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Low-Volatility Anomaly in the U.S. Stock Markets 1975 - 2019

School of Accounting and Finance

Master's thesis in Finance

Master's Degree Programme in Finance

Vaasa 2022

University of Vaasa

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The topic of the thesis:	Low-volatility Anomaly in US Equities 1975 - 2019 : [Subject]		
Degree:	Master of Science in Economics and Business Administration		
Master's Program:	Master's Degree Programme in Finance		
Supervisor:	Sami Vähämaa		
Year of completing the thesis:	2022	**Pages:**	85

ABSTRACT:

This Master's thesis focuses on the risk-adjusted performance of low-volatility strategies. This thesis examines if the previously documented anomaly is still present and if the strategy compensates the investor with above-market risk-adjusted returns. Since the low-volatility effect is an umbrella term used by the financial literature, this thesis focuses specifically on the trailing low-volatility and trailing low-beta strategies. Previous literature has found that using these relatively simple risk measures to sort portfolios compensates investors with larger returns and lower risk. Such a risk-return relationship makes the strategy especially compelling for investors.

A positive relationship between risk and return is a central paradigm in finance. However, the findings of the low-volatility studies show that such a relationship is not observed empirically. Instead, the relationship is flat or even reversed in some cases. While many anomalies have been documented, the low-volatility anomaly's direct implication to the financial theory makes the anomaly even more fascinating. Earlier studies have found that it does not matter what volatility-related risk measure is used; the results indicate that the investor is compensated with higher returns for lower risk.

Since low-volatility strategies are complex to form and upkeep, the strategy is more directed to professional investors with access to ample data storage and computing power. The strategy requires substantial capital and is subject to transaction costs as portfolios are balanced monthly. While multiple studies have argued that the transaction costs for a low-volatility strategy are insignificant, they are still large compared to a buy-and-hold strategy that many non-professionals use. Therefore, the strategy works only on a large scale. A retail investor that wishes to gain access to a low-volatility strategy should seek out low-volatility funds or ETFs instead of trying to mimic the portfolios themselves.

The thesis differs from the prior studies by using newer data and three-factor and five-factor models that earlier studies have not used. Additionally, the CAP-model is also used to determine whether earlier findings can be replicated. Furthermore, previous studies lack necessary robustness tests implemented in this thesis.

This thesis finds that the low-volatility and low-beta strategies generate significant alpha in U.S. stocks from 1975 to 2019 when the CAPM or Fama-French three-factor model is used. Furthermore, the alpha disappears when the newer Fama-French five-factor model is used. The results show that these returns are generated with a much lower standard deviation indicating a reversed risk-return relationship. The main conclusion is that the low-volatility anomaly exists in the US stocks, but the profitability and investment factor can largely explain it.

KEYWORDS: Low-Volatility, Low-Beta, Low-Risk

VAASAN YLIOPISTO
Laskentatoimen ja rahoituksen laitos

Tekijä:	Sami Aho
Tutkielman nimi:	Low-volatility Anomaly in US Equities 1975 - 2019 : [Subject]
Tutkinto:	Master of Science in Economics and Business Administration
Oppiaine:	Master’s Degree Programme in Finance
Työn ohjaaja:	Sami Vähämaa
Vuosi:	2022 **Sivumäärä:** 85

TIIVISTELMÄ:

Tutkielman tarkoitus on tutkia alhaisen volatiliteetin strategioiden riskikorjattua suorituskykyä. Aikaisemmat tutkimukset ovat löytäneet, että alhaisen volatiliteetin strategioiden riskikorjattu suorituskyky on erinomainen. Tutkielma pyrkii löytämään vastauksen voiko aiempien tutkimuksien havainnot vahvistaa uudemmalla osake aineistolla sekä kaikkein uusimmilla osakehinnoittelu malleilla. Alhaisen volatiliteetin strategia on sateenvarjo termi, jota käytetään akateemisessa rahoituksessa. Tämän tutkielman keskiössä on kaksi alhaisen volatiliteetin strategiaa: pieni volatiliteetti ja pieni beta. Aikaisemmat tutkimukset ovat todenneet, että käyttämällä näitä yksinkertaisia tapoja alhaisen volatiliteetin mittaamiseen ja portfolioiden rakentamiseen, sijoittajat ovat voineet saada korkeampia tuottoja alhaisemmalla riskillä. Tämä alhaisen riskin ja korkean tuoton välinen suhde tekee strategiasta erityisen mielenkiintoisen kaikille sijoittajille.

Rahoitusteorian mukaan riskin ja tuoton pitäisi olla positiivisesti korreloituneita. Aikaisemmat alhaisen volatiliteetin tutkimukset ovat kuitenkin osoittaneet, ettei senkaltaista suhdetta voida empiirisesti vahvistaa. Sen sijaan nämä aiemmat tutkimukset ovat löytäneet, että riskin ja tuoton suhde on pikemminkin tasainen tai jopa käänteinen. Vaikka monet tutkimukset muista strategioista ovat tehneet saman löydöksen, alhaisen volatiliteetin strategian suora implikaatio riskin ja tuoton väliseen tasaiseen tai negatiiviseen suhteeseen tekee strategiasta erittäin mielenkiintoisen. Lisäksi aikaisempi kirjallisuus alhaisen volatiliteetin strategioista on osoittanut, että käytettäessä mitä tahansa volatiliteetti liitännäistä riski mittaria sijoittaja voi odottaa korkeampia tuottoja matalammalla riskitasolla.

Volatiliteetti liitännäiset strategiat ovat monimutkaisia rakentaa ja ylläpitää. Lisäksi strategian toimeenpano vaatii merkittävän määrän pääomaa sekä sen transaktiokulut ovat suuret suhteessa osta-ja-pidä-strategiaan. Näin ollen strategiaa voi suositella ainoastaan ammattilaissijoittajille, joilla on kokemusta kvantitatiivisten strategioiden rakentamisesta. Tästä johtuen strategia toimii ainoastaan suuressa mittakaavassa, käyttäen merkittäviä määriä osakkeita jokaisessa portfoliossa. Tavanomaisen sijoittajan, joka haluaa päästä osaksi strategiaa, kannattaa etsiä strategiaan erikoistunut osakerahasto tai ETF-rahasto ja sijoittaa sen kautta strategiaan.

Tämä tutkielma osoittaa, että alhaisen volatiliteetin strategioiden avulla voidaan saavuttaa positiivisia riskikorjattuja tuottoja Yhdysvaltain osakemarkkinoilla. Lisäksi tutkimus osoittaa, että nämä tuotot saavutetaan alhaisemmalla tuottojen keskihajaumalla implikoiden käänteistä riskin ja tuoton suhdetta. Alhaisen volatiliteetin strategian kannattavuutta selittää pääosin tuotto- ja investointi-faktorit.

AVAINSANAT: Alhainen volatiliteetti, Alhainen beta, Alhainen riski

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1. INTRODUCTION

The outperformance of low-volatility stocks relative to high-volatility stocks is commonly referred to as the low-volatility anomaly. According to Baker, Bradley, and Wurgler (2011), the low-volatility anomaly is one of the “greatest anomalies in finance.” The anomaly is persistent and offers an excellent risk-return relationship with small drawdowns. The rational view is that with risk, returns should increase. The evidence presented by Baker et al. (2011) (and others, introduced in chapter three) contradicts that view. Their findings indicate that low-volatility portfolios perform better than their benchmarks.

For the reader, the term low-volatility anomaly is misleading as stock volatility refers to the dispersion of returns. The low-volatility anomaly refers to a range of different volatility-related anomalies. Another term for the low-volatility anomaly is the low-risk anomaly. However, the former term is more common among industry practitioners and academic researchers. The low-volatility anomalies include, e.g., trailing low-volatility, trailing low-beta, and minimum variance, to name a few. All low-volatility anomalies have in common that the returns increase with lower risk (measured by volatility, beta, or minimum variance). This is a puzzling contradiction to the general financial theory, where larger risk should be compensated with larger returns. (Baker et al. 2011; Carvalho, Lu & Moulin 2011; Li, Sullivan & Garcia-Feijóo 2014.)

Fama (1998) claims that the abnormal returns should dissipate over time once anomalies are detected. Shiller (2003) disputes Fama’s (1998) argument. According to him, anomaly dissipation is not evidence for the efficient market hypothesis (EMH) to hold. He claims that the volatility anomaly is one of the most well-known and long-lasting anomalies, yet there is no evidence of it disappearing. Furthermore, Shiller (2003) argues that it would be eccentric for inefficient markets to bear consistent and longliving patterns. The expectation is that anomaly returns fluctuate and even change signs over time in inefficient markets.

The academic literature has two general reasons for the long-term presence of anomalies. First, there are barriers to why anomalies are not exploited to the fullest (Shleifer & Vishny 1997, Baker et al. 2011) or anomaly authors have arrived at conclusions by chance (Fama 1998). The first reason contradicts the financial theory of efficient markets. The second reason would be evidence of a naïve approach to conclusions made from research or p-hacking. While researchers may be gullible at times, it is fair to expect that in case the latter is true, it is intentional.

In theory, information is thought to flow through markets instantaneously, and as news arrives, it is reflected immediately in security prices. The assumption implies that using any past information will not result in abnormal returns. The implication means that security prices follow a random walk where only the latest news can move prices, and any past information is irrelevant for future development. As further information arrives, prices react to it. In case security prices can be predicted, there exist arbitrage opportunities. The theory says that any such arbitrage opportunity will be dealt with by arbitrageurs since such opportunities are entirely risk-free. However, in practice, arbitrage carries risk. (Malkiel 2003, Shleifer & Vishnya 1997.)

Market efficiency and the prevalence of anomalies are a contradiction. The academic literature is divided on this matter. The efficient market critics argue that the concept is merely a theoretical framework, and anomaly findings are empirical proofs against it. The advocates of efficient markets demonstrate that anomalies do dissipate over time, and efficient markets exist in different forms. It is enigmatic that the EMH advocates have not determined in what time frame anomalies should dissipate.

The efficient market advocates argue that return and risk are positively correlated, and there should be no “free lunches left.” However, Multiple studies have benchmarked their returns on the Capital Asset Pricing Model (CAPM) and shown that returns increase even if the risk does not. The CAPM tests a positive linear dependence between risk and market return. The evidence suggests that the CAPM is unable to explain anomalies. This serves as additional evidence that the EMH does not hold. However, academics have identified that market risk is not the only risk present in the market. Therefore, they have enhanced the CAPM by including known anomalies turned into factors to the old model to decompose returns to capture effects common to all assets. The predictors may vary from macroeconomic factors to region-specific or asset class-specific factors. Unlike the CAPM, the multi-factor models differ from each other. The models have in common that the overall market return and the risk-free rate are included. The two most well-known multi-factor models are Fama and French’s (1993) three-factor model (FF-3) and Fama and French’s (2015) five-factor model (FF-5). Yet even after the decomposition of returns with multifactor models, abnormal returns are still present. This serves as evidence against the efficient market hypothesis.

The low-volatility anomaly has been known for a long time. Fama and French (2015) showed that their five-factor models’ profitability and investment factor could explain a large part of the low-volatility anomaly. This thesis has, therefore, several goals. The first goal is to replicate previous studies with new data and compare the findings. The study will follow the approach presented in Baker et al.’s (2011) paper. The second goal is to decompose the returns of the low-volatility strategy using the well-known multi-factor model, the FF-3. The third goal is conditional. If the results are the same as in previous literature, then an explanation for the anomaly is sought by using the FF-5 model. Additional robustness tests missing from the previous literature are implemented to find if p-hacking played a part in earlier findings.

1.1. Purpose of the study

This study investigates whether the low-volatility anomaly exists in the U.S. stock markets from 1975 to 2019. While the anomaly has been acknowledged for a long time, there is convincing evidence that the anomaly is still present. Previous academic papers have constructed the strategy with new data and found that after controlling for different factor loadings, there are still unexplained returns left with a low risk to return relationship. Furthermore, hedge funds have created financial products around the anomaly providing empirical support that the strategy can be monetized even after transaction costs. (Baker et al. 2011)

The study is implemented by constructing two portfolios with the chosen strategies. The cut-offs are determined by allocating the same number of stocks to each portfolio. The portfolio performances are then compared and tested for factor loadings. Data mining issues are relevant as the benchmark study's period ended just before the financial crisis. The crisis might have positively affected the benchmark study's outcome as low-volatility strategies are commonly thought to be defensive.

1.2. Hypotheses

This study has three main hypothesizes as follows:

H_1: *The low-volatility anomaly still exists.*

The evidence suggests that the low-volatility anomaly is a persistent anomaly that arbitragers have not traded out. The papers have in common that investors can earn a premium for holding lower risk using the spread between low- and high-volatility portfolios. This is inconsistent with the financial theory.

H_2: *The market, size, or value factor can explain the low-volatility anomaly.*

Previous studies have found that the strategy yields excellent returns even after accounting for the common risk factors included in the FF-3 model. Therefore, a secondary goal is to test whether these claims are valid with a new data set.

H_3: *The profitability or investment factor can explain the low-volatility anomaly.*

According to Fama and French (2015), profitability and investment factor, which they added to form the FF-5 model, can explain the low-volatility anomaly. Thus, the third and final goal is to test whether their claim is confirmed with a new data set.

1.3. Structure of the study

The structure of the study is as follows. The first chapter introduces the motivation and an overview of the theories involved. The second chapter discusses efficient markets and behavioral finance, and general anomalies. The third chapter focuses on low-volatility anomalies. Additionally, the third chapter will introduce studies using different variants of low-volatility anomaly. The fourth chapter discusses the portfolio performance measures, namely the capital asset pricing model (CAPM) and its extensions, the FF-3 and the FF-5. The fifth chapter discusses the data, methodology, and chosen strategies. The sixth chapter presents the results and how they compare to previous studies. The seventh and closing chapter summarizes and concludes the thesis.

2. EFFICIENT MARKET AND STOCK MARKET ANOMALIES

This chapter discusses the efficient capital market, behavioral finance, and anomalies. The chapter will introduce the efficient market hypothesis and behavioral finance, a competing theory for explaining how the markets behave. The EMH and behavioral finance are essential parts of the academic literature and often discuss security market anomalies and their explanations.

2.1. Efficient Capital Markets, Rational Expectations, and Random Walk

The financial market's primary function is to offer a funnel where resources are allocated efficiently from those who have a surplus to those who have a deficit. When resources are allocated accordingly, economies can thrive. In such a state, any project, investment, or idea with a positive expected return will find the required resources in a reasonable time. The EMH argues that the markets are efficient when certain assumptions are met. When all assumptions are met, the market is efficient in the strongest form.

The hypothesis states that the market is strong-form efficient when security prices reflect all prior, current, and known future information. At this state, no investment strategy will result in abnormally large returns given the amount of risk taken as all information is already priced in. The EMH is an essential building block for the financial literature and has molded how financial markets are seen. A critical hypothesis of EMH is the rational expectation theory, where agents weigh each option and make the optimal choice based on gained utility, often measured in monetary gain, or return. (Fama 1970; Fama 1995.)

Another important concept related to EMH is the random walk assumption. The assumption says that all subsequent security prices are independent of each other with identical distributions. Thus, past prices or movements do not contain any information that can be used to predict future price movements. When news arrives, rational agents will react to them, and the market will find an equilibrium price that reflects the information's value. However, as the actual value of information is ambiguous, two critical implications can be drawn. First, overreaction and underreaction to news are equally typical. Second, the time it takes for prices to adjust to the information is also a random variable. This all leads to a situation where subsequent price movements are independent of each other – a random walk. (Fama 1970; Fama 1995.)

$$E\left(p_{j,t+1} \middle| \Phi_t\right) = \left[1 + E\left(r_{j,t+1} \middle| \Phi_t\right)\right] p_{j,t} \tag{1}$$

Equation one (1) introduced by Fama (1970) is a mathematical representation of the EMH. The E is the expected value operator, $p_{j,t}$ is the price of security j at time t. The $p_{j,t+1}$ is the price of security j at time t + 1 with cash flows re-invested, $r_{j,t+1}$ is the one period percentage return ($p_{j,t+1}$ – $p_{j,t}$ / $p_{j,t}$) of security j at time t+1, the Φ_t is the conditional term that replicates which information is reflected in the security prices . The $p_{j,t+1}$ and $r_{j,t+1}$ are random variables at time t. The value of the expected return $E(r_{j,t+1}|\Phi_t)$ conditioned by the information Φ_t, is meant to be interpreted that whatever expected return model is applied, the information of Φ_t is mirrored in the prices of securities. If the EMH holds, Fama (1970) argues that there should be no arbitrage or abnormal return opportunities. This would result in the following equations:

$$x_{j,t+1} = p_{j,t+1} - E\left(p_{j,t+1} \middle| \Phi_t\right) \tag{2}$$

$$E\left(x_{j,t+1} \middle| \Phi_t\right) = 0 \tag{3}$$

In equation two (2) $x_{j,t+1}$ is the excess return of security j at time t + 1, the difference between the observed security price and expected security price conditioned on the information factor Φ_t. Equations two and three state that given the information Φ_t, there should not be any excess returns as security prices should already reflect all the available information. (Fama 1970.)

