0.09	-0.04	-0.12	0.12	**2**	-0.29	-0.15	-0.20	0.37
3	0.04	0.09	0.24	0.13	**3**	0.13	0.20	0.89	0.34
Big	-0.09	-0.08	-0.06	-0.06	**Big**	-0.36	-0.33	-0.22	-0.22
	β					**t(β)**			
Small	0.80	0.60	0.69	0.73	**Small**	**8.96**	**9.68**	**8.97**	**8.95**
2	0.76	0.73	0.44	0.84	**2**	**12.66**	**11.84**	**3.51**	**12.63**
3	0.88	0.75	0.77	0.89	**3**	**12.96**	**8.74**	**14.07**	**11.19**
Big	0.77	0.66	0.78	0.72	**Big**	**16.04**	**13.00**	**13.24**	**13.77**
	s					**t(s)**			
Small	0.46	0.32	0.54	0.42	**Small**	**3.71**	**3.70**	**5.03**	**3.69**
2	0.37	0.25	1.85	0.40	**2**	**4.48**	**2.92**	**10.78**	**4.39**
3	0.07	0.16	0.14	0.13	**3**	0.72	1.33	1.85	1.19
Big	-0.36	-0.46	-0.53	-0.42	**Big**	**-5.48**	**-6.64**	**-6.51**	**-5.81**
	h					**t(h)**			
Small	0.01	-0.03	0.23	0.04	**Small**	0.09	-0.31	1.75	0.29
2	-0.15	0.07	0.93	-0.04	**2**	-1.43	0.65	**4.39**	-0.33
3	0.08	0.02	0.03	0.13	**3**	0.71	0.16	0.32	0.93
Big	0.00	0.20	0.30	-0.03	**Big**	0.02	**2.27**	**3.03**	-0.31
	r					**t(r)**			
Small	-0.28	-0.20	-0.31	-0.29	**Small**	**-2.88**	**-2.94**	**-3.72**	**-3.29**
2	-0.20	-0.08	2.24	-0.15	**2**	**-3.10**	-1.28	**16.73**	**-2.13**
3	-0.08	-0.12	-0.03	-0.05	**3**	-1.13	-1.31	-0.43	-0.56
Big	0.21	0.23	0.31	0.16	**Big**	**4.01**	**4.18**	**4.94**	**2.86**
	c					**t(c)**			
Small	0.25	0.19	-0.07	-0.19	**Small**	1.52	1.60	-0.46	-1.22
2	0.54	-0.01	-0.16	-0.44	**2**	**4.85**	-0.09	-0.68	**-3.52**
3	0.08	-0.17	-0.16	-0.30	**3**	0.63	-1.04	-1.61	**-1.99**
Big	0.41	0.09	-0.10	-0.45	**Big**	**4.58**	0.91	-0.94	**-4.53**
	m					**t(m)**			
Small	0.07	-0.03	0.10	-0.07	**Small**	0.56	-0.40	0.99	-0.66
2	-0.08	-0.04	-0.12	-0.03	**2**	-0.99	-0.48	-0.72	-0.28
3	0.15	-0.15	0.06	-0.28	**3**	1.67	-1.32	0.85	**-2.61**
Big	-0.01	-0.08	-0.17	0.02	**Big**	-0.15	-1.10	**-2.17**	0.22

	Adj. R^2			
Small	0.37	0.43	0.40	0.43
2	0.57	0.55	0.89	0.62
3	0.55	0.43	0.62	0.59
Big	0.71	0.67	0.70	0.70

Table 6 presents results for the Fama-French three-, five-, and six-factor models on Size-Inv double-sorted portfolios. Panel A shows estimated intercepts, factor exposures, and adjusted R^2 for the three-factor model. Panel B presents corresponding values for the five-factor model. Panel C includes the results for the six-factor model. Stocks are sorted into four size groups (Small to Big) and four investment groups (Low to High). Instead of median market capitalization as a breakpoint for size each of these value-weighted portfolios use sample quartiles of size and investment character as break-points. Bolded t-statistics indicate significance at the 5% level.

