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The forecasting performance of implied volatility

Does the level of volatility matter?

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ABSTRACT:
This thesis examines the forecasting accuracy of implied volatility and investigates whether the expected volatility of implied volatility or the market volatility level affects the forecasting performance. Therefore, the OLS and quantile regressions are used to test the relationship between future realised volatility and the VIX index on various levels of market volatility. In addition, the impact of the VVIX index on the absolute percentage deviation between S&P 500 realised volatility and one-month lagged VIX is examined. The realised volatility is estimated using the range-based volatility estimator.

Consistent with prior literature, the results show that implied volatility is a reasonable but biased forecast of future volatility over a one-month forecasting horizon. The conclusion remains the same regardless of whether overlapping data, non-overlapping data or log-transformed variables are used. The forecasting accuracy seems to increase during periods of high market volatility. However, the results for the quantile regression indicate that the forecasting accuracy does not significantly vary within the level of realised volatility. Moreover, the implied volatility of the VIX index seems to be negatively associated with the forecasting performance of VIX.

KEYWORDS: Implied volatility, volatility forecasting, VIX, range-based volatility

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TIIVISTELMÄ:

Tämä tutkielma tarkastelee implisiittisen volatiliteetin kykyä ennustaa tulevaisuuden realisoitunutta volatiliteettia. Tavoitteena on selvittää, vaikuttaako implisiittisen volatiliteetin odotettu volatiliteetti tai markkinavolatiliteetin taso ennustetarkkuuteen. Pienimmän neliösumman menetelmää ja kvantaaliregressiota käytetään tulevaisuuden realisoituneen volatiliteetin ja VIX-indeksin välisen yhteyden analysointiin volatiliteetin eri tasoilla. Lisäksi tarkastellaan, vaikuttaako VVIX-indeksi S&P 500 -indeksin ja yhden kuukauden takaisen VIX-indeksin absoluuttiseen prosentuaalisen hajontaan. Realisoitunut volatiliteetti estimoidaan käyttäen päivittäisten avaus- ja päätösarvojen lisäksi päivän korkeinta sekä matalinta arvoa.

Aiempien tutkimusten mukaisesti tutkimustulokset osoittavat, että implisiittinen volatiliteetti kykenee ennustamaan tulevaa 30 päivän volatiliteettia, mutta se ei ole harhaton estimaattori. Johtopäätökset pysyvät muuttumattomina riippumatta siitä, käytetäänkö päällekkäisiä havaintoja tai muuttujien logaritmisia muunnoksia. Ennustetarkkuus näyttää kasvavan korkean markkinavolatiliteetin aikoina. Toisaalta kvantaaliregression tulosten mukaan ennustetarkkuudessa ei ole tilastollisesti merkittäviä eroja realisoituneen volatiliteetin eri tasojen välillä. VIX-indeksioptioista johdetulla implisiittisellä volatiliteetilla näyttää puolestaan olevan negatiivinen vaikutus VIX-indeksin ennustekykyyn.

AVAINSANAT: Implied volatility, volatility forecasting, VIX, range-based volatility

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1 Introduction

Volatility plays a significant role in finance (Andersen and Bollerslev, 1998; Dumas et al., 1998). It is applied to explain price movements (Schwert, 1989; Grullon et al., 2012), manage risk (Fleming et al., 2001), value financial derivatives (Black & Scholes, 1973; Cox et al., 1979), and construct profitable trading strategies (Cremers et al., 2015; Moreira & Muir, 2017). Therefore, both volatility estimation and forecasting have received rather unsurprisingly considerable attention in the literature over the past decades (e.g., Engle, 1982; Bollerslev, 1986; Figlewski, 1997; Christensen & Prabhala, 1998; Britten-Jones & Neuberger, 2000; Seo & Kim, 2015).

Financial volatility is forecastable (Engle, 1993; Poon & Granger, 2005), but there seems to be no agreed consensus on the most efficient forecasting model (e.g., Figlewski, 1997; Taylor et al., 2010). Since most studies focus on comparing the performance of various models, little attention is given to investigate the factors affecting the forecasting performance of implied volatility. Instead, commonly found biases in the implied volatility forecasts are entirely explained by market inefficiency. Hence, this thesis aims to provide a more comprehensive analysis of the forecasting performance of implied volatility.

1.1 Purpose of the paper

Even though implied volatility is often found as the most accurate forecast of future volatility (e.g., Latané & Rendleman, 1976; Jorion, 1995; Poon & Granger, 2005), the forecast accuracy appears to be inconsistent and imperfect (Figlewski, 1997; Christensen & Prabhala, 1998). Furthermore, implied volatility tends to be a biased estimate of future volatility despite the calculation method (Jiang & Tian, 2005; Biktimirov & Wang, 2017). Consequently, the purpose of this thesis is to examine how accurately implied volatility can forecast future realised volatility. The addressed research question leads naturally to the following hypothesis:

H_1: Implied volatility forecasts future realised volatility.

The topic is widely discussed in the literature, and it seems that implied volatility is a reasonable predictor. However, in contrast to the majority of previous studies, which commonly use squared returns (e.g., Canina & Figlewski, 1993; Jorion, 1995; Christensen & Prabhala, 1998; Corrado & Miller, 2005; Biktimirov & Wang, 2017) or high-frequency data (e.g., Blair et al., 2001; Busch et al., 2011; Seo & Kim, 2015), the Garman-Klass (1980) range-based estimator is applied to estimate realised volatility. In addition, the recent behaviour of the VIX index, especially the high values during 2020, produce data that may reveal unrecognised features of volatility forecasting.

As the first hypothesis provides limited evidence of the forecasting accuracy, the information content of implied volatility requires further analysis. In the context of informationally efficient financial markets, implied volatility should be an unbiased estimate that contains all the available information regarding future volatility (Figlewski, 1997). If implied volatility was not the most accurate forecast of future realised volatility, profitable trading strategies based on mispriced options could be built (Jorion, 1995). Hence, the second hypothesis is:

H_2: Implied volatility is an unbiased estimate of future volatility.

To avoid the joint hypothesis problem (e.g., Fama, 1991; Jorion, 1995), the VIX index is used to estimate implied volatility in this thesis. The VIX is based on the model-free implied volatility (Jiang & Tian, 2005; Biktimirov & Wang, 2017), which facilities a direct test of the informational efficiency instead of a joint test (Jiang & Tian; 2005).

In prior research, the forecasting accuracy is found to vary over time (e.g., Christensen & Prabhala, 1998; Seo & Kim, 2015; Wang & Wang, 2016; Plíhal & Lyócsa, 2021). For instance, Plíhal and Lyócsa (2021) find the forecasting power to increase during periods of high market volatility. Furthermore, implied volatility tends to over-forecast high volatility and under-forecast low volatility (Poon & Granger, 2005). For investigating the potential changes in the forecasting performance, the third hypothesis is:

H_3: The forecasting performance of implied volatility is not affected by the level of market volatility.

In the same way, the forecasting accuracy appears to be affected by the level of investor sentiment (Seo & Kim, 2015). Since implied volatility is considered the market's assessment of future volatility (Mayhew, 1995), and volatility is inconstant (Schwert, 1989; Andersen & Bollerslev, 1997), the expected volatility of implied volatility could affect the forecasting accuracy. Therefore, the fourth hypothesis is:

H_4: The forecasting performance of implied volatility is not affected by the expected volatility of implied volatility.

For this purpose, the VVIX index is used as an estimate of the expected volatility (implied volatility) of implied volatility.

1.2 Structure of the paper

The rest of this thesis is organised as follows. Section 2 discusses the theoretical background of option valuation and volatility estimation. Section 3 reviews the prior literature related to the forecasting performance of implied volatility. Section 4 describes the data used, descriptive statistics, and the methodology for conducting the thesis. Section 5 presents the empirical results. Section 6 concludes the paper.

2 Theoretical background

As implied volatility is derived from option prices, and volatility is an unobservable variable, both option valuation and volatility estimation are discussed in this section. Furthermore, the VIX and VVIX indices are examined since they represent implied volatilities in this thesis.

2.1 Option value

Despite the long history of options trading (e.g., Franklin & Colberg, 1958; Kairys & Valerio, 1997; Mixon, 2009; Haug & Taleb, 2011), not until 1973 was a listed options exchange introduced (Cox et al., 1979; Kairys & Valerio, 1997). In the same year, Black and Scholes (1973) presented the option pricing formula that has substantially impacted option pricing theory (Jarrow, 1999). Since then, both the options market and the literature relating to option valuation have expanded rapidly (Cox et al., 1979; Mixon, 2009; Hull, 2015).

Before the introduction of exchange-traded options, option pricing remained somewhat mysterious as the price quotations were not published (Franklin & Colberg, 1958). Nevertheless, options seem to have been overpriced relative to theoretical valuation models (Kairys & Valerio, 1997), and the difference between the market prices and theoretical values have since decreased (Mixon 2009). Mixon (2009) argues that the shift in option prices toward their theoretical values stems mainly from the opening of the exchange rather than from the publication of the Black-Scholes (1973) model, and option pricing in practice has not considerably changed over time. In addition, Haug and Taleb (2011) suggest that options have been priced based on sophisticated heuristics and tricks, at least since 1902.

Even though traders price options arguably the same way as before (Mixon, 2009; Haug & Taleb, 2011), the option pricing theory has evolved over the years (Jarrow, 1999). The origin of the theory arises from Bachelier's (1900) work, where essential mathematics related to Brownian motions and option valuation are derived (Jarrow, 1999). These findings are since applied and extended in many studies that attempt to discover a theoretical value for an option (e.g., Boness, 1964; Samuelson, 1965; Stoll, 1969; Black & Scholes, 1973).

In the early literature, it is well recognised that option values vary depending on the exercise price, volatility, expected growth rate of the underlying asset, and time to expiration (Boness, 1964). The option value is still determined by these variables, including the initial value of the underlying asset and potential dividends (Hull, 2015). However, in the more recent literature (e.g., Black & Scholes, 1973), option values are often calculated under the assumption of risk-neutral valuation. Thus, the expected return from the underlying asset is assumed to be the risk-free interest rate (Hull, 2015).

The option value can alternatively be considered in relation to its *intrinsic value* and (time) premium (Carr & Jarrow, 1990; Quigg, 1993). Because an option gives the right to buy or sell the underlying asset, its intrinsic value must be equal to the difference between the underlying asset's current price and the exercise price but not less than zero (Carr & Jarrow, 1990; Hull, 2015). The difference between the option price and its intrinsic value is referred to as *time value* (Carr & Jarrow, 1990). Hence, the value of a call option is

$$c = \max(S_0 - K, 0) + time\ value, \tag{1}$$

and the value of a put option is

$$p = \max(K - S_0, 0) + time\ value, \tag{2}$$

where S_0 is the current price of the underlying asset, and K is the exercise price (Carr & Jarrow, 1990; Hull, 2015).

For the relationship between European put and call option values, Stoll (1969) formalises the *put-call parity*. When the risk-free interest rate is continuously compounded, the put-call parity is given by

$$c + Ke^{-rT} = p + S_0, \tag{3}$$

where c is the call option price, r is the risk-free rate, T is the time to expiration, and p is the put option price (Hull, 2015). If the relationship is considered for American options or dividend-paying stocks, the put-call parity requires adjustments (Hull, 2015). The empirical results show that, while the put-call parity holds on average, it frequently misprices options (Stoll, 1969; Klemkosky & Resnick, 1979; Evnine & Rudd, 1985; Kamara & Miller, 1995). However, Kamara and Miller (1995) suggest that the mispricing may happen less frequently and on a smaller scale with European than American options.

