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On co-dependent power-law behavior across cryptocurrencies

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Using daily returns on large-cap altcoins, this paper uses power-law functions to model cryptocurrency-specific exposure to events exhibiting potentially large standard deviations. Since our analysis provides evidence for power-law behavior in the returns on cryptocurrencies, co-fractality analysis is employed to explore potential co-dependencies in the heavy-tailed part of return distributions. The findings indicate that the potential arrival of events exhibiting large standard deviations in Bitcoin returns can hardly be diversified using other sample altcoins. Other altcoins exhibit very similar features in terms of co-dependencies. Further results show that co-fractal behavior is not specific to any subsample.

1. Introduction

In contrast to traditional asset markets, cryptocurrencies carry different risks. Especially altcoins—which are typically understood as 'Bitcoin-like' cryptocurrencies—involve the risk of hacking attacks (e.g., Rauchs and Hileman, 2017; Grobys et al., 2022) or illiquidity risk (Grobys and Sapkota, 2020). Furthermore, Baur and Dimpfl (2021) highlight that altcoins are about 10 times more volatile than traditional currencies—a feature that rules out using altcoins as an alternative means of payment. Another manifestation of extraordinary riskiness is the presence of reoccurring extreme events—a feature manifested in excess kurtosis indicating the presence of heavy tails. Mandelbrot (1963) was the first to model heavy tails evidenced in cotton price changes via power laws. Specifically, cotton price changes are that 'wild' that the variance is statistically undefined (Mandelbrot, 1963 & 2008). If variances are undefined, correlation-based methods cannot be applied to measure co-dependencies between returns. In a recent study, Grobys (2023a) proposes the concept of co-fractality measuring co-dependencies of power-law behavior in the presence of infinite variances. Surprisingly, there is no study available exploring whether reoccurring extreme events in the market for altcoins are diversifiable—despite of the well-documented 'wildness' of altcoins and the recent emerge of crypto-based investment funds. This study attempts to fill this important gap in the literature. In doing so, this study (a) assesses the tail exponents of a sample of large cap cryptocurrencies, and (b) investigates to which extent power-law behavior coincides across cryptocurrencies.

This study considers a sample of the top-10 altcoins exhibiting the highest market capitalizations as of January 1, 2016, and tracks

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which of those remained in the top-20 towards the end of the sample. This procedure leaves us with five large-cap altcoins comprising 80.42 percent of the overall relevant market capitalization as of October 3, 2023. Using daily data, we estimate power-law functions for the return data and identify the fraction of observations exhibiting power-law behavior. Then we compute the matrix of coinciding observations and calculate the co-fractality matrix in terms of its weak form—which appears to be practically more relevant than co-fractality in its strong form. Finally, we split the sample into two non-overlapping subsamples of equal length and investigate whether the results are subject to sample-specificity.

This study contributes to the existing literature in some important aspects. First, this study takes a fractal view on altcoins. Whereas the vast majority of studies focuses on exploring multifractal behavior of cryptocurrencies by using multifractal detrended fluctuation analysis (i.e., Stosic et al., 2019; Gunay and Kaşkaloğlu, 2019; Cheng et al., 2019), this study follows Grobys (2023b) by using power laws to model potential exposures to extreme events. As pointed out in Taleb (2020), power-law exponents capture via extrapolation low probability events—that is, extreme events—not seen in the empirical data. Thereby, this study answers the important question: Are the variances of large-cap altcoins statistically defined? This is an important question to clarify because Fama (1963, p. 421) emphasized that if the theoretically variance is undefined, correlation-based methods will “give very misleading answers.”

