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Low-risk investing in the Finnish stock market

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ABSTRACT:
Positive linear relationship between risk and return is one of the more fundamental concepts of traditional finance theory. However, there exists extensive literature that documents that the reality of the relationship consistently diverges from what is proposed in theory. Previous literature has shown that low-risk stocks tend to perform better than expected, whereas high-risk stocks tend to perform worse than expected by theory. Additionally, it has been consistently shown that in risk-adjusted terms, low-risk stocks outperform not only their high-risk counterparts but also the market index the stocks are extracted from. These findings collectively form what is known as the low-risk anomaly.

This thesis documents how risk-return relationship manifests in the Finnish stock market and in addition, it explores the validity of a low-risk investing strategy in context of said market. In addition to focusing on a highly specific market this thesis distinguishes itself from other literature on the subject, by utilizing multiple methodological approaches and by making transaction cost related considerations in the final evaluation. Stocks in the Finnish stock market are divided into quintiles based on their past 5-year, 3-year and 1-year rolling standard deviation of returns and beta measure. The subsequent returns are evaluated by mainly two methods, by CAPM regression and by Sharpe ratios. The anomaly is additionally further evaluated in different size sections of the Finnish stock market.

This thesis finds that the low-risk anomaly is present in the Finnish stock market, specifically that high-risk stocks consistently generate statistically significant negative alpha and that low-risk stocks consistently and statistically significantly outperform their high-risk counterparts. However, the results also reveal that the strength of the anomaly is affected considerably by the length of the risk estimation period, which raises concerns over the robustness of the anomaly. The results in different size sections of the market further suggest that explanations related to market microstructure are persuasive in explaining the anomaly.

This thesis finds that low-risk stocks do not outperform the market index on a risk adjusted basis making long-only low-risk strategies ineffective. Transaction cost related considerations additionally raise critical observations against both long-only and long-short strategies based on the low-risk anomaly in the Finnish stock market.

KEYWORDS: factor investing, low-risk, low volatility, low beta, excess returns, anomaly

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TIIVISTELMÄ:

Positiivisesti korreloiva suhde riskin ja tuoton välillä on yksi perinteisen rahoitusteorian keskeisimmistä periaatteista. Tästä huolimatta on olemassa huomattava määrä tutkimusnäyttöä, joka dokumentoi riskin ja tuoton suhteen säännönmukaisesti poikkeavan tästä periaatteesta. Aikaisempi tutkimus on osoittanut, että matalan riskin osakkeet tuottavat paremmin kuin niiden teoriassa pitäisi, kun taas korkean riskin osakkeet tuottavat huonommin kuin niiden teoriassa pitäisi. Tämän lisäksi on säännönmukaisesti osoitettu, että matalan riskin osakkeet tuottavat keskimäärin paremmin kuin korkean riskin osakkeet ja, että riskikorjatusti matalan riskin osakkeet tuottavat paremmin kuin markkinaindeksi, josta osakkeet on valittu. Ilmiötä kutsutaan matalan riskin anomaliaksi.

Tämä tutkielma käsittelee riskin ja tuoton suhdetta Suomen osakemarkkinoilla. Lisäksi tutkielma arvioi matalan riskin sijoitusstrategian toimivuutta kyseisillä markkinoilla. Spesifiin markkinaan keskittymisen lisäksi, tutkielma erottautuu muista aiheeseen liittyvistä tutkimuksista hyödyntämällä useita tutkimusmenetelmiä ja ottamalla huomioon transaktiokustannusten vaikutuksen strategian arvioimisessa. Osakkeet jaetaan tutkimuksessa viidesosaportfolioihin perustuen näiden viimeisen viiden, kolmen tai yhden vuoden tuottojen keskihajontaan tai beta-kertoimeen. Portfolioiden tuottoja arvioidaan tutkimuksessa CAPM regression ja Sharpe-kertoimen avulla. Lisäksi anomaliaa arvioidaan tutkimuksessa kahdessa eri osakemarkkinan kokoluokassa.

Tutkielman tulokset osoittavat, että matalan riskin anomalia on läsnä Suomen osakemarkkinoilla. Tulokset osoittavat, että korkean riskin osakkeet tuottavat säännönmukaisesti negatiivista CAPM alfaa ja, että matalan riskin portfolioiden riskikorjattu tuotto on säännönmukaisesti tilastollisesti merkittävällä tavalla parempi kuin korkean riskin osakkeiden riskikorjattu tuotto. Tulokselliset erot riskin arviointitavasta riippuen kuitenkin kyseenalaistavat anomalian luotettavuuden. Tulokset osakemarkkinan eri kokoluokissa viittaavat siihen, että markkinoiden mikrostruktuuriin liittyvät selitykset anomalialle ovat vakuuttavia verrokkeihin nähden.

Tutkielman tulokset osoittavat, että matalan riskin osakkeet eivät tuota riskikorjatusti paremmin kuin markkinaindeksi, mikä tekee matalan riskin ”long-only” strategioista toimimattomia Suomen osakemarkkinoilla. Transaktiokustannusten vaikutuksiin liittyvät huomiot nostavat esiin lisäseikkoja, jotka ovat on kriittisiä anomaliaan perustuvia ”long-only” ja ”long-short” strategioita kohtaan.

AVAINSANAT: faktorisijoittaminen, matala riski, matala volatiliteetti, matala beta, ylituotot, anomalia

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1 Introduction

One of the key questions in the field of finance has been what explains the stock market returns. In the traditional and early finance theories, there was a strong principal conclusion that returns were essentially random due to implications of the efficient market hypothesis. However, later research has proved that this efficiency does not manifest in the real financial markets as expected. Therefore, it has also become evident that returns in the stock market are not purely random either.

This change in the evolution of finance theory has given more reason to research stock market returns and this research has led to the emergence of factor investing. Factor investing is an investing approach in which a specific driver of returns that should not be achieved if the markets were efficient, is identified and portfolios are constructed to take advantage of this driver or anomaly. Research over the years has identified many of these factors, some of the more well-known being size, value, momentum, quality, and minimum and low volatility.

This thesis focuses especially on the low volatility factor, also known as the low-risk factor, that has become popular after the financial crisis. The low risk factor is based on the low-risk anomaly, which is an observation that low-risk assets tend to outperform high risk assets in the long term. This observation violates the traditional risk and return relationship, in which there is expected to be a positive relationship between risk assumed and returns gained in exchange for assuming the risk.

The terms relating to the anomaly and the low-risk investing are complicated, so it is reasonable to explain some of them in this part of the thesis (see Baltussen et al., 2020). The difference between the terms low and minimum is the following. Low risk stands for portfolio construction method in which only stocks with low individual risk are chosen into portfolio. Minimum risk stands for portfolio construction method, in which individual stocks that have high risk are allowed, if they help to reduce the overall risk of the portfolio through low or negative correlations with other assets in the portfolio. As stated earlier, this thesis concerns the prior portfolio construction method. The term low risk investing can be seen as similar or equivalent to the terms low volatility investing, low variance investing and low beta investing. What unites these terms is that they all concern low values of different measures of risk in relation to return. Terms are also interlinked through mathematical theory. Volatility, also known as standard deviation, is the square root of variance. Similarly, beta is the covariance between returns of an asset and a market index divided by the variance of the market index. As can be inferred, the differences in use of these risk terms are mostly semantic. For the sake of avoiding further confusion discussing these terms in this thesis, low-risk term is used as an umbrella term and low volatility, low variance, and low beta terms are specifically used according to the measure of risk used in the reference material. Another relevant distinction that should be made is the difference between systematic risk, idiosyncratic risk, and total risk. Total risk of an investment can be seen as a sum of systematic and idiosyncratic risk. Systematic risk is part of the risk that affects all investment, for example the market. Idiosyncratic risk refers to risk that is investment specific, such as a company characteristic.

1.1 Purpose of the study

This thesis aims to comprehensively study the low-risk anomaly and viability of low-risk investing in context of the Finnish stock market on a period between July 1998 and June 2023. The methods of research for the low-risk anomaly tend to vary, and therefore this thesis evaluates the different approaches. The thesis analyses portfolios sorted by rolling estimates of volatility and beta. The risk is estimated with 5-year, 3-year and 1-year estimation periods. There are two major methods the subsequent results are evaluated with, standard CAPM regression and by examining the differences in portfolio Sharpe ratios with Jobson & Korkie (1981) z-test with Memmel (2003) correction. Robustness of the anomaly is also evaluated in accordance with the results. In addition, this thesis analyses whether transaction costs challenge the real financial market implementation of low-risk strategies. Methodology is further explained in detail in the chapter concerning data and methodology.

1.2 Hypotheses of the study

Past research has shown that the low-risk anomaly has been documented in developed and developing markets alike (see Blitz et al., 2013). Research has also shown that strategies constructed around the low-risk anomaly tend to outperform the market index from which the low-risk portfolios are extracted from on a risk adjusted basis and, in some cases, in absolute terms (see Blitz & Van Vliet, 2007). However, it is documented in the literature that the application of transaction costs in factor investing studies considerably hinder the potential returns, especially in the case of long-short strategies (see Novy-Marx & Velikov, 2022). Based on the prior literature the three hypotheses of the study are formulated as follows.

H1: The low-risk anomaly exists in the Finnish stock market.

H2: The low-risk portfolios outperform the market index on a risk adjusted basis.

H3: The outperformance declines when transaction costs are imposed on the portfolios.

The hypotheses closely relate to in which steps the analysis is conducted in the thesis. In the first part of the analysis, portfolios of different risk levels are constructed, and their returns are observed and analysed. If the low-risk portfolios outperform or match the performance of the higher risk portfolios or in other words, the security market line is flatter than expected, it is concluded that the low-risk anomaly exists in the Finnish stock market. In the second part of the analysis, the risk adjusted performance of risk portfolios, and the market index is examined. If the low-risk portfolios outperform the benchmark index in risk adjusted basis, it implies that the long only low-risk strategy is a viable investment strategy in the Finnish stock market. Finally, the effects of transaction costs are examined. Transaction cost related considerations are expected to negatively influence the returns observed in exposure optimized portfolios. What remains to be seen is whether the performance is affected to an extent that makes strategies constructed around the low-risk anomaly unviable to implement in practice, at least in the context of the Finnish stock market.

1.3 Intended contribution

Although the body of literature on the low-risk anomaly and low-risk investing is extensive, this thesis contributes to it in several ways. Existing research predominantly focuses on large and well-established financial markets such as United States and larger European markets (see Baker et al., 2011; Dimson et al, 2017). While studies focusing on lesser markets such as emerging markets or markets of individual countries are not completely unprecedented (see Blitz et al., 2013), it remains true that research focused on the Nordic markets and specifically on the Finnish stock market is insufficient. Additionally, in cases where the Finnish stock market is a part of broader sample, the research is often limited to a singular methodological approach or low-risk factor is studied as one of multiple factors (see Baker & Haugen, 2012; Silvasti et al., 2021; Grobys et al., 2024). Additionally, the Finnish results specifically are rarely gone into in detail in such studies. For these reasons, there exists a gap for research focusing only on the low-risk anomaly and investing in the Finnish stock market and for a study that compares different methodologies.

Factor investing studies often ignore the possible effects of transaction costs (See Baker et al., 2011; Frazzini & Pedersen, 2014), and therefore the implementation of such strategies in the real financial markets varies from what is portrayed in these studies. The omission of transaction costs is a major point of criticisms for such studies, especially when the majority of results showcased may disappear as transaction costs are implemented (see Novy-Marx & Velikov, 2022). Therefore, there exists a general gap for factor investing studies that consider the relevance of transaction costs.

This thesis aims to contribute to existing research by approaching the research gaps raised earlier. This thesis comprehensively focuses on the low-risk anomaly and investing in the context of the Finnish stock market and explores multiple methods for risk estimation and result evaluation. This thesis also adds the layer of transaction cost related considerations, which additionally enhances the significance of the thesis results in the context of real financial markets.

2 Theoretical background

The theoretical background of this thesis is organized into three subchapters. The first chapter discusses traditional finance theories and efficient market hypothesis. The second chapter discusses behavioural finance theories. Understanding the difference between traditional financial theories, especially the efficient markets hypothesis, and the behavioural finance theories is essential background for understanding the thesis topic. The third chapter focuses on how the risk-return relationship has been depicted in traditional finance theory.

2.1 Traditional finance and efficient markets

Earlier concepts of traditional finance theories could be traced to introduction of the concept of "economic man" by John Stuart Mill in 1844 (see Mill, 2011), whose aim is to maximize his utility in decisions he faces. To achieve the goal of maximized utility, the economic agent has to act rationally in the decision environment he faces. This also implies that to make rational decisions, the decision maker must account for all the available information regarding the decision. The concept of rational decision-making is at the center of the expected utility hypothesis proposed by Von Neumann and Morgenstern in 1944 (see Von Neumann & Morgenstern, 2007), in which economic agents assign weights to their decision options, according to the expected utility of them.

Evolution from these basic concepts eventually led to the birth of efficient market hypothesis, which was comprehensively studied and presented by Fama (1970). According to Fama, markets are efficient when market prices fully reflect all the available fundamental information. This also implies that the prices efficiently represent a good estimate of the intrinsic value of the underlying securities.

There are a few implications to the efficient market hypothesis. For prices to reflect all the fundamental information accurately or in other words to be in equilibrium, most of the market participants must act rationally in the sense of correctly pricing investments according to the level of their expected risk and return. Prices can only change substantially when new fundamental information is incorporated into the prices. As investors collectively act rationally, irrational phenomena in the markets should not exist. As an example, this means that mispricing of stocks should not occur. As the prices represent good estimates of the intrinsic value of the underlying securities, there is no way to predict in which direction prices will go in the future. This implies that stock prices follow a random walk model in which stock prices cannot be efficiently forecasted and they move randomly (see Fama, 1995).

Fama (1970) however concedes that the hypothesis of markets that fully reflect all the available information is extreme. To examine different levels of market efficiency, Fama studies the efficient market hypothesis in three different levels. In the weak form test, the prices reflect all the information of historical prices. In the semi-strong test, the prices reflect all the information that is publicly available. In the strong-form test, the prices reflect information that only a subsection of investors have access to. Fama concludes that on weak and semi-strong basis, multiple tests have supported the efficient market hypothesis. On a strong-form basis, Fama concludes that this level of efficiency is best viewed as somewhat of a benchmark.

2.2 Behavioural Finance

According to Shiller (2003), the efficient market hypothesis became a subject of criticism in the 1980s due to many observations of anomalies on real financial markets that contradicted the more fundamental ideas of the efficient markets. Due to these contradictions, academics began researching other theories to explain the behaviour of the financial markets. Behavioural finance was a theoretical orientation that particularly rose from this schism.

One of the important theories of what would become the foundation of behavioural finance was the prospect theory, presented by Kahneman and Tversky (1979). Prospect theory argues that when making decisions, individuals are often not as rational as proposed in traditional finance theories. In prospect theory, decision weights are assigned according to perceived prospects they present for the individual making the decisions. These prospects often vary from the theoretical reward or utility that they mathematically offer. According to Kahneman and Tversky, this is because individuals utilize mental shortcuts and have different tendencies when making decisions. The authors also present that the point of reference is an important contributor to decision making process presented in the model. The feelings of an individual who has a wealth of 10 million vary depending on if the individual previously had a wealth of 7 or 12 million. In this context, past wealth is a reference point that influences how the decision maker feels about the decision to be made in the present moment. Because of these factors, Kahneman and Tversky present among other things that individuals tend to weigh wins and losses differently. When a decision-making process described in the prospect theory is extended to the theory regarding capital markets and pricing, it can be inferred that capital markets could also function inefficiently, when decisions like these are made en masse.

