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A Regret-based Stochastic Bi-level Framework for Scheduling of DR Aggregator under Uncertainties

Homa Rashidizadeh-Kermani, Mostafa Vahedipour-Dahraie, Miadreza Shafie-khah, Senior Member, IEEE, and Pierluigi Siano, Senior Member, IEEE

Abstract—A regret-based stochastic bi-level framework for optimal decision making of a demand response (DR) aggregator to purchase energy from short term electricity market and wind generation units is proposed. Based on this model, the aggregator offers selling prices to the customers, aiming to maximize its expected profit in a competitive market. The clients’ reactions to the offering prices of aggregators and competition among rival aggregators are explicitly considered in the proposed model. Different sources of uncertainty impressing the decisions made by the aggregator are characterized via a set of scenarios and are accounted for by using stochastic programming. Conditional value-at-risk (CVaR) is used for minimizing the expected value of regret over a set of worst scenarios whose collective probability is lower than a limitation value. Simulations are carried out to compare CVaR-based approach with value-at-risk (VaR) concept and traditional scenario based stochastic programming (SSBP) strategy. The findings show that the proposed CVaR strategy outperforms the SSBP approach in terms of making more risk-averse energy biddings and attracting more customers in the competitive market. The results show that although the aggregator offers the same prices in both CVaR and VaR approaches, the average of regret is lower in the VaR approach.

Keywords—Aggregator, bi-level stochastic programming, demand response (DR), regret, risk-averse, wind generation unit.

NOMENCLATURE

Sets and indices

\( \{ \} \) At time \( t \) and at scenario \( \omega \).
\( \{ \} \) At time \( t \) and at scenario \( \phi \).
\( t \) Time period indices.
\( D/Ch \) Demand of loads/Charge process.
\( A, A' \) Indices (set) of aggregators.
\( \phi(\Omega) \) Scenario index (set) of rival retailers' prices.
\( N_t \) Number of aggregators.
\( t (T) \) Index (set) of time periods.
\( \omega (\Omega) \) Index (set) of scenario associated with market prices, demand loads and charge of EVs.

Variables

\( B_t(E_{D,\omega}^{D}) \) Income of customers after implementing DR programs (€).
\( S_t(E_{D,\omega}^{D}) \) Benefit of customers after implementing DR programs (€).
\( E_t^{D,\text{int}} \) Initial value of required demand of loads (MWh).
\( E_t^{D,\text{adj}} \) Adjusted demand of loads (MWh).
\( \Delta E_t^{D} \) Difference energy between the initial and adjusted value of required demand of loads (MWh).
\( E_{\text{up/down}}^{\text{wind}} \) Energy traded in up/down regulation markets (MWh).
\( E_t^{\text{DA}} \) Energy transaction in day-ahead (DA) market (MWh).
\( E_t^{\text{Wind}} \) Total energy of wind generation unit (MWh).
\( v/c \) The revenue/cost of the aggregator (€).
\( P_{A_k} \) Price signals offered by the under study aggregator (€/MWh).
\( \chi_{\omega}^{wind} \) The percentage of purchased wind energy.

Parameters

\( \rho^{DA} \) Price of DA market (€/MWh).
\( \rho^{DA,\text{int}} \) The average of DA market price (€/MWh).
\( \rho^{\text{Wind}} \) The price offered by the wind generation unit (€/MWh).
\( \rho_{r/c}^{\text{Wind}} \) The perfect price offered by the wind generation unit (€/MWh).
\( \rho_{A_k}^{\text{Wind}} \) Price signals offered by rival aggregator (€/MWh).

