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No reward—no effort: Will Bitcoin collapse near to the year 2140?

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ABSTRACT

This paper explores whether the overall evolution of Bitcoin log-prices would manifest a log-period power-law singularity (LPPLS) signature, eventually resulting in the arrival of a finite-time singularity. Calibrating the LPPLS model using daily data on Bitcoin covering the 2011—2023 period, this study indeed finds evidence for a strong LPPLS signature suggesting the arrival of a spontaneous singularity in the year 2129. Further striking evidence suggests that Bitcoin will experience what we term a *close-to-singularity-condition* near to the year 2050—a remarkable coincidence with the recently documented arrival of a finite-time singularity in U.S. equities.

1. Introduction

Reoccurring bubble formations appear to be a stylized fact of cryptocurrencies (Kyriazis et al., 2020). Kyriazis et al. (2020) highlight that the log-period power-law singularity (LPPLS) model is a typical tool to identify such bubble formations. Also, Wheatley et al. (2019) model universal super-exponential unsustainable growth manifested in the price evolution of Bitcoin with LPPLS models which parsimoniously capture diverse positive feedback phenomena, such as herding and imitation. Wheatley et al. (2019) show that the LPPLS model provides an ex-ante warning of market instabilities. In their study, the authors use hourly data on Bitcoin from January 1, 2012 to January 8, 2018 and identify five bubble formations which ended on 18 Aug 2012, 11 Apr 2013, 23 Nov 2013, 18 Dec 2017, and 18 Dec 2017.

Relatedly, Shu and Zhu (2020) propose a novel adaptive multilevel time series detection methodology based on the LPPLS model to detect bubble formations in the cryptocurrency market and demonstrate that their method provides real-time detection of bubbles. Interestingly, Wang et al. (2022) examine price bubbles in the NFT and DeFi markets and conclude that these markets exhibit some intrinsic value and should not be dismissed as simply bubbles. Further, Chaim and Laurini's (2019) study—entitled "Is Bitcoin a bubble?"—uses daily price data between January 2013 and September 2018 and finds mixed results. Specifically, their results suggest the existence of a bubble in Bitcoin prices from early 2013 to mid-2014, but, interestingly, not in late 2017. Is Bitcoin in the process of a long-lasting bubble formation? This is a question which still remains unanswered. The present study attempts to fill this gap in the literature.

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Whereas earlier studies focused on exploring temporary episodes of bubble formations, this study exclusively takes a coarse-grained perspective like Johansen and Sornette (2001), who were the first to fit the LPPLS model to U.S. equity data using the overall available data from 1790 to 1999. The authors found that the dynamics of equities are compatible with a spontaneous singularity occurring around 2052, signaling an abrupt transition into a new regime. Likewise, we consider the whole information set derived from all available data on Bitcoin log-prices to explore whether Bitcoin is in the process of a long-lasting bubble formation. In doing so, we add to the existing literature by answering the following important question: Does Bitcoin serve as a meaningful tool for long-term investments? This is not a trivial question because recent literature documented that Bitcoin rather serves as an investment tool as opposed to serving as a medium of exchange (Baur et al., 2018).

Employing daily data on Bitcoin over the January 1, 2011 to September 17, 2023 period, we calibrate the LPPLS model using the methodological approach recently proposed by Grobys (2023). To test whether the LPPLS signature is statistically significant, we implement the residual test proposed by Lin et al. (2014). As an additional robustness check, we also explore whether log-prices of Bitcoin exhibit explosiveness (Grobys, 2023).

