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# Refund policies and core classification errors in the presence of customers' choice behaviour in remanufacturing

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## ABSTRACT

In light of a circular economy, to encourage core returns, the remanufacturer charges a deposit and refund it to the customer based on quality inspection of cores. Generally, two types of classification errors exist and interact with each other during the inspection process: either low-quality cores are sorted as remanufacturable, or high-quality cores are sorted as non-remanufacturable. The remanufacturer needs to choose refund policies and determine a reasonable deposit value, considering customers' potential responses. This paper firstly develops analytical solutions for these issues within a game theory framework. The effect of inspection information transparency is evaluated by comparing two settings: the information of inspection errors is available to customers or not. The study results show the advantage of inspection information transparency from the remanufacturer's perspective. The analysis indicates the importance of avoiding overestimating customers' payoff of products and the significance of inspection accuracy. The study also highlights that the salvage value of different cores significantly influences the remanufacturer's profits, and the improvement of inspection accuracy does not necessarily reduce the customer's return of low-quality cores.

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Remanufacturing; refund policies; core quality; inspection errors; game theory



## 1. Introduction

In the circular economy, remanufacturing has been proposed as an important business strategy to manage and improve resource loops (Jensen et al. 2019). Compared with traditional manufacturing, the process of remanufacturing is more complicated due to the quantity and quality uncertainties of the used products to be returned (Sitcharangsie, Ijomah, and Wong 2019). These returned used products are often defined as cores, which are essential material resources for remanufacturing. Efficient and successful management of the core return process is a prerequisite for sustaining the remanufacturing activities.

Research efforts have been made to improve the process of core returns and remanufacturing (Yang, Wang, and Ji 2015; Mitra 2016; Yang et al. 2016; Kurilova-Palisaitiene, Sundin, and Poksinska 2018; Arredondo-Soto et al. 2019). In the collection and recycling of used electronic products, Esenduran, Kemahlioğlu-Ziya, and Swaminathan (2017) studied the competition between an original equipment manufacturer (OEM) and an independent manufacturer (IR), considering remanufacturing levels, consumer surplus, and the OEM profit. Despite the progress, significant challenges remain in

core acquisition and returns. Mismatching core supply and demand is one serious barrier that damages the profitability of both IRs and OEMs (Liao et al. 2020). To encourage the core return quantity and quality in practice, OEMs apply different take-back policies. A deposit-refund system is claimed to be cost-efficient and operationally effective compared to other take-back incentives in e-waste management, such as disposal fees and recycling subsidies (Esenduran, Kemahlioğlu-Ziya, and Swaminathan 2012). This policy is then the focus of the current study.

A deposit-refund system is widely used in collecting beverage bottles, and such a system is mandatory in the beverage industry in many EU countries (CM Consulting 2016). It can be viewed as an additional tax on the selling product and a subsidy on the return one. The same principle is also applied in recollecting auxiliary equipment in logistics, such as pallets and maritime containers. According to Pappis, Rachaniotis, and Tsoulfas (2005), nearly 25% of containers in a Greek container line company can be classified in the category of deposit-refund. The customers, namely container fleets, can return those containers, which will be inspected, maintained, and

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reused for five years, or trade the containers in an open market.

Caterpillar is a leading remanufacturer in the construction equipment business, with annual reprocessing of parts and components equivalent to 150 million pounds of iron. To keep material resources in circulation for multiple lifetimes, and in the meantime, to guarantee the remanufactured products, Caterpillar adapts an exchange business, in which customers pay the price plus a deposit and receive a refund in returning the core (Caterpillar 2020). Another example is in the tire retreading business, where worn-out tires (often the ones for heavy-duty vehicles) are reprocessed by laying a new layer of rubber and resell to the market. Between the remanufacturer and tire dealers (who could be viewed as the customers), a deposit-refund mechanism is used to close the loop and enhance the material collection (Debo and Van Wassenhove 2005). In all the above examples, the deposit-refund policy plays a vital role in improving core return opportunities by binding the remanufacturer and customer relationship.

Cores are often sorted based on the quality before remanufacturing. A deposit-refund policy is also used to improve the management of core quality. In practice, many remanufacturers provide refunds according to the inspection quality. The quality will affect the recoverable rate and manufacturing opportunity of cores, and therefore the remanufacturing costs. As in previous Caterpillar and tire retreading cases, the core quality has a considerable variation after products have been used. For instance, using tires as an example, typically returned cores from a fleet have better quality and often the quality is also more consistent than individual users (Debo and Van Wassenhove 2005).

Even though a manufacturer could classify cores into different quality categories, both overestimation (qualifying a low-quality core) and underestimation (disqualifying a high-quality core) errors are common in remanufacturing, such as automotive parts and electronic returns. Sometimes a crack or material fault in an engine can only be revealed after the core is fully disassembled and reprocessed (Tang, Grubbström, and Zanoni 2007). Thus, inspection quality could be somewhat different from the actual quality of the core. Such errors (over- and underestimation) can be adjusted using different inspection technologies (such as visual inspection, testing, or even partly disassembly) and inspection standards. Whether cores will be centrally or decentrally inspected, the sorting strategy should also impact inspection errors (Sakao and Sundin 2019). Nevertheless, it is still unclear how the deposit-refund policy, together with inspection errors, should affect the core collecting system and remanufacturer performance.

On the other hand, the customers' responses to the refund policies could be complicated. The customers could return cores to OEMs to receive the refund or sell cores to other collectors for more attractive prices. In the previous cases of maritime containers and used tires retreading, there are returns reported to the third-party remanufacturers (Debo and Van Wassenhove 2005; Pappis, Rachaniotis, and Tsoulias 2005). However, it remains unclear whether the result is initiated only by the price advantage or caused by fear of receiving less or no refund due to the distrust of the inspection process. The customer responses are also affected by various factors such as the competition of cores in the market, the customers' information about core quality. It is critical to understand the customers' possible responses, define appropriate deposit-refund policies, and design the inspection process for the successful core collecting process and correct remanufacturing operations. The above background motivates the current study.

Therefore, this study aims to develop a model to describe the customers' payoff and possible responses concerning both the deposit-refund policies and inspection errors. The decision processes of the remanufacturer and the customer are described using a game theory framework to decide the good deposit-refund policies that suit the specific remanufacturing environments. More specifically, influences of the quality classification errors and benefits/shortcomings of quality classification accuracy improvements are investigated. Also, quality classification errors are integrated into the refund policies to maximise the remanufacturer's profits.

To describe the inspection errors, we consider two types of classification errors  $\alpha$  (non-rejection of a false) and  $\beta$  (rejection of a true) and consider their dependence. The inter-dependency of the two errors is important, as we cannot reduce both errors in principle. This is also applied in core inspection processes and eventually is the essential features of its operation. We need to note that in previous work studying core quality, Van Wassenhove and Zikopoulos (2010) considered only quality overestimation error (error  $\alpha$ ), while others (Tagaras and Zikopoulos 2008; Zikopoulos and Tagaras 2008) considered both but assumed errors to be independent. In the studies mentioned above, the deposit-refund policy is not included either. In short, with a deposit-refund policy and dependent core inspection errors, this study investigates the customer's response and remanufacturer's decision making and also considers the impact of asymmetric information. The study results should provide a better understanding and guideline to improve the core acquisition process, and subsequently, the performance of a remanufacturing process.

The remainder of this paper is organised as follows. Brief literature is presented in Section 2. The models are established in Section 3, where notations and detailed assumptions are also introduced. In Section 4, solutions of customer's pure strategies are studied, assuming that the customer has the information of inspection errors. When a customer does not have access to such information, mixed strategies could be used by the customer, and the analysis is carried out in Section 5. In Section 6, we use numerical examples to illustrate the possible solutions developed in the previous sections, and demonstrate the value of announcing the information of inspection errors. We also show the advantage of underestimating the customer's payoff of the product and the benefit of improving the inspection accuracy. Section 7 concludes the study.

## 2. Literature review

In recent years, due to stricter regulations and widespread social concerns on environmental protection, remanufacturing has received increasing attention from both academia and industry, which can drive the circular economy and retain value from used products (Sakao and Sundin 2019; Shi 2019; Silva Teixeira et al. 2019; Kleber et al. 2020; Pazoki and Samarghandi 2020). Significant research efforts have been conducted on core acquisition management to deal with the uncertainties of core quality, quantities, and returning time. Contributions of these works are highlighted below.

To manage core acquisition, the influences on the core acquisition prices and quantities have been analysed using stochastic dynamic programming (Cai et al. 2014). A combined optimisation model has been used to obtain optimal acquisition prices and selling prices of remanufactured products (Bulmuş, Zhu, and Teunter 2014). Managing return time is also a concern. For example, in automotive parts remanufacturing, a series of hazard rate models have been developed for modelling core return delay durations (Kumar, Chinnam, and Murat 2017). For more complicated cases, core acquisitions in the planned, reactive, or sequential acquisition manners have been investigated (Mutha, Bansal, and Guide 2016). Additionally, a literature review of the core acquisition management has been presented, which mainly focuses on acquisition control (Wei, Tang, and Sundin 2015).

As inspection error is the primary concern in this paper, we now focus on the core acquisition quality issue. In relevant core acquisition management research, most research assumes that the condition of the returned end of life (EOL) cores is constant. In contrast, others assume it as a discrete series (Liao, Deng, and Shen 2019) as the quality classification of cores is already given and

accurate so that the remaining decisions are to determine the acquisition volume and production schedule. For example, Galbreth and Blackburn (2009) dealt with the optimal acquisition volume when the quality sorting process is free of errors. A larger acquisition volume provides a remanufacturer more flexibility to only remanufacture the high-quality cores to meet the demand, and in the meantime decreases the production cost. Ferguson et al. (2009) studied a production planning problem where the returns have different quality levels. They observed that grading the cores and keeping them in separate inventories for each quality grade can increase the remanufacturer's profits. In another study, refund policies for multiple quality classes and principles for quality partition have been developed with the assumption that quality can be precisely defined (Wei, Tang, and Liu 2015). Similarly, in Galbreth and Blackburn (2009), the grading process was assumed to be accurate without errors. Such an assumption is widely used in dealing with the quality uncertainty in remanufacturing systems, with additional examples in Heydari and Ghasemi (2018), Denizel, Ferguson, and Souza (2010), Teunter and Flapper (2011), Su and Xu (2014), among others.

Uncertainty is a particular issue in the remanufacturing process, including the core acquisition/collecting process. Not only the core quality varies, but also the inspection accuracy can be very different in classifying qualities. The remanufacturer's active choice of such accuracy should be important, but it is much less noticed by researchers studying remanufacturing. Below we present some relevant studies. Yanıkoğlu and Denizel (2020) proposed a robust remanufacturing planning optimisation model considering variability in quality levels and quality grading of available cores. Robotis, Boyaci, and Verter (2012) compared two extreme settings of the inspection environment when the remanufacturer cannot inspect the core quality so that all collected cores are remanufactured. As the remanufacturer can inspect the core without errors, only cores with low remanufacturing cost are further processed. Souza, Ketzenberg, and Guide (2009) simulated a multiclass queueing system to model the remanufacturing system with multiple workstations.

Cores with different quality classes have different operation costs and processing times at different workstations. Consequently, classification errors will lead to inferior system performance. Tagaras and Zikopoulos (2008) considered two types of classification errors and developed the optimal core replenishment policy for a remanufacturer. In their study, sorting of the cores can happen at multiple collection sites, but system performance may differ depending on whether the sorting decision is made centrally or locally. Nevertheless, in this most relevant study, the remanufacturer still

passively accepts the quality classification errors and adjusts the production or replenishment policy accordingly, rather than actively choosing the proper inspection standards according to the customer's behaviour. Zikopoulos and Tagaras (2008) considered a similar problem with a single collection site. A crucial difference is the random remanufacturable yield, which was considered a constant in Tagaras and Zikopoulos (2008). In another relevant study, Van Wassenhove and Zikopoulos (2010) investigated the loss that a remanufacturer suffers from suppliers' classification errors (quality overestimation) in a system where multiple quality classes exist.

