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# An Integrated Framework for Dynamic Capacity Withholding Assessment Considering Commitment Strategies of Generation Companies

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## **Abstract**

This paper addresses an integrated framework for the dynamic capacity withholding assessment of an independent system operator that determines the mid-term maintenance scheduling of generation companies and day-ahead scheduling of wholesale market participants. The main contribution of this research is that two dynamic capacity-withholding indices are proposed for mid-term and day-ahead scheduling of generation companies that estimate the dynamic capacity withholding opportunities of generation units in an ex-ante manner. The proposed framework is another contribution of this research that uses a four-stage optimization process that the system operator can detect and prevent the formation of withholding groups. The optimal maintenance scheduling from the generation companies viewpoint is assessed in the first-stage problem that considers different mid-term withholding opportunities. The optimal mid-term maintenance scheduling is carried out in the second-stage problem that recognizes and rejects the dynamic capacity withholding of generation companies. The optimal scheduling of day-ahead generation companies considering their dynamic capacity withholding is the third contribution of this paper that optimizes the scheduling of generation units for day-ahead horizon considering responsive loads. The proposed method is applied to 30-bus, 57-bus and 118-bus IEEE test systems. A full competition algorithm is also carried out to evaluate the competition states of generation companies. The proposed algorithm detected that the dynamic capacity withholding might lead to increase of nodal price by about 279.22%, 764.43%, and 851.2% for 30-bus, 57-bus, and 118-bus IEEE test systems with respect to the non-capacity withholding conditions, respectively.

**Keywords:** Dynamic Capacity Withholding, Generation Company, Optimization, Maintenance Scheduling, Market Power.

## **Nomenclature**

### **Abbreviations**

DCW	Dynamic Capacity Withholding
DCWG	Dynamic Capacity Withholding Group
DCWI	Dynamic Capacity Withholding Index
GENCO	Generation Company
KKT	Karush-Kuhn-Tucker
MPEC	Mathematical Programming with Equilibrium Constraints

### **Indices**

$i$	Index of dynamic capacity withholding group generation company
$j$	Index of non-dynamic capacity withholding group generation company
$l$	Index of load bus
$k$	Index of generation bus consists of $i$ and $j$ indices
$m$	Index of bus
$t, t'$	Hour index

### **Parameters**

$NDCWG$	Number of dynamic capacity withholding groups
$NGU$	Number of generation units
$NB$	Number of buses
$NGB$	Number of generation buses
$NLB$	Number of load buses
$c$	Maintenance cost of unit (\$)
$MC$	Marginal cost of generating unit (\$/MWh)
$\underline{p}$	Minimum generating capacity of unit (MW)
$\bar{p}$	Maximum generating capacity of unit (MW)
$RU$	Ramp-up of unit (MW/h)
$RD$	Ramp-down of unit (MW/h)
$SU$	Start-up cost of unit (\$)
$SD$	Shut-down cost of unit (\$)
$T^{off}$	Minimum up-time of unit (Hour)
$T^{on}$	Minimum downtime of unit (Hour)
$X$	Maximum maintenance time duration of unit (Hour)
$v$	Slope of demand function

$\zeta$	y-intercept of demand function
$Y$	Admittance matrix

### ***Variables***

$\lambda$	Nodal price of generation bus (\$/MWh)
$\xi$	Active power consumption in demand bus (MW)
$p$	Active power generation of unit (MW)
$p^{FC}$	Active power generation of unit in full competition market (MW)
$p^{NC}$	Active power generation of unit in multi-polar market (MW)
$\gamma$	Nodal price of demand bus (\$/MWh)
$\varphi, \mu, \kappa, \varphi', \mu', \kappa'$	Lagrange multipliers matrix
$V$	Voltage of bus
$\theta$	Angle of voltage

### ***Integer variables***

$u$	Binary decision variable for maintenance of unit: 1 if the unit is on maintenance; otherwise 0
$I$	Binary decision variable for self-commitment of unit: 1 if the unit is committed; otherwise 0
$K$	Binary decision variable for independent system operator commitment of unit: 1 if the unit is committed; otherwise 0

## **1. Introduction**

The power system restructuring has led to the complexity of the operational paradigms and maintenance scheduling procedures. The generation companies may adopt strategic behaviour, impose market power, and maximize their profits that may lead to very high prices of the electricity market. A generation company may either have market power based on its market share or may withhold its capacity from the wholesale market [1]. In an Oligopoly market, it is possible for generation companies to change the market price by increasing their bid price that this process is known as economic withholding [2].

Further, the generation companies can meet this goal by reducing their output that this procedure is called capacity withholding [1]. The economic withholding can be easily detected by the market monitoring units based on the fact that the marginal costs of generation companies data are available for the market monitoring units. However, the capacity withholding detection is very complicated



based on the fact that this process can be dynamically implemented by multiple generation companies for different durations, and more procedures and indices are needed to detect and analyse these process.

The generation companies may withhold their generations statically or dynamically in both economic withholding and capacity withholding manners. The static withholding procedure may be exercised for a “snapshot of the system state by a generation company” without forming collusive groups, while the dynamic withholding procedure may be applied by the generation companies to form groups and withhold their capacity from markets [1]. In the dynamic capacity withholding condition, it may be exercised in an implicitly or explicit manner for a specific period and generation companies may present their bids based on their hidden group formations. In an implicit capacity withholding, there is no direct relationship between the generation companies, whereas, in explicit capacity withholding, the generation companies are forming groups that are controlling the market price for long-term or short-term periods. The term of “dynamic” indicates that the generation companies dynamically consider their objective functions and constraints, and system conditions in their capacity withholding group formations to maximize their profits. However, in the “static” capacity withholding, none of these group formations and dynamic constraints of generation companies are considered. The volume, pattern, and time of dynamic capacity withholding can be highly changed based on the system and generation companies’ objective functions and constraints [1].

Examples of capacity and economic withholding have been seen in the electricity markets around the world. In the UK and Wales’ electricity markets, two major companies kept prices above the real market costs without competition in the mid of 90’s [3]. In California in the years 2000 and 2001, the capacity withholding procedure of many companies led to the crisis [4]. In Spain, two major companies refused to compete with each other and made no effort to reduce market prices [5].

Many researchers have assessed these procedures in recent years to detect capacity and/or economic withholding procedures. The capacity or economic withholding assessment can be performed by employing ex-ante or ex-post methods. As shown in Table 1, different methods have been utilized to assess the economic and capacity withholding process of generation companies in mid-term and day-ahead horizons and the literature can be categorized into static and dynamic capacity withholding assessment methods.

Table 1: Comparison of proposed dynamic capacity withholding assessment with other researches.

		[1]	[2]	[6]	[7]	[8]	[9]	[10]	[11]	[12]	[13]	[14]	[15]	[16]	[17]	[18]	[19]	[20]	[21]	[22]	[23]	[24]	[25]	[26]	[27]	Present paper	
Withholding	Capacity	Static	✓	✓	×	×	×	×	×	✓	×	✓	×	×	×	×	×	×	✓	✓	✓	×	✓	✓	×	×	
		Dynamic	×	×	×	×	×	×	✓	×	×	✓	×	✓	×	×	×	×	×	×	×	×	✓	×	×	✓	✓
	Economic	Static	×	×	✓	✓	✓	✓	×	✓	×	×	×	×	✓	✓	✓	✓	✓	×	×	×	×	×	×	×	×
		Dynamic	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×
Withholding Simulation	Game Theory	✓	✓	×	×	×	✓	✓	×	✓	×	✓	×	×	×	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	×	×
	Agent Based	×	×	✓	✓	✓	×	✓	✓	×	×	✓	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×
	Withholding Group Formation	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	✓	
Withholding Assessment	Ex-ante	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	✓
	Ex-post	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	×
Withholding Index		✓	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	✓	×	×	×	✓
Optimization	Nonlinear	×	×	×	×	×	×	×	×	✓	×	✓	×	×	×	×	×	×	×	×	×	×	✓	×	✓	×	×
	Linear	✓	✓	✓	✓	✓	✓	✓	✓	×	✓	×	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	×	✓	×	×	✓
GENCO Optimization		✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	×	×	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	×	✓
System Operator Optimization		✓	✓	×	×	×	×	✓	×	×	×	×	×	×	×	×	✓	×	✓	×	✓	×	✓	✓	✓	×	✓
GENCO Constraints		✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	×	×	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	×	✓
Network Constraints		✓	✓	×	×	×	×	✓	×	×	×	×	×	×	×	×	✓	×	✓	×	✓	×	×	×	✓	×	✓
Responsive Loads		✓	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	✓	×	×	×	×	×	×	✓

Ref. [6] analyzed the market power in terms of generation companies' behaviour with regard to the market information and proposed the behavioural-based model of rivals. Ref. [7] described the market equilibrium point and the possibility of withholding of generation companies and proposed a framework to analyse the impact of market structural variables on the competition of generation companies. Ref. [8] investigated the withholding process of generation companies based on a simulation procedure and the behavioural model of rivals. Refs. [6-8] did not propose any index for dynamic capacity withholding assessment.

Ref. [9] simulated the effect of economic withholding on the market prices and the profit of generation companies was evaluated without introducing any withholding index. Ref. [10] proposed a procedure for economic withholding assessment using market price parameters in the simulation-based competitive market and the non-competitive market. It concluded that the increased market

price could be interpreted as the capacity withholding of generation companies that might be led to an increase in their profit. Ref. [11] proposed that economic withholding could be assumed as a kind of monopoly considering the fact that the market power of generation companies could be evaluated by their profit function. Further, it concluded that the increase in market price might be due to the withholding of all generation companies or a large part of them to increase their profit.

Ref. [12] proposed that economic withholding of generation companies could be detected by analyzing the market price and generation companies' profit margins. Ref. [13] presented a withholding assessment procedure according to Marco's model that was performed for two generation company markets. Ref. [14] summarized that the economic withholding can be increased by structural factors such as network structure and congestion, strategic biddings of generation companies, and demand elasticity. None of the mentioned references presented any index for ex-ante capacity withholding assessment. Ref. [15] presented a repetitive game model where each scenario of generation companies' participation was evaluated and their role in economic withholding was assessed. Ref. [16] evaluated the effects of forward contracts on economic withholding based on the historical data recovered from the electricity market. It concluded that long-term contracts highly reduced the probability of economic withholding in the spot market.

Ref. [17] proposed a profit-based Nash-Cournot model that analyzed the ex-post economic withholding of generation companies in a non-competitive market. All these references considered ex-post analysis method for economic withholding assessment. Ref. [18] presented a procedure for fast calculation of equilibrium of markets considering tacit economic withholding using game theory using an elementary model of the market. However, the detailed model of dynamic capacity withholding was not modelled. Ref. [19] developed the model of fossil-based thermal units' economic withholding against renewable units in accordance with the regulatory framework, but it did not propose any index. Ref. [20] examined the possibility of generation units in an economic withholding and used incomplete information from other units for this study. However, the capacity-withholding index was not proposed in this reference.

Ref. [21] introduced a static capacity-withholding index for analyzing of generation companies behaviour in non-competitive electricity market using Cournot game theory. However, the dynamic capacity withholding analyzes was not performed. Ref. [22] presented an equilibrium based optimization procedure to assess the market power of prosumers. The Cournot based game model optimized the benefits of prosumers and system operator. Ref. [23] explored the market power of renewable energy resource on the wholesale market and assessed their strategic behaviour using a bi-level optimization model. At the first and second levels, the day-ahead and real-time revenue optimization of resources were modelled, respectively. This reference utilized the simulation-based

method to assess the bidding process of energy resources. However, Ref. [21-23] did not propose any dynamic capacity withholding indices.

Ref. [24] introduced a repeated game model to evaluate the ex-post dynamic capacity withholding process. The bidding problem of generation companies was modelled by an optimal control process that maximized their profits. The capacity withholding assessment was performed using reinforcement algorithm. However, the ex-ante dynamic capacity withholding analysis was not carried out in this reference. Ref. [25] proposed a static capacity-withholding index for analyzing of generation companies strategic behaviour. Nevertheless, the network constraints and dynamic behaviour of rivals were not modelled.

Ref. [26] presented a Nash equilibrium model for analyzing hydrothermal generation companies in the electricity market and a supply function model was utilized to assess the bidding strategies of rivals. Ref. [27] explored the capacity withholding procedures of generation companies in German-Austrian electricity markets and investigated the relationship between failures of generation companies and the spot market prices. Refs. [26-27] did not present any capacity withholding indices.

Ref. [1] proposed a static capacity-withholding index that utilized the nodal prices distortion as an indicator of withholding. The introduced algorithm used the game-theory framework to assess the behaviour of generation companies in full competition and non-competitive environments. Ref. [2] presented a bi-level optimization procedure for static capacity withholding assessment that modelled the profit maximization of generation companies in the first stage and cost minimization of energy procurement of system operator in the second level. The optimization algorithm utilized mixed-integer linear programming process to find the optimal solutions of the bi-level problem. Refs. [1, 2] did not consider the dynamic capacity withholding procedures of generation companies.

As shown in Table 1, all of the above-mentioned references assessed the static capacity or economic withholding of generation companies in an ex-post procedure and did not consider the dynamic capacity withholding analysis in an ex-ante procedure.

In this paper, for the first time, the ex-ante mid-term and day-ahead dynamic capacity withholding assessment based on two indices is proposed to detect and prevent the formation of generation companies' groups by the independent system operator. Further, an analysis process for capacity withholding group formation is proposed to detect the probable dynamic capacity withholding opportunities of generation companies.

The main contributions of this paper are:

- The proposed framework investigates the mid-term and day-ahead dynamic capacity withholding possible opportunities of generation companies and estimates the mid-term and day-ahead generation companies withheld power and increase of nodal price for the first time.