The Black-Sholes (1973) option pricing model is "the first completely satisfactory equilibrium" model for option valuation (Cox et al., 1979). Since the model is expanded by Merton (1973), it is commonly known as the Black-Scholes-Merton model (Jarrow, 1999). The Black-Scholes formula for a European call option is

$$c = S_0N(d_1) - Ke^{-rT}N(d_2), \tag{4}$$

and for a European put option

$$p = Ke^{-rT}N(-d_2) - S_0N(-d_1), \tag{5}$$

where

$$d_1 = \frac{\ln(S_0/K) + (r + \sigma^2/2)T}{\sigma\sqrt{T}}, \quad (6)$$

$$d_2 = \frac{\ln(S_0/K) + (r - \sigma^2/2)T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T}, \quad (7)$$

S_0 Is the current stock price, K Is the exercise price, r is the continuously compounded risk-free rate, T is the time to expiration, σ is the stock price volatility, $N(d_1)$ and $N(d_2)$ are the cumulative probability distributions of standard normal distributions for the areas below d_1 and d_2, respectively (Black & Scholes, 1973; Hull, 2015).

Along with the Black-Scholes (1973) model's high impact on the literature (e.g., Rubinstein, 1994; Jarrow, 1999), its failure to price options correctly is well recognised (Black, 1975; Macbeth & Merville 1979; Rubinstein, 1985; Hull & White, 1987; Lauterbach & Schultz, 1990; Mayhew, 1995). Even though the strict assumptions behind the Black-Scholes (1973) model are regularly violated in the real market (Lauterbach & Schultz, 1990), many of these assumptions, such as the absence of riskless arbitrage opportunities, seem to be rather necessary for option valuation (Hull, 2015).

The binomial option pricing model (Cox et al., 1979) is a more simplified approach for valuing options. In contrast to the Black-Scholes (1973) model, the binomial model allows for calculating the underlying asset and the option for multiple periods (Cox et al., 1979). The option value is solved by constructing a binomial tree, which represents the possible outcomes of the stock price in each time step until the expiration (Cox et al., 1979; Hull, 2015). Because the binomial model can incorporate the changes at different periods, it is also suitable for valuing American options (Hull, 2015).

A general two-step binomial tree is illustrated in Figure 1. Since the price of the underlying asset is assumed to have two possible outcomes, up to u times its initial value and down to d times its initial value, the option price, f, in each step is calculated as

$$f = e^{-r\Delta t}[qf_u + (1-q)f_d], \quad (8)$$

where the probability of an up movement, q, in a risk-neutral world is

$$q = \frac{e^{r\Delta t} - d}{u - d}, \quad (9)$$

r Is the risk-free interest rate, Δt is the length of the time step, f_u is the option if the stock moves up, andf_d is the option value if the stock moves down (Hull, 2015). S_0 in Figure 1 denotes the initial value of the underlying asset.

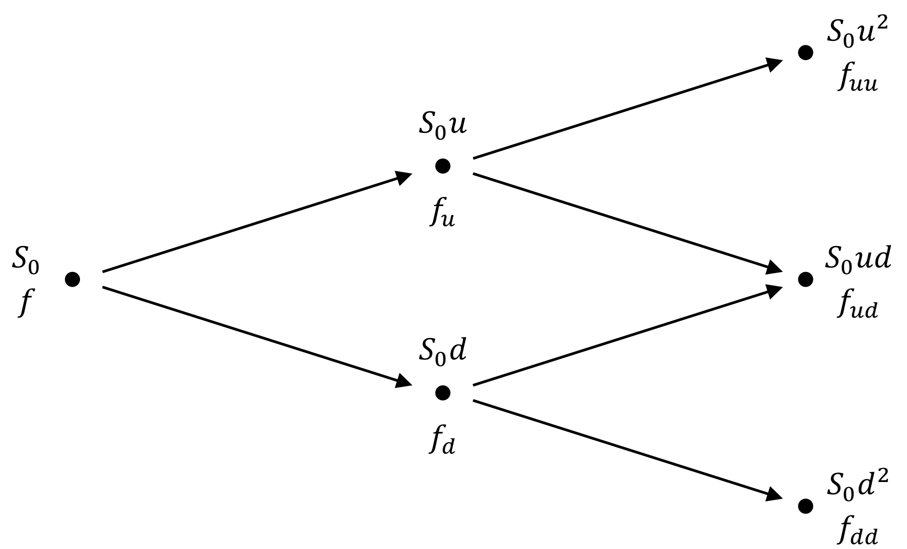

Figure 1. A general two-step binomial tree (Hull, 2015).

In searching for a realistic result, the parameters u and d should be chosen to match the underlying asset's volatility (Cox et al., 1979; Hull, 2015). Therefore, the proportional up movement of the underlying asset is

$$u = e^{\sigma\sqrt{\Delta t}}, \tag{10}$$

and the proportional down movement is

$$d = e^{-\sigma\sqrt{\Delta t}}, \tag{11}$$

where σ is the volatility of the underlying asset (Cox et al., 1979; Hull, 2015). Furthermore, when considering the real world instead of the risk-neutral world, the probability of the up move is given by

$$q^* = \frac{e^{\mu\Delta t} - d}{u - d}, \tag{12}$$

where μ is the expected return of the underlying asset (Cox et al., 1979; Hull, 2015).

Out of all the factors that directly impact option value, volatility is the only one that is not observable in the market (Mayhew, 1995; Hull, 2015). Thus, option valuation is closely related to the estimation of volatility, and this relationship depends on the assumptions underlying the option valuation formula (Dumas et al., 1998). For instance, as the assumption of constant volatility is often unrealistic (Schwert, 1989; Andersen & Bollerslev, 1997), the option pricing models may misprice options relative to their market prices (Black, 1975; Macbeth & Merville 1979; Lauterbach & Schultz, 1990). Consequently, several other option pricing models and extensions of the Black-Scholes (1973) and binomial (Cox et al., 1979) models are developed in the literature (e.g., Boyle, 1977; Hull & White, 1987; Heston, 1993; Rubinstein, 1994).

2.2 Volatility

Volatility is a measure of the dispersion of possible outcomes around the expected value, i.e., the uncertainty about the returns (Hull, 2015; Bodie et al., 2018). It is widely used to estimate investment risk among practitioners and academic research (Schwert, 1990). Even though volatilities of financial assets are assumed to be constant in numerous models (e.g., Black & Scholes, 1973), empirical evidence shows that volatility is usually highly variable, persistent and changes over time (Schwert, 1989; Andersen & Bollerslev, 1997; Moreira & Muir, 2017). Furthermore, actual volatility is unobservable and must be estimated over a specified period (Molnár, 2012; Corsi et al., 2013).

Volatility is generally measured by calculating the standard deviation of returns (Schwert, 1990), although there is no agreed consensus on the most efficient measure (Engle & Gallo, 2006). The sample standard deviation of returns is given by

$$\hat{\sigma} = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(R_i - \bar{R})^2}, \tag{13}$$

where R_i is the return at time interval i, $\bar{R}$ is the average return (sample mean), and n denotes the number of observations (Ederington & Guan, 2006; Hull, 2015). Since stock prices are widely assumed to follow a geometric Brownian motion,

$$dS = \mu\, S\, dt + \sigma\, S\, dz \tag{14}$$

or

$$\frac{dS}{S} = \mu\, dt + \sigma\, dz, \tag{15}$$

where dS is the asset price change over a time interval dt, μ is the mean return, σ is the volatility, and dz is a Wiener process, a stock return over any period i can be calculated as

$$R_i = \ln\left(\frac{S_i}{S_{i-1}}\right), \tag{16}$$

where S_i is the stock price at the end of the interval i (Figlewski, 1997; Hull, 2015).

When volatility is computed from historical returns, the estimated volatility refers to *historical volatility* (Figlewski, 1997; Poon & Granger, 2005). In estimating historical volatility, asset prices are commonly observed at fixed intervals of time, e.g., daily closing prices (Molnár, 2012; Hull, 2015). However, because asset prices do not follow the geometric Brownian motion exactly, historical volatility varies depending on the length of intervals in the estimation (Figlewski, 1997). Moreover, the sample mean is a noisy estimate of the true mean in financial time series, particularly over short horizons (Figlewski, 1997; Poon & Granger, 2005). Thus, instead of calculating the deviation from the average return, the sample mean is sometimes set to zero as average short-term (e.g., daily or weekly) returns tend to be close to zero (Figlewski, 1997; Poon & Granger, 2005; Molnár, 2012).

Even though the mean estimate cannot be improved by sampling data more frequently (Poon & Granger, 2005), the use of high-frequency data increases the accuracy of estimates of actual volatility (Andersen & Bollerslev, 1998; Poon & Granger, 2005; Molnár, 2012). For instance, Andersen and Bollerslev (1998) find that volatility measures based on high-frequency returns reduce noise and improve temporal stability relative to measures based on daily returns. However, volatility estimation from intraday high-frequency data is complex in practice because of the issues with data accessibility and market microstructure features (Andersen & Bollerslev, 1998; Molnár, 2012). Furthermore, sampling at longer intervals can limit the effect of serial dependence (Figlewski, 1997).

As discussed above, the assumption of constant volatility is often unrealistic. Consequently, Engle (1982) introduces the autoregressive conditional heteroscedasticity (ARCH) model to recognise the time-varying volatility. In the ARCH model, the conditional variance of the error term at a particular time point is described as a function of past residuals, which implies that the volatility of the time series varies depending on the data point (Engle, 1982). The ARCH model and its extensions are widely applied in volatility modelling (Bollerslev et al., 1992; Figlewski, 1997; Hansen et al., 2012).

The generalized autoregressive conditional heteroscedasticity (GARCH) model, proposed by Bollerslev (1986), is an extension of the ARCH model. The GARCH model includes past conditional variances in the current conditional variance equation (Bollerslev, 1986). Thus, the volatility at a certain time point is dependent not only on the past squared residuals but also on the past conditional variances (Bollerslev, 1986). The GARCH models seem to work better with short estimating horizons (Lamoureux & Lastrapes, 1993; Figlewski, 1997). However, the GARCH models based on daily returns cannot precisely capture volatility (Hansen et al., 2012; Molnár, 2012). For instance, Andersen et al. (2003) find that the standard GARCH model is unsuitable for situations where volatility changes rapidly to a new level.

Since high-frequency data is not always available for every financial asset, and volatility measures based only on low-frequency closing prices may be inaccurate and inefficient, range-based volatility estimators are proposed in the literature (e.g., Parkinson, 1980; Garman & Klass, 1980; Rogers & Satchell, 1991; Alizadeh et al., 2002; Brandt & Diebold, 2006; Yang & Zhang, 2000). These estimators use the price range, defined as the difference between the highest and lowest log asset prices over a fixed sampling interval, in contrast to the daily return-based volatility measures that use only the information contained in opening and closing prices (Alizadeh et al., 2002; Brandt & Diebold, 2006).

Because range-based approaches recognise volatility information from the entire intraday price path, they can improve the estimation accuracy and even challenge the estimators based on high-frequency data (Alizadeh et al., 2002; Brandt & Diebold, 2006; Molnár, 2012). For instance, Molnár (2012) show that the daily returns normalised by the standard deviations calculated from Garman-Klass (1980) formula are almost normally distributed, and this result is similar to the results that Andersen et al. (2001) obtain from high-frequency data. In addition, the range-based estimator is robust to market microstructure noise arising from bid-ask bounce (Alizadeh et al., 2002; Brandt & Diebold, 2006).

The Garman-Klass (1980) range-based estimator is an improvement on the Parkinson (1980) estimator as, in addition to the high and low prices, the opening and closing prices are included in the formula (Alizadeh et al., 2002). The Garman-Klass (1980) volatility estimator is given by

$$\hat{\sigma}^2_{Garman-Klass} = 0.5(h-l)^2 - (2\ln 2 - 1)c^2, \tag{17}$$

where the open-to-close return, c, is

$$c = \ln(C) - \ln(O), \tag{18}$$

the open-to-high return, h, is

$$h = \ln(H) - \ln(O), \tag{19}$$

the open-to-low return, l, is

$$l = \ln(L) - \ln(O), \tag{20}$$

C is the daily closing price, O is the opening price, H is the highest price of the day, and L is the lowest price of the day (Garman & Klass, 1980; Molnár, 2012).