Later, two more “building blocks” were added to behavioural finance theory (see Ritter, 2003). These were limits to arbitrage and behavioural biases of individuals. Shleifer and Vishny (1997) use the following characterization for arbitrage. Arbitrage is a simultaneous purchase and sale of the same or sufficiently same security in two different markets at different prices. They also conclude that theoretical arbitrage requires no capital and is risk neutral. These factors mean that theoretical arbitrage is essentially a way to profit from mispricing in the market with no risk. This mechanic is one of the key drivers of efficient markets described by Fama (1970). Shleifer and Vishny however raise several real limitations that hinder the ability of arbitrageurs to arbitrage, such as lack of arbitrageurs who have the means and know how to arbitrage, incentives not to arbitrage and barriers of arbitrage. The implication from the perspective of behavioural finance is the following. The failure of arbitrage in driving market efficiency gives room for temporal inefficiency, that is driven by factors that the efficient market hypothesis cannot explain. From the perspective of behavioural finance, these factors and irrational outcomes result from cognitive biases of investors. Hirshleifer (2015) divides psychological biases into three categories, which can be used to explain most of the phenomena studied from the perspective of behavioural finance. The categories are overconfidence and self-esteem maintenance, limited attention and cognitive processing and lastly, feelings. Overconfidence is characterized by Hirshleifer as a tendency to be self-deceptive about the level of one's achievement and skills. Self-esteem maintenance refers to one's tendency to give strong weight to events that support one's opinions and actions from before, while disregarding information that does not. The author raises that these biases explain phenomenon such as overconfidence driving aggressive trading, price over- and underreactions, over optimism and overpricing in the markets. According to the author, limited attention and cognitive processing refer especially to common heuristics that people tend to use in decision making to make decisions more efficiently. These heuristics are useful in everyday life but tend to lead into misjudgements in decision environments where the variables are more complex. The author raises that these heuristics explain failure to process relevant signals of investing environment, contribute to category thinking, lead to pattern seeking and overextrapolation of future. The author characterizes feelings as actions taken quickly because of intuition in contrast to decision making after more careful assessment. The author raises that change in feelings cause investors to seek familiarity and liking in the stocks they own and cause shifts in sentiment, optimism, and risk tolerance.

Additionally, another important concept of behavioural finance is noise. The concept of noise was first introduced by Black (1986). Black does not concretely explain what noise is, but it can be seen as actions, for example investing decisions, taken due to fundamentally unimportant events. Black states that noise enables trading in financial markets but also slightly limits their efficiency. De Long et al. (1990) derive meaning for the term "noise trader" from Black (1986). The authors raise that a noise trader is an investor who has no access to inside information and acts on noise information as if it was information that gives them an edge in the market. In more concrete terms, a noise trader is an investor who acts on information that does not fundamentally affect the value of their investments. The authors argue that noise trader risk is responsible for many of the anomalies in the market. Relevantly for this thesis, excess volatility in the markets mentioned by (Shiller, 2003) is also potentially explained by noise trader risk. The authors also conclude that against the assumption of efficient market hypothesis, noise traders can profit from their actions in the market. This implies that noise is a constant factor in the markets, as noise traders do not disappear because of losses served to them by rational market participants.

2.3 Risk-return relationship in traditional finance

The invention of the risk-return relationship concept cannot be attributed to any singular academic, although the works of Markowitz (1952), Sharpe (1964) and Lintner (1965) are often mentioned as more influential works on the subject. Based on their work, risk in financial markets is often divided into two parts that are idiosyncratic risk and systematic risk. Idiosyncratic risk is also known as specific and unsystematic risk, whereas systematic risk is also known as market risk and undiversifiable risk. Generally, idiosyncratic risk refers to investment specific risk that is only relevant for the investment in question, whereas market risk refers to risk that affects an entirety of different investments. Furthermore, idiosyncratic risk is often defined as excess variance or standard deviation of returns, notated by ε. Whereas systematic risk, is defined as beta (β), which is the covariance between returns of an investment and a market index, divided by the variance of the market index. Additionally, the total risk of the investments can also be portrayed as the variance or standard deviation of returns.

$$Systematic\ and\ idiosyncratic\ risk = \beta_i(r_m) + \varepsilon_t, \qquad (1)$$

$$Variance = \sigma^2 = \frac{\sum(x_r - \mu)^2}{n}, \qquad (2)$$

$$Standard\ deviation\ or\ volatility = \sigma = \sqrt{\frac{\sum(x_r - \mu)^2}{n}}, \qquad (3)$$

$$Beta = \beta = \frac{Cov(r_p, r_m)}{\sigma^2(r_m)}, \qquad (4)$$

The pioneering of the modern portfolio theory can be attributed to Markowitz (1952). Markowitz argues that most investments are either high risk and high return or low risk and low return and investors could achieve best return for their risk tolerance by optimizing the ratio of these two types of investments in a single portfolio. This type of optimization is referred as the mean-variance optimization, in which the mean refers to a mean expected return, and the variance refers to an expected level of risk. When this type of optimization is conducted on portfolios of different risk levels, these portfolios create an efficient frontier. Findings of Markowitz also led to the popularization of the concept of diversification, a process in which the overall risk of the portfolio is controlled by owning different types of assets each exposed to different types of risks.

Markowitz's portfolio theory is the foundation for the capital asset pricing model (CAPM) later introduced by Sharpe (1964) and contributed by Lintner (1965). Key development from the portfolio theory is that the authors present that the idiosyncratic risk can be eliminated by diversification and therefore, systematic risk is the only relevant measure of risk after sufficient diversification. Sharpe argues that there exists a linear relationship between systematic risk and expected return of a portfolio. This linearity is derived from a beta of portfolio's assets and the risk-free rate. The risk-free rate is the minimum return investor would accept for holding a risky asset and beta is a measure of how sensitive the asset is to the systematic risk. By deriving a line from the risk-free rate to the tangent of an efficient frontier, this relationship is modelled by the capital market line. The line is a tangent because it is assumed that no rational mean-variance optimizing investor would create portfolios other than those that reside on an efficient frontier. The efficient frontier can also be rearranged to illustrate this trade-off for individual assets. This relationship is modelled by the security market line. This means that individual investment's risk can be estimated by using their respective betas. Both lines are illustrated at figure 1.

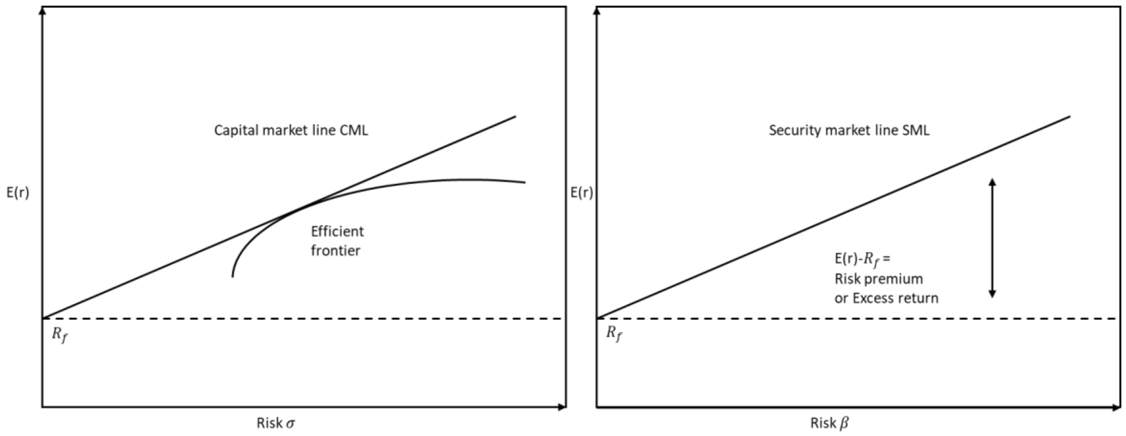

Figure 1. Capital market line and security market line.

There are several problems related to the use of CAPM in investing practice that are discussed in later chapters. These problems are part of the reason why there is still a lot of research being conducted on asset pricing models. Models such as arbitrage pricing theory (APT) by Ross (1976) and various CAPM extensions by Fama and French, aim to eliminate these problems. Ross argues that asset returns are determined by multiple macroeconomic factors such as interest rates and inflation opposed to singular factor as in CAPM. Ross also argues that arbitrage is the driving force that holds the risk-return trade-off intact. Fama and French have conducted research into CAPM extensions, where factors are added to CAPM for it to represent reality better. In first one of these studies (Fama & French, 1992), authors introduce a three-factor model which includes market, size, and value factors. More recently, Fama and French (2015; 2018) have introduced a five-factor and six-factor model which add profitability, investment, and momentum factors into the original three factor model. Fama and French conclude that their model explains variation in stock returns better than the CAPM, although the absolute explanatory ability of both models remains a subject of debate.

Interestingly, even though research has highlighted the problems of these models and proposed supposed improved models to describe risk in the financial markets, the CAPM and mean-variance optimization remain in popular use and are widely taught in business schools. Pinto et al. (2019) conducted a survey for CFA Institute members with equity analysis job responsibilities. In the survey, the authors observe that 68,2% of respondents use the CAPM to estimate required return, while the APT and Fama-French related models are used only by 4,8% and 4% of the respondents respectively. Other methods such as bond yield plus risk premium and judgementally determined hurdle rates are used by 42,7% and 47,5% of respondents respectively. What is observed is that the CAPM and mean-variance optimization continues to be a significant factor on how risk is approximated in the financial markets.

3 The low-risk effect

The low-risk anomaly and studies concerning low-risk investing are the more fundamental parts of the theoretical background. Therefore, they are discussed more comprehensively in this chapter, which is divided into seven subchapters. In the first chapter, the origins of low-risk anomaly are discussed. In the second chapter, how the risk-return relationship continues to break down in different ways is discussed. In the third chapter, the major points of criticism towards low-risk anomaly are presented. In the fourth chapter, potential reasons for the low-risk anomaly are discussed. In the fifth chapter, the studies concerning investing strategies constructed around the anomaly and their potential limitations are discussed. In the sixth chapter, the effects of transaction costs and cost mitigation techniques in investing studies are discussed. Finally, this section includes a part discussing the size anomaly and its relation to Finnish stock market. This is due to the Finnish stock market having size related differences from the typical markets tested in previous literature. Comprehensive articles (Baltussen et al., 2020; Traut 2023) and a presentation focusing on what sustains the low-risk anomaly by Ilmanen (2016) provided many of the sources cited in this chapter and are good further readings for the topic.

3.1 The origins of low-risk anomaly

The CAPM's ability to accurately model risk was relatively quickly dismantled by research in the 1970s. Study by Black et al. (1972) shows that the relationship between beta and return is flatter than the relationship implied by the CAPM. In other words, they find that low beta stocks exhibit significant positive excess returns whereas the opposite is true for high beta stocks. In the study by Haugen and Heins (1975), it is noted that the relationship between variance and return is not merely flat, it is inverted. Blitz and van Vliet (2007) further observe that the relationship between volatility and return is also flat. They find that the difference in Sharpe ratios between the high and low volatility portfolios is statistically significant and additionally, that the Sharpe ratio of the riskiest portfolio is significantly lower than the Sharpe of the market index. In the case of idiosyncratic volatility, Ang et al. (2006) observe that the relationship is flatter than implied by CAPM and that stocks with high idiosyncratic volatility tend to have exceptionally poor returns. Therefore, it has been shown with all measures of risk in the mean-variance space that low-risk stocks tend to outperform their high-risk counterparts. Baltussen et al. (2020) discuss whether the anomaly is stronger or weaker depending on whether the risk is measured by volatility or beta. The authors raise that as the measures are interlinked, the choice essentially comes down to the added value of correlations. The authors conclude that the anomaly tends to be slightly stronger with volatility as the measure.

3.2 Some old wine in new bottles

Past research has shown that mean-variance related risk-return relationship systematically breaks down in the real financial markets. However, the original finding is around 50 years old and therefore it is reasonable to discuss what new information has surfaced on the anomaly and have newer and better measures for risk managed to depict trade-off better.

Blitz and Van Vliet (2007) document that the anomaly is present in multiple developed countries and Blitz et al. (2013) further document the anomaly existing in emerging markets. The authors find that the anomaly has seemingly grown stronger as the time has passed and propose that this could be due to increased portfolio management operations in emerging markets. The anomaly has also been documented in other asset classes. Pilotte and Sterbenz (2006) document the anomaly in government bonds. Van Zundert (2017) document it in corporate bond markets. Jordan and Riley (2015) document it in mutual funds. These findings imply that investors seem to be rewarded from taking less risk in multiple different economic zones and across different asset classes.

Ever since the introduction of CAPM, a lot of research has been conducted on uncovering new risk factors in addition to market risk, most well-known studies being conducted by Fama and French. Fama and French (1992) note that the relationship between returns and beta remain flat, even when the effects of the size factor are accounted for. In the study by Ang et al. (2006), the authors further study the relationship between returns and the Fama-French 3-factor model by using zero-cost portfolio approach. The authors also control the subsequent regression analysis for multiple factors and examine it through multiple time periods. The authors find that both the CAPM and the Fama-French model fail to accurately portray the risk-return relationship and find that equities with a high idiosyncratic risk tend to have exceptionally low average returns. Ang et al. (2009) further makes the same observation in G7 countries and argue that in United States the effect cannot be explained by trading frictions, information dissemination or by higher moments. Grobys and Kolari (2022) find in their study that 3-factor model fails to outperform CAPM in explanatory power. Grobys and Kolari raise that 3-factor model remains in popular use especially in academic research concerning asset-pricing, which makes the aforementioned results significant in the context of present moment. Fama and French themselves raise many imperfections in their newer models and gained improvements are more often in comparison to previous model rather than major improvements on the absolute level of explanatory ability (see Fama & French, 2015; 2018). Moreover, as concluded by Grobys and Kolari (2022), explanatory ability and relevance of factor models tends to vary with the sample and can be country specific. Overall, what can be concluded is that the nature of risk-return relationship continues to be a subject of debate and research. Old problems prevail, at least when it comes to the risk-models in the mean-variance space, making new models some old wine in new bottles as originally coined by Haugen and Heins (1975).

3.3 Criticism of the low-risk anomaly

The topic of low-risk anomaly is naturally controversial and some of the studies have later encountered criticism for varied reasons. This chapter introduces a few of the more prominent points of critique towards the anomaly.

One of the major studies that has been criticised is (Ang et al., 2006). Fu (2009) raises problems with the methodology used in the study. He notes that due to short estimation period of idiosyncratic risk, the study is prone to look-ahead bias. He argues that a longer-term perspective on idiosyncratic volatility, which reflects its variability over time, provides a more accurate depiction of how risk affects future returns and that the original study methodology crucially fails to account for this. He tests his argument by incorporating a GARCH model to account for time-varying volatility. Fu finds that the relationship between idiosyncratic volatility and expected returns is positive, when the time variability of risk is taken into account. He additionally notes that the results in original study are driven by a subset of small firms with high idiosyncratic volatilities and that these firms have high returns in the month of high idiosyncratic volatility. He then argues that the results in the original study are driven by these high returns reversing in the subsequent month. Aabo et al. (2017) further extrapolate this criticism to studies concerning low-risk anomaly. They raise that different measures for risk often lead to differentiating results and that the estimation period for risk has significant effects on the results. They note that shorter risk estimation periods tend to overstate the strength of the anomaly. They assert that the variability in results based on different risk and estimation measures casts doubt on the robustness of the anomaly.

Another significant criticism that relates to low-risk anomaly study methodologies is how they account for liquidity. As Fu (2009) raised, the anomaly is partly driven by small firms and small firms naturally tend to have less liquidity than larger firms. Han et al. (2015) especially approaches the topic from this angle. They document that the negative relationship between idiosyncratic volatility and stock returns is due to liquidity biases. They show that stocks with high idiosyncratic volatility tend to have high liquidity costs or in other words wide bid-ask spreads. They point out that in such cases the closing price, which is commonly used in investing studies, might not present accurate price point as it might differ from the price that would have been obtainable for investor. This in turn implies that price data points used for such stock has significant effect on the results. The authors test this in multiple markets by using quote-midpoint prices, which measure prices based on the midpoint of the bid-ask spread, instead of closing prices. Their analysis yields that the negative relation between idiosyncratic volatility and future returns weakens or even disappears in some cases when liquidity adjustments are made.

Overall, while the low-risk anomaly is consistently documented, it remains a subject to criticism. The methodologies used to study the anomaly vary and thus there exists a dispersion in results obtained, depending on the choices made with methodology. It is also shown that the anomaly tends to be driven by a subsection of stocks, namely small and illiquid ones. Lastly, the reasons for the anomaly remain a subject of contention. Some argue there is a fundamental flaw in how risk and return are viewed in finance, while others argue that the anomaly is driven by factors outside the risk-return trade-off.

3.4 Potential reasons for the low-risk effect

There is no consensus on what exactly causes the low-risk anomaly, but several theories have been proposed. This section introduces the most prominent of these theories. Theories generally relate to limitations of risk portraying models, to behavioural explanations and to market structure and agency problems.