The three-, five-, and six-factor model regressions do not find significant α_i, values for portfolios double sorted on size and investment at the 5% significance level. Market factor β_i exposures range from 0.44 to 1.02 without any observable decreasing or increasing patterns, therefore size and investment sorted portfolios join the group of value and size, as well as value and operating profitability, sorted portfolios. The coefficients for s_i decrease with increasing size with significant t-statistics among each model. Exceptions are similar to the size and operating profitability sorted portfolios. The coefficients for h_i are mostly significant among the biggest stocks for the three-, five-, and six-factor models. The coefficients for r_i tend to be negative and statistically significant for the smaller stocks but turn positive and stay significant for the highest size quartile. Additionally, the coefficients for c_i exhibit positive and mostly significant for the lowest investment quartile but turn negative and significant for the highest investment quartile. Finally, the factor exposures for m_i are like for size and value-double sorted portfolios without any universal patterns. The average adjusted R^2 is largest for each asset pricing models' highest size quartile, whereas adjusted R^2 values range from 0.34 to 0.70 for the three-factor model and 0.37 to 0.89 for both the five-factor model and the six-factor model.

Table 7. Regressions for 16 value-weight Size-Mom portfolios

Panel A: The Fama-French three-factor model

	α					t(α)			
	Low	**2**	**3**	**High**		**Low**	**2**	**3**	**High**
Small	0.29	-0.13	0.22	1.01	**Small**	0.51	-0.07	0.71	**2.33**
2	2.27	-0.48	0.01	0.84	**2**	0.97	-1.76	0.05	**2.67**
3	0.53	-0.18	-0.31	0.33	**3**	0.58	-0.62	-1.26	1.27
Big	-0.43	-0.67	0.01	0.09	**Big**	-0.71	**-2.18**	0.02	0.23
	β					t(β)			
Small	1.01	0.57	0.69	0.70	**Small**	**10.22**	1.93	**12.63**	**9.31**
2	0.99	0.72	0.61	0.66	**2**	**2.44**	**15.17**	**13.13**	**11.94**
3	0.90	0.80	0.70	0.68	**3**	**5.58**	**16.21**	**16.19**	**15.10**
Big	0.85	0.63	0.58	0.43	**Big**	**8.07**	**11.65**	**11.83**	**6.28**
	s					t(s)			
Small	0.25	6.22	0.18	0.17	**Small**	**2.18**	**17.76**	**2.83**	1.89
2	0.51	0.17	0.12	0.11	**2**	1.07	**3.01**	**2.12**	1.65
3	0.26	0.08	0.02	0.12	**3**	1.37	1.34	0.43	**2.26**
Big	0.09	-0.04	0.07	-0.10	**Big**	0.70	-0.57	1.25	-1.21
	h					t(h)			
Small	-0.20	2.69	0.16	-0.20	**Small**	-1.05	**4.75**	1.56	-1.39
2	-0.55	-0.09	-0.06	0.05	**2**	-0.71	-1.02	-0.68	0.46
3	-0.02	-0.06	0.01	0.05	**3**	-0.05	-0.66	0.09	0.53
Big	0.16	-0.10	-0.03	0.19	**Big**	0.79	-1.00	-0.27	1.42
	Adj. R^2								
Small	0.39	0.67	0.50	0.35					
2	0.03	0.59	0.51	0.46					
3	0.15	0.61	0.61	0.58					
Big	0.27	0.45	0.45	0.20					

Panel B: The Fama-French five-factor model

	α					t(α)			
	Low	**2**	**3**	**High**		**Low**	**2**	**3**	**High**
Small	0.25	-0.09	0.21	0.99	**Small**	0.46	-0.08	0.70	**2.33**
2	2.28	-0.47	0.00	0.84	**2**	0.98	-1.75	0.02	**2.67**
3	0.52	-0.20	-0.32	0.33	**3**	0.56	-0.70	-1.26	1.29
Big	-0.44	-0.67	0.01	0.09	**Big**	-0.74	**-2.21**	0.04	0.24
	β					t(β)			
Small	0.98	0.09	0.70	0.71	**Small**	**9.57**	**8.47**	**12.51**	**9.09**
2	1.09	0.74	0.61	0.66	**2**	**2.55**	**14.86**	**12.53**	**11.40**
3	0.89	0.78	0.70	0.68	**3**	**5.24**	**15.05**	**15.31**	**14.33**