- The algorithm detects the mid-term dynamic capacity withholding group formation between different generation companies that can be implemented in maintenance scheduling using mid-term dynamic capacity withholding index and optimizes the maintenance scheduling of generation companies.
- The process detects the day-ahead dynamic capacity withholding group formation of generation companies using da-ahead dynamic capacity withholding index and schedules the day-ahead unit commitment of generation companies considering responsive loads.
- The overall four-stage solution method is another contribution of this research that solves the problem for the 118-bus IEEE tests system with 5843311 equations in about 67 seconds.

The paper has been organized as follows: Section 2 presents the problem modelling and formulation. In Section 3, two dynamic capacity-withholding indices are proposed. Solution methodology is proposed in Section 4. Section 5 presents the simulation results. Finally, Section 6 presents the conclusions.

## **2. Problem Modelling and Formulation**

A four-stage optimization method is presented as shown in Fig. 1. At the first stage, the mid-term maintenance scheduling of Dynamic Capacity Withholding Groups (DCWGs) of generation companies is simulated by the independent system operator.

As shown in Fig. 1, at the first-stage problem, the independent system operator simulates the dynamic capacity withholding groups' formations of generation companies in their midterm maintenance scheduling and estimates their best capacity withholding groups' formations. At the second stage, the independent system operator optimizes the mid-term maintenance scheduling of generation companies by adding transmission system and security constraints; and rejects the mid-term maintenance scheduling of generation companies that lead to capacity withholding. At the third stage, the optimal day-ahead scheduling of dynamic capacity withholding groups are simulated by the independent system operator and the different states of short-term capacity withholding groups are generated. At the fourth stage, the independent system operator maximizes the social welfare of the system in the day-ahead horizon considering its system and generation companies constraints. In this case, the commitment of generation companies units may change due to the system security and transmission system constraints.

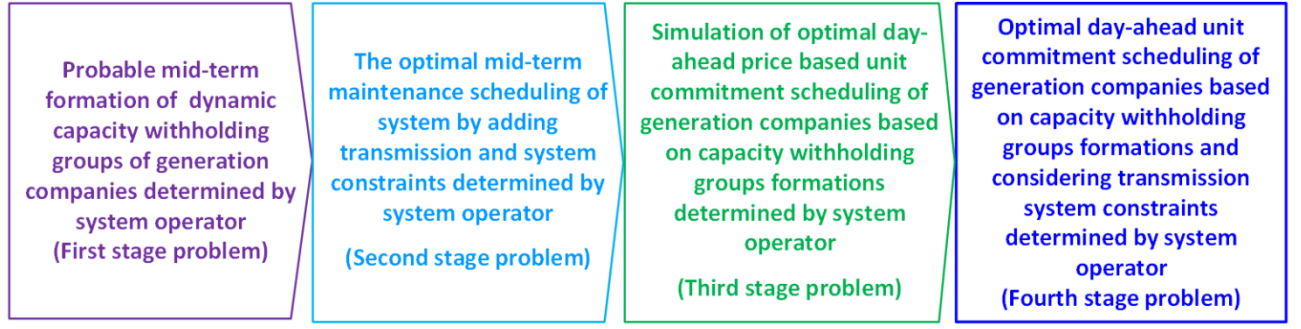


Fig. 1. Proposed framework for dynamic capacity withholding analysis.

Based on the above description of four-stage problem, the detailed formulation of each stage is presented as follows.

## 2.1. First Stage Problem

At the first stage, the independent system operator simulates and finds the mid-term formation of possible dynamic capacity withholding groups' states. The objective function of the first stage problem maximizes the mid-term profit of dynamic capacity withholding groups considering their constraints that can be presented as (1):

$$\text{Max } Z_{DCWG}^{MT} = \sum_{t=1}^{168} \sum_{i=1}^{NDCWG} ((\lambda_{i,t} \cdot p_{i,t} - MC_{i,t} \cdot p_{i,t})(1 - u_{i,t}) - u_{i,t} \cdot c_{i,t}) \quad (1)$$

The first stage objective function is decomposed into following terms: 1) the dynamic capacity withholding groups' revenue of electricity sold to the electricity market ( $\lambda_{i,t} \cdot p_{i,t}$ ); 2) the costs of electricity generation ( $MC_{i,t} \cdot p_{i,t}$ ); and 3) the maintenance costs ( $u_{i,t} \cdot c_{i,t}$ ).

Thus, the output of the first stage problem is the dynamic capacity group formations that maximize the generation companies' profits.

The objective function is maximized for 168 hours of a week.

The objective function of (1) is constrained by multiple constraints that can be written as:

The lower and upper limits of dynamic capacity withholding groups electricity generation:

$$\underline{p}_i \cdot (1 - u_{i,t}) \leq p_{i,t} \leq \overline{p}_i \cdot (1 - u_{i,t}) \quad \forall i, \forall t \quad (2)$$

Minimum downtime constraints of dynamic capacity withholding groups:

$$\sum_{t'=t}^{t+T_i^{off}-1} u_{i,t'} \geq T_i^{off} \cdot (u_{i,t-1} - u_{i,t}) \quad \forall i, \forall t \in \{1, \dots, 24 - T_i^{off}\} \quad (3)$$

Minimum up-time constraints of dynamic capacity withholding groups:

$$\sum_{t'=t}^{t+T_i^{on}-1} u_{i,t'} \geq T_i^{on} \cdot (u_{i,t} - u_{i,t-1}) \quad \forall i, \forall t \in \{1, \dots, 24 - T_i^{on}\} \quad (4)$$

Maintenance time constraints of dynamic capacity withholding groups for 168 hours of week:

$$\sum_{t=1}^{168} u_{i,t} = X_i \quad \forall i \quad (5)$$

Continuity of maintenance time constraints of dynamic capacity withholding groups:

$$u_{i,t} - u_{i,t-1} \leq u_{i,(t+X_i-1)} \quad \forall i, \forall t \quad (6)$$

## 2.2. Second Stage Problem

At the second stage problem, the independent system operator optimizes the mid-term maintenance scheduling of generation companies considering transmission system and security constraints. The independent system operator utilizes the first stage problem outputs to detect dynamic capacity groups' formation. Then, at the second stage, he/she optimizes the mid-term maintenance scheduling of system and rejects the maintenance scheduling of generation companies that may lead to capacity withholding. Thus, only the maintenance scheduling of competitive generation companies are accepted by the independent system operator. Hence, the first stage problem is a pre-processing optimization problem that determines withholding groups and delivers its outputs to the second stage problem.

The second stage optimization problem is a bi-level problem that the upper-level subproblem tries to find the optimal maintenance strategy of generation companies considering their profit maximization in maintenance scheduling [1, 28]. The lower-level subproblem minimizes the energy procurement costs that the optimal solution is determined by the system operator. The detailed formulation of bi-level problem is presented in [28] and is not repeated for the sack of space.

The objective function of the lower-level subproblem of second stage problem can be presented as (7):

$$\text{Min } M_{ISO}^{MTDCW} = \sum_{t=1}^{168} \sum_{j=1}^{NGU} ((MC_{j,t} \cdot p_{j,t} + SU_{j,t} + SD_{j,t})(1 - u_{j,t}) + u_{j,t} \cdot c_{j,t}) \quad (7)$$

The second stage objective function is optimized for 168 hours of a week and decomposed into following terms: 1) the costs of electricity generation ( $MC_{j,t} \cdot p_{j,t}$ ) for all of the generation units; 2) start-up costs ( $SU_{j,t}$ ); 3) shut-down costs ( $SD_{j,t}$ ); and 4) the maintenance costs ( $u_{j,t} \cdot c_{j,t}$ ).

Thus, the output of the second stage problem is the optimal "system" mid-term maintenance scheduling considering generation companies capacity withholding opportunities.

The constraints of the second stage problem can be written as:

The lower and upper limits of generation companies:

$$\underline{p}_j \cdot I_{j,t} \leq p_{j,t} \leq \overline{p}_j \cdot I_{j,t} \quad \forall j, \forall t \quad (8)$$

Ramp-up constraints of generation companies:

$$p_{j,t} - p_{j,t-1} \leq RU_j \cdot I_{j,t} \quad \forall j, \forall t \quad (9)$$

Ramp-down constraints of generation companies:

$$p_{j,t-1} - p_{j,t} \leq RD_j \cdot I_{j,t-1} \quad \forall j, \forall t \quad (10)$$

Minimum downtime constraints of generation companies:

$$\sum_{t'=t}^{t+T_j^{off}-1} I_{j,t'} \geq T_j^{off} \cdot (I_{j,t-1} - I_{j,t}) \quad \forall j, \forall t \in \{1, \dots, 24 - T_j^{off}\} \quad (11)$$

Minimum up-time constraints of generation companies:

$$\sum_{t'=t}^{t+T_j^{on}-1} I_{j,t'} \geq T_j^{on} \cdot (I_{j,t} - I_{j,t-1}) \quad \forall j, \forall t \in \{1, \dots, 24 - T_j^{on}\} \quad (12)$$

Committing constraint of generation companies:

$$I_{j,t} \leq (1 - u_{j,t}) \quad \forall j, \forall t \quad (13)$$

The maintenance time constraints of generation companies (5) and continuity of maintenance time constraints of generation companies (6) from the first stage problem are considered and not presented for the sack of space. The AC load flow constraints and voltage limits of buses are considered as the second stage problem constraints.

### 2.3. Third Stage Problem

At the third stage, the independent system operator simulates the day-ahead price based unit commitment of dynamic capacity withholding groups and estimates their day-ahead offers. A generation company may dynamically withhold its capacity to gain more profit. Further, it can form dynamic capacity withholding groups with other generation companies to increase the market price. Thus, the dynamic capacity withholding groups optimize their profits by determining their generation scheduling and dynamic capacity withholding strategies in the specified intervals.

The objective function of the third stage problem is to maximize the day-ahead profit of dynamic capacity withholding groups considering their constraints and possible dynamic capacity withholding procedures.

The objective function of the third-stage problem can be presented as (14):

$$\text{Min } \mathbf{Z}_{DCWG}^{DA} = \sum_{t=1}^{24} \sum_{i=1}^{NDCWG} (\lambda_{i,t} \cdot p_{i,t} - MC_{i,t} \cdot p_{i,t} - SU_{i,t} - SD_{i,t}) \cdot I_{i,t} \quad (14)$$

The third stage objective function is decomposed into following terms: 1) the dynamic capacity withholding groups' revenue of electricity sold to the electricity market ( $\lambda_{i,t} \cdot p_{i,t}$ ); 2) the costs of electricity generation ( $MC_{i,t} \cdot p_{i,t}$ ); 3) start-up costs ( $SU_{i,t}$ ); and 4) shut-down costs ( $SD_{i,t}$ ).



Thus, the output of the third stage problem is the estimated optimal day-ahead price-based unit commitment of generation companies that maximize their day-ahead profits.

The objective function of (14) is constrained by (2), (9), (10), (11), and (12) are considered as the third stage optimization problem.

#### 2.4. Fourth Stage Problem

It is assumed that the systems loads are decomposed into: 1) the critical loads that their electricity consumptions are fixed, and 2) the responsive loads that their active power consumption at bus  $l$  can be formulated as (15) [1]:

$$\xi = -v \cdot \gamma + \zeta \quad \forall l \quad (15)$$

At the fourth stage optimization problem, the independent system operator maximizes day-ahead social welfare that can be formulated as (16) [1]:

$$\text{Max } \mathbf{A}_{ISO}^{DA} = \sum_{t=1}^{24} \sum_{l=1}^{NLB} \left( -\frac{1}{2} v \cdot \gamma^2 + \zeta \cdot \gamma \right) - \sum_{t=1}^{24} \sum_{k=1}^{NGB} (MC_{i,t} \cdot p_{i,t} + SU_{i,t} + SD_{i,t}) \cdot K \quad (16)$$

The fourth stage objective function is decomposed into following terms [1]: 1) the aggregated surplus of loads that equals  $\xi \cdot \gamma$  that the objective function tries to maximize it; and 2) the generation units costs that the objective function tries to minimize it. The detailed formulations of social welfare maximization are available in [1] and are not presented for the sack of space.

Thus, the output of the fourth stage problem is the optimal day-ahead unit commitment of generation companies considering system constraints.

The constraints of fourth stage can be presented as:

The supply-demand constraints must be considered for each interval of simulation.

$$\sum_{k=1}^{NGB} p_{k,t} - \sum_{l=1}^{NLB} |V_{n,t}| \cdot |V_{m,t}| \cdot |Y_{nm}| \cdot \cos(\theta_n - \theta_m) = 0 \quad \forall t \quad (17)$$

$$\sum_{k=1}^{NGB} q_{k,t} + \sum_{l=1}^{NLB} |V_{n,t}| \cdot |V_{m,t}| \cdot |Y_{nm}| \cdot \sin(\theta_n - \theta_m) = 0 \quad \forall t \quad (18)$$

The voltage limit constraints, generation unit constraints and power flow constraints should be considered in the optimization process that can be presented as  $\mathbf{\Omega}$ ,  $\mathbf{\Theta}$ , and  $\mathbf{\Psi}$  in compact forms, respectively.