Especially in the early studies on the anomaly, the proposed reasons related to flaws in risk portraying models. In the case of the low beta anomaly, the study by Blitz et al. (2014) highlights unrealistic assumptions related to CAPM. The authors note that the CAPM and the efficient market hypothesis assume that there are no factors such as constraints on credit availability, short selling, and regulatory constraints. In financial markets, all these factors are present. Investors have access to limited amount of credit due to lender requirements. The cost of credit is also much higher than risk-free rate, which should be the rate available for all investors in perfectly efficient markets. The cost and lack of credit availability means that investors cannot move along the security market line with leverage as proposed in efficient markets theory. This is the reason for low beta anomaly as proposed by Black et al. (1972). They suggest that due to leverage constraints, investors tend to overweight risky investments instead of levering up less risky investments, which leads into the security market line appearing flat in the study. Frazzini and Pedersen (2014) further document in their data that investors facing leverage constraints leverage constraints tend to buy into riskier stocks, meaning their data supports leverage constraints as the reason for the anomaly.

Short-sell constraints are another proposed theoretical reason for failure of CAPM by Blitz et al. (2014). As discussed in chapter on theoretical background, short sellers often have barriers and incentives not to arbitrage. Baker et al. (2011) observe that highest volatility stocks tend to be small and illiquid, implying that often it is not possible to take short positions in them. This in turn implies that arbitraging the anomaly in a way that involves taking short positions in the riskier stocks is not realistic. Given that many of the high-risk stocks tend to be stocks with low market capitalization, one very concrete barrier of arbitraging them is that there is a limited supply of stocks to sell short. In some situations, short selling might not be possible at all due law or temporary ban on it. Beta, volatility and variance all come from the same mean-variance theory base, which means that same limitations generally apply in all the measures. Each of these along with many other factors can be seen as limit to arbitrage that is supposed to hold the risk-return relationship together as proposed in efficient market hypothesis. On more fundamental level, the question whether traditional mean-variance approach is the right approach to measure risk at all is also raised in the literature. Conventional variance related study methodologies tend to be sample specific due to the nature of financial markets as raised in (Grobys, 2021). These methods also generally assume that stock market returns follow a normal distribution. Instances of extreme market events or returns however, are more prominent than could be expected if the returns were normally distributed as raised by Grobys. As mean-variance related risk-models ignore this, it is possible that they leave at least some components of the risk in financial markets completely unaccounted for. Critics of risk model related explanations argue that the risk models or risk-return relationship, does not necessarily have to be flawed for low-risk anomaly to exist. They argue that low-risk anomaly and especially the high-risk underperformance could be related to reasons varied from investor behaviour to market level frictions. If the low-risk anomaly can be explained by such factor, there is an argument that rather than the traditional risk-return relationship being flawed, the anomaly is driven by non-related factors that are necessarily not in contradiction with risk-return trade-off.

Behavioural explanations are a popular category when the persistence of low-risk effect is discussed. Blitz et al. (2014) raise some of the better known biases as a potential reason for the anomaly. The biases mentioned are limited attention bias, representativeness bias, overconfidence bias and mental accounting bias. The general idea behind presenting these biases as potential contributors for the low-risk anomaly is that they encourage investors into buying high-risk stocks and into neglecting low-risk stocks. In the case of limited attention bias, the authors discuss that investors are susceptible to buy stocks that have been experiencing attention grabbing events or returns, which tend to be riskier. Opposite is true for less risky stocks. The authors raise that representativeness bias affects investors in somewhat similar way, by increasing the investors willingness to invest in stocks that share similar characteristics with stocks that have experienced significant events or returns in the past. Overconfidence is often mentioned as potential reason for many anomalies in addition to low-risk effect and is a common trait for majority in any population. The authors discuss that overconfidence tends to cause excessive demand for high-risk stocks as many investors tend to believe that they have the skills necessary to participate successfully in more riskier parts of the market. Finally, the authors raise mental accounting. Mental accounting is closely related to preference for “lottery like stocks”, which is often mentioned among behavioural reasons for the anomaly. Lottery like stocks tend to be characterised by positive skewness and high idiosyncratic volatility. The authors discuss that investors might purposefully invest a certain portion of their portfolio into stocks that have lottery-like qualities, which creates an excess demand for such stocks. Hou and Loh (2016) tests this hypothesis, they find that although preference for lottery like stocks does not completely explain the anomaly, it is considerably more promising when compared with many other potential explanations. Bali et al. (2011) further study the preference for lottery like stocks, showing that stocks with lottery-like characteristics, are consistently overvalued. This demand for high-risk, high-reward investments artificially inflates their prices and leads to underperformance in the future, reinforcing the low-risk anomaly.

Agency problems and market structures have also been raised as potential cause for the persistence of the low-risk anomaly. Perhaps most prominent of these explanations is the observation that professionally managed capital is often on the other side of the low-risk trade (see Baltussen et al., 2020). Christoffersen and Simutin (2017) find that in an effort to maintain tracking error against fund benchmark, mutual fund managers tend to increase high beta stocks in their portfolios. Blitz (2018) finds that hedge funds are not arbitraging against the low-risk anomaly, meaning that they too tend to be on the other side of the low-risk trade. Stambaugh et al. (2015) raise that investors in general are more willing to buy underpriced stocks than to short overpriced stocks due to risks and costs associated with short-selling. They raise that the arbitrage asymmetry that results from this is one reasons for the anomaly sustaining in the financial markets. Baker and Haugen (2012) argue that the institutional tendency to overweight high-risk stocks is due to option-like compensation structure. In such structure, portfolio managers are incentivised to maximise the expected value of their payoff, which can be achieved by taking excessive risk.

Hou and Loh (2016) test a multitude of potential reasons for the anomaly, among them market frictions. They find that although a lot of the anomaly remains unexplained, market frictions such as one-month reversal effect and illiquidity explain a sizable portion.

The authors find that one month reversal effect and specifically liquidity as measured by bid-ask spread capture sizeable portion of the anomaly. As mentioned before, Fu (2009) also notes that especially with short risk estimation period, the underperformance in high-risk stocks is mostly explained by one month reversal effect after a month of high-risk measurement and high returns. Han et al. (2015) similarly raise that illiquidity biases are a major driver of the anomaly. Avramov et al. (2013) find that especially the under-performance of high-risk stocks is driven by financial distress as distressed firms tend to have high idiosyncratic volatility. They argue that as credit conditions continue to deteriorate the returns of the stock tends to eventually culminate in extreme negative returns, which creates the illusion of the anomaly. Aabo et al. (2017) raise noise traders as a reason for the anomaly. They find that both the level of market volatility and the level of idiosyncratic volatility caused by noise trading are generally associated with mispricing. The excess volatility is typically related to high-risk stocks, meaning they are prone to mispricing, which in turn could explain the low returns in high-risk stocks.

Overall, there are multiple explanations that can be connected with and proved to be contributing to the anomaly. Whether it is the unrealistic or even fundamentally wrong assumptions of the risk models, limited rationality of investors, agency problems or market microstructure, it is evident that investors seem to systematically make faulty approximations on the nature of risk in the financial markets when mean-variance related methods are used. When the findings of questionnaire in (Pinto et al., 2019) are raised in this context, it becomes compelling to think that analysts using these models will systematically fail to accommodate relevant factors in their assessment of risk.

3.5 The low-risk trade

In this chapter an extensive literature review is concluded on studies concerning the low-risk investment strategy. Previous chapters have established existence of the low-risk anomaly and how it seems to have persisted despite continuing research into nature of risk. This chapter will answer questions such as how portfolios constructed to take advantage of the low-risk anomaly have performed? What kind of methodologies have been used to study the effect? What these studies potentially do not take into account and what other details should be noted when concluding this type of study?

In the study by Blitz and Van Vliet (2007), the authors construct low volatility portfolios with a simple construction method, in which stocks are ranked based on their past levels of volatility. The data of the study was collected from FTSE World Developed index, which consisted of approximately 2 000 large-cap stocks on average, from a period between December 1985 and January 2006. The authors rank stocks in the index at the end of each month based on their past three-year volatility of weekly returns. These stocks are then divided into equally weighted decile portfolios. Portfolios are rebalanced monthly, and transaction costs are ignored. Returns over the country specific risk-free rates are calculated for each portfolio monthly. Authors find that low-risk decile portfolios generally outperform the portfolios in the higher end of the spectrum in raw returns, although there are also exceptions. In risk-adjusted basis, low-risk portfolios outperform the high-risk portfolios and the market in statistically significant way.

Comprehensive study by Baker et al. (2011) show that portfolios constructed from bottom beta and volatility quintiles significantly outperform top quintile portfolios of same measurement of risk. The stocks chosen in the study were from two different groups, the first group included all the stocks listed in U.S. and the second group included largest 1 000 companies listed in the U.S. Portfolios in the study were constructed by dividing stocks into five equal quintiles according to their trailing volatility and beta. Volatility and beta were estimated by using up to 60 months of monthly return data. These stocks were then sorted into quintile portfolios, which were market-cap weighted. Portfolios were rebalanced monthly, and the authors assumed no transaction costs. On a period between 1968-2008, authors show that the 1$ invested in low volatility and beta portfolios grew to 59,55$ and 60,46$ respectively. At the same period the respective returns for high volatility and beta were 0,58$ and 3,77$. When inflation is considered, results were

10,12$, 10,28$, circa 0,10$ and 0,64$. The results are similar for 1 000 largest companies. In the same period 1$ invested in similar portfolios from this group grew to 53,81$ for low-volatility portfolio and 78,66$ for low-beta portfolio. The authors note that the similar results with size restricted and non-restricted portfolios imply that the low-risk trade is not size-dependent.

Baker and Haugen (2012) investigate the low-risk effect in 33 countries, including Finland and other Nordic countries. They use a simple and easily replicable method. The authors calculate 2-year volatility of monthly returns for country specific stocks. These stocks are then divided into deciles. The performance of each decile is then followed monthly after which the portfolio is rebalanced, and the process repeats itself. Transaction costs are ignored. The authors again find that low-volatility stocks consistently outperform their high-volatility counterparts, including in Finland.

Frazzini and Pedersen (2014) propose a long-short strategy to take advantage of risk anomaly, called betting against beta. The authors present a model which involves going long leveraged low-beta assets and going short on high beta assets. The authors observe significant positive risk adjusted returns with the strategy. In the study, betting against beta factor is constructed by ranking all securities in ascending order according to their estimated beta. The ranked securities are then assigned to one of two portfolios, which are low-beta and high-beta. The asset class median beta acts as the distributing factor. In both portfolios, securities are weighted according to their ranked betas. Portfolios are rebalanced monthly. The authors find that betting against beta factors ability to deliver excess returns exist in almost every stock exchange that was examined in the study. The authors also find that same is true for different types of debt assets, equity indices and commodities. They note that decreasing funding liquidity seems to incur losses for the factor and that the increased funding liquidity risk seems to compress all betas towards one.

The findings of Frazzini and Pedersen have however recently been criticised by Novy-Marx and Velikov (2022). Novy-Marx and Velikov raise that when betting against beta portfolio building procedure is critically examined, it is very similar to simple beta arbitrage strategy. The authors raise that Frazzini and Pedersen deviate from standard portfolio construction procedure in 3 ways that mystify the underlying idea behind betting against beta and contribute to the results showcased in the original paper. The first non-standard procedure is rank-weighted portfolio construction method, in which stocks are ranked and weighted according to their cross-sectional deviation of the stock's estimated beta rank from the median rank. The second non-standard procedure is the hedging by leveraging the low-beta part of the portfolio. The final non-standard procedure the authors note is the novel beta estimation technique used in betting against beta, in which instead of estimating beta as slope of CAPM regression, the beta is estimated by combining market correlations estimated using five years of overlapping three-day returns with volatilities estimated using one year of daily data. In essence, the time frames between the measures differ drastically. The authors critically raise that as a consequence of these non-standard procedures, betting against beta returns are driven by dramatically overweighting the markets smallest and most illiquid stocks, in which transaction costs are high. When transaction costs are taken into account, authors conclude that the real-life implementation of the strategy is far less profitable than in theory.

Dimson et al. (2017) conduct a study on multiple different investment factors, including low volatility. The authors study the low volatility effect on U.K. and U.S. markets on periods between 1984-2016 and between 1963-2016 respectively. Datasets for each markets include all the stocks listed in London Stock Exchange and in U.S. respectively. The residual risk level for each stock is calculated from approximately 3 months of daily return data for both countries. The U.K. stocks are divided into three portfolios on 30/40/30 division whereas U.S. stocks are divided into quintiles. Portfolios for each are balanced monthly and returns are value weighted. Transaction costs are ignored. The authors find that the performance of portfolios outside the highest risk classification are on a similar level, whereas the returns of highest risk classification are poor. The authors also discuss practical limitations of a long-short strategy based on the anomaly, as the stocks in highest risk classes tend to be very small and therefore, going short on them would be unrealistic and expensive due to inflated transaction costs.

There are also low-risk studies that are more closely related to Finnish stock market. Silvasti et al. (2021) study smart beta strategies in Nordic stock markets between 1991 – 2019. Although not especially focused on low-risk nor Finland, low-risk is one of the factors examined and Finland is part of the wider sample which additionally includes Sweden, Denmark, and Norway. Using equally weighted quintiles and monthly rebalancing, the authors find that low beta quintile outperforms the high beta quintile both in small and large segments of the sample. They find that large low beta stocks generally do better than small low beta stocks, while small high beta stocks do worse than large high beta stocks. Grobys et al. (2024) combine low-volatility with momentum in Nordic stock markets. Their study period is between 1999 – 2022 and equally weighted tercile portfolios are used. They use CAPM, Fama-French 3 and 5 factor models in their regression. They find that low-volatility produces alpha when regressed against CAPM and Fama-French 3 factor model, but the alpha subsumes when 5 factor model is used. Contradictory evidence also exists. Bradrania et al. (2023) finds that low-risk anomaly does not exists in Finland. They examine low beta stocks across 22 different markets between 1990 – 2021, using equally weighted portfolios and risk estimation between 1 to 5 years. They find that low beta stocks do not outperform high beta stocks in the Finnish stock market.

Van Vliet (2018) takes a slightly different approach to low volatility portfolio building. Van Vliet notes that low volatility portfolios require regular rebalancing, as risk of stocks varies over time, and this naturally has real implications for the return of the strategy through transaction costs. Therefore, Van Vliet raises the question of how much turnover is required to decrease the risk of low volatility portfolio. He argues that low volatility is typically associated with lower turnover overall and the stocks traded tend to have larger market capitalization than high volatility stocks. Main conclusion of the study is that the effect of turnover on risk reduction diminishes for each unit of turnover. Van Vliet states that first 10% turnover decreases risk by 22,5% whereas the second 10% decreases it by only 3,5%. Van Vliet concludes that 30% turnover is enough to create efficient low-volatility portfolio, when 11 basis point transaction cost is assumed.

Baltussen et al. (2020) comprehensively discusses this topic. The authors raise that transaction costs are obvious limitation for low-risk strategies and therefore strategies tend to require low turnover and are practically long-only for majority of investors. Findings by Blitz et al. (2020) however raise that this is not necessarily a problem for factor investing, as they raise that returns are largely driven by the long legs of different factor trades. Baltussen et al. (2020) also discuss what differentiates the low-risk and minimum risk strategies and which is better in real financial markets. The authors elaborate that low-risk portfolio construction stands for construction method in which only stocks with low individual risk are chosen. In minimum risk portfolio construction, individual stocks that have high risk are allowed, if they help to reduce the overall risk of the portfolio through low correlations with other stocks in the portfolio. The authors conduct literature review into the issue and conclude that there are minimal differences in outcomes of the two styles, meaning that the differences lie mostly in semantics. Another relevant issue that authors raise is the currency risk. Currencies can have effects on the overall risk of portfolio. The authors conclude that risk should be minimized in some way, either by home currency bias or currency risk hedging. In addition to these considerations, authors raise that low-risk investing is most efficient if portfolio is actively managed and that low-risk tactic can efficiently be combined with other factor investing types.

From literature review, following conclusions can be inferred. There are multiple low-risk portfolio construction and study methods, most of which generate similar results. The research on the effects of transaction costs is limited, although what can be inferred is that they significantly hinder the performance of long-short strategies and therefore, low-risk strategies tend to be long only in practice. Although research on transaction costs with long only strategies is limited, findings that low-risk generally needs little trading and effect remains robust with large-cap stocks give reason to believe, that excess returns generated by long only low-risk strategies could remain significant even after transaction costs are accounted for. Additionally, the effects of currency fluctuations should be taken into account when constructing low-risk portfolios.

3.6 Transaction costs and cost mitigation techniques

As the literature shows, many of the studies concerning low-risk anomaly, and investing studies as a group overall, ignore the effects of transaction costs. Moreover, the critics of such studies raise that the excess returns of these types of strategies may diminish completely when transaction costs are accounted for. Therefore, the realism of implementing an investing strategy from theory to practice depends on whether the factor exposure can be maintained with a reasonable level of transaction costs.