Big	0.89	0.65	0.59	0.45	**Big**	**8.18**	**11.49**	**11.39**	**6.19**
		s					**t(s)**		
Small	0.39	2.94	0.35	0.34	**Small**	**2.44**	**9.32**	**3.92**	**2.76**
2	1.00	0.25	0.19	0.15	**2**	1.49	**3.21**	**2.45**	1.65
3	0.31	0.09	0.03	0.08	**3**	1.17	1.09	0.37	1.12
Big	0.40	0.08	0.08	-0.02	**Big**	**2.32**	0.96	0.96	-0.15
		h					**t(h)**		
Small	-0.05	1.83	0.23	-0.13	**Small**	-0.26	**4.74**	**2.09**	-0.83
2	-0.49	-0.09	-0.02	0.06	**2**	-0.60	-0.95	-0.20	0.55
3	0.03	0.00	0.01	0.02	**3**	0.10	0.02	0.13	0.26
Big	0.25	-0.08	-0.04	0.20	**Big**	1.19	-0.70	-0.44	1.43
		r					**t(r)**		
Small	-0.14	3.65	-0.18	-0.19	**Small**	-1.13	**14.81**	-2.58	-1.93
2	-0.54	-0.09	-0.08	-0.05	**2**	-1.04	-1.50	-1.26	-0.65
3	-0.05	0.00	0.00	0.04	**3**	-0.25	-0.07	-0.09	0.69
Big	-0.34	-0.13	-0.01	-0.09	**Big**	**-2.57**	-1.94	-0.12	-1.04
		c					**t(c)**		
Small	-0.39	0.18	-0.08	-0.12	**Small**	-1.82	0.42	-0.68	-0.70
2	0.20	0.06	-0.09	-0.01	**2**	0.22	0.59	-0.83	-0.07
3	-0.12	-0.22	-0.01	0.05	**3**	-0.35	**-1.98**	-0.11	0.46
Big	-0.05	0.01	0.07	0.03	**Big**	-0.20	0.05	0.62	0.19
		Adj. R^2							
Small	0.40	0.86	0.51	0.36					
2	0.02	0.59	0.51	0.45					
3	0.14	0.62	0.61	0.58					
Big	0.30	0.46	0.45	0.19					

Panel C: The Fama-French six-factor model

		α					**t(α)**		
	Low	**2**	**3**	**High**		**Low**	**2**	**3**	**High**
Small	0.83	-0.01	0.26	0.89	**Small**	1.51	-0.01	0.83	**2.02**
2	3.21	-0.43	-0.08	0.70	**2**	1.34	-1.53	-0.29	**2.16**
3	0.89	0.00	-0.33	0.06	**3**	0.94	0.02	-1.29	0.25
Big	-0.14	-0.33	-0.02	-0.37	**Big**	-0.23	-1.11	-0.08	-0.96
		β					**t(β)**		
Small	0.76	0.06	0.69	0.75	**Small**	**6.90**	**8.28**	**10.74**	**8.46**
2	0.74	0.72	0.64	0.72	**2**	1.54	**12.78**	**11.62**	**10.95**
3	0.75	0.70	0.71	0.78	**3**	**3.92**	**12.24**	**13.60**	**15.19**
Big	0.78	0.52	0.60	0.62	**Big**	**6.37**	**8.58**	**10.25**	**8.04**
		s					**t(s)**		
Small	0.34	2.93	0.34	0.35	**Small**	**2.22**	**9.26**	**3.86**	**2.83**
2	0.92	0.25	0.19	0.16	**2**	1.37	**3.15**	**2.54**	1.80

3	0.28	0.07	0.03	0.11	**3**	1.05	0.88	0.40	1.51
Big	0.37	0.05	0.08	0.02	**Big**	**2.19**	0.64	1.00	0.23
		h					**t(h)**		
Small	-0.16	1.82	0.22	-0.11	**Small**	-0.83	**4.66**	**1.99**	-0.70
2	-0.66	-0.10	0.00	0.09	**2**	-0.80	-1.03	-0.04	0.79
3	-0.04	-0.04	0.01	0.07	**3**	-0.11	-0.36	0.17	0.83
Big	0.20	-0.14	-0.04	0.29	**Big**	0.93	-1.36	-0.37	**2.16**
		r					**t(r)**		
Small	-0.13	3.65	-0.18	-0.19	**Small**	-1.07	**14.81**	**-2.57**	**-1.96**
2	-0.52	-0.09	-0.08	-0.05	**2**	-1.00	-1.49	-1.29	-0.70
3	-0.04	0.00	-0.01	0.07	**3**	-0.21	0.01	-0.10	0.83
Big	0.20	-0.14	-0.04	0.29	**Big**	0.93	-1.36	-0.37	**2.16**
		c					**t(c)**		
Small	-0.31	0.19	-0.07	-0.13	**Small**	-1.52	0.45	-0.62	-0.78
2	0.33	0.07	-0.10	-0.03	**2**	0.37	0.64	-0.94	-0.24
3	-0.07	-0.19	-0.01	0.01	**3**	-0.20	-1.75	-0.13	0.09
Big	0.00	0.05	0.06	-0.04	**Big**	-0.02	0.48	0.57	-0.24
		m					**t(m)**		
Small	-0.61	-0.09	-0.05	0.11	**Small**	**-4.07**	-0.28	-0.57	0.95
2	-0.99	-0.05	0.09	0.16	**2**	-1.51	-0.64	1.17	1.74
3	-0.39	-0.21	0.02	0.29	**3**	-1.51	**-2.74**	0.27	**4.09**
Big	-0.32	-0.37	0.04	0.50	**Big**	-1.90	**-4.49**	0.47	**4.70**
		Adj. R^2							
Small	0.45	0.86	0.51	0.36					
2	0.03	0.58	0.51	0.46					
3	0.15	0.63	0.60	0.61					
Big	0.31	0.51	0.45	0.29					