### 3. Mid-term and Short-term Dynamic Capacity Withholding Indices

#### 3.1 Mid-term Dynamic Capacity Withholding Index

A mid-term Dynamic Capacity Withholding Index ( $DCWI_{MT}$ ) is proposed as (19):

$$DCWI_{MT} = \frac{\left| M_{ISO}^{MTDCW} - M_{ISO}^{MTFC} \right|}{M_{ISO}^{MTFC}} \quad (19)$$

$M_{ISO}^{MTDCW}$  is calculated in the second stage problem. The  $M_{ISO}^{MTFC}$  is the objective function of full competition condition that is calculated by the independent system operator and can be written as (20):

$$\text{Min } M_{ISO}^{MTFC} = \sum_{t=1}^{168} \sum_{j=1}^{NGU} ((MC_{j,t} \cdot p_{j,t} + SU_{j,t} + SD_{j,t})(1 - u_{j,t}) + u_{j,t} \cdot c_{j,t}) \quad (20)$$

Eq. (20) consists of the same terms as Eq. (7). However, it is assumed that the market is fully competitive. Eq. (20) is constrained by the second stage constraints.

The  $DCWI_{MT}$  is defined as the relative difference of objective functions of the complete competition and dynamic capacity withholding condition. The described index is calculated for all of the possible states of dynamic capacity withholding groups and dynamic capacity withholding opportunities are determined.

#### 3.2 Short-term Dynamic Capacity Withholding Index

A day-ahead Dynamic Capacity Withholding Index ( $DCWI_{DA}$ ) is proposed as (21):

$$DCWI_{DA} = \frac{\sum_{i=1}^{NDCWG} \left( \frac{v}{a_i} (p_i^{NC}) + \frac{1}{a_i} (\mu_i^{FC} - \mu_i'^{FC} + \mu_i^{NC} - \mu_i'^{NC}) \right)}{\sum_{i=1}^{NGU} \left( \frac{v}{a_i} (p_i^{NC}) + \frac{1}{a_i} (\mu_i^{FC} - \mu_i'^{FC} + \mu_i^{NC} - \mu_i'^{NC}) \right)} \quad (21)$$

$DCWI_{DA}$  shows the ratio of the hourly-withheld capacity generation of the generation companies in the non-competitive market with respect to the competitive market. Higher values of  $DCWI$  show the higher ability of dynamic capacity withholding groups for dynamic capacity withholding. The proof of Eq. (21) is presented in Appendix I. The independent system operator calculates the hourly values of  $DCWI_{DA}$  and rejects the offers of generation companies that may increase the estimated  $DCWI_{DA}$ .

## 4. Solution Methodology

The optimization process assumes:

- The Karush-Kuhn-Tucker (KKT) conditions that are necessary for optimality of the second problem.
- The second-stage problem is a Mathematical Programming with Equilibrium Constraints (MPEC) problem. The proposed algorithm of [28] is utilized the bi-level optimization process of second stage problem. The second stage MPEC problem can be recast as a mixed-integer linear optimization problem.
- The 1<sup>st</sup>, 2<sup>nd</sup>, 3<sup>rd</sup>, and 4<sup>th</sup> problems are mixed-integer programs that are solved by the CPLEX solver [29] and linearization techniques are adopted to linearize the problems [30].
- The MPEC solving procedure is presented in [28] and is not presented for the sack of space.
- The overall proposed procedure is presented in Fig. 2. The simulation was carried out on a PC (Intel Core i7-870 processor, 4\*2.93 GHz, 8 GB RAM).

## 5. Simulation Results

Three test systems IEEE 30-bus, IEEE 57-bus, and IEEE 118-bus were utilized to evaluate the proposed method. The data of the modified IEEE 30-bus, 57-bus, and 118-bus test system are available in [31]. The inverse demand function of each demand was assumed as  $\xi = -v \cdot \gamma + 45$  for all systems. The slope of inverse demand function was chosen in a way that  $\xi=35$  \$/MW for the given value of  $\gamma$  in [31].

### 5.1. 30-bus IEEE test system

Fig. 3 shows the IEEE 30-bus system topology and its responsive load locations. It was assumed that six load buses of the 30-bus system were responsive. Fig. 4 presents the daily load curve of the system. The independent system operator performed the fourth-stage optimization process. At first, the independent system operator carried out the first and second stage of simulations, considered different mid-term states of dynamic capacity withholding groups and calculated their corresponding values of  $DCWI_{MT}$ . Table 2 presents the maximum values of  $DCWI_{MT}$  for different dynamic capacity withholding groups that their withholding indices were at the highest value for the mid-term optimization horizon. The first and second ranks of dynamic capacity withholding groups are marked in orange and yellow, respectively.

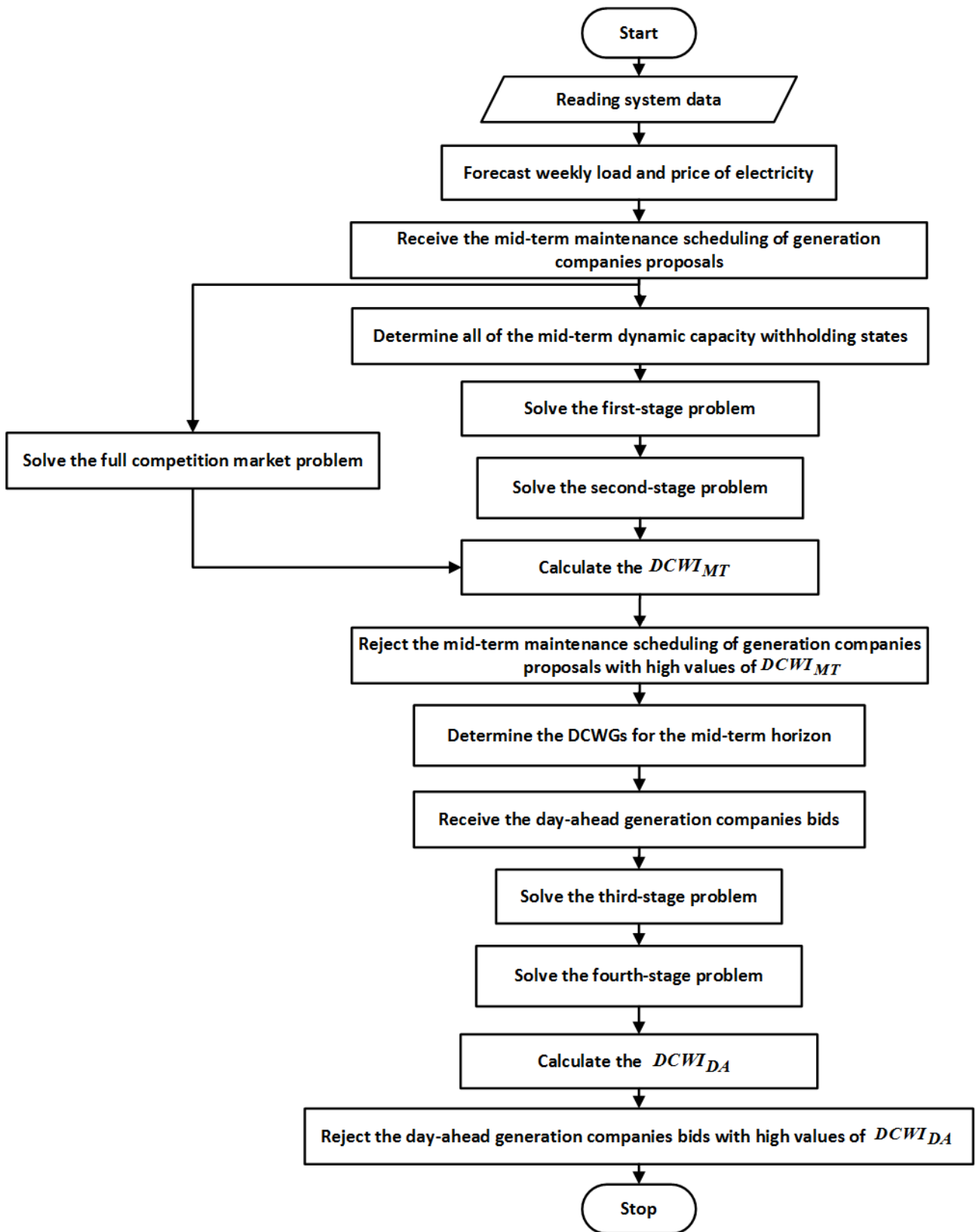


Fig. 2. Flowchart of the proposed algorithm.

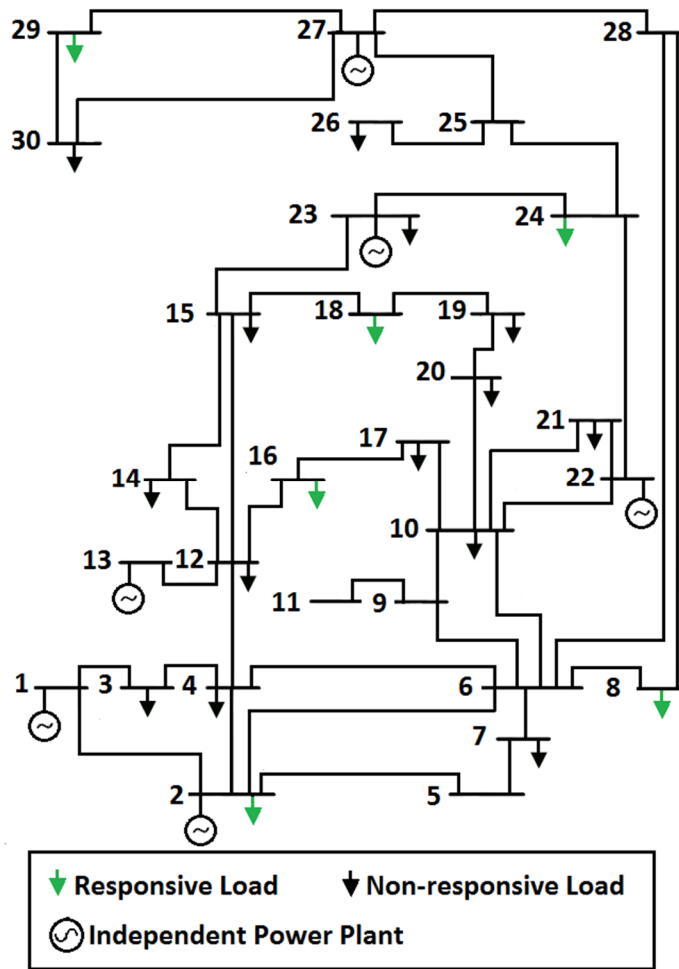


Fig. 3. The topology of 30-bus IEEE test system.

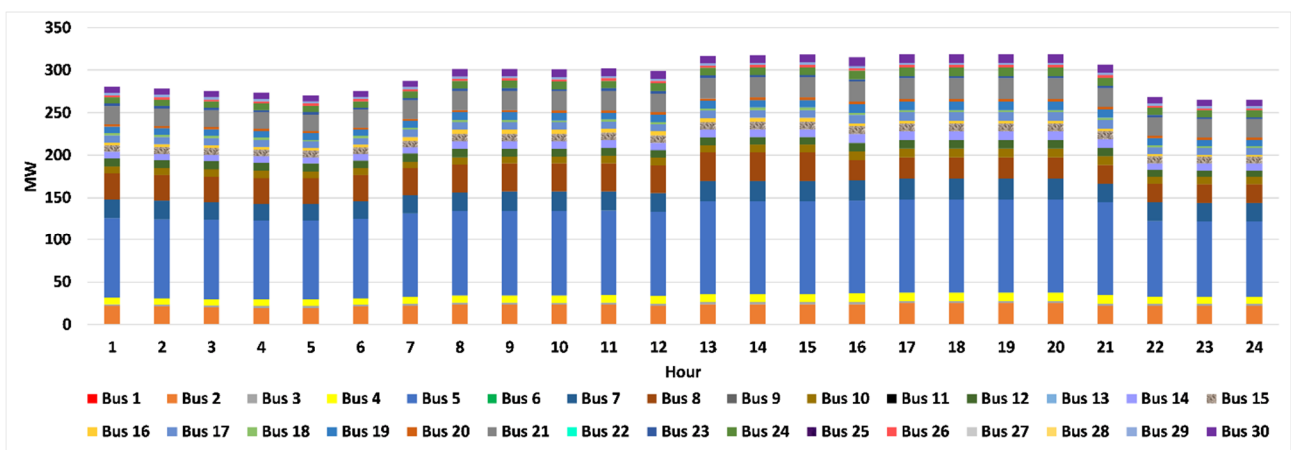


Fig. 4. Estimated day-ahead load forecasting of 30-bus IEEE tests system.

Table 2. The mid-term dynamic capacity withholding indices for the 30-bus system.

NDCWG=2				NDCWG=3		NDCWG=4				NDCWG=5	
DCWG	DCWI <sub>MT</sub>	DCWG	DCWI <sub>MT</sub>	DCWG	DCWI <sub>MT</sub>	DCWG	DCWI <sub>MT</sub>	DCWG	DCWI <sub>MT</sub>	DCWG	DCWI <sub>MT</sub>
(1,2)	0.213	(3,5)	0.101	(1,2,5)	0.294	(1,2,3,4)	0.387	(1,4,5,6)	0.301	(1,2,3,4,5)	0.451
(1,5)	0.187	(3,6)	0.123	(1,2,8)	0.276	(1,2,5,11)	0.311	(2,5,8,11)	0.275	(1,2,5,8,13)	0.504
(1,8)	0.181	(4,5)	0.103	(1,2,11)	0.224	(1,2,5,13)	0.391	(2,5,11,13)	0.287	(1,2,5,11,13)	0.401
(1,11)	0.161	(4,6)	0.121	(1,2,13)	0.239	(1,2,8,11)	0.298	(2,8,11,13)	0.295	(1,2,8,11,13)	0.411
(1,13)	0.177	(5,6)	0.117	(2,5,8)	0.203	(1,2,8,13)	0.303	(5,8,11,13)	0.269	(1,5,8,11,13)	0.394
(2,5)	0.184			(2,5,11)	0.198	(1,2,11,13)	0.287			(2,5,8,11,13)	0.381
(2,8)	0.173			(2,5,13)	0.21	(1,5,8,11)	0.283				
(2,11)	0.121			(5,8,11)	0.153	(1,5,8,13)	0.295				
(2,13)	0.135			(5,8,13)	0.167	(1,5,11,13)	0.292				
(5,8)	0.118			(8,11,13)	0.131						

The highest values of mid-term dynamic capacity withholding for the (1, 2) and (1, 13) groups were 0.213 and 0.177, respectively. The highest values of  $DCWI_{MT}$  for the three members, four members, and five members were 0.294, 0.391, and 0.504, respectively. The  $DCWI_{MT}$  was highly increased when the number of capacity withholding members were increased.