Despite the strong results of producing accurate volatility estimates (Molnár, 2012), the Garman-Klass (1980) estimator has limitations. For instance, it does not recognise opening jumps and depends on the continuous-time geometric Brownian motion with zero drift (Yang & Zhang, 2000; Shu & Zhang, 2006). If the drift term is significant, the Garman-Klass (1980) estimator overestimate the actual variance (Shu & Zhang, 2006). Nonetheless, the suitability of range estimators in historical volatility estimation is supported by the empirical results (Shu & Zhang, 2006; Molnár, 2012).

2.3 Implied volatility

In contrast to backwards-looking historical volatility, *implied volatility* is considered the market’s assessment of the volatility derived from option prices, i.e., the expected volatility of the underlying asset over the option’s maturity (Mayhew, 1995; Hull, 2015). Implied volatility is calculated by inverting the given option pricing formula to determine the volatility implied by the option market prices (Mayhew, 1995). The interpretation of implied volatility depends on the assumption of volatility (Mayhew, 1995). Under the strict assumptions of the Black-Scholes (1973) model, implied volatility represents a market’s estimate of the constant volatility, whereas, under the assumption of time-varying volatility, it is the market’s expectation of the average volatility over the remaining life of the option (Mayhew, 1995).

As a forward-looking estimate, implied volatility is widely used to forecast future realised volatility (e.g., Poon & Granger, 2005). However, since implied volatility is derived from market prices, it is influenced by the noisy forces of supply and demand in the market (Figlewski, 1997). Consequently, the forecasting accuracy of implied volatility is related to market efficiency (Jorion, 1995; Figlewski, 1997). In an efficient market, implied volatility should contain all the available information and provide the most accurate forecast of future volatility (Figlewski, 1997).

One issue with implied volatility is its dependency on the assumptions underlying the option valuation formula (Dumas et al., 1998). Conversely, the model-free implied volatility, derived by Britten-Jones and Neuberger (2000), is independent of any option pricing formulas and extracts information from the full range of available strike prices. Jiang and Tian (2005) establish the validity of the model-free implied volatility and present a more straightforward derivation method. However, the empirical results for comparing model-based and model-free implied volatilities regarding forecasting performance seem inconsistent (e.g., Jiang & Tian, 2005; Cheng & Fung, 2012; Biktimirov & Wang, 2017).

2.4 The VIX and VVIX

The CBOE Volatility Index, more commonly known as VIX, measures the 30-day expected volatility of the S&P 500 index implied by the real-time prices of the S&P 500 call and put options (Whaley, 2009; Cboe, 2019). In other words, the VIX index is an estimate of the implied volatility of the S&P 500 index over the following 30 calendar days (Cboe, 2019). The VIX was introduced in 1993 by the Chicago Board Options Exchange, and its values are quoted in percentage points and annualised terms (Whaley, 2009; Cboe, 2019).

The methodology for the VIX calculation was switched from model-based to model-free approach in 2003 (Biktimirov & Wang, 2017). The generalised formula for the VIX is

$$\sigma^2 = \frac{2}{T}\sum_{i=1}^{n}\frac{\Delta K_i}{K_i^2}e^{rT}Q(K_i) - \frac{1}{T}\left(\frac{F}{K_0}-1\right)^2, \tag{21}$$

where T is time to expiration, F is the forward index level derived from index option prices, K_0 is the first strike below F, K_i is the strike price of ith out-of-the-money option (a call if $K_i > K_0$, a put if $K_i < K_0$ and both put and call if $K_i = K_0$) and r is the risk-free interest rate based on U.S. Treasury yield curve rates (Cboe, 2019).

The forward index level F is derived from the out-of-the-money S&P 500 index options centred around an at-the-money strike price (Cboe, 2019). Thus, the forward index level is given by

$$F = Strike\ price + e^{rT}(Call\ price - Put\ price), \tag{22}$$

where $Strike\ price$ is the price at which the absolute difference between the call and put prices is the smallest (Cboe, 2019). The variable ΔK_i measures the interval between strike prices above and below K_i and is calculated as (Cboe, 2019)

$$\Delta K_i = \frac{K_{i+1} - K_{i-1}}{2}. \tag{23}$$

The value $Q(K_i)$ is the midpoint of the bid-ask spread for each option with the strike price K_i (Cboe, 2019). The time to expiration T is given by

$$T = \frac{M_{Current\ day} + M_{Settlement\ day} + M_{Other\ day}}{Minutes\ in\ a\ year}, \tag{24}$$

where $M_{Current\ day}$ denotes the minutes remaining until midnight of the current day, $M_{Settlement\ day}$ denotes the minutes from midnight until 9.30 a.m., and $M_{Other\ day}$ denotes the minutes between current and expiration day (Cboe, 2019).

The VIX is calculated from the put and call options with more than 23 days and less than 37 days to expiration (Cboe, 2019). All the options with a bid price of zero are excluded from the calculation (Cboe, 2019). Furthermore, if there are two call (put) options with consecutive strike prices and zero bid prices, no call (put) options with higher (lower) strike prices are considered for inclusion (Cboe, 2019). In the VIX calculation, options with a remaining time between 23 and 30 days are distinguished as near-term options, and options with a remaining time between 30 and 37 as next-term options (Cboe, 2019).

Since, at any given time, there are no options that expire precisely in 30 days, the VIX value is an interpolation between the results, σ_1^2 and σ_2^2, given by Equation 21 for the near- and next term options (Cboe, 2019). Thus, the VIX index value is obtained as follows:

$$VIX = 100 * \sqrt{\left[T_1\sigma_1^2\left(\frac{N_{T_2} - N_{30}}{N_{T_2} - N_{T_1}}\right) + T_2\sigma_2^2\left(\frac{N_{30} - N_{T_1}}{N_{T_2} - N_{T_1}}\right)\right] * \frac{N_{365}}{N_{30}}}, \tag{25}$$

where T_1 and T_2 denote the time to expiration of the near- and next term options respectively, N_{T_1} is the number of minutes to a settlement of the near-term options, N_{T_2} is the number of minutes to a settlement of the next-term options, N_{30} is the number of minutes in 30 days, and N_{365} is the number of minutes in a 365-day year (Cboe, 2019).

In the same way as the VIX, The CBOE VVIX index is a volatility measure derived from option prices (Cboe, 2012). However, the VVIX is derived from the VIX options, making it a volatility of volatility measure (Cboe, 2012). In that sense, the VVIX estimates the implied volatility of the VIX index over the following 30 days, or in other words, it represents the expected volatility of the VIX (Cboe, 2012). The calculation method is similar to the VIX index (Cboe, 2012). The correlation between the VIX and VVIX tends to be low, except at extreme values of VIX (Cboe, 2012).

3 Literature review

After the publication of the Black-Scholes (1973) model, the research concerning option valuation and implied volatility has expanded. A large part of the literature examines the forecasting ability of implied volatility and compares implied volatility to other forecasting models. In this section, the literature relating to the forecasting accuracy of implied volatility is reviewed.

Analysing the options of 24 companies traded on the Chicago Board Options Exchange, Latané and Rendleman (1976) compare the implied volatilities derived from the Black-Scholes (1973) model to the actual volatilities. To address the Black-Scholes (1973) implied volatilities varying among the exercise prices in the real market, Latané and Rendleman (1976) use a weighted average implied standard deviation that considers the moneyness of options. They find that the weighted average implied volatility is significantly correlated with the actual standard deviation and generally a more accurate predictor of future volatility than historical volatilities (Latané & Rendleman, 1976).

Schmalensee and Trippi (1978), Chiras and Manaster (1978) and Beckers (1981) confirm the forecasting performance of implied volatility by examining options listed on the Chicago Board Options Exchange. All these studies suggest that implied volatility outperforms historical volatility as a predictor of future realised volatility. In particular, Schmalensee and Trippi (1978) find the expectations of future volatility to be not influenced at all by historical volatility. Furthermore, Beckers (1981) shows that the information content of option prices depends on the moneyness of options, and the at-the-money options include the most relevant information.

Unlike the previous studies (cf. Latané & Rendleman, 1976; Schmalensee & Trippi, 1978; Chiras & Manaster, 1978; Beckers, 1981), Gemmil (1986) and Vasilellis and Meade (1996) use the data from the London Traded Options Market and London Stock Exchange. The former finds similarly to Beckers (1981) that the moneyness of an option affects the forecasting performance. However, Gemmil (1986) suggests that the in-the-money implied volatility is the most accurate forecast but only slightly better than the forecasts based on past share prices. Moreover, the out-of-the-money implied volatilities contain no information relevant to forecasting future volatility (Gemmil, 1986). Vasilellis and Meade (1996) report opposite results as the combination of time series forecast and implied volatility is found to outperform either of its components.

Investigating S&P 100 index options, Canina and Figlewski (1993) find completely different results than previous studies as implied volatility is found to have no statistically significant correlation with future volatility. More specifically, neither implied volatility nor historical volatility provides accurate forecasts of future volatility (Canina & Figlewski, 1993). On the other hand, the findings of Christensen and Prabhala (1998) and Fleming (1998) indicate that the volatility implied by S&P 100 option prices outperforms historical based volatility predictors. Concerning prior studies (e.g., Canina & Figlewski, 1993), Christensen and Prabhala (1998) argue that implied volatility is a more biased estimate before the regime shift around the stock market crash of 1987.

Lamoureux and Lastrapes (1993) apply the Hull and White (1987) stochastic volatility option pricing model for analysing informational efficiency in the options market. They find that, even though implied volatility tends to unpredict future volatility, it still contains valuable information not contained in the historical price process (Lamoureux & Lastrapes, 1993). However, as explained in Figlewski (1997), tests of the information content of implied volatility may suffer from the joint hypothesis problem. Jorion (1995) recognises the efficiency test results to have two possible interpretations: either the test procedure is faulty, or markets are inefficient.

As an alternative to the model-based implied volatilities, the tests based on the model-free implied volatility (Britten-Jones & Neuberger, 2000) are direct tests of market efficiency instead of the joint test (Jiang & Tian, 2005). However, neither approach is confirmed to provide better forecasts of future volatility. For instance, Jiang and Tian (2005) find the model-free implied volatility to outperform the Black-Sholes (1973) implied volatility. In contrast, Biktimirov and Wang (2017) suggest that both model-based and model-free implied volatilities contain efficient information, but the Black-Scholes (1973) implied volatility provides a more accurate forecast. Furthermore, Cheng and Fung (2012) conclude that the Black-Scholes (1973) implied volatility subsumes all the information in the model-free implied volatility over one to six weeks forecasting horizons.

Consistent with the majority of previous studies on equity options, implied volatilities of currency (Jorion, 1995; Xu & Taylor, 1995; Busch et al., 2011; Plíhal & Lyócsa, 2021) and crude oil options (Day & Lewis, 1993; Martens & Zein, 2004) are found superior compared to the historical predictors. Poon and Granger (2005) summarise the findings of volatility forecasting literature in their review of 93 studies. First of all, they conclude that financial market volatility is forecastable (Poon & Granger, 2005). Second, based on the results of stock indexes, individual stocks, exchange rates, and interest rates from both developed and emerging financial markets, implied volatility seems to provide the most accurate forecasts of future volatility (Poon & Granger, 2005).