This study has some important contributions. First, it extends the literature on bubble formations in cryptocurrencies (i.e., Kyriazis et al., 2020; Wheatley et al., 2019). Whereas earlier studies focused on specific temporary episodes, this study makes use of the whole information embedded in the log-prices of Bitcoin. Next, another strand of literature is devoted to analyzing the underlying mechanism that could explain the pricing of Bitcoin. For instance, de la Horra et al. (2019) argue that whereas Bitcoin is not demanded as a safe-haven commodity or a medium of exchange, in the short term, the demand for Bitcoin is mainly driven by speculation. We argue that evidence for a finite-time singularity in the distant future could indicate that Bitcoin's price evolution is driven by herding and imitation, in line with the theory of the LPPLS methodology, as detailed in Sornette (2017). Finally, the present study complements the literature on investigating the potential arrival of finite-time singularities in the dynamics of the socio-economic data. Johansen and Sornette (2001) were the first to explore the potential arrival of finite-time singularities in the dynamics of the world population and some important financial indices. In a recent study, Grobys (2023) performs a scientific replication of Johansen and Sornette's (2001) study and finds virtually identical results—that is, the arrival of a spontaneous singularity occurring around the year 2050 signaling an abrupt transition into a new regime. What do the data on Bitcoin tell us? This is the first study that sheds light on this issue.

2. Data

Data on Bitcoin covering the period January 1, 2011 until September 17, 2023 were downloaded from investing.com.[2] The overall sample comprises 4643 daily observations. Descriptive statistics of Bitcoin log-returns are reported in Table A.1 in the appendix.

3. Methodology

3.1. Implementing the LPPLS model using daily Bitcoin log-prices

Following Sornette (2017), a simple power-law model for financial log-prices is given by

$$\ln[p(t)] = A + B(t_c - t)^{\beta}, \tag{1}$$

where ln $[p(t)]$ is the logarithm of a financial asset (e.g., Bitcoin) at time t, t_c is the critical time, A is the expected value of the financial asset as it approaches t_c, B defines the exposure to faster-than-exponential growth, and β is the power-law exponent controlling faster-than-exponential price growth. The critical time t_c indicates a regime change, manifested in a change from super-exponential growth to a lower growth and the end of the accelerating oscillations (Zhang et al., 2016). Following Sornette (2017), this model specification requires the following constraints:

$$A > 0,$$

$$B < 0,$$

$$0.1 \leq \beta \leq 0.9.$$

Furthermore, Sornette (2017) highlights that the simple power-law model of Eq. (1) needs to be extended by accounting for periodic oscillations:

$$\ln[p(t)] = A + B(t_c - t)^{\beta}[1 + C\cos(\omega \ln(t_c - t) + \phi)], \tag{2}$$

where C denotes the exposure the log-periodic oscillations around the power-law singular growth, ω denotes the angular log-frequency of oscillations during the formation of the bubble, ϕ is the phase parameter, and all other notations are as previously defined. Whereas Sornette (2017) documents that ϕ cannot be meaningfully restricted, the present research requires $|C| < 1$, and imposes the constraint $5 \leq \omega \leq 15$ (Grobys, 2023). Implementing the LPPLS model of Eq. (2) requires several model parameters $\mathbf{\Phi} = (A, B, t_c, \beta, C, \phi)$ to be

[2] Since investing.com did not provide any longer data on Bitcoin for the ex-ante January 1, 2012 period, we obtained data for January 1, 2011 until December 31, 2011 from Grobys (2021), who retrieved those data from investing.com when it was still available.

estimated using a highly non-linear model. This study follows a recently proposed approach to calibrate the LPPLS model (Grobys, 2023): First, we set $A = \ln[p(T)]$, where $p(T)$ denotes the logarithm of the financial asset at time t, and optimize four models by using Eq. (1), where only the starting values for β vary; that is, $\beta \in \{0.2, 0.4, 0.6, 0.8\}$. Moreover, we set $B = -1$, and $t_c = T + 1$. Then, we use the non-linear solver provided from Microsoft Excel to find optimal solutions for each model specification. Since we use daily data on Bitcoin and 4643 observations, $t_c = T + 1 = 12.7180$.