Next, this study adds to literature on investigating risk co-dependencies of cryptocurrencies. Surprisingly, the vast majority of studies employs GARCH-type models (Kyriazis, 2021), although correlation-based methods cannot be used in the presence of undefined variances (Fama, 1963). Another relevant study in this research stream is the one of Beneki et al. (2019) that uses a multivariate BEKK-GARCH model in association with impulse response analysis to analyze whether volatility spillovers and hedging abilities exist between Bitcoin and Ethereum. Also, Katsiampa et al. (2019) use multivariate BEKK models to investigate the conditional volatility dynamics along with interlinkages and conditional correlations between three pairs of cryptocurrencies—Bitcoin-Ether, Bitcoin-Litecoin, and Ether-Litecoin. While these studies explore risk co-dependencies emerging from volatility transmission, the present study is the first to examine return co-dependencies in power-law behavior using co-fractality as relevant tool. The advantage of using co-fractality analysis is that it is defined even in the presence of infinite variances (Grobys, 2023a). Finally, this study adds to the wide strand of literature on power laws in financial economics. An extensive review on some relevant literature is provided in Lux and Alfarano (2016).

The study is organized as follows: The next section describes the data, the third section presents the methodology, the fourth section discusses the results, and the last section concludes.

2. Data

In line with Liu et al. (2020) we download daily price data on the following top-10 altcoins as of January 1, 2016 from coinmarketcap.com: Bitcoin (BTC), Ripple (XRP), Litecoin (LTC), Ethereum (ETH), Dogecoin (DOGE), Peercoin (PPC), BitShares (BTS), Stellar Lumen (XLM), Nxt (NXT), and MaidSafeCoin (MAID).[2] This homogenous sample of altcoins has also been used in earlier studies (Grobys et al., 2020). Our sample is from January 1, 2016 to October 3, 2023. The market capitalizations as of January 1, 2016 and October 3, 2023 are reported in Table A.1 in the appendix. We observe that only BTC, XRP, LTC, ETH, and DOGE remained in the top-20 towards the end of the sample. Since these five altcoins alone comprise 80.42 percent of the overall market capitalization for altcoins as of October 3, 2023, we regard our sample of coins as representative.[3] Descriptive statistics are reported in Table 1. Note from Table A.1 that, for instance, the market capitalizations of NXT and MAID are close to zero towards the end of the sample—suggesting that these coins are illiquid, and hence, lack representativeness. Furthermore, these sample restrictions concerning selected coins are necessary to reliably estimate statistical models.

3. Methodology

3.1. Power laws

Since we observe from Table 1 that all altcoins in the sample exhibit heavy tails, as manifested in kurtosis values > 3 and reoccurring extreme events are stylized facts of cryptocurrencies, we model heavy tails for altcoin returns using the following power-law function:

$$p(x) = Cx^{-\alpha}, \tag{1}$$

where $C = (\alpha - 1)x_{MIN}^{\alpha-1}$ with $\alpha \in \{\mathbb{R}_+ | \alpha > 1\}$, x denotes the respective absolute amount of some altcoin return, provided that $x \in \{\mathbb{R}_+ | x_{MIN} \leq x < \infty\}$, x_{MIN} is the cutoff, and α is the magnitude of the corresponding tail exponent.[4] Furthermore, it can be shown that the conditional expectation is defined as:

[2] Following, Grobys et al. (2020) we excluded Dash from the sample to keep our sample homogenous and therefore we only account for non-privacy cryptocurrencies.

[3] The market capitalization of the overall market for cryptocurrencies was USD 1.09 trillion as of October 3, 2023. However, the relevant market capitalization—that is the market for altcoins—is 966.25 billion.

[4] Following Clauset et al. (2009), to simplify notation, index *i*, which denotes the respective individual altcoin return in terms of its absolute amount is neglected.

Table 1
Descriptive statistics.

Cryptocurrency	BTC	XRP	LTC	ETH	DOGE
Mean	0.22	0.41	0.25	0.41	0.51
Median	0.14	−0.03	0.01	0.06	−0.07
Maximum	25.56	179.55	66.77	34.33	335.76
Minimum	−39.18	−47.95	−36.18	−45.02	−40.01
Std. Dev.	3.80	7.71	5.51	5.48	9.56
Skewness	−0.16	6.40	1.42	0.42	16.67
Kurtosis	10.79	121.79	19.22	9.76	547.12
Jarque-Bera	7169.92	1,685,596.00	32,027.50	5475.83	35,091,741.00
Probability	0.00	0.00	0.00	0.00	0.00
Observations	2834	2834	2834	2834	2834
Share top-20 %/total	0.56	0.60	0.56	0.56	0.63