For the forecasting performance of the VIX index, Fleming et al. (1995) find a strong relationship between VIX and future realised stock market volatility, implying that the VIX performs well as a volatility forecast. Furthermore, Blair et al. (2001) show that the VIX provides more accurate forecasts than the forecasts based on high-frequency index returns, regardless of the calculation method of realised volatility or the forecasting horizon. Since the calculation of the VIX index was switched from model-based to model-free approach in 2003 (Biktimirov & Wang, 2017), the VIX in early studies refers to the VXO index.

The forecasting accuracy of VIX and VXO indices is examined by Corrado and Miller (2005). They find that both indices provide more accurate forecasts of the corresponding stock indices than historical volatilities over a one-month forecasting horizon (Corrado & Miller, 2005). Conversely, Becker et al. (2008) suggest that the VIX index forecasts are inferior to model-based forecasts. Moreover, Han and Park (2013) compare the information content of VIX to several realised measures constructed from high-frequency data. In the out-of-sample forecast, implied volatility is found more informative than the realised measures (Han & Park, 2013).

Some studies detect the forecasting ability of implied volatility varying over time. For instance, Seo and Kim (2015) find that the forecasting performance depends on the level of investor sentiment. In addition, the forecasting models that recognise investor sentiment are shown to improve the forecasting ability (Seo & Kim, 2015). Wang and Wang (2016) verify the time-varying forecasting performance of implied volatility as the information content of the intraday VIX index is found to vary during a day. More specifically, they conclude that the most accurate forecasts are provided around noon (Wang & Wang, 2016). Furthermore, Plíhal and Lyócsa (2021) find evidence of the predictive power of implied volatility to increase during periods of high market volatility.

The behaviour of implied volatility indices around macroeconomic news is examined in several studies. Nikkinen and Sahlström (2004) show that the VIX increases prior U.S. macroeconomic news announcements and drops after the announcement. However, the findings by Chan and Gray (2018) indicate that implied and realised volatilities behave very differently over the days surrounding news announcements. In particular, the realised volatility increases sharply, while implied volatility tends to decline (Chan & Gray, 2018). Similarly, the OVX index seems to decrease after the release of EIA's weekly petroleum status report (Nikkinen & Rothovius, 2019).

In summary, implied volatility is regularly found to forecast future realised volatility, but it is not an unbiased estimate. Instead, some findings suggest that the forecasting performance varies over time. In addition, implied volatility indices are found to be affected by macroeconomic news announcements, which may imply that they provide biased forecasts if the corresponding future realised volatilities do not behave identically. This thesis contributes to the existing literature in two ways. First, for addressing the time-varying forecasting ability, this thesis examines whether the level of market volatility influences the forecasting accuracy of implied volatility. Second, the effect of the expected volatility of implied volatility on the forecasting performance is investigated to reveal further details behind the forecast bias.

4 Data and methodology

The data and methodology for examining the forecasting performance of implied volatility are presented in this section. The VIX index is used as an estimate of implied volatility to diminish the probability of measurement errors. Thus, the relation between the VIX and future realised volatility of the S&P 500 index is tested.

4.1 Data

All the data is obtained from the Thomson Reuters Datastream. The data consist of the daily opening, closing, high and low values of the S&P 500 index, the daily closing values of the VIX index, and the daily closing values of the VVIX index. The time series covers the period from June 2006 to April 2021.

In order to test the forecasting performance of the VIX index, the actual volatility of the S&P 500 index needs to be measured. Since the Garman-Klass (1980) volatility estimator produces accurate results from daily data (Molnár, 2012), and it is suitable for the S&P 500 index (Shu & Zhang, 2006), the realised volatility of the S&P 500 index is measured by applying the Garman-Klass (1980) formula. Hence, the realised volatility estimate for each day is calculated as follows:

$$\hat{\sigma}_t = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\frac{1}{2}\left(\ln\left(\frac{H_{t+1-i}}{L_{t+1-i}}\right)\right)^2 - (2\ln 2 - 1)\left(\ln\left(\frac{C_{t+1-i}}{O_{t+1-i}}\right)\right)^2}, \quad (26)$$

where n denotes the number of days in the estimation horizon and O_{t+1-i}, H_{i+1-i}, L_{t+1-i} and C_{t+1-i} are the open, high, low and close values on day $t+1-i$.

The VIX represents the implied volatility of the S&P 500 index over the following 30 calendar days (Whaley, 2009). Assuming that 30 calendar days correspond to 21 trading days on average (e.g., Figlewski, 1997), the realised volatility estimate for each day is calculated from the previous 21 observations, resulting in $n = 21$ in Equation 26. Because the VIX index is quoted in annual terms and as a percentage (Whaley, 2009), the realised volatility estimate is annualised and multiplied by 100. As explained in Fleming et al. (1995) and Corrado and Miller (2005), the annualised volatility requires an additional adjustment of $\sqrt{30/21}$ when compared to the VIX. Under the general assumption of 252 trading days per year, the annualised 21-day realised volatility is given by

$$\hat{\sigma}_{Realised,t} = 100 * \hat{\sigma}_t \sqrt{\frac{30}{21} * 252}, \tag{27}$$

which is used as an estimate of the actual 21-day volatility of the S&P 500 index in this thesis.

Table 1. Descriptive statistics of the S&P 500 realised volatility, VIX and VVIX from June 2006 to April 2021.

	$\hat{\sigma}_{Realised}$	VIX	$VVIX$
Mean	14.87	19.75	91.37
Median	11.95	16.91	88.99
Maximum	80.26	82.69	207.59
Minimum	4.05	9.14	36.14
Standard deviation	9.75	9.60	15.98
Skewness	3.01	2.40	1.32
Kurtosis	15.15	10.93	7.08
Jarque-Bera	28693.83***	13422.43***	342356.8***
Number of observations	3747	3747	3747

The S&P 500 realised volatility is measured by the Garman-Klass (1980) estimator. *** indicates that the Jarque-Bera test rejects the null hypothesis of normality at the 1% significance level.

The descriptive statistics for the S&P 500 realised volatility, VIX and VVIX are presented in Table 1. The VIX has a 4.88 higher mean than the realised volatility, but the standard deviations are almost equal. However, the VIX is less skewed (2.40 vs. 3.01) and not as leptokurtic (10.93 vs. 15.15) as the realised volatility. The VVIX has skewness (1.32) and kurtosis (7.08) much lower than the VIX and realised volatility. Nonetheless, the hypothesis of normality is rejected for all the variables. Furthermore, the minimum is significantly lower for the realised volatility (4.05) compared to the minimum of the VIX (9.14), whereas the difference between the maximum values (80.26 vs. 82.69) is not as significant.

Figure 2 illustrates the VIX index and 21-day realised volatility measure of the S&P 500 index for the whole sample period. The graph indicates a positive correlation between the VIX and the realised volatility. Furthermore, a visual inspection of the figure reveals several major spikes simultaneously in both times series, most notably, during 2008–2009 and 2020.

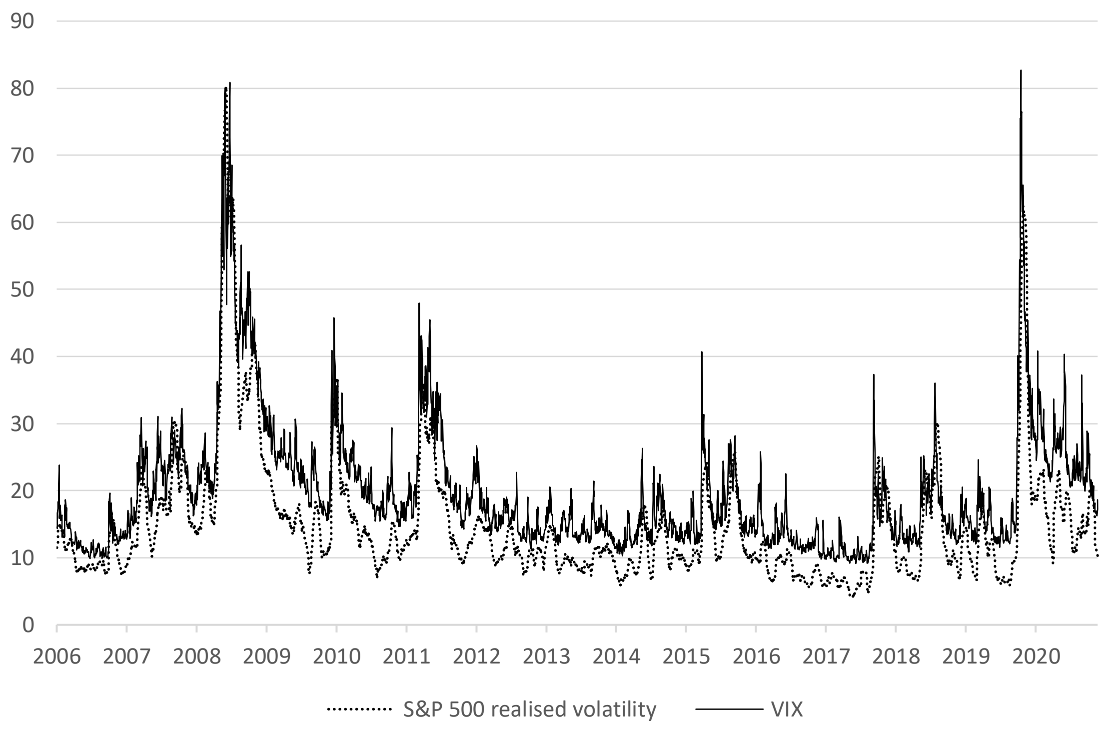

Figure 2. The S&P 500 realised volatility and VIX from June 2006 to April 2021.

4.2 Methodology

If implied volatility is an informationally efficient forecast of future volatility, then

$$\sigma_{Actual} = \sigma_{IV} + \varepsilon, \qquad E(\varepsilon) = 0, \tag{28}$$

where σ_{Actual} is the actual volatility, σ_{IV} is the implied volatility, and ε is the random error with a zero mean (Figlewski, 1997). Hence, following Canina and Figlewski (1993), Jorion (1995), Figlewski (1997), Christensen and Prabhala (1998), and Corrado and Miller (2005), the forecasting performance of implied volatility is tested by running the OLS regression of the realised volatility on the 21-day lagged VIX values,

$$\hat{\sigma}_{Realised,t} = \alpha + \beta_1 VIX_{t-21} + \varepsilon_t, \tag{29}$$

where $\hat{\sigma}_{Realised,t}$ is the 21-day realised volatility of the S&P 500 index observed on day t, and VIX_{t-21} is the VIX value on day $t - 21$. Non-zero β_1 indicates that the VIX contains some information about future volatility (Christensen & Prabhala, 1998). Furthermore, the VIX is an unbiased estimate of the future volatility if $\alpha = 0$ and $\beta_1 = 1.0$ (Figlewski, 1997; Christensen & Prabhala, 1998). In addition, some information regarding the predictive power is indicated by R^2 (Corrado & Miller, 2005).

To examine whether the forecast accuracy is affected by historical volatility, the regression model is extended by adding the 21-day lagged realised volatility measure as follows:

$$\hat{\sigma}_{Realised,t} = \alpha + \beta_1 VIX_{t-21} + \beta_2 \hat{\sigma}_{Realised,t-21} + \varepsilon_t, \tag{30}$$

where $\hat{\sigma}_{Realised,t-21}$ is the 21-day realised volatility of the S&P 500 index observed on day $t - 21$. If the VIX contains all the information involved in the historical volatility, the coefficient estimates should be $\alpha = 0$, $\beta_1 = 1.0$ and $\beta_2 = 0$ (Figlewski, 1997).