Table 7 presents results for the Fama-French three-, five-, and six-factor models on Size-Mom double-sorted portfolios. Panel A shows estimated intercepts, factor exposures, and adjusted R^2 for the three-factor model. Panel B presents corresponding values for the five-factor model. Panel C includes the results for the six-factor model. Stocks are sorted into four size groups (Small to Big) and four momentum groups (Low to High). Instead of median market capitalization as a breakpoint for size, these value-weighted portfolios use sample quartiles of size and prior performance as breakpoints. Bolded t-statistics indicate significance at the 5% level.

Regarding size and momentum double-sorted portfolios, the three, five-, and six-factor model regressions find the largest number of significant pricing errors with 3, 3, and 2 significant estimates at a 5% significance level for the corresponding models. Therefore, the six-factor regressions show only slight improvement. The coefficients for the market factor β_i, which range from 0.43 to 1.09, do not indicate any universal patterns concerning size and momentum sorted portfolios. Thus, the phenomenon is identical to size and value, operating profitability, and investment double-sorted portfolios. The coefficients for s_i decrease with increasing size and turn basically insignificant. The factor exposures for h_i show statistical significance mainly only in the lowest size quartile for each asset pricing model. The coefficients for r_i are mostly negative for the five- and six-factor models but give no clear decreasing or increasing patterns. Additionally, the coefficients for c_i are mostly negative and insignificant for both models. Finally, the coefficients for m_i are positive and significant solely for the highest momentum quartile. The average adjusted R^2 is largest for each asset pricing models' smallest size quartile, an opposite to other portfolio sorts. Adjusted R^2 values range from 0.03 to 0.67 for the three-factor model, 0.02 to 0.86 for the five-factor model, and 0.03 to 0.86 for the six-factor model.

Table 8. Test statistics of the Fama-French factor models

		χ^2	$A\|\alpha\|$	A Adj. R^2
Size-B/M	3-factor	20.892	0.08	0.55
	5-factor	21.314	0.08	0.58
	6-factor	19.906	0.14	0.59
Size-OP	3-factor	20.001	0.21	0.55
	5-factor	19.995	0.07	0.58
	6-factor	17.615	0.02	0.59
Size-Inv	3-factor	9.733	0.02	0.54
	5-factor	10.166	0.09	0.58
	6-factor	5.813	0.03	0.58
Size-Mom	3-factor	**39.071**	0.21	0.43
	5-factor	**38.863**	0.21	0.44
	6-factor	**34.871**	0.21	0.46
Degrees of freedom		16		
Significance level		5 %		
Critical value		26.296		

Table 8 shows the Wald test (the GRS test) statistics, the average intercepts ($A|\alpha|$), and the average adjusted R^2. Every test contains 16 portfolios which is the corresponding number for degrees of freedom, and at the 5% significance level critical value stands for 26.296. Bolded values indicate a smaller p-value than 5%. Chi-square values are above critical value and statistically significant only for Size-Mom double-sorted portfolios, indicating that the three-, five-, and six-factor models must be rejected in testing portfolios sorted by Size-Mom. Portfolios double sorted on size and value, investment and profitability have Chi-square values under the critical value but considering the p-values, these results are insignificant. In contrast, Fama and French (2018) as well as Grobys and Kolari (2021) present that the Fama-French six-factor model is outstanding at capturing the average stock returns compared to the other nested Fama-French models. Anyway, Fama and French (2015; 2017) as well as Huynh (2018) state that purely relying on the GRS statistics (the Wald test) is not a proper way to demonstrate the ultimate performance of a certain model.