This table reports the descriptive statistics of the following cryptocurrency returns: Bitcoin (BTC), Ripple (XRP), Litecoin (LTC), Ethereum (ETH), and Dogecoin (DOGE). Publicly available daily price data on BTC, XRP, LTC, ETH, and DOGE were downloaded from coinmarketcap.com. The sample period is from January, 1, 2016 to October, 4, 2023.

$$E[x|x > x_{MIN}] = \int_{x_{MIN}}^{\infty} xd\ p(x)x = \frac{(\alpha - 1)}{(\alpha - 2)} x_{MIN}, \tag{2}$$

whereas higher moments of order k are defined as:

$$E\left[X^k | x > x_{MIN}\right] = \frac{(\alpha - 1)}{(\alpha - 1 - k)} x_{MIN}^k. \tag{3}$$

From Eqs. (2) and (3), it becomes evident that the (conditional) theoretical mean only exists for $\alpha > 2$, whereas the (conditional) theoretical variance only exists for $\alpha > 3$.

3.2. Maximum likelihood estimation

Following Clauset et al. (2009), power-law exponents are estimated as:

$$\widehat{\alpha} = 1 + N\left(\sum_{i=1}^{N} \ln\left(\frac{x_i}{x_{MIN}}\right)\right)^{-1}, \tag{4}$$

where $\widehat{\alpha}$ denotes the MLE estimator, N is the number of observations exceeding x_{MIN}, and other notations are as previously defined.[5] Furthermore, Clauset et al. (2009) show that the corresponding standard deviation of the estimated power-law exponent is given by:

$$\widehat{\sigma} = \frac{\widehat{\alpha} - 1}{\sqrt{N}} + O\left(\frac{1}{N}\right). \tag{5}$$

In line with Clauset et al. (2009), x_{MIN} is selected with respect to the optimized Kolmogorov-Smirnov (KS) distance D, which measures the maximum distance between the cumulative density functions (CDFs) of the data and the fitted power-law model as defined by:

$$D = MAX_{x \geq x_{MIN}} |S(x) - P(x)|, \tag{6}$$

where $S(x)$ is the CDF of the data for the observation with a value of at least x_{MIN}, and $P(x)$ denotes the CDF for the power-law model that best fits the data in the region $x \geq x_{MIN}$. The estimate for x_{MIN} $(\widehat{x}_{MIN})$ is then the value of x_{MIN} that minimizes D. Note that the power-law model implies the following functional form for the data-generating return processes:

$$x_t = \begin{cases} s_{1,t} x_{1,t} \\ s_{2,t} x_{2,t} \end{cases}, \text{ with } \boldsymbol{P} = \begin{pmatrix} s_{1,t} | s_{1,t-1} & s_{1,t} | s_{2,t-1} \\ s_{2,y} | s_{1,t-1} & s_{2,t} | s_{2,t-1} \end{pmatrix},$$

with $s_{1,t}, s_{2,t} \in (0, 1)$, where s_1 denotes a state where returns are governed by some thin-tailed process (e.g., Gaussian-type), s_1 denotes a power law regime, $x_{1,t} \sim N(\mu, \sigma^2)$, $x_{2,t} \sim PL(\alpha, x_{MIN})$, and $\boldsymbol{P}$ is the transition probability matrix.

[5] Note that Clauset et al. (2009, p. 693, proof 3.2) show that the exponent is normally distributed.

3.3. Goodness-of-fit tests

To test the power law model, we employ the goodness-of-fit (GoF) test, as proposed in Clauset et al. (2009) that generates a p-value that quantifies the plausibility of the power-law null model by comparing D from the Eq. (6) with distance measurements for comparable synthetic data sets drawn from the hypothesized model. The p-value is calculated as the fraction of synthetic distances that exceed D. The power-law null model is not rejected for p-values > 5 percent.[6]