OMENCLATURE

\( X_{A} \) Percentage of loads supplied by rival aggregator \( A \).
\( X_{A,\omega} \) Percentage of loads supplied by the under study aggregator.
\( Y_{A,\omega} \) Percentage of loads shifted among the aggregators.
\( R/\eta \) Auxiliary variables for CVaR calculations.
\( \text{profit}_{\omega} \) Profit of aggregator at scenario \( \omega \).
owners minimize their payments. A bottom-up model for demand response (DR) aggregators in electricity markets has been presented in [3], where the DR aggregator considers the technical constraints of customers in developing an optimal trading strategy in the wholesale electricity market. In addition, the stepwise functions for the participation of customers in DR programs are used, although such modeling of customers’ response to the retail price may not explicitly model the competition among rival aggregators. In [4], a decision making strategy is proposed for a retailer to optimize both the time-of-use price settings and the purchase allocation in the multiple channels of purchase. In that work, only the participation of consumers in price-based DR programs has been investigated while the effect of EV owners as a dynamic source of uncertainty has been neglected. The competition among DR aggregators to sell energy stored in the residential storage systems has been provided in [5], where independent energy producers, and anyone capable of storing energy with the desire to sell it can participate in the market through a DR aggregator program. However, the interactions of EVs as storage devices as well as their uncertainties were not considered. Moreover, an optimization model is provided in [6] for the participation of an aggregator of distributed energy resources in the DA market in the presence of demand flexibility. Although, a decision support tool for the energy and financial interactions of the aggregator with its customers and the wholesale market has been provided, no measurement tool was applied to handle the effect of uncertainties. The uncertain resources introduce risks in the problem of decision making of the aggregator. On this basis, the authors of many research works employed different risk measurement tools in stochastic optimization models to deal with the effects of the uncertainties. In [7] and [8], a risk-averse stochastic bi-level programming approach to solve the decision-making of a retailer in a competitive market under different sources of uncertainty is presented. Although, in [7] and [8], CVaR tool is applied to the problem owing to the uncertainties associated with market prices, offering prices by rivals and demand of EVs and DRs, the effect of renewable resources on the decision making problem of the aggregator is neglected. Furthermore, in [9], a methodology to maximize Plug-in EV aggregator profit taking decisions in DA and balancing markets with considering risk aversion has been developed without considering the preferences of EV owners and responsive loads. Other risk-averse instruments such as bilateral contracts are also designed to reduce the effects of uncertainties [10]. In bilateral contracts, the aggregator may sign a long term contract with generating companies to purchase energy, which points out buying power quantity through a pre-specified time period [11]. However, in the conditions that the aggregator misses signing any predefined bilateral contracts, it should control the risk of outcome volatility using appropriate risk measures. With increasing the penetration of wind power generation in power systems, a challenging problem for aggregator is making short term decisions [12]. Meanwhile, both wind power and its associated selling prices are considered volatile and hard to predict in the short term. A stochastic short term decision-making problem for a wind power producer in DA and balancing markets is proposed in [13]. In [14], a comprehensive stochastic decision-making model for the coordinated operation of wind power producers and DR aggregators participating in the DA market is provided in which a minimum CVaR term has been included in the model. In that model, the wind power producer participates in DA market while arranging DR contracts with DR aggregator to lessen their risk via a bilateral agreement. In [15], the authors have presented a stochastic optimization model for an optimal bidding strategy of EV aggregator in DA energy and ancillary service markets where CVaR approach has been utilized for measuring EV aggregators’ risks caused by the uncertainties. Moreover, in [16], a bidding strategy model for an EV aggregator for a smart demand-side management has been presented, in which the conditional expectation of electricity purchase cost was minimized to optimally determine DA and real-time flexible adjustment bids including quantities and prices submitted by the EV aggregator. However, the role of demand-side participation is not identified in [15]-[16], and the customers’ utility is not taken into account in the presented methods. Although in most of the stochastic programming problems, the risk measurement tool (e.g., CVaR) has been incorporated into the problem; in some others, researchers used the regret concept in the stochastic process. Regret of each scenario is defined as the difference between the value of the objective function given by the overall compromise solution and the value of the optimal solution for that single scenario [17]. In [18], the minimax regret criterion is applied to the problem of unit commitment model aiming to minimize the maximum regret of the DA decision from the actual realization of the uncertain real-time wind power generation. A generic model to characterize a variety of flexible demand-side resources is presented in [19] in which multiple stochastic scenarios are evaluated to show key sources of uncertainty. Then, a risk-averse optimal bidding formulation based on CVaR is applied considering the expected regret value over an endogenously selected set of the worst scenarios, whose summation of probabilities is minimized. However, the interaction between the aggregator and customers through a bi-level model has not been addressed in [19]. In [20], a stochastic bi-level scheduling model for decision-making of a load serving entity in DA and regulating markets with uncertainties is proposed. In this model, LSE as the main interacting player of the market sells electricity to end-use customers and plug-in EVs to maximize its expected profit. In [21], a wind power producer participates in short term market to compete against other rival agents to supply the aggregators. In this model, the aggregators are able to choose the most competitive WPP in such a way that their energy payments be minimized in the scheduling horizon. In order to compare the highlights and important aspects of this paper, Table I is also added to show the contributions of the works in view of the existing state of the art literature. In prior researches [7], [8] and [13], the authors presented different stochastic models for optimal scheduling of aggregators and wind power producers in a competitive environment. In the mentioned studies, the inclination of EV owners and loads toward cost minimization of the energy requirement lead them to select the most competitive aggregator for their energy purchases. In this regard, the distinctive feature of the proposed approach with respect to [22]-[24] in which the competition among the aggregators is not considered, is that in our study, the customers’ response to rivals’ prices and competition among rival aggregators are both completely modeled via a bi-level programming framework. In that sense, the bi-level models in [22] and [25] do not provide the aspects of aggregator competition and selection of aggregators by loads and EVs in a retailing market. Authors in [22], proposed a mathematical program with equilibrium
constraints optimizing the aggregator’s decisions. It endogenously determines the profit-optimal price level subject to the cost minimizing charging schedule of the final customers, who are reacting to a combination of retail price signals and distribution use-of-system network charges. This active response follows an affine demand-price relationship, which is individually parametrized only for vehicles without considering the responsive loads. Moreover, the demand-price relationship does not show the clients' response to prices offered by all aggregators and even does not provide competition among the aggregators to attract them. However, the bi-level decision making scheme of an aggregator and its interactions with the final customers are completely modeled in this paper. Also, the aggregator plays the leader role in the upper level while loads and EV owners are the followers in the lower level.

The uncertain resources in decision making problem of the aggregator introduce risks. To hedge against these uncertainties, the authors of many research works employed different risk measurement tools in stochastic optimization models to deal with the effects of uncertainties. In some works, such as [7]- [8] and [14], CVaR as an effective way is applied to limit the risk on profit variability for decision maker while in [24], DR is used to lessen the risk of wind power uncertainties. In other studies, as in [18] and [19], regret concept is introduced by the difference between the solution without knowing realized uncertain parameters and the solution with perfect information. Therefore, in this paper, regret concept is used to reduce the risk natured from uncertainties and allows the aggregator to compare different decision makings to bid in the electricity market and to offer prices to the customers in different strategies such as CVaR, VaR and SBSP approaches. In these strategies, the trade-off between expected profit and offering price to the individuals is given to the aggregator. For instance, based on different values of regret from these strategies, the expected profit, the contribution of the aggregator in supplying loads and EVs and its participation in electricity market would be provided for the aggregator. This paper extends our prior work and proposes a regret-based stochastic bi-level framework for optimal decision making of a demand response (DR) aggregator to purchase energy from short term electricity market and from wind generation units. Based on this bi-level model, in the upper level, the aggregator offers selling prices to the customers, aiming to maximize its expected profit in a competitive market, while in the lower level, the customers tend to supply their load from the fairest aggregator such that to minimize their costs. In this regard, the clients' reactions to the offering prices of aggregators and competition among rival aggregators are explicitly considered in the proposed model. In this regret-based bidding strategy, CVaR concept is used to explicitly quantify the risks of aggregator's bidding based on the difference between the solution without being aware of the realized uncertain parameters and the solution with perfect information; • A decision-making approach for a DR aggregator is developed based on bi-level stochastic programming to determine its optimal involvement in the wholesale market and its trading energy with wind generation units. Also, the reaction of customers to the selling prices of all aggregators and the competition among all rival aggregators to attract the loads and EV owners are explicitly accounted for at the retailing layer; • The effectiveness of the proposed bidding strategy is evaluated in hedging risks of uncertainties via different case studies against the SBSP approach and VaR concept.