The EMH is divided into three levels of efficiency: 1. weak form, 2. semi-strong for,m 3. strong form. Each efficiency level comes with baseline assumptions that must be met. The assumptions must be tested to determine the level of efficiency. The tests are sequentially ordered and must always start with the weak form test. The weak form tests if historical information can be used to predict returns. The semi-strong form tests if prices react accordingly to publicly available information. The strong form tests if some market participant has monopolistic information unavailable to other participants. (Fama 1970.)

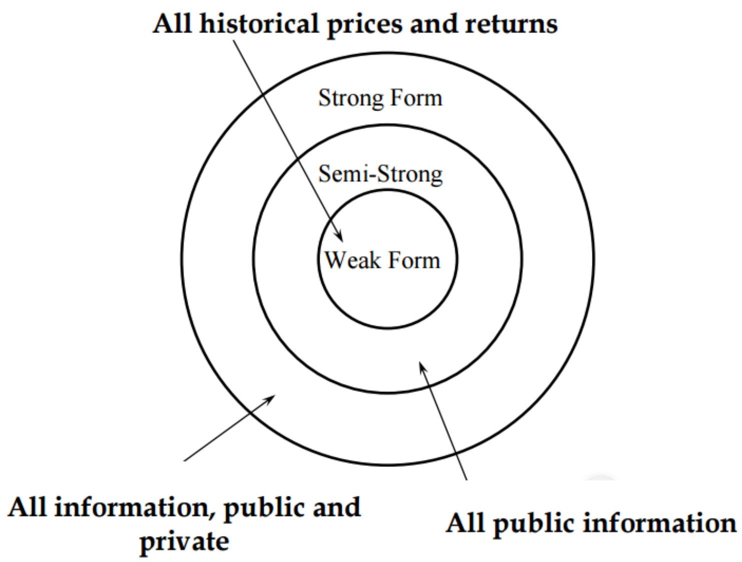

Figure 1 Three forms of the EMH

Figure 1 depicts the three forms of EMH. The weak form of market efficiency includes only past prices, the semi-strong form consists of the weak form and all public information, and the strong form consists of the weak form, the semi-strong form, and all private information.

Fama (1970) notes that their hypothesis of prices reflecting all information entirely is a bold statement and should not be taken literally. The study cannot find evidence to reject the weak form or semi-strong form, while there is limited evidence to reject the strong form state.

2.2. Weak form

The weak form tests if historical information can predict future prices. The market is weakly efficient when past information cannot be used for future return prediction. Fama (1970) argues that the weak form hypothesis can be tested by examining if autocorrelation exists in security prices. The null hypothesis is that price changes are random, independent, and identically distributed. Using 30 stocks from the Dow Jones index for approximately four years, from the end of 1957 to September 1962, Fama (1970) studies if there is autocorrelation between different daily lags. He finds no evidence for substantial linear dependence. While some of the results are significant, the absolute level is relatively small.

Opposite to Fama's (1970) view, evidence exists that stock market returns are serially correlated. Cambell, Grossman, and Wang (1993) study serial correlation, volatility, and trading volume in stock markets. Their study uses daily data from the New York Stock exchange (NYSE) from July 1962 to December 1988. They find that autocorrelation is higher on low-volume days than on high-volume days, with autocorrelation dropping with higher degrees of autocorrelation. Berglund and Liljeblom (1988) find supporting evidence from the Finnish stock market for Cambell et al.'s (1993) study. They find that index returns and individual stocks are serially correlated. Their study further finds evidence that index return serial correlation exists even after controlling for the Fisher effect and individual stock serial correlation.

Jegadeesh and Titman (1993) studied a strategy from 1965 to 1989 where NYSE stocks are ranked into deciles based on their past 3-to-12-month return. The strategy then waits for a week before buying the "winner" portfolio – stocks that returned the most – and selling the "loser" portfolio. The zero-cost portfolio is then held for 3-to-12 months. They find that stocks that performed well in the lookback period tend to perform well in the holding period and vice versa. The strategy that they considered has been named as "Momentum" strategy.

Stock market autocorrelation is evidence against the weak form state. There also exists evidence that stock market performance can be predicted from measures of uncertainty derived from historical price information. Pettengill, Sundaram, and Mathur (1995) show a positive relationship between risk (measured by market beta) and return. Their study suggests that high beta stocks compensate the investor for higher risk. Like Pettengill et al. (1995), Fama and French (1992) show an increasing relationship between returns and market beta. However, they estimated the relationship to be much smaller than suggested by other studies.

Baker et al. (2011) repeat the Pettengill et al. (1995) study partially and confirm their findings. As opposed to them, they find that the low-beta anomaly is present when beta-formed portfolios are regressed against market returns. Their evidence implies that low-beta portfolios compensate more for each risk point taken. Furthermore, Baker et al.'s (2011) main findings are that low-beta and low-volatility portfolios tend to overperform their high peers. Ang, Hodrick, Xing, and Zhang (2006) study how portfolios based on idiosyncratic volatility (IVOL) perform. Instead of having a single period when the strategy is reviewed, they divide their main period into subsamples. The strategy's performance is evaluated during expansion, recession, volatile and stable periods. They find that low (high) IVOL portfolios perform better (worse) in all assessed periods. Thus, the high-risk portfolio is not rewarded with the returns suggested by the efficient market theory. The evidence indicates that low-beta, low-volatility, and other low-volatility strategies tend to be favorable for investors. These strategies fall under the umbrella of "low-volatility anomaly" and are studied comprehensively in chapter three.

While a weak form state is in effect, all past price information should be included in prices. Previous research by Jegadeesh and Titman (1993) and Baker et al. (2011), as well as others (see, e.g., Ang et al. (2006); Pettengill et al. (1995); Li, Sullivan & Garcia-Feijóo (2016)), show that anomaly strategies generate excess returns. Low-volatility and momentum strategies are based purely on past information, and the evidence suggests that the market is not efficient in the weakest form.

2.3. Semi-Strong form

The semi-strong state is achieved when all available public information is reflected in security prices. Such general information generation events refer to stock splits, new security issues, announcements of financial reports, etc. The events are evaluated separately, and each test brings only supporting or contradicting evidence for the EMH's semi-strong state. The tests conducted for this level of efficiency are event studies. Event studies use the information before, at, and after an event to see if the event is priced accordingly, e.g., a stock split announcement should not increase the stock price since it does not affect future cash flows (Grinblatt, Masulis & Titman 1984). Multiple event studies suggest that prices respond accordingly to public events, suggesting that markets are semi-strong. (Fama 1970.)

However, some studies disagree with the studies presented by Fama (1970). Grinblatt et al. (1984) studied stock price behavior around stock split and dividend announcement date. According to Grinblatt et al. (1984), previous event studies conducted around announcements fail to address the issue of having important concurrent announcements. They find that stock splits and dividend announcements significantly affect stock price behavior once contemporaneous announcements are accounted for. Lakonishok and Lev (1987) also studied stock splits and stock dividends. They confirm the findings of Grinblatt et al. (1984). However, they find that companies whose earnings and stock price have recently experienced significant growth tend to do stock splits. For stock dividends, they cannot find similar evidence. They hypothesize that the stock split is an action to get the stock price to a more “normal” level to appear appealing to investors. Furthermore, if stock splits are associated with higher earnings growth, investors’ reaction to stock split announcements can be justified to signal improved growth prospects.

Financial report analysis should not result in abnormal market returns when the markets are semi-strong. Yet, there exist anomalies such as the value anomaly where high book-to-market stocks tend to perform their low peers. Zhang (2005) shows that value portfolios outperform growth portfolios. He argues that the value premium is due to low book-to-market stocks being riskier than high book-to-market stocks. His view contradicts other studies where low book-to-market is associated with less risk. Zhang (2005) explains his view:

> In my application, a popular interpretation of the value effect, suggested by Fama and French (1993, 1996), is that book-to-market as a proxy for a state variable associated with relative financial distress. As value stocks are typically in distress, if a credit crunch comes along, these stocks will do very badly and hence are risky. A sizable literature has since developed to test this distress hypothesis, but the evidence is mixed at best.

Assness, Moskowitz, and Pedersen’s (2013) study found similar results to Zhang’s when studying eight different markets and asset classes over 30 years. Their study concludes that value and momentum are still prevalent anomalies in markets, adding contradicting evidence to the EMH. The earlier discussed studies are evidence against the semi-strong form.

2.4. Strong form

The highest efficiency level is reached when the strong form tests are passed. These tests are concerned with whether all available information is reflected in prices. The strong form includes the weak and semi-strong form but adds that even privately held information used for trading should be reflected in the prices. The assumptions of strong form efficiency are hard to meet precisely, and there is strong evidence that the strong form does not hold. However, for practical reasons, it is essential to assess whether the privately held information is valuable and can be exploited to such a level that it is worth the search. Furthermore, is the average profit of the average user of such information positive, negative, or zero? And finally, who are the people in possession of such information? (Fama 1970.)

When Fama (1970) published the article regarding market efficiencies, there had already been published studies showing evidence that private information is used for trading purposes. Niederhoffer and Osborne (1966) discuss in their research that NYSE traders tend to use their knowledge of the limit order book to execute personal trades and gain abnormal returns using such information. On average, the NYSE trader sells above his last purchase on 83% of the cases and buys below his previous sale on 81% of the cases. It is doubtful that such averages are the result of a random draw.

While insider trading is illegal in most countries, there is evidence that insiders trade using non-public information. Syed, Liu, and Smith (1989) studied how stocks react to Wall Street Journal's column "Heard on the Street" and if early column leakage before its publishing influenced the concerning stocks. Their results show that before the day of publishing, there was a significant jump in stock prices and that the column had a statistically significant impact on prices on the day of publication. Their study finds that the insider trade group earns significantly higher abnormal returns than the control group.

Sharpe (1966) studies the performance of 34 open-end mutual funds for ten years. He argues that the performance gap between the funds emerges from either management's capability to access confidential information or funds expense structure. The fund's size determines the manager's access to concealed information, and thus larger funds can finance more security analysis. His findings indicate that the performance gap occurs primarily due to expense structure, leverage, and risk profile. Yet, Sharpe cannot rule out that funds accessing confidential information might overperform due to possessing insider information. However, the obtained expense data does not include all costs, e.g., broker fees, and thus the manager's capabilities might be optimistically weighted in the results.

Like Sharpe (1966), Jensen (1968) studied 115 mutual funds between 1955 and 1964. His findings indicate that mutual funds cannot outperform the buy-and-hold market strategy. Even if the mutual funds are exempt from expenses, the funds cannot beat the market on average. While some funds did outperform the market, there isn't enough data to separate whether such funds are outperforming due to skill, private information, or merely by chance. The author himself expects that funds that generated abnormal returns were just lucky.

Jaffe (1974) studies if illegal insider trading exists and how profitable it is. The results indicate that insiders are highly likely to possess special information and utilize it. While trading on private information is illegal and regulated by the SEC (in the U.S.), insider trading isn't illegal. Jaffe points out that since insiders know that trading on private information is illegal, they may make non-informed trades to cause noise and mask their true intentions. Additionally, other insiders who know that the public is following insider trades may utilize this knowledge by causing pump-and-dump schemes. Such schemes do not require any special information. It requires the public to trust that the insider knows something that the public does not. While the results show that insiders have private information, only one of the studied samples yields profits after commissions. However, the author clarifies that this does not indicate that trading in private information does not generate profits. Insiders probably share or trade their private information to parties that are not followed by the SEC and get similar information back to trade with. This way, insiders may monetize their private knowledge by selling it for untraceable illicit information.

Finnerty's (1976) study shows similar results as Jaffe's (1974). He studied stocks listed on the New York Stock Exchange for three years with 30 000 insider transactions. His study extends Jaffe's (1974) distinguishing sell and buy transactions. The results indicate that insiders outperform the market and can determine profitable opportunities, at least in the short term.

More recent research by Aboody and Lev (2002) investigate information asymmetry arising from research and development (R&D) investments. There is a significant difference in insider gains in companies that invest heavily in R&D versus companies that do not. The authors show that the information asymmetry gap is not closed before the R&D investments are made public, and the insiders have already traded with the information. The profits made by the insiders from this asymmetry are large and statistically significant.

The subchapter has introduced the EMH's strongest form and its research. There is contradicting evidence of whether the market is in a strong form state. Fama (1970) argues that even if individuals hold private information, can they monetize it? Sharpe's (1966) and Jensen's (1968) evidence indicates that finance professionals do not have such information. On the other hand, Jaffe's (1974) study clearly shows that stakeholders possess private knowledge and use it. Finnerty (1976) and Aboody and Lev (2002) show that insiders have private information and use it. The current literature agrees that the markets are not in a strong form state. The following subchapter will discuss more literature that challenges the efficient market hypothesis and proposes alternative explanations for market anomalies and inefficiencies.

2.5. Behavioral finance

This subchapter will discuss behavioral finance. The advocates of behavioral finance have challenged efficient market theorists for the assumptions that must hold for the EMH to work.

Behavioral finance is a collaboration between finance, psychology, and sociology, and it disagrees with the EMH's rational-agent model. The EMH's picture of the world consisting of rational optimizers that can solve complex stochastic probability matrixes within seconds is far from reality. A more realistic explanation, while unlikely, for EMH's assumption is that the market consists of "smart money" and "dumb money." In such a scenario, one of the "smart money's" functions is to offset the "dumb money's" actions, i.e., when dumb money pushes a stock out of its fundamental price, smart money joins in and makes a reverting action until the price converges back to the fundamental level. However, there is doubt that smart money would have the power and will to take such actions. The research shows that such behavior is not observed empirically. Instead, the smart money tried to benefit by buying stocks ahead of dumb money, thus amplifying the feedback effect. The evidence also indicates that rational traders are unwilling to offset the irrational traders fully as the amount of risk carried increases for the rational traders. (Shiller 2003.)

Kahneman (2003) argues that from a psychological point of view, the rational agent model and thus the efficient market theory is unrealistic. The psychologically more realistic assumption is that agents acting in markets maximize their satisfaction instead of utility. These agents are boundedly rational, facing biases and time constraints. The biases and limitations separate the suboptimal boundedly rational agent from the optimal rational agent.

De Bondt and Thaler (1985) study investors' overreaction to the news. According to psychological theory, individuals put more weight on recent events than on older information. The authors' results confirm what the theory predicts. They construct portfolios consisting of prior losers and prior winners and hold the portfolios for 36 months. The loser portfolios outperformed the market within their study period by 19.6%. In comparison, the winner portfolio lost to the market by an average of 5% three years after the portfolio formation. Daniel, Hirshleifer & Subrahmanyam (1998) propose an explanation for the overreaction found by DeBondt and Thaler (1985). Their theory uses two psychological biases to explain stock market over- and underreaction: investor overconfidence and self-attribution bias. Overconfident investors tend to misjudge their prediction error variances, following in investors believing that they are superior in forecasting relative to others. Furthermore, investors suffer intensely from biased self-attribution. When public information agrees with the investor's market view, his confidence grows. However, when public information disagrees with his view, there is no change in confidence. Previous findings suggest that investors are likely to take credit for their past accomplishments while blaming external factors for their losses.