Because of the overlapping data, the possibility of serial correlation needs to be considered. Therefore, the Newey and West (1987) standard errors are used to correct for heteroskedasticity and autocorrelation. In some studies (e.g., Christensen & Prabhala, 1998; Corrado & Miller, 2005; Cheng & Fung, 2012), log-transformed data is used instead of the absolute values of volatility measures as the distributions of these transformed values are closer to normal. Thus, the OLS regressions are performed with the log-transformed values in addition to the original volatility measures. Furthermore, 30 calendar days (or one month) are sometimes assumed to match 22 trading days instead of 21 (e.g., Corrado & Miller, 2005; Seo & Kim, 2015). However, this adjustment does not significantly affect the conclusions of this study.

As explained in Christensen and Prabhala (1998), non-overlapping data increases the reliability of regression estimates. Therefore, following Christensen and Prabhala (1998), Corrado and Miller (2005) and Biktimirov and Wang (2017), the OLS regressions defined by Equations (29) and (30) are run with monthly non-overlapping observations in addition to the full sample. Despite the smaller number of observations, diminishing serial correlation may decrease the probability of invalid test statistic (Jian & Tian, 2005). Hence, the non-overlapping sample is also used in further analysis.

The quantile regression (Koenker & Bassett, 1978) approach is applied to investigate whether the forecasting performance varies among realised volatility levels. In contrast to the OLS regression that estimates the average relationship between dependent and independent variables based on the conditional mean, the quantile regression provides estimates in different points of the conditional distribution of a dependent variable (Koenker & Bassett, 1978). Therefore, the following quantile regression is constructed to estimate the relationship between realised volatility and VIX in five different quantiles $(\tau) \in (0.1, 0.25, 0.5, 075, 0.9)$:

$$Q_{\tau}\left(\tau | \hat{\sigma}_{Realised,m}\right) = \alpha(\tau) + \beta_1(\tau) VIX_{m-1} + \beta_2(\tau) \hat{\sigma}_{Realised,m-1} + \varepsilon_m, \quad (31)$$

where $Q_\tau(\tau|\hat{\sigma}_{Realised,m})$ is the τ-th conditional quantile of the 21-day S&P 500 realised volatility in month m, VIX_{m-1} is the VIX index value in month $m-1$, and $\hat{\sigma}_{Realised,m-1}$ is the realised volatility in month $m-1$. The standard errors are obtained using the bootstrap method with ten thousand replications to consider heteroskedasticity and serial correlation.

The percentage difference between the realised volatility and lagged VIX is calculated for each month as

$$D_m = 100 * \left|\frac{VIX_{m-1} - \hat{\sigma}_{Realised,m}}{\hat{\sigma}_{Realised,m}}\right|, \quad (32)$$

where D_m is the absolute percentage difference between the matched realised volatility and VIX value in month m. The smaller the deviation, the more accurate is the forecast. The effect of volatility level on the forecast accuracy is examined by running the following OLS regression:

$$D_m = \alpha + \beta_1 VVIX_{m-1} + \beta_2 Dummy_{High} + \varepsilon_m, \quad (33)$$

where $VVIX_{m-1}$ is the VVIX index value in month $m-1$. The dummy variable $Dummy_{High}$ represents high market volatility and, adapting the method that Dutta et al. (2017) use for specifying the extreme values of the OVX index, is defined as $Dummy_{High} = 1$ if both $VIX_{m-1} > Q_{VIX,3}$ and $\hat{\sigma}_{Realised,m} > Q_{Realised,3}$, and $Dummy_{High} = 0$ otherwise. $Q_{i,3}$ indicates the third quartile of the corresponding variable i. A positive (negative) β_1 implies that the increase in VVIX (i.e., the expected volatility of VIX) decreases (increases) the forecasting accuracy. Similarly, a significant β_2 indicates the forecasting accuracy to change during the high market volatility.

5 Empirical results

In this section, the forecasting performance of implied volatility is analysed. Implied volatility is shown to be able to forecast future realised volatility over a one-month forecasting horizon. However, the results indicate that implied volatility is a biased estimate, and the forecasting accuracy varies depending on the level of volatility. Furthermore, some evidence of the implied volatility of implied volatility affecting the forecasting ability is found.

5.1 The relation between implied and realised volatility

The results for the OLS regression of the realised volatility on the 21-day lagged VIX (Equation 29) and additionally on the 21-day lagged realised volatility (Equation 30) are presented in Table 2. The regression parameter estimates are reported in Columns (1) and (2). The Newey-West (1987) heteroskedasticity and autocorrelation consistent standard errors are reported in parentheses. Column t indicates the t-statistic on the null hypothesis of $\beta_1 = 1$. The chi-square χ^2 corresponds to the null hypothesis of $\alpha = 0$ and $\beta_1 = 1$ with p-value in parentheses.

The statistically significant slope coefficient of 0.758 for the 21-day lagged VIX, reported in Column (1), indicates that the VIX forecasts future realised volatility. The coefficient decreases slightly in the multiple regression (0.660), reported in Column (2), but remains significant at the 1% level, while the coefficient for the 21-day lagged historical volatility is much lower (0.107) and insignificant. In addition, neither of the intercept terms differs significantly from zero. The R-squared of 0.557 in Column (1) indicates that the VIX can explain 56% of the variation in the future volatility. Furthermore, R-squared values for both regressions are almost equal (0.557 vs. 0.559), implying that the explanatory power of the regression is not improved by adding the historical volatility.

Table 2. OLS regressions of the realised volatility on the lagged VIX.

	(1)	t	(2)	t
α	-0.107		0.249	
	(1.366)		(1.095)	
VIX_{t-21}	0.758***	-2.931	0.660***	-3.133
	(0.082)		(0.109)	
$\hat{\sigma}_{Realised,t-21}$			0.107	
			(0.137)	
χ^2 (p-value)	118.47		10.44	
	(0.000)		(0.005)	
Adjusted R^2	0.557		0.559	
Number of observations	3726		3726	

The dependent variable is $\hat{\sigma}_{Realised,t}$. Newey-West (1987) standard errors are reported in parentheses. *** indicates significance at the 1% level. Column t reports the t-statistic on the null hypothesis of $\beta_1 = 1$. χ^2 (p-value) corresponds to the Wald test for $\alpha = 0$ and $\beta_1 = 1$ and indicates its p-value in parentheses.

The reported t-values of -2.931 and -3.133 in Table 2 reject the null hypothesis of $\beta_1 = 1$, which suggests that the coefficient for the lagged VIX is significantly less than one in both models. Moreover, the chi-square statistics of 118.47 and 10.44 reject the joint null hypothesis of $\alpha = 0$ and $\beta_1 = 1$ at the 1% significance level. According to these results, implied volatility does not satisfy the conditions for an unbiased estimate of realised volatility.

As shown in Table 3, the OLS regression results for the log-transformed volatilities are consistent with the results for the original measures. The log-transformed 21-day lagged VIX coefficient is significantly different from zero at the 1% level despite the inclusion of log-transformed lagged realised volatility in the model. Moreover, the coefficient for VIX (0.971), reported in Column (3), satisfies the null hypothesis of $\beta_1 = 1$, and the intercept term (-0.254) is insignificant. However, the joint hypothesis of zero intercept and unit slope coefficient for $\ln VIX_{t-21}$ is still rejected at the 1% level.

Table 3. OLS regressions with the log-transformed volatility measures.

	(3)	t	(4)	t
α	-0.254		-0.174	
	(0.168)		(0.153)	
$\ln VIX_{t-21}$	0.971***	-0.494	0.844***	-1.765
	(0.059)		(0.089)	
$\ln \hat{\sigma}_{Realised,t-21}$			0.113	
			(0.079)	
χ^2 (p-value)	215.85		10.56	
	(0.000)		(0.005)	
Adjusted R^2	0.557		0.559	
Number of observations	3726		3726	

The dependent variable is $\ln \hat{\sigma}_{Realised,t}$. Newey-West (1987) standard errors are reported in parentheses. *** indicates significance at the 1% level. Column t reports the t-statistic on the null hypothesis of $\beta_1 = 1$. χ^2 (p-value) corresponds to the Wald test for $\alpha = 0$ and $\beta_1 = 1$ and indicates its p-value in parentheses.

In Column (4), the one-month lagged realised volatility is included in the regression. In this case, the t-statistic shows that the VIX coefficient of 0.884 is significantly different from one, implying a bias in the forecasting performance. However, the insignificant coefficient for log-transformed lagged realised volatility (0.113) implies that the lagged realised volatility does not contain any information regarding future volatility beyond the VIX. In addition, the almost equal R-squared values (0.557 vs. 0.559) indicate that both models can explain roughly 56% of the variability in the realised volatility.

Table 4 presents the results for the OLS regressions with non-overlapping monthly observations. These results are consistent with the full sample as the coefficient for VIX is significantly positive in both models and the R^2 values indicate no substantial differences in the explanatory power. As an exception, the t-statistic of -1.246 for Column (6) implies that the coefficient for VIX is not significantly different from one. The null hypothesis of $\alpha = 0$ and $\beta_1 = 1$ is still rejected at the 5% level even though the intercept and historical volatility are both insignificant. Similarly, the chi-square in Column (5) suggests a rejection of the joint null hypothesis, implying that the VIX is a biased estimate. The results for the corresponding regression with the log-transformed variables show negligible dissimilarities besides the higher forecasting accuracy in the simple linear regression (Appendix 1).

Table 4. OLS regressions for the non-overlapping sample.

	(5)	t	(6)	t
α	-0.951		-0.824	
	(1.629)		(1.686)	
VIX_{t-21}	0.795***	-2.084	0.761***	-1.246
	(0.098)		(0.192)	
$\hat{\sigma}_{Realised,t-21}$			0.037	
			(0.181)	
χ^2 (p-value)	141.34		6.41	
	(0.000)		(0.041)	
Adjusted R^2	0.571		0.567	
Number of observations	178		178	

The OLS regressions of the realised volatility on the lagged VIX for the non-overlapping monthly data. The dependent variable is $\hat{\sigma}_{Realised,t}$. Newey-West (1987) standard errors are reported in parentheses. *** indicates significance at the 1% level. Column t reports the t-statistic on the null hypothesis of $\beta_1 = 1$. χ^2 (p-value) corresponds to the Wald test for $\alpha = 0$ and $\beta_1 = 1$ and indicates its p-value in parentheses.

The results in Tables 2, 3 and 4 are in line with the previous findings of Christensen and Prabhala (1998), Jian and Tian (2005), Corrado and Miller (2005), and others as the VIX seems to be a biased estimate of future volatility but still provides more accurate forecast than historical volatility. More specifically, the coefficients for implied volatility and values of R^2 with both full and non-overlapping sample are similar to those obtained by Corrado and Miller (2005). In order to examine whether the forecasting performance of VIX differs from other volatility indices, corresponding OLS regressions are performed with the DAX and VDAX. The results appear to be analogous besides the significantly positive but low log-transformed lagged DAX, implying no substantial differences between the indices regarding their forecasting accuracy (Appendix 2 and 3). According to these results, H_1 is accepted and H_2 is rejected.

5.2 Forecasting performance in different quantiles

Table 5. Quantile regression estimates.

	Quantile (τ)				
	0.1	0.25	0.5	0.75	0.9
α	0.347	-0.214	1.589	-0.095	-1.020
	(0.989)	(0.688)	(1.178)	(1.888)	(4.721)
$VIX_{\mathrm{m}-1}$	0.485***	0.566***	0.494***	0.642***	1.257**
	(0.084)	(0.094)	(0.110)	(0.231)	(0.496)
$\hat{\sigma}_{Realised,\mathrm{m}-1}$	-0.000	0.014	0.148	0.248	-0.225
	(0.108)	(0.123)	(0.133)	(0.178)	(0.388)
Pseudo R^2	0.321	0.348	0.379	0.404	0.418

Quantile slope equality test	$Q_{0.1} = Q_{0.5}$	$Q_{0.9} = Q_{0.5}$	$Q_{0.1} = Q_{0.9}$	$Q_{0.1} = Q_{0.25} = Q_{0.75} = Q_{0.9}$
χ^2	0.01	2.55	2.55	3.95
p-value	0.93	0.11	0.28	0.41

The quantile regression estimates according to the model defined by Equation (31). The bootstrapped standard errors are reported in parentheses. *** and ** denotes significance at the 1% and 5% levels, respectively.