Besides, the remanufacturer can quantify the advantage of increasing the classification accuracy and decide the improvement effort. However, only one type of classification error was often considered. Actually, in a core inspection process, both errors – overestimation and underestimation of core qualities could exist. The trade-off between these errors indeed affects the system performance and thus the choice of classification accuracy. Such a concern will be the focus of the current study. Another stream of research closely related to our study is the inspection game theory, which has various applications in spectrum sensing and sharing (Kim 2017), on-site inspections (Deutsch, Goldberg, and Perlman 2019), among other fields. In a typical inspection game, inspection is used against the 'moral hazard' that the inspectee may take advantage of information asymmetry. In contrast, inspection errors may produce random and systematic errors. For a more detailed review of this subject area, we refer to Rasmusen (2006) and the recent work of Deutsch (2020).

The main contributions of this paper to the remanufacturing research literature includes two aspects: (1) the inspection game theory is used to investigate the deposit-refund policy in remanufacturing environments, and therefore the study provides a modelling framework for relevant topics; and (2) the limited inspection accuracy and the tradeoff between two types of errors are thoroughly analysed, instead of the commonly used assumption of error-free inspection. Thereby the study has developed new knowledge in the relevant area.

### 3. Model basics

In this section, we describe the basics of the model. The notations are firstly listed in Section 3.1. The tradeoff between the two types of errors is explained in detail in Section 3.2. The decision process and essential assumptions are presented in Section 3.3.

#### 3.1. Notations

The notations are listed below.

$v$ ,	unit profit for the remanufacturer ( $R$ ) selling one product, without considering the benefit of collecting cores and deposit
$x$ ,	deposit charged when selling a product
$q_H$ ,	the value of a high-quality core to the remanufacturer
$q_L$ ,	the value of a low-quality core to the remanufacturer
$s_H$ ,	salvage value of a high-quality core to the customer
$s_L$ ,	salvage value of a low-quality core to the customer
$u$ ,	the payoff of the customer ( $C$ ) when buying the product, without the consideration of deposit, refund, and a salvage value of the core
$\pi_C$ ,	the customer's total profit of buying the product
$P(L)$ ,	the probability that a used product has a low-quality
$P(H)$ ,	the probability that a used product has a high-quality $P(H) = 1 - P(L)$
$p_H$ ,	the probability that a customer is returning a core, given it is of high-quality. For pure strategies, the customer can only choose $p_H$ to be either 0 or 1.
$p_L$ ,	the probability that a customer is returning a core, given it is of low-quality. For pure strategies, the customer only chooses $p_L$ to be either 0 or 1.
$P(r L)$ ,	( $\alpha$ for short), the probability that a low-quality core is mistakenly categorised as remanufacturable ( $r$ ) by the remanufacturer
$P(n H)$ ,	( $\beta$ for short), the probability that a high-quality core is categorised as non-remanufacturable ( $n$ ) by the remanufacturer

#### 3.2. Quality classification errors

We assume that there are two quality classes of the cores: high-quality core with value  $q_H$  and low-quality core with value  $q_L$ , both are defined for the remanufacturer. The same cores have salvage values to the customer,  $s_H$  and  $s_L$  respectively, which could be different from the quality values. An intuitive assumption regarding their relationship is  $q_H > q_L$ ,  $s_H > s_L$ .  $q_H$  denotes the value of a high-quality core to the remanufacturer, while  $s_H$  denotes the salvage value of a high-quality core to the customer. Thus,  $q_H > s_H$ .  $q_L$  denotes the value of a low-quality core to the remanufacturer, while  $s_L$  denotes the salvage value of a low-quality core to the customer. Therefore,  $q_L > s_L$ .

The relation between  $q_L$  and  $s_H$  is not described here, as it is less evident in practice.

For the remanufacturer, the actual quality of a core can only be revealed after it is reprocessed, such as being fully disassembled. Upon accepting the core, the remanufacturer should evaluate its quality by some preliminary examination such as visual inspection and simple physical inspection. Nevertheless, there could be a quality classification error due to the difference between the judged quality and its actual quality. For example, a nasty looking rusty core may have parts that are actually in good condition; on the other hand, a core with a good appearance may have serious cracks inside. We use two conditional probabilities  $\alpha = P(r|L)$  and  $\beta = P(n|H)$  to describe such classification errors. These two errors are also called non-rejection of a false and rejection of a true, respectively (Ross 2019). The value of  $\alpha$  is controllable. The remanufacturer can adjust  $\alpha$  by selecting different inspection technologies, using inspectors with different skill levels, or altering the logistics network for core collection (centralised sorting vs decentralised sorting). Again it should be noted that as  $\alpha$  changes,  $\beta$  also varies.

The dependent probabilities  $\alpha$  and  $\beta$  cannot be reduced simultaneously with a given technology (Ross 2019). However, with a technology improvement or change of the inspection process, such errors can be reduced to another level, but their relationship still exists. Writing the relationship between  $\alpha$  and  $\beta$  in the form of  $\beta = \beta(\alpha)$ , we have the first-order derivative of  $\beta$  with

respect to  $\alpha$ ,  $\beta'(\alpha) < 0$ . There is also a requirement that  $\beta(\alpha)$  should be convex thus  $\beta''(\alpha) \geq 0$ , so that  $\alpha + \beta \leq 1$ . An extreme case is that  $\beta(\alpha) = 1 - \alpha$ . We also use the notation later  $\beta^{-1}(\alpha)$  as the inverse function of  $\beta(\alpha)$ , and  $\beta'^{-1}(\alpha)$  the inverse function of  $\beta'(\alpha)$ .

Both errors have effects on remanufacturer and customers. Besides, the remanufacturer needs to define a deposit value  $x$  to ensure the desired return of cores. This paper investigates the interaction of the above issues in the presence of the customers' choice behaviour.

### 3.3. The decision process and assumptions

The remanufacturer and customer are risk-neutral and try to maximise their expected profits. Both the remanufacturer and customer have access to the information of core quality distributions  $P(H)$  and  $P(L)$ , i.e. the customer and remanufacturer have experience with the product and the knowledge of the related products is publicly available.

Concerning the classification errors  $\alpha$  and  $\beta$ , while the remanufacturer will gain knowledge through experiments to make a deliberate choice, we have to be careful about the customer's related knowledge. In this paper, we first assume that the information is symmetric for the remanufacturer and the customer about such errors (Section 4), then we further extend to the asymmetric case (Section 5). The decision process of the remanufacturer and the customer is described as follows (Figure 1).

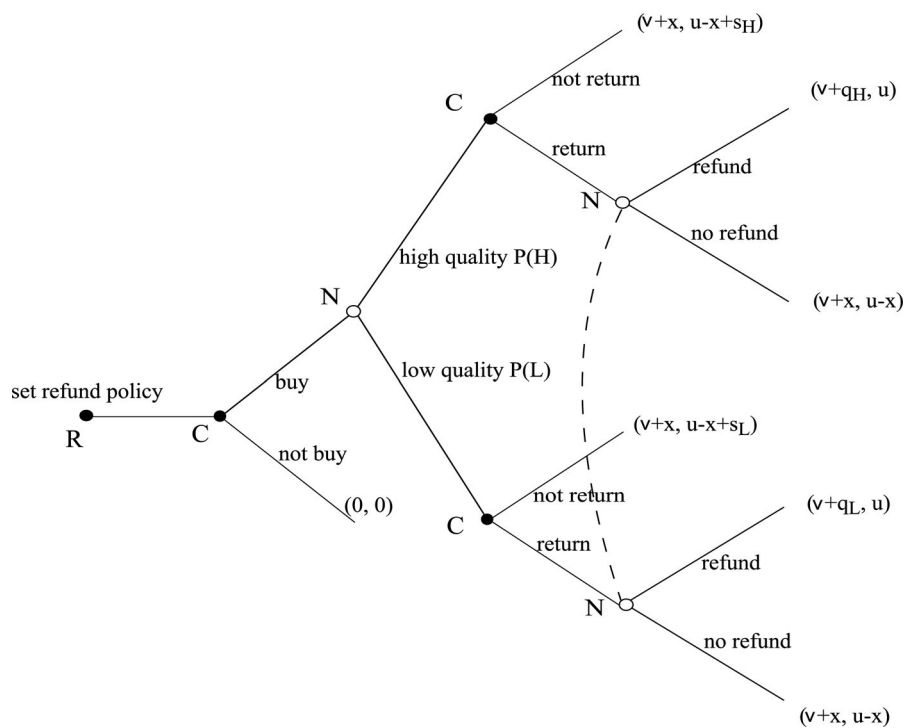


Figure 1. The decision process of the refund game.

**Step 1**

The remanufacturer sets the refund policy with a deposit  $x$ . In the meantime, to estimate the core quality upon receiving the returns, the remanufacturer should choose quality inspection standards with inevitable classification errors ( $\alpha$  and  $\beta$ ). The customer knows the deposit value  $x$ ; however, access to the classification errors depends on whether the remanufacturer announces it or not.

At this stage, the remanufacturer will make the refund policy clear to its customers: based on the inspection results, remanufacturable cores will get full refund  $x$ ; non-remanufacturable cores will receive no refund. To optimise the refund policy, the remanufacturer has all the information needed:

- the distribution of the core quality  $P(H)$  and  $P(L)$ ;
- the customer's utility and decision behaviour.

**Step 2**

The customer decides whether to buy the product. If the expected payoff of buying the product is positive:  $E(\pi_C) > 0$ , the customer buys the product, otherwise not. The customer has the following information at this stage:

- the distribution of the core quality  $P(H)$  and  $P(L)$
- the remanufacturer's classification errors  $\alpha$  and  $\beta$ , if it is announced by the remanufacturer.

**Step 3**

The product is used, and the actual quality of an individual core is realised as  $q_H$  with probability  $P(H)$  and  $q_L$  with probability  $P(L)$ .

**Step 4**

The customer decides whether to return the core. The customer returns the core if the expected profit of returning the core is larger than not returning, and vice versa. The customer knows the actual quality of the core  $q$ , and makes the return decision based on the core quality and the remanufacturer's classification errors  $\alpha$  and  $\beta$  (if it is announced).

**Step 5**

The returned cores are classified after inspection as remanufacturable ( $r$ ) or non-remanufacturable ( $n$ ), and received full refund  $x$  or no refund, respectively. We note here that the refund is based on the inspection results, which could deviate from the actual quality of the cores.

The payoffs of the remanufacturer ( $R$ ) and the customer ( $C$ ) are described in the order ( $R, C$ ) as in the game tree as in Figure 1.

We need to note the decisions follow the above steps. However, in order to analyse and solve the above game problems, a backward induction method is often used

(Raiffa 1997; Rasmusen 2006), i.e. the analysis begins at the end of the problem, to determine a sequence of optimal actions in a subproblem, and then the analysis is conducted backwards till the beginning of the decision so that the optimal decision of the entire problem is obtained. For instance, in our problem in Section 4, as Steps 3 and 5 have no impact on the decision, the subsections are respectively dedicated to analysing the customer's return choice (Step 4 and Section 4.1), the impact of inspection errors (Step 2 and Section 4.2) and the choice of deposit (Step 1 and Section 4.3). Such an approach has been widely used in decision analysis, game theory, dynamic programming.

## 4. Symmetric information of inspection

In this section, we present the model with symmetric information of inspection error, i.e. the remanufacturer announces the inspection error, so both the remanufacturer and the customer know the accuracy of inspection. In this case, only pure strategies will be adapted by the customer, i.e.  $p_H = 0$  or  $1$ , and  $p_L = 0$  or  $1$ . Later in Section 5, we extend the discussion to mixed strategies when the customer does not have the inspection error information, and thus the information is asymmetric.

### 4.1. Customer's pure strategy

We denote these strategies as follows.

Strategy A: Customer: *return*| $H$ , *not return*| $L$ , i.e.  $(p_H, p_L) = (1, 0)$

Strategy B: Customer: *not return*| $H$ , *not return*| $L$ , i.e.  $(p_H, p_L) = (0, 0)$

Strategy C: Customer: *return*| $H$ , *return*| $L$ , i.e.  $(p_H, p_L) = (1, 1)$

Strategy D: Customer: *not return*| $H$ , *return*| $L$ , i.e.  $(p_H, p_L) = (0, 1)$

For the customer, given that the core has a low-quality, the expected profit of the customer returning the core is  $\alpha u + (1 - \alpha)(u - x) = u - (1 - \alpha)x$  and the expected profit of not returning the core is  $u - x + s_L$ . So the customer will return the core if  $\alpha x > s_L$ , i.e.  $\alpha > (s_L/x)$ ; the customer will not return the core if  $\alpha < (s_L/x)$ . Notice that if  $x < s_L$ , the customer will never return the low-quality core.