We select the model generating the minimum sum of squared residuals (SSR) with corresponding optimal parameter vector $\boldsymbol{\Phi}_1^* = (A^*,\ B^*,\ t_c^*,\ \beta^*)$. The numerical values of $\boldsymbol{\Phi}_1^*$ are then used as the initial values in the second step of the optimization procedure. Specifically, employing $\boldsymbol{\Phi}_1^*$ as initial values for model 2 in Eq. (2) and setting $C = \phi = 0$, model 2 is then optimized for varying values ω; that is, $\omega \in \{5, 6, \ldots, 14, 15\}$. Optimizing model 2, requires the following constraints:

$$t_c \geq T + 1,$$

$$0.1 \leq \beta \leq 0.9,$$

$$5 \leq \omega \leq 15.$$

As initial values for the second-stage optimization, $\boldsymbol{\Phi}_2^* = (\boldsymbol{\Phi}_1^*, C, \omega,\ \phi)$ is used with starting values $\boldsymbol{\Phi}_1^* = (A^*,\ B^*,\ t_c^*,\ \beta^*)$ and $C = \phi = 0$, for each given $\omega \in \{5, 6, \ldots, 14, 15\}$. Again, model 2 in Eq. (2) is optimized using Microsoft Excel's non-linear solver. Thus, we obtain eleven optimized parameter vectors $\boldsymbol{\Phi}_{2,j}^{**} = (A_j^{**},\ B_j^{**},\ t_{c,j}^{**},\ \beta_j^{**}, C_j^{**},\ \omega_j^{**},\ \phi_j^{**})$ with $j = 1, \ldots, 11$. We select the optimal model with respect to the minimum SSR generated.

3.2. Robustness checks

3.2.1. Residuals tests

To address the so-called *spurious regression problem*, we investigate whether the residual process exhibits stationarity (Lin et al., 2014). Hence, the stationarity of the LPPLS model residuals is tested employing the standard augmented Dickey-Fuller (ADF) test (Grobys 2023). Calibrations with the 99 % confidence level of stationarity of the residuals derived from the parametrization $\boldsymbol{\Phi}_{2,j}^{**}$ are considered statistically significant (Jiang et al., 2010). The ADF test is implemented by running the regression:

$$\Delta e_t = \delta_0 + \delta_1 t + \delta_2 u_{t-1} + \gamma_1 \Delta u_{t-1} + \ldots + \gamma_p \Delta u_{t-p} + \epsilon_t, \tag{3}$$

where $e_t = p_t - m_t$ defines the difference between the log-prices of Bitcoin and the calibrated optimal model using the parameter vector

Table 1
Calibrating the simple power-law model for Bitcoin log-prices using daily data from January 2011—September 2023
A simple power-law model for Bitcoin log-prices is given by:
$\ln[p(t)] = A + B(t_c - t)^{\beta}$,
where $\ln[p(t)]$ is the logarithm of Bitcoin at time t, t_c is the critical time, A is the expected value of Bitcoin as it approaches t_c, B defines the exposure to faster-than-exponential growth, and β is the power-law exponent controlling faster-than-exponential price growth. This model specification requires the following constraints:
$A > 0$,
$B < 0$,
$0.1 \leq \beta \leq 0.9$.
Following Grobys (2023), we set $A = \ln[p(T)]$, where $p(T)$ denotes the logarithm of Bitcoin at time t. Then, four models are optimized by using the Equation above. For each model, the starting values for β vary; that is, $\beta \in \{0.2, 0.4, 0.6, 0.8\}$. Moreover, we set $B = -1$, and $t_c = T + 1$. Then we use the non-linear solver provided from MS Excel to find optimal solutions for each model specification. Since we use daily data on Bitcoin and 4643 observations, $t_c = T + 1 = 12.7180$. We select the model generating the minimum sum of squared residuals (SSR) with corresponding optimal parameter vector $\boldsymbol{\Phi}_1^* = (A^*,\ B^*,\ t_c^*,\ \beta^*)$. This table reports the corresponding model estimates. The optimal model is highlighted in **bold** figures.

Panel A. Initial parameter values for model 1.

Model	1	2	3	**4**
A	10.1862	10.1862	10.1862	**10.1862**
B	−1	−1	−1	**−1**
t_c	12.7180	12.7180	12.7180	12.7180
β	0.2	0.4	0.6	**0.8**
SSR	52,141.8505	36,406.2981	19,749.2681	**13,692.9510**

Panel B. Optimized parameter values for model 1.