Moreover, in order to study the question further a country specific level of transaction costs has to be estimated. There are relatively few studies or information sources that disclose the level of transaction costs in the Finnish stock market. The working paper by Frazzini et al. (2018) discusses the topic in context of different economic zones, company sizes, trade types and trade sizes. In the paper, they find that the value weighted average cost of trade in the Finnish stock market is around 15 basis points. The data they used is from 1998 to 2016, which corresponds well with the thesis study period. However, more Finland oriented researchers Rinne and Vähämaa (2011) use considerably higher estimation in their study that uses data from 1988 to 2008. The authors calculate the cost by calculating the yearly turnover for portfolio, in terms of number of stocks that were replaced during the year and multiplying this with a conservative 1% round-term transaction cost. In their study, the portfolio size was 10 stocks and the yearly turnover on average was 4,9 stocks, therefore the yearly performance penalty they assumed was 0,49% or 49 basis points.

The question whether strategy specific balancing between exposure and costs is possible depends on the properties of the investing strategy. Generally, this can be pursued with the use of cost mitigation techniques. The different cost mitigation techniques are discussed in (Novy-Marx & Velikov, 2019). The authors introduce and compare three different cost mitigation techniques that are limiting investments to cheap-to-trade securities, reducing rebalancing frequency and trade “banding”. The main idea behind each is either limit the turnover of the strategy or limit the cost of turnover. Cheap-to-trade securities can be characterized as typically large and liquid and therefore cheaper to trade. Reducing rebalancing frequency naturally reduces the turnover. Trade banding is a process of imposing higher hurdles for buying into new positions. For example, such hurdle could be that a new position is only bought into, if the incoming stock is within the best 10% of stocks already in the portfolio by chosen measure. The authors state that all cost mitigation strategies reduce the transaction costs but also affect factor exposure.

3.7 A few notes on size and the Finnish stock market

As discussed earlier, the low-risk effect is only one of many factors that have been identified as a source of excess returns. The size effect was first documented by Banz (1981). Banz finds that smaller firms tend to have higher risk adjusted returns than larger firms. Banz notes that the main effect seems to be present in only the very small firms whereas the effect disappears as the size grows. Finding additionally led to the size factor being included in the Fama-French 3-factor model (Fama & French, 1992).

The size effect does present a few interesting questions for the study. As previously mentioned, low-volatility stocks tend to be larger stocks whereas high-risk stocks tend to be smaller. Additionally, much of the past literature limits the investing universe used in the studies to large caps. However, the average market capitalization of a stock listed in the Finnish stock market is notably small when compared with more established markets. Therefore, it makes sense to evaluate how the anomaly manifests in different sections of the Finnish stock market similarly with Baker et al. (2011), who conducted their analysis in the whole sample and in a sample solely restricted to large caps.

Size of the Finnish stock market is additionally related to potential reasons for the low-risk anomaly. Since some of the potential reasons proposed for the anomaly are related to institutional investors and liquidity it is interesting to see whether the anomaly manifests as expected in a market, or a section of the market, in which large institutional traders are not overly present and stocks trading volumes are generally low.

4 Data and methodology

In this section of the thesis the data of the study and the methodology are discussed. The first chapter discusses data and its limitations. The second chapter comprehensively discusses methodology. Finally, the third chapter discusses how the results are to be evaluated.

4.1 Description of data

The original data is retrieved from Thomson Reuters Datastream database, and it consists of monthly price observations for stocks listed in Helsinki stock exchange between the period from 30.04.1988 to 31.10.2023. All observations are converted to Euros. From this data, monthly observations of returns, 5-year rolling volatility and 5-year rolling beta are calculated for each stock. The period in which these measures are studied is from 31.07.1998 to 30.06.2023. The analysing period begins over 10 years after the dataset begins. The long gap is due to there not being enough requirement meeting stocks that are listed for a long enough period to construct low-risk portfolios. The gap also provides enough observations to estimate the rolling values for volatility and beta. There are 300 monthly observation dates overall for the study period, which translates to 25 years.

Additionally, the data includes monthly observations for multiple Finnish market indexes. Similarly to (Rinne & Vähämaa, 2011), OMXH CAP Total return index is chosen as the benchmark of the study. As the authors raise, the composition of market capitalization weighted index would be very Nokia dominated for extensive periods of time and therefore, the use of OMXH CAP is more suitable as the weight of individual company in it is limited to 10% of the index. The chosen benchmark is also used to estimate the beta in the study. To proxy the risk-free rate, the 12-month Helibor and Euribor are used according to which rate was used at the time. The 12-month rate is also used in other literature related to the Finnish stock market (see Ahmed et al, 2019; Davydov et al, 2016).

During the study period, securities enter and leave the stock exchange and therefore, the data sample is not prone to survivorship bias. Some of these securities are different classes of shares in the same companies or rights issues. It is noteworthy that different classes in public shares are increasingly rare in the Finnish stock exchange and most of such shares are present at the beginning of the study period. Additionally, the rights issues do not have a significant effect on the study as they typically exist for a short period of time in the data, which means that they are automatically excluded due to the chosen methodology. As per data, 195 of these securities are traded in the Helsinki stock exchange at the present time and a handful of them are present for the whole period.

One considerable data limitation relates to the size and liquidity of the Finnish stock market. When contrasted with the number of stocks traded in the U.S. for example, the 195 equities in the dataset at present moment is small. In addition, some of these companies are considered extremely small and thinly traded. These facts have implications on the feasibility of constructing cost-efficient low-risk portfolios and therefore, a market capitalization restriction is imposed on the data. Only stocks that have a market capitalization of over 70 million are included in the potential investment space. This is the same minimum bound used in (Jokipii & Vähämaa, 2006), which deals with a similar Finnish dataset. Table 1. shows the company size related descriptive statistics for uncontrolled sample and size restricted sample. The numbers are expressed in averages over the 12-month period for each year. Observations correspond to the number of securities in the sample for the given year. Numbers are expressed in millions.

Table 1. Company size related descriptive statistics.

The table represents yearly average descriptive statistics for market capitalization of companies in the sample. The left-hand side presents the whole sample. The right-hand side presents the sample when 70 million size restriction is imposed.

Size related descriptive statistics										
	Yearly averages for whole sample					Yearly averages for size-restricted sample				
	OBS	MIN	MEDIAN	AVERAGE	MAX	OBS	MIN	MEDIAN	AVERAGE	MAX
1998*	156	0,6	132,9	949,9	36 565,3	93	73,0	386,6	1 581,9	36 565,3
1999	162	0,6	118,5	1 234,4	106 214,0	94	72,4	344,4	2 111,9	106 214,0
2000	173	1,1	126,7	2 171,4	238 956,2	104	71,2	366,6	3 571,3	238 956,2
2001	174	0,1	86,5	1 297,5	128 042,9	92	72,4	316,9	2 412,1	128 042,9
2002	169	0,2	70,8	1 053,6	85 205,4	85	71,3	330,5	2 070,5	85 205,4
2003	164	0,5	76,4	1 056,2	67 023,8	83	71,8	314,2	2 051,4	67 023,8
2004	158	1,6	104,5	1 299,4	59 687,1	93	71,6	308,1	2 197,0	59 687,1
2005	157	3,0	123,9	1 527,6	59 592,4	96	72,7	404,9	2 478,3	59 592,4
2006	155	3,8	156,5	1 857,9	66 962,9	99	73,5	476,0	2 869,2	66 962,9
2007	154	3,5	182,2	2 217,6	87 042,0	107	72,3	473,5	3 156,7	87 042,0
2008	148	0,6	129,3	1 614,5	65 411,4	95	71,8	413,6	2 472,7	65 411,4
2009	143	0,9	114,5	1 240,2	35 541,4	85	71,3	406,0	2 047,3	35 541,4
2010	141	1,2	166,9	1 568,8	33 157,1	94	72,6	503,2	2 351,6	33 157,1
2011	139	1,4	165,0	1 526,2	29 215,7	91	73,0	479,6	2 307,9	29 215,7
2012	137	3,2	137,0	1 374,4	28 663,4	88	72,3	426,7	2 133,5	28 663,4
2013	138	3,1	139,1	1 608,6	36 716,4	90	71,7	436,1	2 458,1	36 716,4
2014	143	2,4	145,9	1 775,5	41 204,8	96	71,3	462,5	2 642,0	41 204,8
2015	150	1,5	176,2	1 832,4	44 505,5	102	71,5	433,3	2 689,9	44 505,5
2016	157	3,0	179,2	1 679,3	36 203,1	102	73,1	452,2	2 561,8	36 203,1
2017	160	2,7	208,3	1 859,1	44 076,9	110	71,4	487,0	2 704,3	44 076,9
2018	165	4,3	155,6	1 830,5	34 994,9	112	70,9	434,8	2 684,9	34 994,9
2019	170	3,8	136,7	1 740,4	27 997,1	109	71,7	461,4	2 702,7	27 997,1
2020	171	3,0	138,2	1 743,4	32 059,5	106	72,1	511,7	2 799,3	32 059,5
2021	180	4,7	199,4	2 106,0	41 243,0	129	72,0	481,5	2 913,1	41 243,0
2022	196	3,6	140,3	1 692,6	36 878,7	128	71,8	399,5	2 576,3	36 878,7
2023*	196	2,1	138,0	1 588,6	37 171,6	125	71,6	374,4	2 475,3	37 171,6

The market capitalization restriction and the chosen study methodology have implications on the universe of investments available to invest for each period. As touched upon earlier, the years before the change of millennia do not have enough stocks to sensibly construct low-risk portfolios that meet the requirements. For example, the number of securities listed on 30.07.1993, which is the earliest possible date to begin the study with 5-year rolling risk measures, is 81 and when the market capitalization restriction is imposed the number of investable equities becomes 55. Since the calculation of rolling values for volatility and beta require continuous observations of monthly returns, the timeframe in which these rolling values are calculated has a significant further effect on how many equities are available for invest. In the case of 30.07.1993, there would be only 18 securities that meet all these requirements, which practically means that portfolio construction would be impossible. On the chosen start date of 31.08.1998, there are 57 such securities as seen in table 2. Although this is not optimal either, it is enough for the purpose of the study. It is also noteworthy that the number of securities that meet all the demands is at its lowest at the beginning of the study period and steadily progresses upwards as the time progresses. Table 2. describes the yearly averages for different descriptive statistics for volatility and beta measures. The observation column corresponds to average yearly of stocks that meet all the requirements. Min and max columns correspond to average of min and max values over 12 months for a given year. Similar interpretations apply for median and average columns.

Table 2. Average yearly descriptive statistics for volatility and beta measures.

The table represents the descriptive statistics for average yearly risk measures in the size-restricted sample for stocks that have over five years of past returns the risk can be estimated from.

	Yearly averages for all volatility values					Yearly averages for all Beta values				
	OBS	MIN	MEDIAN	AVERAGE	MAX	OBS	MIN	MEDIAN	AVERAGE	MAX
1998*	57	20 %	33 %	34 %	61 %	57	0,19	0,87	0,86	1,51
1999	57	18 %	33 %	34 %	67 %	57	0,07	0,86	0,82	1,49
2000	58	16 %	32 %	34 %	72 %	58	0,06	0,70	0,72	1,43
2001	54	15 %	32 %	35 %	70 %	54	0,10	0,60	0,64	1,55
2002	59	14 %	34 %	36 %	66 %	59	0,10	0,54	0,58	1,60
2003	62	14 %	33 %	36 %	86 %	62	0,04	0,45	0,56	2,02
2004	75	14 %	30 %	35 %	85 %	75	0,01	0,39	0,57	2,26
2005	84	15 %	30 %	35 %	87 %	84	0,05	0,53	0,70	2,82
2006	86	14 %	28 %	31 %	78 %	86	0,06	0,64	0,78	2,63
2007	92	14 %	26 %	28 %	84 %	92	0,03	0,70	0,81	2,20
2008	83	13 %	27 %	28 %	55 %	83	0,15	0,89	0,92	1,90
2009	75	17 %	32 %	33 %	57 %	75	0,18	0,83	0,88	1,96
2010	83	18 %	35 %	35 %	65 %	83	0,14	0,84	0,88	2,03
2011	83	19 %	36 %	35 %	66 %	83	0,15	0,85	0,88	1,96
2012	83	17 %	37 %	37 %	68 %	83	0,16	0,84	0,91	1,96
2013	85	16 %	36 %	36 %	67 %	85	0,24	0,84	0,89	1,97
2014	87	14 %	29 %	30 %	59 %	87	0,21	0,80	0,89	2,02
2015	90	14 %	27 %	29 %	56 %	90	0,09	0,80	0,89	2,02
2016	85	14 %	26 %	29 %	62 %	85	0,11	0,77	0,88	2,01
2017	86	16 %	25 %	27 %	60 %	86	0,04	0,74	0,82	1,85
2018	82	15 %	24 %	26 %	57 %	82	0,08	0,75	0,80	1,82
2019	82	15 %	26 %	27 %	58 %	82	0,11	0,86	0,91	2,34
2020	82	15 %	27 %	29 %	60 %	82	0,07	0,95	0,96	2,30
2021	98	15 %	29 %	31 %	68 %	98	0,07	1,02	1,03	2,28
2022	93	16 %	31 %	33 %	79 %	93	0,13	1,03	1,03	2,11
2023*	97	18 %	32 %	35 %	81 %	97	0,21	1,02	1,03	2,07

As discussed, the OMXH Cap Total return index is the benchmark of the study. When the benchmark is compared with indexes constructed from the sample, a few additional observations about the Finnish stock market can be made. Figure 2. shows cumulative returns of 4 market indexes over the study period that are OMXH Cap total return index, equally weighted index of the sample, equally weighted index of the size-restricted sample and index of sample limited to large caps, or in other words stocks with over 1 billion market capitalization. There exists a large gap in performance between the sample and the benchmark. The gap slightly improves when size restriction is imposed on the sample. Overall, this strongly implies that small stocks tend to perform poorly when compared to larger, more established stocks. It seems that the returns in the Finnish stock market are mostly generated by a small group of large blue-chip stocks. The effect can be further evidenced in table 2. as the mean and median values for beta are lower than the market beta of 1 for large proportion of the study period, meaning that the performance of the stocks in the sample on average lags behind the performance of the benchmark. The observation can be tested by restricting the sample to only large cap stocks, which according to Nasdaq (2024), are defined in the Finnish stock market as stocks with over 1 billion market cap. When such a change is made, the new size restricted sample ends up performing the best.

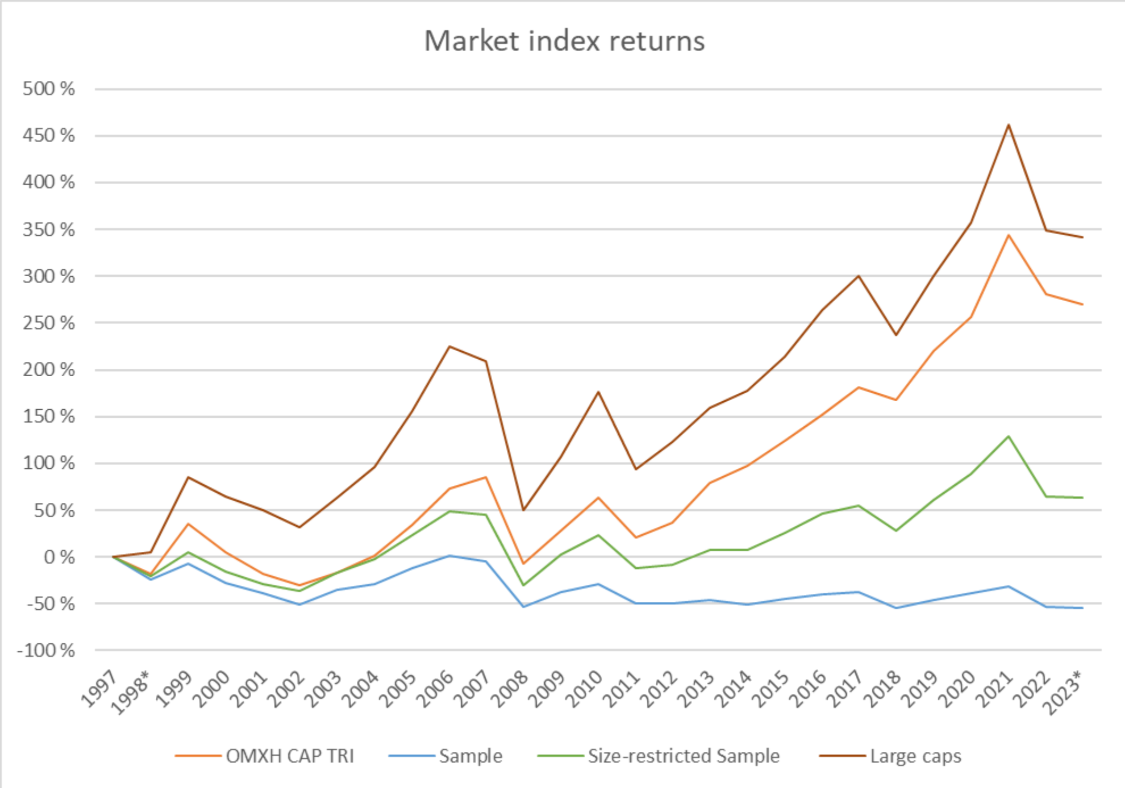

Figure 2. Benchmark of the study and sample indexes.