3.4. Co-fractality analysis

Following Grobys (2023a), the co-fractality coefficient λ is defined in its weak form as :[7]

$$\lambda = \frac{\boldsymbol{x}_i^{*'}\boldsymbol{x}_j^*}{MIN\left(\boldsymbol{x}_i^{*'}1, \boldsymbol{x}_j^{*'}1\right)}, \tag{7}$$

where 1 is a $Tx1$ vector of ones, $\boldsymbol{x}_i^*$ and $\boldsymbol{x}_j^*$ are $Tx1$ vectors having values of 1 if $x_{\mathrm{i,t}} \geq x_{\mathrm{i,MIN}}$ or $x_{\mathrm{j,t}} \geq x_{\mathrm{j,MIN}}$, and values of 0 if $x_{\mathrm{i,t}} < x_{\mathrm{i,MIN}}$ or $x_{\mathrm{j,t}} < x_{\mathrm{j,MIN}}$ and $i, j \in \{BTC, XRP, LTC, ETH, DOGE\}$. Note that Grobys (2023a) highlights that (weak) co-fractality requires that $\lambda > 0.5$ and for testing statistical significance the following test statistic is required:

$$t = \frac{\sqrt{n}(\widehat{p} - p)}{\sqrt{\widehat{p}(1 - \widehat{p})}}, \tag{8}$$

where $p = 0.5$, $n = MIN(\boldsymbol{x}_i^{*'}1, \boldsymbol{x}_j^{*'}1)$, and $\widehat{p}$ is the fraction of coinciding observations in power-law regimes.[8]

4. Results

Our results reported in Table 2 show that estimated power-law exponents vary across our sample of altcoins between $\widehat{\alpha} = 2.77$ and $\widehat{\alpha} = 5.91$. Because $\widehat{\alpha} < 3$ for XRP and DOGE, the theoretical variance for these cryptocurrencies is statistical undefined.[9] Furthermore, between 1.44 (BTC) and 52.36 (XRP) percent of the return distributions are governed by a power-law, suggesting that the fractions of the cryptocurrency-specific distributions allowing for the generation of extreme events varies between altcoins by a substantial margin. Since XRP and DOGE exhibit undefined—respectively, infinite—theoretical variances, we cannot rely on using correlation-based methods to model co-dependencies between the returns. Whereas BTC is least exposed to extreme events—as implied by $\widehat{\alpha} = 5.91$, which is considerably larger than $\widehat{\alpha}$ for any other sample altcoin—DOGE appears to be most exposed to extreme events. Interestingly, Grobys (2023b) used weekly data on BTC covering a sample from June 2018 to May 2023 period and found that $\widehat{\alpha} = 4.09$. Implementing a two-sample z-test shows that the point estimates $\widehat{\alpha} = 5.91$ and $\widehat{\alpha} = 4.09$ are statistically not different from each other. This is in line with Mandelbrot (2008) highlighting that fractality implies that power-law behavior does not change across time frequencies.[10] Specifically, the power-law behavior of BTC is the same on daily frequency as it is on a weekly frequency.

Furthermore, the results from GoF tests (see Table 2) imply that the power law model accurately models the returns on most altcoins. Next, we transform the GoF p-values into implied values of drawings from the $\chi^2(1)$ distribution enabling us to test for market-wide power-law behavior. The sum of implied drawings follows under the market-wide power-law null model a $\chi^2(5)$ distribution. From Table 2 we observe that $\widehat{\lambda} = 9.21$ (p-value 0.10) implying that we cannot reject the null hypothesis using a significance level of 5 percent.

Can we diversify the arrival of extreme events? Table 3 reports the co-fractality matrix for our set of cryptocurrencies. We observe that $\widehat{\lambda}$ varies between $\widehat{\lambda} = 0.91$ and $\widehat{\lambda} = 0.94$ for BTC, implying that substantial fractions of BTC returns that are subject to power-law behavior coincide with other cryptocurrencies' observations governed by some power law.[11] Since potential arrivals of extreme events are generated in power-law regimes, the evidence here suggests that extreme events in BTC returns can hardly be diversified by adding other sample altcoins.

Next, co-fractality analysis for XRP provides very similar evidence, as $\widehat{\lambda}$ varies between $\widehat{\lambda} = 0.63$ and $\widehat{\lambda} = 0.91$. Overall, the evi-

[6] The GoF test is detailed in Clauset et al. (2009, p. 675—678).