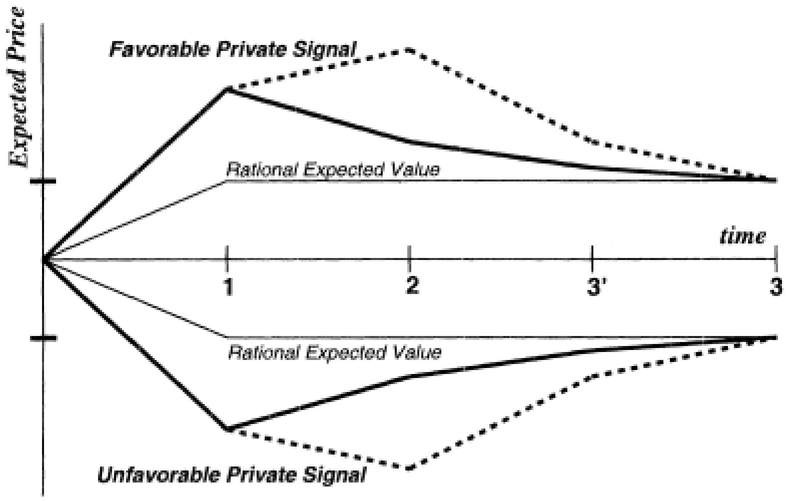

Figure 2 Average price as a function of time with overconfident investors. (Daniel et al. 1998.)

Figure 2 depicts the overconfident investor's reaction. The thin line is the stock's fundamental value, while the thick line is the reaction of the overconfident investor. The investor overreacts to a positive (negative) private signal on day one. On day two, public information arrives, which causes the investor to correct his overreaction closer to the fundamental value. On day three, the second-day correction continues closing the gap between the overreaction and the fundamental value. This process continues until the investor's overreaction, and fundamental value meet. (Daniel et al. 1998.)

Fama (1998) challenges DeBondt and Thaler (1985), and Daniel et al. (1998) view. He argues that anomaly findings are results of chance as an overreaction to news is as common as underreaction, and post-event anomalies are as common as pre-event anomalies. The intuition is that since both reactions are as common as the other, only chance can generate them. Fama (1998) argues that researchers just identify spurious patterns that are mistaken as anomalies, and such patterns dissipate over time since rational traders take advantage of the situation.

Mclean and Pontiff (2016) confirm Fama's (1998) anomaly dissipation argument. They studied 97 stock market anomalies after the anomaly had been published in a journal. The general result is that anomalies' abnormal returns diminish or wipe out after publication in a journal, suggesting that most anomalies are due to mispricing. Their findings imply two things: the publication makes markets more efficient, or the anomaly never existed.

While overreaction might be a partial explanation and only apply to certain anomalies, Stambaugh, Yu, and Yuan (2011) discuss investor constraints and sentiment. While neutral on whether the rational investor exists, they claim that any investor will face short sale impediments, limits to arbitrage, and trading costs. Their evidence shows that anomaly returns are higher when the investor sentiment is high. During high sentiment periods, the short leg of the portfolio is more overpriced than the long leg, i.e., the short leg returns are lower when the sentiment is high. Why doesn't the rational investor step in and trade out the arbitrage? Because they face specific rules about what they can and can't do. Stambaugh et al. (2011) argue that the rational investor (institutional investor, according to the authors) might recognize the mispricing. However, they are prohibited from taking such short positions. Furthermore, those that do not face such short sale impediments can be discouraged by the risks involved in arbitrage. Finally, shorting stocks is expensive. There is a limited amount of stock loans circulating the market at any given moment, increasing the lending price.

3. LITERATURE REVIEW

The following chapter will present academic literature written on low-volatility anomalies. The subchapters will discuss four low-volatility anomalies separately to give the reader a picture of the earlier studies. While the thesis will focus on the trailing volatility and beta, it is necessary to present the minimum variance and IVOL strategies. The previous literature has found that these strategies co-move significantly and are closely related. Furthermore, when discussing the low-volatility anomaly, the earlier papers often discuss these four anomalies or a combination of them. While the chapter aims to present the research extensively, it is not exhaustive and focuses on the prominent literature.

The low-volatility anomaly is an exciting concept compared to other anomalies. The term “low-volatility anomaly” refers to low-risk anomalies and therefore is an umbrella term that includes a range of volatility-related anomalies found in the stock markets. For the reader, it can be confusing that low-risk and low-volatility terms are used interchangeably. To clarify this dilemma, volatility or volatility-related measures are often used as risk proxies. The anomaly should be called a low-risk anomaly, as volatility could be thought to be only one of multiple risk measures. However, the literature has called the anomaly “low-volatility anomaly” while referring to a range of volatility-related anomalies. (Baker et al. 2011; Li et al. 2014.) These anomalies have in common that low-risk stocks tend to outperform high-risk stocks, contrary to the efficient market hypothesis, where risk should be compensated with the return. The contradiction leads to an exciting topic: Are volatility or volatility-related measures good proxies for risk? There is considerable debate regarding this subject. Finally, these low-volatility strategies include, but are not limited to, trailing volatility, trailing beta, and minimum variance portfolio, to name a few. This chapter will discuss the relevant literature concerning the low-volatility anomaly, present different low-volatility strategies, and explain them.

For a long time, the financial theory has argued that volatility and beta are proxies for risk, and risk and return should be positively correlated. However, the puzzling fact is that the U.S. equity returns have had a reversed relationship – with the lower risk, you gain a higher return. The reversed risk-return relationship is contrary to what the EMH and CAPM predict. Whether we use beta or volatility as a proxy for risk, the result is the same. Lower risk leads to higher returns. Unlike many other anomalies, the low-volatility anomaly has been documented by many authors in different asset classes and markets. Baker et al. (2011) called the low-volatility anomaly “one of the greatest anomalies in finance.” (Baker et al. 2011.)

3.1. Minimum Variance

The minimum variance strategy differentiates itself from the low-volatility, -beta, and IVOL strategies. While these three strategies sort stocks into portfolios based on the preferred risk measure, the minimum variance strategy considers all stocks in question. The strategy then finds the optimal combination and weights of stocks that minimize the portfolio’s variance in the preceding period and then invests in these stocks for the holding period. Once the holding period is over, the process is repeated. (Walkshäuls 2014.)

In 1991, Baker and Haugen researched how a capitalization-weighted minimum variance portfolio composed of the 1000 largest U.S. equities performed against the Wilshire 5000 index (an index considered as a broad-based U.S. index, capitalization-weighted) from 1972 to 1989. The authors constructed their minimum variance portfolio by investigating what combination of stocks would have minimized the portfolio’s volatility in the preceding 24 months. Once they find the optimal combination, the strategy is held for a quarter, and then the process is repeated. The optimal portfolio had weights in 100 to 150 stocks meaning that most of the 1000 stocks had zero weight at any given quarter. The results show that the minimum variance portfolio is much more efficient than the Wilshire 5000 index. The geometric mean return is 14.8 % with a volatility of 16.5 %, while the Wilshire 5000 had a geometric mean return of 11.8 % with a volatility of 18.5 %. Furthermore, it is shown that the low-volatility portfolio has consistently lower five-year rolling volatility and higher five-year rolling return than the benchmark index. Finally, like Baker et al. (2011), the authors argue that there are restrictions to short-selling, which causes deviations in market efficiency.

Chan, Karceski, and Lakonishok (1999) also investigated the minimum variance portfolio. They compared multiple different minimum variance portfolio construction methods. Like Baker and Haugen (1991), they restrict stock shorting, as there are limitations to shorting for even large funds, and cap the weight of any given stock in the portfolio to 2%. Their minimum variance portfolio construction method differs significantly from Baker and Haugen's (1991). Their period of study starts in 1972 and ends in 1997. They draw a random sample of 250 stocks from NYSE and AMEX each year. Using the past 60 months of data, they calculate seven different covariance models for forecasting stock covariance for the following year. Then they hold the portfolio for a year, and the stock randomization and minimum variance calculations are repeated for the subsequent year, and a new portfolio is formed. Intriguingly, choosing a more complex covariance model instead of a standard one does not improve the results. However, all the minimum variance portfolios result in larger returns with lower standard deviation indicating a minimum variance portfolio is more efficient than the value-weighted index. Furthermore, the market beta for the minimum variance portfolios ranged from 0.50 to 0.65, indicating a defensive strategy and a strong link to market beta. The overall results are what Baker and Haugen discovered in 1991, while the methods used are novel.

Jagannathan and Ma (2003) studied how imposing short sale impediments and upper bound weights to minimum variance portfolios affects the results found by Baker and Haugen (1991) and Chan et al. (1999). The authors follow the approach of Chan et al.

(1999) in their study. Each April, from 1968 to 1998, they randomly choose 500 large-capitalization stocks to their minimum variance portfolio from the NYSE and AMEX. Following the randomization, they construct three minimum variance portfolios from the selected stocks using return data from the past 60 months. The first portfolio does not have any restrictions. In contrast, the second portfolio restricts stock weights non-negative (short-sale impediment). Finally, the third portfolio, in addition to the short-sale constraint, cannot invest more than 2 % in any stock. Whether monthly or daily return data is used, all three minimum variance portfolios outperform their benchmark. The standard deviation ranges from 10.52 % to 12.85 %, with a mean return ranging from 12.65 % to 14.30 %. In contrast, the benchmark index has a standard deviation range from 15.60 % to 17.48 %, with a mean return range of 13.39 % to 14.52 %. The unrestricted portfolios had a short interest ranging from 50.7 % to 128.5 %. However, with short sale restrictions and upper bounds, the general result was a slight improvement in the average return with a minor increase in the standard deviation. Additionally, the more constrained portfolios had much fewer stocks at any given year than the unrestricted portfolio indicating high trading costs for the latter. The authors' findings are consistent with Baker and Haugen's (1991) and Chan et al.'s (1999).

Clarke, de Silva, and Thorley(2006) follow the footsteps of Baker and Haugen (1991) by constructing a similar capitalization-weighted minimum variance portfolio of the largest thousand U.S. stocks from 1968 to 2005. The authors impose short-sale and upper bound restrictions for their portfolios. As opposed to their benchmark study, past 60 months' return data is used for the minimum variance portfolio estimation with monthly rebalancing. The results resemble what was found by earlier studies: the portfolio outperforms the market with lower volatility. The returns are then decomposed against three well-known factors in the market (at the time): size, value, and momentum. The exposures indicate that the higher return realized by the portfolio may be due to small-cap and value premium as the portfolio behaves in a remarkably equivalent manner to those risk factors. Finally, the authors liberate the short-sale restriction to assess whether the portfolio's return improves. Like Jagannathan and Ma (2003), there was an increase in return, but contrary to them the volatility decreased. Additional evidence for the minimum variance portfolio performance in the global equity markets is provided by De Carvalho, Lu, and Moulin (2012). Contrary to Clarke et al. (2006) they do not find that the minimum variance portfolio generates any significant alpha, even when the short-sale restriction is liberated. However, Carvalho et al. (2012) document that the low-beta anomaly largely explains the minimum variance anomaly, with the beta strategy having small drawdowns. These findings strongly link further the minimum variance anomaly and low-beta anomaly together.

Walkshäuls (2014) studies the minimum variance, low-volatility, and low-beta strategies in an international setting. Specifically, he focuses on developed and emerging markets. Like earlier studies (see, e.g., Jagannathan and Ma 2003; Clarke et al. 2006; Baker et al. 2011; Blitz, Pang & van Vliet2013), he finds that the three low-volatility strategies do generate statistically significant alpha after CAPM and FF-3. Since he considers multiple markets and strategies separately, it is necessary to clarify his findings. For all strategies, the alpha is significant for the developed market, EU, and emerging market in both CAPM and FF-3 regression. The market explains the anomaly when considering the U.S. and Japan separately since the alpha is statistically insignificant. His findings are contrary to what the earlier studies have found about the U.S. market. Like the earlier studies, the general result is that these strategies do offer an excellent risk-return relationship compared to the market.

3.2. Trailing Low-Volatility and Trailing Low-Beta

The paper by Baker et al. (2011) is the benchmark study of this thesis. Their study focuses on the U.S. equities downloaded from the CRSP database spanning 41 years, starting in January 1968 and ending in December 2008. The authors constructed capitalization-weighted portfolios by sorting stocks into quintiles based on their 5-year trailing volatility or 5-year trailing beta. The academic literature considers both measures as proxies for risk. Furthermore, they studied two datasets where the other was restricted to one thousand largest stocks based on market capitalization, and the other had no restrictions. On both risk measures, and whether limited to large caps or not, the low-risk portfolio outperforms the high-risk portfolio. The authors discovered that the low-risk outperformance was even larger when the unrestricted sample was studied. Additionally, the low-risk path to glory was much smoother, and the portfolio was rebalanced much less than its high-risk comparison.

So, an important question comes up: why has the low-volatility anomaly succeeded for so long? And why have arbitragers not traded it out? Baker et al. (2011) offer two explanations for the low-risk anomaly. First, investors' behavioral biases affect how they approach the stock market. Specifically, the mentioned biases are preferences for lotteries, better known as loss aversion, representativeness, and overconfidence. Second, they argue that mutual fund benchmarking to indexes causes limitations on arbitrage.

According to Baker et al.'s (2011) data, 94.6 % of all mutual funds have been benchmarked to some popular U.S. index. But benchmarking does not restrict a fund to a particular strategy; fund traders can still take on arbitrage opportunities. So why don't the managers construct a low-volatility strategy and trade out the anomaly? One reason is that the low quintile of the strategy consists of small stocks which are costly to trade and expensive to borrow for shorting. However, the main reason is the fund benchmarking. When writing, U.S. funds had to be benchmarked to some index. Furthermore, funds show KPIs (Key Performance Indicators) to their investors, which judge the funds' performance based on the displayed figures. One of these KPIs is the IR (Information Ratio), which considers the tracking error (standard deviation of the difference between portfolio and benchmark returns) as the denominator, and the fund's and benchmarks' return difference as the numerator. The higher the IR is, the better for the fund. Yet, the authors show that the IR is low compared to other strategies (such as value and momentum). Thus, the authors argue that the average investor compares funds based on their KPIs, and if one fund is reporting lower IRs than others, it reduces interest for the fund – whether the returns are higher or not. Consequently, fund managers are unwilling to partake in arbitrages that can be costly, risky, and deter investors by reducing important KPIs. Finally, the behavioral biases were introduced as explanations for the low-volatility anomaly. While the authors argue that the behavioral biases are not required for the benchmarking, which flattens the CAPM relationship, they are facts that just enhance the low-volatility anomaly. The higher the portion of market participants willing to buy lottery tickets instead of steady stocks, the larger the low-volatility anomaly will become. (Baker et al. 2011.)

The low-volatility anomaly was also identified by Blitz and van Vliet (2007) in the U.S. and global markets. The data used for the study consists of large-cap developing world stocks that include roughly 2000 stocks. The authors base their study around the low-volatility anomaly by sorting stocks into deciles based on the past three-year weekly volatility with monthly rebalancing. Additionally, to account for the size and value effect, they construct portfolios that are first sorted on these known factors and then resorted based on the three-year volatility. They furthermore compare the performance of their portfolios against the size, value, and momentum sorted portfolios and test the risk-adjusted returns of all constructed portfolios against the FF-3 model. Their results are consistent with the earlier papers showing that the global low-volatility decile earns on average 7.3 % annually in excess versus the high-volatility portfolio earning 1.4 % annually in excess to the market. However, the most intriguing finding is that the Sharpe ratio of the low-decile is 0.72 while the high-deciles is 0.05, indicating an abysmal risk-adjusted return for the high-volatility portfolio. They also study Europe, the U.S., and Japan separately, where they uncover that the results of these regions follow a similar pattern of low-volatility portfolios earning higher returns than the high-volatility comparison. When comparing the low-volatility performance to value, size, or momentum sorted portfolios, the results imply that low-volatility enjoys a higher Sharpe ratio than any of the anomalies mentioned above while ranking second in alpha generation only after the momentum sorted portfolios. Moreover, the authors show that portfolio volatility is positively related to portfolio beta and that low-beta sorted portfolios behave similarly to low-volatility sorted portfolios. However, the low beta sorted portfolios generated slightly lower alpha than low-volatility portfolios. Blitz and van Vliet's (2007) study's portfolio formation method is like Baker et al.'s (2011), with minor differences in the portfolio formation method. Interestingly, both studies results are parallel, indicating that the simple trailing low-volatility strategy is an effortless way to capture "low hanging fruits" in U.S. and global markets.