The independent system operator might prevent the formation of dynamic capacity withholding groups in mid-term maintenance scheduling that their  $DCWI_{MT}$  values were high. Then, the independent system operator performed the third and fourth stages of simulations and calculated their corresponding values of  $DCWI_{DA}$ . Table 3 and Table 4 present the outputs of the third and fourth stages of the optimization process for the IEEE 30-bus test system that their  $DCWI_{DA}$  withholding indices were at the highest values for the day-ahead optimization horizon. The maximum values of  $DCWI_{DA}$  for the  $NDCWG=2$  were estimated for the first, seventh, eleventh and seventeenth hours that belonged to the (1, 2) group. Further, the first rank of day-ahead dynamic capacity withholding group for  $NDCWG=3$  was for the (1, 2, 4) group that the corresponding values of  $DCWI_{DA}$ , withheld power and changes of nodal price were 0.9413, 44.071 MW and 4.31 \$/MWh, respectively. The  $DCWI_{DA}$  values were highly increased for  $NDCWG=4$ ,  $NDCWG=5$  and  $NDCWG=6$  with their corresponding values of  $DCWI_{DA}=0.966$ ,  $DCWI_{DA}=0.9793$  and  $DCWI_{DA}=1$ , respectively. The highest value of nodal price changes was 8.55 \$/MWh that was for the  $NDCWG=6$ . The maximum nodal price for the  $NDCWG=6$  was reached to 190.977 \$/MWh that corresponded to 279.22% increase of the nodal price. The independent system operator should reject the electricity generation bids of dynamic capacity withholding groups that their  $DCWI_{DA}$  values are high and penalize them.

Table 3. The day-ahead dynamic capacity withholding indices for the 30-bus system.

Hour	NDCWG=2					NDCWG=3					NDCWG=4				
	DCWG	DCWI <sub>DA</sub>	$\Delta p^{withheld}$ (MW)	$\Delta \lambda^{distortion}$ (\$/MWh)	$\lambda$ (\$/MWh)	DCWG	DCWI <sub>DA</sub>	$\Delta p^{withheld}$ (MW)	$\Delta \lambda^{distortion}$ (\$/MWh)	$\lambda$ (\$/MWh)	DCWG	DCWI <sub>DA</sub>	(MW)	$\Delta \lambda^{distortion}$ (\$/MWh)	$\lambda$ (\$/MWh)
1	(1,2)	0.9131	42.773	2.89	78.181	(1,2,4)	0.9413	44.071	4.31	116.597	(1,2,4,5)	0.966	45.23	5.73	142.937
2	(4,5)	0.0524	2.457	2.84	50.36	(3,5,6)	0.059	2.769	4.24	80.19	(1,2,4,5)	0.966	45.23	5.73	142.937
3	(1,3)	0.8697	40.742	2.86	77.847	(3,5,6)	0.059	2.769	4.24	80.19	(1,2,4,5)	0.966	45.23	5.73	142.937
4	(2,6)	0.0777	3.641	2.81	60.1	(3,5,6)	0.059	2.769	4.24	80.19	(1,2,3,6)	0.9479	44.383	5.71	139.87
5	(2,6)	0.0777	3.641	2.81	60.1	(3,5,6)	0.059	2.769	4.24	80.19	(1,2,3,6)	0.9479	44.383	5.71	139.87
6	(2,6)	0.0777	3.641	2.81	60.1	(1,2,4)	0.9413	44.071	4.31	116.597	(1,2,3,6)	0.9479	44.383	5.71	139.87
7	(1,2)	0.9131	42.773	2.89	78.181	(1,2,4)	0.9413	44.071	4.31	116.597	(1,2,4,5)	0.966	45.23	5.73	142.937
8	(4,5)	0.0524	2.457	2.84	50.36	(1,2,4)	0.9413	44.071	4.31	116.597	(1,2,4,5)	0.966	45.23	5.73	142.937
9	(1,3)	0.8697	40.742	2.86	77.847	(1,2,3)	0.9269	43.398	4.29	114.2	(1,2,4,5)	0.966	45.23	5.73	142.937
10	(1,3)	0.8697	40.742	2.86	77.847	(1,2,3)	0.9269	43.398	4.29	114.2	(1,2,4,5)	0.966	45.23	5.73	142.937
11	(1,2)	0.9131	42.773	2.89	78.181	(1,5,6)	0.9021	42.261	4.29	109.8	(1,2,3,6)	0.9479	44.383	5.71	139.87
12	(4,6)	0.0487	2.283	2.84	48.67	(1,5,6)	0.9021	42.261	4.29	109.8	(1,2,3,6)	0.9479	44.383	5.71	139.87
13	(1,3)	0.8697	40.742	2.86	77.847	(1,2,3)	0.9269	43.398	4.29	114.2	(1,2,3,6)	0.9479	44.383	5.71	139.87
14	(2,5)	0.0814	3.815	2.81	65.9	(1,2,3)	0.9269	43.398	4.29	114.2	(1,2,4,5)	0.966	45.23	5.73	142.937
15	(2,5)	0.0814	3.815	2.81	65.9	(1,2,3)	0.9269	43.398	4.29	114.2	(1,2,4,5)	0.966	45.23	5.73	142.937
16	(2,5)	0.0814	3.815	2.81	65.9	(1,2,4)	0.9413	44.071	4.31	116.597	(1,2,4,5)	0.966	45.23	5.73	142.937
17	(1,2)	0.9131	42.773	2.89	78.181	(3,5,6)	0.059	2.769	4.24	80.19	(1,2,4,5)	0.966	45.23	5.73	142.937
18	(3,4)	0.041	1.923	2.18	48.2	(3,5,6)	0.059	2.769	4.24	80.19	(1,2,3,6)	0.9479	44.383	5.71	139.87
19	(1,6)	0.8774	41.102	2.77	77.9	(3,5,6)	0.059	2.769	4.24	80.19	(1,2,3,6)	0.9479	44.383	5.71	139.87
20	(2,5)	0.0814	3.815	2.81	65.9	(3,5,6)	0.059	2.769	4.24	80.19	(1,2,3,6)	0.9479	44.383	5.71	139.87
21	(2,5)	0.0814	3.815	2.81	65.9	(1,2,4)	0.9413	44.071	4.31	116.597	(1,2,4,6)	0.966	45.23	5.71	139.87
22	(1,3)	0.8697	40.742	2.81	77.847	(1,2,4)	0.9413	44.071	4.31	116.597	(1,2,4,6)	0.966	45.23	5.71	139.87
23	(2,4)	0.0844	3.954	2.81	67.2	(1,3,5)	0.8944	41.901	4.26	98.17	(1,2,4,6)	0.966	45.23	5.71	139.87
24	(2,4)	0.0844	3.954	2.81	67.2	(1,3,5)	0.8944	41.901	4.26	98.17	(1,2,4,6)	0.966	45.23	5.71	139.87

Table 4. The day-ahead dynamic capacity withholding indices for the 30-bus system.

Hour	NDCWG =5					NDCWG =6				
	DCWG	DCWI <sub>DA</sub>	$\Delta p^{withheld}$ (MW)	$\Delta \lambda^{distortion}$ (\$/MWh)	$\lambda$ (\$/MWh)	DCWG	DCWI <sub>DA</sub>	$\Delta p^{withheld}$ (MW)	$\Delta \lambda^{distortion}$ (\$/MWh)	$\lambda$ (\$/MWh)
1	(1,2,3,4,5)	0.9793	45.855	7.13	166.957	(1,2,3,4,5,6)	1	50.2	8.55	190.977
2	(1,2,3,4,5)	0.9793	45.855	7.13	166.957	(1,2,3,4,5,6)	1	50.2	8.55	190.977
3	(1,2,3,4,5)	0.9793	45.855	7.13	166.957	(1,2,3,4,5,6)	1	50.2	8.55	190.977
4	(1,2,3,4,5)	0.9793	45.855	7.13	166.957	(1,2,3,4,5,6)	1	50.2	8.55	190.977
5	(1,2,3,4,5)	0.9793	45.855	7.13	166.957	(1,2,3,4,5,6)	1	50.2	8.55	190.977
6	(1,2,3,4,6)	0.9756	45.681	7.11	165.9	(1,2,3,4,5,6)	1	50.2	8.55	190.977
7	(1,2,3,4,6)	0.9756	45.681	7.11	165.9	(1,2,3,4,5,6)	1	50.2	8.55	190.977
8	(1,2,3,4,6)	0.9756	45.681	7.11	165.9	(1,2,3,4,5,6)	1	50.2	8.55	190.977
9	(1,2,3,4,6)	0.9756	45.681	7.11	165.9	(1,2,3,4,5,6)	1	50.2	8.55	190.977
10	(1,2,3,4,6)	0.9756	45.681	7.11	165.9	(1,2,3,4,5,6)	1	50.2	8.55	190.977
11	(1,2,3,4,5)	0.9793	45.855	7.13	166.957	(1,2,3,4,5,6)	1	50.2	8.55	190.977
12	(1,2,3,4,5)	0.9793	45.855	7.13	166.957	(1,2,3,4,5,6)	1	50.2	8.55	190.977
13	(1,2,3,4,5)	0.9793	45.855	7.13	166.957	(1,2,3,4,5,6)	1	50.2	8.55	190.977
14	(1,2,3,4,5)	0.9793	45.855	7.13	166.957	(1,2,3,4,5,6)	1	50.2	8.55	190.977
15	(1,2,3,4,5)	0.9793	45.855	7.13	166.957	(1,2,3,4,5,6)	1	50.2	8.55	190.977
16	(1,2,3,4,6)	0.9756	45.681	7.11	165.9	(1,2,3,4,5,6)	1	50.2	8.55	190.977
17	(1,2,3,4,6)	0.9756	45.681	7.11	165.9	(1,2,3,4,5,6)	1	50.2	8.55	190.977
18	(1,2,3,4,6)	0.9756	45.681	7.11	165.9	(1,2,3,4,5,6)	1	50.2	8.55	190.977
19	(1,2,3,4,6)	0.9756	45.681	7.11	165.9	(1,2,3,4,5,6)	1	50.2	8.55	190.977
20	(1,2,3,4,6)	0.9756	45.681	7.11	165.9	(1,2,3,4,5,6)	1	50.2	8.55	190.977
21	(1,2,3,4,5)	0.9793	45.855	7.13	166.957	(1,2,3,4,5,6)	1	50.2	8.55	190.977
22	(1,2,3,4,5)	0.9793	45.855	7.13	166.957	(1,2,3,4,5,6)	1	50.2	8.55	190.977
23	(1,2,3,4,5)	0.9793	45.855	7.13	166.957	(1,2,3,4,5,6)	1	50.2	8.55	190.977
24	(1,2,3,4,5)	0.9793	45.855	7.13	166.957	(1,2,3,4,5,6)	1	50.2	8.55	190.977

It is assumed that the independent system operator rejects the bid of generation companies that their estimated values of  $DCWI_{DA} = 0.15$ . Fig. 5 presents the optimal day-ahead scheduling of generation companies and responsive loads that was performed by the independent system operator in the fourth stage of the optimization process. The corresponding value of  $DCWI_{DA}$  for the optimal day-ahead generation companies scheduling was about 0.091. As shown in Fig 5, the optimization

procedure committed the entire generation companies to reduce the  $DCWI_{DA}$  and their corresponding capacity withholding opportunities.

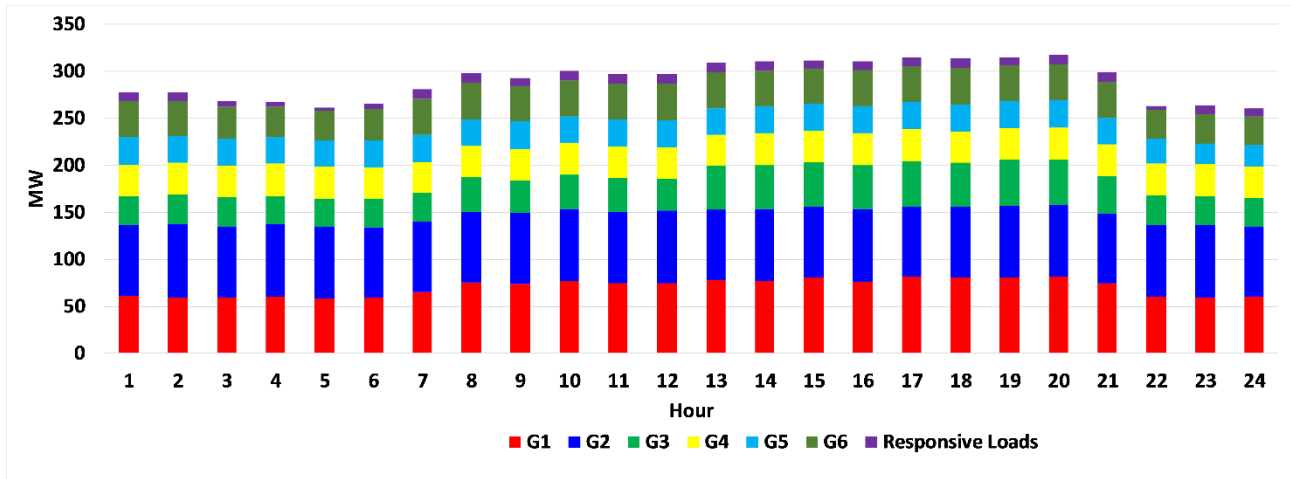


Fig 5. The optimal day-ahead unit commitment of generation units and responsive loads for the 30-bus system.