[7] Note that co-fractality in its strong form is perhaps less relevant because the fraction of power-law observations considerably varies in empirical data.

[8] The differences between risk diversification in mean-variance space and co-fractality analysis are detailed in Grobys (2023a).

[9] Note that Taleb (2020) highlights that $\widehat{\alpha} < 3$ and $\widehat{\alpha} < 4$ have qualitatively the same implications: Even if $\widehat{\alpha} \approx 3$, the law of large number works too slow and $N = 10^6$ is required to have the same convergence as for $N = 30$ in the Gaussian case.

[10] The z-test statistic is defined as: $z = (\widehat{\alpha}_{DAILY} - \widehat{\alpha}_{WEEKLY})/(\sqrt{n_1\widehat{\sigma}^2_{DAILY} + n_2\widehat{\sigma}^2_{WEEKLY}})$. Because Grobys (2023b) documents that $\widehat{\sigma}_{WEEKLY} = 1.8760$, whereas $N_1 = 17$ weekly observations satisfy $x > x_{MIN}$, and our figures are $\widehat{\sigma}_{DAILY} = 0.85$ (see Table 2), whereas $N_2 = 33$ daily observations satisfy $x > x_{MIN}$, $z = (5.91 - 4.09)/(\sqrt{(0.34 \cdot 1.876^2 + 0.66 \cdot 0.85^2)})$. Hence, $z = 1.41 < 1.65$, implying that the estimate derived from daily data is statistically not larger than the estimate derived from weekly data.

[11] Future studies could employ rolling windows estimates for the co-fractality statistic and develop statistical tests in line with Cai and Juhl (2023).

Table 2
Estimated power-law exponents for daily cryptocurrency data.

Estimated power-law exponents for cryptocurrencies					
	BTC	XRP	LTC	ETH	DOGE
$\hat{\alpha}$	5.91	2.83	3.90	4.72	2.77
$\hat{\sigma}$	0.85	0.05	0.10	0.14	0.05
$\hat{x}_{min}$	12.67	6.15	11.10	14.59	6.63
N (absolut)	33	1196	842	710	1176
N (percent)	1.44 %	52.36 %	36.87 %	31.09 %	51.49 %
p-value of GoF test	0.97	0.04	0.73	0.67	0.03
$\chi^2(1)$ (*implied*)	0.00	4.22	0.16	0.12	4.71

Absolute amounts of cryptocurrency returns are modeled using the following power-law function:
$p(x) = Cx^{-\alpha}$,
where $C = (\alpha - 1)x_{MIN}^{\alpha-1}$ with $\alpha \in \{\mathbb{R}_+ | \alpha > 1\}$, x denotes the respective absolute amount of cryptocurrency returns provided $x \in \{\mathbb{R}_+ | x_{MIN} \leq x < \infty\}$, x_{MIN} is the minimum value governed by the power-law process, and α is the magnitude of the corresponding power-law exponent. The tail exponents are estimated as: $\hat{\alpha} = 1 + N\left(\sum_{i=1}^{N} \ln\left(\frac{x_i}{x_{MIN}}\right)\right)^{-1}$, where $\hat{\alpha}$ denotes the MLE estimator, N is the number of observations exceeding x_{MIN}, and other notations are as previously defined. The estimate $\hat{\alpha}$ is selected based on the optimal Kolmogorov–Smirnov (KS) distance D measuring the maximum distance between the cumulative density functions (CDFs) of the data and the fitted power-law model as defined by:
$D = MAX_{x \geq x_{MIN}} |S(x) - P(x)|$,
where $S(x)$ is the CDF of the data for the observation with a value of at least x_{MIN}, and $P(x)$ is the CDF for the power-law model that best fits the data for $x \geq x_{MIN}$. The estimate $\hat{x}_{MIN}$ is the corresponding value of x_{MIN} minimizing D. This table reports the estimates $\hat{\alpha}$, $\hat{x}_{min}$, $\hat{\sigma}$, and N in absolute and relative terms. The sample period is from January, 1, 2016 to October, 4, 2023.

Table 3
Co-fractality matrix for cryptocurrencies.