Blitz et al. (2013) provide evidence that the low-volatility and -beta anomalies are present in the emerging markets. Their study follows a similar structure as Blitz and van Vliet's (2007) with minor changes. Instead of weekly volatility, they use monthly volatility, just like Baker et al. (2011), and instead of capitalization-weighting, they use equal-weighting. Their study period starts in 1988, ends in 2010, and covers stocks from 30 emerging markets. At the start of their period, there were only 200 stocks in the total sample increasing gradually to 1800 stocks at the end of 2010. Interestingly, the authors show that past risk is highly predictive of future risk. The connection is a key assumption in constructing the low-volatility and -beta portfolios. The whole concept relies on the idea that the constructed portfolios have lower risk during the holding period. Furthermore, they find evidence that the size, value, and momentum premiums are present in the emerging markets and note that the low-volatility and -beta premiums are larger than the size premium and equal to the value premium. The momentum premium is larger with a much larger turnover than low-volatility and -beta, and thus the outperformance is eaten away by transaction costs in practice. They find no evidence supporting the positive risk-return relationship. Instead, the results show a flattened or even a negative relationship.

Frazzini and Pedersen invented a more complex low-beta strategy in 2014. They called their strategy “Betting against Beta,” or BAB, showing that low-beta assets tend to outperform high-beta assets. Their construction method is novel. Like Baker et al. (2011), they calculate the past beta of the asset. However, they use a one-year rolling standard deviation of daily returns for the volatility calculation and five-year rolling daily returns for the correlation. Furthermore, the security weights are unusually determined by the beta. In the low-beta portfolio, the lower the beta’s rank is, the higher the weight is, and conversely, in the high-beta portfolio, the higher the beta’s rank is, the larger the weight is. Finally, the portfolio is not a simple zero-cost. Instead, the portfolio is 1.4 $ long in low-beta stocks and 0.7 $ short in high-beta stocks. This is essentially done by increasing the portfolio leverage by short selling more the risk-free bond in addition to the high-beta stocks. The authors’ goal is to increase the low- and high-beta portfolios’ beta to match the market. Their data period covers a maximum of 60 years (the range depends on the country and asset class) from 1962 to 2012, with 20 markets, and four asset classes. There is a monotonic decrease in alpha from low-beta to high-beta assets in every asset class and country. The flattened risk-return relationship found by Blitz et al. (2013) in emerging markets is also discovered by Frazzini and Pedersen (2014) in developed countries and different asset classes.

The findings of Blitz et al. (2013) about the predictive nature of past risk on future risk were confirmed by Baker, Bradley, and Taliaferro (2014). The research focuses on the low-beta effect and shows that it is highly likely that the stock stays in the same quantile as it was in the previous month. They find that the probability is 95 %. The structure follows a similar method as Baker et al. (2011) and Blitz et al. (2013) by estimating betas based on the past 60 months of return and then sorting the stocks into quintiles. Over the period from 1968 to 2012, the authors find strong evidence for the low-beta anomaly in both the U.S. and the developed world. The standard beta strategy spread was 6.76 % and 9.24 % per year and statistically significant at the 1 % level. The high probability of the stock staying in the same quintile month over month indicates that the turnover for the portfolios is small. Thus, the transaction costs of the strategy should be insignificant. The study adds to the beta discussion by considering double sorted beta portfolios based on their stock market and industry betas. The industry beta sort follows a different approach than the market beta. The industry beta is the capitalization-weighted average of the betas in any given industry, and thus for any industry, the betas are the same. The industry portfolios are then formed by allocating the industries into quintiles like the market-beta formed portfolios. The results show that the industry sorting does not improve the results much. The alpha of the industry beta sorted portfolio is insignificant. Therefore, a simple beta or volatility sorting is preferable over a double-sorted, industry-beta, and market-beta sorted portfolio. The authors provide a quick look at the IVOL strategy as well. Like Ang et al. (2006) and Ang, Hodrick, Xing, and Zhang (2009), Baker et al. (2014) confirm that IVOL anomaly is present. When performing a double sorting within the industry and all stocks, they find that most of the IVOL strategy's alpha is generated by the overall riskiness of the stock compared to other stocks, instead of the industry's riskiness compared to other sectors.

The preference for lotteries discussed by Baker et al. (2011) was also investigated by Bali, Brown, Murray, and Tang (2017). They argue that the beta anomaly disappears once the preference for lotteries is accounted for. Therefore, the driving force behind the anomaly is investors' preference for lotteries. They study U.S. stocks from 1963 to 2012. The general portfolio construction method is similar to Blitz and van Vliet (2007), Baker et al. (2011), and Blitz et al. (2013). However, the authors use daily returns instead of monthly and require only 12 months of data for the beta estimation. To proxy the lottery preference, the authors construct a MAX measure, the average of the five highest daily returns of a given stock in each month. The authors show that market beta is positively related to the MAX factor, market capitalization, momentum, and IVOL while negatively associated with the book-to-market ratio. Further results show that the MAX factor is negatively correlated with future returns (statistically significant). Therefore, the authors claim that the MAX factor is a good proxy for the lottery demand. The main finding is that the beta effect is no longer present with a double sorting based on the MAX factor and secondly on the beta. Nine of the ten portfolio alphas based on the double sorting are insignificant. Previous studies have not yet offered a way to measure the investors' demand for lotteries. Therefore, the MAX measure is the first one to do so, and there is no guarantee that the measure is the right way to do so. However, it clarifies the low-risk anomaly if it is even a proxy for lottery demand. If correct, Bali et al. (2017) provide strong evidence that earlier studies (see, e.g., Baker et al. 2011) claim that investors' demand for lotteries is correct, and the low-risk anomaly is explained by it.

A recent paper by Novy-Marx and Velikov (2022) heavily criticizes Frazzini and Pedersen's (2014) BAB paper. The original article is one of the most read papers published by the Journal of Financial Economics. Yet, Novy-Marx and Velikov (2022) argue that it is also one of the least understood papers. Their critique focuses on the original paper's portfolio construction method. The standard way of constructing portfolios is by weighting each stock by market capitalization (e.g., Baker et al. 2011; Blitz et al. 2013). Instead, the original paper uses beta rank weighting. Novy-Marx and Velikov (2022) show that essentially beta rank weighting equates to equal-weighted stock portfolios, which is uncommon in recent literature. Another critical difference is portfolio hedging. The standard method is to construct zero-cost portfolios where the strategy goes short on one side to finance the long side. Frazzini and Pedersen (2014) deviate from this standard procedure by leveraging the long side and deleveraging the short side to achieve market neutrality. Finally, the original paper uses a novel method in beta estimation. Instead of market beta estimation by regression, they estimate betas by combining market correlations approximated from five years of overlapping three-day returns with volatilities estimated using one year of daily returns. Novy-Marx and Velikov (2022) note that while the strategy performs exceptionally well on paper, the returns are unachievable. The BAB's premium decreases by 60 % when transaction costs are accounted for. The critique doesn't stop there. The low-volatility anomaly's long portfolio is often composed of small stocks. Arguments for why the anomaly exists usually revolve around this widely known fact. Institutions cannot arbitrage the phenomenon since it would require them to take disproportionately large positions in relatively small stocks. However, with the BAB strategy, this phenomenon is even more prominent. Novy-Marx and Velikov (2022) show that the BAB strategy has extreme weights on micro and nano-cap stocks. For every invested dollar on the long side (low beta), 60 % goes to nano-caps and 10 % to micro-caps. For the short side (high beta), 37 % shorts the nano-caps and 13 % micro caps. While nano- and micro-caps make 3.3 % of the whole market in combination, the strategy leans heavily toward the small-cap stocks. Novy-Marx and Velikov (2022) construct a simple beta arbitrage strategy by more standard methods to compare the original BAB strategy. They use equal-weighting, long minus short strategy, and CAPM for beta estimation. They show that their simple beta arbitrage has a high correlation of 84 % with the original BAB. The conclusion is that the original BAB method is just a backdoor to construct an equal-weighted beta arbitrage that performs phenomenally on paper but is unachievable in reality.

3.3. IVOL

The EMH states that investors should be rewarded for holding risk. Risk can be divided into diversifiable and undiversifiable risks, and according to economic theory, returns should only increase for having undiversifiable risk. Ang et al. (2006) try to uncover if aggregate volatility is a risk factor and how sensitivity difference to innovations (by innovation is meant the unpredictable component of our expectation for the subsequent period, mathematically: $\sigma_{t+1} = \sigma_{t+1}+\varepsilon_{t+1}$, the ε_{t+1} is the innovation) in aggregate volatility affect the expected return. Those stocks with a larger exposure to aggregate volatility should have larger IVOLs and thus a greater expected return. The authors argue that VIX (Volatility Index) is a good proxy for aggregate volatility as it represents the market's view of the 30-day forward expectation of S&P 500 volatility. The exposure to aggregate volatility innovations is done by measuring stock loadings on the daily first differences of VIX for the past month with an extended CAPM (in addition to the market beta, the first difference VIX beta is calculated). Five portfolios are formed using previous months' first difference VIX loadings and grouping the low loadings to the first and high loadings to the last portfolio. The process is then repeated monthly from 1986 to 2000 using AMEX, NASDAQ, and NYSE stocks. The authors develop their study by extending the FF-3 model and including the VIX factor. The results show that whether using the extended CAPM or FF3-model, the low-volatility portfolios earn larger alphas than their high-volatility peers. For example, the NYSE stocks' low minus high volatility portfolio earned an alpha of 0.66 % each month with a t-statistic of 4.85 when regressed against the FF-3 model. This pattern continues whether the authors use double sorting, control various effects (such as value or size) or regress the portfolio against the extended CAPM. Their findings are inconsistent with the theory where stocks with large IVOLs should earn high returns but consistent with other low-volatility studies.

To test if the results found by Ang et al. (2006) were due to an actual low-volatility anomaly instead of data snooping, Ang et al. (2009) revisit their study with a more global view. In a similar setting as the 2006 study, the authors study if IVOL can predict future returns in 23 developed markets from 1980 to 2003. With a new global outlook, the authors confirm their earlier made findings. The results are statistically significant in all 23 developed markets with negative loading on IVOL factor, even while controlling for the size, value, and momentum factors. To further tackle any doubts, the authors use returns from the past three, six, and twelve months for the portfolio construction time. The results are the same: the expected return and IVOL have a negative relationship. Furthermore, the long-short portfolios (longing the high- and shorting the low-volatility) alphas are negative and statistically significant for all markets. Additional contribution is made by finding that the global high- and low-IVOL spread co-moves with the U.S. IVOL spread, implying that the U.S. effect can partially explain the global effect.

The findings of Ang et al. (2006; 2009), Blitz and van Vliet (2007), Baker et al. (2011) were criticized by Li et al. (2014) as hypothetical. The authors test the low-volatility (or low-risk) anomaly using many known low-volatility measures, namely IVOL, trailing low-volatility, and trailing low-beta. Consistent with the earlier mentioned studies, the authors document that the low-volatility anomaly is present using quantile sorts for the three low-volatility strategies. However, they show that value-weighted zero-cost portfolios returns are wiped out when excluding low-priced stocks and accounting for trading costs. With equal-weighted portfolios, the authors do not document the anomaly. Considering the previous studies, the study’s findings are very much different. Although Li et al. (2014) exclude low-priced stocks (stocks less than 5 dollars), it is not necessarily the same exclusion that the criticized studies do. The earlier mentioned studies exclude stocks with low market capitalization. There is a considerable difference in excluding low-priced stocks as stocks with high market capitalization can have low prices (in absolute dollars, less than 5). Furthermore, any earlier studies have not documented the trading costs having such a large effect on the anomaly. While some have considered them, the general consideration has been that the high-volatility side has had a higher turnover than the low counterpart, indicating that the spread should be greater with trading costs. Additionally, Carvalho et al. (2012) argue that the transaction costs for a minimum variance portfolio are minor and should not affect the returns. Their evidence is contradictory to earlier studies. The minimum variance portfolio does not earn any significant alpha. However, the authors note that it may be due to the minimum variance portfolio being explained mainly by the low-beta anomaly and IVOL.

Recent research by Stambaugh, Yu, and Yuan (2015) compares how IVOL sorted portfolios perform when mispricing and investor sentiment is accounted for. The hypothesis is that arbitrage risk and asymmetry deter investors from exploiting mispricing. As investors are not willing to short (long) overpriced (underpriced) stocks to eliminate the mispricing, overpriced (underpriced) stocks should have a negative (positive) risk-return relationship. Moreover, arbitrage asymmetry conceptually means that investors’ long and short positions are highly skewed towards long positions. Thus, the authors argue that during high (low) sentiment periods, the overpricing (underpricing) of overpriced (underpriced) stocks should be the highest following into a more significant negative (positive) IVOL effect. Since mispricing is merely a concept, the mispricing is proxied by 11 known return anomalies. Stocks are given a generalized mispricing rank, according to these return anomalies. The mispricing ranks are then used to sort the stocks into quintiles and then again sorted into quintiles by their IVOLs, resulting in 25 portfolios. To start with, the general result is that among overpriced stocks, the risk-return pattern is reversed, while among underpriced stocks, the risk-return pattern is what the theory predicts. The authors argue that for overpriced (underpriced) stocks that are harder to short, the negative (positive) IVOL relationship should be more substantial. To differentiate between harder to short stocks, they use institutional ownership. The thought is that stocks are relatively easier to short with high institutional ownership due to the availability of stock lending. The findings confirm their expectations. The stock returns with low institutional ownership, large overpricing, and high IVOL are worse than stocks with high institutional ownership. Finally, the authors show that during high (low) sentiment regimes, the negative (positive) IVOL effect is larger among overpriced (underpriced) stocks. These findings are robust even when excluding small companies. The research gives strong evidence in addition to Ang et al. (2006; 2009) and Baker et al. (2011) that the low-volatility anomaly is still present in U.S. stocks. Finally, the authors show that IVOL and beta have a strong positive correlation, and therefore, the IVOL is perhaps a measure that can explain the long-known beta anomaly.

The IVOL's involvement in the beta anomaly proposed by Stambaugh et al. (2015) was investigated by Liu, Stambaugh, and Yuan (2018). Their study uses all CRSP stocks from 1963 to 2013. They confirm that the beta and IVOL effects are found proposed by Ang et al. (2006;2009), Stambaught et al. (2015), Blitz and van Vliet (2007), and Baker et al. (2011), and Blitz et al. (2013). For the beta construction, they follow a similar method as Baker et al. (2011), and for the IVOL construction, their method is identical to Ang et al. (2006). They also use the mispricing concept introduced by Stambaugh et al. (2015). They hypothesize that the IVOL anomaly largely explains the beta anomaly as they find that the beta and IVOL are positively correlated. Their explanation for the beta anomaly contradicts Bali et al. (2017), who propose that the preference for lotteries is the explanation. So why would IVOL and beta be correlated? There are two offered explanations. First, it has been shown that levered equities are more sensitive to news, increasing the volatility, implying that volatility increases with leverage. Furthermore, there exists evidence that volatility is proportional to equity beta. The argument is that higher beta stocks bear more leveraged investors. Secondly, a behavioral explanation is offered. High IVOL stocks are more sensitive to mispricing determined by market-wide sentiment, correlated with market returns. Therefore, high IVOL stocks are more susceptible to mispricing, increasing the high IVOL stocks' beta. The authors focus on two ways to determine the connection. First, they exclude 20 % of stocks with the largest mispricing and 25 % of stocks with the highest IVOL and then sort by mispricing and beta. With the simple exclusion, they find that the beta anomaly gains strength. Their second method sorts stocks into beta deciles and IVOL quantiles. The results reveal that the beta spread is insignificant in four out of five spreads indicating that IVOL partially explains the beta anomaly. The conclusion is that the beta anomaly is present, but its explanation is its positive correlation with the IVOL anomaly.

The academic literature has not settled on the explanations behind these four low-volatility anomalies. Baker et al. (2011) claim that the benchmarking and preference for lotteries are the reasons. Bali et al. (2017) investigated the lottery preference and found out that the anomaly disappears once the lottery demand is accounted for. Liu et al. (2018) offer an alternative explanation. They argue that the IVOL anomaly explicitly explains the beta anomaly. What is common across these anomalies is that they are highly correlated. Specifically, beta and volatility are very related measures and for example. Baker et al. (2011) show that whether sorting by beta or volatility, the low-side of the portfolio outperforms the high-side with minor differences in the choice of risk measure. Blitz, van Vliet, and Baltussen (2019) discuss in their meta-study that volatility and beta are related measures as the stock's beta is the stock's volatility times its correlation with the market divided by market volatility. Furthermore, they argue that the IVOL is closely related to beta and volatility effects but note that the strategy's main drawback is its high transaction costs.