4.2 Portfolio construction methodology

The literature review showed that there are multiple ways to determine the level of risk, and the measurement periods varied considerably. However, in studies where the volatility and beta were estimated from monthly data the measurement periods were typically longer. When it comes to studies in the literature review, in which the measurement of risk is estimated from monthly data, the Baker et al. (2011) study had the longest risk measurement period at 5 years. As discussed earlier, the length of the risk estimation period decreases the number of securities available to invest. On the other side of the argument is that the longer estimation period also means less portfolio turnover as the risk estimation values change less quickly. As one of the aims of the study is to examine the viability of a low-risk investing strategy, the level of turnover has significant implications for it. Therefore, the 5-year rolling estimation period is used as the primary estimation period of the study. However, the results with shorter estimation periods of 3 and 1 years are additionally tested and analysed for robustness.

There are two types of risk measured portfolios inferred from the return data, which are volatility and beta portfolios. To estimate the volatility and beta, 60 months or in other words 5 years of prior monthly return data is required. The estimation method is not prone to look-ahead bias as the estimates are based on past return data. For volatility portfolio, 60-month rolling annualized standard deviations for all the stocks that meet the study requirements are calculated and ranked according to their respective values. The stocks are subsequently ranked and sorted into five different portfolios according to these values. The portfolios are equally weighted. The same process is conducted to construct beta portfolios, except the betas are estimated by dividing the five-year covariance between each security and the benchmark index with the 5-year variance of benchmark indexes returns. The decision to use 5 different portfolios is in line with Baker et al. (2011). For this thesis, similar division is used due to the lack of securities that meet all the requirements. Further division would mean that the number of securities in individual portfolios would be considerably under 10 in many instances and that would imply significant loss in diversification. Furthermore, more conservative portfolio division such as the 30-40-30 proposed in Dimson et al. (2017), risks that the effect becomes invisible as properties of each portfolio begin to converge.

In the second phase of the thesis, the portfolios are studied further. The original portfolios are in theory the most exposed to the low-risk factor. However, the original portfolio construction strategy largely ignores the possible effect of transaction costs. To account for transaction costs, the amount of turnover is calculated for each portfolio and transaction costs are estimated. After which, a cost mitigation technique of reducing rebalancing period is applied to the portfolios. The second portfolio construction method aims to optimize for the effects of transaction costs. The optimal target turnover for such portfolio is around 30% turnover as suggested by Van Vliet (2018). As discussed, (Novy-

Marx & Velikov, 2019) introduces three different cost mitigation techniques that are limiting investments to cheap-to-trade securities, reducing rebalancing frequency and trade "banding". Since the Finnish investing space is fundamentally limited and stocks with market capitalization over 1 billion are scarce, the further use of cheap-to-trade cost mitigation strategy is expected to be impractical. As the study uses long estimation periods for volatility and beta, the relative change in their values is expected to happen slowly. Therefore, reducing rebalancing periods is the relevant cost mitigation method to use.

As discussed, there are relatively few studies or sources that estimate the transaction costs in the Finnish stock market. Since there exist a level of uncertainty on the sufficiently accurate level for transaction costs, this thesis follows the more conservative estimation method proposed by Rinne and Vähämaa (2011), which calculates the transaction penalty as yearly turnover, as measured by number of stocks, multiplied with a conservative 1% round-term transaction cost. As the portfolios are equally weighted, the turnover can effectively be calculated as position changes in the portfolio divided by the number of stocks in the portfolio. In this sense, there are two types of turnover in the study set up. In the first type a stock enters the portfolio as another leaves the portfolio, therefore this type of turnover is calculated as two position changes. In the second type of turnover, the number of total positions in the portfolio either increases or decreases, in which case a new stock enters, or an existing stock leaves the portfolio. In such a case there is one position change.

These portfolios and methodology allow for more than superficial analysis into the low-risk anomaly and investing strategy in Finnish stock market. Through quintile portfolios, it is possible to observe how risk-return relationship in stocks holds up in the Finnish stock market and whether low-risk stocks outperform their high-risk counterparts and the market in risk adjusted basis. By comparing volatility and beta-based portfolios, it is possible to examine if the results between the measures vary. Lastly, through the application of transaction cost relative considerations in portfolio construction, it is possible to observe whether factor exposure is significantly limited by smart portfolio upkeep methods and what are the return relative implications of each portfolio construction method when transaction costs are accounted for.

4.3 Measuring performance and statistical significance of results

The following metrics are calculated for each quintile portfolio. Alpha, beta, total return, annualized total return, excess return, annualized excess return, annualized standard deviation of excess returns and based on the two aforementioned metrics, the Sharpe ratio. Excess returns are estimated in accordance with (Blitz & Van Vliet, 2007), in which they are calculated as the product of monthly observations of return over risk-free rate. The estimation method for Sharpe ratio is also based on the same study.

In the literature, there are typically two different ways to measure the performance of portfolios, which are portfolio alpha, and risk adjusted returns. Jensen's alpha is the return that is not explained by the model the returns are regressed against and risk adjusted returns are excess returns that have been adjusted to some form of risk measure.

In this thesis, the portfolio Jensen's alpha is measured as the alpha not predicted by CAPM as per the equation below

$$R_p - R_f = \alpha_p + \beta_p\left(R_m - R_f\right) + \varepsilon_p, \qquad (5)$$

In which R_p corresponds to portfolio return, R_f is the risk-free rate, α_p is the alpha generated by the portfolio, β_p is the beta of the portfolio, R_m is the return of market index and ε_p is the error term or in other words, the idiosyncratic risk.

In essence, the alpha is the return generated that is not explained by the model it is regressed against. In a situation where returns are explained by the systematic risk, or in other words the CAPM holds, the alpha and the error term for all the portfolios is zero or very close to it. The returns over risk-free rate of each portfolio are regressed against the returns over the risk-free rate by the market index. The statistical significance of the results is evaluated according to standard methods of regression analysis. If statistically significant alpha is generated by a portfolio, it can be concluded that the portfolio in question consistently produce returns not explained by the level of systematic risk and therefore the low-risk anomaly exists.

In this thesis, the risk adjusted returns are measured by Sharpe ratio

$$Sharpe\ ratio = \frac{R_p - R_f}{\sigma_p}, \qquad (6)$$

In which $R_p - R_f$ refers to excess returns, or in other words returns exceeding the return of the risk-free rate, and σ_p refers to the standard deviation, or in other words volatility of returns of the portfolio.

In essence, the returns for each portfolio over the risk-free rate are adjusted to the corresponding level of standard deviation of the portfolio. Statistical significance of risk adjusted returns can be tested with z-test as in (Blitz and Van Vliet, 2007). The model used by the authors was first introduced by Jobson and Korkie (1981) and was later corrected by Memmel (2003) into the form used by the authors.

$$z = \frac{SR_1 - SR_2}{\sqrt{\frac{1}{T}\left[2\left(1-\rho_{1,2}\right) + \frac{1}{2}\left(SR_1^2 + SR_2^2 - SR_1 SR_2\left[1+\rho_{1,2}^2\right]\right)\right]}}, \qquad (7)$$

In which SR refers to Sharpe ratio for portfolios 1 and 2, ρ refers to correlation between the returns of portfolios 1 and 2, T is the number of observations and z is the test statistic, which asymptotically follows a standard normal distribution.

The test measures whether the risk adjusted returns are significantly different from each other. Z value indicates how many standard deviations the observed difference in risk adjusted returns between portfolios 1 and 2 is from zero, the statistical significance of the difference is determined according to standard methods of a two-tail z-test. The significance of the difference can be tested between two constructed portfolios or between a portfolio and the market index. If there exists a statistically significant difference between the Sharpe ratio of the lowest and the highest risk portfolio and the risk adjusted return of the low-risk portfolio is higher, it can be concluded that the low-risk anomaly exists in the market. If a statistically significant difference of same direction is observed between a low-risk portfolio and the market index, it can be concluded that the low-risk portfolio outperforms the market on a risk-adjusted basis.

Both test methods are illustrated in figure 3. In traditional finance theory, risk and return are portrayed by the upward sloping line in the figures. In a situation in which the low-risk anomaly exists the portfolios of different risk levels are expected to arrange similarly to the points of the graph. The relationship is then flatter than expected by traditional theory. In such situation, the low-risk portfolios generate high Sharpe ratios and positive alpha, whereas high risk portfolios generate low Sharpe ratios and negative alpha. In contrast, if CAPM and theoretical risk-return relationship hold the Sharpe ratios are equal in all the risk quintiles, as risk and return grow in tandem. Alphas on the other hand, would be zero or close to zero as CAPM serves as a good estimate for risk and subsequent return. Z-test determines whether the difference between the Sharpe ratios of two portfolios is statistically robust. In CAPM regression, the alpha needs to be large enough in order to rule out the possibility of it being caused by a random chance before similar conclusion on statistical robustness can be reached.

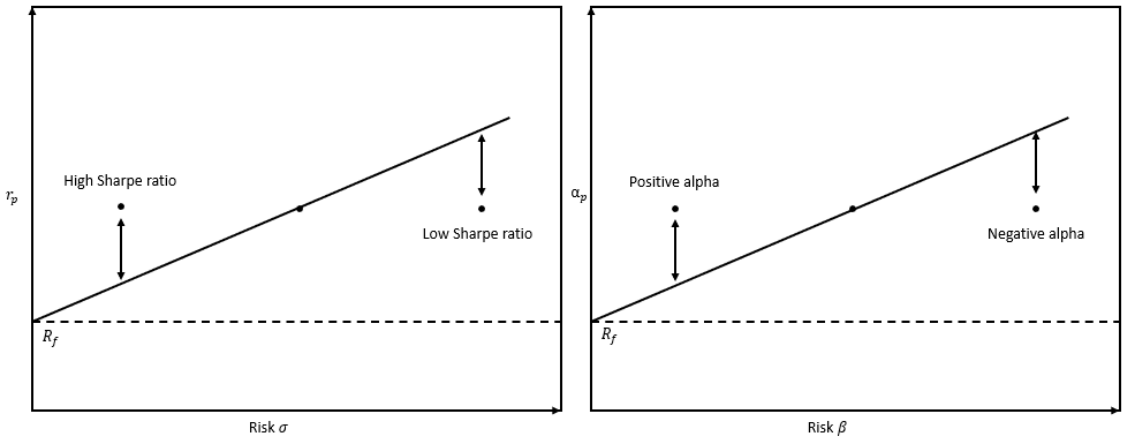

Figure 3. Testing framework.

5 Empirical results

This chapter is divided into four subchapters. The first chapter concerns the empirical results produced by the original study set up. The second chapter and further subchapters explore the anomaly further by altering the set up in small ways with variations in line with previous literature. Additionally, the empirical results produced by variations are evaluated. In the third chapter hypotheses one and two are evaluated and conclusion regarding them is made. In the fourth chapter, turnover and transaction cost related tests are conducted and a conclusion for the third and final hypothesis is presented.

5.1 The results with the primary study methodology

The chosen methods to test the low-risk anomaly yield somewhat surprising results. In panel A of table 3, which showcases the CAPM regression results for Volatility quintiles, it can be seen that the Jensen's alpha is negative for all the quintiles. The two highest risk quintiles generate worst alpha, followed by the two lowest risk quintiles. The middle quintile produces the smallest underperformance in terms of alpha. The observed alphas are not statistically significant, meaning that the results are not statistically reliable.

In panel B of table 3, the results follow a similar trajectory. During the study period, the risk-free rate yields a cumulative return of 57% or 1,8% in annual basis. The two highest and the lowest volatility quintiles produce a cumulative return less than the risk-free rate, meaning their excess returns and resulting Sharpe ratios are negative. The difference in Sharpe ratios between market and most of the quintile portfolios is statistically significant at 0,1% level, meaning the benchmark index significantly outperforms all the quintiles in risk adjusted basis. The Sharpe ratio of the lowest volatility quintile is better than the Sharpe ratio of highest quintile, although the difference is not statistically significant. It can additionally be observed that most of the stocks in the investment universe seem to be on average less risky than market. Only the fourth and the highest quintiles experience volatility higher than the market and none obtain higher beta value than the market.

Table 3. Summary statistics for Volatility quintiles.

In the panel A, the quintile specific results of CAPM regression are presented. The table presents annualized Jensen's alpha, corresponding beta, and r squared values of regression. In the panel B, total returns, annualized total and excess returns $(R_p - R_f)$, annualized standard deviation of excess returns and the resulting Sharpe ratios are presented. The results of Jobson and Korkie (1981) two-sample z-test with Memmel (2003) correction are presented in parentheses. *S(portfolio – market)* denotes the difference in Sharpe ratios between the quintile and the benchmark, whereas *S(low – high)* denotes the difference in Sharpe ratios between the lowest quintile and the highest quintile. Statistical significance at 5%, 1% and 0,1% levels are denoted by *, ** and *** respectively.

Volatility portfolios	**Lowest quintile**	**2nd quintile**	**Middle quintile**	**4th quintile**	**Highest quintile**	**Market**
Panel A: CAPM regression						
Jensen's alpha	-2,18 %	-1,61 %	-0,92 %	-3,13 %	-3,82 %	
Beta intercept	0,51	0,69	0,79	0,88	0,99	
R squared	0,57	0,65	0,69	0,71	0,67	
Panel B: Performance evaluation						
Total return	43,3 %	83,8 %	133,2 %	36,2 %	9,8 %	269,5 %
Annualized total return	1,4 %	2,5 %	3,4 %	1,2 %	0,4 %	5,4 %
Annualized excess return	-0,4 %	0,6 %	1,6 %	-0,6 %	-1,4 %	3,5 %
Standard deviation	13,0 %	16,5 %	18,4 %	20,3 %	23,4 %	19,4 %
Sharpe ratio (*S*)	-0,03	0,04	0,09	-0,03	-0,06	0,18
S(portfolio - market)	(-5,03)***	(-3,88)***	(-2,75)**	(-6,23)***	(-6,67)***	
S(low - high)	(0,74)				(-0,74)	

For beta quintiles, the results can be seen in table 4. Generally, the results are slightly more consistent with theory of the low-risk anomaly. As seen in the panel A, all the quintiles produce negative alphas. However, the observed alphas are mostly in coherent order as the underperformance gradually increases with the level of risk, although the 4th beta quintile is a notable exception to this trend. The negative alpha generated by the highest beta quintile is statistically significant at 5% level.

The results at panel B change modestly. The benchmark outperforms all but the 4th beta quintile in risk-adjusted terms at 0,1% significance level. This time, there exist a statistically significant difference between Sharpe ratios of the lowest and highest quintile at 0,1% significance level. Additionally, the beta of highest quintile exceeds the market beta of 1. The results with beta quintiles suggest that low-risk anomaly is present in the Finnish stock market as the highest quintile generates statistically significant negative alpha and the difference in Sharpe ratios between the lowest and the highest quintile is statistically significant.

Table 4. Summary statistics for Beta quintiles.

In the panel A, the quintile specific results of CAPM regression are presented. The table presents annualized Jensen's alpha, corresponding beta, and r squared values of regression. In the panel B, total returns, annualized total and excess returns $(R_p - R_f)$, annualized standard deviation of excess returns and the resulting Sharpe ratios are presented. The results of Jobson and Korkie (1981) two-sample z-test with Memmel (2003) correction are presented in parentheses. *S(portfolio – market)* denotes the difference in Sharpe ratios between the quintile and the benchmark, whereas *S(low – high)* denotes the difference in Sharpe ratios between the lowest quintile and the highest quintile. Statistical significance at 5%, 1% and 0,1% levels are denoted by *, ** and *** respectively.