Similarly, given that the core is of high-quality, the expected profit of returning the core is  $(1 - \beta)u + \beta(u - x) = u - \beta x$  and the expected value of not returning the core is  $u - x + s_H$ . So the customer will return the core if  $u - \beta x > u - x + s_H$ , i.e.  $\beta(\alpha) < 1 - (s_H/x)$ ; the customer will not return the core if  $\beta(\alpha) > 1 - (s_H/x)$ .

The total expected payoff of the customer buying the product is

$$E(\pi_C) = P(L) \max(u - (1 - \alpha)x, u - x + s_L) + P(H) \max(u - \beta x, u - x + s_H) \quad (1)$$

Combined the above cases, the customer will choose:

Strategy A:  $(p_H, p_L) = (1, 0)$ , if  $\alpha < (s_L/x)$  and  $\beta < 1 - (s_H/x)$

Strategy B:  $(p_H, p_L) = (0, 0)$ , if  $\alpha < (s_L/x)$  and  $\beta > 1 - (s_H/x)$

Strategy C:  $(p_H, p_L) = (1, 1)$ , if  $\alpha > (s_L/x)$  and  $\beta < 1 - (s_H/x)$

Strategy D:  $(p_H, p_L) = (0, 1)$ , if  $\alpha > (s_L/x)$  and  $\beta > 1 - (s_H/x)$

The above expressions can be plotted in Figure 2, in which the values of  $x$ ,  $s_H$  and  $s_L$ , together with the selection of  $\alpha$  and  $\beta$  will define the preferable strategy applied by the customer. Notice that the crossing point of two critical lines  $\alpha = (s_L/x)$  and  $\beta = 1 - (s_H/x)$  can only move along a straight (deposit) line with slope  $-(s_H/s_L)$ . This deposit line also passes points  $(0, 1)$  and  $(s_L/s_H, 0)$ .

The relation between  $\alpha$  and  $\beta$  is not independent with a given inspection process, and the remanufacturer cannot freely choose  $\alpha$  and  $\beta$ . If we add the curve of  $\beta(\alpha)$  in the figure, we can further see the customer's choice behaviour.

For the convenience of discussion, we define that a specific pure strategy is *Reachable* when the customer

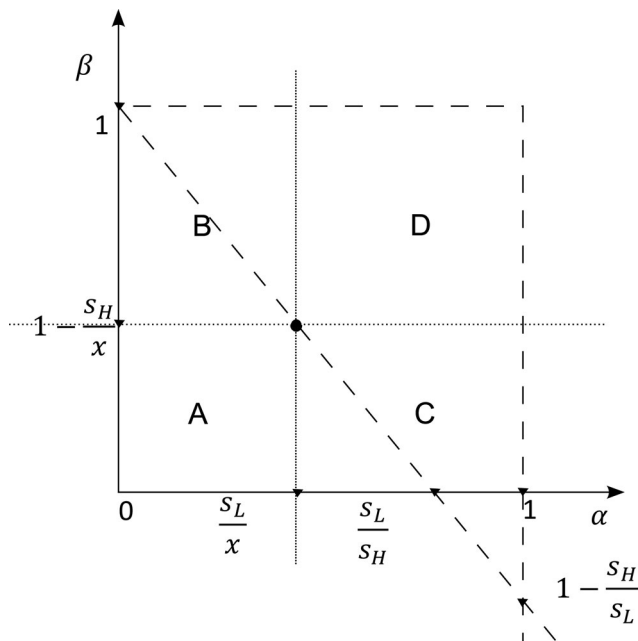


Figure 2. An illustration of the choice of deposit  $x$ .

can choose this strategy if the remanufacturer adjusts its choices appropriately. On the other hand, we state that a specific pure strategy is *Not reachable* when the customer will not choose this pure strategy no matter how the remanufacturer adjusts its choices of  $\alpha$  (or the combination of  $\alpha$  and  $x$ , if clearly stated).

If the remanufacturer fixes the deposit  $x$  and chooses  $\alpha$ , the customer cannot choose each of the four strategies, i.e. not all four strategies are reachable. As explained in Figure 3, Strategy D will not happen, as the curve  $\beta(\alpha)$  is not crossing the upper right Strategy D region. On the other hand, Strategy A, B, and C are reachable by setting  $\beta^{-1}(1 - (s_H/x)) < \alpha < (s_L/x)$ ,  $\alpha < \beta^{-1}(1 - (s_H/x))$ ,  $\alpha > (s_L/x)$ , respectively.

In Figure 3, we note that if  $\beta^{-1}(1 - (s_H/x)) < (s_L/x)$ , the crossing point will be above the  $\beta(\alpha)$  curve, and consequently, Strategy A is reachable; if  $\beta^{-1}(1 - (s_H/x)) > (s_L/x)$ , Strategy A is not reachable.

Figure 4 illustrates the case when Strategy A is not reachable by only adjusting  $\alpha$ . This is also the case when it is never possible to collect only high-quality cores, whereas Strategies B, C, and D are reachable by setting  $\alpha < (s_L/x)$ ,  $\alpha > \beta^{-1}(1 - (s_H/x))$ ,  $(s_L/x) < \alpha < \beta^{-1}(1 - (s_H/x))$ , respectively.

In case both  $x$  and  $\alpha$  can be changed by the remanufacturer, as illustrated in Figure 5, the straight (deposit) line may lay under the  $\beta(\alpha)$  curve when  $-(s_H/s_L) < \beta'(0)$ , and the crossing of critical lines is never above the  $\beta(\alpha)$  curve. Thus, even if the remanufacturer can choose both deposit  $x$  and  $\alpha$  at the same time, Strategy A is not reachable, i.e. the customer will not only return high-quality cores.

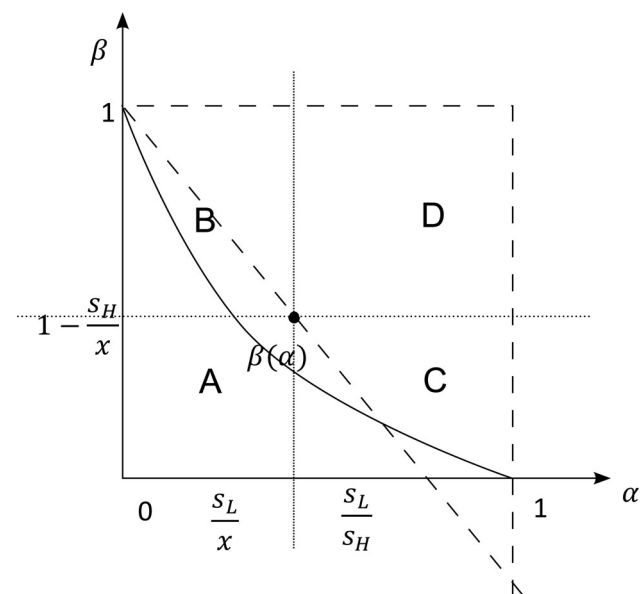


Figure 3. Strategy D is not reachable by changing  $\alpha$ .



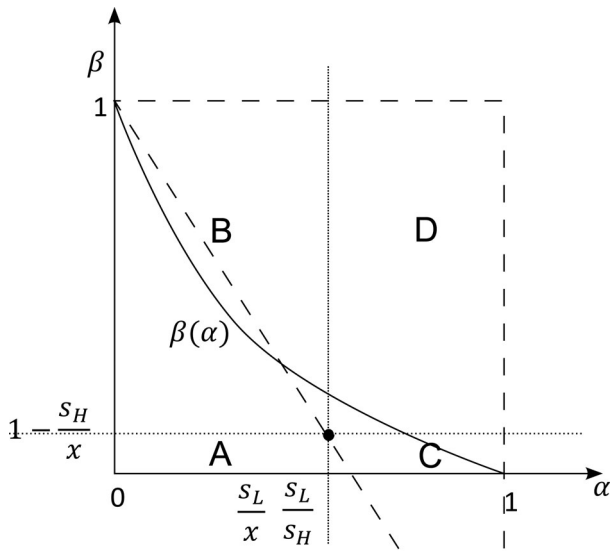


Figure 4. Strategy A is not reachable by changing  $\alpha$ .

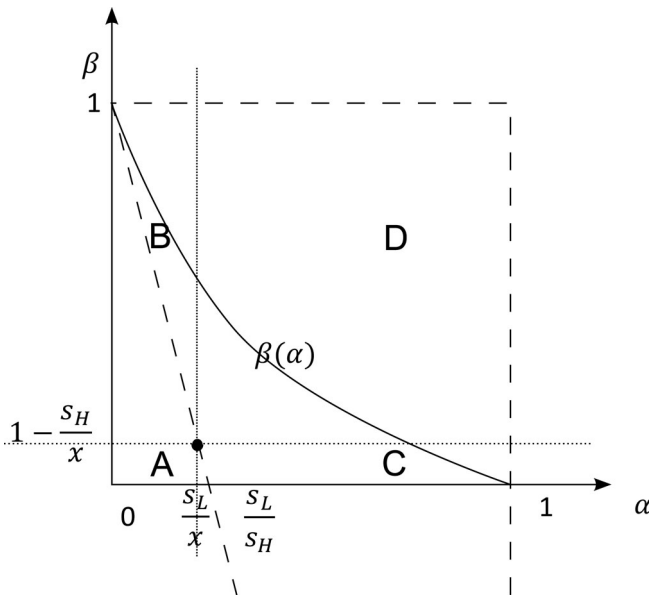


Figure 5. Strategy A is not reachable by changing deposit  $x$  and classification error  $\alpha$ .

To summarise, we have the proposition regarding the presence of the four possible strategies with deposit  $x$  fixed and varying, respectively.

**Proposition 4.1:** (a) The remanufacturer chooses  $\alpha$ , while  $x$  is fixed: i. Strategies A and D cannot both be reachable; ii. Strategies B and C are always reachable.

(b) The remanufacturer chooses both  $\alpha$  and  $x$ : i. Strategies B, C, and D are always reachable; ii. Strategies A is reachable if and only if  $(s_H/s_L) < -\beta'(0)$ .

In Strategy A, the customer returns high-quality cores but not low-quality cores, while in Strategy D, the

customer returns low-quality cores but not high-quality cores. It can be seen that these two strategies cannot coexist in case deposit  $x$  is fixed (Proposition 1a). If the salvage value of high-quality cores is very close to the deposit and/or the conditional probability  $\alpha$  is high enough, the customer prefers Strategy D than Strategy A for a better profit, i.e. to return low-quality cores to take advantage of large inspection error  $\alpha$  and sell high-quality cores as salvages instead of returning. The opposite condition brings the preference for Strategy A.

Given both higher deposit  $x$  and conditional probability  $\alpha$ , the customer prefers Strategy C for a better profit, i.e. to return high-quality cores instead of selling as salvages and return low-quality cores to take advantage of large inspection error  $\alpha$ . If the deposit  $x$  is lower or close to the salvage value of high-quality cores, the customer prefers strategies B or D to sell high-quality cores as salvages instead of returning them.

With Proposition 1b, we have: given that specific inspection technology ( $\beta'(0)$  keeps the same), for Strategy A to be reachable,  $s_H/s_L$  needs to be small enough; in other words,  $s_L/s_H$  needs to be close enough to 1. This indicates that if the salvage value of a low-quality core is too low compared with the high-quality core, the customer will return the low-quality core anyway no matter what the refund policy is (to take advantage of classification error). Thus, it will be impossible for the remanufacturer to collect only high-quality cores.

#### 4.2. The impact of inspection errors

In this section, we analyse how the choice of classification error  $\alpha$  affects the customers return behaviour, with  $x$  fixed and assuming that the customer already had the product. In the next section, we further consider the case when deposit  $x$  is a decision variable along with classification error  $\alpha$ .