Model	**1**	2	3	4
A^*	**76.7464**	78.8064	52.7676	127.4716
B^*	**−1.4604**	−1.9965	−1.2762	−2.5204
t_c^*	**86.1646**	83.4791	59.7431	131.2962
β^*	**0.8828**	0.8245	0.9000	0.8010
SSR	**4912.0205**	4937.7198	4924.3482	4915.8808

$\mathbf{\Phi}_{2,j}^{**}$, t denotes a time trend, and ε_t is assumed to be a white noise process. Note that Eq. (3), $\delta_0 = \delta_1 = 0$ defines a model specification with no deterministic terms, whereas $\delta_1 = 0$ defines a model with a constant term only, and the parameters $\gamma_1,\ldots,\gamma_p$ measure exposures to $\Delta u_{t-1},\ldots,\Delta u_{t-p}$. Note that the parametrization $\delta_0 = \delta_1 = 0$ corresponds to a random walk as the null model, whereas the parametrization $\delta_1 = 0$ corresponds to a random walk with a drift as the null model. The optimal lag-order p is selected in line with the Schwarz Criterion (Grobys, 2023).

3.2.2. *Testing for explosiveness in log-prices of Bitcoin*

Grobys (2023) notes that faster-than-exponential growth in the underlying price series of a financial asset should be able to detect using the regression model of Eq. (3) fitted to differenced log-prices of Bitcoin, that is:

$$\Delta p_t = \varphi_0 + \varphi_1 t + \varphi_2 p_{t-1} + \varphi_3 \Delta p_{t-1} + \ldots + \varphi_{p+2} \Delta p_{t-p} + e_t, \tag{4}$$

where p_t denotes the log-prices of Bitcoin, t denotes a time trend and e_t is assumed to be a white noise process. Faster-than-exponential growth should be manifested in $\varphi_2 < 0$ in association with $\varphi_1 > 0$. Again, the optimal lag-order p is selected in line with the Schwarz Criterion.

4. Results

Panel A of Table 1 documents the initial parameter values for each model specification using Eq. (1), whereas Panel B of Table 1 reports the optimized parameters. Interestingly, regardless of the initial parametrization, the optimized β is $\beta^* > 0.80$ for all optimized models which is indeed a strong commonality. Next, $\mathbf{\Phi}_1^*$ is given by $(A^*, B^*, t_c^*, \beta^*) = (76.7464, -1.4604, 86.1646, 0.8828)$ and used in the next stage of the optimization.

We use $\mathbf{\Phi}_1^*$ as starting values for $(A, B, t_c, \beta, C, \phi)$ in association with $C = \phi = 0$ and optimize model 2 (e.g., Eq. (2)) for varying values of ω; that is, $\omega \in \{5, 6, \ldots, 14, 15\}$. To compute optimal values for the parameter vector $\mathbf{\Phi}_2^{**}$, we employ Microsoft Excel's non-linear solver. Panel A of Table 2 reports the initial parameter values used in optimization procedure for model 2, whereas Panel B of Table 2 reports the optimized parameter values. We observe from Panel A of Table 2 that the input parameterization, given by $\mathbf{\Phi}_2^* = (A^*, B^*, t_c^*, \beta^*, C, \omega, \phi) = (76.7464, -1.4604, 86.1646, 0.8828, 0, 11, 0)$, results in the optimized parameterization for model 2 giving the least SSR (e.g.,$SSR = 2615.2660$), where $A^{**} = 126.8880$, $B^{**} = -4.0553$, $t_c^{**} = 118.1294$, $\beta^{**} = 0.7511$, $C^{**} = 0.1498$, $\omega^{**} = 8.3886$, and $\phi^{**} = 7.5415$. Strikingly, the optimal model 2 suggests that the finite-time singularity occurs at $t_c^{**} = 118.1294$, which corresponds in the notation here to 38,501 units of time (e.g., days) in the future. Since our sample ends at time $t = 12.71527$, our model estimates a finite-time singularity to occur in the year 2129.1945—that is, mid-March 2129.