### 5.2. 57-bus IEEE test system

The second case study was carried out for IEEE 57-bus system. Fig. 6 and Fig. 7 depict the IEEE 57-bus system topology and daily load curve, respectively. At first, the first and second stage of simulations were carried out. Table 5 presents the maximum values of  $DCWIMT$  for the entire mid-term optimization horizon.

As shown in Table 5, the high values of  $DCWIMT$  indicated the dynamic capacity withholding groups could withhold power generation from the market by maintenance scheduling. The (1, 3) and (1, 2, 3) groups were 0.198 and 0.287, respectively. The highest values of  $DCWIMT$  for the three-member and four-member were 0.349 and 0.493, respectively. The  $DCWIMT$  was highly increased when the number of capacity withholding members were increased. The formation of dynamic capacity withholding groups that their  $DCWIMT$  values were high should be prevented. Then, the third and fourth stages of simulations were carried out. Table 6 and Table 7 show the outputs of the third and fourth stages of the optimization process for the day-ahead optimization horizon. The first and second ranks of dynamic capacity withholding groups are marked in orange and yellow, respectively.

The maximum values of  $DCWI_{DA}$  for the  $NDCWG=2$  were estimated about 0.9226 that were for first, ninth and tenth hours and belonged to the (1, 7) group. The  $DCWI_{DA}$  took on a value 0.9531 for  $NDCWG=3$  that it was about 103.3% of its corresponding value for  $NDCWG=2$ . The  $DCWI_{DA}$  values were highly increased for  $NDCWG=4$ ,  $NDCWG=5$ ,  $NDCWG=6$  and  $NDCWG=7$  with their corresponding values of  $DCWI_{DA}=0.968$ ,  $DCWI_{DA}=0.9846$ ,  $DCWI_{DA}=0.9917$  and  $DCWI_{DA}=1$ , respectively. The highest value of nodal price changes was 8.6 \$/MWh that was for the  $NDCWG=7$ . The maximum nodal price for the  $NDCWG=7$  was reached to 200.55 \$/MWh that corresponded to 764.43% increase of the nodal price.



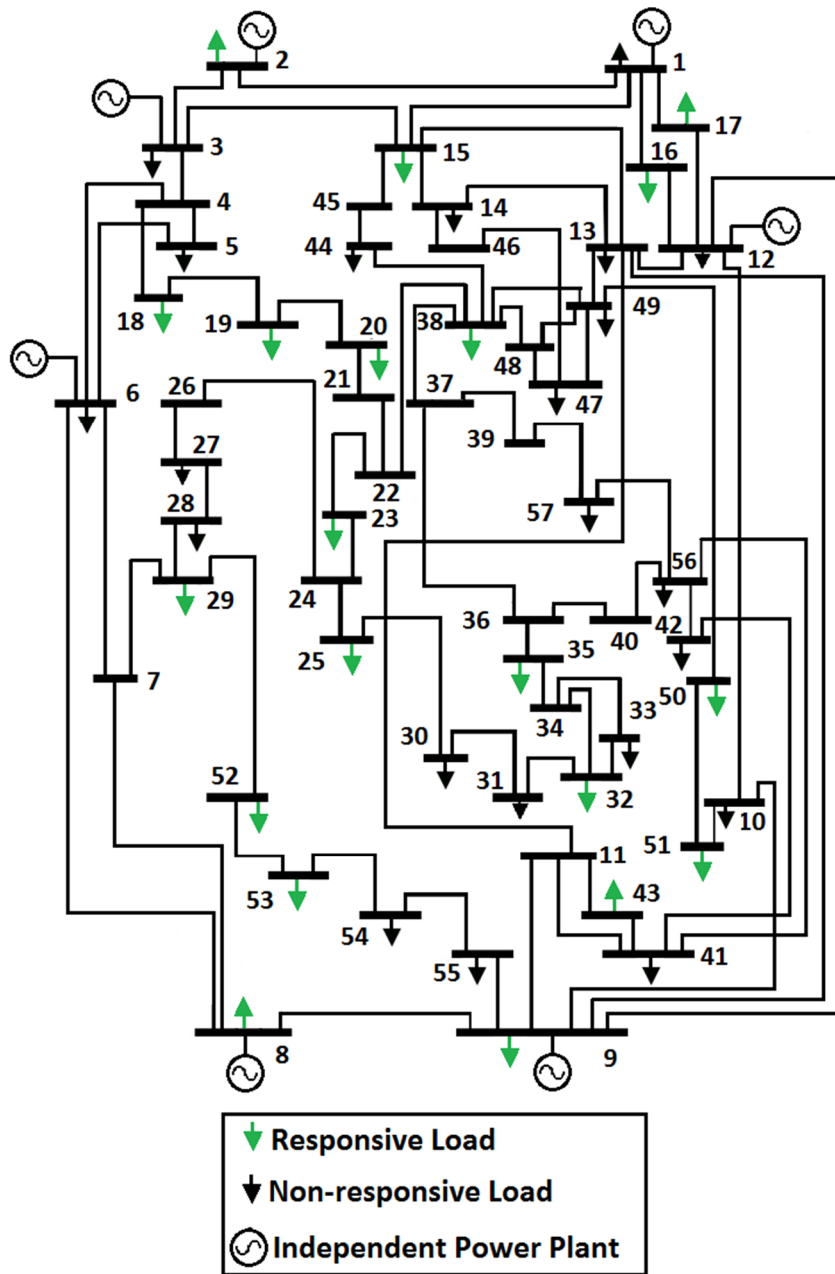


Fig. 6. The topology of 57-bus IEEE test system.

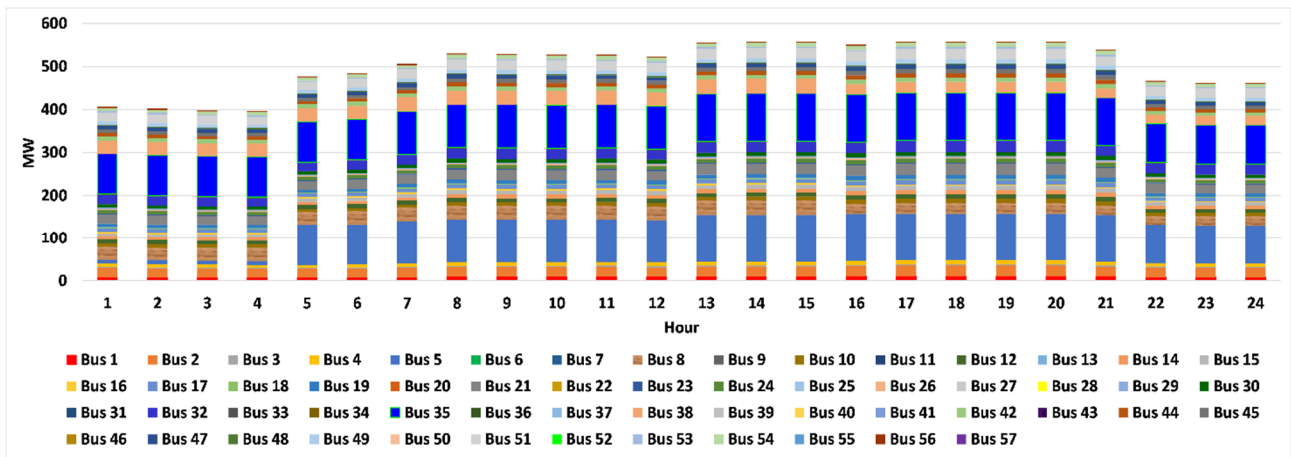


Fig. 7. Estimated day-ahead load forecasting of 30-bus IEEE tests system.

Table 5. The mid-term DCWI for the 57-bus system.

<i>NDCWG</i>	<i>DCWGs</i>	<i>DCWI<sub>MT</sub></i>
2	1,3	0.198
3	1,2,3	0.287
4	1,2,3,8	0.349
5	1,2,3,8,9	0.493

Table 6. The day-ahead dynamic capacity withholding indices for the 57-bus system.

<i>Hour</i>	<i>NDCWG=2</i>					<i>NDCWG=3</i>					<i>NDCWG=4</i>				
	<i>DCWG</i>	<i>DCWI<sub>DA</sub></i>	<i>A<sub>p</sub><sup>withheld</sup></i> (MW)	<i>λ<sub>c</sub><sup>distortion</sup></i> (\$/MWh)	<i>λ</i> (\$/MWh)	<i>DCWG</i>	<i>DCWI<sub>DA</sub></i>	<i>A<sub>p</sub><sup>withheld</sup></i> (MW)	<i>λ<sub>c</sub><sup>distortion</sup></i> (\$/MWh)	<i>λ</i> (\$/MWh)	<i>DCWG</i>	<i>DCWI<sub>DA</sub></i>	(MW)	<i>λ<sub>c</sub><sup>distortion</sup></i> (\$/MWh)	<i>λ</i> (\$/MWh)
1	(1,7)	0.9226	80.234	2.47	57.14	(1,2,7)	0.9531	82.89	3.66	85.5	(1,2,4,7)	0.968	84.188	4.87	114.2
2	(2,4)	0.0454	3.954	2.36	24.04	(3,4,5)	0.0386	1.9379	3.17	36.2	(1,2,4,7)	0.968	84.188	4.87	114.2
3	(1,5)	0.4779	41.276	2.45	36.8	(3,4,5)	0.0386	1.9379	3.17	36.2	(1,3,5,6)	0.4963	42.886	4.22	96.9
4	(3,6)	0.0184	1.61	2.37	23.2	(1,2,6)	0.5031	43.758	3.5	66.9	(1,3,5,6)	0.4963	42.886	4.22	96.9
5	(1,2)	0.4918	42.773	2.44	37.9	(1,2,6)	0.5031	43.758	3.5	66.9	(1,3,5,6)	0.4963	42.886	4.22	96.9
6	(1,2)	0.4918	42.773	2.44	37.9	(1,2,6)	0.5031	43.758	3.5	66.9	(1,3,5,6)	0.4963	42.886	4.22	96.9
7	(1,4)	0.4762	41.415	2.45	36.9	(1,2,4)	0.5067	44.071	3.56	38.1	(1,2,3,4)	0.5138	44.696	4.56	100.8
8	(2,3)	0.0376	3.281	2.42	23.8	(1,2,4)	0.5067	44.071	3.56	38.1	(1,2,3,4)	0.5138	44.696	4.56	100.8
9	(1,7)	0.9226	80.234	2.47	57.14	(1,5,7)	0.9392	81.393	3.14	67.1	(1,2,3,4)	0.5138	44.696	4.56	100.8
10	(1,7)	0.9226	80.234	2.47	57.14	(1,5,7)	0.9392	81.393	3.14	67.1	(1,2,5,7)	0.9679	84.049	4.54	100.8
11	(1,2)	0.4918	42.773	2.44	37.9	(1,2,3)	0.4989	43.398	3.6	39.3	(1,2,5,7)	0.9679	84.049	4.54	100.8
12	(3,6)	0.0184	1.61	2.37	23.2	(1,2,3)	0.4989	43.398	3.6	39.3	(1,3,4,6)	0.4946	43.025	4.17	95.5
13	(1,5)	0.4779	41.276	2.45	36.8	(1,4,6)	0.4875	42.4	3.15	37.9	(1,3,4,6)	0.4946	43.025	4.17	95.5
14	(2,6)	0.0418	3.641	2.46	24.02	(1,4,6)	0.4875	42.4	3.15	37.9	(1,3,4,6)	0.4946	43.025	4.17	95.5
15	(2,6)	0.0418	3.641	2.46	24.02	(1,2,5)	0.5084	43.932	3.57	38.1	(1,3,4,6)	0.4946	43.025	4.17	95.5
16	(1,3)	0.4684	40.742	2.34	35.88	(1,2,5)	0.5084	43.932	3.57	38.1	(1,2,3,5)	0.5155	44.557	4.43	100.2
17	(2,7)	0.4918	42.773	2.19	37.2	(1,3,7)	0.9297	80.859	3.18	64.3	(1,2,3,5)	0.5155	44.557	4.43	100.2
18	(1,4)	0.4762	41.415	2.45	36.9	(1,3,7)	0.9297	80.859	3.18	64.3	(1,2,3,5)	0.5155	44.557	4.43	100.2
19	(1,4)	0.4762	41.415	2.45	36.9	(1,2,4)	0.5067	44.071	3.56	38.1	(1,2,6,7)	0.968	84.188	3.9	99.8
20	(1,2)	0.4918	42.773	2.44	37.9	(1,2,4)	0.5067	44.071	3.56	38.1	(1,2,4,7)	0.968	84.188	4.87	114.2
21	(3,5)	0.0173	1.784	2.28	23.9	(1,3,5)	0.485	41.901	3.33	37.5	(1,2,4,7)	0.968	84.188	4.87	114.2
22	(1,6)	0.4726	41.102	2.24	35.8	(1,3,5)	0.485	41.901	3.33	37.5	(1,2,3,5)	0.5155	44.557	4.43	100.2
23	(2,4)	0.0454	3.954	2.36	24.04	(1,2,6)	0.5031	43.758	3.5	66.9	(1,2,3,5)	0.5155	44.557	4.43	100.2
24	(2,4)	0.0454	3.954	2.36	24.04	(1,2,6)	0.5031	43.758	3.5	66.9	(1,2,3,5)	0.5155	44.557	4.43	100.2

Table 7. The day-ahead dynamic capacity withholding indices for the 57-bus system.