Beta portfolios	**Lowest quintile**	**2nd quintile**	**Middle quintile**	**4th quintile**	**Highest quintile**	**Market**
Panel A: CAPM regression						
Jensen's alpha	-1,35 %	-1,67 %	-2,56 %	-0,33 %	-5,61 %*	
Beta intercept	0,52	0,63	0,76	0,88	1,07	
R squared	0,54	0,59	0,66	0,73	0,71	
Panel B: Performance evaluation						
Total return	75,7 %	72,7 %	48,7 %	181,3 %	-28,2 %	269,5 %
Annualized total return	2,3 %	2,2 %	1,6 %	4,2 %	-1,3 %	5,4 %
Annualized excess return	0,4 %	0,4 %	-0,2 %	2,3 %	-3,1 %	3,5 %
Standard deviation	13,7 %	15,9 %	18,1 %	20,0 %	24,5 %	19,4 %
Sharpe ratio (*S*)	0,03	0,02	-0,01	0,12	-0,13	0,18
S(portfolio - market)	(-3,43)***	(-3,90)***	(-5,31)***	(-1,94)	(-8,91)***	
S(low - high)	(3,45)***				(-3,45)***	

The cumulative returns for quintiles and the benchmark are illustrated in figures 4 and 5. As can be seen, on both occasions the outperformance of OMXH CAP TRI is considerable and the riskiest quintiles consistently produce the worst results. With other quintiles, the order of performance varies considerably. It is also notable that apart from the highest risk quintiles, most of the other quintiles keep up with the benchmark until a few years after the great financial crisis of 2008, after which the benchmark starts to move ahead. Although this might be a coincidence, the same period is characterized by zero interest rate policy.

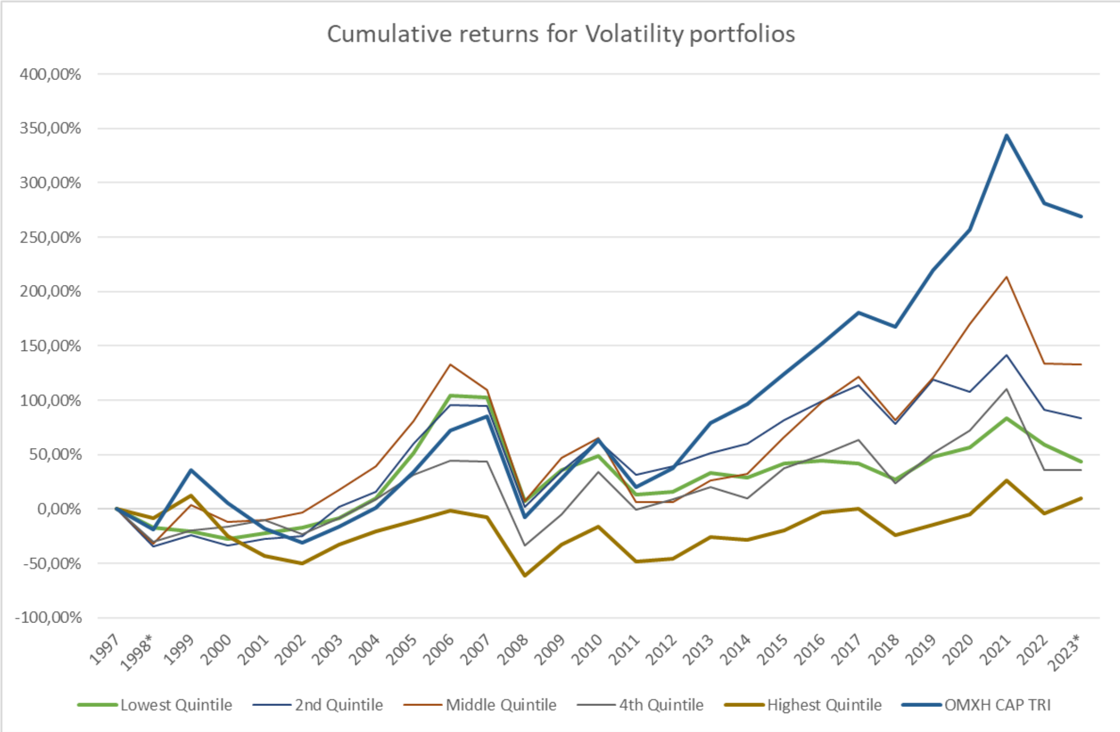

Figure 4. Performance of Volatility quintiles.

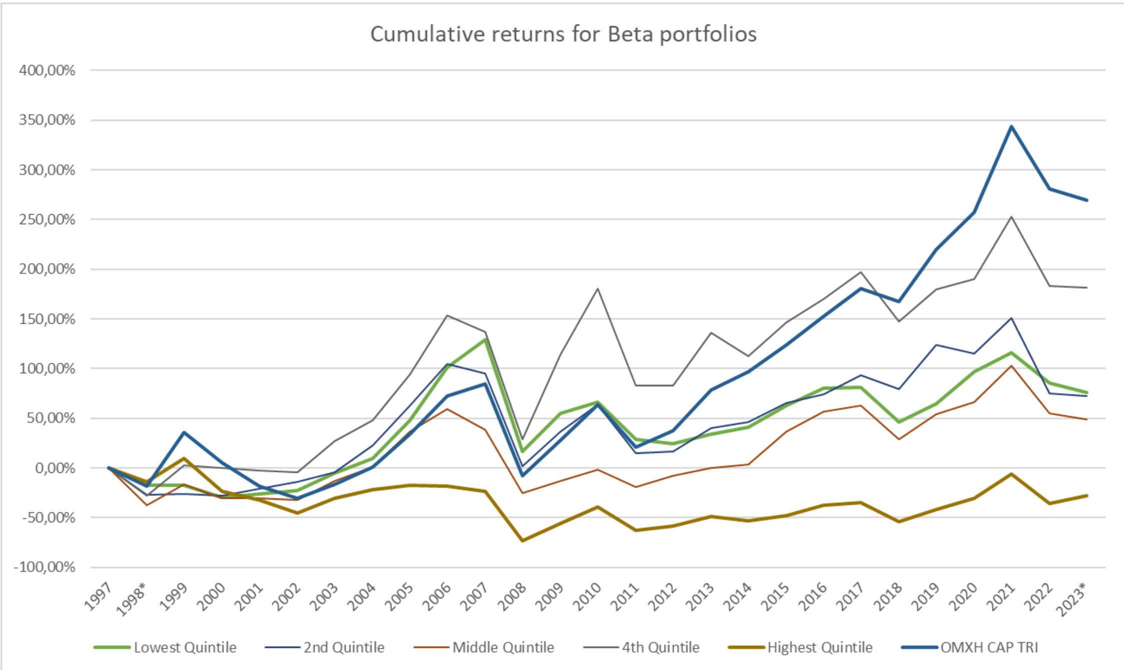

Figure 5. Performance of Beta quintiles.

5.2 The differentiating study methods and further tests

As discussed in the literature review, the methods which are used to study low-risk vary considerably and therefore, it makes sense to test results by altering relevant aspects of the original set up. The estimation period for risk varies considerably between studies. Therefore, the observation period for risk is changed to 3 and 1 years in order to see whether the estimation period significantly alters the results. Similarly with Baker et al. (2011), the anomaly is tested in different size sections of the stock market. This is especially relevant, since the Finnish stock market differentiates considerably from the larger and more established markets in related studies. All the changes will be made on a ceteris paribus basis.

Additionally, there are other ways study set ups differ in literature. The division of portfolios varies from deciles (Blitz & Van Vliet, 2007) to 30-40-30 division (Dimson et al., 2017). However, since the sample is narrow, the changing of division would not yield value adding results. In most of the studies portfolios are market cap weighted. However, since there are stocks in the data, namely Nokia and Sonera, which at times constitute for over 50% of the market capitalization of the Finnish stock market, the testing of market cap weighted portfolios would not yield value adding nor consistent results. Additionally, the sample is used as the benchmark in some studies instead of a real market index (see Blitz & Van Vliet, 2007). However, since the Finnish investor is able to invest in OMXH CAP TRI tracking fund, the importance of results obtained through such change would be irrelevant.

5.2.1 Changing risk estimation period

The effects of decreasing the risk estimation period from five years to 3 years can be seen in table 5. With volatility quintiles, the magnitude of negative alpha decreases in the two low-volatility quintiles. With high-volatility quintiles, it increases in the 4th volatility quintile while decreasing in the highest quintile. The middle quintile produces a positive but highly insignificant alpha. The 4th volatility quintile produces statistically significant negative alpha at 5% significance level. The beta values do not change in a material way, although the highest volatility quintile achieves a beta slightly over 1. In panel B of the volatility table, the total return for all the volatility quintiles slightly improves except for the 4th quintile. Other than that, there are no material differences to the results yielded by the original set up. The risk adjusted overperformance of market remains statistically impressive. The Sharpe ratio of low-volatility quintile is again better than the ratio for high-volatility quintile, however the difference is not statistically significant.

In beta quintiles, the negative alphas for the two low-beta and the highest beta quintile escalate, while the middle and the 4th quintile improve. The 4th quintile produces statistically insignificant but positive alpha in contrast to negative alphas generated by the rest of the portfolios. The statistical significance of negative alpha for highest beta quintile increases to 1% significance level. In terms of total return, the improvement in the middle quintile is noteworthy as its cumulative total return increases from 48,7% to 105,1%. The beta value for said quintile is also considerably higher than in the original set up. Despite the increase in the underperformance of highest beta quintile, the decrease in the performance of lowest beta quintile means that the statistical significance of difference between the Sharpe ratios of the two decreases to 1% significance level.

Table 5. Summary statistics for risk quintiles with 3-year risk estimation.

In the panel A, the quintile specific results of CAPM regression are presented. The table presents annualized Jensen's alpha, corresponding beta, and r squared values of regression. In the panel B, total returns, annualized total and excess returns $(R_p - R_f)$, annualized standard deviation of excess returns and the resulting Sharpe ratios are presented. The results of Jobson and Korkie (1981) two-sample z-test with Memmel (2003) correction are presented in parentheses. *S(portfolio – market)* denotes the difference in Sharpe ratios between the quintile and the benchmark, whereas *S(low – high)* denotes the difference in Sharpe ratios between the lowest quintile and the highest quintile. Statistical significance at 5%, 1% and 0,1% levels are denoted by *, ** and *** respectively.

3-year risk estimation

Volatility portfolios	Lowest quintile	2nd quintile	Middle quintile	4th quintile	Highest quintile	Market
Panel A: CAPM regression						
Jensen's alpha	-1,97 %	-1,55 %	0,05 %	-4,58 %*	-3,65 %	
Beta intercept	0,50	0,65	0,82	0,88	1,03	
R squared	0,56	0,66	0,73	0,70	0,68	
Panel B: Performance evaluation						
Total return	50,7 %	86,0 %	205,9 %	-6,8 %	16,4 %	269,5 %
Annualized total return	1,7 %	2,5 %	4,6 %	-0,3 %	0,6 %	5,4 %
Annualized excess return	-0,2 %	0,7 %	2,7 %	-2,1 %	-1,2 %	3,5 %
Standard deviation	13,0 %	15,6 %	18,6 %	20,4 %	24,2 %	19,4 %
Sharpe ratio (*S*)	-0,01	0,04	0,14	-0,10	-0,05	0,18
S(portfolio - market)	(-4,63)***	(-3,79)***	(-1,09)	(-8,14)***	(-6,46)***	
S(low - high)	(0,81)				(-0,81)	

Beta portfolios	Lowest quintile	2nd quintile	Middle quintile	4th quintile	Highest quintile	Market
Panel A: CAPM regression						
Jensen's alpha	-2,00 %	-2,08 %	-1,29 %	0,14 %	-6,29 %**	
Beta intercept	0,52	0,65	0,75	0,80	1,17	
R squared	0,56	0,64	0,66	0,67	0,78	
Panel B: Performance evaluation						
Total return	50,6 %	60,3 %	105,1 %	199,2 %	-35,9 %	269,5 %
Annualized total return	1,7 %	1,9 %	2,9 %	4,5 %	-1,8 %	5,4 %
Annualized excess return	-0,2 %	0,1 %	1,1 %	2,6 %	-3,6 %	3,5 %
Standard deviation	13,5 %	15,7 %	18,0 %	18,8 %	25,7 %	19,4 %
Sharpe ratio (*S*)	-0,01	0,00	0,06	0,14	-0,14	0,18
S(portfolio - market)	(-4,59)***	(-4,65)***	(-3,36)***	(-1,17)	(-10,26)***	
S(low - high)	(2,79)**				(-2,79)**	

The consequences of reducing the risk estimation period to 1-year are seen in table 6. In volatility quintiles, the negative alpha considerably decreases in low-volatility quintiles when compared to original set up. Simultaneously, the negative alpha in the highest volatility class considerably escalates, becoming significant at 1% level. For the first time, there is also a notable increase in both the level of volatility and beta in the highest volatility quintile. In panel B of volatility table, the total returns for low-volatility quintiles increase. By extension the excess returns become positive for all but the highest volatility quintile, which cumulative return turns negative to a considerable degree. The highest volatility quintile yields a -74,3% negative return. The difference between the Sharpe ratios of the lowest and the highest volatility quintile also turns from statistically insignificant to statistically significant at 0,1% level. Despite better results, the benchmark significantly outperforms all the low-risk quintiles in risk adjusted basis.

The observations are mostly similar with beta quintiles. The most notable difference comes from the escalation in negative alpha for the highest beta quintile, increasing its statistical significance to 1% level. Although the negative alpha for lowest beta quintile decreases, there exists no clear trend with the other quintiles as the negative alpha expands for the 2nd and the 4th quintile, while decreasing for the middle quintile. This similarly transfers to total returns, where the lowest beta quintile improves and achieves the highest return of all the quintiles. Similarly to the highest volatility quintile, the highest beta quintile yields a considerably weak total return of -63,6%. Also similarly, there is a considerable increase in both the volatility and beta values for the high beta quintile. For the first time, the Sharpe ratio of the benchmark is not larger in a statistically significant way when compared with the Sharpe ratio of the low beta quintile. Between the lowest and highest beta quintile, the difference is significant at 0,1% confidence level.

Table 6. Summary statistics for risk quintiles with 1-year risk estimation.

In the panel A, the quintile specific results of CAPM regression are presented. The table presents annualized Jensen's alpha, corresponding beta, and r squared values of regression. In the panel B, total returns, annualized total and excess returns $(R_p - R_f)$, annualized standard deviation of excess returns and the resulting Sharpe ratios are presented. The results of Jobson and Korkie (1981) two-sample z-test with Memmel (2003) correction are presented in parentheses. *S(portfolio – market)* denotes the difference in Sharpe ratios between the quintile and the benchmark, whereas *S(low – high)* denotes the difference in Sharpe ratios between the lowest quintile and the highest quintile. Statistical significance at 5%, 1% and 0,1% levels are denoted by *, ** and *** respectively.

1-year risk estimation

Volatility portfolios	Lowest quintile	2nd quintile	Middle quintile	4th quintile	Highest quintile	Market
Panel A: CAPM regression						
Jensen's alpha	-0,92 %	-0,65 %	-1,45 %	-2,54 %	-9,19 %**	
Beta intercept	0,52	0,65	0,78	0,95	1,19	
R squared	0,58	0,66	0,72	0,78	0,69	
Panel B: Performance evaluation						
Total return	99,5 %	133,1 %	105,4 %	69,2 %	-74,3 %	269,5 %
Annualized total return	2,8 %	3,4 %	2,9 %	2,1 %	-5,3 %	5,4 %
Annualized excess return	1,0 %	1,6 %	1,1 %	0,3 %	-7,0 %	3,5 %
Standard deviation	13,1 %	15,5 %	17,9 %	20,8 %	27,7 %	19,4 %
Sharpe ratio (*S*)	0,07	0,10	0,06	0,01	-0,25	0,18
S(portfolio - market)	(-2,65)**	(-2,14)*	(-3,69)***	(-5,76)***	(-11,55)***	
S(low - high)	(6,47)***				(-6,47)***	

Beta portfolios	Lowest quintile	2nd quintile	Middle quintile	4th quintile	Highest quintile	Market
Panel A: CAPM regression						
Jensen's alpha	-0,46 %	-2,44 %	-2,31 %	-1,55 %	-8,08 %**	
Beta intercept	0,55	0,67	0,76	0,85	1,25	
R squared	0,57	0,62	0,71	0,73	0,75	
Panel B: Performance evaluation						
Total return	126,3 %	46,4 %	63,4 %	103,8 %	-63,6 %	269,5 %
Annualized total return	3,3 %	1,5 %	2,0 %	2,9 %	-4,0 %	5,4 %
Annualized excess return	1,5 %	-0,3 %	0,1 %	1,0 %	-5,7 %	3,5 %
Standard deviation	14,1 %	16,5 %	17,5 %	19,3 %	28,2 %	19,4 %
Sharpe ratio (*S*)	0,10	-0,02	0,01	0,05	-0,20	0,18
S(portfolio - market)	(-1,84)	(-5,12)***	(-5,13)***	(-3,92)***	(-11,31)***	
S(low - high)	(6,40)***				(-6,40)***	

The general consequences of decreasing the estimation period seem to be that underperformance of high-risk stocks escalates. The finding is in line with Aabo et al. (2017) suggestion that shorter estimation period tends to overstate the strength of the anomaly at the high-risk tail. It seems that the shorter estimation period better captures the subsequent risk levels as evidenced by consistent increase in volatility and beta values when estimation period is decreased. Although the performance of low-risk quintiles seems to improve, the improvements generally remain marginal relative to the performance of the benchmark.