The rest of this paper is organized as follows: the proposed risk-averse optimal decision-making strategy of the aggregator is presented in Section II. The problem is formulated in Section III. Section IV discusses numerical results utilizing CVaR, VaR and SBSP approaches. Finally, Section V provides concluding remarks.

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Table I. The contribution of literature in view of existing state of the art.
II. PROBLEM FRAMEWORK

A. Assumptions
In the proposed methodology, different assumptions are taken into account as follows:

- It is assumed that only unidirectional bids are allowed for the aggregator in the electricity market. Therefore, it can only purchase energy from the grid but cannot sell the excess energy back to the market [19];
- It is supposed that the aggregator plays as a price taker meaning that its bids cannot affect the clearing price of the wholesale market. This is deemed a reasonable assumption for an aggregator representing a small to medium-sized fleet;
- The structure of liberalized power markets is considered which includes a DA market and a real-time market where unforeseen events can be balanced [26]. Also, there exists the possibility of signing contracts between the aggregator and wind generation unit as a supplier outside an organized market place [27].

B. Risk-averse optimal decision making of aggregator
Here, it is assumed that the clients including a number of EVs and several loads are equipped with smart energy management controllers (SEMC) and are able to respond to the electricity prices by adjusting their consumption levels to reduce their energy costs. Therefore, based on the offered electricity prices from different aggregators, EV owners can change their behavior and demand level, while SEMC in responsive loads can participate in DR programs, automatically and adjust the customer consumption to reduce energy costs. To this end, SEMC of each responsive load can choose proper aggregator by monitoring real-time prices and can switch to the most competitive aggregator in short-term scheduling. This is feasible by developing a fast communication media with bidirectional data transfer between the aggregators and smart loads and the EV charging stations. It should be noted that the clients have not gotten involved each day in the process, but this act is done by SEMC system and therefore it is not difficult and burdensome in practice for the clients [28]. Also, EVs participate in DR program and are motivated to mitigate their charging energy with regard to the prices offered by the aggregator. It is also assumed that consumers are categorized into responsive and non-responsive loads. The consumption of responsive loads can be adjusted according to the price signal such that the customers' payment is minimized. The under study aggregator deals with two markets including DA and regulation markets. First, it submits its bids in DA market and then updates them in the regulation market close to the real time to compensate the energy deviations. As seen from Fig. 1, due to the competition among the aggregators, the under study aggregator estimates the prices offered by rivals. Also, the aggregator requires to forecast the expected demand of customers. Once each aggregator offers a selling price, then in the lower level of the problem, clients choose which aggregator to supply their electricity demand during the planning horizon. Therefore, the profit maximization problem considers the reaction of clients to the prices offered by aggregators. This reaction is given in a bi-level problem in which the demand share supplied by each aggregator is obtained via minimization of the procurement cost of clients at the lower level. In a competitive market, the aggregator competes to augment its share to supply customers by offering proper prices to them. In this problem, the aggregator deals with different sources of uncertainty including DA and regulation market prices, the randomness of the responsive loads and charging demand of EVs fleet as well as the inherent changes of wind power generation and its associated prices. Here, a set of possible scenarios are generated using associated predicted values within probability distribution function (PDF) considering each uncertainty. Then, the standard deviation (SD) for scenario generation action is considered based on historical forecasting errors. Different sources of uncertainty involved in decision-making problem may result in high profit volatility in the offering strategy. Since, the aggregator may not be willing to face such a high profit volatility, here, the proposed risk-averse decision making strategy is adopted based on the regret concept as a widely applied measurement in decision making problems under uncertainty. As Zeelenberg discussed in [29], regret is a negative, cognitively based emotion experienced by people when realizing that their present conditions would have been better if they had decided differently. Therefore, the foresight of regret could potentially persuade people to make decisions more rationally. In the context of the optimal decision making problem of the aggregator, the regret associated with each scenario under the given wind energy prices can be defined by the difference between the objective function value when the wind energy prices are chosen to be optimal and the objective function value with the given decision making strategy. Fig. 2 illustrates the flowchart of the implemented procedure step by step. As owners and responsive loads to maximize its expected profit. On the other hand, since the aggregator does not know the offering prices of its competitors, it should estimate the prices offered by them. Considering only the competition among aggregators and ignoring the role of customers and their goals is far from reality. In fact, response of customers to the prices offered by the aggregator affects its revenue. In this regard, decision-making problem from the aggregator's viewpoint should be considered as a stochastic bi-level model in which in the upper-level, the objective of the aggregator is to maximize its expected profit through its energy interactions while in the lower-level, the loads and EV owners tend to minimize their payments. The schematic of the proposed bi-level framework is depicted in Fig. 1. In the upper level of the optimization problem, the aim of the aggregator is to maximize its expected profit. To this end, the aggregator submits its hourly energy blocks to DA market several hours before the operating day. Also, it may purchase its required energy from wind generation unit. Then, during the operating day, depending on actual conditions of loads, the aggregator may participate in the regulating market to compensate the energy deviations. As seen from Fig. 1, due to the competition among the aggregators, the under study aggregator estimates the prices offered by rivals. Also, the aggregator requires to forecast the expected demand of customers. Once each aggregator offers a selling price, then in the lower level of the problem, clients choose which aggregator to supply their electricity demand during the planning horizon. Therefore, the profit maximization problem considers the reaction of clients to the prices offered by aggregators. This reaction is given in a bi-level problem in which the demand share supplied by each aggregator is obtained via minimization of the procurement cost of clients at the lower level. In a competitive market, the aggregator competes to augment its share to supply customers by offering proper prices to them. In this problem, the aggregator deals with different sources of uncertainty including DA and regulation market prices, the randomness of the responsive loads and charging demand of EVs fleet as well as the inherent changes of wind power generation and its associated prices. Here, a set of possible scenarios are generated using associated predicted values within probability distribution function (PDF) considering each uncertainty. Then, the standard deviation (SD) for scenario generation action is considered based on historical forecasting errors. Different sources of uncertainty involved in decision-making problem may result in high profit volatility in the offering strategy. Since, the aggregator may not be willing to face such a high profit volatility, here, the proposed risk-averse decision making strategy is adopted based on the regret concept as a widely applied measurement in decision making problems under uncertainty. As Zeelenberg discussed in [29], regret is a negative, cognitively based emotion experienced by people when realizing that their present conditions would have been better if they had decided differently. Therefore, the foresight of regret could potentially persuade people to make decisions more rationally. In the context of the optimal decision making problem of the aggregator, the regret associated with each scenario under the given wind energy prices can be defined by the difference between the objective function value when the wind energy prices are chosen to be optimal and the objective function value with the given decision making strategy. Fig. 2 illustrates the flowchart of the implemented procedure step by step. As
shown, plausible realizations of stochastic parameters are generated based on the forecasted data and the scenarios related to the forecasting errors are generated with their mean values and standard deviation based on their associated PDF [11]. Then, this PDF is divided into discrete intervals with different probability levels. Forecasted errors of each uncertain parameter are modelled by generating a large enough number of scenarios by implementing Monte Carlo simulation (MCS) and Roulette wheel mechanism (RWM). Then, the reduction approach is applied to the problem. With the obtained scenarios, in the upper level problem, the aggregator tends to maximize its expected profit while in the lower level, the customers try to choose the most competitive aggregator to supply their demand so that to minimize their payments. The two levels are combined using Karush-Kuhn-Tucker (KKT) optimality conditions. The final decisions include DA energy biddings and TBR offering prices as well as the energy deviations compensated in the regulation market.