Blitz et al.'s (2019) meta-study compile the reasons for the low-volatility anomaly's existence. The first reason is short-selling constraints argued by Baker et al. (2011). Investors are deterred from arbitraging away the anomaly. Furthermore, Frazzini and Pedersen (2014) found convincing evidence for the explanation as a low-volatility anomaly is more substantial when leverage constraints are tighter (option to short stocks). Secondly, they argue that investors tend to compare strategies instead of focusing on absolute measures. The explanation was also discussed by Blitz and van Vliet (2007) and Baker et al. (2011). Thirdly, agency issues play an essential role, as asset managers create high-beta products to maximize profits. Fourthly, the investors' preference for highly skewed stocks partially explains the anomaly. Blitz and van Vliet (2007), Baker et al. (2011), and Bali et al. (2017) all argue that part of the explanation is that investors join the stock market to seek lottery-like payoffs. Therefore, investors prefer highly volatile stocks instead of low volatile stocks. For them, the stock market is a casino. Finally, the fifth explanation is behavioral biases that investors face. Investors look for attention-grabbing stocks, face representativeness, and overconfidence bias discussed by Daniel et al. (1998) and Kahneman (2003).

This chapter has introduced four different low-volatility strategies that have a strong link between them. While most research finds that these strategies generate significant returns even after decomposing them to known risk factors, the most intriguing thing is that they strongly challenge the EMH and CAPM. Not only in the sense that "anomalies do exist," but showing a flattened risk-return relationship. Such findings are groundbreaking for the financial theory and prove that the existing models are inadequate in explaining how the financial markets work. Finally, since the anomaly debate often revolves around potential data mining concerns, these studies have shown that the low-volatility effect is found in different countries, asset classes, and periods, proving that the effect is a constant and a potential risk factor. It is doubtful that all the previous studies' findings are due to data snooping.

4. ASSET PRICING MODELS

This chapter will discuss asset pricing models with wide recognition among researchers and practitioners. The introduced models are the Capital Asset Pricing Model (CAPM), Fama-French three- (FF-3), and five-factor (FF-5) models. These models are often used to measure a strategy's performance. The foundation of asset pricing models is the CAPM. The FF-3 and FF-5 are extensions of the CAPM to account for additional market-wide systematic risk.

4.1. Capital Asset Pricing Model (CAPM)

The Capital Asset Pricing Model, or CAPM, is the most known and widely accepted asset pricing model to date. The model was developed collectively by Sharpe (1964), Lintner (1965), and Mossin (1966). The CAPM's foundation is in the work of Harry Markovic, who developed the Modern Portfolio theory a decade earlier. The model's key property measures a stock's (or a portfolio's) risk and returns relationship. The model uses a single factor to capture an asset's exposure to market-wide, systematic risk. The factor is often represented as the Greek letter Beta. The CAPM formula is the following:

$$E(r_i) = r_f + \beta_i\left[E(r_m) - r_f\right] \tag{4}$$

Where $E(r_i)$ is the expected return of asset (or portfolio) i, r_f is the risk-free rate (in practice, this is often proxied by an economically strong country's shortest maturity bond's rate, e.g., T-bill in the U.S.), β_i is the beta coefficient. It measures the portfolio's exposure to systematic risk, and $E(r_m)$ is the market portfolio's expected return.

The beta coefficient measures the portfolios loading on the whole market. The overall market portfolio always has a beta of one. When a portfolio's beta is below one, it is called a defensive portfolio, while a portfolio with a beta above one is regarded as an aggressive portfolio. As shown in equation four, the expected return of a portfolio i depends on the risk-free rate and the portfolio's loading to the overall market.

The beta coefficient is the covariance of portfolio i and market divided by the variance of the market. Mathematically:

$$\frac{cov(r_i, r_m)}{\sigma_m^2} \tag{5}$$

The intuition is that the expected return should increase when beta increases, and while it decreases, the expected return should decrease. The positive relationship is derived from the idea that the expected return should increase by increasing risk. When a portfolio is priced at its fundamental value, it lies on the security market line. Yet, multiple studies have found that stocks tend to deviate from the security market line. This can indicate a couple of things. First, the model may be inadequate in explaining asset returns. Second, assets may be mispriced. Third, both explanations happen at the same time. This is a crucial concept of this thesis. The previous low-volatility literature has shown that the relationship is reversed, and investors earn higher returns with lower-volatility stocks. (Baker et al. 2011; Fama & French 1992.)

4.2. Three-factor Model

At its inception, the CAPM was a revolutionary model that strengthened the foothold of the theoretical financial literature. While some critiqued it for its simplicity, it was still a model with sound logic, and more importantly, the CAPM's accuracy could be measured. Since the introduction, multiple studies found that the CAPM did not explain the stock market returns well (Banz 1981). This led to Fama and French (1993) constructing an extended version of the CAPM, the Fama-French three-factor model, including two well-known anomalies as factors: size and value.

In 1981 Banz found the size anomaly. He studied stocks listed in the NYSE from 1926 to 1975 and found that small stocks tend to outperform large stocks. While he did not find the explanation for why such behavior occurs, he noted that there is no theoretical explanation for such an anomaly. Another key concept of the FF-3 model was the value anomaly. Rosenberg, Reid, and Lanstein discovered it in 1985. Their finding indicated that high book-to-market ratio stocks outperform low book-to-market ratio stocks. High book-to-market stocks are often called value stocks, while low book-to-market ratio stocks are growth stocks. Multiple other studies have confirmed both findings.

Using the findings of Banz (1981) and Rosenberg et al. (1985), Fama and French (1993) extended the CAPM by producing a model that has three explanatory factors in it. The beta, small-minus-big (SMB, size factor), and high-minus-low (HML, value factor). The authors argue that the size and value factor capture systematic risk left uncaptured by the CAPM. The SMB factor is constructed by ranking stocks based on their market capitalization, buying the small market capitalization portfolio, and selling the large market capitalization portfolio. The HML factor is constructed by ranking the stocks by the book-to-market ratio. The strategy then buys the high book-to-market ratio stocks and sells the low book-to-market ratio stocks. To test the Fama-French three-factor model, a following time series regression can be run:

$$r_{it} - r_{ft} = \alpha_i + \beta_i\left(r_{mt} - r_{ft}\right) + s_i SMB_t + h_i HML_t + e_{it} \tag{6}$$

Where α is the excess return on portfolio i or the y-axis intersect.

r_{it} is the return on of portfolio i for period t.

r_{ft} is the risk-free rate for period t.

r_{mt} is the return on the value-weighted market portfolio for period t.

SMB_t is the return on size for period t.

HML_t is the return on value for period t.

B_i, s_i, and h_i are the loadings for market, size, and value factors.

e_{it} is the zero-mean residual for period t.

The FF-3 model captures three known risk factors on the market: the market factor, the size factor, and the value factor. For the FF-3 to be a model that thoroughly explains the stock market, the alpha of any strategy regressed against the FF-3 model should be zero. When the alpha is zero and the coefficient loadings are statistically significant, it can be said that the model explains the given strategy. However, there is proof that the low-volatility and low-beta strategies earn statistically significant alpha after the regression (Blitz and van Vliet 2007; Blitz and van Vliet 2013). Yet the evidence is for capitalization-weighted U.S. stocks is narrow and should be further examined.

4.3. Five-factor Model

Mounting evidence such as Blitz and van Vliet (2007), and Blitz and van Vliet (2013) showed that the FF-3 was unsuccessful in explaining all anomalies. Further evidence was provided by Novy-Marx (2013) who showed that company profitability could be used to predict returns. The author used the gross profits to assets ratio to proxy for profitability. While the profitability strategy is close to the value strategy, it is different. The value strategy intends to buy cheap assets by selling expensive assets while the profitability buys productive assets, financing them by selling unproductive assets. The evidence showed that after the FF-3 regression, the zero-cost profitability portfolio earned a statistically significant alpha of 0.52 % monthly.

Titman, Wei, and Xie (2004) showed that companies that were conservative with their investments tended to outperform companies investing heavily. The former companies are called conservative and the latter aggressive. Their strategy bought the conservative stocks and sold the aggressive stocks. A four-factor regression (FF-3 and added momentum factor proposed by Carhart) showed that the strategy could not be explained by market, size, value, or momentum factor.

Building on the work of Novy-Marx (2013) and Titman et al. (2004) Fama and French (2015) enhanced their previous FF-3 model by adding two new factors to it robust-minus-weak (RMW, profitability factor) and conservative-minus-aggressive (CMA, investment factor). The RMW factor is constructed by buying profitable stocks measured by the gross profit to asset ratio and selling unprofitable stocks. The CMA factor is constructed by buying stocks that are conservative in their investments and selling stocks that invest aggressively. The Fama and French five-factor model regression is as follows:

$$r_{it} - r_{ft} = \alpha_i + \beta_i(r_{mt} - r_{ft}) + s_i SMB_t + h_i HML_t + k_i RMW_t + c_i CMA_t + e_{it} \qquad (7)$$

Where α is the excess return on portfolio i or the y-axis intersect.

r_{it} is the return on of portfolio i for period t.

r_{ft} is the risk-free rate for period t.

r_{mt} is the return on the value-weighted market portfolio for period t.

SMB_t is the return on size for period t.

HML_t is the return on value for period t.

RMW_t is the return on profitability for period t.

CMA_t is the return on investment for period t.

B_i, s_i, h_i, k_i, and c_i are the loadings for the market, size, value, profitability, and investment factors.

e_{it} is the zero-mean residual for period t.

According to Fama and French (2015) the model's biggest flaw is found in explanations of small stocks which have negative loadings to RMW and CMA factors. These stocks consist of small companies that invest aggressively while having low profitability. According to the authors, the expected return on such stocks is zero or negative. The puzzling discovery is that large stocks investing aggressively with weak profitability have a positive expected return.

5. Data and Methodology

This chapter discusses the empirical part of the thesis. The chapter will introduce the data and methodology used in the thesis. The empirical methods will follow the technique used by Baker et al. (2011). According to the presented literature, multiple other studies have used Baker et al.'s (2011) method. Thus, there is reason to believe that the method is a robust way to construct low-volatility portfolios (e.g., Blitz and van Vliet 2007; Blitz et al. 2013; Bali et al. 2017). Furthermore, the portfolio's performance will be evaluated similarly to the earlier studies. In other terms, the portfolio returns are regressed against the CAPM. This thesis will extend the previous literature by examining how the volatility and beta sorted portfolios perform against newer asset pricing models, the FF-3 and FF-3. According to Fama and French (2015), their five-factor model explains the low-volatility effect. Their claims will be tested. Finally, the thesis will find if the found alphas are jointly zero using the Gibbon, Ross, and Shanken (1989) test, known as GRS, and if the estimated loadings are priced using the Fama and Macbeth (1973) regression.

5.1. Data

The sample consists of CRSP 1970 – 2019 equity data from all U.S. stock exchanges. The first portfolio was formed in 1975. The total obtained sample spans 50 years. The monthly regression data is obtained from Kenneth French's website for the asset pricing models (CAPM, FF-3, and FF-5). Previous research has resorted to the freely available asset pricing data extensively, and thus, there is no argument to deviate from the previous data sources. The equity data consists of 15 882 companies and approximately 2.9 million monthly observations. However, not all companies are included at every point in time, as many enter or leave the public market during the examined period.

5.2. Methodology

Two different strategies will be formed from the return data: the low-volatility and low-beta. For all stocks used to create the portfolios, 60 months of prior return data is required to calculate the volatility and beta estimates. A rolling 60-month standard deviation is calculated each month for the volatility estimation from the return data. The beta is estimated with a CAPM regression. Once both estimates are found, the stocks are sorted into quintiles using NYSE cut-offs. NYSE cut-offs are a standard method in previous literature. Each quintile is then value-weighted to not overweight small stocks that enhance the low-volatility and low-beta effect. The portfolios are then held for a month, after which the process repeats itself. At the start of every month, each of the five portfolios will buy (sell) stocks if the stock belongs (does not belong) to the portfolio for the coming month. Furthermore, at the beginning of the month, the portfolios will rebalance the stock weights inside the portfolio to have a proper value-weighted portfolio. Since the portfolios use NYSE cut-offs, there is an unequal number of stocks in each portfolio for any given month.

Since the dataset is large, there are minor flaws within it. When estimating the beta, 357 monthly observations receive a beta of above ten or below -10. 2183 monthly observations had 5-year rolling volatility above 100 % when calculating the volatility. It is safe to say that with a 5-year rolling beta or 5-year rolling volatility, such beta or volatility is not realistic. These observations were excluded from the dataset.

A zero-cost portfolio approach will be used as the benchmark study by Baker et al. (2011). The portfolio will go long in the low-beta and low-volatility portfolio and short in the high-beta and high-volatility portfolio. This will result in the investor not investing any capital in the portfolio formation.

The resulting portfolios are evaluated using earlier introduced portfolio performance measures. Each portfolio is regressed against the capital-asset-pricing model, Fama-French three-factor model, and Fama-French five-factor model. While the benchmark study of Baker et al. (2011) used only CAPM to regress the results, there is reason to believe that the low-volatility and low-beta might have high correlations with the value, size, profitability, and investment factors. Thus, the thesis adds to the current literature by extending the analysis of low-volatility and low-beta. While Blitz and van Vliet (2006) used the FF-3 regression, they used equal-weighting, considered a non-standard way for portfolio weighting. Additional portfolio performance measures are shown, such as the Sharpe and Sortino ratios.

Once the portfolio measures are obtained, the GRS (1989) test will be used to determine if the alphas are jointly zero. The time-series regressions will provide insight if the used factors that can explain the low-volatility phenomenon. However, there is a 1 in 20 chance that the alphas are statistically significant by chance for each of the five regressions (if we use the 5 % statistical significance level). Therefore, the GRS tests if the estimated alphas are jointly zero. Furthermore, a Fama-Macbeth regression must be run to assess the factors' actual loadings. The regression determines the correct factor loadings for the portfolios. Previous low-volatility papers have used neither of these methods to determine if the alphas are jointly zero or the actual factor loadings. Therefore, the study will extend the previous literature by uncovering if the earlier found alphas are just by random chance and what the actual factor loadings for the low-volatility and low-beta portfolios are.

6. RESULTS

The sixth chapter will present the results of the empirical part. The tables will be divided into general characteristics of the portfolios and the market, the CAPM regression results, FF-3 regression results, FF-5 regression results, and GRS and Fama-Macbeth regression results. The chapter will also present figures related to the study.

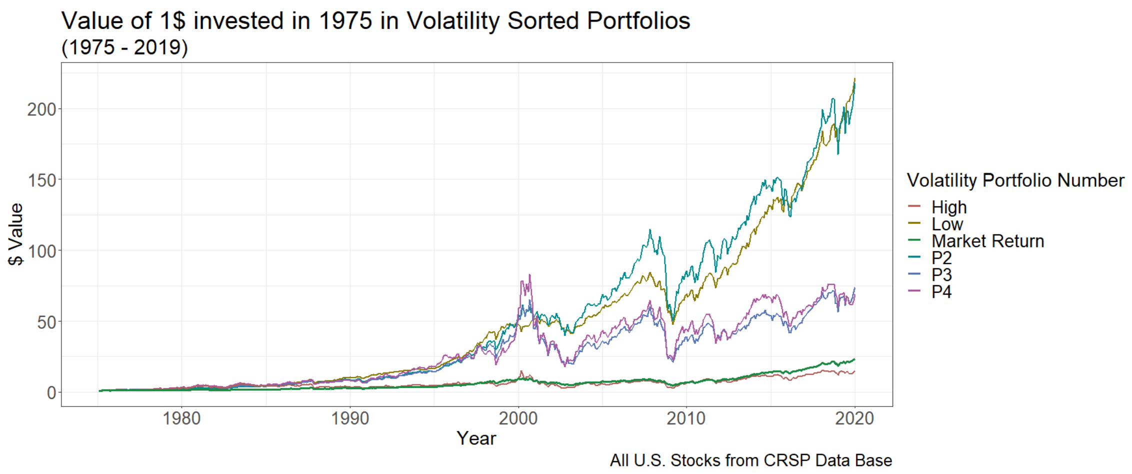

Figure 3 $1 Invested in Volatility Sorted Portfolios.