As excess returns of all estimation periods are plotted against the corresponding risk levels, the anomaly is illustrated at figures 6 and 7. Excess returns are on a similar level for the majority of the risk quintiles with slight upward slope until after a certain point, the relationship reverses, and additional risk leads to increasing underperformance.

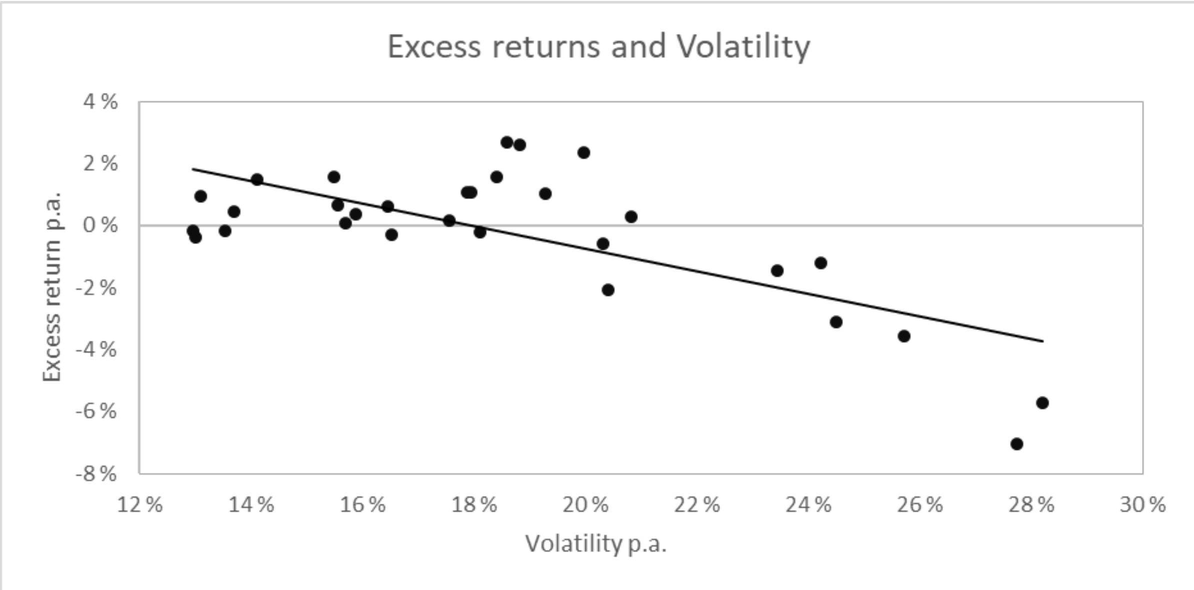

Figure 6. Volatility and excess returns of Volatility quintiles.

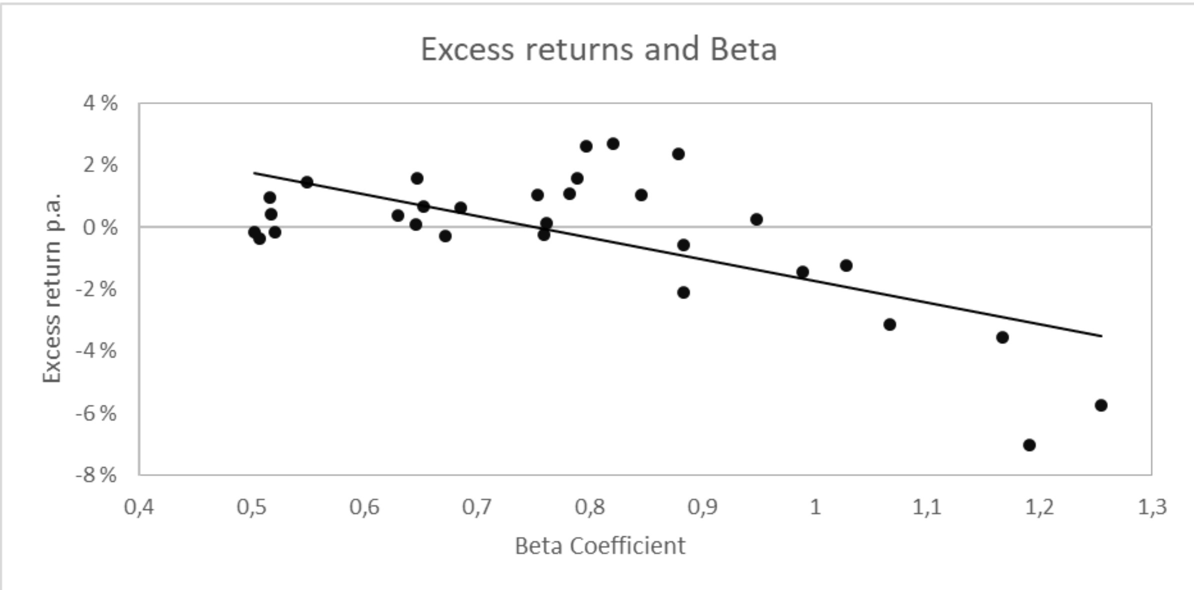

Figure 7. Beta and excess returns of Beta quintiles.

5.2.2 Size and risk

As discussed before, the size of the Finnish stock market considerably differentiates from more established markets examined in the literature. Additionally, it seems that a few large companies have an outsized effect on the return and risk profile of the Finnish stock market, while an average Finnish stock is considerably different from this profile. Average Finnish stock tends to be smaller and less traded than the ones in more established markets. It is therefore relevant to test how the performance of risk quintiles is affected by this setting. In previous literature, low-risk stocks are generally described as stable large caps whereas high-risk stocks are described as less liquid small caps. To test this observation, the median market cap of a stock in each risk quintile is estimated for each year of the study period. The risk quintiles are then ranked according to this median market cap in each year and the cumulative times the risk quintile ranked in each position is also observed. The results can be seen in tables 7 and 8.

Table 7. Median size of a stock in Volatility quintiles.

Table presents yearly estimate of median company size in each volatility quintile. Numbers are expressed in millions. The heat map at the bottom represents how the median size in each risk quintile ranked relative to other quintiles over the study period.

Median size of a stock in Volatility quintiles					
	Lowest quintile	2nd quintile	Middle quintile	4th quintile	Highest quintile
1998*	468	433	617	542	714
1999	308	374	599	539	582
2000	225	387	665	868	424
2001	227	381	484	1071	367
2002	187	456	459	1237	403
2003	126	471	474	784	256
2004	164	527	440	682	283
2005	348	589	804	370	255
2006	529	651	634	273	274
2007	371	912	653	439	204
2008	348	708	280	528	328
2009	244	348	369	652	600
2010	233	391	668	983	578
2011	226	388	642	879	718
2012	208	395	437	878	663
2013	401	311	457	802	711
2014	384	757	427	517	808
2015	396	905	370	394	668
2016	391	924	465	525	538
2017	524	888	325	779	427
2018	1066	406	445	900	585
2019	988	637	379	1195	404
2020	567	643	1085	825	665
2021	822	677	1025	563	385
2022	664	1115	618	470	466
2023*	532	1060	409	376	465
Smallest	13	3	4	2	4
Second smallest	4	7	4	3	8
Middle	4	5	6	6	5
Second largest	4	3	8	4	7
Largest	1	8	4	11	2

As can be seen, the results are not too coherent but generally the smallest stocks reside in low and high volatility portfolios and larger stocks are generally more in the middle of the spectrum when it comes to volatility quintiles. Notably, the lowest volatility quintile had the smallest median stock size in 13 years, while it had the largest median market capitalization once.

Table 8. Median size of a stock in Beta quintiles.

Table presents yearly estimate of median company size in each beta quintile. Numbers are expressed in millions. The heat map at the bottom represents how the median size in each risk quintile ranked relative to other quintiles over the study period.

Median size of a stock in Beta quintiles					
	Lowest quintile	2nd quintile	Middle quintile	4th quintile	Highest quintile
1998*	329	366	373	881	1158
1999	223	286	467	672	990
2000	232	300	515	536	1371
2001	243	389	407	766	895
2002	154	432	491	626	1162
2003	119	451	297	541	779
2004	163	319	455	721	350
2005	278	350	541	1100	317
2006	196	484	643	1361	496
2007	181	359	773	788	1089
2008	158	442	484	519	826
2009	156	226	557	562	941
2010	219	344	593	767	1449
2011	217	433	573	420	1544
2012	203	302	494	641	1099
2013	170	322	420	773	1075
2014	234	329	300	637	1295
2015	231	351	476	549	1425
2016	256	335	1134	620	1592
2017	198	542	801	799	2413
2018	187	586	759	1013	2315
2019	310	831	921	598	1092
2020	338	567	539	824	1175
2021	267	570	437	1266	962
2022	387	314	707	764	781
2023*	296	359	550	643	1059
Smallest	25	1	0	0	0
Second smallest	1	18	4	2	1
Middle	0	7	15	2	2
Second largest	0	0	7	18	1
Largest	0	0	0	4	22

For beta portfolios, the results are coherent to an extreme. Smaller stocks reside exclusively in the low beta portfolios and larger stocks in the high beta portfolios. With beta portfolios, the results are against the observations from previous literature. This raises an interesting question of whether risk quintiles are affected by exposure to the size classes they are concentrated in. To test this, the sample is divided into control groups. Similarly to Baker et al. (2011), the first group contains stocks with market capitalization between 70 million and 1 billion. The second group only contains stocks with over 1 billion market cap. The first group essentially represents small and medium sized stocks as defined by Nasdaq (2024). Similarly, the second group represents large caps.

When investment universe is limited to only small and mid-caps with 5-year risk estimation, the results change considerably as evidenced in table 9. All the quintiles perform poorly to a degree in which all the beta quintiles generate statistically significant negative alpha. Curiously, beta values for all the quintiles remain under 1, whereas volatility values are more in line with theory. Although all of the risk quintiles generate poor results, it can be inferred that the underperformance in high-risk quintiles is further escalated when large caps are excluded. High-volatility quintile yields a total return of -95,4% while the high beta quintile yields a total return of -91,8%. The negative alphas for both become significant at 0,01% level. Although all the stocks perform poorly, the low-risk quintiles still do relatively better than their high-risk counterparts. The differences in Sharpe ratios between low-risk and high-risk portfolios is significant at 0,1% level for volatility quintiles and at 1% level for beta quintiles. The results strongly suggest that investors should on average avoid smaller stocks in the Finnish stock market and especially the high-risk kind.

Table 9. Summary statistics for risk quintiles in small and medium sized companies.

In the panel A, the quintile specific results of CAPM regression are presented. The table presents annualized Jensen's alpha, corresponding beta, and r squared values of regression. In the panel B, total returns, annualized total and excess returns $(R_p - R_f)$, annualized standard deviation of excess returns and the resulting Sharpe ratios are presented. The results of Jobson and Korkie (1981) two-sample z-test with Memmel (2003) correction are presented in parentheses. *S(portfolio – market)* denotes the difference in Sharpe ratios between the quintile and the benchmark, whereas *S(low – high)* denotes the difference in Sharpe ratios between the lowest quintile and the highest quintile. Statistical significance at 5%, 1% and 0,1% levels are denoted by *, ** and *** respectively.

Volatility portfolios	**Lowest quintile**	**2nd quintile**	**Middle quintile**	**4th quintile**	**Highest quintile**	**Market**
Panel A: CAPM regression						
Jensen's alpha	-4,57 %	-5,43 %	-2,65 %	-7,64 %**	-14,64 %***	
Beta intercept	0,45	0,62	0,72	0,85	0,92	
R squared	0,46	0,53	0,59	0,58	0,52	
Panel B: Performance evaluation						
Total return	-27,9 %	-37,5 %	37,6 %	-62,8 %	-95,4 %	269,5 %
Annualized total return	-1,3 %	-1,9 %	1,3 %	-3,9 %	-11,6 %	5,4 %
Annualized excess return	-3,1 %	-3,6 %	-0,5 %	-5,6 %	-13,3 %	3,5 %
Standard deviation	13,0 %	16,6 %	18,2 %	21,7 %	24,7 %	19,4 %
Sharpe ratio (*S*)	-0,24	-0,22	-0,03	-0,26	-0,54	0,18
S(portfolio - market)	(-9,41)***	(-9,93)***	(-5,62)***	(-11,40)***	(-15,63)***	
S(low - high)	(5,99)***				(-5,99)***	

Beta portfolios	**Lowest quintile**	**2nd quintile**	**Middle quintile**	**4th quintile**	**Highest quintile**	**Market**
Panel A: CAPM regression						
Jensen's alpha	-5,72 %**	-5,27 %*	-6,18 %*	-5,15 %**	-12,85 %***	
Beta intercept	0,48	0,60	0,72	0,78	0,97	
R squared	0,46	0,51	0,54	0,58	0,57	
Panel B: Performance evaluation						
Total return	-46,7 %	-35,2 %	-46,7 %	-27,9 %	-91,8 %	269,5 %
Annualized total return	-2,5 %	-1,7 %	-2,5 %	-1,3 %	-9,5 %	5,4 %
Annualized excess return	-4,2 %	-3,5 %	-4,3 %	-3,1 %	-11,2 %	3,5 %
Standard deviation	13,7 %	16,3 %	19,1 %	20,1 %	24,8 %	19,4 %
Sharpe ratio (*S*)	-0,31	-0,21	-0,22	-0,15	-0,45	0,18
S(portfolio - market)	(-10,39)***	(-9,40)***	(-10,31)***	(-9,01)***	(-15,10)***	
S(low - high)	(2,89)**				(-2,89)**	

A similar test with large caps encounters a problem. At the start of the study period, there are only 12 stocks on average with over 1 billion market capitalization. Therefore, it is sensible to shorten the study period for the test. In 2010, there were 28 such stocks whereas in 2009 there were only 21. Although not optimal, mid-2010 seems to be the earliest starting point in which there are at least 5 stocks for each portfolio. This returns a 13-year study period, which is notably shorter than in most of the comparable studies and additionally occurs almost exclusively on a period in which the interest rates were at zero. Results can nevertheless be seen at table 10. The results are more in line with traditional theory on risk and return. The higher risk quintiles generate larger returns than the lower risk quintiles almost in completely consistent order. Curiously, the 2nd and the middle volatility quintiles generate statistically significant negative alpha for the period. Other than that, the CAPM regression results lack statistical significance. It is also notable that all the quintiles still underperform the market index in total return terms in all, but for one instance. There is also notable change in the risk characteristics as larger stocks replicate the risk profile of the benchmark better than the smaller stocks. This is evidenced by the quintile specific values for beta being coherently scattered around the market beta 1. Curiously, only the highest risk quintiles perform considerably better in annualized basis than their counterparts in the original setup. The annualized return for highest volatility quintile is 0,4% in the original setup, whereas it is 4,8% in the latter. The same values for highest beta quintile are -1,3% and 3,9%. For the rest of the quintiles the trend varies. For example, lowest volatility quintile produces 1,4% annual return in the original set up and 1,8% in the latter. The lowest beta quintile produces 2,3% annual return in the original set up and 1,2% in the latter. This finding does suggest that there exists a link between characteristics of small stocks and the underperformance of high-risk stocks, which may lend support to market microstructure related explanations for the anomaly. The finding is particularly in line with observation that the underperformance of high-risk stocks tends to be driven by small and illiquid stocks (see Hou & Loh, 2016; Novy-Marx & Velikov, 2018).

Table 10. Summary statistics for risk quintiles in large caps.