Fig. 1. The schematic of the proposed problem.

III. MATHEMATICAL FORMULATION

A. Upper Level: Aggregator scheduling problem

The regret associated with each scenario is defined as the difference between the value of objective function achieved under perfect wind power price information (i.e., the profit obtained if the aggregator had known the electricity price before making any decisions) and the expected profit corresponding to the decisions made under the realized scenarios of wind energy prices. In other words, the regret gives the loss of profit due to incomplete information of the wind energy prices. The value of the objective function with the hourly wind energy prices of a specific scenario \( \omega \), is defined as below:

\[
\text{profit}_{\omega} = \max \sum_{t \in T} \left[ P_{DA}^{t} - P_{D,\omega}^{t} - P_{wind}^{t} - V_{1,\omega}^{t} - A_{2,\omega}^{t} + P_{D}^{t} - P_{D,\omega}^{t} - P_{wind}^{t} - V_{2,\omega}^{t} - A_{3,\omega}^{t} \right]
\]  

(1)

where, equation (1) investigates the objective of the aggregator to maximize its profit in scenario \( \omega \) and in a certain time period. The first term of the objective function represents the costs due to the purchase of the energy from the DA market and from wind generation unit. Due to the deviations between the real time energy consumption and the DA bid, the second line provides the costs to cover such deviations in the regulation market. It is rational that those consumers who incur excess consumption than the scheduled one, should pay for it and those who reduce their consumption when the system encounters with low production and high consumption, should buy the energy requirement with lower prices (or be paid for the volume of energy injected). The last line represents the revenues that the aggregator obtains from selling energy to loads and EVs.
Constraint (2) represents the energy balance for each scenario and at each period. The under-study aggregator contributes to supply loads and EVs demand according to (3). Constraint (4) imposes a limitation on the energy constraint from wind generation unit for each scenario $\omega$ and at time period $t$.

$$
E_{t,\omega}^D + E_{t,\omega}^{Ch} = E_{t,\omega}^{DA} + E_{t,\omega}^{Ch} - \sum_{\Phi} x_t \ E_{t,\omega}^{\text{wind} \ \text{wind}} (2)
$$

Subject to: (2) - (6)

where, $R_{\omega}$ and $\eta$ are auxiliary variables with respect to CVaR. It is worth noting that $R_{\omega}$ is a non-negative variable.

Another approach to measure the risks of decision making under uncertainty is VaR. In this paper, the maximum regret over an endogenously selected subset of scenarios, whose collective probability of occurrence is at least $\alpha$ is minimized. Through minimizing the VaR of the regret, the aggregator can be 100% certain that the realized regret will be no more than the VaR value found by the model. Mathematically, the VaR minimization model is formulated as follows [19]:

$$
\text{(VaR)} \min \ W (11)
$$

$$
\text{Subject to:} \sum_{\omega} \pi_{\omega} V_{\omega} \geq \alpha (12)
$$

$$
W - \text{regret} + M(1-V_{\omega}) \geq 0 (13)
$$

$$
V_{\omega} \in [0,1] (14)
$$

And the expressions in (2)-(6) (15)

where, $M$ is a sufficiently large enough constant and $V_{\omega}$ is a binary variable to show whether the scenario $\omega$ is selected in the reliability set or not. Constraint (12) explains that the collective probability of the reliability set has to be no less than $\alpha$. Constraints (13) states that the value of $W$ has to be no less than regret values of the scenarios that are included in the reliability set.

C. Lower Level: Customers’ cost minimization

The final customers including responsive loads and EV owners tend to minimize their payments as stated in (16):

$$
\text{Min} \left[ \sum_{A_{\omega},J} E_{t,\omega}^{D} \left( p_{A_{\omega},J}^{D} X_{A_{\omega},J}^{D} + \sum_{A_{\omega},J} p_{A_{\omega},J}^{Ch} X_{A_{\omega},J}^{Ch} \right) ight] + 
$$

$$
\sum_{A_{\omega},J} \sum_{\omega} E_{t,\omega}^{Ch} \left( p_{A_{\omega},J}^{Ch} X_{A_{\omega},J}^{Ch} + \sum_{A_{\omega},J} p_{A_{\omega},J}^{Ch} X_{A_{\omega},J}^{Ch} \right) \right] (16)
$$

The first two lines of the objective function in (16) denotes the costs of purchasing energy from the aggregator and the rivals by loads and EVs, respectively. The last two lines describe the reluctance of the customers to shift among the aggregators. Fictitious cost denotes that there are some customers or EVs that are not willing to switch among aggregators to choose the cheapest one. The reluctance by
customers and EVs to change their aggregator causes that each aggregator has at least some demand to supply at any time. Although, it is forecastable that when the aggregator offers high prices to the customers, it loses some of its loads. In other words, the customers of an aggregator will decrease, suddenly, if its offering prices be higher at all times. Practically, if an aggregator wants to stay in the game, it should offer fair prices to the customers so they do not leave it. Therefore, this behavior of customers does not lead to very risky and unstable conditions for the aggregators.