The quantile regression results are presented in Table 5. The results demonstrate that the VIX has a considerable forecasting power regardless of the volatility level as the VIX coefficients appear significantly positive in every quantile. Furthermore, the pseudo R^2 values indicate the ability of VIX to explain some variability in the realised volatility. However, the coefficients, as well as the R^2 values, seem to vary among the quantiles. More specifically, the estimates are higher (lower) in the upper (lower) quantiles, implying an increasing pattern of the forecasting performance (Appendix 4). The increasing forecasting accuracy of implied volatility during periods of high market uncertainty is documented in the recent study of Plíhal and Lyócsa (2021) as well.

To further examine whether the forecasting performance of implied volatility depends on the market volatility level, the equality of coefficients is tested. Table 5 presents the results for the slope equality test for multiple quantile pairs and show that none of the tested quantiles are significantly unequal. Moreover, the joint test with all the observed quantiles does not reject the null hypothesis of equal coefficient estimates. The largest chi-square statistic is obtained for the test between $Q_{0.9}$ and $Q_{0.5}$, but it still does not indicate a significant difference. However, these results are highly dependent on the standard errors, and for instance, the ordinary (IID) covariances instead of the bootstrap resampling leads to the rejection of equal coefficients.

5.3 The impact of volatility on the forecasting accuracy

Table 6 presents the results for the OLS regression defined by Equation (33). The dependent variable is the absolute percentage deviation between the lagged VIX and realised volatility. The regression is performed with the monthly non-overlapping sample to avoid serial correlation occurring. Consequently, as the Durbin-Watson statistic shows no evidence of autocorrelation, the White (1980) heteroscedasticity-consistent standard errors are used.

Table 6 reports a positive coefficient for the VVIX at the 5% level in the simple regression, and 1% level after the dummy variable is included in the model. Hence, the absolute percentage difference is expected to increase by 0.415 or 0.551 percentage points if the VVIX value increases by one. This relation implies that the expected volatility of the VIX has a negative impact on the forecasting accuracy of VIX. In other words, an increase (decrease) in the VVIX decreases (increases) the accuracy of the VIX forecast for the following 30-day volatility of the S&P 500 index.

In addition, the effect of the high market volatility on the forecasting accuracy is reported in Table 6. The significantly negative coefficient for $Dummy_{High}$ indicates that the absolute percentage deviation between the 21-day realised volatility and one-month lagged VIX is -18.82 percentage points lower during the periods of high volatility. Therefore, the forecasting performance of implied volatility seems to be more accurate when the market is highly volatile. These results lead to the rejection of H_3 and H_4. However, the low R^2 values in Table 6 indicate that further research is required.

Table 6. The effect of volatility on forecasting performance.

Dependent variable: $D_m = 100 * \left\| \frac{VIX_{m-1} - \hat{\sigma}_{Realised,m}}{\hat{\sigma}_{Realised,m}} \right\|$		
α	16.032	7.182
	(15.70)	(15.32)
$VVIX_{m-1}$	0.415**	0.551***
	(0.180)	(0.178)
$Dummy_{High}$		-18.823***
		(5.678)
Adjusted R^2	0.026	0.061
DW	1.90	1.91
Number of observations	178	178

The results for the OLS regression defined by Equation (33). White (1980) heteroscedasticity-consistent standard errors are reported in parentheses. DW corresponds to the Durbin-Watson statistic for measuring autocorrelation. *** and ** denotes significance at the 1% and 5% levels, respectively.

Figure 3 illustrates the forecasting accuracy of VIX measured by the percentage deviation between the one-month lagged VIX and realised volatility of the S&P 500 index. The deviation shows considerable variation over time, ranging from -50.10% to 190.66%. Most of the time, the deviation is positive, implying that the VIX regularly over-forecasts future volatility. The highlighted periods represent high market volatility defined by the lagged VIX and realised volatility, as explained in Section 4. The mean of highlighted values (33.59%) is closer to zero than the mean of non-highlighted values (51.35%), implying growing forecast accuracy during high market volatility.

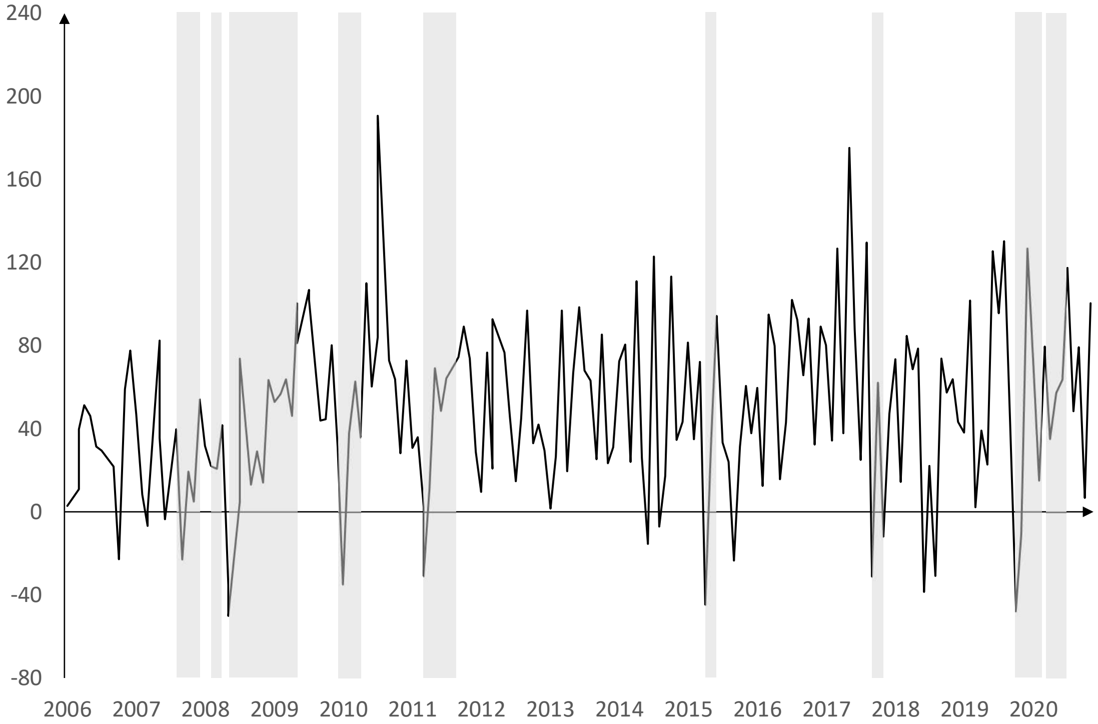

Figure 3. Percentage deviation between the lagged VIX and realised volatility.

5.4 Robustness check

The robustness of the findings is investigated by repeating the analysis after replacing the range volatility estimator with the sample standard deviation. Following Figlewski (1997), Corrado and Miller (2005) and others, the 21-day realised volatility of the S&P 500 index is calculated for each day as

$$\hat{\sigma}_{RV,t} = 100 * \sqrt{\frac{30}{21} * \frac{252}{n-1} \sum_{i=1}^{n} \left(R_{t+1-i} - \frac{1}{n} \sum_{j=1}^{n} R_{t+1-j} \right)^2}, \quad (34)$$

where R_{t+1-i} represents an index return on day $t+1-i$, and $n = 21$, which corresponds to the number of trading days in a month. Monthly realised volatility $\hat{\sigma}_{RV,m}$ is obtained by including only non-overlapping observations in the equation.

As shown In Table 7, the results do not change remarkably with the alternative volatility estimator. The lagged VVIX is significantly positive in both models, although the magnitude of impact is marginally smaller than the corresponding values in Table 6. Similarly, the dummy variable remains statistically significant and negative. These results support the findings of time-varying forecasting ability.

Table 7. Results of the robustness test.

	Dependent variable:	$D_m = 100 * \left\lvert \frac{VIX_{m-1} - \hat{\sigma}_{RV,m}}{\hat{\sigma}_{RV,m}} \right\rvert$
α	11.746	2.298
	(10.94)	(10.95)
$VVIX_{m-1}$	0.269**	0.409***
	(0.124)	(0.130)
$Dummy_{High}$		-19.928***
		(4.566)
Adjusted R^2	0.011	0.056
DW	2.11	2.17
Number of observations	178	178

The results for the OLS regression defined by Equation (33). The dependent variable D_m is calculated from the $\hat{\sigma}_{RV,m}$ realised volatilities defined by Equation (34). White (1980) heteroscedasticity-consistent standard errors are reported in parentheses. DW corresponds to the Durbin-Watson statistic for measuring autocorrelation. *** and ** denotes significance at the 1% and 5% levels, respectively.

6 Conclusions

This thesis examines the forecasting accuracy of implied volatility. For this purpose, the performance of the VIX index to predict the one-month future realised volatility of the S&P 500 index is analysed. In contrast to the majority of previous studies, the realised volatility is estimated using the range-based volatility. Furthermore, the effect of market volatility level and VVIX index on the forecasting accuracy is investigated. The quantile regression approach is applied to examine the forecasting power on different volatility levels.

Consistent with prior empirical results, implied volatility is found to forecast future realised volatility over a one-month forecasting horizon but not to pass the unbiased test. The conclusion remains the same regardless of whether overlapping, non-overlapping data or log-transformed variables are used. However, a notable serial correlation occurs with the overlapping sample, which may invalidate the test statistic despite the heteroskedasticity and autocorrelation consistent standard errors.

As a contribution to the research of volatility forecasting, some evidence of the time-varying forecasting performance of implied volatility is found. The level of market volatility appears to affect the forecast accuracy as the absolute percentage deviation between one-month lagged VIX and S&P 500 realised volatility decreases during periods of high market volatility. Hence, the VIX index seems to forecast future volatility more accurately when the market volatility is high. However, the quantile slope equality test indicates that the forecasting accuracy does not vary within the level of realised volatility.

In addition, prior work seems to not focus on the reasons for the VIX forecast bias beyond the explanation of market inefficiency. For extending the research further, the impact of the VVIX index (i.e., implied volatility of VIX options) on the forecasting performance of VIX is examined. The VVIX appears to have a negative influence on the VIX forecasting power. In particular, an increase (decrease) in the expected volatility of VIX decreases (increases) the forecasting accuracy of VIX.

Due to the nature of the VIX index, only a one-month forecasting horizon is investigated. Thus, additional research is needed for shorter and longer horizons. Furthermore, a more sophisticated research methodology and use of high-frequency data could improve the test results. As volatility is unobservable and inconstant, both the forecasted realised volatility and forecasting implied volatility are estimates of the actual volatilises. Therefore, the effect of volatility behaviour on the forecasting accuracy in different market conditions could create possibilities for further research. Especially, a better understanding of the volatility of volatility could improve volatility forecasting.

Despite the limitations, these findings can be implemented by investors and financial practitioners. For instance, the implied volatilities of liquid and actively traded index options may provide valuable information for investors and risk management, especially during periods of high market volatility, as the VIX index seems to be a proper estimate of the 30-day future realised volatility. Furthermore, the time-varying forecasting accuracy implies that the markets are occasionally inefficient, which may benefit trading strategies that consider the time-varying forecasting performance of implied volatility.