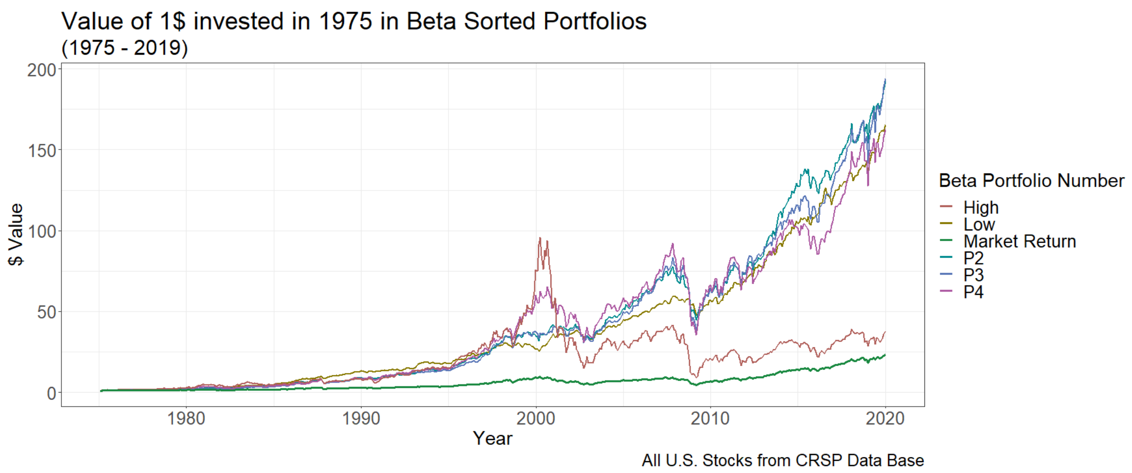

Figure 4 $1 Invested in Beta Sorted Portfolios.

Figures three and four show how investing $1 in each portfolio and the market would have grown from 1975 to 2019. The $1 invested in the low-volatility portfolio would have grown to $221.28 in 45 years. During the same time, if the investor were to invest $ 1 to the high-volatility portfolio, he would have gained only $15.29. The $1 in the low-beta portfolio grew to $165.51, and the high-beta portfolio rose to $42.38. At the same time, the market would have yielded the investor $23.35. Whether the risk is defined by volatility or beta, the low-risk portfolio outperforms the high risk over the examined period. These preliminary results have the same story as Baker et al. (2011) in the benchmark study. While their strategy started eight years before and ended ten years earlier, their low-volatility portfolio grew to $59.55 and low-beta to $60.46. In contrast, the high volatility reduced to 0.58 cents (a loss), and the high-beta increased to $3.77. The significant spike for the high-volatility or high-beta near 2000 is due to both high portfolios holding many internet bubble stocks. In the beta plot, the portfolios Low to P4 all grow steadily to above $150. However, it is noticeable that the low-beta portfolio has a significantly smaller return variance than P2 to P4.

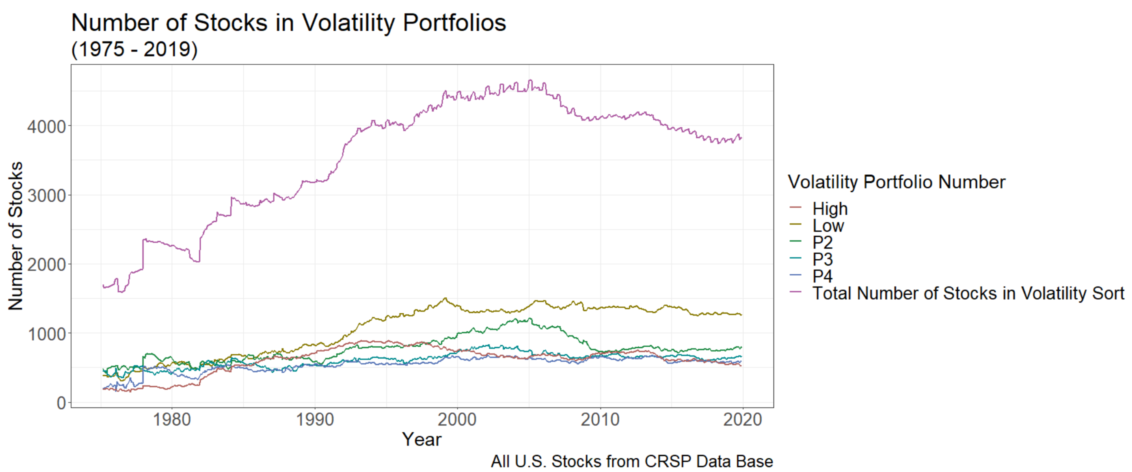

Figure 5 Number of Stocks in Volatility Portfolios.

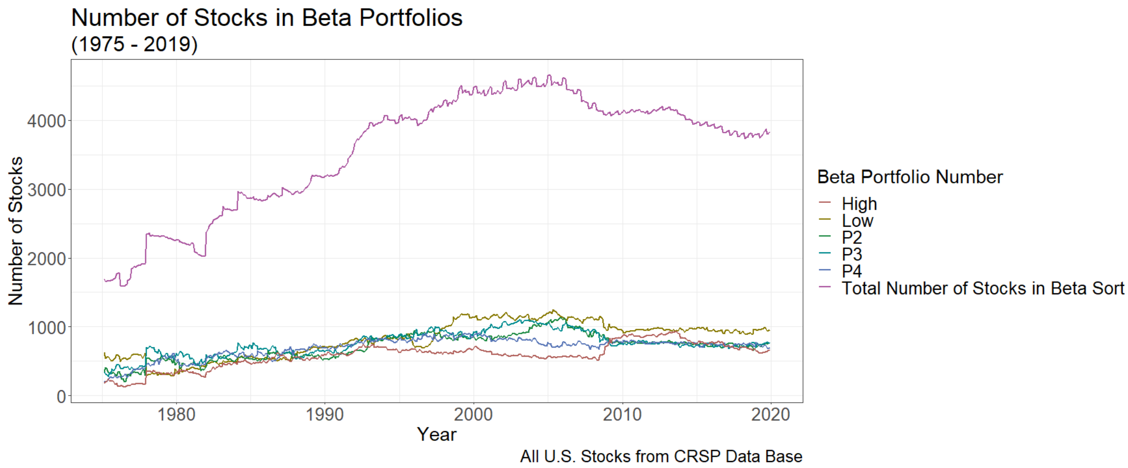

Figure 6 Number of Stocks in Beta Portfolios.

Figures five and six show the number of stocks in volatility and beta-sorted portfolios. In volatility sort, it is noticeable that the number of stocks in low-volatility portfolios diverts from the other portfolios significantly after 1993. The beta portfolios seem to have quite an equal number of stocks at any given moment in the portfolios—furthermore, the total number of stocks peaks in 2005 at around 4 500 stocks.

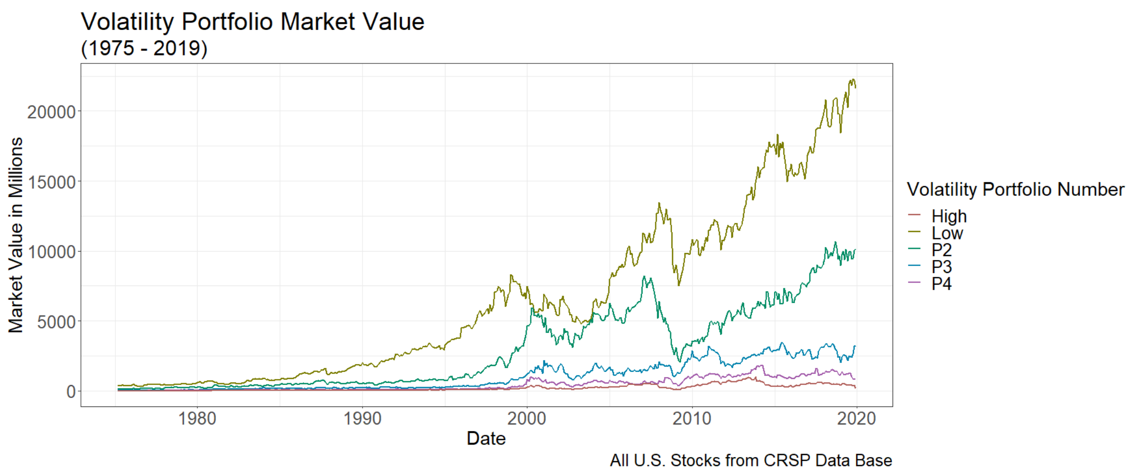

Figure 7 Volatility Portfolio Market Value (in Millions)

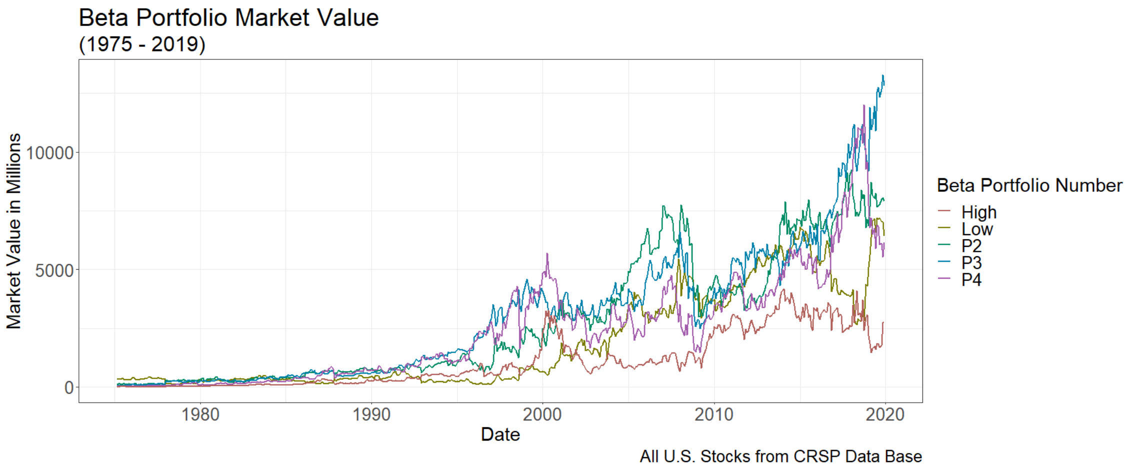

Figure 8 Beta Portfolio Market Value (in Millions)

Figures seven and eight show the market value of stocks in each portfolio every month from January 1975 to December 2019. The volatility sorted portfolios have much larger market cap differences in the later part of the data set than the beta portfolio. The figures reveal that neither of the anomalies is driven by size anomaly. If they were, the low portfolio of both strategies should be substantially below all other portfolios.

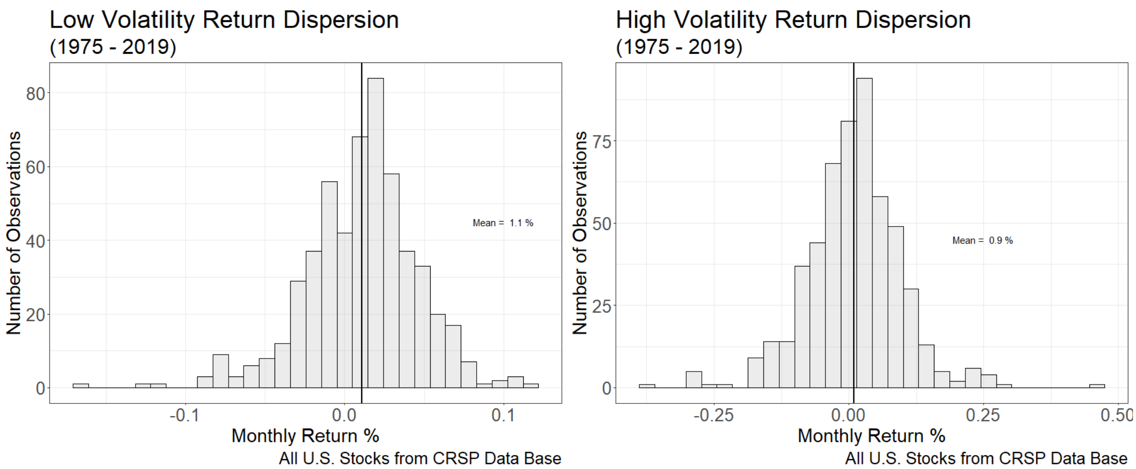

Figure 9 Low and High Volatility Portfolio Dispersion

Figure nine shows the dispersion of returns for low and high-volatility portfolios. While the pictures seem identical, the x-axis is not. Most of the low-volatility monthly returns are between -10 % and 10 %, with a minimum of -16.8 % and an average of 1.07 %. The high-volatility returns are mostly between -25 % and 25 %, with a minimum of -38.3 % and an average of 0.91 %. The significant difference in the returns shows that the low-volatility portfolio's returns are much more predictable with smaller drawdowns monthly.

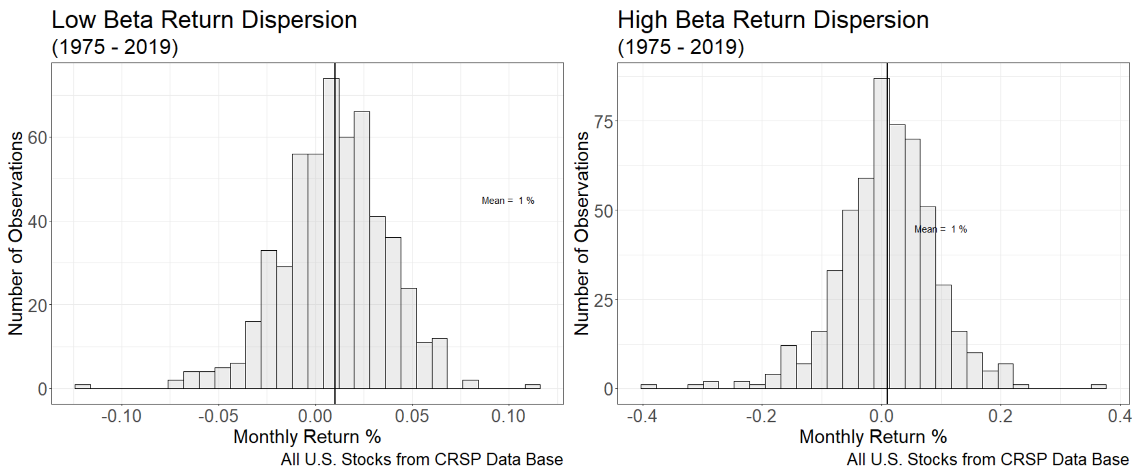

Figure 10 Low and High Beta Portfolio Dispersion

Figure ten shows the dispersion of returns for low and high-volatility portfolios. As with the volatility, the low beta returns are much denser compared to high beta returns. The low-beta returns are mostly between -5 % and 5 %, with a minimum of -12.12 % and an average of 0.99 %. The majority of high-beta returns are between -20 % and 20 %, with a minimum of –39.8 % and an average of 1.02 %.

6.1. Low-Volatility portfolios

Table one below presents general portfolio characteristics of the volatility sorted portfolios. The mean excess return hovers between 0.9 % and 1.13 % per month, showing no increasing or decreasing pattern going from Low to High volatility. However, the standard deviation tends to rise from Low to High, starting from 3.5 % and ending at 8.9 %. The pattern in the standard deviation reveals that the previous 60-month volatility is a good predictor for the following month's volatility, i.e., if volatility was low in the preceding 60 months, it is highly likely that it will be low in the next month. This is an essential finding as the goal is to invest in stocks with low volatility and short stocks with high volatility. If the results were the opposite, the strategy would lose ground with preceding low 60-month volatility following a month of high volatility. Since the low-volatility portfolio has approximately half of the standard deviation as the high-volatility portfolio, the low-volatility portfolio's Sharpe ratio is three times as large as the High comparison. A similar pattern can be seen in the Sortino ratio, which measures the excess return divided by downside volatility. The CAGR of the high-volatility portfolio is nearly half of the low-volatility portfolio. Furthermore, the low-volatility portfolios' worst monthly return is the lowest (-16.8 %) of all the portfolios. The statistics also show that the low-volatility portfolio consists of large stocks. The average low-volatility portfolios market capitalization is 6.4 billion, whereas the high-volatility portfolio's average market capitalization is 0.2 billion. The earlier claims that the low-volatility anomaly is driven by size anomaly seem unfounded considering these portfolio characteristics. The results are like those reported by Baker et al. (2011), except their paper's results were stronger. This can be due to having a new data set, a larger sample, or both.

Table 1 General Statistics of Volatility Portfolios

The table below reports general monthly portfolio measures of the volatility sorted portfolios. The cut-offs for the portfolios are calculated each month from the NYSE exchange after which each stock is assigned to a portfolio. The sample period is Jan-1975 to Dec-2019.