The customers can select their supplying aggregator for buying electricity. Therefore, the total demand of customers including loads and EVs should be supplied by the aggregators as stated in (17):

$$
\sum_{A \in N_A} X^{D/Ch}_{A,J,\phi} = 1; \mu_{A,\phi}
$$

(17)

The aggregators compete with each other to retain and increase their customers by optimizing their prices. Therefore, the customers may move among aggregators to choose the most competitive aggregator with the lowest offering prices. Therefore, the movement of customers among the aggregators is modeled as represented in (18):

$$
X^{D/Ch}_{A,J,\phi} = X^{D/Ch}_{A,J,\phi} + \sum_{A \in A_A} Y^{D/Ch}_{A,J,\phi} - \sum_{A \in A^C} Y^{D/Ch}_{A,J,\phi}; \lambda_{A,J,\phi}
$$

(18)

Equation (18) expresses the share of each aggregator to supply the required energy of demand loads. From this relation, it is seen that a percentage of the total load is supplied by each aggregator and also, another percentage is transferred among the aggregators. In other words, the demand supplied by each aggregator consists of the initial demand supplied by the aggregator plus the customers who transfer from another aggregator to this aggregator minus those clients who leave the aggregator and go to the rivals [30]. Moreover, the customers are concerned about the technical constraints of their loads and EVs that should be satisfied as investigated in [7]. Moreover, constraints (19)-(20) impose limits on EVs' battery at each period that should be considered in the problem [20].

$$
S_{OC,t,\omega} = S_{OC,t,\omega} + \eta_{Ch} F_{t,\omega} / E_{Cap}; \varphi_{t,\omega}
$$

(19)

$$
S_{OC} \leq S_{OC,t,\omega} \leq S_{OC}: \mu_{t,\omega} R_{Cap}
$$

(20)

Equation (19) shows that the state of charge of each EV in period $t$ depends on the state of charge in period $t-1$ and the charging power absorbed from the upstream grid during period $t$. Also, based on (20), the state of charge of each EV is restricted between certain levels at each hour and in each scenario and cannot exceed this bound. The dual variables are separated by a colon after equations (17)-(20).

D. Model of Responsive Loads

The customers' energy consumption behavior can be adjusted in response to the incentives received based on electricity prices. To obtain maximum benefit, end-use consumers adjust their energy usage pattern in period $t$ from an initial value, $E^{D/int}_{t,\omega}$ to $E^{D}_{t,\omega}$ as below:

$$
E^{D}_{t,\omega} = E^{D/int}_{t,\omega} + \Delta E^{D}_{t,\omega}
$$

(21)

After managing the pattern of energy usage, the benefit of customers is achieved as below:

$$
S(E^{D}_{t,\omega}) = B(E^{D}_{t,\omega}) - E^{D}_{t,\omega} \rho_{t,\omega}
$$

(22)

where, $S(E^{D}_{t,\omega})$ and $B(E^{D}_{t,\omega})$ represent the benefit and income of customers at period $t$ after the implementation of DR programs, respectively. In order to maximize the benefit of customers, equation (23) must be satisfied.

$$
\frac{\partial S(E^{D}_{t,\omega})}{\partial E^{D}_{t,\omega}} = \frac{\partial B(E^{D}_{t,\omega})}{\partial E^{D}_{t,\omega}} - \rho_{t,\omega} = 0
$$

(23)

In this study, a quadratic utility function, is used to incentivize the participation of responsive loads in DR programs. Based on the model, the utility of customers is obtained. Finally, the consumption of customers is obtained as follows:

$$
E^{D}_{t,\omega} = E^{D/int}_{t,\omega} \left( \frac{\rho_{t,\omega}}{\rho_{t,\omega}^{D/int}} + \frac{1}{1 + Ela_{s,h}} \right) E^{s/h,\omega}_{t,\omega}
$$

(24)

Additionally, based on cross-elasticity coefficients [31], which are defined as demand sensitivity of the $h$th period with respect to the price elasticity at $h$th period, the amount of demand after the DR would be obtained. Then, by combining (21)-(24), the economic model of responsive loads would be obtained as bellow [31]:

$$
E^{D}_{t,\omega} = E^{D/int}_{t,\omega} \exp \sum_{h=0}^{T} Ela_{s,h} \ln \left[ \frac{\rho_{t,\omega}}{\rho_{t,\omega}^{D/int}} + \frac{1}{1 + Ela_{s,h}} \right]
$$

(25)

where, $E^{D/int}_{t,\omega}$ is the initial demand before participating in DR and $E^{D}_{t,\omega}$ is the total demand after DR participants.

Note that based on constraint (26), the supplied demand by the under study aggregator at each hour and each scenario should be less than the total required demand of loads as bellow:

$$
E^{D}_{t,\omega} \leq E^{D}_{t,\omega}_0
$$

(26)

E. Combining the upper and lower levels

The presented stochastic model is finally formulated as a bi-level problem that the upper level problem represents the maximization of the expected profit of the aggregator while the lower level problem states the minimization of energy procurement costs of loads and EV owners.

In order to solve the obtained bi-level programming problem by a commercially available optimization solver, it should be converted to an equivalent mixed-integer linear programming (MILP) problem with the following steps:

- Lagrange function of the lower level for a vector of the variable of the upper level is obtained;
- The KKT optimality conditions of the lower level problem is obtained by partial derivatives of the Lagrange function;
- The non-linear complementary slackness conditions are equivalently expressed as a set of linear constraints based on the approach explained in [37];
- The bi-linear products of $E^{D/int}_{t,\omega}$ and $E^{Ch}_{t,\omega}$ are replaced by the related equivalent linear expressions using duality theory [30].