Now suppose that the deposit  $x$  is decided already, the remanufacturer needs to decide the choice of  $\alpha$  (therefore  $\beta$  is determined based on  $\alpha$ ). As discussed before, depending on the relation of  $x$  and  $q_L, q_H$ , the customer has four strategies. Thus, by comparing the remanufacturer's profits in the four strategies, we have the following proposition which provides the solutions for choosing classification error  $\alpha$  when deposit  $x$  is pre-determined and the customer has already bought the product. The detailed development of the solutions is presented in Appendix A.

**Proposition 4.2:** When deposit  $x$  is pre-determined, the remanufacturer's preference of customer's choice is:

- (i) Given  $x > q_H$ , Strategy B;

**Table 1.** Summary of solutions when  $x$  has already been determined.

$x > q_H$	$\alpha = 0, \beta = 1.$		
$x < q_L$	$\alpha = 1, \beta = 0.$		
$q_L < x < q_H$	If A is reachable	$\alpha = \frac{s_L}{x}, \beta = \beta \left( \frac{s_L}{x} \right)$	
	If A is not reachable	If $\pi_R^b > \pi_R^c$	$\alpha = 0, \beta = 1.$
		If $\pi_R^b < \pi_R^c$	$\alpha = 1, \beta = 0.$

(ii) Given  $x < q_L$ , Strategy C;

(iii) Given  $q_L < x < q_H$ , Strategy A if it is reachable;

Strategies B or C, depending on respective profits  $\pi_R^b = v + x$  and  $\pi_R^c = v + x + P(H)(1 - \beta(\alpha))(q_H - x) + P(L)\alpha(q_L - x)$ , otherwise.

In practice, by setting  $\alpha = 0, \beta = 1$ , both high-quality cores and low-quality cores are categorised as non-remanufacturable. Thus, the remanufacturer would not prefer Strategy D as the customer's best choice since there will be no core returns for remanufacturing. The solutions of all three cases mentioned above are summarised in Table 1, which, together with the discussion in the next section, provide solution procedures in Appendix C.

The above solutions are valid when  $x$  is already given, and the customer has the product. To study the deposit decision and the customer's purchasing behaviour, we need to consider the upper bounds of  $x$ , which is addressed in the next section.

### 4.3. The impact of the deposit value $x$

In this section, we analyse how the choice of deposit  $x$  affects a customer's buying decision in the four strategies. The first thing to notice is that, similarly with the analysis in the last section, Strategy D will not be the optimal solution for the remanufacturer.

**Proposition 4.3:** *If the remanufacturer can choose deposit  $x$  and inspection error  $\alpha$ , Strategy D will not be optimal.*

The proof is given in Appendix B. This means Strategy D is not encouraged for the customer only to return low-quality cores but not to return high-quality cores. For example, by choosing a higher deposit  $x$  and a lower conditional probability  $\alpha$ , the customer prefers Strategy A or C to Strategy D for a better profit, i.e. to return high-quality cores instead of selling as salvages. In each strategy, the profit of the remanufacturer increases along with the deposit value  $x$ . However, there is an upper bound beyond which the customer stops buying the product. By examining the customer's payoff, we derive the conditions

that the customer buys the product in different strategies.

$$\text{Strategy A : } x \leq \frac{u + P(L)s_L}{P(L) + P(H)\beta} \quad (2)$$

$$\text{Strategy B : } x \leq u + P(L)s_L + P(H)s_H \quad (3)$$

$$\text{Strategy C : } x \leq \frac{u}{P(L)(1 - \alpha) + P(H)\beta} \quad (4)$$

$$\text{Strategy D : } x \leq \frac{u + P(H)s_H}{P(L)(1 - \alpha) + P(H)} \quad (5)$$

Combined the above constraints of deposit and Proposition 2, we can develop the procedures to determine the optimal solutions when the inspection error is announced. The solution procedure is explained in Appendix C.

## 5. Asymmetric information of inspection

In section 4, the remanufacturer decides the deposit and inspection error and subsequently announces the information. It is a leadership version of the inspection game, and the remanufacturer should always choose the strategy to its advantage so that the customer would stay with a pure strategy. In this section, we assume that the inspection error is not announced to the customer. Therefore, the inspection error becomes asymmetric information. In this case, the customer can use mixed strategies to improve his payoffs (Rasmusen 2006). This section investigates the impact of such asymmetric information.

Thus, we investigate the customer's choice behaviour using a mixed strategy, assuming that the customer has already bought the product. The customer's decision is about to return the core or not, given it is high- or low-quality.

The customer can use mixed strategies for a high-quality core ( $p_H \neq 0$  and  $p_H \neq 1$ ), only when the payoff of return the core ( $u - \beta x$ ) equals the payoff of not return the core ( $u - x + s_H$ ), i.e.  $\beta(\alpha) = 1 - (s_H/x)$ . Similarly, the customer can use mixed strategies for a low-quality core ( $p_L \neq 0$  and  $p_L \neq 1$ ), only when the payoff of return equals not return, which leads to  $\alpha = (s_L/x)$ .

There will be no mixed strategies for the customer when  $x > q_H$  or  $x < q_L$ : if  $x > q_H$ , the best choice for the remanufacturer is to return no refund at all, thus set  $\alpha = 0, \beta = 1$ , and the customer should know this, thus no core will be returned; on the other hand, if  $x < q_L$ , the best choice for the remanufacturer to return all refunds, and let  $\alpha = 1, \beta = 0$ , therefore all cores will be returned.

If  $q_L < x < q_H$ , the best outcome for the remanufacturer is to refund only high-quality cores. There will be two cases depending on whether Strategy A is reachable. Furthermore, depending on the values

**Table 2.** Summary of four scenarios of different equilibrium solutions with symmetric and asymmetric inspection information.

$q_L < x < q_H$		Inspection error Announced	Inspection error Not announced
If A is reachable	If $\beta'^{-1} \left( \frac{P(L)(q_L - x)}{P(H)(q_H - x)} \right) < \beta^{-1} \left( 1 - \frac{s_H}{x} \right)$	Pure strategy	Mixed strategy
		Different equilibriums*	
If A is not reachable	If $\beta'^{-1} \left( \frac{P(L)(q_L - x)}{P(H)(q_H - x)} \right) > \beta^{-1} \left( 1 - \frac{s_H}{x} \right)$	Pure strategy	Pure strategy
		Different equilibriums	
	If $\beta'^{-1} \left( \frac{P(L)(q_L - x)}{P(H)(q_H - x)} \right) > \beta^{-1} \left( 1 - \frac{s_H}{x} \right)$	Pure strategy	Pure strategy
		Same equilibrium**	
	If $\beta'^{-1} \left( \frac{P(L)(q_L - x)}{P(H)(q_H - x)} \right) < \beta^{-1} \left( 1 - \frac{s_H}{x} \right)$	Pure strategy	Pure strategy
		Different equilibriums	

\*The equilibrium solutions are different between the cases that inspection error is announced or not.

\*\*The equilibrium solutions are the same regardless of whether inspection error is announced or not.

of  $\beta'^{-1}((P(L)(q_L - x))/(P(H)(q_H - x)))$  and  $\beta^{-1}(1 - (s_H/x))$ , the customer will respond in two different cases (see details in Appendix D).

Summarising the above four combinations, we obtain the type of solutions as illustrated in Table 2 in the case that the remanufacturer does not announce the inspection error. The table also includes the solutions when the inspection error is announced for comparison purposes. Later in Section 6, numerical examples are conducted to illustrate these different scenarios.

### 6. Numerical examples

In Section 6.1, we first illustrate various scenarios of deposit solutions and classification errors, i.e. when pure strategies A, B, and C are optimal, respectively. Besides, we discuss the influence of customer’s product payoff  $u$  on the optimal solutions. In Section 6.2, we present the different equilibriums when the inspection error is not announced, with the customer’s possible mixed strategies. When the remanufacturer has limited inspection technology/effort, the tradeoffs between two inspection errors need to be considered. Section 6.3 studies the influences of improving inspection technology/effort by adjusting  $\beta(\alpha)$  function.

In the following numerical examples, we set the relation of classification errors in the simple form of  $\beta(\alpha) = (1 - \alpha)^n, n > 1$ , so there is  $\beta(0) = 1$  and  $\beta(1) = 0$ , and also  $\beta'(\alpha) = -n(1 - \alpha)^{n-1} \leq 0$ , and  $\beta''(\alpha) = n(n - 1)(1 - \alpha)^{n-2} > 0$ . In Sections 6.1 and 6.2, we expressly set  $n = 2$  for the numerical illustration, while in Section 6.3,  $n$  is varying to show the impact.

#### 6.1. On the influences of the customer’s net income

In the basic setting, the unit product profit for the remanufacturer is  $v = 20$ , and the probability of high-quality cores is  $P(H) = 0.5$ . The other parameters are

**Table 3.** Numerical examples of optimal solutions.

No.	Examples					Solutions				
	$u$	$q_H$	$q_L$	$s_H$	$s_L$	Optimal strategy	$\alpha$	$\beta$	$x^*$	$\pi_R$
1	4					C	1	0	$[6, \infty)$	28.0
2	6	10	6	3.5	2	A	0.23	0.60	8.8	29.0
3	8					B	$[0, 0.19]$	$(0.67, 1]$	10.8	30.8

summarised in Table 3, where the customer’s payoff  $u$  increases from examples No.1 to No. 3. Note that the parameters are selected in such a way that the optimal solutions belong to different strategies, as shown in the table.

In Table 3, with a low value  $u$ , the remanufacturer will choose  $\alpha = 1$  and  $\beta = 0$ , thus both high- and low-quality cores will be accepted and therefore refunded. The customer’s best response will be pure Strategy C, which returns both high and low qualities. In this case, the deposit  $x^*$  will be paid on purchasing the products and fully refunded when returning the cores. Without considering any time value of the money,  $x^*$  will be balanced with in- and out- transactions, and therefore will not change the profit of either remanufacturer or customer. Thus, it is an optimal value as long as it is larger than or equal to 6.

Generally, with the increase of the customer’s product payoff  $u$ , the customer cares more about the product itself and has less concern about losing the deposit so that the remanufacturer can raise the deposit. Thus, the remanufacturer can gain more profit  $\pi_R$  by using a small  $\alpha$  to reduce low-quality cores (but at the same time a high probability  $\beta$  for rejecting some high-quality cores). This results that more deposits are kept by the remanufacturer, whereas fewer cores are returned. In this case, the remanufacturer is more interested in the deposit than the returned cores.

In example No. 1, when the customer’s product payoff  $u$  is very low, the remanufacturer sets an extreme value of

inspection error  $\alpha = 1$ , i.e. giving refunds to all returned cores so that all cores are collected, and the remanufacturer keeps no deposit. In example No. 2, with the increase of customer's product payoff  $u$ , the remanufacturer can keep some deposit. Thus, the remanufacturer gradually reduces the inspection error  $\alpha$ , and consequently reduces the possibility of refunding low-quality cores. In example No. 3, when  $\alpha$  is further reduced and  $\beta$  is further increased, the probability of rejecting a high-quality core becomes very high. This prevents the return of any type of cores (Strategy B).

From Table 3, we can see that the customer's product payoff  $u$  has an important impact on the remanufacturer's choice of the optimal strategy. However, the exact value of  $u$  can be difficult for the remanufacturer to estimate in practice. The consequence of overestimating this value needs to be investigated since an overcharge of deposit may prevent the customer from buying the product

In Figure 6, the vertical axis indicates the remanufacturer's optimal profit  $\pi_R$ , whereas the horizontal axis indicates the customer's actual product payoff  $u$ . For comparison, we investigate 4 cases with different estimations of customer's product payoff  $u_0 = 5, 5.5, 6, 6.5$ . With the estimation of  $u_0 = 6$  as an example, the remanufacturer will assign  $\alpha = 0.23$  and  $\beta = 0.60$ , in order to obtain the optimal  $\pi_R = 29.0$  (Table 3, Example No.2). However, this will be true if the actual product payoff  $u$  is large than or equal to 6, as the customer will choose the optimal Strategy A. However, when the value  $u$  is smaller than 6 (or the remanufacturer overestimates  $u_0 > u$ ), the customer will not return any core. The remanufacturer's profit is reduced to the unit product profit  $v = 20$ . The same explanation is valid for  $u_0 = 5.5, 6.5$ . With a high level of estimation  $u_0$ , the remanufacturer can potentially reach a high profit if the true value  $u > u_0$ . Nevertheless, there is also a large interval  $u$  that the remanufacturer's profit is reduced to  $v = 20$ .