Portfolio	Low	P2	P3	P4	High
Excess return (%)	1.07	1.13	1.00	1.09	0.91
CAGR (%)	1.01	1.01	0.80	0.78	0.51
Std. (%)	3,5 %	5,1 %	6,3 %	7,7 %	8,9 %
Sharpe	0,30	0,22	0,16	0,14	0,10
Sortino	0,27	0,20	0,13	0,13	0,09
Worst Monthly Return (%)	-16.80	-27.16	-26.69	-31.65	-38.28
Average Number of Stocks	948	692	547	487	546
Average Market Cap (in Millions)	6437	2992	1040	446	196

Table two below reports the CAPM regression results of volatility sorted portfolios. The reported alpha is the monthly percentage return. As shown in table two, there is a clear pattern that the alpha decreases from low-volatility portfolios to high-volatility portfolios. The low-, high-, and low – high alphas are all statistically significant at the 1 % level, while the P3 and P4 alphas are statistically significant at the 5 % level. The only portfolio alpha that is not statistically significant is the P2. There is a clear increasing tendency for the beta from low to high portfolios, starting from 0.75 and ending at 1.68, with all betas being statistically significant at the 1 % level. Comparing the results with the Baker et al. (2011) paper, the results are similar. The difference between alphas in the original article is a bit larger. At the same time, the betas are very close to being equal (only minor deviations). These CAPM results indicate that the preliminary data sorting and regression have been done correctly and that the first hypothesis can be confirmed partially (the low-volatility anomaly is present today).

Table 2 CAPM regression results of Volatility Portfolios

The table below reports the monthly excess return and CAPM factor loadings of the Volatility sorted portfolios. Stocks are assigned to their quantiles based on NYSE cut-offs for the month. The portfolios are market capitalization-weighted and rebalanced at the start of each month.

Portfolio	Low	P2	P3	P4	High	Low - High
Alpha (%)	0.19***	0.00	-0.26**	-0.33**	-0.61***	0.80***
t-stat	3.21	0.07	-2.28	-2.06	-2.76	3.02
Beta	0.75***	1.12***	1.31***	1.54***	1.68***	-0.93***
t-stat	56.11	76.31	50.95	42.32	33.70	-15.52
R-Squared	0.85	0.92	0.83	0.77	0.68	0.31

The table shows CAPM regressions results of Volatility sorted portfolios from Jan-1975 to Dec-2019. * marks the 10 % significance level, ** marks the 5 % significance level, and *** marks the 1 % significance level

Figure eleven depicts the alpha and beta relationship of the volatility-sorted portfolios. While there are only five observations (five portfolios), the pattern is unmistakable. The efficient market hypothesis dictates that with risk, the returns should increase. However, the figure below shows an inverted relationship opposite to what the EMH states. The inverted alpha and beta relationship were also found by Baker et al. (2011), Blitz et al. (2013), and Frazzini and Pedersen (2014). The pattern seems not to have changed with new data.

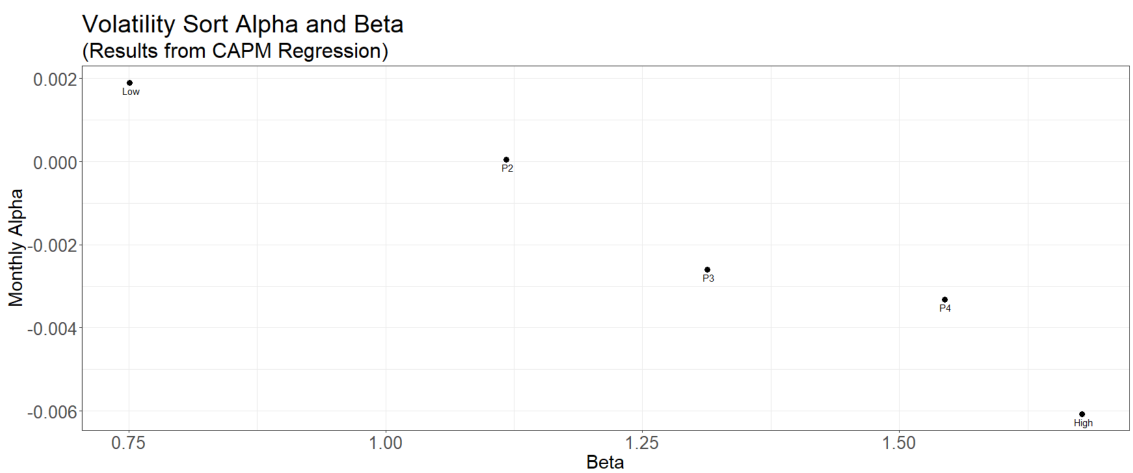

Figure 11 Alpha and Beta relationship of Volatility sorted portfolios.

Table three shows the Fama-French three-factor regression results for the volatility sorted portfolios. The benchmark study of Baker et al. (2011) did not use the Fama-French three-factor regression for unknown reasons, even though the required data was already available. The table shows that the low, high, and low-high alphas are statistically significant at the 1 % level. The low and low-high portfolios alphas are the only positive alphas. The P3 and P4 alphas are statistically significant at the 5 % level, while the P2 alpha is insignificant as in the CAPM regression. All the betas are statistically significant at the 1 % level, with the low portfolio being defensive and the high portfolio being aggressive. The SMB factor for the low portfolio is negative and statistically significant at the 1 % level. This indicates that the low portfolio co-moves with large stocks. The SMB factor increases consistently from low to high portfolio, with the high portfolio having a 1.18-factor loading on the SMB. The P2 of the SMB factor is statistically significant at 10 %, and the rest of the portfolios are statistically significant at the 1 % level. All HML loadings are statistically significant at the 1 % level, while only low and P2 loadings are positive, meaning that those two portfolios co-move with value stocks (high book-to-market stocks). The adjusted r-squared increases uniformly for each portfolio, meaning the FF-3 is a better fit model. The second regression indicates that the study’s second hypothesis can be rejected partially. While the SMB and HML loadings are mostly statistically significant and large in magnitude, there is no significant difference between the CAPM alphas and FF-3 alphas levels. Furthermore, the alpha’s significance level does not change from CAPM regression to FF-3 regression. Some alphas’ p-levels even increase.

Table 3 Fama-French three-factor regression results of Volatility Portfolios

The table below reports the monthly excess return and Fama-French three-factor loadings of the Volatility sorted portfolios. Stocks are assigned to their quantiles based on NYSE cut-offs for the month. The portfolios are market capitalization-weighted and rebalanced at the start of each month.

Portfolio	Low	P2	P3	P4	High	Low - High
Alpha (%)	0.15***	-0.02	-0.24**	-0.26**	-0.60***	0.75***
t-stat	3.25	-0.36	-2.36	-2.02	-3.76	4.04
Beta	0.82***	1.12***	1.23***	1.37***	1.43***	-0.61***
t-stat	74.57	73.02	50.08	44.68	38.08	-14.00
SMB	-0.24***	0.04*	0.37***	0.68***	1.18***	-1.42***
t-stat	-14.78	1.71	10.20	14.88	21.35	-22.00
HML	0.19***	0.06***	-0.16***	-0.41***	-0.41***	0.59***
t-stat	11.38	2.83	-4.35	-8.96	-7.28	9.09
R-Squared	0.91	0.92	0.86	0.85	0.84	0.67

The table shows Fama-French three factor regressions results of Volatility sorted portfolios from Jan-1975 to Dec-2019. * marks the 10 % significance level, ** marks the 5 % significance level, and *** marks the 1 % significance level

Table four shows the Fama-French five-factor regression results for the volatility-sorted portfolios. Only the high portfolios alpha is statistically significant at the 10 % level from the three-factor regression. Furthermore, the level of all alphas is nearly zero, with only the high portfolio's alpha being -0.24 % and the low-high portfolio's alpha being 0.23 %. The beta structure stays the same, with beta increasing from low to high and all beta loadings being statistically significant at the 1 % level. The SMB factor structure remains the same, with a minor drop in magnitude. All SMB loadings are still statistically significant at the 1 % level. The HML structure also stays the same, but the magnitude of the factors for the low, P2, and P3 goes to nearly 0. The only non-significant HML loading is for portfolio 3. The rest of the HML loadings are statistically significant at the 1 % level. The RMW loading for the low-volatility portfolio is positive and consistently dropping towards the high portfolio, with the high portfolio having a negative loading on RMW. The positive loading indicates that the low-volatility portfolio co-moves with stocks that have robust operating profitability. All RMW loadings are statistically significant at the 1 % level. The CMA loadings do not show a clear pattern as the low-portfolios loading is negative, and then the loading for the factor starts to drop towards portfolio four and goes negative. However, the high-volatility portfolio has a slight positive loading on the factor. The CMA loadings are statistically significant at a 1 % level for the low and P3, significant at the 5 % level for P2 and P4, and insignificant for the high portfolio. The adjusted r-squared increases uniformly further from the FF-3 regression, indicating that the FF-5 regression is a better fit for the analysis. The results confirm the claims made by Fama and French (2015) that their five-factor model can explain the low-volatility anomaly. The third hypothesis of the thesis can be confirmed considering the regression results as the alphas level drops significantly, and there is no statistical significance left.

Table 4 Fama-French five-factor regression results of Volatility Portfolios

The table below reports the monthly excess return and Fama-French five-factor loadings of the Volatility sorted portfolios. Stocks are assigned to their quantiles based on NYSE cut-offs for the month. The portfolios are market capitalization-weighted and rebalanced at the start of each month.

Portfolio	Low	P2	P3	P4	High	Low - High
Alpha (%)	-0.01	-0.05	-0.05	0.03	-0.24*	0.23
t-stat	-0.25	-0.79	-0.49	0.21	-1.68	1.43
Beta	0.86***	1.12***	1.17***	1.30***	1.36***	-0.49***
t-stat	89.08	69.64	46.84	43.01	38.40	-12.40
SMB	-0.17***	0.08***	0.29***	0.52***	0.94***	-1.12***
t-stat	-11.99	3.22	7.76	11.65	18.05	-19.02
HML	0.07***	0.10***	-0.02	-0.28***	-0.36***	0.42***
t-stat	3.72	3.39	-0.39	-4.82	-5.33	5.66
RMW	0.27***	0.12***	-0.33***	-0.58***	-0.86***	1.13***
t-stat	14.40	3.88	-6.81	-9.90	-12.51	14.68
CMA	0.22***	-0.11**	-0.26***	-0.20**	0.05	0.17
t-stat	7.75	-2.41	-3.57	-2.23	0.50	1.46
R-Squared	0.94	0.92	0.87	0.88	0.87	0.76

The table shows Fama-French five factor regressions results of Volatility sorted portfolios from Jan-1975 to Dec-2019. * marks the 10 % significance level, ** marks the 5 % significance level, and *** marks the 1 % significance level

The GRS and Fama-Macbeth regressions are added to the study for further robustness tests shown in table five. The GRS test uncovers if the volatility sorted portfolios alphas are jointly zero. The GRS CAPM result rejects the hypothesis at the 5 % significance level that all CAPM alphas are jointly zero implying that the CAPM regression results can be trusted and that the market beta does not fully explain the low-volatility anomaly. The GRS FF-3 test also rejects the test that all alphas are zero at the 1 % significance level indicating that the beta, HML, and SMB cannot explain the volatility anomaly. The GRS test for the FF-5 cannot be rejected. Therefore, it is highly likely that the alphas for the Fama-French five-factor model are jointly zero, and the RMW and CMA factors do capture the low-volatility anomaly.

The Fama-Macbeth regression determines the risk premium for each factor. Only the CAPM and FF-3 models are used as it would require more portfolios to perform the Fama-Macbeth regression for the FF-5. The Fama-Macbeth regression shows that the CAPM alpha is statistically significant at the 1 % level. Furthermore, the insignificant beta for the CAPM in the Fama-Macbeth regression indicates that the beta factor does not capture returns. The Fama-Macbeth FF-3 regression shows that none of the risk factors is significant, nor the alpha is significant, indicating that none of the risk factors capture the volatility returns.

Table 5 GRS and Fama-Macbeth regression results of Volatility Portfolios

The table reports the GRS and Fama-Macbeth regression results for Volatility sorted portfolios. On the left table are reported the GRS-test statistics of $\alpha_1 = 0$ for all i for all three factor regressions. On the right table are the Fama-Macbeth regression results of the factor risk premiums for CAPM and Fama-French three-factor regressions.

GRS Test Statistic			Fama-Macbeth Regression				
Regression	t-stat	p-value	Regression	Alpha	Beta	SMB	HML
CAPM	2.34	0.04	CAPM	0.008***	-0.001		
FF-3	3.52	0.00	t-stat	3.07	-0.41		
FF-5	1.11	0.35	FF-3	0.003	0.004	-0.004	-0.002
			t-stat	0.79	0.83	-1.21	-0.37

The CAPM regression combined with the GRS test and Fama-Macbeth regression confirms the evidence found by Baker et al. (2011) that the low-volatility anomaly is present today. The Fama-French three-factor regression, the GRS test, and the Fama-Macbeth three-factor regression point in the direction that the study’s second hypothesis should be partially rejected. The results indicate that the Fama-French three-factor model cannot capture the returns of the volatility anomaly. The Fama-French five-factor regression and the GRS test imply that the third hypothesis should be partially confirmed. The claims of Fama and French (2015) seem trustworthy with the results at hand. While there are no results for the Fama-Macbeth regression for the five-factor model, both the GRS test and the five-factor regression are already strong evidence to confirm the third hypothesis.

6.2. Low-Beta portfolios

Table six shows the general characteristics of the beta-sorted portfolios. The mean excess return increases slightly from low-beta to high-beta, but the pattern is unclear. The cumulative average growth is relatively stable over the portfolios low to P4, with the high portfolio CAGR dropping significantly. A similar pattern of standard deviation found in the volatility sorted portfolios is found in the beta sorted portfolios. The standard deviation of beta sorted portfolios increases monotonically from low portfolio to high portfolio. The Sharpe ratio of the beta sorted portfolio decreases going from low beta portfolios to high beta portfolios. The Sortino ratio also behaves like the Sharpe ratio. However, the magnitude of the Sortino ratio difference is much more subtle than in the volatility sort. The worst monthly return percentage for the low-beta portfolio is -12.12 %, while the high-beta portfolio's worst monthly return is -39.76 %. When comparing the beta-sorted portfolios to volatility-sorted portfolios, the beta-sorted portfolios had a lesser standard deviation across all portfolios. Furthermore, the past 60-month beta seems to be a good predictor for the following months' beta as the standard deviation is smaller for the low-beta portfolio and larger for the high-beta portfolio. As in the case of volatility-sorted portfolios, this is an essential finding for the beta-sorted portfolios. The beta-sorted portfolios' market capitalization is relatively evenly distributed across all portfolios. The high-beta portfolio has the lowest average market capitalization of 1.1 billion, and the low-beta portfolio's average market capitalization is in the middle of all market capitalizations at 1.9 billion. The beta-sorted portfolios' general characteristics are close to the results of Baker et al. (2011) except that their mean excess return decreases going from low to high. In turn, the mean excess return of this new data set is flat across the portfolios.

Table 6 General Statistics of Beta Portfolios

The table below reports general monthly portfolio measures of the Beta sorted portfolios. The cut-offs for the portfolios are calculated each month from the NYSE exchange after which each stock is assigned to a portfolio. The sample period is Jan-1975 to Dec-2019.

Portfolio	Low	P2	P3	P4	High
Excess return (%)	0.98	1.05	1.09	1.11	1.02
CAGR (%)	0.95	0.98	0.98	0.94	0.67
Std. (%)	2,71 %	3,82 %	4,76 %	5,74 %	8,16 %
Sharpe	0,36	0,28	0,23	0,19	0,12
Sortino	0,34	0,29	0,28	0,28	0,25
Worst Monthly Return (%)	-12.12	-21.29	-24.68	-23.82	-39.76
Average Number of Stocks	819	711	757	695	586
Average Market Cap (in Millions)	1882	2745	2994	2391	1100

Table seven below reports the CAPM regression results of beta-sorted portfolios. The reported alpha is the monthly percentage return. As shown in table seven, there is a clear pattern that the alpha decreases from low-beta portfolios to high-beta portfolios. The low-, high-, and low – high alphas are all statistically significant at the 1 % level, while portfolio two is statistically significant at the 5 % level. The alpha of portfolios three and four are statistically insignificant. There is a clear increasing tendency for the beta from low to high portfolios, starting from 0.38 and ending at 1.70, with all betas being statistically significant at the 1 % level. Comparing the results with the Baker et al. (2011) paper, the results are similar. Though, the difference between the alphas is smaller in the benchmark study. The level of betas is equivalent except for low-beta portfolios beta, which in the article has a beta of 0.6 (compared to the tables beta of 0.38). The results indicate that the beta-sorted portfolios have been formed accordingly and the regression is done correctly. The findings further suggest that the first hypothesis (The low-volatility anomaly is present today) can be confirmed fully. The beta- and volatility-sorted CAPM results show that using a low-risk measure to sort stocks leads to abnormal returns that the market cannot explain. These findings confirm what Baker et al. (2011) and Blitz et al. (2013) have reported.