Based on the mentioned steps, the lower level problem is replaced by its associated first order optimality conditions, the KKT necessary optimality conditions, and the equivalent linear form of complementary slackness conditions. Finally, the obtained bi-level problem is provided based on the objective function of the upper level and the constraints associated with it as well as the constraints of KKT conditions of the lower level, the equivalent expression
obtained for the bi-linear production of $E^{D}_{t,0}P_{A,t}^{D}$ and $E^{Ch}_{t,0}P_{A,t}^{Ch}$ and the linear form of the complementary slackness conditions [7].

IV. CASE STUDY AND NUMERICAL RESULTS

A. Case Study

In this simulation, an aggregator operating in an electricity market during a time horizon of one day is considered. It is assumed that the under-study aggregator competes against three rival aggregators in order to supply the energy of loads and a number of EVs. The uncertainties of prices offered by all rival aggregators are extracted based on the given PDFs which is divided into three discrete intervals with different probability levels with a uniform random error of ±10% for hourly rivals’ prices [21]. Moreover, the forecasted values of customers’ and EVs’ demand are extracted from [32] and [22], respectively. The forecast values of DA market prices, up/down regulation market prices [33] as well as wind energy prices are shown in Fig. 3 (a). Also, the forecasted prices of rival aggregators offered to the customers is given in Fig. 3 (b). The amount of wind energy, loads demand and EVs charging power are depicted in Fig. 4. The scenarios related to demand of the responsive loads and EVs are correlated to DA prices based on the relation explained in [30].

Here, the forecast errors associated with DA price, down regulation price and wind energy forecast errors are considered equal to 10% [34]. The forecast errors of up regulation and wind energy prices are also 0.05 and 0.15, respectively [7] and [13]. It is assumed that in the base case, 50% of customers participate in price-based DR programs and others remain non-responsive. Moreover, the confidence level (α) is considered to be equal to 0.95 in the base case and varies in the simulation. Finally, a set of 2500 scenarios is generated based on their associated PDF to model the uncertainties of wind power, electricity prices and demand of loads and EVs. In the next step, by implementing K-means algorithm as an efficient scenario reduction algorithm [35], a set of 243 scenarios are selected to represent well enough the uncertainties. The reduced scenarios are applied to the proposed regret based bi-level optimization model to determine optimal bidding profile under the two strategies including CVaR and SBSP. The optimization process is carried out on a PC with 4 GB of RAM and Intel Core i7 @ 2.60 GHz processor using CPLEX solver and GAMS software [36]. The optimality gap of different cases of the optimization algorithm is set to 0.0, and the running times of the mentioned algorithms under different operating conditions are less than 118 seconds (~2 min) in worst cases, which are indeed small values if compared with the typical 24-h (even 1-h) time resolution of the simulated scenarios. Also, total of iterations in this model are 39356.

B. Numerical results

Fig. 5 shows the bidding profile under the two strategies including CVaR and SBSP for $\alpha=0.95$, that is considered as the base case. With respect to the lower average prices of wind energy than that of DA market, the energy purchased through wind unit is higher on average than that of from the DA. As observed under both strategies, the aggregator prefers to arbitrage in the price differences to maximize its expected profit. Moreover, since the peak values of DA market prices happen at the same time with peak electricity demand at night, the aggregator supplies more energy from wind generation unit with lower prices. In this regard, it submits large energy bids to the wind energy unit during low wind energy price periods (e.g., 18:00-24:00) and small energy bids during high prices (e.g., 10:00-17:00). Compared with the SBSP strategy, the CVaR criteria is more conservative to hedge against uncertainties. In fact, high price volatility associated with wind energy prices prevents CVaR-based strategy to arbitrage aggressively with wind unit and, as a result, the energy traded in the DA market is higher. The energy bidding of the aggregator in both up and down regulation markets are illustrated in Fig. 6. As can be seen, in both strategies, the expected up deviation incurred by the aggregator is higher than that in the down regulation market. Since, the aggregator can obtain some revenue from trading in a less volatile up regulation. However, the aggregator purchases less energy from down regulation market in both CVaR and SBSP methods since it is more volatile and often expensive. Although, the aggregator in both CVaR and SBSP strategies participates in the regulation market to cover the lack or surplus of consumption, based on CVaR strategy, it is concerned to reduce its requirement for usual expensive balancing energy. In order to better evaluate the proposed approach, the impact of different confidence levels (i.e. $\alpha =0.90$ to $\alpha =0.99$) as well as the SD associated with the wind energy price on the regret values is evaluated in Fig. 7. CVaR criteria with a confidence level $\alpha$ can be defined as

![Fig. 3. The forecasted hourly prices, (a) market and wind price (b) offering by rival aggregators](image-url)

![Fig. 4. The forecasted values of wind energy, loads demand and EVs charging power.](image-url)
the expected regret of the scenarios which belong to the lower tail of the regret distribution, i.e., scenarios whose regret values are lower than or equal to (1-α) quantile of the regret distribution. When α=0.90, the risk-taker aggregator selects the higher risk level in the hope to obtain higher profit values. This might lead to higher energy procurement from cheaper and even more volatile resources which results in experiencing low levels of regret. In α=0.99, the aggregator is willing to trade energy in less volatile floors that are usually more expensive. Therefore, the aggregator deals with more regret values.

Accurate estimation of the prices offered by wind generation units is a key role in decision making strategy of the aggregator. To handle this issue suitably, the probable deviations around the estimated SD (SD=0.15 as the base case) are considered for wind energy prices. The lowest deviation might be very risky leading to very high regret amounts as in the case with SD=0.3. Therefore, the better estimation of the wind energy prices results in a better performance that yields a better decision making of the aggregator. To assess the effectiveness and computational efficiency of the proposed bidding strategy in hedging risks under uncertainty, Table II shows three benchmarks, including stochastic programming approach, VaR principles and CVaR.