<i>Hour</i>	<i>NDCWG=5</i>					<i>NDCWG=6</i>					<i>NDCWG=7</i>				
	<i>DCWG</i>	<i>DCWI<sub>DA</sub></i>	<i>A<sub>p</sub><sup>withheld</sup></i> (MW)	<i>λ<sub>c</sub><sup>distortion</sup></i> (\$/MWh)	<i>λ</i> (\$/MWh)	<i>DCWG</i>	<i>DCWI<sub>DA</sub></i>	<i>A<sub>p</sub><sup>withheld</sup></i> (MW)	<i>λ<sub>c</sub><sup>distortion</sup></i> (\$/MWh)	<i>λ</i> (\$/MWh)	<i>DCWG</i>	<i>DCWI<sub>DA</sub></i>	(MW)	<i>λ<sub>c</sub><sup>distortion</sup></i> (\$/MWh)	<i>λ</i> (\$/MWh)
1	(1,2,4,5,7)	0.9846	85.347	6.06	142.5	(1,2,3,4,5,7)	0.9917	85.972	7.26	171.9	(1,2,3,4,5,6,7)	1	86.957	8.6	200.55
2	(1,2,4,5,7)	0.9846	85.347	6.06	142.5	(1,2,3,4,5,7)	0.9917	85.972	7.26	171.9	(1,2,3,4,5,6,7)	1	86.957	8.6	200.55
3	(1,2,4,5,7)	0.9846	85.347	6.06	142.5	(1,2,3,4,5,7)	0.9917	85.972	7.26	171.9	(1,2,3,4,5,6,7)	1	86.957	8.6	200.55
4	(1,2,4,5,7)	0.9846	85.347	6.06	142.5	(1,2,3,4,5,7)	0.9917	85.972	7.26	171.9	(1,2,3,4,5,6,7)	1	86.957	8.6	200.55
5	(1,2,4,5,7)	0.9846	85.347	6.06	142.5	(1,2,3,4,5,7)	0.9917	85.972	7.26	171.9	(1,2,3,4,5,6,7)	1	86.957	8.6	200.55
6	(1,2,3,4,6)	0.5251	45.681	5.89	121.2	(1,2,3,4,5,7)	0.9917	85.972	7.26	171.9	(1,2,3,4,5,6,7)	1	86.957	8.6	200.55
7	(1,2,3,4,6)	0.5251	45.681	5.89	121.2	(1,2,3,4,5,6)	0.5417	46.84	7.01	140.8	(1,2,3,4,5,6,7)	1	86.957	8.6	200.55
8	(1,2,3,4,6)	0.5251	45.681	5.89	121.2	(1,2,3,4,5,6)	0.5417	46.84	7.01	140.8	(1,2,3,4,5,6,7)	1	86.957	8.6	200.55
9	(1,2,3,4,6)	0.5251	45.681	5.89	121.2	(1,2,3,4,5,6)	0.5417	46.84	7.01	140.8	(1,2,3,4,5,6,7)	1	86.957	8.6	200.55
10	(1,2,3,5,7)	0.9768	84.674	5.95	140.6	(1,2,3,4,5,6)	0.5417	46.84	7.01	140.8	(1,2,3,4,5,6,7)	1	86.957	8.6	200.55
11	(1,2,3,5,7)	0.9768	84.674	5.95	140.6	(1,2,3,4,5,6)	0.5417	46.84	7.01	140.8	(1,2,3,4,5,6,7)	1	86.957	8.6	200.55
12	(1,2,3,5,7)	0.9768	84.674	5.95	140.6	(1,2,3,4,5,6)	0.5417	46.84	7.01	140.8	(1,2,3,4,5,6,7)	1	86.957	8.6	200.55
13	(1,2,3,5,7)	0.9768	84.674	5.95	140.6	(1,2,3,4,5,7)	0.9917	85.972	7.26	171.9	(1,2,3,4,5,6,7)	1	86.957	8.6	200.55
14	(1,2,3,4,6)	0.5251	45.681	5.89	121.2	(1,2,3,4,5,7)	0.9917	85.972	7.26	171.9	(1,2,3,4,5,6,7)	1	86.957	8.6	200.55
15	(1,2,3,4,6)	0.5251	45.681	5.89	121.2	(1,2,3,4,5,7)	0.9917	85.972	7.26	171.9	(1,2,3,4,5,6,7)	1	86.957	8.6	200.55
16	(1,2,3,4,6)	0.5251	45.681	5.89	121.2	(1,2,3,4,5,7)	0.9917	85.972	7.26	171.9	(1,2,3,4,5,6,7)	1	86.957	8.6	200.55
17	(1,2,3,4,6)	0.5251	45.681	5.89	121.2	(1,2,3,4,5,7)	0.9917	85.972	7.26	171.9	(1,2,3,4,5,6,7)	1	86.957	8.6	200.55
18	(1,2,3,5,7)	0.9768	84.674	5.95	140.6	(1,2,3,4,5,7)	0.9917	85.972	7.26	171.9	(1,2,3,4,5,6,7)	1	86.957	8.6	200.55
19	(1,2,3,5,7)	0.9768	84.674	5.95	140.6	(1,2,3,4,5,6)	0.5417	46.84	7.01	140.8	(1,2,3,4,5,6,7)	1	86.957	8.6	200.55
20	(1,2,3,5,7)	0.9768	84.674	5.95	140.6	(1,2,3,4,5,6)	0.5417	46.84	7.01	140.8	(1,2,3,4,5,6,7)	1	86.957	8.6	200.55
21	(1,2,3,5,7)	0.9768	84.674	5.95	140.6	(1,2,3,4,5,6)	0.5417	46.84	7.01	140.8	(1,2,3,4,5,6,7)	1	86.957	8.6	200.55
22	(1,2,3,4,6)	0.5251	45.681	5.89	121.2	(1,2,3,4,5,6)	0.5417	46.84	7.01	140.8	(1,2,3,4,5,6,7)	1	86.957	8.6	200.55
23	(1,2,3,4,6)	0.5251	45.681	5.89	121.2	(1,2,3,4,5,6)	0.5417	46.84	7.01	140.8	(1,2,3,4,5,6,7)	1	86.957	8.6	200.55
24	(1,2,3,4,6)	0.5251	45.681	5.89	121.2	(1,2,3,4,5,6)	0.5417	46.84	7.01	140.8	(1,2,3,4,5,6,7)	1	86.957	8.6	200.55

Fig. 8 presents the optimal day-ahead scheduling of generation companies and responsive loads that was performed by the independent system operator in the fourth stage of the optimization process. The corresponding value of  $DCWI_{DA}$  for the optimal day-ahead generation companies scheduling was about 0.085. As shown in Fig 8, the fourth stage optimization process committed the generation companies to reduce the  $DCWI_{DA}$  values.

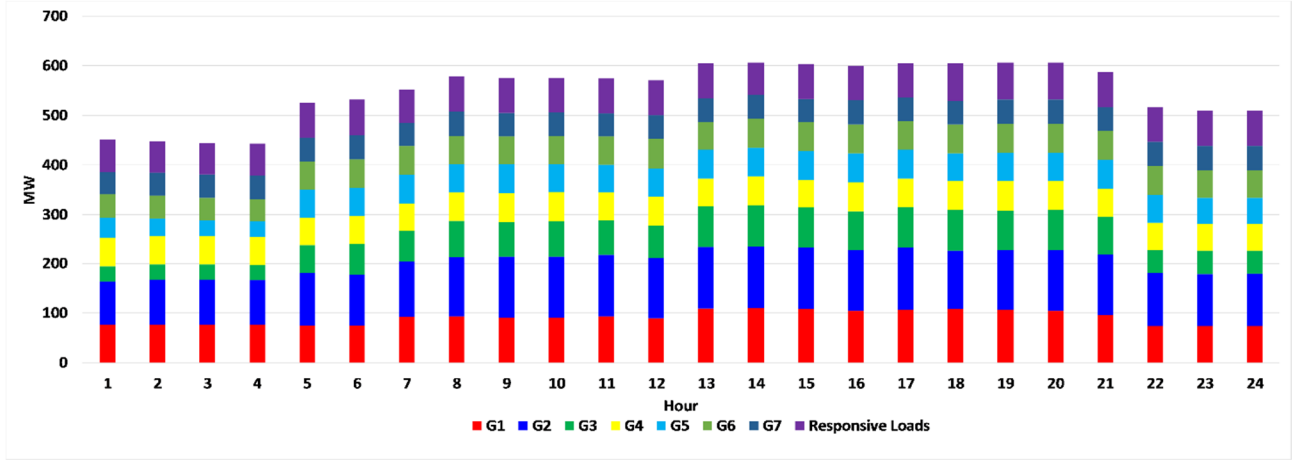


Fig 8. The optimal day-ahead unit commitment of generation units and responsive loads for the 57-bus system.

### 5.3. 118-bus IEEE test system

The simulation of the proposed algorithm was carried out for the IEEE 118-bus system. Fig. 9 shows the IEEE 118-bus system topology that 30 buses of the 118-bus system are responsive loads. Fig. 10 presents the daily load curve of the 118-bus system. The first and second stages of simulations were performed and the values of  $DCWI_{MT}$  were calculated. Table 8 presents the maximum values of  $DCWI_{MT}$  for the mid-term optimization horizon.

Table 8. The mid-term DCWI for the 118-bus system.

$NDCWG$	$DCWGs$	$DCWI_{MT}$
2 Unit	12,13	0.136
3 Unit	1,12,13	0.165
4 Unit	1,2,12,13	0.246
5 Unit	1,2,12,13,14	0.298
6 Unit	1,2,6,12,13,14	0.367
7 Unit	1,2,3,6,12,13,14	0.401
8 Unit	1,2,3,6,10,12,13,14	0.459
9 Unit	1,2,3,6,10,12,13,14,19	0.489
10 Unit	1,2,3,6,10,12,13,14,16,19	0.529
11 Unit	1,2,3,6,8,10,12,13,14,16,19	0.557
12 Unit	1,2,3,6,8,10,12,13,14,16,17,19	0.576
13 Unit	1,2,3,6,8,10,12,13,14,15,16,17,19	0.587
14 Unit	1,2,3,5,6,8,10,12,13,14,15,16,17,19	0.601

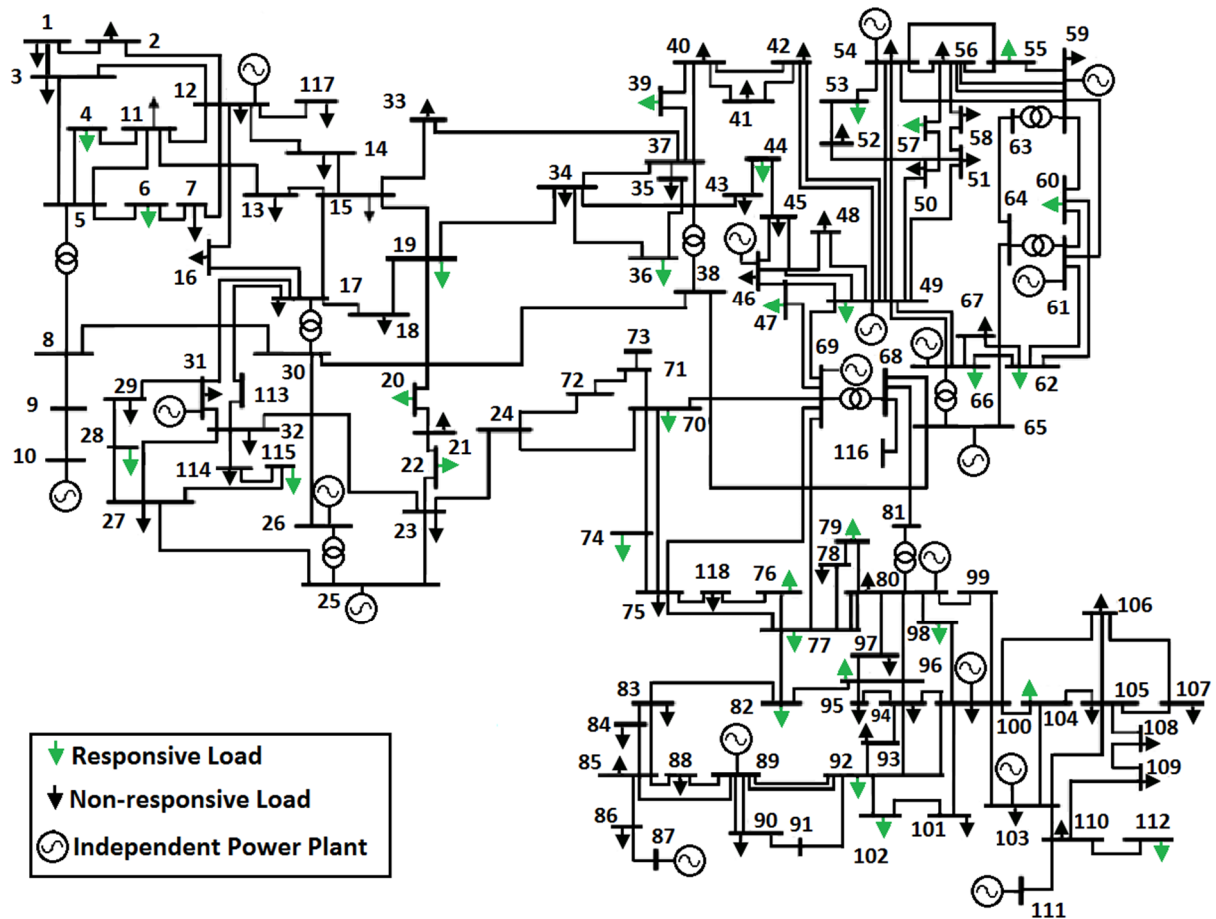


Fig. 9. The topology of 118-bus IEEE test system.