In the panel A, the quintile specific results of CAPM regression are presented. The table presents annualized Jensen's alpha, corresponding beta, and r squared values of regression. In the panel B, total returns, annualized total and excess returns $(R_p - R_f)$, annualized standard deviation of excess returns and the resulting Sharpe ratios are presented. The results of Jobson and Korkie (1981) two-sample z-test with Memmel (2003) correction are presented in parentheses. *S(portfolio – market)* denotes the difference in Sharpe ratios between the quintile and the benchmark, whereas *S(low – high)* denotes the difference in Sharpe ratios between the lowest quintile and the highest quintile. Statistical significance at 5%, 1% and 0,1% levels are denoted by *, ** and *** respectively.

Volatility portfolios	**Lowest quintile**	**2nd quintile**	**Middle quintile**	**4th quintile**	**Highest quintile**	**Market**
Panel A: CAPM regression						
Jensen's alpha	-2,11 %	-5,50 %*	-5,15 %*	-3,51 %	0,30 %	
Beta intercept	0,71	0,95	1,01	1,10	1,37	
R squared	0,64	0,77	0,74	0,68	0,68	
Panel B: Performance evaluation						
Total return	57,9 %	21,8 %	33,2 %	73,1 %	224,4 %	176,6 %
Annualized total return	1,8 %	0,8 %	1,2 %	2,2 %	4,8 %	4,2 %
Annualized excess return	1,6 %	0,5 %	0,9 %	1,9 %	4,5 %	3,9 %
Standard deviation	13,9 %	16,9 %	18,3 %	20,7 %	25,9 %	15,6 %
Sharpe ratio (*S*)	0,11	0,03	0,05	0,09	0,18	0,25
S(portfolio - market)	(-2,63)**	(-5,23)***	(-4,56)***	(-3,20)**	(-1,51)	
S(low - high)	(-0,91)				(0,91)	

Beta portfolios	**Lowest quintile**	**2nd quintile**	**Middle quintile**	**4th quintile**	**Highest quintile**	**Market**
Panel A: CAPM regression						
Jensen's alpha	-2,57 %	-3,77 %	-3,96 %	-4,13 %	-1,80 %	
Beta intercept	0,63	0,89	0,98	1,20	1,41	
R squared	0,46	0,73	0,76	0,76	0,72	
Panel B: Performance evaluation						
Total return	34,3 %	47,1 %	54,8 %	74,0 %	157,4 %	176,6 %
Annualized total return	1,2 %	1,6 %	1,8 %	2,2 %	3,9 %	4,2 %
Annualized excess return	0,9 %	1,3 %	1,5 %	2,0 %	3,6 %	3,9 %
Standard deviation	14,5 %	16,3 %	17,7 %	21,5 %	25,9 %	15,6 %
Sharpe ratio (*S*)	0,06	0,08	0,08	0,09	0,14	0,25
S(portfolio - market)	(-2,84)**	(-3,83)***	(-3,91)***	(-3,77)***	(-2,44)*	
S(low - high)	(-0,98)				(0,98)	

The results do imply that the Finnish stock market seems to be divided to two sections. The large caps do relatively well, and the return-risk profile of the whole market is dominated by them. Smaller stocks on the other hand perform poorly on average.

5.3 Discussing the results and hypothesis evaluation

The results leave room for some philosophical discussion. The most prominent of which is the observations that on average a stock listed in the Finnish stock market is considerably different in profile from the qualities of the benchmark index. This is evidenced by the fact that in most cases, all but the beta of the highest risk quintiles is less than the market beta. If low risk is defined as investment containing less risk than the market, most of the quintiles in this thesis and most of the Finnish stock market could be considered as such. This further implies that the risk and return characteristics of the benchmark or the Finnish stock market are mostly defined by a handful of large companies.

It is additionally noteworthy, that a considerable degree of quintiles in the study underperform the risk-free rate. The generally poor results can be partly explained by the study period. The beginning of the period exists at a time with relatively high valuation levels. This in combination with post tech bubble depression and the great financial crisis of 2008 effectively means that many of the quintiles do not have meaningful cumulative returns after the first 10 years of the study period. While the subsequent years are generally better, the period ends with two relatively weak post-covid years, leading to the situation described earlier. The observations can be further verified by cumulative returns of quintiles portrayed in figures 4 and 5.

Despite these challenges, the results do yield valuable information and can be used to make judgement on the hypothesis of the study. Based on the results provided by the original testing framework and further tests of previous chapters, the following can be concluded on hypotheses 1. Based on the evidence that only the high risk quintiles consistently generate statistically significant negative alpha in majority of the test variations, and by the observation that the Sharpe ratio produced by the lowest risk quintiles is consistently and statistically significantly better than the Sharpe ratio produced by the highest risk quintiles, the hypothesis 1 is accepted and the low-risk anomaly can be said to exist in the Finnish stock market. In quintiles other than the high-risk group, the results are generally not consistent. Overall, it can be said that the relationship between the risk and return in the Finnish stock market is a gently upward sloping line to a certain point, after which the relationship reverses when risk is increased. This can be observed in figures 6 and 7.

However, the results with different risk estimation periods and with different size classes raise a few noteworthy considerations. As raised by Aabo et al. (2017), the anomaly's robustness can be criticized based on the variability of results when estimation period is altered. The findings of this thesis especially support the observation that shorter estimation period tends to amplify the strength of the anomaly. The results also suggest that the market microstructure related explanations for the anomaly might be the more convincing ones. In the context of this thesis, the results suggest that size and by extension liquidity could be seen as potential factors. Most of the stocks in the Finnish stock market are small and illiquid. When the sample is limited to small and midcaps, the negative results for high-risk stocks are amplified, although it is noteworthy that the performance in other quintiles also weakens considerably. However, when the sample is limited to large caps, the risk-return relationship turns positive, and the performance of highest risk quintiles considerably improves. The combination of these two observations gives reason to believe that there exists a link between characteristics of smaller stocks and high-risk underperformance. The finding is in line with the previous observation that underperformance in high-risk stocks tends to be driven by small and illiquid firms (see Hou & Loh, 2016; Novy-Marx & Velikov, 2018) which supports market microstructure related explanations. However, it is also noteworthy that the results with the large cap stocks are not comparable and not as reliable as the other results of this thesis, since the study period for them is different and the number of large caps in the sample is small.

The majority of the sample in the study consists of smaller firms that perform on average poorly when compared with the benchmark. Therefore, the second hypothesis 2 is inevitably rejected. The low-risk portfolios consistently underperform the market on a risk adjusted basis and this thesis finds little evidence to suggest otherwise. However, as discussed it is noteworthy that the chosen benchmark considerably outperforms the sample as a whole. More compelling research into the effect in large caps which replicate the characteristics of the benchmark better would require larger sample, which the Finnish stock market as of now is unable to provide. This could be achieved by expanding the research into Nordic markets.

5.4 The effects of turnover management on the results

At this point this thesis encounters an awkward reality. Since there are no consistent and significant excess returns produced by the low risk stocks the testing of the final hypothesis becomes obsolete. However, as a curiosity the overall turnover in the lowest and highest quintiles is nevertheless tested and the effects of cost-optimizing methods on performance are observed.

The results of comparing monthly and yearly rebalancing periods within the original test framework can be seen in table 11. The panel A shows that with the standard monthly rebalancing period and 5-year risk estimation, the yearly turnover varies from 162% to 218%. Even with a more cost-efficient risk estimation period of 5-years, the yearly turnover figures can confidently be said to diminish "paper returns". When a yearly rebalancing interval is applied, the turnover and return relative effects are seen at panel B. The turnover figures do remain relatively high, although more reasonable at 53% to 65%. When compared how the appliance of rebalancing period affects the performance, a weak trend becomes visible. For the low-risk portfolios, the performance seems to be affected negatively and for the performance of high-risk portfolios, the effects are positive.

In the case of 5-year risk estimation and yearly rebalancing, the potential performance penalty is calculated as follows. At the lowest there are on average 11 stocks in a portfolio and at most 20 during the study period. 53% and 64% turnover would therefore mean between 5,83 to 12,8 transactions per year, which in turn would translate from 0,58% or

58 basis points to 1,3% or 130 basis point performance penalty per year. The performance penalty is in the typical range of operating expenses charged by mutual funds, although higher than for example an index tracking ETF. Although the level is not unreasonable the loss in performance is also a relevant input for the calculation.

Table 11. Effects of yearly rebalancing on turnover and performance (5-year).

The table shows the effects of reduced rebalancing intervals on portfolio turnover and performance in both low-risk and high-risk portfolios with 5-year risk estimation period. The panel A represents the 1-month and yearly turnover for portfolios, when the monthly rebalancing of original study set up is applied. The panel B represents the turnover and how performance is affected, when yearly rebalancing is applied. The performance differential values are interpreted as percentage point differential to original results, negative value representing decrease in performance and vice versa.

5-year risk estimation	**Panel A**				**Panel B**			
Rebalancing interval	monthly				yearly			
Portfolio	Low-volatility	Low beta	High volatility	High beta	Low-volatility	Low beta	High volatility	High beta
1-month turnover	14 %	18 %	13 %	14 %	4 %	5 %	5 %	5 %
Yearly turnover	170 %	218 %	162 %	163 %	53 %	64 %	65 %	63 %
Performance differential					2 %	-20 %	13 %	15 %
Performance differential p.a					0,1 %	-0,8 %	0,5 %	0,6 %

The results of comparing monthly and yearly rebalancing periods within the 1-year risk estimation framework, which yielded the strongest results, can be seen in panel A of table 12. The yearly turnover varies from 341% to 632%. With similar inputs as previously, the potential performance penalty would be 3,8% at its lowest and potentially 12,6% at its highest. As can be inferred, replicating this level of turnover would be unrealistic in the real financial markets. With the introduction of one year rebalancing, the level remains relatively high at 120% to 147%. It is also noteworthy that the effect weakening trend is even more magnified in the low-risk portfolios with a shorter risk estimation period, which makes intuitive sense. With yearly rebalancing the 1-year risk estimation set up counterintuitively changes to test how short-term risk estimation predicts long-term performance. The observation of weakening effect is critical for investment thesis built around the low-risk anomaly. It shows that with reasonable portfolio turnover both building blocks of the anomaly, namely low-risk overperformance and high-risk underperformance, tend to be affected negatively. This is especially relevant for long-short strategies as the performance gap between the long and short component diminishes, ergo the performance of such strategy as a whole is set to diminish. When the costs of shorting smaller stocks are additionally taken into account the findings of this thesis suggest similarly to (Novy-Marx & Velikov, 2022), that a long-short strategy designed to take advantage of low-risk anomaly is unrealistic to be efficiently implemented in the real financial markets.

Table 12. Effects of yearly rebalancing on turnover and performance (1-year).

The table shows the effects of reduced rebalancing intervals on portfolio turnover and performance in both low-risk and high-risk portfolios with 1-year risk estimation period. The panel A represents the 1-month and yearly turnover for portfolios, when the monthly rebalancing of original study set up is applied. The panel B represents the turnover and how performance is affected, when yearly rebalancing is applied. The performance differential values are interpreted as percentage point differential to original results, negative value representing decrease in performance and vice versa.

1-year risk estimation	Panel A				Panel B			
Rebalancing interval	monthly				yearly			
Portfolio	Low-volatility	Low beta	High volatility	High beta	Low-volatility	Low beta	High volatility	High beta
1-month turnover	34 %	53 %	28 %	34 %	10 %	12 %	11 %	12 %
Yearly turnover	409 %	632 %	341 %	410 %	120 %	147 %	131 %	142 %
Performance differential					-64 %	-38 %	5 %	-6 %
Performance differential p.a					-2,6 %	-1,5 %	0,2 %	-0,2 %

The second one of the cost management methods suggested by Novy-Marx and Velikov (2019) is limiting the investments to cheap-to-trade stocks. The method was indirectly tested in the previous chapter. Generally, limiting the investment universe to large caps, or in other words cheapest to trade stocks in the Finnish stock market, is unrealistic since there are not enough large caps to construct effectively factor exposed yet large enough portfolio. Therefore, limiting the investment universe to cheap-to-trade stocks can be ruled out as a relevant cost mitigation technique in the Finnish stock market. More robust testing of the anomaly in cheap to trade stocks could potentially be concluded by expanding the investment universe to the rest of the Nordic stock markets.

The final method suggested by Novy-Marx and Velikov (2019) is trade banding. Although this thesis does not directly test the banding method, its effects can in general be theorised. In combination with decreasing the rebalancing periods, the method is expected to further decrease transaction costs to that closer of an ETF fund for example. However, there is no consistent order between the quintile returns in a sense that lowest risk consistently yields better results than the next quintile or vice versa. Therefore, it cannot be further deduced that most of the potential returns would be generated by for example 3 stocks with the lowest risk in the low-risk portfolio. In other words, the stocks with strongest propositions from theoretical point of view do not necessarily yield the strongest results. Therefore, the return related effects of trade banding on low-risk investment strategy cannot be estimated based on theory. In a case in which there is a clear pattern that strongest signal typically yields the strongest results, trade banding could be especially good turnover management method.

Overall, this thesis finds that cost mitigation methods generally alter the results of low-risk investment strategy in a negative way. The results are especially critical for potential long-short strategies. Based on these observations, hypothesis three is accepted although not in the form originally proposed in introduction. The performance is generally decreased by the introduction of transaction costs in two ways. In order to achieve the strongest factor exposure with one-year risk estimation, an investor would have to pay unreasonably high transaction costs. On the other hand, if investor were to limit turnover and by extension factor exposure, the performance would weaken considerably.

6 Conclusions and suggestions for further research

Positive linear relationship between risk and return is one of the more fundamental concepts of traditional finance theory, yet this thesis joins the extensive literature that documents that the reality of the relationship diverges from what is proposed in theory. This thesis documents how risk-return relationship manifests in the Finnish stock market and in addition, it explores the validity of an investment strategy constructed around the anomaly. Distinguishing from other literature on the subject, the effect is studied with multiple methodological approaches and transaction cost related considerations are made in the final evaluation.

This thesis finds that the low-risk anomaly is present in the Finnish stock market. High-risk stocks consistently generate statistically significant negative alpha and low-risk stocks consistently and significantly outperform their high-risk counterparts. However, the results also reveal that the strength of the anomaly is affected considerably by the length of the risk estimation period, which raises concerns over the robustness of the anomaly.

Past theory has raised many potential reasons for the low-risk anomaly, the findings of this thesis suggest that the reasons could be different for each component of the anomaly, namely low-risk overperformance and high-risk underperformance. For high-risk underperformance, the findings of this thesis in the different size sections of the Finnish stock market suggest that the market microstructure related explanations for the anomaly could be more compelling than the other proposed explanations. When the sample is limited to generally illiquid small and midcaps, the negative results for high-risk stocks are amplified. On the contrary, when the sample is limited to large caps, the risk-return relationship turns positive. The combination of these two observations gives reason to believe that there exists a link between characteristics of smaller stocks and high-risk underperformance. The absence of low-risk overperformance raises the question whether the component could be driven by some factor that is not prominent in the Finnish stock market.

Opposed to the findings of the past research, this thesis finds that low-risk stocks do not outperform the market index on a risk adjusted basis. It is however noteworthy, that the benchmark of the study considerably outperforms all the quintile portfolios of the study. This implies that a few larger companies have an outsized effect on the performance of the whole market, while the average stock in the sample has a very different risk-return profile than the benchmark. One of the aims of this thesis has been to evaluate the feasibility of an investment strategy constructed around the low-risk anomaly and although the rejection of the second hypothesis could be seen as making this evaluation obsolete, the findings are relevant for different investing strategies and for investing in the Finnish stock market as a whole. This thesis finds that low-risk strategies, both long only and long-short, encounter a multitude of potential problems. Primarily, this thesis finds that factor exposure optimized portfolios experience level of turnover that is unreasonable to try and replicate in the real financial markets, this is despite the original study set up being reasonably cost considerate. Secondly, this thesis finds that when turnover is reduced, performance is affected in two critical ways. Cost management techniques tend to hinder both the overperformance of low-risk stocks and the underperformance of high-risk stocks.

Overall, this thesis offers a few potentially usable guidelines for an investor in the Finnish stock market. Primarily, investors are generally better off when avoiding investing in high risk-stocks, especially in the smaller segments of the market. Secondly, investors should be aware that the return-risk profile of an average stock listed in the Finnish stock market is considerably different from the profile of the benchmark index. The findings raise a few potentially interesting directions for future research. Since the results of this thesis suggest that market microstructure related explanations for the anomaly are persuasive but none are directly tested, a further investigation into them would serve as a natural follow up on this thesis. Similarly, a study focusing on the potential causes for the absence of the low-risk overperformance in the Finnish stock market would also enrich the understanding on the subject.

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