Accurate estimation of the prices offered by wind generation units is a key role in decision making strategy of the aggregator. To handle this issue suitably, the probable deviations around the estimated SD (SD=0.15 as the base case) are considered for wind energy prices. The lowest value of the regret is obtained in the lowest SD and by increasing the SD, the amount of regret grows, monotonically. On the other hand, overestimation of the
the regulation market to decrease the energy imbalances. As a result, higher regret value occurs in the case with a lower confidence level.

### Table III. Energy bidding in different SD and α values

<table>
<thead>
<tr>
<th>Case</th>
<th>SD=0.05</th>
<th>SD=0.15</th>
<th>SD=0.30</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.9</td>
<td>4.52</td>
<td>40.90</td>
<td>23.10</td>
</tr>
<tr>
<td>0.95</td>
<td>5.34</td>
<td>40.40</td>
<td>21.63</td>
</tr>
<tr>
<td>0.99</td>
<td>4.14</td>
<td>38.17</td>
<td>14.78</td>
</tr>
</tbody>
</table>

For better assessing the performance of the CVaR strategy, energy bidding of the aggregator with different values of SD is compared with that of in the SBSP strategy as presented in Table III. A lower wind price forecast error (e.g., SD=5%), the expected energy purchased from wind generation units is higher than that in the DA market due to lower prices of wind energy if compared with DA market prices (see Fig. 3). In this condition, as the aggregator becomes more risk-averse (i.e. α=0.99), it bids in a more conservative way in the DA and wind generation unit. While increasing the forecast error of wind prices (i.e. SD=0.15), the aggregator tends to bid more energy in the DA market if compared with the case of SD=0.05, this is due to more volatile prices of wind generation units. In the case with SD=0.15, while increasing the values of α, the aggregator is discouraged to arbitrage more aggressively in the DA and with wind generation units. In the case with SD=0.3, if compared with the other cases, the aggregator deals more energy in the DA market due to its lower price forecast error if compared with that of wind energy prices. Moreover, in α=0.9, the risky aggregator purchases more energy from cheap wind energy (see Fig. 3). With increasing α up to 0.95, the purchases from both DA and wind generation units decreases due to their uncertainties. With α=0.99, the aggregator behaves completely risk averse, and based on the policy that it adopts to hedge against regret variability, it trades more energy in the up regulation market with lower price error forecast to achieve some revenue, and deals more energy in the down regulation market due to its lower average prices if compared to the DA ones. In addition, since the aggregator tends to keep its current load, it supplies most of them from the cheap wind generation units to avoid regret losses. In the SBSP approach, with increasing SD, the energy purchases from wind generation units decreases and the aggregator deals more with the DA market to hedge against volatilities. It also participates more
in the down regulation market to supply the loads with the prices lower than the DA ones. Furthermore, the participation of the aggregator in the up regulation market decreases. This is because when increasing the SD, it buys more energy from the expensive DA market and, thus, offers high prices to the customers. It results that the aggregator loses its loads, so the decrement in the up regulation market participation occurs.

Table IV shows the percentage of loads demand and EVs charge power supplied by each aggregator in different values of SD and confidence levels. In the CVaR strategy, in the case of $SD=0.05$, as the aggregator becomes more risk-averse, it supplies more loads and EVs demand. Because it purchases more energy from wind units with lower prices and, therefore, it can sell energy to the customers with lower prices and consequently, it attracts them. However, with increasing the SD, when the aggregator becomes more risk-averse, it loses some of its customers since it sells more expensive energy to compensate its purchases from the expensive DA market. In other words, when increasing the SD, the demand supplied by the aggregator is higher than that in the SBSP strategy, which means the outperformance of the CVaR criteria. The competition among the aggregators to supply total load is also provided in Table IV. It is reasonable that the aggregator who offers lower prices can attract more customers. Since the under study aggregator offers medium prices, it attracts a moderate percentage of customers. Therefore, the share of the aggregators to supply loads depends on the prices offered by them. Moreover, in different standard deviations and confidence levels, the understudy aggregator behaves differently. In lower confidence levels, the aggregator supplies almost the same percentage of loads. But, as the standard deviation increases, it supplies a different percentage of loads in different confidence levels. That is because it can buy energy from wind generation unit with lower prices in some scenarios, then, it may offer lower prices to attract the customers and vice versa for the scenarios with higher prices. Therefore, its risky behavior shows more variable characteristics.

Fig. 8 depicts the hourly price offered to the customers by the understudy aggregator in CVaR and SBSP strategies. The prices offered in VaR strategy are the same as CVaR one that is not shown in the figure to prevent crowding data. The aggregator modifies the selling prices to encourage customers to reduce or shift their load during the hours with high DA prices to the periods associated with lower DA ones. Instead, it decreases the selling prices in off-peak hours to determine the opposite behavior of customers. Moreover, in the CVaR strategy, the aggregator usually offers lower prices since the average of regret is lower than that of the SBSP strategy. Therefore, due to the regret reduction value in the CVaR strategy, the aggregator can offer lower dealing prices to the customers to attract them and compete with rival aggregators (e.g., at 5:00 and 14:00-16:00). In addition, the aggregator offers higher prices at 7:00 and 18:00 since an increase in the selling price is also a way of mitigating the profit volatility by decreasing the amount of customers’ demand that is supplied. Although the aggregator does not increase the price during the whole day, it may lose its customers in such competitive environment.

This paper presented a stochastic decision making model for an aggregator to determine how to provide its loads from the electricity market or wind generation units. A bi-level problem was formulated to model the reaction of loads to the retailing prices as well as the competition among aggregators. Due to the uncertain nature of the problem, a regret-based optimal bidding strategy for the aggregator was presented in which the value of expected regret over a set of worst scenarios, whose collective probability is not higher than a given threshold, is minimized. To better evaluate the effectiveness of the CVaR strategy, its performance is compared against the SBSP approach. In the CVaR approach, when increasing the SD and the confidence level ($α$), the aggregator makes risk-averse energy biddings and usually reduces its purchases from wind generation units and increases its participation in the DA market to hedge against volatility. Compared with the CVaR, in the SBSP approach, the aggregator prefers to participate more in the down regulation market, which leads to higher average regret values. Also, results show that the proposed risk-averse model based on CVaR criteria is more efficient in offering more fair prices to the customers and, as a result, attracting more loads in the competitive environment.