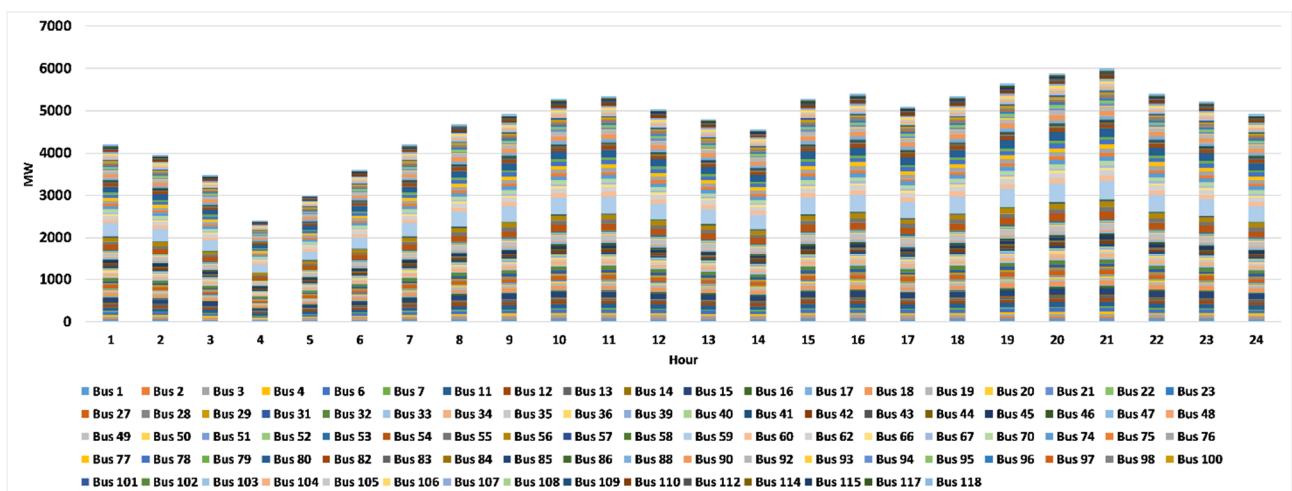


Fig. 10. Estimated day-ahead load forecasting of 118-bus IEEE tests system.

As shown in Table 8, the  $DCWI_{MT}$  values for (13, 23) and (1, 12, 13) groups were 0.136 and 0.165, respectively. The highest values of  $DCWI_{MT}$  for the four, five and six members were 0.246, 0.298, and 0.367, respectively. The  $DCWI_{MT}$  was highly increased when the number of capacity withholding members were increased. The highest value of  $DCWI_{MT}$  took on a value 0.601 for  $NDCWG=14$ . Then, the independent system operator performed the third and fourth stages of simulations and calculated their corresponding values of  $DCWI_{DA}$ .

Tables AII.1-AII.4 present the outputs of the third and fourth stages of the optimization process for cases that their  $DCWI_{DA}$  were at the highest values for the day-ahead optimization horizon. The first ranks of dynamic capacity withholding groups are marked in orange. The maximum values of  $DCWI_{DA}$  for the  $NDCWG=2$  were about 0.136 that was for (12, 13) group. The  $DCWI_{DA}$  took on a value 0.165 for  $NDCWG=3$ . The  $DCWI_{DA}$  values were highly increased when the dynamic capacity withholding group members increased. The  $DCWI_{DA}$  values for  $NDCWG=4$ ,  $NDCWG=5$ ,  $NDCWG=6$  and  $NDCWG=7$  were  $DCWI_{DA}=0.246$ ,  $DCWI_{DA}=0.298$ ,  $DCWI_{DA}=0.367$  and  $DCWI_{DA}=0.401$ , respectively. The dynamic capacity withholding groups for the 118-bus system had the same pattern for multiple hours based on the fact that the ability of generation companies to form a DCWG was increased for large scale power system and multiple generation companies and system constraints increased the number of possible withholding opportunities. The  $DCWI_{DA}$  values for  $NDCWG=8$ ,  $NDCWG=9$ ,  $NDCWG=10$  and  $NDCWG=11$  were  $DCWI_{DA}=0.459$ ,  $DCWI_{DA}=0.489$ ,  $DCWI_{DA}=0.529$  and  $DCWI_{DA}=0.557$ , respectively. Further, the  $DCWI_{DA}$  values for  $NDCWG=12$ ,  $NDCWG=13$ ,  $NDCWG=14$ ,  $NDCWG=15$ ,  $NDCWG=16$ ,  $NDCWG=17$ ,  $NDCWG=18$  and  $NDCWG=19$  were  $DCWI_{DA}=0.576$ ,  $DCWI_{DA}=0.587$ ,  $DCWI_{DA}=0.601$ ,  $DCWI_{DA}=0.712$ ,  $DCWI_{DA}=0.856$ ,  $DCWI_{DA}=0.892$ ,  $DCWI_{DA}=0.943$ , and  $DCWI_{DA}=1$ , respectively. The ability of capacity withholding of generation companies was highly dependent on their constraints, system operational states and constraints.

Table 9 presents a brief summary of Table 9-Table 12. As shown in Table 9, the highest value of nodal price changes was 9.158 \$/MWh that was for the  $NDCWG=19$ . The maximum nodal price for the  $NDCWG=19$  was reached to 220.01 \$/MWh that corresponded to 851.2% increase of the nodal price.

Table 9. The brief summary of day-ahead dynamic capacity withholding indices for the 118-bus system.

<i>NDCWG</i>	<i>DCWG</i>	<i>DCWI<sub>DA</sub></i>	<i>Withheld Power (MW)</i>	$\Delta\lambda^{distortion}$ (\$/MWh)	$\lambda$ (\$/MWh)
2	12,13	0.136	264	0.45	23.15
3	1,12,13	0.165	420	0.536	34.73
4	1,2,12,13	0.246	504	0.597	46.31
5	1,2,12,13,14	0.298	564	1.203	57.89
6	1,2,6,12,13,14	0.367	606	1.588	69.47
7	1,2,3,6,12,13,14	0.401	678	1.667	81.05
8	1,2,3,6,10,12,13,14	0.459	754	2.03	92.63
9	1,2,3,6,10,12,13,14,19	0.489	802	2.504	104.21
10	1,2,3,6,10,12,13,14,16,19	0.529	838	3.073	115.78
11	1,2,3,6,8,10,12,13,14,16,19	0.557	880	3.407	127.36
12	1,2,3,6,8,10,12,13,14,16,17,19	0.576	922	3.925	138.94
13	1,2,3,6,8,10,12,13,14,15,16,17,19	0.587	964	4.552	150.52
14	1,2,3,5,6,8,10,12,13,14,15,16,17,19	0.601	1000	4.654	162.10
15	1,2,3,4,5,6,8,10,12,13,14,15,16,17,19	0.712	1036	4.746	173.68
16	1,2,3,4,5,6,7,8,10,12,13,14,15,16,17,19	0.856	1080	5.089	185.26
17	1,2,3,4,5,6,7,8,9,10,12,13,14,15,16,17,19	0.892	1110	6.411	196.84
18	1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,19	0.943	1150	7.21	208.42
19	1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19	1	1180	9.158	220.01

Fig. 11 presents the optimal day-ahead scheduling of generation companies and responsive loads that was performed by the independent system operator in the fourth stage of the optimization process. The corresponding value of  $DCWI_{DA}$  for the optimal day-ahead generation companies scheduling was about 0.053. The algorithm committed the entire generation companies to reduce  $DCWI_{DA}$  values and limited withholding opportunities. The CPU time required for solving the four-stage problem for the 118-bus system is presented in Table 10. The number of equations for the day-ahead problem is 5112898 that show the curse of dimensionality and the CPU time was about 67 seconds for solving the four-stage problem.

Table 10. The number of equations and discrete variable for the 118-bus system.

Problem	Single equations	Single variables	Discrete variables	CPU time (sec)
First and second stages problems	730413	365069	4246	14
Third and fourth stages problems	5112898	2555490	29729	53

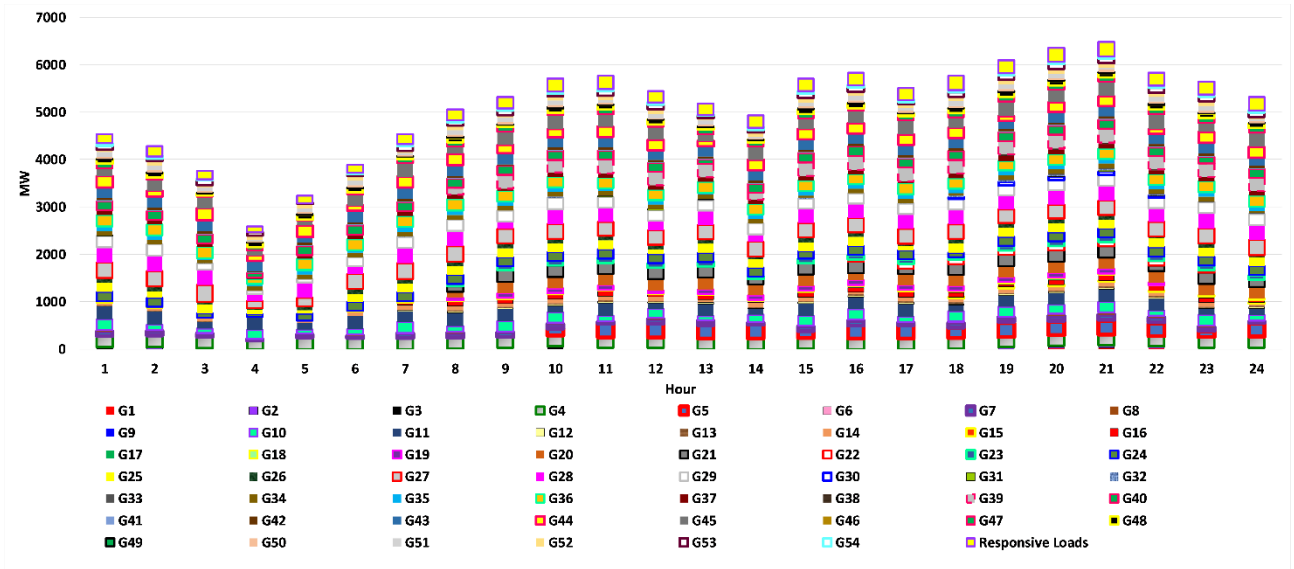


Fig 11. The optimal day-ahead unit commitment of generation units and responsive loads for the 118-bus system.

## 6. Conclusion

This paper proposed a mixed-integer linear optimization algorithm for mid-term and day-ahead scheduling of generation companies that formed multiple dynamic capacity withholding groups, withheld power from the market and changed the nodal prices of system. The four-stage optimization algorithm was proposed that at the first stage the generation companies' maintenance scheduling to maximize their profits was explored. At the second stage, the system operator evaluated the maintenance scheduling of generation companies and optimized the mid-term scheduling of units considering the system and resources constraints. At the third stage, the day-ahead formation of capacity withholding groups was explored and at the fourth stage, the optimal scheduling of generation companies in the day-ahead horizon was carried out. Two mid-term and day-ahead capacity withholding indices were introduced to assess the possible withholding groups in an ex-ante manner. Three test systems were assessed the proposed algorithm that were 30-bus, 57-bus and 118-bus IEEE systems. The proposed algorithm detected that the dynamic capacity withholding might lead to an increase of nodal price by about 279.22%, 764.43%, and 851.2% for the 30-bus, 57 bus and 118 bus test systems with respect to the full competition of generation companies, respectively. The proposed algorithm solved the four-stage problem in about 67 seconds that consisted of 5843311 equations. In conclusion, the adoption of the proposed dynamic capacity withholding assessment method can detect the possible formation of groups that should be prevented in an ex-ante manner by the system operator. The authors are investigating the use of other withholding indices to find the capacity withholding opportunities.