Next, Fig. 1 plots the optimal optimized model 2 denoted as Model* and the second best fit for model 2 (Model**) along with the natural logarithm of Bitcoin prices, ln $[p(t)]$. Moreover, Fig. 2 plots the residuals covering the in-sample period. The residuals, ε_t, are defined as $\varepsilon_t = \ln[p(t)] - \ln[\widehat{p(t)}]$, where $\ln[\widehat{p(t)}]$ is the prediction of model 2 using the optimal parameter vector $\mathbf{\Phi}_2^{**}$. From Fig. 1 we observe that both optimized models suggest that Bitcoin prices reach a close-to-zero-price condition in approximately the year 2045.

Next, Table 3 reports the results from three different ADF tests. Table 3 shows that in models 2 and 3 the parameters modeling the constant, or constant and time trend are statistically not different from zero. Hence, we infer that model 1 is the relevant model specification used for statistical inference. We observe from Table 3 that $\widehat{\lambda}_{ADF} = -2.8629$ and statistically significant on at least a 1 % level (p-value is 0.0041) suggesting a statistically significant LPPLS signature (Lin et al., 2014).

Table 4 reports the results from analyzing the presence of faster-than-exponential growth in the underlying log-price process (Grobys, 2023). Table 4 shows that $\widehat{\varphi}_2$ is statistically significantly negative on a 1 % level, whereas $\widehat{\varphi}_1$ is statistically significantly positive on a 10 % level. Since Grobys (2023) requires that $\widehat{\varphi}_1 > 0$ and $\widehat{\varphi}_2 < 0$, we test the joint hypothesis:

$$H_0: \ \widehat{\varphi}_1 = \widehat{\varphi}_2 = 0 \text{ versus } H_1: \ \widehat{\varphi}_1 > 0 \text{ and } \widehat{\varphi}_2 < 0.$$

The test statistic λ is under the null hypothesis asymptotically distributed as $\chi^2(2)$. We find that $\widehat{\lambda} = 14.5645 > 5.99146 = \chi^2_{0.95}(2)$; hence, we strongly reject the null hypothesis (p-value is 0.0007). Moreover, from Table 4 it becomes evident that the estimated ADF test statistic corresponding to $\widehat{\lambda}_{ADF} = -2.7883$ does not reject the null model as indicated by the p-value corresponding to 0.2016. Thus, the overall evidence suggests here that log-prices of Bitcoin indeed exhibit explosive random walk behavior—a necessary condition for log-periodicity (Grobys, 2023).

5. Discussion and concluding remarks

Our findings indicated a strong LPPLS signature predicting the arrival of a singularity condition by March 2129. Accounting for uncertainty in point estimation, we cannot reject the hypothesis that the arrival of the finite-time singularity coincides with the time when the last Bitcoin will be mined—that is by 2140.[3] It is interesting to note that some scholars noted that when the revenues do not any longer cover the costs, miners might have no further incentives to put effort in maintaining the blockchain—a situation where Bitcoin could be expected to go bust Taleb (2021).

[3] Using the approach in Grobys (2023), the standard deviation of t_c^{**} is estimated at $\widehat{\sigma}_{t_c^{**}} = 11.2321$.

Table 2
Calibrating the log-periodic power-law singularity model for daily Bitcoin data over the 2011—2023 period
In line with Sornette (2017, p. 335), the log-period power-law singularity (LPPLS) model is given by,
$\ln\,[p(t)] = A + B(t_c - t)^{\beta}[1 + C\cos(\omega\ln\,(t_c - t) + \phi)]$,
with $A > 0$, $B < 0$, and $0.1 \leq \beta \leq 0.9$, and A is the expected value of Bitcoin in logarithm, B measures the exposure to faster-than-exponential growth, β is the power-law exponent controlling faster than exponential price growth, C measures the exposure responsible for periodic oscillations, ω is the angular frequency of the log-periodic oscillations during the bubble formation, and ϕ is the phase parameter. We require $|C| < 1$ and impose the constraint $5 \leq \omega \leq 15$. We use the optimal values for $\boldsymbol{\Phi}_1^* = (76.7464,\ 1.4604,\ 86.1646,\ 0.8828)$ from the first estimation step. Using in addition $C = \phi = 0$, the log-period power-law singularity model (model 2) is then optimized for varying values ω, that is, $\omega \in \{5, 6, \ldots, 14, 15\}$. Whereas Panel A of Table 2 reports the corresponding input parameters used to start the optimization procedure, Panel B of Table 2 reports the optimized parameters using the following constraints:
$t_c \geq T + 1$,
$0.1 \leq \beta \leq 0.9$,
$5 \leq \omega \leq 15$.
Note that the parameter ϕ remains unconstrained which is in line with Sornette (2017, p. 336). The parameters are obtained using Microsoft Excel's non-linear solver. The sample period is from January 1, 2011 to September 17, 2023 comprising 4643 daily observations. The optimal model is highlighted in **bold** figures.