References

Alizadeh, S., Brandt, M. W., & Diebold, F. X. (2002). Range-based estimation of stochastic volatility models. *The Journal of Finance*, 57(3), 1047-1091. https://doi-org.proxy.uwasa.fi/10.1111/1540-6261.00454

Andersen, T. G., & Bollerslev, T. (1997). Heterogeneous information arrivals and return volatility dynamics: Uncovering the long-run in high frequency returns. *The Journal of Finance*, 52(3), 975-1005. https://doi-org.proxy.uwasa.fi/10.2307/2329513

Andersen, T. G., & Bollerslev, T. (1998). Answering the skeptics: Yes, standard volatility models do provide accurate forecasts. *International Economic Review*, 39(4), 885-905. https://doi-org.proxy.uwasa.fi/10.2307/2527343

Andersen, T. G., Bollerslev, T., Diebold, F. X., & Ebens, H. (2001). The distribution of realized stock return volatility. *Journal of Financial Economics*, 61(1), 43-76. https://doi.org/10.1016/S0304-405X(01)00055-1

Andersen, T. G., Bollerslev, T., Diebold, F. X., & Labys, P. (2003). Modeling and forecasting realized volatility. *Econometrica*, 71(2), 579-625. https://doi-org.proxy.uwasa.fi/10.1111/1468-0262.00418

Bachelier, L. (1900). Théorie de la spéculation. *Annales scientifiques de l'École Normale Supérieure* 17, 21-86. https://doi.org/10.24033/asens.476

Becker, R., & Clements, A. E. (2008). Are combination forecasts of S&P 500 volatility statistically superior? *International Journal of Forecasting*, 24(1), 122-133. https://doi.org/10.1016/j.ijforecast.2007.09.001

Beckers, S. (1981). Standard deviations implied by options prices as predictors of future stock price variability. *Journal of Banking & Finance*, 5(3), 363-381. https://doi.org/10.1016/0378-4266(81)90032-7

Biktimirov, E. N., & Wang, C. (2017). Model-based versus model-free implied volatility: Evidence from North American, European, and Asian index option markets. *Journal of Derivatives*, 24(3), 42-68. http://dx.doi.org.proxy.uwasa.fi/10.3905/jod.2017.24.3.042

Black, F. (1975). Fact and fantasy in the use of options. *Financial Analysts Journal*, 31(4), 36–72. https://doi-org.proxy.uwasa.fi/10.2469/faj.v31.n4.36

Black, F., & Scholes, M. (1973). The pricing of options and corporate liabilities. *Journal of Political Economy*, 81(3), 637-654. https://doi-org.proxy.uwasa.fi/10.1086/260062

Blair, B. J., Poon, S., & Taylor, S. J. (2001). Forecasting S&P 100 volatility: The incremental information content of implied volatilities and high-frequency index returns. *Journal of Econometrics*, 105(1), 5-26. https://doi.org/10.1016/S0304-4076(01)00068-9

Bodie, Z., Kane, A., & Marcus, A., (2018). *Investments* (11th ed.). McGraw-Hill Education.

Bollerslev, T. (1986). Generalized autoregressive conditional heteroskedasticity. *Journal of Econometrics*, 31(3), 307-327. https://doi.org/10.1016/0304-4076(86)90063-1

Bollerslev, T., Chou, R. Y., & Kroner, K. F. (1992). ARCH modelling in finance: A review of the theory and empirical evidence. *Journal of Econometrics*, 51(1), 5-59. https://doi.org/10.1016/0304-4076(92)90064-X

Boness, A. (1964). Elements of a theory of stock-option value. *Journal of Political Economy*, 72(2), 163-175. https://doi-org.proxy.uwasa.fi/10.1086/258885

Boyle, P. P. (1977). Options: A Monte Carlo approach. *Journal of Financial Economics*, 4(3), 323-338. https://doi.org/10.1016/0304-405X(77)90005-8

Brandt, M. W., & Diebold, F. X. (2006). A no-arbitrage approach to range-based estimation of return covariances and correlations. *Journal of Business*, 79(1), 61-73. https://doi-org.proxy.uwasa.fi/10.1086/497405

Britten-Jones, M., & Neuberger, A. (2000). Option prices, implied price processes, and stochastic volatility. *The Journal of Finance*, 55(2), 839-866. https://doi-org.proxy.uwasa.fi/10.1111/0022-1082.00228

Busch, T., Christensen, B. J., & Nielsen, M. Ø. (2011). The role of implied volatility in forecasting future realized volatility and jumps in foreign exchange, stock, and bond markets. *Journal of Econometrics*, 160(1), 48-57. https://doi.org/10.1016/j.jeconom.2010.03.014

Canina, L., & Figlewski, S. (1993). The informational content of implied volatility. *The Review of Financial Studies*, 6(3), 659-681. https://doi.org/10.1093/rfs/5.3.659

Carr, P., & Jarrow, R. (1990). The stop-loss start-gain paradox and option valuation: A new decomposition into intrinsic and time value. *The Review of Financial Studies*, 3(3), 469-492.

Cboe. (2012). Double the fun with CBOE's VVIX index [White paper]. https://ww2.cboe.com/micro/vvix/documents/vvix-termstructure.pdf

Cboe. (2019). The CBOE Volatility Index VIX [White paper]. https://cdn.cboe.com/resources/vix/vixwhite.pdf

Chan, K. F., & Gray, P. (2018). Volatility jumps and macroeconomic news announcements. *Journal of Futures Markets*, 38(8), 881-897. https://doi-org.proxy.uwasa.fi/10.1002/fut.21922

Cheng, X., & Fung, J.K. (2012). The information content of model-free implied volatility. *The Journal of Futures Markets*, 32(8), 792-806. https://doi-org.proxy.uwasa.fi/10.1002/fut.21548

Chiras, D. P., & Manaster S. (1978). The information content of option prices and a test of market efficiency. *Journal of Financial Economics*, 6(2,3), 213-234. https://doi.org/10.1016/0304-405X(78)90030-2

Christensen, B. J., & Prabhala, N. R. (1998). The relation between implied and realized volatility. *Journal of Financial Economics*, 50(2), 125-150. https://doi.org/10.1016/S0304-405X(98)00034-8

Corrado, C. J., & Miller Jr, T., W. (2005). The forecast quality of CBOE implied volatility indexes. *The Journal of Futures Markets*, 25(4), 339-373. https://doi-org.proxy.uwasa.fi/10.1002/fut.20148

Corsi, F., Fusari, N., & La Vecchia, D. (2013). Realizing smiles: Options pricing with realized volatility. *Journal of Financial Economics*, 107(2), 284-304. https://doi.org/10.1016/j.jfineco.2012.08.015

Cox, J. C., Stephen A. R., & Rubinstein M. (1979). Option pricing: A simplified approach. *Journal of Financial Economics*, 7(3), 229-263. https://doi.org/10.1016/0304-405X(79)90015-1

Cremers, M., Halling, M., & Weinbaum, D. (2015). Aggregate jump and volatility risk in the cross-section of stock returns. *The Journal of Finance*, 70(2), 577-614. https://doi-org.proxy.uwasa.fi/10.1111/jofi.12220

Day, T. E., & Lewis, C. M. (1993). Forecasting futures market volatility. *The Journal of Derivatives*, 1, 33-50. https://doi.org/10.3905/jod.1993.407876

Dumas, B., Fleming, J., & Whaley, R.E. (1998). Implied volatility functions: Empirical tests. *The Journal of Finance*, 53(6), 2059-2106. https://doi-org.proxy.uwasa.fi/10.1111/0022-1082.00083

Dutta, A., Nikkinen, J., & Rothovius, T. (2017). Impact of oil price uncertainty on Middle East and African stock markets. *Energy*, 123, 189-197. https://doi.org/10.1016/j.energy.2017.01.126

Ederington, L. H., & Guan, W. (2006). Measuring historical volatility. *Journal of Applied Finance*, 16(1), 5-14.

Engle, R. (1982). Autoregressive conditional heteroscedasticity with estimates of the variance of United Kingdom inflation. *Econometrica*, 50(4), 987-1007. https://doi-org.proxy.uwasa.fi/10.2307/1912773

Engle, R. F. (1993). Statistical models for financial volatility. *Financial Analysts Journal*, 49(1), 72-78. https://doi-org.proxy.uwasa.fi/10.2469/faj.v49.n1.72

Engle, R. F., & Gallo, G. M. (2006). A multiple indicators model for volatility using intra-daily data. *Journal of Econometrics*, 131(1,2), 3-27. https://doi.org/10.1016/j.jeconom.2005.01.018

Evnine, J., & Rudd, A. (1985). Index options: The early evidence. *The Journal of Finance*, 40(3), 743-756. https://doi-org.proxy.uwasa.fi/10.2307/2327798

Fama, E. F. (1991). Efficient capital markets: II. *The Journal of Finance*, 46(5), 1575-1617. https://doi-org.proxy.uwasa.fi/10.2307/2328565

Figlewski, S. (1997). Forecasting volatility. *Financial Markets, Institutions & Instruments* 6(1), 1-88. https://doi-org.proxy.uwasa.fi/10.1111/1468-0416.00009

Fleming, J. (1998). The quality of market volatility forecasts implied by S&P 100 index option prices. *Journal of Empirical Finance*, 5(4), 317-345. https://doi.org/10.1016/S0927-5398(98)00002-4

Fleming, J., Kirby, C., & Ostdiek, B. (2001). The economic value of volatility timing. *The Journal of Finance*, 56(1), 329-352. https://doi-org.proxy.uwasa.fi/10.1111/0022-1082.00327

Fleming, J., Ostdiek, B., & Whaley, R. E. (1995). Predicting stock market volatility: A new measure. *The Journal of Futures Markets*, 15(3), 265-302. https://doi.org/10.1002/fut.3990150303

Franklin, C., & Colberg, M. (1958). Puts and calls: A factual survey. *The Journal of Finance*, 13(1), 21-34. https://doi-org.proxy.uwasa.fi/10.2307/2975999

Garman, M. B., & Klass, M. J. (1980). On the estimation of security price volatilities from historical data. *Journal of Business*, 53(1), 67-78. https://doi-org.proxy.uwasa.fi/10.1086/296072

Gemmill, G. (1986). The forecasting performance of stock options on the London Traded Options Market. *Journal of Business Finance & Accounting*, 13(4), 535-546. https://doi-org.proxy.uwasa.fi/10.1111/j.1468-5957.1986.tb00516.x

Grullon, G., Lyandres, E., & Zhdanov, D. (2012). Real options, volatility, and stock returns. *The Journal of Finance*, 67(4), 1499-1537. https://doi-org.proxy.uwasa.fi/10.1111/j.1540-6261.2012.01754.x

Han, H., & Park, M. D. (2013). Comparison of realized measure and implied volatility in forecasting volatility. Journal of Forecasting, 32(6), 522-533. https://doi-org.proxy.uwasa.fi/10.1002/for.2253

Hansen, P. R., Huang, Z., & Shek, H. H. (2012). Realized GARCH: A joint model for returns and realized measures of volatility. *Journal of Applied Econometrics*, 27(6), 877-906. https://doi-org.proxy.uwasa.fi/10.1002/jae.1234

Haug, E. G., & Taleb, N. N. (2011). Option traders use (very) sophisticated heuristics, never the Black–Scholes–Merton formula. *Journal of Economic Behavior & Organization*, 77(2), 91-106. https://doi.org/10.1016/j.jebo.2010.09.013

Heston, S. (1993). A closed-form solution for options with stochastic volatility with applications to bond and currency options. *The Review of Financial Studies*, 6(2), 327-343. https://doi.org/10.1093/rfs/6.2.327

Hull, J. (2015). *Options, futures and other derivatives* (9th ed.). Pearson Education.