On the other hand, if  $u_0$  is further reduced to  $u_0 = 5$ , the remanufacturer will assign  $\alpha = 1$  and  $\beta = 0$  (similar

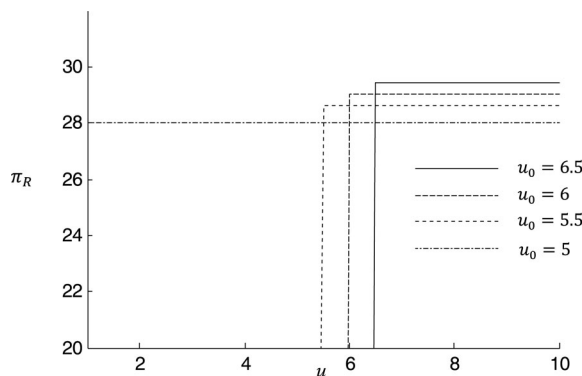


Figure 6. The influences of the customer's product payoff  $u$ .

to Example No.1, Table 3). Regardless of the actual product payoff  $u$ , the customer will choose Strategy C (return all cores), thus the profit  $\pi_R$  is constant with a value of 28. In short, if possible, the remanufacturer should correctly estimate the actual value of  $u$ . If such information is not available, an underestimation ( $u_0 > u$ ) will bring a better profit, or more conservatively, the remanufacturer should find  $u_0$  value to entice customers to return all cores (Strategy C) and ensure a stable profit for a larger range of variation of  $u$ .

## 6.2. On the influences of announcing the inspection error

As indicated in Section 5, different equilibriums exist and thus different profits, depending on whether the remanufacturer announces the inspection error or not (Table 2). In this section, we illustrate such differences. Similar to Section 5, we focus on the cases when the customer already has the product, and  $q_L < x < q_H$ .

The four possible scenarios are illustrated by numerical examples No. 4, 5, 6, and 7 in Table 4. Strategy A is reachable in No. 4 and No. 5, and not reachable in No. 6 and No. 7. Furthermore, we have

$$\beta'^{-1} \left( \frac{P(L)(q_L - x)}{P(H)(q_H - x)} \right) < \beta^{-1} \left( 1 - \frac{s_H}{x} \right)$$

in No. 4 and No. 7, whereas

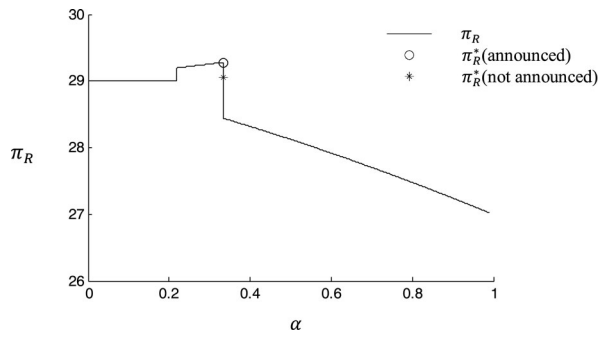
$$\beta'^{-1} \left( \frac{P(L)(q_L - x)}{P(H)(q_H - x)} \right) > \beta^{-1} \left( 1 - \frac{s_H}{x} \right)$$

in No. 5 and No. 6. The examples show that the customer will use a mixed strategy only in No.4 and a pure strategy in other examples. However, even though a pure strategy is used, the equilibriums can be different from the case of announcing the inspection error, with the only exception in No. 6. The numerical examples exactly support the statement in Table 2.

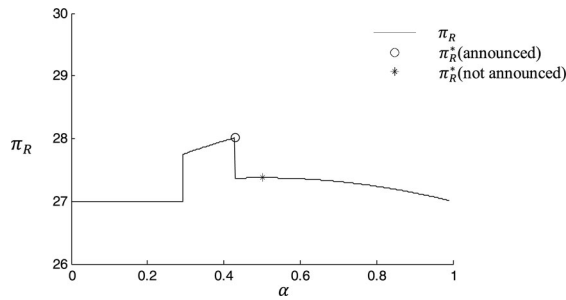
The details of the solutions are shown in Figures 7–10, in which the remanufacturer's profit is illustrated by varying values of inspection error  $\alpha$  (solid lines). In addition, the marked spots refer to the equilibrium solutions.

Table 4. Numerical examples for the influences of announcing the inspection error.

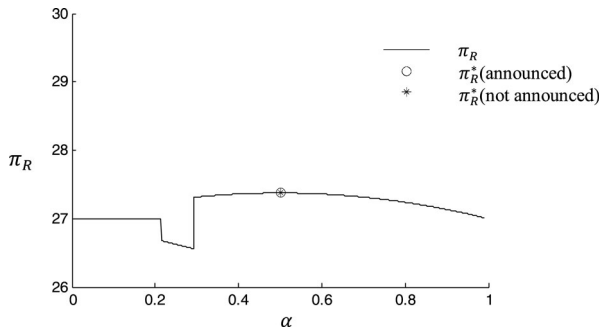
No.	Settings					Solutions (not announced)					Solutions (announced)		
	$u$	$q_H$	$q_L$	$s_H$	$s_L$	$x$	$\alpha$	$\beta$	$p_L$	$p_H$	$\pi_R^*$	$\pi_R^*$	
4						3	0.33	0.44	0.27	1	29.1	29.3	
5	5.5	10	4	3.5		7	0.5	0.25	1	1	27.4	28.0	
6						1.5	7	0.5	0.25	1	1	27.4	27.4
7						7.6	0	1	0	0	27.6	27.7	



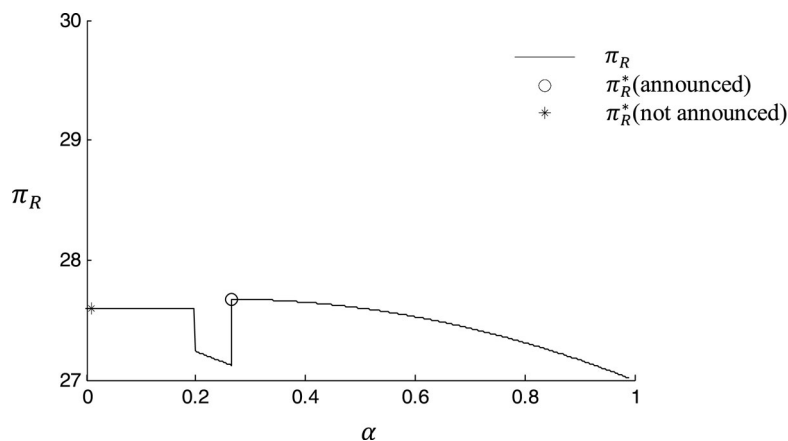
**Figure 7.** Strategy A is reachable with a mixed strategy in equilibrium (Example 4).



**Figure 8.** Strategy A is reachable with a pure strategy in equilibrium (Example 5).



**Figure 9.** Strategy A is not reachable with Strategy C in equilibrium (Example 6).



**Figure 10.** Strategy A is not reachable with Strategy B in equilibrium (Example 7).

Figures 7 and 8 are the cases when A is reachable; the remanufacturer’s profit lines consist of three segments, representing three pure strategies that customer could choose. With an increasing  $\alpha$ , the customer will change from Strategy B to Strategy A, and finally, Strategy C. Here the profit of Strategy A is the highest because the remanufacturer should collect only the high-quality cores when  $q_L < x < q_H$  and if Strategy A is reachable. Thus, when the values of  $s_L$  and  $s_H$  are closer (Strategy A is more likely to be reachable), setting a high value of  $\alpha$  encourages the customer to return many low-quality cores and reduces the remanufacturer’s profit abruptly. Such a choice of  $\alpha$  should be carefully avoided.

In Figure 7, when the inspection error is not announced, the customer uses a mixed strategy to return the low-quality cores with a probability 0.27 (See Table 4), and return all high-quality cores. If the inspection error is announced, the customer would only return high-quality cores, and the remanufacturer can achieve a higher profit. When the inspection error is not announced, the remanufacturer’s profit, marked ‘\*’, is lower than the announced case. This phenomenon is called the first-mover advantage (Rasmusen 2006). For the player who makes the first move in a game, its payoff is often improved (for instance, the open player in a chess game often has some advantages). When inspection errors are not announced, two players are making decision simultaneously. Therefore, the advantages mentioned above may disappear. In Figure 7, the equilibria for both announced and not announces cases are happened to be obtained at  $\alpha = 0.33$ . However,  $\alpha$  in both cases will not necessarily be the same, cf. Figure 8.

In Figure 8, the remanufacturer still achieves a higher profit by announcing the inspection error. In this setting, when the inspection error is not announced, the customer will not use a mixed strategy, but both high- and low-quality cores will be returned (Strategy C). It is

still preferable for the remanufacturer to announce the inspection error.

The above phenomenon explains that if the remanufacturer does not announce the inspection error, the customer is likely to take advantage of the remanufacturer's possible choice of a high value of  $\alpha$ , with the speculation of receiving refunds for a low-quality core. This results in a lower profit for the remanufacturer. By announcing the exact information of inspection error to be committed, the remanufacturer prevents such speculations and achieves a higher profit (again the first-mover advantage).

Figures 9 and 10 are the cases when A is not reachable; the remanufacturer's profit again consists of three segments. As  $\alpha$  increases, the customer will change from Strategy B to Strategy D, and finally, Strategy C. Here the manufacturer receives the lowest profit in Strategy D, with which only low-quality cores are returned, but the remanufacturer still needs to refund some due to the inspection error. It is also important to indicate that when  $s_L$  is too small compared with  $s_H$  (Strategy A is more likely not reachable), setting a low value of  $\alpha$  prevents the customer from return any high-quality core, and subsequently, it reduces the remanufacturer's profit abruptly.

Figure 9 shows no difference whether the inspection error is announced or not. The customer always returns both high- and low-quality cores. While in Figure 10, the customer returns nothing if the inspection error is not announced, and return both high and low-quality cores otherwise. The announcement eliminates the customer's worry of not receiving refunds of high-quality cores, and it results in a relatively higher profit for the remanufacturer.

### 6.3. On the influences of improving classification accuracy

This section investigates the influences of classification accuracy. Let  $\beta(\alpha) = (1 - \alpha)^n$ ,  $n > 1$ . As the value of  $\beta(\alpha) = (1 - \alpha)^n$  decreases with  $n$ , the inspection accuracy improves with an increasing  $n$ .

Intuitively the remanufacturer's profit is expected to improve with a better inspection accuracy (large  $n$ ), except when  $\alpha$  is too small. Thus, no core is returned (Strategy B), and the profit remains constant (Figure 11 with different values  $n = 2, 3$  and 4). This figure uses the basic cost settings as in example No. 3, where Strategy A is reachable. We note that when Strategy A is not reachable, the improvement is similar.

We further study the cases that the inspection error is not announced. Interestingly, with the improvement of classification accuracy, the remanufacturer remains

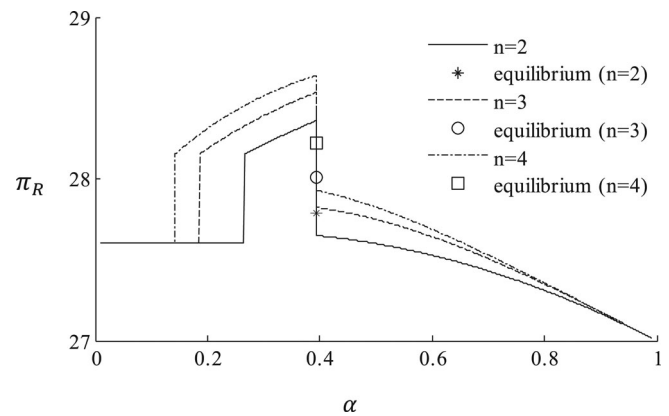


Figure 11. The impact of inspection accuracy.