Panel A. Total observations governed by a power law.					
Cryptocurrency	**BTC**	**XRP**	**LTC**	**ETH**	**DOGE**
Observations	33	1196	842	710	1176
Panel B. Matrix of coinciding observations.					
Cryptocurrency	**BTC**	**XRP**	**LTC**	**ETH**	**DOGE**
BTC	33	30	31	30	30
XRP		1196	623	527	746
LTC			842	428	601
ETH				710	489
DOGE					1176
Panel C. Co-fractality matrix.					
Cryptocurrency	**BTC**	**XRP**	**LTC**	**ETH**	**DOGE**
BTC		0.91*** (6.10)	0.94*** (8.34)	0.91*** (6.10)	0.91*** (6.10)
XRP			0.74*** (9.49)	0.74*** (8.73)	0.63*** (4.26)
LTC				0.60*** (2.83)	0.71*** (7.04)
ETH					0.69*** (6.21)

This table reports in Panel A the total number of cryptocurrency absolute return observations governed by power laws for the following cryptocurrencies: Bitcoin (BTC), Ripple (XRP), Litecoin (LTC), Ethereum (ETH), and Dogecoin (DOGE). Panel B reports the matrix of coinciding observations, whereas Panel C reports the co-fractality matrix. Publicly available daily price data on BTC, XRP, LTC, ETH, and DOGE were downloaded from coinmarketcap.com. The sample period is from January, 1, 2016 to October, 4, 2023.
*** Statistically significant on a 1 % level.

dence suggests a high level of co-fractality between the returns on large-cap altcoins and ascertained co-fractality is statistically significant, as indicated by t-statistics > 2.80. The results are in line with Grobys (2023a) who documented the presence of co-fractality across the realized variances of foreign exchange (FX) rates. Unlike realized FX rate variances, co-fractality is present to a higher degree in the cryptocurrency market.

Are the results sample-specific? To explore this issue, we split the sample into two non-overlapping subsamples of equal length. Table 4 (5) reports the co-fractality matrix for the first (second) subsample from January 1, 2016 to November 17, 2019 (November 18, 2019 to October 4, 2023). Strikingly, the results are qualitatively the same regardless the sample. If anything, co-fractality is even more pronounced in the second subsample implying that risk diversification benefits are virtually non-existent in the market for large-cap altcoins.

Table 4
Co-fractality matrix for cryptocurrencies in the earlier subsample.

Panel A. Total observations governed by a power law.					
Cryptocurrency	**BTC**	**XRP**	**LTC**	**ETH**	**DOGE**
Observations	22	625	424	415	592

Panel B. Matrix of coinciding observations.					
Cryptocurrency	**BTC**	**XRP**	**LTC**	**ETH**	**DOGE**
BTC	22	19	20	19	20
XRP		625	305	294	374
LTC			424	219	281
ETH				415	257
DOGE					592

Panel C. Co-fractality matrix.					
Cryptocurrency	**BTC**	**XRP**	**LTC**	**ETH**	**DOGE**
BTC		0.86*** (3.36)	0.91*** (22.39)	0.86*** (3.36)	0.91*** (22.39)
XRP			0.72*** (5.91)	0.71*** (5.46)	0.63*** (3.51)
LTC				0.53 (0.59)	0.66*** (3.23)
ETH					0.62*** (2.67)

This table reports in Panel A the total number of cryptocurrency absolute return observations governed by power laws for the following cryptocurrencies: Bitcoin (BTC), Ripple (XRP), Litecoin (LTC), Ethereum (ETH), and Dogecoin (DOGE). Panel B reports the matrix of coinciding observations, whereas Panel C reports the co-fractality matrix. Publicly available daily price data on BTC, XRP, LTC, ETH, and DOGE were downloaded from coinmarketcap.com. The sample period is from January, 1, 2016 to November, 17, 2019.
*** Statistically significant on a 1 % level.

Table 5
Co-fractality matrix for cryptocurrencies in the later subsample.