Panel A. Initial parameter values for model 2.

Specification	1	2	3	4	5	6	7	8	9	10	11
A^*	76.7464	76.7464	76.7464	76.7464	76.7464	76.7464	76.7464	76.7464	76.7464	76.7464	76.7464
B^*	−1.4604	−1.4604	−1.4604	−1.4604	−1.4604	−1.4604	−1.4604	−1.4604	−1.4604	−1.4604	−1.4604
t_c^*	86.1646	86.1646	86.1646	86.1646	86.1646	86.1646	86.1646	86.1646	86.1646	86.1646	86.1646
β^*	0.8828	0.8828	0.8828	0.8828	0.8828	0.8828	0.8828	0.8828	0.8828	0.8828	0.8828
C	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
ω	5.0000	6.0000	7.0000	8.0000	9.0000	10.0000	11.0000	12.0000	13.0000	14.0000	15.0000
ϕ	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
SSR	4912.0205	4912.0205	4912.0205	4912.0205	4912.0205	4912.0205	4912.0205	4912.0205	4912.0205	4912.0205	4912.0205

Panel B. Optimized parameter values for model 2.

Specification	1	2	3	4	5	6	7	8	9	10	11
A^{**}	59.2430	29.0586	70.8339	61.8564	70.8738	70.5542	**126.8880**	76.7677	94.9734	57.4199	78.2214
B^{**}	−1.5858	−1.8182	−1.5858	−1.5720	−1.4470	−1.4768	**−4.0553**	−1.4491	−2.3051	−1.4320	−1.4613
t_c^{**}	76.8495	94.2558	78.6440	84.9817	81.7461	81.5568	**118.1294**	85.6672	90.1272	86.1470	87.6251
β^{**}	0.9000	0.7197	0.9000	0.8545	0.9000	0.8912	**0.7511**	0.9000	0.8341	0.8279	0.8930
C^{**}	0.2964	−0.5683	−0.1562	0.1769	0.1091	−0.0959	**0.1498**	0.0707	−0.0593	0.0833	0.0506
ω^{**}	5.1862	6.0020	7.3373	7.9839	9.4543	10.1893	**8.3886**	12.2070	12.7084	13.6729	15.0000
ϕ^{**}	−0.0032	−1.3802	−0.0023	−0.1235	−0.0049	−0.0020	**7.5415**	0.0073	0.1503	0.1979	0.0038
SSR	2616.7693	2632.7546	2619.3739	2631.4499	2628.9593	2634.0421	**2615.2660**	2637.4825	2628.9567	2700.7138	2653.9617