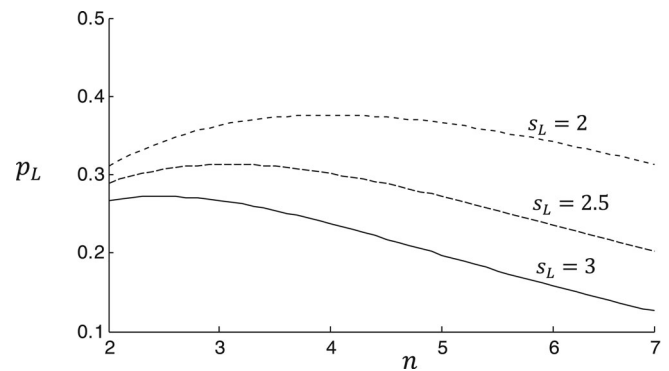


Figure 12. The change of  $p_L$  in mixed strategies with various inspection accuracies.

the same inspection error  $\alpha = s_L/x$  in the equilibrium. Meanwhile, since  $\beta(s_L/x) = (1 - s_L/x)^n$  reduces with the increase of inspection accuracy ( $n$ ), the remanufacturer can reject less high-quality cores and consequently improve the profit.

In the equilibrium, the customer changes the probability of returning low-quality cores. The result is not intuitive since the chance of receiving refunds for low-quality cores is the same when the remanufacturer keeps the same value of  $\alpha = s_L/x$ . As explained in Appendix D,  $p_L^* = \beta'(s_L/x)((P(H)(q_H - x))/(P(L)(q_L - x)))$  decreases (increases) if  $\beta'(s_L/x)$  increases (decreases) with  $n$ , since  $((\partial\beta'(s_L/x))/\partial n) > 0 (< 0)$ . In Figure 12,  $p_L$  first increases with  $n$  and then decrease with  $n$ , and therefore its change is not monotonic. The improvement of inspection accuracy not necessarily reduces the probability of returning low-quality cores. The change of  $p_L$  depends on the change of  $\beta'(s_L/x)$ , therefore not only on the inspection error.

## 7. Conclusions

This paper investigates the deposit-refund policy in remanufacturing environments and develops a game

theory modelling framework for relevant topics. Specifically, instead of the commonly used assumption of error-free inspection, the limited inspection accuracy and the tradeoff between two types of classification errors are considered during inspection processes, i.e. either low-quality cores are defined remanufacturable, or high-quality cores are defined non-remanufacturable.

It is observed that the customer's access to the information on inspection errors is essential. When the information is accessible, there are four possible pure strategies for describing the customer's return behaviour. The equilibrium depends on the remanufacturer's choices of inspection errors and deposit. The conditions for when each pure strategy should be used and when it is reachable are developed. Furthermore, we analyse the remanufacturer's optimal profits under each pure strategy and present a solution procedure for choosing the combination of deposit value and inspection errors from the remanufacturer's perspective.

When the customer does not have the information about inspection errors, i.e. when the remanufacturer does not announce the inspection errors, the customer will use mixed strategies for returning low-quality cores. In this case, optimal solutions and conditions for different solutions are developed, given that the deposit is decided externally and the customer already obtained the product.

It is also shown quantitatively that lacking inspection error information will reduce the remanufacturer's profit. Thus, it is essential to improve such information transparency. To achieve high information transparency, the remanufacturer should carefully choose quality inspection criteria and state the inspection error clearly, if possible.

The remanufacturer should also pay special attention to the value structure of the cores and the products. More specifically, the difference of salvage values between high-quality and low-quality cores determines whether it is possible to collect only high-quality cores, which in turn affects the remanufacturer's decision of the inspection error. The estimation of the product's payoff to the customer is another crucial factor influencing the remanufacturer's choices of the inspection error and deposit. A relatively conservative estimation (underestimation) of such a payoff reduces the chance of profit losses. It also secures the remanufacturer's profit even if the wrong value of such a payoff is used to a certain extent.

For future research directions of this work, more influence factors of the core quality inspection errors will be explored in the return process. Besides, other types of models will be investigated and developed to improve the quality inspection accuracy and encourage core returns.

## Disclosure statement

No potential conflict of interest was reported by the author(s).

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## References

- Arredondo-Soto, K. C., A. Realyvasquez-Vargas, A. A. Maldonado-Macias, and J. Garcia-Alcaraz. 2019. "Impact of Human Resources on Remanufacturing Process, Internal Complexity, Perceived Quality of Core, Numerosity, and Key Process Indicators." *Robotics and Computer-Integrated Manufacturing* 59 (October): Elsevier Ltd: 168–176. doi:10.1016/j.rcim.2019.04.004.
- Bulmuş, Serra Caner, Stuart X. Zhu, and Ruud H. Teunter. 2014. "Optimal Core Acquisition and Pricing Strategies for Hybrid Manufacturing and Remanufacturing Systems." *International Journal of Production Research* 52 (22): Taylor and Francis Ltd.: 6627–6641. doi:10.1080/00207543.2014.906073.
- Cai, Xiaoqiang, Minghui Lai, Xiang Li, Yongjian Li, and Xianyi Wu. 2014. "Optimal Acquisition and Production Policy in a Hybrid Manufacturing/ Remanufacturing System with Core Acquisition at Different Quality Levels." *European Journal of Operational Research* 233 (2): Elsevier BV.V.: 374–382. doi:10.1016/j.ejor.2013.07.017.
- Caterpillar. 2020. "The Benefits of Remanufacturing." Accessed 1 February 2021. <https://www.caterpillar.com/en/company/sustainability/remanufacturing/benefits.html>.
- CM Consulting. 2016. *Deposit Systems for One-Way Beverage Containers: Global Overviews*.
- Debo, Laurens G., and Luk N. Van Wassenhove. 2005. "Tire Recovery: The RetreadCo Case." In *Managing Closed-Loop Supply Chains*, 119–128. Springer. doi:10.1007/3-540-27251-8\_11.
- Denizel, Meltem, Mark Ferguson, and Gilvan Gil C. Souza. 2010. "Multiperiod Remanufacturing Planning with Uncertain Quality of Inputs." *IEEE Transactions on Engineering Management* 57 (3): 394–404. doi:10.1109/TEM.2009.2024506.
- Deutsch, Yael. 2020. "A Polynomial-Time Method to Compute All Nash Equilibria Solutions of a General Two-Person Inspection Game." *European Journal of Operational Research*, June. Elsevier B.V. doi:10.1016/j.ejor.2020.06.032.
- Deutsch, Yael, Noam Goldberg, and Yael Perlman. 2019. "Incorporating Monitoring Technology and On-Site Inspections Into an n-Person Inspection Game." *European Journal of Operational Research* 274 (2): Elsevier BV.V.: 627–637. doi:10.1016/j.ejor.2018.10.012.
- Esenduran, Gökçe, Eda Kemahlioğlu-Ziya, and Jayashankar M. Swaminathan. 2012. "Product Take-Back Legislation and Its Impact on Recycling and Remanufacturing Industries." In *International Series in Operations Research and Management Science* 174: 129–148. Springer New York LLC. doi:10.1007/978-1-4419-6105-1\_9.
- Esenduran, Gökçe, Eda Kemahlioğlu-Ziya, and Jayashankar M. Swaminathan. 2017. "Impact of Take-Back Regulation on the Remanufacturing Industry." *Production and Operations Management* 26 (5): Wiley-Blackwell: 924–944. doi:10.1111/poms.12673.
- Ferguson, Mark, V. Daniel Guide, Eylem Koca, and Gilvan C. Souza. 2009. "The Value of Quality Grading in Remanufacturing." *Production and Operations Management* 18 (3): John Wiley & Sons, Ltd: 300–314. doi:10.1111/J.1937-5956.2009.01033.X.
- Galbreth, Michael R., and Joseph D. Blackburn. 2009. "Optimal Acquisition and Sorting Policies for Remanufacturing." *Production and Operations Management* 15 (3): Wiley: 384–392. doi:10.1111/j.1937-5956.2006.tb00252.x.
- Heydari, Jafar, and Maryam Ghasemi. 2018. "A Revenue Sharing Contract for Reverse Supply Chain Coordination under Stochastic Quality of Returned Products and Uncertain Remanufacturing Capacity." *Journal of Cleaner Production* 197 (October): Elsevier Ltd: 607–615. doi:10.1016/j.jclepro.2018.06.206.
- Jensen, Jonas P., Sharon M. Prendeville, Nancy M.P. Bocken, and David Peck. 2019. "Creating Sustainable Value through Remanufacturing: Three Industry Cases." *Journal of Cleaner Production* 218 (May): Elsevier Ltd: 304–314. doi:10.1016/j.jclepro.2019.01.301.
- Kim, Sungwook. 2017. "Inspection Game Based Cooperative Spectrum Sensing and Sharing Scheme for Cognitive Radio IoT System." *Computer Communications* 105 (June): Elsevier BV.V.: 116–123. doi:10.1016/j.comcom.2017.01.015.
- Kleber, Rainer, Marc Reimann, Gilvan C. Souza, and Weihua Zhang. 2020. "Two-Sided Competition with Vertical Differentiation in Both Acquisition and Sales in Remanufacturing." *European Journal of Operational Research* 284 (2): Elsevier BV.V.: 572–587. doi:10.1016/j.ejor.2020.01.012.
- Kumar, Akhilesh, Ratna Babu Chinnam, and Alper Murat. 2017. "Hazard Rate Models for Core Return Modeling in Auto Parts Remanufacturing." *International Journal of Production Economics* 183 (January): Elsevier BV.V.: 354–361. doi:10.1016/j.ijpe.2016.07.002.
- Kurilova-Palisaitiene, Jelena, Erik Sundin, and Bonnie Poksin-ska. 2018. "Remanufacturing Challenges and Possible Lean Improvements." *Journal of Cleaner Production* 172 (January): Elsevier Ltd: 3225–3236. doi:10.1016/j.jclepro.2017.11.023.
- Liao, Haolan, Qianwang Deng, and Neng Shen. 2019. "Optimal Remanufacture-up-to Strategy with Uncertainties in Acquisition Quality, Quantity, and Market Demand." *Journal of Cleaner Production* 206 (January): Elsevier Ltd: 987–1003. doi:10.1016/j.jclepro.2018.09.167.
- Liao, Haolan, Qingyu Zhang, Neng Shen, and Lu Li. 2020. "Stochastic Analysis of Quality Uncertainty and Optimal Acquisition Strategies for Engine Remanufacturing." *Journal of Cleaner Production* 261 (July): Elsevier Ltd: 121088. doi:10.1016/j.jclepro.2020.121088.
- Mitra, Subrata. 2016. "Optimal Pricing and Core Acquisition Strategy for a Hybrid Manufacturing/Remanufacturing System." *International Journal of Production Research* 54 (5): Taylor and Francis Ltd.: 1285–1302. doi:10.1080/00207543.2015.1067376.
- Mutha, Akshay, Saurabh Bansal, and V. Daniel R. Guide. 2016. "Managing Demand Uncertainty through Core Acquisition in Remanufacturing." *Production and Operations Management* 25 (8): Wiley-Blackwell: 1449–1464. doi:10.1111/poms.12554.
- Pappis, Costas P., Nikos P. Rachaniotis, and Giannis T. Tsoulfas. 2005. "Recovery and Reuse of Maritime Containers: The Blue Container Line Case." In *Managing Closed-Loop Supply Chains*, 57–63. Springer. doi:10.1007/3-540-27251-8\_5.