APPENDIX A

In order to incorporate the upper level and lower level of the problem, the following steps are applied to the problem [21]. For a given vector of upper-level variables, the Lagrangian function ($L$) of the lower-level problem is obtained as bellow:

$$L = \tilde{E}_t^D \left( \rho_{A,t}^D X_{A,t}^D + \sum_{n=1}^{N} \rho_{A,t,n}^D X_{A,t,n}^D \right) + \tilde{E}_t^Ch \left( \rho_{A,t}^Ch X_{A,t}^Ch + \sum_{n=1}^{N} \rho_{A,t,n}^Ch X_{A,t,n}^Ch \right) + \sum_{n=1}^{N} \sum_{A=1}^{A_{max}} \sum_{D=1}^{D_{max}} \tilde{E}_t^D K_{A,D}^{D,t} Y_{A,D}^{D,t} + \sum_{n=1}^{N} \sum_{A=1}^{A_{max}} \sum_{D=1}^{D_{max}} \tilde{E}_t^Ch K_{A,D}^{Ch,t} Y_{A,D}^{Ch,t} - \alpha \left( \sum_{n=1}^{N} \rho_{A,t,n}^D X_{A,t,n}^D - 1 \right) - \mu_{A,D} \left( \sum_{n=1}^{N} \rho_{A,t,n}^Ch X_{A,t,n}^Ch - X_{A,D}^{Ch,t} - \sum_{A'=1}^{A_{max}} \sum_{D'=1}^{D_{max}} y_{A',D'}^{D,t} - \sum_{D'=1}^{D_{max}} y_{A,D'}^{D,t} \right) - \gamma \left( SoC_{t,0} - SoC_{t-1,0} - \delta C_t \left( E_{t,0} + E_{t,0}^P \right) \right) + \mu_{t,0} \left( SoC_{t,0} - SoC_t - \pi_{t,0} \left( SoC_t - SoC_{t-1,0} \right) \right)$$

$$\text{Fig. 8. Hourly prices offered by understudy aggregator in two strategies.}$$

V. CONCLUSION

Transactions on Smart Grid
Then, in addition to the primal feasibility constraints of the lower level, the KKT necessary optimality conditions of the lower-level problem would be obtained by partial derivative of the Lagrangian function as in [30].

\[
\begin{align*}
\frac{\partial L}{\partial \xi^{D/CH}_{A,i,t}} &= -\rho^{D/CH}_{A,i,t} - \mu_{A,i,t} - \lambda_{\phi} = 0 \\
\frac{\partial L}{\partial \lambda_{A,i,t}} &= \rho^{D/CH}_{A,i,t} - \mu_{A,i,t} - \lambda_{\phi} = 0 \\
\frac{\partial L}{\partial \mu_{A,i,t}} &= -\rho^{D/CH}_{A,i,t} - \mu_{A,i,t} - \lambda_{\phi} = 0 \\
\frac{\partial L}{\partial \chi} &= 0 \\
A \neq A' \\
\frac{\partial L}{\partial \lambda_{C,i,t}} &= \lambda_{\phi} - \lambda_{\phi} - \mu^{t}_{A,i,t} - \bar{p}^{t}_{A,i,t} = 0
\end{align*}
\]

where, \( \mu_{A,i,t} \), \( \lambda_{\phi} \), \( \lambda_{A,i,t} \) and \( \mu^{t}_{A,i,t} \) are the Lagrangian multipliers. It should be noted that the multiplication of \( g \) and \( x \) in the complementary slackness conditions can be replaced with the linearized expression by introducing new binary variable \( u \) and large constant \( M \) [37]. Therefore, the nonlinear complementary slackness conditions are equivalently expressed as a set of linear constraints as:

\[
\begin{align*}
\frac{\partial L^{D/CH}_{i}}{\partial \xi^{D/CH}_{A,i,t}} &= -\rho^{D/CH}_{A,i,t} - \mu_{A,i,t} - \lambda_{\phi} \\
\frac{\partial L^{D/CH}_{i}}{\partial \lambda_{A,i,t}} &= \rho^{D/CH}_{A,i,t} - \mu_{A,i,t} - \lambda_{\phi} \\
\frac{\partial L^{D/CH}_{i}}{\partial \mu_{A,i,t}} &= -\rho^{D/CH}_{A,i,t} - \mu_{A,i,t} - \lambda_{\phi} \\
\frac{\partial L^{D/CH}_{i}}{\partial \chi} &= 0 \\
\frac{\partial L^{D/CH}_{i}}{\partial \lambda_{C,i,t}} &= \lambda_{\phi} - \lambda_{\phi} - \mu^{t}_{A,i,t} - \bar{p}^{t}_{A,i,t} = 0
\end{align*}
\]

Then, duality theory is applied to the problem and the bilinear product of terms \( E^{D/CH}_{i,t} / \rho_{A,i,t}^{D/CH} \) are replaced with their linear expressions as follow:

\[
\begin{align*}
\frac{Re}{\partial V_{i,t}} &= \frac{E_{i,t}^{D/CH}}{E_{i,t}^{D/CH}} \\
\sum_{\phi \in \Phi} \left[ - \sum_{A \in N_{\phi}} \sum_{N_{\phi}} \frac{E^{D/CH}_{i,t} / \rho_{A,i,t}^{D/CH}}{\chi_{A,i,t}} - \sum_{A \in N_{\phi}} \sum_{N_{\phi}} \frac{E^{D/CH}_{i,t} / \rho_{A,i,t}^{D/CH}}{\chi_{A,i,t}} - \sum_{A \in N_{\phi}} \sum_{N_{\phi}} \frac{E^{D/CH}_{i,t} / \rho_{A,i,t}^{D/CH}}{\chi_{A,i,t}} \right]
\end{align*}
\]

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