## 7. Appendix I

The Lagrangian function of the second stage problem can be written as:

$$\begin{aligned}
\Lambda = & \sum_{t=1}^{24} \sum_{l=1}^{NLB} \left( -\frac{1}{2} v \cdot \gamma^2 + \zeta \cdot \gamma \right) - \sum_{t=1}^{24} \sum_{k=1}^{NGB} (MC_{i,t} \cdot p_{i,t} + SU_{i,t} + SD_{i,t}) \cdot K \\
& - \sum_{k=1}^{NGB} (\mu(P_i - P_i^{\max})) - (\mu'(P_i^{\min} - P_i)) \\
& - \sum_{m=1}^{NB} (\varphi(V_i - V_i^{\max})) - (\varphi'(V_i^{\min} - V_i)) \\
& - \left( \sum_{k=1}^{NGB} \kappa p_{k,t} + \sum_{l=1}^{NLB} \kappa |V_{n,t}| \cdot |V_{m,t}| \cdot |Y_{nm}| \cdot \cos(\theta_n - \theta_m) \right) \\
& - \left( \sum_{k=1}^{NGB} \kappa' q_{k,t} + \sum_{l=1}^{NLB} \kappa' |V_{n,t}| \cdot |V_{m,t}| \cdot |Y_{nm}| \cdot \sin(\theta_n - \theta_m) \right)
\end{aligned} \tag{AI.1}$$

Equation (AI.1) can be written for non-competitive and full-competitive markets. By derivation of  $p$  variable in Eq. (AI.1) and replacement of  $\frac{\partial \gamma}{\partial p_i} = v$ , the following formulation can be written

[AI.2]:

$$\begin{bmatrix} \Delta p_1^{withheld} \\ \vdots \\ \vdots \\ \vdots \\ \Delta p_{NGU}^{withheld} \end{bmatrix} = v \begin{bmatrix} \frac{P_1^{NC}}{a_1} \\ \vdots \\ \vdots \\ \vdots \\ \frac{P_{NGU}^{NC}}{a_{NGU}} \end{bmatrix} + \begin{bmatrix} \frac{\mu_i^{NC} - \mu_i^{NC} + \mu_i^e - \mu_i^{e'}}{a_1} \\ \vdots \\ \vdots \\ \vdots \\ \frac{\mu_{NGU}^{NC} - \mu_{NGU}^{NC} + \mu_{NGU}^{NC} - \mu_{NGU}^{NC}}{a_{NGU}} \end{bmatrix} \tag{AI.2}$$

The  $DCWI_{DA}$  can be presented as (AI.3) that is the ratio of active power generation of generation unit in dynamic capacity withholding condition and full-competition condition. The FC and NC superscripts present the full competition and non-competition (or dynamic capacity withholding) conditions, respectively.

$$DCWI_{DA} = \frac{\sum_{i=1}^{NDCWG} \left( \frac{v}{a_i} (p_i^{NC}) + \frac{1}{a_i} (\mu_i^{FC} - \mu_i'^{FC} + \mu_i^{NC} - \mu_i'^{NC}) \right)}{\sum_{j=1}^{NGU} \left( \frac{v}{a_j} (p_j^{NC}) + \frac{1}{a_j} (\mu_j^{FC} - \mu_j'^{FC} + \mu_j^{NC} - \mu_j'^{NC}) \right)} \tag{AI.3}$$



## 8. Appendix II

Table AII.1. The day-ahead dynamic capacity withholding indices for the 118-bus system.

Hour	NDCWG=2			NDCWG=3			NDCWG=4			NDCWG=5			NDCWG=6		
	DCWG	DCWI <sub>DA</sub>	Withheld Power (MW)	DCWG	DCWI <sub>DA</sub>	Withheld Power (MW)	DCWG	DCWI <sub>DA</sub>	Withheld Power (MW)	DCWG	DCWI <sub>DA</sub>	Withheld Power (MW)	DCWG	DCWI <sub>DA</sub>	Withheld Power (MW)
1	(12,13)	0.136	264	(1,12,13)	0.165	420	(1,2,12,13)	0.246	504	(1,2,12,13,14)	0.298	564	(1,2,6,12,13,14)	0.367	606
2	(1,2)	0.11	240	(2,6,14)	0.14	186	(3,6,10,14)	0.155	250	(3,6,10,16,19)	0.231	274	(3,8,10,16,17,19)	0.209	316
3	(12,13)	0.136	264	(1,12,13)	0.165	420	(1,2,12,13)	0.246	504	(1,2,12,13,14)	0.298	564	(1,2,6,12,13,14)	0.367	606
4	(1,2)	0.11	240	(2,6,14)	0.14	186	(3,6,10,14)	0.155	250	(3,6,10,16,19)	0.231	274	(3,8,10,16,17,19)	0.209	316
5	(12,13)	0.136	264	(1,12,13)	0.165	420	(1,2,12,13)	0.246	504	(1,2,12,13,14)	0.298	564	(1,2,6,12,13,14)	0.367	606
6	(1,2)	0.11	240	(2,6,14)	0.14	186	(3,6,10,14)	0.155	250	(3,6,10,16,19)	0.231	274	(3,8,10,16,17,19)	0.209	316
7	(12,13)	0.136	264	(1,12,13)	0.165	420	(1,2,12,13)	0.246	504	(1,2,12,13,14)	0.298	564	(1,2,6,12,13,14)	0.367	606
8	(1,2)	0.11	240	(2,6,14)	0.14	186	(3,6,10,14)	0.155	250	(3,6,10,16,19)	0.231	274	(3,8,10,16,17,19)	0.209	316
9	(12,13)	0.136	264	(1,12,13)	0.165	420	(1,2,12,13)	0.246	504	(1,2,12,13,14)	0.298	564	(1,2,6,12,13,14)	0.367	606
10	(1,2)	0.11	240	(2,6,14)	0.14	186	(3,6,10,14)	0.155	250	(3,6,10,16,19)	0.231	274	(3,8,10,16,17,19)	0.209	316
11	(12,13)	0.136	264	(1,12,13)	0.165	420	(1,2,12,13)	0.246	504	(1,2,12,13,14)	0.298	564	(1,2,6,12,13,14)	0.367	606
12	(1,2)	0.11	240	(2,6,14)	0.14	186	(3,6,10,14)	0.155	250	(3,6,10,16,19)	0.231	274	(3,8,10,16,17,19)	0.209	316
13	(12,13)	0.136	264	(1,12,13)	0.165	420	(1,2,12,13)	0.246	504	(1,2,12,13,14)	0.298	564	(1,2,6,12,13,14)	0.367	606
14	(1,2)	0.11	240	(2,6,14)	0.14	186	(3,6,10,14)	0.155	250	(3,6,10,16,19)	0.231	274	(3,8,10,16,17,19)	0.209	316
15	(12,13)	0.136	264	(1,12,13)	0.165	420	(1,2,12,13)	0.246	504	(1,2,12,13,14)	0.298	564	(1,2,6,12,13,14)	0.367	606
16	(1,2)	0.11	240	(2,6,14)	0.14	186	(3,6,10,14)	0.155	250	(3,6,10,16,19)	0.231	274	(3,8,10,16,17,19)	0.209	316
17	(12,13)	0.136	264	(1,12,13)	0.165	420	(1,2,12,13)	0.246	504	(1,2,12,13,14)	0.298	564	(1,2,6,12,13,14)	0.367	606
18	(1,2)	0.11	240	(2,6,14)	0.14	186	(3,6,10,14)	0.155	250	(3,6,10,16,19)	0.231	274	(3,8,10,16,17,19)	0.209	316
19	(12,13)	0.136	264	(1,12,13)	0.165	420	(1,2,12,13)	0.246	504	(1,2,12,13,14)	0.298	564	(1,2,6,12,13,14)	0.367	606
20	(1,2)	0.11	240	(2,6,14)	0.14	186	(3,6,10,14)	0.155	250	(3,6,10,16,19)	0.231	274	(3,8,10,16,17,19)	0.209	316
21	(12,13)	0.136	264	(1,12,13)	0.165	420	(1,2,12,13)	0.246	504	(1,2,12,13,14)	0.298	564	(1,2,6,12,13,14)	0.367	606
22	(1,2)	0.11	240	(2,6,14)	0.14	186	(3,6,10,14)	0.155	250	(3,6,10,16,19)	0.231	274	(3,8,10,16,17,19)	0.209	316
23	(12,13)	0.136	264	(1,12,13)	0.165	420	(1,2,12,13)	0.246	504	(1,2,12,13,14)	0.298	564	(1,2,6,12,13,14)	0.367	606
24	(1,2)	0.11	240	(2,6,14)	0.14	186	(3,6,10,14)	0.155	250	(3,6,10,16,19)	0.231	274	(3,8,10,16,17,19)	0.209	316

Table AII.2. The day-ahead dynamic capacity withholding indices for the 118-bus system.

Hour	NDCWG=7			NDCWG=8			NDCWG=9			NDCWG=10		
	DCWG	DCWI <sub>DA</sub>	Withheld Power (MW)	DCWG	DCWI <sub>DA</sub>	Withheld Power (MW)	DCWG	DCWI <sub>DA</sub>	Withheld Power (MW)	DCWG	DCWI <sub>DA</sub>	Withheld Power (MW)
1	(1,2,3,6,12,13,14)	0.401	678	(1,2,3,6,10,12,13,14)	0.459	754	(1,2,3,6,10,12,13,14,19)	0.489	802	(1,2,3,6,10,12,13,14,16,19)	0.529	838
2	(5,8,10,15,16,17,19)	0.2	322	(4,5,7,8,15,16,17,19)	0.397	326	(4,5,7,8,9,11,15,16,17)	0.454	348	(4,5,7,8,9,11,12,15,17,18)	0.509	420
3	(1,2,3,6,12,13,14)	0.401	678	(1,2,3,6,10,12,13,14)	0.459	754	(1,2,3,6,10,12,13,14,19)	0.489	802	(1,2,3,6,10,12,13,14,16,19)	0.529	838
4	(5,8,10,15,16,17,19)	0.2	322	(4,5,7,8,15,16,17,19)	0.397	326	(4,5,7,8,9,11,15,16,17)	0.454	348	(4,5,7,8,9,11,12,15,17,18)	0.509	420
5	(1,2,3,6,12,13,14)	0.401	678	(1,2,3,6,10,12,13,14)	0.459	754	(1,2,3,6,10,12,13,14,19)	0.489	802	(1,2,3,6,10,12,13,14,16,19)	0.529	838
6	(5,8,10,15,16,17,19)	0.2	322	(4,5,7,8,15,16,17,19)	0.397	326	(4,5,7,8,9,11,15,16,17)	0.454	348	(4,5,7,8,9,11,12,15,17,18)	0.509	420
7	(1,2,3,6,12,13,14)	0.401	678	(1,2,3,6,10,12,13,14)	0.459	754	(1,2,3,6,10,12,13,14,19)	0.489	802	(1,2,3,6,10,12,13,14,16,19)	0.529	838
8	(5,8,10,15,16,17,19)	0.2	322	(4,5,7,8,15,16,17,19)	0.397	326	(4,5,7,8,9,11,15,16,17)	0.454	348	(4,5,7,8,9,11,12,15,17,18)	0.509	420
9	(1,2,3,6,12,13,14)	0.401	678	(1,2,3,6,10,12,13,14)	0.459	754	(1,2,3,6,10,12,13,14,19)	0.489	802	(1,2,3,6,10,12,13,14,16,19)	0.529	838
10	(5,8,10,15,16,17,19)	0.2	322	(4,5,7,8,15,16,17,19)	0.397	326	(4,5,7,8,9,11,15,16,17)	0.454	348	(4,5,7,8,9,11,12,15,17,18)	0.509	420
11	(1,2,3,6,12,13,14)	0.401	678	(1,2,3,6,10,12,13,14)	0.459	754	(1,2,3,6,10,12,13,14,19)	0.489	802	(1,2,3,6,10,12,13,14,16,19)	0.529	838
12	(5,8,10,15,16,17,19)	0.2	322	(4,5,7,8,15,16,17,19)	0.397	326	(4,5,7,8,9,11,15,16,17)	0.454	348	(4,5,7,8,9,11,12,15,17,18)	0.509	420
13	(1,2,3,6,12,13,14)	0.401	678	(1,2,3,6,10,12,13,14)	0.459	754	(1,2,3,6,10,12,13,14,19)	0.489	802	(1,2,3,6,10,12,13,14,16,19)	0.529	838
14	(5,8,10,15,16,17,19)	0.2	322	(4,5,7,8,15,16,17,19)	0.397	326	(4,5,7,8,9,11,15,16,17)	0.454	348	(4,5,7,8,9,11,12,15,17,18)	0.509	420
15	(1,2,3,6,12,13,14)	0.401	678	(1,2,3,6,10,12,13,14)	0.459	754	(1,2,3,6,10,12,13,14,19)	0.489	802	(1,2,3,6,10,12,13,14,16,19)	0.529	838
16	(5,8,10,15,16,17,19)	0.2	322	(4,5,7,8,15,16,17,19)	0.397	326	(4,5,7,8,9,11,15,16,17)	0.454	348	(4,5,7,8,9,11,12,15,17,18)	0.509	420
17	(1,2,3,6,12,13,14)	0.401	678	(1,2,3,6,10,12,13,14)	0.459	754	(1,2,3,6,10,12,13,14,19)	0.489	802	(1,2,3,6,10,12,13,14,16,19)	0.529	838
18	(5,8,10,15,16,17,19)	0.2	322	(4,5,7,8,15,16,17,19)	0.397	326	(4,5,7,8,9,11,15,16,17)	0.454	348	(4,5,7,8,9,11,12,15,17,18)	0.509	420
19	(1,2,3,6,12,13,14)	0.401	678	(1,2,3,6,10,12,13,14)	0.459	754	(1,2,3,6,10,12,13,14,19)	0.489	802	(1,2,3,6,10,12,13,14,16,19)	0.529	838
20	(5,8,10,15,16,17,19)	0.2	322	(4,5,7,8,15,16,17,19)	0.397	326	(4,5,7,8,9,11,15,16,17)	0.454	348	(4,5,7,8,9,11,12,15,17,18)	0.509	420
21	(1,2,3,6,12,13,14)	0.401	678	(1,2,3,6,10,12,13,14)	0.459	754	(1,2,3,6,10,12,13,14,19)	0.489	802	(1,2,3,6,10,12,13,14,16,19)	0.529	838
22	(5,8,10,15,16,17,19)	0.2	322	(4,5,7,8,15,16,17,19)	0.397	326	(4,5,7,8,9,11,15,16,17)	0.454	348	(4,5,7,8,9,11,12,15,17,18)	0.509	420
23	(1,2,3,6,12,13,14)	0.401	678	(1,2,3,6,10,12,13,14)	0.459	754	(1,2,3,6,10,12,13,14,19)	0.489	802	(1,2,3,6,10,12,13,14,16,19)	0.529	838
24	(5,8,10,15,16,17,19)	0.2	322	(4,5,7,8,15,16,17,19)	0.397	326	(4,5,7,8,9,11,15,16,17)	0.454	348	(4,5,7,8,9,11,12,15,17,18)	0.509	420





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