Panel A. Total observations governed by a power law.					
Cryptocurrency	**BTC**	**XRP**	**LTC**	**ETH**	**DOGE**
Observations	11	571	418	295	584

Panel B. Matrix of coinciding observations.					
Cryptocurrency	**BTC**	**XRP**	**LTC**	**ETH**	**DOGE**
BTC	11	11	11	11	10
XRP		571	318	233	372
LTC			418	209	320
ETH				295	232
DOGE					584

Panel C. Co-fractality matrix.					
Cryptocurrency	**BTC**	**XRP**	**LTC**	**ETH**	**DOGE**
BTC		1.00 (∞)	1.00 (∞)	1.00 (∞)	0.91*** (3.52)
XRP			0.76*** (7.61)	0.79*** (7.72)	0.65*** (3.47)
LTC				0.71*** (4.60)	0.77*** (6.88)
ETH					0.79*** (7.71)

This table reports in Panel A the total number of cryptocurrency absolute return observations governed by power laws for the following cryptocurrencies: Bitcoin (BTC), Ripple (XRP), Litecoin (LTC), Ethereum (ETH), and Dogecoin (DOGE). Panel B reports the matrix of coinciding observations, whereas Panel C reports the co-fractality matrix. Publicly available daily price data on BTC, XRP, LTC, ETH, and DOGE were downloaded from coinmarketcap.com. The sample period is from November, 18, 2019 to October, 4, 2023.
*** Statistically significant on a 1 % level.

5. Concluding remarks

The market for cryptocurrencies comprises different risks than other traditional asset markets. Whereas the presence of reoccurring extreme events is a well-established feature for this market, the present study answered the question as to whether the arrival of extreme events can be diversified using a sample of selected large-cap altcoins. Since the market for cryptocurrencies is highly concentrated, we argued that our sample is representative because our five selected altcoins alone comprise 80.42 (99.43) percent of the overall market capitalization for altcoins at the end (beginning) of the sample. Our findings demonstrated that some altcoins are so 'wild' that the theoretical variance is infinite. This is an important issue because correlation-based methods should not be used for evaluating links between return processes in the presence of infinite variances. Using co-fractality in its weak form as a tool for identifying co-dependencies between potential arrivals of extreme events, the present study found that power-law behavior tends to coincide—making diversification of extreme risks across altcoins difficult. Furthermore, our findings suggest the existence of some altcoin-specific risk component which could be diversifiable using other assets. Future research is encouraged to explore whether other assets such as stocks, commodities, or traditional currencies can remedy the risk diversification problem we documented for the market for altcoins. Moreover, future research could also investigate to which extent the thin-tailed part of cryptocurrency returns is diversifiable.

CRediT authorship contribution statement

Klaus Grobys: Writing – review & editing, Writing – original draft, Visualization, Validation, Supervision, Software, Resources, Project administration, Methodology, Investigation, Formal analysis, Data curation, Conceptualization.

Declaration of Competing Interest

The author declares that he has no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data are available online for free.

Appendix

Table A.1

Table A.1
Market capitalization.

Cryptocurrency	Rank as of Jan 1, 2016	Rank as of Oct 3, 2023	Market cap as of Jan 1, 2016 (in Million $)	Market cap as of Oct 3, 2023 (in Million $)
BTC	1	1	6529.30	538,846.47
XRP	2	5	199.72	27,973.45
LTC	3	15	153.91	4761.36
ETH	4	2	71.98	196,871.99
DOGE	5	9	15.82	8618.69
PPC	6	784	9.51	8.45
BTS	7	509	8.78	27.58
XLM	8	23	8.46	3090.92
NXT	9	4435	6.68	0.00
MAID	10	5059	6.51	0.00
*As of October 3, 2023.				Total: 780,198.09

Publicly available daily price data on Bitcoin (BTC), Ripple (XRP), Litecoin (LTC), Ethereum (ETH), Dogecoin (DOGE), Peercoin (PPC), BitShares (BTS), Stellar (XLM), Nxt (NXT), and MaidSafeCoin (MAID) were downloaded from coinmarketcap.com. The sample period is from January 1, 2016 to October 3, 2023. This table reports the rank of each cryptocurrency measured in terms of market capitalization and denoted in USD as of January 1, 2016 compared to October 3, 2023.

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