Hull, J., & White, A. (1987). The pricing of options on assets with stochastic volatilities. *The Journal of Finance*, 42(2), 281-300. https://doi-org.proxy.uwasa.fi/10.2307/2328253

Jarrow, R. A. (1999). In honor of the Nobel laureates Robert C. Merton and Myron S. Scholes: A partial differential equation that changed the world. *Journal of Economic Perspectives*, 13(4), 229-248. https://doi-org.proxy.uwasa.fi/10.1257/jep.13.4.229

Jiang, G., & Tian, Y. (2005). The model-free implied volatility and its information content. *The Review of Financial Studies*, 18(4), 1305-1342. https://doi-org.proxy.uwasa.fi/10.1093/rfs/hhi027

Jorion, P. (1995). Predicting volatility in the foreign exchange market. *The Journal of Finance*, 50(2), 507-528. https://doi-org.proxy.uwasa.fi/10.2307/2329417

Kairys, J., & Valerio, N. (1997). The market for equity options in the 1870s. *The Journal of Finance,* 52(4), 1707-1723. https://doi-org.proxy.uwasa.fi/10.2307/2329454

Kamara, A., & Miller, T. (1995). Daily and intradaily tests of European put-call parity. *The Journal of Financial and Quantitative Analysis*, 30(4), 519-539. https://doi-org.proxy.uwasa.fi/10.2307/2331275

Klemkosky, R., & Resnick, B. (1979). Put-call parity and market efficiency. *The Journal of Finance*, 34(5), 1141-1155. https://doi-org.proxy.uwasa.fi/10.2307/2327240

Koenker, R., & Bassett, G. (1978). Regression quantiles. *Econometrica*, 46(1), 33-50. https://doi-org.proxy.uwasa.fi/10.2307/1913643

Lamoureux, C., & Lastrapes, W. (1993). Forecasting stock-return variance: Toward an understanding of stochastic implied volatilities. *The Review of Financial Studies*, 6(2), 293-326. https://doi.org/10.1093/rfs/6.2.293

Latané, H., & Rendleman, R. (1976). Standard deviations of stock price ratios implied in option prices. *The Journal of Finance*, 31(2), 369-381. https://doi-org.proxy.uwasa.fi/10.2307/2326608

Lauterbach, B., & Schultz, P. (1990). Pricing warrants: An empirical study of the Black-Scholes model and its alternatives. *The Journal of Finance*, 45(4), 1181-1209. https://doi-org.proxy.uwasa.fi/10.2307/2328720

Macbeth, J., & Merville, L. (1979). An empirical examination of the Black-Scholes call option pricing model. *The Journal of Finance*, 34(5), 1173-1186. https://doi-org.proxy.uwasa.fi/10.2307/2327242

Martens, M., & Zein, J. (2004). Predicting financial volatility: High-frequency time-series forecasts vis-à-vis implied volatility. *The Journal of Futures Markets*, 24(11), 1005-1028. https://doi-org.proxy.uwasa.fi/10.1002/fut.20126

Mayhew, S. (1995). Implied volatility. *Financial Analysts Journal*, 51(4), 8-20. https://doi-org.proxy.uwasa.fi/10.2469/faj.v51.n4.1916

Merton, R. (1973). Theory of rational option pricing. *The Bell Journal of Economics and Management Science*, 4(1), 141-183. https://doi.org/10.2307/3003143

Mixon, S. (2009). Option markets and implied volatility: Past versus present. *Journal of Financial Economics*, 94(2), 171-191. https://doi.org/10.1016/j.jfineco.2008.09.010

Moreira, A., & Muir, T. (2017). Volatility-managed portfolios. *The Journal of Finance*, 72(4), 1811-1644. https://doi.org/10.1111/jofi.12513

Molnár, P. (2012). Properties of range-based volatility estimators. *International Review of Financial Analysis*, 23, 20-29. https://doi.org/10.1016/j.irfa.2011.06.012

Newey, W., & West, K. (1987). A simple, positive semi-definite, heteroskedasticity and autocorrelation consistent covariance matrix. *Econometrica*, 55(3), 703-708. https://doi.org/10.2307/1913610

Nikkinen, J., & Rothovius, T. (2019). The EIA WPSR release, OVX and crude oil internet interest. *Energy*, 166, 131-141. https://doi.org/10.1016/j.energy.2018.10.061

Nikkinen, J., & Sahlström, P. (2004). Impact of the federal open market committee's meetings and scheduled macroeconomic news on stock market uncertainty. *International Review of Financial Analysis*, 13(1), 1-12. https://doi.org/10.1016/j.irfa.2004.01.001

Parkinson, M. (1980). The extreme value method for estimating the variance of the rate of return. *The Journal of Business*, 53(1), 61-65.

Plíhal, T., & Lyócsa, S. (2021). Modeling realized volatility of the EUR/USD exchange rate: Does implied volatility really matter? *International Review of Economics and Finance*, 71, 811-829. https://doi.org/10.1016/j.iref.2020.10.001

Poon, S., & Granger, C. (2005). Practical issues in forecasting volatility. *Financial Analysts Journal*, 61(1), 45-56. https://doi-org.proxy.uwasa.fi/10.2469/faj.v61.n1.2683

Quigg, L. (1993). Empirical testing of real option-pricing models. *The Journal of Finance*, 48(2), 621-640. https://doi-org.proxy.uwasa.fi/10.2307/2328915

Rogers, L., & Satchell, S. (1991). Estimating variance from high, low and closing prices. *The Annals of Applied Probability*, 1(4), 504-512. https://doi.org/10.1214/aoap/1177005835

Rubinstein, M. (1985). Nonparametric tests of alternative option pricing models using all reported trades and quotes on the 30 most active CBOE option classes from August 23, 1976 through August 31, 1978. *The Journal of Finance*, 40(2), 455-480. https://doi-org.proxy.uwasa.fi/10.2307/2327895

Rubinstein, M. (1994). Implied binomial trees. *The Journal of Finance*, 49(3), 771-818. https://doi-org.proxy.uwasa.fi/10.2307/2329207

Samuelson, P. A. (1965). Rational theory of warrant pricing. *Industrial Management Review*, 6(2), 13-31.

Schmalensee, R., & Trippi, R. (1978). Common stock volatility expectations implied by option premia. *The Journal of Finance*, 33(1), 129-147. https://doi-org.proxy.uwasa.fi/10.2307/2326355

Schwert, G. W. (1989). Why does stock market volatility change over time? *The Journal of Finance,* 44(5), 1115-1153. https://doi.org/10.1111/j.1540-6261.1989.tb02647.x

Schwert, G. W. (1990). Stock market volatility. *Financial Analysts Journal*, 46(3), 23-34. https://doi-org.proxy.uwasa.fi/10.2469/faj.v46.n3.23

Seo, S. W., & Kim, J. S. (2015). The information content of option-implied information for volatility forecasting with investor sentiment. *Journal of Banking & Finance*, 50, 106-120. https://doi.org/10.1016/j.jbankfin.2014.09.010

Shu, J., & Zhang, J. E. (2006). Testing range estimators of historical volatility. *Journal of Future Markets*, 26(3), 297-313. https://doi-org.proxy.uwasa.fi/10.1002/fut.20197

Stoll, H. (1969). The relationship between put and call option prices. *The Journal of Finance*, 24(5), 801-824. https://doi-org.proxy.uwasa.fi/10.2307/2325677

Taylor, S., Yadav, P. K., & Zhang, Y. (2010). The information content of implied volatilities and model-free volatility expectations: Evidence from options written on individual stocks. *Journal of Banking & Finance*, 34(3), 871-881. https://doi.org/10.1016/j.jbankfin.2009.09.015

Vasilellis, G. A., & Meade, N. (1996). Forecasting volatility for portfolio selection. *Journal of Business Finance & Accounting*, 23(1), 125-143. https://doi-org.proxy.uwasa.fi/10.1111/j.1468-5957.1996.tb00407.x

Wang, Y.-H., & Wang, Y.-Y. (2016). The information content of intraday implied volatility for volatility forecasting. *Journal of Forecasting*, 35(2), 167-178. https://doi-org.proxy.uwasa.fi/10.1002/for.237

Whaley, R. E. (2009). Understanding the VIX. *Journal of Portfolio Management*, 35(3), 98-105. https://doi.org/10.3905/JPM.2009.35.3.098

White, H. (1980). A heteroskedasticity-consistent covariance matrix estimator and a direct test for heteroskedasticity. *Econometrica*, 48(4), 817-838. https://doi-org.proxy.uwasa.fi/10.2307/1912934

Xu, X., & Taylor, S. J. (1995). Conditional volatility and the informational efficiency of the PHLX currency options market. *Journal of Banking & Finance*, 19(5), 803-821. https://doi.org/10.1016/0378-4266(95)00086-V

Yang, D., & Zhang, Q. (2000). Drift-independent volatility estimation based on high, low, open, and close prices. *The Journal of Business*, 73(3), 477-492. https://doi-org.proxy.uwasa.fi/10.1086/209650

Appendices

Appendix 1. OLS Regressions with log-transformed non-overlapping data

		t		t
α	-0.322		-0.226	
	(0.200)		(0.196)	
$\ln VIX_{m-1}$	0.991***	-0.016	0.842***	-1.166
	(0.071)		(0.135)	
$\ln \hat{\sigma}_{Realised,m-1}$			0.131	
			(0.105)	
χ^2 (p-value)	186.69		11.14	
	(0.000)		(0.004)	
Adjusted R^2	0.595		0.597	
Number of observations	178		178	

The OLS regressions with the S&P 500 21-day realised volatility and lagged VIX for the non-overlapping sample. The data consist of the daily index values from June 2006 to April 2021. The dependent variable is $\ln \hat{\sigma}_{Realised,m}$. Newey-West (1987) standard errors are reported in parentheses. *** indicates significance at the 1% level. Column t is the t-statistic on the null hypothesis of $\beta_1 = 1$. χ^2 (p-value) corresponds to the Wald test for $\alpha = 0$ and $\beta_1 = 1$ and indicates its p-value in parentheses.

Appendix 2. OLS regressions with DAX and VDAX

		t		t
α	1.152		1.537	
	(1.209)		(1.066)	
$VDAX_{t-21}$	0.788***	-3.487	0.655***	-2.910
	(0.061)		(0.118)	
$\hat{\sigma}_{DAX,t-21}$			0.137	
			(0.129)	
χ^2 (p-value)	54.58		8.92	
	(0.000)		(0.012)	
Adjusted R^2	0.539		0.543	
Number of observations	3752		3752	

The OLS regressions with the DAX 21-day realised volatility and lagged VDAX. The data consist of the daily index values from June 2006 to April 2021. The dependent variable is $\hat{\sigma}_{DAX,t}$. Newey-West (1987) standard errors are reported in parentheses. *** indicates significance at the 1% level. Column t reports the t-statistic on the null hypothesis of $\beta_1 = 1$. χ^2 (p-value) corresponds to the Wald test for $\alpha = 0$ and $\beta_1 = 1$ and indicates its p-value in parentheses.

Appendix 3. OLS regressions with log-transformed DAX and VDAX

		t		t
α	-0.003		0.121	
	(0.050)		(0.133)	
$\ln VDAX_{t-21}$	0.933***	-1.325	0.665***	-1.765
	(0.051)		(0.092)	
$\ln \hat{\sigma}_{DAX,t-21}$			0.243***	
			(0.088)	
χ^2 (p-value)	108.35		13.78	
	(0.000)		(0.001)	
Adjusted R^2	0.572		0.584	
Number of observations	3752		3752	

The OLS regressions with the log-transformed DAX 21-day realised volatility and lagged VDAX. The data consist of the daily index values from June 2006 to April 2021. The dependent variable is $\ln \hat{\sigma}_{DAX,t}$. Newey-West (1987) standard errors are reported in parentheses. *** indicates significance at the 1% level. Column t reports the t-statistic on the null hypothesis of $\beta_1 = 1$. χ^2 (p-value) corresponds to the Wald test for $\alpha = 0$ and $\beta_1 = 1$ and indicates its p-value in parentheses.

Appendix 4. VIX coefficient in different quantiles