Table 9. Factor-spanning tests, estimated coefficients, intercepts, t-statistics, and the Adj. R^2

	α	***MRF***	***SMB***	***HML***	***RMW***	***CMA***	***MOM***	**Adj. R^2**
MRF	0.83		-0.27	-0.03	0.21	-0.37	-0.65	0.29
	(2.14)		**(-2.51)**	(-0.25)	**(2.53)**	**(-2.53)**	**(-6.76)**	
SMB	0.34	-0.14		-0.12	0.54	-0.11	-0.08	0.48
	(1.21)	**(-2.51)**		(-1.21)	**(12.14)**	(-1.02)	(-1.02)	
HML	0.22	-0.01	-0.08		0.05	0.34	-0.11	0.10
	(0.95)	(-0.25)	(-1.21)		(1.08)	**(4.06)**	(-1.74)	
RMW	-0.01	0.18	0.89	0.14		0.30	0.03	0.49
	(-0.03)	**(2.53)**	**(12.14)**	(1.08)		**(2.23)**	(0.34)	
CMA	-0.15	-0.11	-0.06	0.28	0.10		0.07	0.17
	(-0.73)	**(-2.53)**	(-1.02)	**(4.06)**	**(2.23)**		(1.20)	
MOM	0.93	-0.35	-0.08	-0.17	0.02	0.13		0.25
	(3.33)	**(-6.76)**	(-1.02)	(-1.74)	(0.349)	(1.20)		

Table 9 results indicate significant intercepts for the MRF and MOM at the critical 5% significance level. The average market excess return is captured by SMB, RMW, CMA, and MOM factors as the corresponding slopes are significant at the 5% significance level. Similarly, the average SMB return is explained by the MRF and RMW. The average HML return is captured by CMA and the regression adjusted R^2 is the lowest, approximately 10%. As anticipated from a positive correlation, both SMB and RMW slopes in SMB and RMW regressions are positive and statistically significant (t-stat 12.14). Moreover, SMB and RMW regressions have also the highest adjusted R^2. Additionally, the average RMW return is captured by SMB. Correspondingly, the average CMA return is captured by the MRF, HML, and CMA. Finally, the average MOM return explained by the MRF and the adjusted R^2 settles on the mid-range at 25%. To sum up, spanning regressions show that only the MRF and MOM increase the mean-variance frontier and other factors are redundant. However, Fama and French (2017) find significant intercepts for MRF, HML, and RMW whereas, SMB and CMA are redundant in Europe. Moreover, Grobys and Kolari (2021) confirm that SMB and CMA do not matter in Europe, whereas MRF and HML matter. The authors also demonstrate that RMW plays a significant role in explaining the cross-section of average stock returns. Anyway, the spanning regressions for the German setting do not expose any other factors than MRF and MOM that would play a large role in expanding the mean-variance frontier.

Thereby, factor spanning inferences must be sensitive to sample-specific data, as concluded by Grobys and Kolari (2021).

7 Conclusions and discussion

Factor models are a very popular method to evaluate portfolio performance even though they are less-than-perfect models. Thereby, this study implemented and measured the performance of the Fama-French three-, five-, and six-factor models for the German stock universe. Each of the nested asset pricing models was examined in explaining the cross-section of average stock returns by using portfolios double sorted on size and value, investment, profitability, or momentum.

Regarding the results of the Fama-French asset pricing models for the time-period of July 2007 – December 2020 in the German equity market, the variation between models seems to be just marginal. The average adjusted R^2 for the asset pricing models are 51%, 53%, and 54%, respectively. Considering the Wald test, none of these nested models can explain the cross-sectional return variation at the 5% significance level. Additionally, spanning regressions reveal that only the market factor and momentum increase the mean-variance frontier. This confirms Grobys and Kolari's (2021) finding that size does not matter in Europe's largest single economy. However, the European evidence by Grobys and Kolari (2021) is only loosely supported by implementing the choosing factors approach on the German setting alone. As such, value, operating profitability, and investment factors do not matter in Germany, whereas Grobys and Kolari (2021) found that value matters in Europe and operating profitability plays an important role in pricing the cross-section of equity returns. Additionally, this study confirms that the investment factor is redundant in Germany, respective to Fama and French (2017) as well as Grobys and Kolari (2021).

This study could motivate future research. One could want to investigate other potential risk-factors beyond Fama and French that are found and proposed by recent financial literature. For instance, Asness, Frazzini, Gormsen, and Pedersen (2020) constructed BAC and $SMAX$ factors in 2020 which are not included to Fama and French's (2018) pool of anomalies, thus we do not yet have empirical evidence about a potential expansion of the mean-variance frontier caused by these risk-factors which may be substitutes for the Fama-French risk-factors. Future research could implement Grobys and Kolari's (2021) block-based boost-strapping methodology that considers factor dependencies and evaluate whether non-nested models outperform nested models in the German stock universe. However, this study does not address this problem because it would consider non-nested models and therefore left for future research.

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