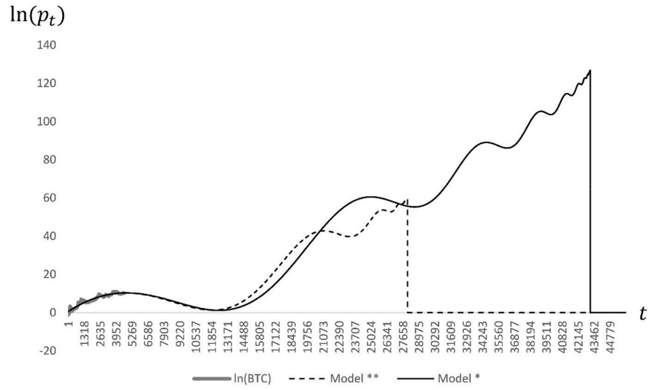

Fig. 1. Bitcoin prices in logarithms and optimized model 2 using daily data for the period January 1, 2011 until September 17, 2023
This figure shows two optimized models along with the natural logarithm of Bitcoin prices denoted as ln [$p(t)$]. The evolution of Bitcoin prices in natural logarithms is highlighted as the grey curve. Model* denotes the model generating the best fit with parametrization:
$\mathbf{\Phi}_2^{**} = (126.8880, \ -4.0553, \ 118.1294, \ 0.7511, \ 0.1498, \ 8.3886, \ 7.5415)$.The corresponding time-series evolution of that model is highlighted as the black curve. Moreover, Model ** denotes the model generating the second best fit with parametrization:
$\mathbf{\Phi}_2^{**} = (59.2430, -1.5858, \ 76.8495, \ 0.9000, \ 0.2964, \ 5.1862, -0.0032)$.
The corresponding time-series evolution of that model is highlighted as the dashed black curve. The in-sample period is from January 1, 2011 until September 17, 2023 comprising 4643 daily observations.

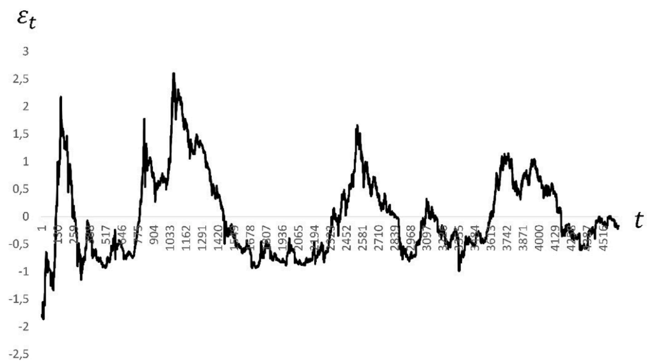

Fig. 2. Residuals of the calibrated optimal log-periodic power-law singularity model
This figure plots the residuals, ε_t, of Model* which offers the best fit. The in-sample period is from January 1, 2011 until September 17, 2023 comprising 4643 daily observations.

Table 3
Testing for unit-roots in the residuals of the optimal model 2
We use the residuals from Model* denoted as e_t and implement the augmented Dickey-Fuller (ADF) test by running the following test regression:
$\Delta e_t = \delta_0 + \delta_1 t + \delta_2 u_{t-1} + \gamma_1 \Delta u_{t-1} + \ldots + \gamma_p \Delta u_{t-p} + \varepsilon_t$,
where $e_t = p_t - m_t$ defines the difference between the log-prices of Bitcoin and calibrated optimal model 2 using the parametrization $\mathbf{\Phi}_2^{**} = (126.8880, \ -4.0553, \ 118.1294, \ 0.7511, \ 0.1498, \ 8.3886, \ 7.5415)$, t denotes a time trend, and ε_t is assumed to be a white noise process. Note that in this regression model, $\delta_0 = \delta_1 = 0$ defines the model specification where no deterministic terms are accounted for, whereas $\delta_1 = 0$ defines the model specification exhibiting a constant term only. Furthermore, the parameters $\gamma_1, \ldots, \gamma_p$ measure the exposure to the autoregressive variables, $\Delta u_{t-1}, \ldots, \Delta u_{t-p}$. It is noteworthy that the parametrization $\delta_0 = \delta_1 = 0$ corresponds to a random walk as the null model, whereas the parametrization $\delta_1 = 0$ corresponds to a random walk with a drift as the null model. Thus, we carry out three different versions of the ADF test. For all model specifications, the optimal lag-order p is selected in line with the Schwarz Criterion. The sample period is from January 1, 2011 to September 17, 2023 comprising 4643 daily observations.