- Pazoki, Mostafa, and Hamed Samarghandi. 2020. "Take-Back Regulation: Remanufacturing or Eco-Design?" *International Journal of Production Economics* 227 (September): Elsevier BV.V.: 107674. doi:10.1016/j.ijpe.2020.107674.
- Raiffa, Howard. 1997. *Decision Analysis: Introductory Lectures on Choices Under Uncertainty*. New York: McGraw-Hill College.
- Rasmusen, Eric. 2006. *Games and Information: An Introduction to Game Theory*. Berlin: Wiley-Blackwell.
- Robotis, Andreas, Tamer Boyaci, and Vedat Verter. 2012. "Investing in Reusability of Products of Uncertain Remanufacturing Cost: The Role of Inspection Capabilities." *International Journal of Production Economics* 140 (1): Elsevier: 385–395. doi:10.1016/j.ijpe.2012.04.017.
- Ross, Sheldon. 2019. *Introduction to Probability Models*. London: Academic Press.
- Sakao, Tomohiko, and Erik Sundin. 2019. "How to Improve Remanufacturing? - A Systematic Analysis of Practices and Theories." *Journal of Manufacturing Science and Engineering, Transactions of the ASME* 141 (2), American Society of Mechanical Engineers (ASME). doi:10.1115/1.4041746.
- Shi, Zhenyang. 2019. "Optimal Remanufacturing and Acquisition Decisions in Warranty Service Considering Part Obsolescence." *Computers and Industrial Engineering* 135 (September): Elsevier Ltd: 766–779. doi:10.1016/j.cie.2019.06.019.
- Silva Teixeira, Evandro Leonardo, Benny Tjahjono, Sadek Crisóstomo Absi Alfaro, and Richard Wilding. 2019. "Extending the Decision-Making Capabilities in Remanufacturing Service Contracts by Using Symbiotic Simulation." *Computers in Industry* 111 (October): Elsevier BV.V.: 26–40. doi:10.1016/j.compind.2019.06.005.
- Sitharangsie, Sakraan, Winifred Ijomah, and T. C. Wong. 2019. "Decision Makings in Key Remanufacturing Activities to Optimise Remanufacturing Outcomes: A Review." *Journal of Cleaner Production*, Elsevier Ltd. doi:10.1016/j.jclepro.2019.05.204.
- Souza, Gilvan C., Michael E. Ketzenberg, and V. Daniel R Guide. 2009. "Capacitated Remanufacturing with Service Level Constraints." *Production and Operations Management* 11 (2): Wiley: 231–248. doi:10.1111/j.1937-5956.2002.tb00493.x.
- Su, Chun, and Aijuan Xu. 2014. "Buffer Allocation for Hybrid Manufacturing/Remanufacturing System Considering Quality Grading." *International Journal of Production Research* 52 (5): Taylor & Francis: 1269–1284. doi:10.1080/00207543.2013.828165.
- Tagaras, George, and Christos Zikopoulos. 2008. "Optimal Location and Value of Timely Sorting of Used Items in a Remanufacturing Supply Chain with Multiple Collection Sites." *International Journal of Production Economics* 115 (2): Elsevier: 424–432. doi:10.1016/j.ijpe.2007.10.026.
- Tang, Ou, Robert W. Grubbström, and Simone Zanoni. 2007. "Planned Lead Time Determination in a Make-to-Order Remanufacturing System." *International Journal of Production Economics* 108 (1–2): Elsevier: 426–435. doi:10.1016/j.ijpe.2006.12.034.
- Teunter, Ruud H., and S. D. P. Flapper. 2011. "Optimal Core Acquisition and Remanufacturing Policies Under Uncertain Core Quality Fractions." *European Journal of Operational Research* 210 (2): North-Holland: 241–248. doi:10.1016/j.ejor.2010.06.015.
- Van Wassenhove, Luk N., and Christos Zikopoulos. 2010. "On the Effect of Quality Overestimation in Remanufacturing." *International Journal of Production Research* 48 (18): Taylor & Francis Group: 5263–5280. doi:10.1080/00207540903150585.
- Wei, Shuoguo, Ou Tang, and Weihua Liu. 2015. "Refund Policies for Cores with Quality Variation in OEM Remanufacturing." *International Journal of Production Economics* 170 (December): 629–640. Elsevier. doi:10.1016/j.ijpe.2014.12.006.
- Wei, Shuoguo, Ou Tang, and Erik Sundin. 2015. "Core (Product) Acquisition Management for Remanufacturing: A Review." *Journal of Remanufacturing* 5 (1): 1–27. SpringerOpen. doi:10.1186/s13243-015-0014-7.
- Yang, Cheng Hu, Hai bo Liu, Ping Ji, and Xin Ma. 2016. "Optimal Acquisition and Remanufacturing Policies for Multi-Product Remanufacturing Systems." *Journal of Cleaner Production* 135 (November): Elsevier Ltd: 1571–1579. doi:10.1016/j.jclepro.2015.10.057.
- Yang, Cheng Hu, Jian Wang, and Ping Ji. 2015. "Optimal Acquisition Policy in Remanufacturing under General Core Quality Distributions." *International Journal of Production Research* 53 (5): Taylor and Francis Ltd.: 1425–1438. doi:10.1080/00207543.2014.944283.
- Yanikoğlu, İhsan, and Meltem Denizel. 2020. "The Value of Quality Grading in Remanufacturing under Quality Level Uncertainty." *International Journal of Production Research*, January. Taylor and Francis Ltd., 1–21. doi:10.1080/00207543.2020.1711983.
- Zikopoulos, Christos, and George Tagaras. 2008. "On the Attractiveness of Sorting before Disassembly in Remanufacturing." *IIE Transactions* 40 (3): Taylor & Francis Group: 313–323. doi:10.1080/07408170701488078.

## Appendices

### Appendix A. The proof of Proposition 2

#### Strategy A

Given the customer buys the product and the deposit is decided already, the expected profit of the remanufacturer is:

$$\begin{aligned}\pi_R &= v + x(P(L) + \beta P(H)) + q_H(1 - \beta)P(H) \\ &= v + xP(L) + (x - q_H)P(H)\beta + q_HP(H)\end{aligned}\quad (A1)$$

The optimisation problem is:

$$\max_{\alpha} v + xP(L) + (x - q_H)P(H)\beta(\alpha) + q_HP(H)\quad (A2)$$

Subject to:

$$\begin{cases} \alpha < \frac{s_L}{x} \\ \beta(\alpha) < 1 - \frac{s_H}{x} \end{cases}\quad (A3)$$

Since  $(\partial\pi_R/\partial\alpha) = (x - q_H)P(H)\beta'(\alpha)$ , we have:

If  $x < q_H$ ,  $(\partial\pi_R/\partial\alpha) > 0$ ,  $\alpha$  should be increased to improve the profit, as long as (A3) is satisfied, i.e.  $\alpha$  is smaller than but as close to  $(s_L/x)$  as possible. For simplicity, we denote it as  $\alpha = (s_L/x)$  (we use similar notations for the other strategies below), if  $(s_L/x) \geq \beta^{-1}(1 - (s_H/x))$ ; otherwise, no solution for Strategy A. (A2) is rewritten as

$$\max v + x - xP(H) + xP(H)\beta\left(\frac{s_L}{x}\right) - q_HP(H)\beta\left(\frac{s_L}{x}\right)$$

$$+ q_H P(H) \\ = v + x + (q_H - x)P(H) \left(1 - \beta \left(\frac{s_L}{x}\right)\right) \quad (A4)$$

If  $x > q_H$ ,  $(\partial\pi_R/\partial a) < 0$ , so  $\alpha$  should be as small as possible, as long as (A3) is satisfied, that is  $\alpha = \beta^{-1}(1 - (s_H/x))$ . The profit of the remanufacturer is

$$\max v + xP(L) + (x - q_H)P(H) \left(1 - \frac{s_H}{x}\right) + q_H P(H) \\ = v + x + P(H)s_H \left(\frac{q_H}{x} - 1\right) \quad (A5)$$

### Strategy B

Given the customer buys the product and the deposit is decided already, the expected profit of the remanufacturer is  $v + x$ . The problem becomes

$$\max_{\alpha} v + x \quad (A6)$$

Subject to

$$\begin{cases} \alpha < \frac{s_L}{x} \\ \beta(\alpha) > 1 - \frac{s_H}{x} \end{cases} \quad (A7)$$

The choice of  $\alpha$  does not affect the profitability, as long as (A7) is satisfied, that is  $\beta^{-1}(1 - (s_H/x)) < \alpha < (s_L/x)$ , or  $0 \leq \alpha < (s_L/x)$  if  $x < s_H$ .

### Strategy C

Given the customer buys the product and the deposit is decided already, the expected profit of the remanufacturer is:

$$E(\pi_R) = v + x + P(H)(1 - \beta(\alpha))(q_H - x) + P(L)\alpha(q_L - x) \quad (A8)$$

The problem is to optimise

$$\max_{\alpha} v + x + P(H)(1 - \beta(\alpha))(q_H - x) + P(L)\alpha(q_L - x) \quad (A9)$$

Subject to:

$$\begin{cases} \alpha > \frac{s_L}{x} \\ \beta(\alpha) < 1 - \frac{s_H}{x} \end{cases} \quad (A10)$$

Since  $(\partial\pi_R/\partial\alpha) = -\beta'(\alpha)P(H)(q_H - x) + P(L)(q_L - x)$ , we have the following:

If  $x > q_H$ ,  $(\partial\pi_R/\partial a) < 0$ , thus  $\alpha$  should reduce, as long as (A10) is satisfied, that is  $\alpha = \max((s_L/x), \beta^{-1}(1 - (s_H/x)))$ . The profit of the remanufacturer is

$$\max v + x + P(H) \left(1 - \beta \left(\frac{s_L}{x}\right)\right) (q_H - x) + P(L) \frac{s_L}{x} (q_L - x) \quad (A11)$$

or

$$\max v + x + P(H) \frac{s_H}{x} (q_H - x) + P(L) \beta^{-1} \left(1 - \frac{s_H}{x}\right) (q_L - x) \quad (A12)$$

If  $x < q_L$ ,  $(\partial\pi_R/\partial\alpha) > 0$ , then  $\alpha$  should increase as long as (A10) is satisfied. Thus  $\alpha = 1$ , i.e. the remanufacturer gives the refund to all cores, and the profit of the remanufacturer is  $v + P(H)q_H + P(L)q_L$ .

If  $q_L < x < q_H$ , let  $(\partial\pi_R/\partial\alpha) = 0$ , there is  $\beta'(\alpha) = (P(L)(q_L - x))/(P(H)(q_H - x))$ , also  $(\partial^2\pi_R/\partial\alpha^2) = -\beta''(\alpha)P(H)(q_H - x) < 0$ . So if  $\beta'^{-1}((P(L)(q_L - x))/(P(H)(q_H - x))) > \max((s_L/x), \beta^{-1}(1 - (s_H/x)))$ ,  $\alpha = \beta'^{-1}((P(L)(q_L - x))/(P(H)(q_H - x)))$  and the remanufacturer's profit is:

$$\max v + x + P(H) \left(1 - \beta \left(\beta'^{-1} \left(\frac{P(L)(q_L - x)}{P(H)(q_H - x)}\right)\right)\right) \\ \times (q_H - x) + P(L) \beta'^{-1} \left(\frac{P(L)(q_L - x)}{P(H)(q_H - x)}\right) (q_L - x) \quad (A13)$$

If

$$\beta'^{-1} \left(\frac{P(L)(q_L - x)}{P(H)(q_H - x)}\right) < \max \left(\frac{s_L}{x}, \beta^{-1} \left(1 - \frac{s_H}{x}\right)\right),$$

then  $\alpha = \max((s_L/x), \beta^{-1}(1 - (s_H/x)))$ . The profit becomes

$$\max v + x + P(H) \left(1 - \beta \left(\frac{s_L}{x}\right)\right) (q_H - x) + P(L) \frac{s_L}{x} (q_L - x) \quad (A14)$$

or

$$\max v + x + P(H) \left(1 - \beta \left(\beta'^{-1} \left(1 - \frac{s_H}{x}\right)\right)\right) (q_H - x) \\ + P(L) \beta'^{-1} \left(1 - \frac{s_H}{x}\right) (q_L - x) \quad (A15)$$

To conclude, we have

$$\alpha = \max \left(\frac{s_L}{x}, \beta^{-1} \left(1 - \frac{s_H}{x}\right), \beta'^{-1} \left(\frac{P(L)(q_L - x)}{P(H)(q_H - x)}\right)\right).$$

### Strategy D

Given the customer buys the product and the deposit is decided already, the expected profit of the remanufacturer is:

$$E(\pi_R) = P(H)(v + x) + P(L)\alpha(v + q_L) \\ + P(L)(1 - \alpha)(v + x) \\ = v + x + P(L)\alpha q_L - P(L)\alpha x \quad (A16)$$

The problem is to solve:

$$\max_{\alpha} v + x + P(L)\alpha q_L - P(L)\alpha x \quad (A17)$$

Subject to:

$$\begin{cases} \alpha > \frac{s_L}{x} \\ \beta(\alpha) > 1 - \frac{s_H}{x} \end{cases} \quad (A18)$$

Since  $(\partial\pi_R/\partial\alpha) = P(L)(q_L - x)$ , we have:

If  $x > q_L$ ,  $(\partial\pi_R/\partial\alpha) < 0$ ,  $\alpha$  should reduce as (A18) is satisfied, that is  $\alpha = (s_L/x)$ . The remanufacturer's profit is

$$\max v + x + P(L) \frac{s_L}{x} (q_L - x) \quad (A19)$$

Similarly if  $x < q_L$ , then  $\alpha = \beta^{-1}(1 - (s_H/x))$  and the profit is

$$\max v + x + P(L) \beta^{-1} \left(1 - \frac{s_H}{x}\right) (q_L - x) \quad (A20)$$

And if  $x < s_H$ ,  $\alpha = 1$  and the profit becomes

$$\max v + x + P(L)(q_L - x) \quad (A21)$$

## Appendix B. The proof of Proposition 3

If the remanufacturer can choose deposit  $x$  and inspection error  $\alpha$ , Strategy D will not be optimal.