Table 7 CAPM regression results of Beta Portfolios

The table below reports the monthly excess return and CAPM factor loadings of the Beta sorted portfolios. Stocks are assigned to their quantiles based on NYSE cut-offs for the month. The portfolios are market capitalization-weighted and rebalanced at the start of each month.

Portfolio	Low	P2	P3	P4	High	Low - High
Alpha (%)	0.36***	0.16**	0.03	-0.11	-0.51***	0.87***
t-stat	3.90	2.03	0.39	-1.54	-3.44	4.16
Beta	0.38***	0.77***	1.03***	1.26***	1.70***	-1.32***
t-stat	18.26	42.44	64.77	76.11	50.68	-27.81
R-Squared	0.38	0.77	0.89	0.92	0.83	0.59

The table shows CAPM regressions results of Beta sorted portfolios from Jan-1975 to Dec-2019. * marks the 10 % significance level, ** marks the 5 % significance level, and *** marks the 1 % significance level

Figure twelve shows the relationship between alpha and beta in the beta-sorted portfolios. The pattern indicates that the relationship between beta and alpha is negative instead of positive, as discussed in the EMH. The inverse relationship was also reported by Baker et al. (2011), Blitz et al. (2013), and Frazzini and Pedersen (2014).

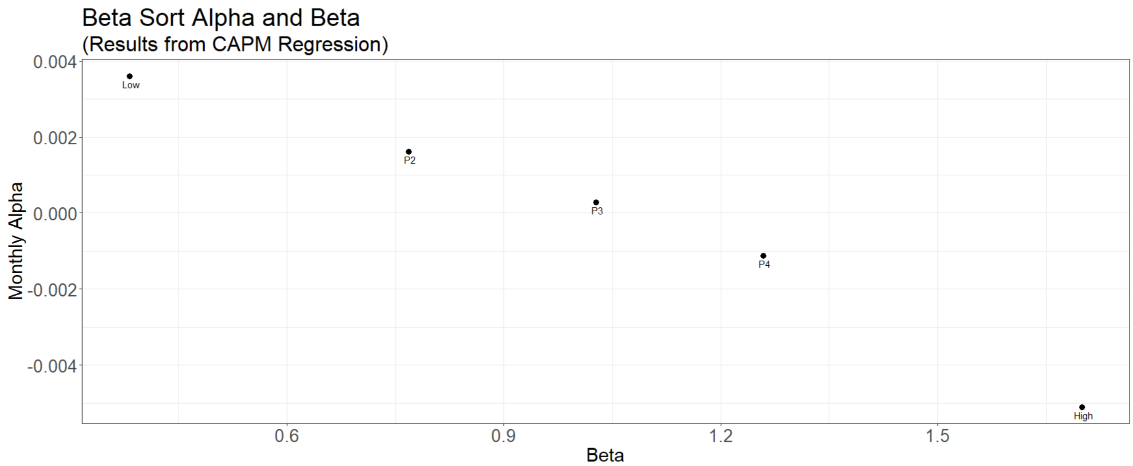

Figure 12 Alpha and Beta relationship of Beta sorted portfolios.

Table eight shows the Fama-French three-factor regression results of the beta-sorted portfolios. Baker et al. (2011) did not either run the beta-sorted portfolio regression on the three proposed market-wide factors. The regression results show that the low-, high-, and low-high portfolios alpha are statistically significant at the 1 % level. The alphas of portfolios two, three, and four are insignificant. The beta of all portfolios is statistically significant at the 1 % level and strictly increases from the low portfolio to the high portfolio. The SMB factor is statistically significant at the 1 % level for all portfolios except portfolio four, for which it is insignificant. The SMB factor does not show a clear increasing pattern from low to high and is small in magnitude for all portfolios but the high-beta and the zero-cost portfolio. The small magnitude of the SMB factor indicates, while statistically significant, that the beta-sorted portfolios do not co-move with the SMB factor (except for the high-beta and low-high-beta portfolios). The HML factor is also statistically significant at the 1 % level for all portfolios except for portfolio four, which is insignificant. The adjusted r-squared is lower across all portfolios when compared with the volatility-sorted portfolios, especially the low-beta portfolio. However, the adjusted r-squared is larger than in the CAPM, indicating that the Fama-French three-factor model captures the returns better. Given the FF-3 regression results, the study’s second hypothesis can now be fully rejected. The market, size, and value factors explain a significant part of the low-beta anomaly. Still, there is very strong statistical significance in the alphas for low, high, and low-high portfolios. Furthermore, the level of the alphas for these portfolios does not change drastically.

Table 8 Fama-French three-factor regression results of Beta Portfolios

The table below reports the monthly excess return and Fama-French three-factor loadings of the Beta sorted portfolios. Stocks are assigned to their quantiles based on NYSE cut-offs for the month. The portfolios are market capitalization-weighted and rebalanced at the start of each month.

Portfolio	Low	P2	P3	P4	High	Low - High
Alpha (%)	0.29***	0.11	-0.04	-0.12	-0.46***	0.75***
t-stat	3.31	1.48	-0.63	-1.62	-3.35	3.92
Beta	0.44***	0.83***	1.08***	1.26***	1.59***	-1.15***
t-stat	21.11	47.23	69.73	72.16	49.08	-25.47
SMB	-0.12***	-0.17***	-0.08***	-0.01	0.41***	-0.52***
t-stat	-3.90	-6.61	-3.48	-0.42	8.45	-7.84
HML	0.23***	0.20***	0.21***	0.02	-0.27***	0.51***
t-stat	7.53	7.71	9.34	0.84	-5.63	7.49
R-Squared	0.46	0.81	0.90	0.92	0.86	0.67

The table shows Fama-French three factor regressions results of Beta sorted portfolios from Jan-1975 to Dec-2019. * marks the 10 % significance level, ** marks the 5 % significance level, and *** marks the 1 % significance level

Table nine shows the Fama-French five-factor regression results for the beta-sorted portfolios. Of all of the portfolios, only portfolio number three's alpha is statistically significant at the 1 % level. All the rest are insignificant at any level. The betas for all portfolios remain significant at the 1 % level or even higher. The lowest beta t-statistic is for the low-high portfolio at -22.72, far above the 1 % statistical significance mark of -2.65. The SMB factor is statistically significant at the 1 % level for portfolios two, high, and low-high and significant at the 5 % level for portfolio low. The rest of the SMB loadings are insignificant. The HML factor is no longer statistically significant for portfolios low and two. In contrast, the statistical significance at the 1 % level remains for portfolios three, high, and low-high. Furthermore, the SMB and the HML factor level drops across all portfolios, indicating that the three-factor model's SMB and HML factors capture most of the RMW and CMA factors' loadings. The RMW loadings are statistically significant at the 1 % level for low, two, three, high, and low-high portfolios. The level of loadings is large and increases when going from low to high portfolio. Portfolio four is an exception, where the RMW loading suddenly drops to near zero. The CMA factor is statistically significant at the 1 % level for all portfolios and decreases when going from low to high. The level of the RMW factor is also large, indicating that the low-side portfolios co-move with profitable firms while the high-side portfolios co-move with firms with low profitability. The adjusted r-squared increases across all portfolios when comparing the five-factor model to the three-factor model, suggesting that the five-factor model fits the analysis better. The results signal that the study's second hypothesis should be rejected, and the third hypothesis confirmed. Since the second hypothesis states that the beta, SMB, or HML factor captures all the returns, table nine results show that once the RMW and CMA factors are introduced, the SMB or HML factors are no longer statistically significant or large in magnitude. Moreover, both the RMW and CMA factors are statistically significant and large, suggesting that the two factors, together with the beta, capture the returns of the low-beta strategy.

Table 9 Fama-French five-factor regression results of Beta Portfolios

The table below reports the monthly excess return and Fama-French five-factor loadings of the Beta sorted portfolios. Stocks are assigned to their quantiles based on NYSE cut-offs for the month. The portfolios are market capitalization-weighted and rebalanced at the start of each month.

Portfolio	Low	P2	P3	P4	High	Low - High
Alpha (%)	0.10	-0.10	-0.20***	-0.10	-0.16	0.26
t-stat	1.13	-1.43	-3.32	-1.30	-1.22	1.42
Beta	0.51***	0.89***	1.12***	1.25***	1.51***	-1.01***
t-stat	24.49	53.11	75.05	67.28	46.66	-22.72
SMB	-0.07**	-0.09***	0.00	0.00	0.27***	-0.34***
t-stat	-2.36	-3.69	0.15	0.09	5.54	-5.16
HML	0.01	0.03	0.13***	0.09**	-0.09	0.10
t-stat	0.35	1.07	4.67	2.46	-1.47	1.24
RMW	0.24***	0.33***	0.32***	0.03	-0.55***	0.78***
t-stat	5.85	10.17	10.92	0.76	-8.67	9.08
CMA	0.46***	0.32***	0.13***	-0.16***	-0.32***	0.78***
t-stat	7.70	6.62	3.00	-2.87	-3.34	6.05
R-Squared	0.53	0.85	0.92	0.92	0.87	0.72

The table shows Fama-French five factor regressions results of Beta sorted portfolios from Jan-1975 to Dec-2019. * marks the 10 % significance level, ** marks the 5 % significance level, and *** marks the 1 % significance level

The GRS CAPM and FF-3 results provide further evidence for the first and second hypotheses. Both GRS t-statistics are statistically significant at the 1 % level revealing that the alphas of these regressions are not jointly zero. The robustness check confirms what was found in tables seven and eight. The low-beta anomaly is present today and is not explained by market beta, SMB, or HML factor. The GRS FF-5 result contradicts what was found in the FF-5 regression results. The GRS FF-5 t-statistic signals that the alphas are not jointly zero, while the FF-5 individual regressions clearly indicate that most of the alphas are zero. The only exception is portfolio three, which is statistically significant at the 1 % level. There is reason to believe that the statistical significance of portfolio three in the FF-5 model is due to chance, as portfolio three's alpha is insignificant in the CAPM and FF-3 models. Thus, the GRS FF-5 t-statistic could be explained solely by portfolio three's result. In that case, there is reason to believe that the five-factor model fully captures the beta anomaly and that portfolio three's alpha is just a result of a random draw.

The Fama-Macbeth regressions confirm what was found in the CAPM, FF-3, and GRS regressions. The alpha for both models is statistically significant (at 1 % for the CAPM and 5 % for the FF-3). Furthermore, all the factors are insignificant, suggesting that the beta, SMB, or HML do not capture the returns of the beta anomaly.

Table 10 GRS and Fama-Macbeth regression results of Beta Portfolios

The table reports the GRS and Fama-Macbeth regression results for Beta sorted portfolios. On the left table are reported the GRS-test statistics of α_1 = 0 for all i for all three factor regressions. On the right table are the Fama-Macbeth regression results of the factor risk premiums for CAPM and Fama-French three-factor regressions.

GRS Test Statistic		
Regression	t-stat	p-value
CAPM	3.84	0.00
FF-3	3.49	0.00
FF-5	2.95	0.01

Fama-Macbeth Regression				
Regression	Alpha	Beta	SMB	HML
CAPM	0.007***	0.000		
t-stat	4.04	0.13		
FF-3	0.005**	0.002	-0.003	0.001
t-stat	2.54	0.81	-0.67	0.12

The CAPM regression combined with the GRS test and Fama-Macbeth regression suggests that the results found by Baker et al. (2011) were correct. The low-beta anomaly is present today. The FF-3 regression with the GRS test and Fama-Macbeth regression suggest that the study’s second hypothesis should be rejected fully as the model cannot explain the low-volatility or the low-beta anomaly. The FF-5 regression combined with the GRS test shows that the third hypothesis of the study should be confirmed. There is still room for interpretation as portfolio three in low-beta is significant. However, it is neither the study's low nor the high side portfolio, and therefore, the strategy would not even invest in the portfolio.

7. SUMMARY AND CONCLUSION

The motivation of this thesis was to study if the well-known low-volatility effect was still present today even though it has been known for at least three decades. Fama and French (2015) provided further motivation as they claimed to be the first to explain the phenomenon fully. The previous low-volatility literature could be described as rudimentary as multiple studies use unconventional methods, leave critical findings unreported, or do not use well-known and understood robustness tests. Interestingly, such behavior occurs as the thesis can confirm previous results while using more rigorous robustness tests.

The thesis confirms that the low-volatility anomaly is present when using either the low-volatility or low-beta strategy. Neither of the strategies is captured by the capital-asset-pricing model nor the Fama-French three-factor model. While the factors in both models do explain a large part of the anomaly, there is a significant part left unexplained. The GRS and Fama-Macbeth regressions confirm and strengthen the conclusion that neither of the models can capture the low-volatility or low-beta strategies returns.

While the thesis confirms that the previous literature was right – CAPM or the three-factor model cannot explain the low-volatility anomaly – it also confirms the findings of Fama and French (2015). They argued that the five-factor model fully captures the low-volatility effect. Given the vast amount of equity data used and a relatively long study period, Fama and French (2015) were correct. After introducing the CMA and RMW factors, neither the low-volatility nor low-beta strategies earn significant alpha.

The simple volatility or beta sorting offers a unique opportunity for hedge funds to construct a quantitative low-risk strategy. The strategy survives strong robustness checks and can be lucrative for the investor. However, there is considerable doubt that the profitability and investment factor explain the anomaly. Thus, the investor may want to use these strategies to capture the low-volatility and low-beta anomaly returns. While previous literature has not used the five-factor model for robustness checks, not all findings are reported publicly. Not publishing such findings may be due to them being unattractive to the public's eye. Only the strategies that do earn abnormal returns are often reported.

The thesis also provides contradicting proof against the theoretical literature claiming that the markets are efficient. Both strategies are based on simple past return data to group stocks into portfolios. Using historical data should not lead to abnormal returns, yet the results indicate otherwise. Why such behavior in markets occurs is left for further studies. The previous low-volatility literature has claimed multiple reasons for the anomaly's existence, presented in chapter three. Still, only a few can provide empirical evidence supporting their claims. This thesis has not claimed any explanations for the anomaly, just the results that it does exist. The question is not left unexplained intentionally. No robust method has been suggested to measure the reasons for the anomaly's existence. Future research with novel methods may be able to address the issue at hand.

The thesis is also implementable in practice. While retail investors should not consider the strategy as it requires a good understanding of the underlying literature, a relatively large amount of capital, and advanced knowledge of programming languages, it should be considered by funds with access to the earlier mentioned requirements. Compared to the most well-performing strategies, the offered risk-return relationship of the low-volatility strategy is competitive and even beats some strategies in traditional portfolio performance measures. Furthermore, those retail investors interested in the strategy should seek out funds that implement the strategy. At least Robeco in Europe offers the low-volatility strategy in one of their funds.

Future research may extend this thesis in multiple ways. First, the low-volatility anomaly should be researched in other markets. A good starting point would be European or emerging stock markets. Previous studies have documented the effect globally, but the methods used have been non-standard. Thus, future research should focus on using standard methods of portfolio construction. Additionally, the previous studies from European and emerging stock markets lack robustness tests and use old asset, pricing models. Therefore, future research may want to use more recent asset pricing models and add robustness checks to increase the level of the study. Second, nearly all "low-volatility" strategies are constructed from return data. It is possible that the low-volatility anomaly is not restricted to stock markets. Future research should consider other asset classes as well. There are considerably fewer studies of the low-volatility anomaly in other asset classes, and therefore, it would be a welcome addition to the literature on the low-volatility anomaly. Third, according to recent studies, the low-beta and low-volatility strategies are primarily explained by the IVOL anomaly. However, the finding is quite recent, and thus the literature is scarce. Future research should consider studying the link between the anomalies. Fourth, the most intriguing and long lingering question remains: what causes the low-volatility anomaly? There have been nearly a dozen explanations for the low-volatility anomaly, yet only one paper has provided empirical evidence for its explanation of the anomaly. Thus, future research should consider finding evidence for or against the provided explanations.

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