Model	$\widehat{\delta}_0$	$\widehat{\delta}_1$	$\widehat{\delta}_2$	$\widehat{\lambda}_{ADF}$	p-value	lags
1			−0.0027*** (−2.8629)	−2.8629***	0.0041	0
2	0.0004 (0.5016)		−0.0028*** (−2.8656)	−2.8656**	0.0495	0
3	0.0016 (1.1031)	−5.29E-07 (−0.9841)	−0.0027*** (−2.8528)	−2.8528	0.1783	0

** Statistically significant on a 5 % level.
*** Statistically significant on a 1 % level.

Table 4
Testing for explosiveness in the evolution of the log-prices of Bitcoin
We use the differenced log-prices of Bitcoin denoted as Δp_t and implement the augmented Dickey-Fuller (ADF) test using the following regression: $\Delta p_t = \varphi_0 + \varphi_1 t + \varphi_2 p_{t-1} + \varphi_3 \Delta p_{t-1} + \ldots + \varphi_{p+2} \Delta p_{t-p} + e_t$,
where t denotes a time trend and e_t is assumed to be a white noise process. Faster-than-exponential growth should be manifested in $\varphi_2 < 0$ in association with $\varphi_1 > 0$. According to Grobys (2023), faster-than-exponential growth should be manifested in $\varphi_2 < 0$ in association with $\varphi_1 > 0$. The optimal lag-order p is selected in line with the Schwarz Criterion. The sample period is from January 1, 2011 to September 17, 2023 comprising 4643 daily observations.

	$\widehat{\varphi}_1$	$\widehat{\varphi}_2$	$\widehat{\lambda}_{ADF}$	p-value	lags
0.0098*** (4.7625)	2.75E-06* (1.7379)	−0.0020*** (−2.7883)	−2.7883***	0.2016	0

*Statistically significant on a 10 % level.
** Statistically significant on a 5 % level.
*** Statistically significant on a 1 % level.

Further, recent literature documented that Bitcoin is mainly used an investment as opposed to a medium of exchange (e.g., Baur et al., 2018; Baur and Dimpfl, 2021). What is the problem with Bitcoin from an investment perspective? The present study adds to the literature by providing evidence that investors, who wish to employ Bitcoin as a long-term investment, may face a fallacy due to the expected arrival of a finite singularity in the future suggesting an expected final value of zero.

Another novel finding is here that the LPPLS model predicts that Bitcoin prices will reach a local minimum by the end of February 2045, coinciding with the spontaneous singularity in U.S. equities around the year 2050 (Grobys, 2023). Further, the evidence suggests that Bitcoin is subject to accumulated herding and imitation. We argue that a finite-time singularity in the distant future supports the literature documenting that the demand for Bitcoin arises from speculation (e.g., Grobys and Junttila, 2021), whereas our findings appear to be contrary to the view that the demand may be driven by expectations regarding Bitcoin's future utility as a medium of exchange (e.g., de la Horra et al., 2019).

Finally, Chaim and Laurini's (2019) finding that Bitcoin prices do not show a bubble formation in late 2017 could be a manifestation of sample-specificity because the results of the present study show that after accounting for all available data, a long-lasting bubble formation in Bitcoin prices—incorporating the 2017 period—is indeed detectable. Likewise, the intrinsic value of the NFT and DeFi markets, as documented in Wang et al. (2022), could be a manifestation of data availability. Do these markets still exhibit intrinsic value when taking a coarse-grained perspective? Future research needs to explore this issue in more detail.

CRediT authorship contribution statement

Klaus Grobys: Writing – review & editing, Writing – original draft, Visualization, Validation, Supervision, Software, Resources, Project administration, Methodology, Investigation, Formal analysis, Data curation, Conceptualization.

Data availability

Data will be made available on request.

Appendix

Table A.1
Descriptive statistics of daily Bitcoin log-returns
This table reports the decretive statistics of Bitcoin log-returns. Data on Bitcoin covering the period January 1, 2011 until September 17, 2023 were downloaded from investing.com.

	BTC log-returns
Mean	0.24537
Median	0.04212
Maximum	42.2857
Minimum	−49.7032
Std. Dev.	4.9138
Skewness	−0.3587
Kurtosis	17.3512
Jarque-Bera	39,934.8000
Probability	0.0000
Observations	4642

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