Proof: Let  $\alpha = 1$ , since  $(u + P(H)s_H)/(P(H)) > s_H$ , there is no constraint to limit  $x$  being smaller than  $s_H$ , and once  $x$  exceeds  $s_H$ , the payoff of Strategy D will be less than either Strategy B if  $q_L < s_H$ , or Strategy C if  $q_L < s_H$ . So, Strategy D will not be optimal.

## Appendix C. The solution procedure for optimising deposit $x$ and inspection error $\alpha$

We discuss the choices of deposit and inspection error in three cases. Firstly we analyse the case when  $q_L < x < q_H$ , then the cases when  $x > q_H$  and  $x < q_L$ .

*Case 1.* When  $q_L < x < q_H$ , the remanufacturer would prefer Strategy A if it is reachable. If it is not reachable, the remanufacturer has to choose from Strategies B and C by comparing:  $\pi_R^b = v + x$ ,  $\pi_R^c = v + x + P(H)(1 - \beta(\alpha))(q_H - x) + P(L)\alpha(q_L - x)$ .

Let us assume that Strategy A is reachable. In this case, only high-quality cores will be returned and  $\alpha = (s_L/x)$ . There is  $x \leq (u + P(L)s_L)/(P(L) + P(H)\beta(s_L/x))$  to ensure the customer buy the product. We further have three conditions for deciding  $x$ .

If  $((u + P(L)s_L)/(P(L) + P(H)\beta)) < q_L$ , Strategy A will not be the solution, because there is  $x < ((u + P(L)s_L)/(P(L) + P(H)\beta(s_L/x))) < q_L$ , which contradicts with  $q_L < x < q_H$ .

If  $q_L < ((u + P(L)s_L)/(P(L) + P(H)\beta(s_L/x^*))) < q_H$ , then  $x^* = (u + P(L)s_L)/(P(L) + P(H)\beta(s_L/x^*))$ , and  $\pi_R = v + u + P(L)s_L + q_H(1 - \beta)P(H)$ . Note that we need to implicitly solve the value of  $x^*$ .

If  $((u + P(L)s_L)/(P(L) + P(H)\beta(s_L/q_H))) > q_H$ , the remanufacture chooses  $x^* = q_H$ ,  $\pi_R = v + q_H$ .

*Case 2.* When  $x > q_H$ , the remanufacturer prefers Strategy B as the best choice. To ensure the customer buy the product, there is  $x \leq u + P(L)s_L + P(H)s_H$ . Similarly, we have conditions indicating the relationship between  $q_H$  and  $u + P(L)s_L + P(H)s_H$ .

If  $u + P(L)s_L + P(H)s_H > q_H$ , the optimal profit  $\pi_R = v + u + P(L)s_L + P(H)s_H$  is obtained by letting  $x^* = u + P(L)s_L + P(H)s_H$ , and

$$\begin{cases} \alpha < \frac{s_L}{u + P(L)s_L + P(H)s_H} \\ \beta(\alpha) > 1 - \frac{s_H}{u + P(L)s_L + P(H)s_H} \end{cases}$$

which is  $\alpha < \min((s_L/(u + P(L)s_L + P(H)s_H)), \beta^{-1}(1 - (s_H/(u + P(L)s_L + P(H)s_H))))$ .

If  $u + P(L)s_L + P(H)s_H < q_H$ , the remanufacturer cannot choose a deposit higher than  $q_H$ , otherwise the condition  $x > q_H$  contradicts. Thus, Strategy B cannot be the best choice.

*Case 3.* When  $x < q_L$ , the remanufacturer prefers Strategy C as the best choice. The remanufacturer will set  $\alpha = 1$  and  $\pi_R = v + P(H)q_H + P(L)q_L$ . The constraint to ensure the customer buy the product becomes  $x \leq (u/P(L)(1 - \alpha) + P(H)\beta)$ , which tends to infinity as  $\alpha$  close to 1. Thus, the constraint is always fulfilled.

To summarise, when deposit  $x$  and classification error  $\alpha$  are optimised simultaneously, the solution procedure should be as follows.

*Step 1.* Calculate  $\pi_R^A$ ,  $\pi_R^B$  and  $\pi_R^C$  as

$$\pi_R^A = \begin{cases} z + q_H & \text{if } \frac{u + P(L)s_L}{P(L) + P(H)\beta(s_L/q_H)} > q_H \\ & \text{and} \\ & \text{Strategy A is reachable} \\ v + u + P(L)s_L & \text{if } \max(q_L, s_H) < x \\ + q_H(1 - \beta)P(H) & \\ = \frac{u + P(L)s_L}{P(L) + P(H)\beta(s_L/x)} \leq q_H & \\ & \text{and Strategy A is reachable} \\ \text{Strategy A cannot} & \text{else} \\ \text{be optimal} & \end{cases}$$

The first two expressions of  $\pi_R^A$  are directly from the derivation in *Case 1*. The third case occurs when the deposit limit is too low, or Strategy A is not reachable so that the remanufacturer chooses either Strategy B or Strategy C.

$$\pi_R^B = \begin{cases} v + u + P(L)s_L & \text{if } u + P(L)s_L + P(H)s_H \geq q_H \\ + P(H)s_H & \\ \text{Strategy A cannot} & \text{else} \\ \text{be optimal} & \end{cases}$$

$$\pi_R^C = v + P(H)q_H + P(L)q_L$$

*Step 2.* Compare  $\pi_R^A$ ,  $\pi_R^B$  and  $\pi_R^C$ , and choose the solution with the highest profit.

## Appendix D. Development of Table 2

If  $q_L < x < q_H$ , the best outcome for the remanufacturer is to refund only high-quality cores. There are two scenarios depending on whether Strategy A is reachable.

### Strategy A is not reachable

In this case, the remanufacturer would prefer either no return of Strategy B, or all return of Strategy C. With Strategy B, there is no mixed strategy for the customer since the remanufacturer simply sets  $\alpha = 0$  and  $\beta = 1$ , i.e. no refund, and it does not influence the remanufacturer's profit. With Strategy C, the remanufacturer chooses  $\alpha = \beta'^{-1}((P(L)(q_L - x))/(P(H)(q_H - x)))$  to maximise the profit. If  $\beta'^{-1}((P(L)(q_L - x))/(P(H)(q_H - x))) > \beta^{-1}(1 - (s_H/x))$ , the customer will return all cores, and no mixed strategy will be used. Still, if  $\frac{s_L}{x} < \beta'^{-1}((P(L)(q_L - x))/(P(H)(q_H - x))) \leq \beta^{-1}(1 - (s_H/x))$ , the customer will return only the low-quality cores, i.e. Strategy D, consequently the remanufacturer chooses  $\alpha$  as small as possible and set  $\alpha = 0$ , and the customer switches to Strategy B, i.e. no core will be returned. This will end as the equilibrium solution. As a result, if Strategy A is not reachable, no mixed strategy will be used by the customer, and in equilibrium, no core is returned.

### Strategy A is reachable

When the customer returns only high-quality cores (Strategy A), the remanufacturer attempts to increase  $\alpha$  and choose  $\alpha = 1$ ,  $\beta = 0$ . In this case, the customer has the motivation to return all low-quality cores as well. The remanufacturer would

reduce  $\alpha$  so that  $\alpha = \beta'^{-1}((P(L)(q_L - x))/(P(H)(q_H - x)))$ , if  $\beta'^{-1}((P(L)(q_L - x))/(P(H)(q_H - x))) > (s_L/x)$ , in which case the customer will return all cores, and this becomes the equilibrium solution since no player attempts to move away from it; however, if  $\beta'^{-1}((P(L)(q_L - x))/(P(H)(q_H - x))) < (s_L/x)$ , the customer will not return any low-quality cores.

As a result, assume that the customer chooses to return the low-quality cores with probability  $p_L$ , and the remanufacturer's profit becomes:

$$\begin{aligned} \pi_R(\alpha, p_L) &= (1 - p_L)(v + x + P(H)(1 - \beta(\alpha))(q_H - x)) \\ &\quad + p_L(v + x + P(H)(1 - \beta(\alpha))(q_H - x)) \\ &\quad + P(L)\alpha(q_L - x) \\ &= v + x + P(H)(1 - \beta(\alpha))(q_H - x) \\ &\quad + P(L)p_L\alpha(q_L - x) \end{aligned} \quad (A22)$$

There is  $((\partial\pi_R^2(\alpha, p_L))/\partial\alpha^2) = -\beta''(\alpha)P(H)(q_H - x) < 0$ . Let  $((\partial\pi_R(\alpha, p_L))/\partial\alpha) = -\beta'(\alpha)P(H)(q_H - x) + P(L)p_L(q_L - x) = 0$ , the remanufacturer chooses

$$\alpha = \beta'^{-1}\left(\frac{P(L)p_L(q_L - x)}{P(H)(q_H - x)}\right) \quad (A22)$$

Combined with the condition of using the mixed strategy for returning low-quality cores  $\alpha^* = s_L/x$ , there is

$$\alpha = \beta'^{-1}\left(\frac{P(L)p_L(q_L - x)}{P(H)(q_H - x)}\right) = \frac{s_L}{x} \quad (A23)$$

$$p_L^* = \beta'\left(\frac{s_L}{x}\right) \frac{P(H)(q_H - x)}{P(L)(q_L - x)} \quad (A24)$$

Thus, the customer chooses to return the low-quality core with probability  $p_L^*$ . In this case, the remanufacturer's profit is

$$\begin{aligned} \pi_R(\alpha, p_L) &= v + x + P(H)(q_H - x) \\ &\quad \times \left( \left(1 - \beta\left(\frac{s_L}{x}\right)\right) + \frac{s_L}{x} \beta'\left(\frac{s_L}{x}\right) \right) \end{aligned} \quad (A25)$$

and the customer's profit is,

$$\begin{aligned} E(\pi_C) &= (1 - p_L^*)(P(L)(u - x + s_L) \\ &\quad + P(H)(u - \beta(\alpha^*)x)) \\ &\quad + p_L^*(P(L)(u - (1 - \alpha^*)x) \\ &\quad + P(H)(u - \beta(\alpha^*)x)) \end{aligned} \quad (A26)$$

$p_L^*$  increases with  $P(H)$  and decreases with  $P(L)$ . When there are more high-quality cores and less low-quality cores, the customer is more likely to return his/her low-quality cores. The explanation is the following: if the customer keeps the same  $p_L^*$ , the remanufacturer tends to increase  $\alpha$ , as  $\alpha = \beta'^{-1}((P(L)p_L(q_L - x))/(P(H)(q_H - x)))$  leads the most profit to the remanufacturer. However, in such a case, the customer has a better choice to return more low-quality cores, instead of keeping the same  $p_L^*$ . Finally, the equilibrium is that the customer increases  $p_L^*$  to keep  $\beta'^{-1}((P(L)p_L(q_L - x))/(P(H)(q_H - x))) = s_L/x$ , and the customer does not receive any additional refund from low-quality cores since the remanufacturer keeps  $\alpha = s_L/x$ .  $p_L^*$  also increases with  $(q_H - x)$  and decreases with  $(x - q_L)$ . The explanation is similar to the above discussion.